DEVELOPMENT OF ANALYTICAL TECHNIQUE FOR THE OPTIMIZATION OF JET ENGINE AND DUCT ACOUSTIC LINERS

By
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1982
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ABSTRACT

This report summarizes the work performed during the NASA LANGLEY research program entitled "Development of an Analytical Technique for the Optimization of Jet Engine and Duct Acoustic Liners." This research program ran for one year (3/1/81-2/28/82) and carries the NASA number NAG 1-133. Detailed results of the work performed during the first six months of this contract are presented in the NASA LANGLEY SEMI-ANNUAL STATUS REPORT (3/1/81-8/31/81) for NAG 1-133 and thus will not be repeated here in its entirety.

During the past six months, a new method was developed for the calculation of optimum constant admittance solutions for the minimization of the sound radiated from an arbitrary axisymmetric body. This method utilizes both the integral equation technique used in the calculation of the optimum non-constant admittance liners and the independent solutions generated as a by product of these calculations. The results generated by both these methods are presented for three duct geometries: (1) a straight duct; (2) the QCSEE inlet; and (3) the QCSEE inlet less its centerbody.
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I. INTRODUCTION

The object of this research program was the development of an analytical technique for the determination of the optimum admittance distribution along the wall of an axisymmetric duct for the minimization of sound radiated from the duct given a specific source of acoustic radiation in the duct. The results of this method were to be checked against calculations performed for constant admittance liners to see if better results could be obtained with the new method. Finally, a parametric study was to be done, based on wave number, for at least two geometries in which the optimum constant and distributed admittance liners were to be calculated.

The formulation of the problem which has been used in the parametric study is presented in detail in Chapter IV of the previous six month status report for this grant (See Reference 1.). This being the case, the precise mathematical formulation of the method will not be repeated. Instead, only a brief overview of the method will be presented here.

The method itself is based upon a special integral formulation of the external solutions of the Helmholtz equation. The basic formulation of the governing equations for three dimensions is given in great detail in Reference 2. This formulation can be specialized for axisymmetric bodies and it is this form of the equations which is used in this study.

These integral equations govern the acoustic quantities on the surface of the body and take into account the Sommerfeld radiation conditions at infinity in the field so that only outgoing, decaying solutions are considered. To solve these equations, the surface of the body is discretized into many small areas and since
the problem is elliptic in nature a boundary condition is applied over each small area. The boundary condition specified may be either the acoustic potential which is directly related to the acoustic pressure, the normal acoustic velocity, or a ratio of these two quantities referred to as the effective acoustic admittance at each point.

When this is done, a system of linear equations can be developed in which the acoustic potential or the normal acoustic velocity is the unknown at each point on the body depending on which boundary condition is specified there. The boundary conditions themselves contribute to the inhomogeneous term in each equation and in some cases the diagonal term of the matrix.

Since the resulting equations are linear, the solutions may be superimposed. Also, if the boundary conditions are chosen appropriately they do not effect the matrix coefficients, only the inhomogeneous vector terms. It is these two characteristics of this formulation which are exploited in both the calculation of the optimum varying admittance for a duct and the optimum constant admittance.

Normally to find the optimum constant admittance for a duct, a parametric study must be done in which the real and imaginary parts of the admittance of the liner are varied. Usually, this means that a complete, separate solution must be generated for each admittance value; however, a method has been developed which utilizes the same independent solutions on the admittance surface which were generated for the calculation of the optimum varying admittance solution. This new method greatly reduces the amount of computing time required for the generation of constant admittance solutions and is presented in detail in the following section of this report.
Having developed both the theory and the computer codes for the generation of both optimum constant and varying admittance liners for general finite axisymmetric ducts, a parametric study was performed on three separate duct geometries. The three duct geometries are: (1) a straight duct with a rounded lip; (2) the NASA QCSEE inlet of Reference 4; and (3) the NASA QCSEE inlet less its centerbody. The results of this parametric study are presented at six wave numbers for each geometry at which both the constant and varying optimum admittance liners are calculated for both constant acoustic potential and constant normal acoustic velocity drivers.
II. CALCULATION OF OPTIMUM CONSTANT ADMITTANCE LINERS

In this section, we will briefly go over the generation of the independent solutions on the surface of the body. Then, the development of constant admittance solutions will be discussed in detail. Since the development of the special integral formulation of the external solutions of the Helmholtz equation is given in References 1-3, only the final form of the equations will be presented here. It will be noted that although this form of the equations has been specialized for axisymmetric geometries, that any cylindrically symmetric acoustic mode may be calculated.

Firstly, let us define the geometrical variables that we will use on a surface of revolution. In Fig. 1, the coordinate system employed on the body S is given \((\rho, Z, \Theta)\) along with an outward normal from the body, \(\hat{n}\), and an element of area on the surface of the body, \(\rho \, ds \, d\Theta\). The variable \(s\) is the distance along the generating line of the surface of revolution and is assumed to go from \(0\) at one end of the body to \(L\) at the other.

We now assume that the acoustic potential on the surface of a body of revolution can be written as

\[
\phi(\rho, Z, \Theta) = \phi(s) \cos(m\Theta)
\]

and similarly that the normal acoustic velocity on the surface of the body can be written as

\[
\frac{\partial \phi(\rho, Z, \Theta)}{\partial n} = V(s) \cos(m\Theta)
\]
In doing this we have incurred no loss in generality. Since all of the equations are linear, any acoustic radiation pattern may be generated as a sum of these simple, cylindrically symmetric patterns. Also, the variable \( m \) is commonly referred to as the tangential acoustic mode number.

In order to write the equation in compact form we now define three sets of functions:

Influence functions:

\[
I_{1 \ pq}(r) = 2 \int_0^\pi G(P, Q) \cos\left(\frac{m - \pi}{\pi} \psi\right) d \psi
\]

\[
I_{2 \ pq}(r) = 2\alpha \int_0^\pi \frac{\partial G(P, Q)}{\partial n_p} \cos\left(\frac{m - \pi}{\pi} \psi\right) d \psi
\]  

Kernel Functions:

\[
K_{1 \ pq}(r) = 2 \int_0^\pi \frac{\partial G(P, Q)}{\partial n_q} \cos\left(\frac{m - \pi}{\pi} \psi\right) d \psi
\]

\[
K_{2 \ pq}(r) = 2\alpha \int_0^\pi \frac{\partial^2 G(P, Q)}{\partial n_p \partial n_q} \cos\left(\frac{m - \pi}{\pi} \psi\right) d \psi
\]  

\[\theta \neq \theta_p\]

Forcing functions:

\[
F_{1 \ pq}(r) = 2\alpha \int_0^\pi G(P, Q) \left(ik \hat{n}_p \cdot \hat{n}_q\right) d \psi
\]

\[
F_{2 \ pq}(r) = 2\alpha \int_0^\pi \frac{\partial^2 G(P, Q)}{\partial n_p \partial n_q} d \psi',
\]  

\[\theta \neq \theta_p\]
where \( r_{pq} \) is the distance between points P and Q and \( \hat{n}_p \) and \( \hat{n}_q \) are the outward normals from the points P and Q, respectively (See Fig. 2.). Also, \( G(P,Q) \) is the free space Green's function

\[
G(P,Q) = \frac{\text{e}^{ikr_{pq}}}{r_{pq}}
\]

where \( k \) is the wave number and \( \alpha \) is the complex coupling constant for this particular formulation which is found to be

\[
\alpha = \frac{i}{\kappa}
\]

It will be noted that in evaluating \( K_2 \) and \( F_2 \) the point at which \( \theta_p = \theta_q \) is excluded from the integration as it constitutes a strong singularity.

Using the above definitions and equations, the special integral formulation of the external solutions of the Helmholtz equation may be written as

\[
\int_0^l \phi(s_q) \left[ K_1 (r_{pq}) + K_2 (r_{pq}) \right] \rho_q \, ds_q
\]

\[
- \phi(s_p) \int_0^l \left[ F_1 (r_{pq}) + F_2 (r_{pq}) \right] \rho_q \, ds_q
\]

\[
- \int_0^l V(s_q) \left[ I_1 (r_{pq}) + I_2 (r_{pq}) \right] \rho_q \, ds_q
\]

\[
= 2\pi \left[ \phi(s_p) + \alpha V(s_p) \right]
\]
In this particular formulation of the problem the s and 0 coordinate directions have been uncoupled so that the solution of the problem has been reduced to the evaluation of line integrals on the surface of the body.

Equation (8) represents a relationship between the acoustic pressure and normal acoustic velocity at any given point on a body (i.e., point P) to all of the values everywhere else on the body (i.e., at the Q points). If this equation is applied at each point on the body, along with the boundary condition at each point, a system of linear algebraic equations is obtained for the unknown variables at each point on the body. Thus, if there are N points on the body, a system of N complex equations in N complex unknowns is developed.

In the numerical integration of the functions (See Eqns. (3)-(5).) a Gauss-Legendre integration formula is used. For the integration in the s direction, a simple two point integration is employed such that the point P is never actually equal to any of the integration points (i.e., the Q points). Also, when the body is divided into N points in the s direction, both the acoustic potential \( \phi \) and the normal acoustic velocity \( V \) are assumed to be constant over each element even though there are two integration points per element.

For the development of the independent solutions on the surface of the body let us assume that the body is divided into three distinct regions as in Fig. 3. These regions do not necessarily have to be contiguous however, for the sake of clarity they are presented as such here. The first solution which we must consider is the driver solution. To calculate it we must solve for the acoustic quantities on the surface of the body subject to the boundary conditions

\[
\phi(Q) = \tilde{\phi}_D(Q) \quad \text{on } S_D
\]

\[
V(Q) = 0 \quad \text{on } S_H \text{ and } S_L
\]

(9)
where \( \phi_D(Q) \) is some specified function of the acoustic potential on the driver. Solving this problem, we obtain the driver solution

\[
V_D(Q) \quad \text{on } S_D \\
\phi_D(Q) \quad \text{on } S_H \text{ and } S_L
\]  

(10)

Next, the liner surface(s) is divided up into \( M \) finite regions as in Fig. 4. Then \( M \) independent solutions are generated which represent the effect of \( M \) simple acoustic velocity sources on the liner using the boundary conditions given below

\[
\phi(Q) = 0 \quad \text{on } S_D \\
V(Q) = 0 \quad \text{on } S_H \\
V(Q_j) = 1 \quad j = 1, \ldots, M \quad \text{on } S_L \\
V(Q_i) = 0 \quad i \neq j
\]  

(11)
The M solutions thus generated are given by

\[ V_j(Q) \quad \text{on} \quad S_D \]
\[ \phi_j(Q) \quad \text{on} \quad S_H \]
\[ \phi_j(Q) \quad \text{on} \quad S_L \]  \hspace{1cm} (12)

If we now sum these solutions multiplied by some arbitrary coupling constants designated by \( a_j \), which we can do as the problem is linear, we generate a general solution which has the form

\[ \phi(Q) = \sum_{j=1}^{M} a_j V_j(Q) \]
\hspace{1cm} (13)

\[ V(Q) = V_D(Q) + \sum_{j=1}^{M} a_j V_j(Q) \quad \text{on} \quad S_D \]

\[ \phi(Q) = \phi_D(Q) + \sum_{j=1}^{M} a_j \phi_j(Q) \]
\hspace{1cm} (14)

\[ V(Q) = 0 \quad \text{on} \quad S_H \]

\[ \phi(Q) = \phi_D(Q) + \sum_{j=1}^{M} a_j \phi_j(Q) \]
\hspace{1cm} (15)

\[ V(Q_j) = a_j \quad j = 1, \ldots, M \quad \text{on} \quad S_L \]
\[ V(Q_i) = 0 \quad i \neq j \]
It will be noted here that the above solution has some interesting properties in that the acoustic potential on the driver surface (See Eqn. (13).) and the normal acoustic velocity on the hard walled surface (See Eqn. (14).) are not dependent upon the choice of the coupling constants $a_j$.

In this study we are interested in the effective acoustic admittance $Y$ which is defined as

$$Y = \frac{\partial\phi}{\partial n} = \frac{V}{\phi}$$

(16)

This being the case, we can now represent the effective acoustic admittance at any point on the admittance surface as

$$Y(Q_j) = \frac{a_j}{\phi_D(Q_j) + \sum_{i=1}^{M} a_i \phi_i(Q_j)}$$

(17)

If we now specify that the effective acoustic admittance at all points on the admittance surface is to be the complex number $C$ we obtain

$$\sum_{i=1}^{M} a_i \phi_i(Q_j) - \frac{1}{C} a_j \phi_D(Q_j) = 0,$$

(18)

$$j = 1, \ldots, M$$
which represents a system of $M$ linear complex equations for the $M$ complex coupling constants, $a_j$. Using this method many constant admittance solutions can be generated very economically once the independent solutions on the surface of the body are known. Since the independent solutions have already been calculated for the generation of the optimum varying admittance, a relatively small amount of extra computing time is required for the determination of the optimum constant admittance solution.

To find the optimum constant admittance solution for a specified geometry, driver and wave number, the values of $C$ are chosen in a grid pattern and a solution is generated for each value. Once the surface solution is known it is an easy job to calculate the acoustic power radiated from the driver and the acoustic power lost to the admittance surface using $^1, 5$

$$E = \iiint_{S_L} \left[ \phi^R(Q) V^I(Q) - \phi^I(Q) V^R(Q) \right] dS(Q) \quad (19)$$

where $E$ is the acoustic energy radiated out of a surface and the superscripts $R$ and $I$ refer to the "real and imaginary part of", respectively. When the solution having the minimum radiated power is found, the region may be further subdivided to "home in" on the optimal value of the admittance.

It is of interest to note here that strictly speaking all possible values of the effective admittance $Y$ are not possible at each point on the liner surface. To demonstrate this, let us look at the point $j=1$ on the liner surface where

$$Y(Q_1) = \frac{a_1}{\phi_D(Q_1) + \sum_{i=1}^{M} a_i \phi_i(Q_1)} \quad (20)$$
Solving for $a_1$ we obtain

$$a_1 = \frac{Y(Q_1) \sum_{i=2}^{M} a_i \phi_i(Q_1)}{1 - Y(Q_1) \phi_1(Q_1)} \quad (21)$$

where it can be seen that if we want $Y(Q_1) = \frac{1}{\phi_j(Q_j)}$ we must have $a_1 \to \infty$. Thus, we cannot generate the solution where the effective admittance $Y(Q_j) = \frac{1}{\phi_j(Q_j)}$ with a finite value for the complex coupling constant, $a_j$. 
III. SOME GENERAL COMMENTS

The problem of acoustic radiation from a duct, as formulated for this study, is strictly elliptic so that only one boundary condition may be specified on any part of the body. Thus, either the acoustic potential (i.e., pressure) or the normal acoustic velocity may be specified on the driver but not both. This leads us to an interesting problem when trying to compare the results of this method to any other as other methods utilize the mathematical artifice of a semi-infinite duct. This artifice allows them to keep the driver power and modal input constant while varying the acoustic properties of a liner. This tends to neglect any possible effect the acoustic properties of the liner could have on the amount or modal content of the power coming out of the driver.

In the problem, as formulated for this study, the driver power and more importantly the radial modal output of the driver cannot be fixed as this would overspecify the problem. This being the case, there are two possible optimum constant admittance liners possible, one a relative measure of the percent of the driver power attenuated by the liner and the other an absolute measure of the power coming out of the duct. Both were calculated at each wave number for each geometry and are presented as such (i.e., Relative and Absolute optimum constant admittances). Also, since either the acoustic potential or the normal acoustic velocity could be specified on the driver runs were done with each and are noted as such. For the runs where the normal acoustic velocity is specified on the driver, the acoustic potential is specified on the admittance (i.e., liner) surface and vice versa (See Eqns. (9) and (11)).
IV. NUMERICAL CONSIDERATIONS

The special integral formulation of the external solutions of the Helmholtz equation\textsuperscript{2,3} which is used as the basis for all of the calculations done in this study requires a closed body. Thus, all three of the ducts used in this study: the straight duct with the rounded lip; the NASA QCSEE inlet; and, the NASA QCSEE inlet less its centerbody were terminated with a 2:1 ellipse (See Figs. 5-7.). Also, for the three geometries investigated the total height to the inner wall of the duct at the driver plane was normalized to one and the outer wall of the duct was 1.15. All of the ducts have an L/a of 2.0

For the numerical calculations, points were spaced evenly along the inner walls of the ducts with a nominal spacing of 0.05a. On the outer walls of the ducts, the points were systematically spaced at larger and larger intervals as it has been found that the outer walls of ducts and their terminations have little effect on the total power radiated and the radiation pattern in the forward half plane. The total number of points used on the three geometries in the s direction for the calculations performed for this study were: 92 points for the straight duct; 108 points for the NASA QCSEE inlet; and, 100 points for the NASA QCSEE inlet less its centerbody. For the \( \theta \) integration, a 32-point Gauss-Legendre integration formula was used in all cases.

For all three of the ducts, the admittance surface consisted of 25 points or intervals over which the optimum admittance distributions were to be generated and ran from 0.4a to 1.6a in the Z direction along the inner walls of the ducts.
Thus, a hard wall or driver solution and 25 independent source solutions were calculated for each geometry, wave number and type of driver specified (i.e., potential or velocity).
V. RESULTS

Each of the geometries was run with a plane wave as input on the driver for non-dimensional wave numbers of 1, 2, 3, 5, 7, and 10. That is, in all of the cases run, the tangential mode number was taken as zero. Although a plane wave was input, a plane wave driver did not necessarily result since only one variable could be specified at a time.

The results for all of the straight duct runs are presented in Tables I-VI and in Figs. 8-13. In the Tables, the power radiated out of the driver and the power radiated into the field are tabulated along with their values, for the optimum distributed admittance and for the optimum absolute and relative constant admittances. In all the Tables, the power values are relative as they have been normalized by the power out of the hard walled configuration. Also, each table contains the results for one wave number for both the constant acoustic pressure and normal acoustic velocity drivers.

It will be of interest to note here that for the lower wave numbers, the power out of the driver is negative (i.e., it is damping). This necessarily means that the liner surface is driving since the formulation of the integral equations only allows for the case where there is a net flow of power out of the body (i.e., no incoming waves). If the imaginary part of the effective admittance $Y$ (See Eqn. (16).) is positive, this denotes driving; that is, an active suppressor. The relative optimum constant admittance must always be a damping admittance since it is determined as the smallest ratio of power out of the driver, to the power lost to the admittance surface.
In general, it is found that the lowest power output is obtained from the optimum admittance distribution. Also, the relative constant admittance usually has the highest power output as measured in the field surrounding the duct.

Each Figure constitutes a set of 6 plots for each wave number. The first group of three plots in each set are for the case where a constant acoustic pressure is specified on the driver and the second group is for the case where a constant normal acoustic velocity was specified. The first plot in each group (e.g., Figs. 8a & d), contains a plot of the optimum admittance distribution on the inner wall of the duct from the driver end $Z=0.4a$ (inner), to the open end, $Z=1.6a$ (outer). As can be seen even at the low wave numbers where there are a more than sufficient number of points on the body to generate an accurate solution, the effective admittance distribution is not very smooth. This is because it is a ratio of two functions on the surface of the body which tends to make it less continuous than either generating function. Of course, more points could be taken on the surface of the body to obtain a smoother function for the effective admittance; however, this would not substantially change the overall accuracy of the solution (i.e., the power output). At the higher wave numbers, the solution does become suspect however, and more points should probably have been used for the cases where $ka=7$ and $10$. This should not detract from the overall validity of the method however.

It will be noted that at the lower wave numbers, the distributed admittance found for the minimum power out of the body is totally driving. As the wave number gets higher, the optimum admittance distribution becomes mixed (i.e., some of the liner surface drives and some of it damps) and finally at some of the higher wave numbers, the distributed admittance is almost totally passive. This
is probably due to the fact that at the higher wave numbers, the wave structure in
the duct becomes more complicated so that interference patterns are more
difficult to set up. Since an active suppressor damps out sound through the setting
up of interference patterns, these types of suppressors are probably only useful at
lower wave numbers where the wave patterns are less complicated. Also, since it
is more difficult to set up interference patterns with the constraint of a constant
admittance liner, the optimum absolute constant admittance liner transition from
driving to damping occurs sooner.

In the second plot in each group of three, is a plot of the absolute power
out of the duct as a function of the admittance (constant) on the liner surface
which is expressed in dB. The admittance value for which the minimum power out
of the duct is obtained is marked with a large dot. Again, these values are
tabulated in the tables (See Tables I-VI.).

In the final plot in each group of three, is a plot of the relative power out
of the duct as a function of admittance (constant) on the liner which is also
expressed in dB. Only negative values of the imaginary part of the admittance are
considered in this case as the power out of the duct is referenced to the power
out of the driver. As with the previous plot, the admittance value, for which the
minimum percent power is radiated, is marked with a large dot and those values
also are tabulated in the Tables.

The results for the QCSEE inlet are presented in Tables VII-XII and in
Figs. 14-19. As with the straight duct, the tables contain the results for the six
wave numbers run, one wave number per table. The results at a non-dimensional
wave number of $ka=7.0$ for the case where the acoustic potential is specified on the
driver are not included since the optimum values for the absolute and relative
constant admittances fell outside of the initial search pattern. This pattern ran from -10 to 10 in increments of 1 for both the real and imaginary parts of the admittance. This is not to imply that they couldn't be calculated, just that they were not, since this would have required modification of the computer programs used for all of the other cases run.

As with the straight duct, each figure for this geometry consists of the six plots done for each wave number. As before, the optimum admittance distribution for both the constant acoustic pressure and the constant normal acoustic velocity drivers are presented along with the contour power plots for the constant absolute and relative admittance liners. Again, the optimum values are marked with dots in these plots and are tabulated in the Tables. It will be noted in Fig. 18a and b that these points are not marked since they fell outside the range of the plots.

The results for the QCSEE inlet less its centerbody are presented in Tables XIII-XVIII and in Figs. 20-25. The reason for running the cases for this particular geometry was to see if any trends could be established in going from the straight duct geometry to the full inlet geometry. At the lower wave numbers, the optimum admittance values calculated for it, seem to fall between those for the other two geometries as one would intuitively expect; however, this trend is not maintained at the higher wave numbers.
VI. SUMMARY AND CONCLUSIONS

During the past year, a method was developed for the calculation of optimum distributed admittance duct liners. This method is based upon a special integral representation of the external solutions of the Helmholtz equation which is valid (i.e., can be used to generate the correct, unique solutions) at all wave numbers. The equations used had been specialized for axisymmetric geometries but this is not a restriction on the method itself.

As a by-product of this method, a procedure was developed for the identification of optimum constant admittance duct liners. This procedure utilizes solutions already developed for the optimum distributed admittance calculation. At present, it entails the use of a simple search pattern for the optimum constant admittance; however, it is believed that this could be refined if time allowed.

To give some idea of the time involved in calculating these results, some typical computing times are presented below. These runs were done on the Georgia Tech CDC CYBER 760 and the programs are written in Fortran V. For the case where 100 points were used on the body in the s direction, a 32 point Gauss-Legendre integration formula was used in the θ direction (See Fig. 1.), and there were 25 points on the liner surface, the calculation of the 26 independent solutions required for the optimization procedure took 185 seconds of CPU time. The generation of the optimum distributed admittance then took an additional 10 seconds and the identification of the optimum constant admittances took 390 seconds. As can be seen, the calculation of the constant admittance solutions is slow compared to the calculation of the optimum distributed admittance. The contour plots of the sound radiated for each constant admittance chosen on the
liner surface were done with the GPCP (General Purpose Contour Plotting) package which we have available here at Georgia Tech. It was developed originally for plotting contour maps but was found to be very useful in this research program.

In conclusion, an effective, efficient method has been developed for the calculation of both optimum distributed and constant admittance liners for general geometries. It was found through the use of this method that even very similar geometries may have vastly different optimum liners associated with them. Also, it was found that at low wave numbers often the most efficient liners for the reduction of the sound radiated are active and not passive. At the higher wave numbers, the optimum distributed admittances are found to be almost always a combination of both active and passive elements.
REFERENCES


TABLE I

STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

\[ ka = 1.0 \]

<table>
<thead>
<tr>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
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<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td><strong>POWER OUT OF THE DRIVER</strong></td>
</tr>
<tr>
<td></td>
<td>-0.57</td>
</tr>
<tr>
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<td>-0.67</td>
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<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td><strong>POWER OUT OF THE DRIVER</strong></td>
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<tr>
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<td>-0.64</td>
</tr>
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<td>-0.53</td>
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<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td><strong>POWER OUT OF THE DRIVER</strong></td>
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<td>0.87</td>
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</table>
|                             | 0.65                           | 0.0012
TABLE II
STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

\( ka = 2.0 \)

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
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<td>OPTIMUM ADMITTANCE</td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.65</td>
<td>-0.61</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00012</td>
<td>0.00014</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABSOLUTE CONSTANT ADMITTANCE</td>
<td>(-2.95, 3.05i)</td>
<td>(-2.70, -2.90i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.89</td>
<td>0.75</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00034</td>
<td>0.00054</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RELATIVE CONSTANT ADMITTANCE</td>
<td>(-2.64, -3.14i)</td>
<td>(-2.65, -3.13i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.91</td>
<td>0.78</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000088</td>
<td>0.00068</td>
</tr>
</tbody>
</table>
TABLE III

STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

\[ ka = 3.0 \]

<table>
<thead>
<tr>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td><strong>TOTAL POWER IN FAR FIELD</strong></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.000075</td>
</tr>
<tr>
<td>(-2.71, -2.38i)</td>
<td>0.00072</td>
</tr>
<tr>
<td>(-2.70, -2.39i)</td>
<td>0.00079</td>
</tr>
</tbody>
</table>

25
TABLE IV
STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

$ka = 5.0$

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.0011</td>
<td>0.0075</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00084</td>
<td>0.000011</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td>(-3.48, -1.66i)</td>
<td>(-4.61, -2.29i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.00</td>
<td>0.043</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.37</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td>(-4.13, -1.77i)</td>
<td>(-4.44, -2.38i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.06</td>
<td>0.043</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.37</td>
<td>0.010</td>
</tr>
</tbody>
</table>
TABLE V
STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

$ka = 7.0$

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>0.066</td>
<td>0.014</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.00064</td>
<td>0.054</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>1.29</td>
<td>0.016</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.43</td>
<td>0.0078</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>1.42</td>
<td>0.019</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.42</td>
<td>0.0086</td>
</tr>
</tbody>
</table>
TABLE VI
STRAIGHT DUCT

Relative power normalized with respect to the hard walled radiated power

$ka = 10.0$

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMUM ADMITTANCE DISTRIBUTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.050</td>
<td>0.00066</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0049</td>
<td>0.00016</td>
</tr>
<tr>
<td>ABSOLUTE CONSTANT ADMITTANCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.02</td>
<td>0.010</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.48</td>
<td>0.0051</td>
</tr>
<tr>
<td>RELATIVE CONSTANT ADMITTANCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.02</td>
<td>0.010</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.48</td>
<td>0.0051</td>
</tr>
</tbody>
</table>
TABLE VII
NASA QCSEE INLET

Relative power normalized with respect to the hard walled radiated power

\[ ka = 1.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-1.91</td>
<td>-2.45</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00012</td>
<td>0.00012</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td>(-0.64, 4.03i)</td>
<td>(-0.65, 4.11i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-1.25</td>
<td>-0.74</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0015</td>
<td>0.00082</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td>(-0.47, -3.78i)</td>
<td>(-0.53, -3.77i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.27</td>
<td>0.79</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0019</td>
<td>0.0011</td>
</tr>
</tbody>
</table>
TABLE VIII
NASA QCSEE INLET

Relative power normalized with respect to the hard walled radiated power

\[ \text{ka} = 2.0 \]

<table>
<thead>
<tr>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td>(-1.11)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>(0.00011)</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>((-2.99, 3.91i))</td>
</tr>
<tr>
<td>(0.82)</td>
<td>(0.59)</td>
</tr>
<tr>
<td>(0.0013)</td>
<td>(0.00094)</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>
TABLE IX

NASA QCSEE INLET

Relative power normalized with respect to the hard walled radiated power

\[ ka = 3.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-3.69</td>
<td>-0.050</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0096</td>
<td>0.000049</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td>(-3.10, -3.20i)</td>
<td>(-3.00, -3.19i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.69</td>
<td>0.18</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00045</td>
<td>0.00020</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td>(-3.04, -3.20i)</td>
<td>(-3.05, -3.18i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.69</td>
<td>0.18</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00061</td>
<td>0.00015</td>
</tr>
</tbody>
</table>
TABLE X
NASA QCSEE INLET

Relative power normalized with respect to the hard walled radiated power

\( \text{ka} = 5.0 \)

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.023</td>
<td>0.00059</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00040</td>
<td>0.000031</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td>(-4.20, -1.80i)</td>
<td>(-4.57, -1.89i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.80</td>
<td>0.040</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.13</td>
<td>0.0065</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td>(-4.26, -1.96i)</td>
<td>(-4.37, -1.87i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.81</td>
<td>0.041</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.13</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

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TABLE XI
NASA QCSEE INLET

Relative power normalized with respect to the hard walled radiated power

\[ \text{ka} = 7.0 \]

<table>
<thead>
<tr>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.56</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.13</td>
</tr>
</tbody>
</table>

| **ABSOLUTE CONSTANT ADMITTANCE** | **ABSOLUTE CONSTANT ADMITTANCE** |
| POWER OUT OF THE DRIVER | ----- | 0.018 |
| TOTAL POWER IN FAR FIELD | ----- | 0.0022 |

| **RELATIVE CONSTANT ADMITTANCE** | **RELATIVE CONSTANT ADMITTANCE** |
| POWER OUT OF THE DRIVER | ----- | 0.018 |
| TOTAL POWER IN FAR FIELD | ----- | 0.0022 |
TABLE XII
NASA QCSEE INLET
-------------

Relative power normalized with respect to the hard walled radiated power

\( ka = 10.0 \)

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMUM ADMITTANCE DISTRIBUTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.29</td>
<td>0.010</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00075</td>
<td>0.000064</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABSOLUTE CONSTANT ADMITTANCE</td>
<td>(-4.32, -3.83i)</td>
<td>(-4.02, -3.56i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.94</td>
<td>0.010</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.22</td>
<td>0.0026</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RELATIVE CONSTANT ADMITTANCE</td>
<td>(-4.27, -3.78i)</td>
<td>(-4.05, -3.60i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.94</td>
<td>0.010</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.22</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

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TABLE XIII
QCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\( ka = 1.0 \)

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMUM ADMITTANCE DISTRIBUTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-1.20</td>
<td>-1.78</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000025</td>
<td>0.000072</td>
</tr>
<tr>
<td>ABSOLUTE CONSTANT ADMITTANCE</td>
<td>(0.81, 4.68i)</td>
<td>(-0.75, 4.72i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-1.19</td>
<td>-1.06</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0021</td>
<td>0.00091</td>
</tr>
<tr>
<td>RELATIVE CONSTANT ADMITTANCE</td>
<td>(-0.73, -3.49i)</td>
<td>(-0.79, -3.44i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>1.71</td>
<td>1.33</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0029</td>
<td>0.0023</td>
</tr>
</tbody>
</table>
## TABLE XIV
CCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\[ ka = 2.0 \]

<table>
<thead>
<tr>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMUM ADMITTANCE DISTRIBUTION</td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.56</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000044</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ABSOLUTE CONSTANT ADMITTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER OUT OF THE DRIVER</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RELATIVE CONSTANT ADMITTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER OUT OF THE DRIVER</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
</tr>
</tbody>
</table>
TABLE XV
QCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\[ \text{ka} = 3.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.41</td>
<td>-0.024</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000084</td>
<td>0.000032</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>-0.67</td>
<td>0.13</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000087</td>
<td>0.000094</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.69</td>
<td>0.13</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.000047</td>
<td>0.000063</td>
</tr>
</tbody>
</table>
TABLE XVI

QCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\[ ka = 5.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTIMUM ADMITTANCE DISTRIBUTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.098</td>
<td>0.0069</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.00077</td>
<td>0.0000071</td>
</tr>
<tr>
<td>ABSOLUTE CONSTANT ADMITTANCE</td>
<td>(-3.89, -1.65i)</td>
<td>(-3.93, -2.39i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.74</td>
<td>0.044</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.20</td>
<td>0.0042</td>
</tr>
<tr>
<td>RELATIVE CONSTANT ADMITTANCE</td>
<td>(-3.87, -1.98i)</td>
<td>(-3.88, -2.24i)</td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.77</td>
<td>0.044</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.21</td>
<td>0.0042</td>
</tr>
</tbody>
</table>
TABLE XVII
QCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\[ ka = 7.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POWER OUT OF THE DRIVER</td>
<td>0.14</td>
<td>0.0091</td>
</tr>
<tr>
<td>TOTAL POWER IN FAR FIELD</td>
<td>0.0016</td>
<td>0.00028</td>
</tr>
</tbody>
</table>

| **ABSOLUTE CONSTANT ADMITTANCE** | (-4.77, -2.07i) | (-7.32, -1.67i) |
| POWER OUT OF THE DRIVER | 1.02            | 0.020            |
| TOTAL POWER IN FAR FIELD | 0.29            | 0.0058           |

| **RELATIVE CONSTANT ADMITTANCE** | (-4.87, -2.06i) | (-6.84, -1.57i) |
| POWER OUT OF THE DRIVER | 1.02            | 0.021            |
| TOTAL POWER IN FAR FIELD | 0.29            | 0.0062           |
TABLE XVIII

QCSEE INLET LESS CENTERBODY

Relative power normalized with respect to the hard walled radiated power

\[ \text{ka} = 10.0 \]

<table>
<thead>
<tr>
<th></th>
<th>Constant Phi on the Driver</th>
<th>Constant Velocity on the Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OPTIMUM ADMITTANCE DISTRIBUTION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>0.33</td>
<td>0.00060</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.16</td>
<td>0.000060</td>
</tr>
<tr>
<td><strong>ABSOLUTE CONSTANT ADMITTANCE</strong></td>
<td>(-5.27, -3.01i)</td>
<td>(-4.38, -3.18i)</td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>0.97</td>
<td>0.010</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.36</td>
<td>0.0039</td>
</tr>
<tr>
<td><strong>RELATIVE CONSTANT ADMITTANCE</strong></td>
<td>(-5.05, -2.91i)</td>
<td>(-4.49, -3.30i)</td>
</tr>
<tr>
<td>Power out of the driver</td>
<td>0.98</td>
<td>0.010</td>
</tr>
<tr>
<td>Total power in far field</td>
<td>0.36</td>
<td>0.0039</td>
</tr>
</tbody>
</table>
Figure 1. \((\rho, Z, \theta)\) coordinate system for a body of revolution
Figure 2. Body $S$ showing $P$ and $Q$ points, the distance between them $r_{pq}$ and their outward normals
Figure 3. The three types of regions on the body
Figure 4. Liner surface divided into $M$ finite regions.
Figure 5. Straight Duct
Figure 6. QCSEE Inlet
Figure 7. QCSEE inlet less centerbody
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 8a
STRAIGHT DUCT, $K_A=1.0$, PHI SPECIFIED ON THE DRIVER
(Absolute Power)

Figure 8b
STRAIGHT DUCT, $KA=1.0$, PHI SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 8c
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 8d
STRAIGHT DUCT, KA=1.0, VEL. SPECIFIED ON THE DRIVER
(ABSOLUTE POWER)

Figure 8e
STRAIGHT DUCT, KA=1.0, VEL. SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 8f
STRAIGHT DUCT

REAL PART

IMAGINARY PART

OPTIMUM ADMITTANCE DISTRIBUTION
Constant Phi on the Driver

Figure 9a
STRAIGHT DUCT, KA=2.0, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 9b
STRAIGHT DUCT, KA=2.0, PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 9c
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 9d
STRAIGHT DUCT, $KA=2.0$, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 9e
STRAIGHT DUCT, $K_A=2.0$, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 9f
STRAIGHT DUCT

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION
Constant Phi on the Driver

Figure 10a
STRAIGHT DUCT, $\kappa A = 3.0$, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 10b
STRAIGHT DUCT, $KA=3.0$, PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 10c
STRAIGHT DUCT

IMAGINARY PART

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 10d
STRAIGHT DUCT, KA=3.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 10e
STRAIGHT DUCT, KA=3.0, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 10f
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 11a
Figure 11b
STRAIGHT DUCT, \( k_a = 5.0 \), PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 11c
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 11d
STRAIGHT DUCT, KA=5.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 11e
STRAIGHT DUCT, KA=5.0, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 11f
STRAIGHT DUCT

ka = 7.0
m = 0

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 12a
STRAIGHT DUCT, $KA = 7.0$, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)
IMRG I RY PART O FR MIT T RNC STRIGHT DUCT, KA=7.0, PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 12c
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 12d
STRAIGHT DUCT, $kA = 7.0$, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 12e
STRAIGHT DUCT, $KA=7.0$, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 13a
STRAIGHT DUCT, KA=10.0, PHI SPECIFIED ON THE DRIVER
(ABSOLUTE POWER)

Figure 13b
STRAIGHT DUCT, $KA=10.0$, PHI SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 13c
STRAIGHT DUCT

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 13d
STRAIGHT DUCT, KA=10.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 13e
Figure 13f
Figure 14a
NASA QGSEE INLET, KA=1.0, PHI SPECIFIED ON THE DRIVER
(ABSOLUTE POWER)

Figure 14b
NASA OCSEE INLET, KA=1.0, PHI SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 14c
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 14d
NASA OCSEE INLET, KA=1.0, VEL. SPECIFIED ON THE DRIVER
(ABSOLUTE POWER)

Figure 14e
NASA QCSEE INLET, KA=1.0, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 14f
Figure 15a

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

QCSEE INLET
NASA QCSEE INLET, KA=2.0, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 15b
NASA OCSEE INLET, KA=2.0, PHI SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 15c
QCSEE INLET

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 15d
Figure 15e

NASA QCSEE INLET, KA=2.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)
NASA QCSEE INLET, KA=2.0, VEL. SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 15f
OPTIMUM ADMITTANCE DISTRIBUTION

Constant $\Phi_1$ on the Driver

Figure 16a
NASA QCSEE INLET, KA=3.0, PHI SPECIFIED ON THE DRIVER
(Absolute Power)

Figure 16b
NASA QCSEE INLET, $KA=3.0$, PHI SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 16c
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 16d
Figure 16e
NASA QCSEE INLET, KA=3.0, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 16f
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 17a
NASA QCSEE INLET, KA=5.0, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 17b
NASA QCSEE INLET, KA=5.0, PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 17c
QCSEE INLET

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 17d
NASA OCSEE INLET, KA=5.0, VEL. SPECIFIED ON THE DRIVER
(Absolute Power)

Figure 17e
NASA OCSEE INLET, $kA=5.0$, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 17f
Figure 18a

Optimum admittance distribution

Constant Phi on the Driver
NASA QCSEE INLET, $\kappa = 7.0$, PHI SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 18b
NASA QCSEE INLET, \( K_A = 7.0 \), PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 18c
QCSEE INLET

**OPTIMUM ADMITTANCE DISTRIBUTION**

*Constant Velocity on the Driver*

Figure 18d
NASA QCSEE INLET, KA=7.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 18e
NASA QSGE INLET, KA=7.0, VEL. SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 18f
QCSEE INLET

IMAGINARY PART

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 19a
NASA QCSEE INLET, $K_a=10.0$, PHI SPECIFIED ON THE DRIVER

(Absolute Power)

Figure 19b
NASA QCSEE INLET, KA=10.0, PHI SPECIFIED ON THE DRIVER (RELATIVE POWER)

Figure 19c
OPTIMUM ADMITTANCE DISTRIBUTION
Constant Velocity on the Driver

Figure 19d
NASA QCSEE INLET, KA=10.0, VEL. SPECIFIED ON THE DRIVER (ABSOLUTE POWER)

Figure 19e
NASA QCSEE INLET, KA=10.0, VEL. SPECIFIED ON THE DRIVER
(RELATIVE POWER)

Figure 19f
INLET LESS CENTERBODY

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 20a

120
QCSEE INLET LESS CENTERBODY, KA=1.0, PHI SPECIFIED
(ABSOLUTE POWER)

Figure 20b
QCSEE INLET LESS CENTERBODY, KA=1.0, PHI SPECIFIED
(RELATIVE POWER)

Figure 20c
Constant Velocity on the Driver

Figure 20d
QCSEE INLET LESS CENTERBODY, KA=1.0, VEL. SPECIFIED
(Absolute Power)

Figure 20e
QCSEE INLET LESS CENTERBODY, KA=1.0, VEL. SPECIFIED
(RELATIVE POWER)

Figure 20f
INLET LESS CENTERBODY

ka = 2.0
m = 0

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 21a
QCSEE INLET LESS CENTERBODY, KA=2.0, PHI SPECIFIED
(ABSOLUTE POWER)

Figure 21b
QCSEE INLET LESS CENTERBODY, KA=2.0, PHI SPECIFIED
(RELATIVE POWER)

Figure 21c
INLET LESS CENTERBODY

\[ \begin{align*}
\text{REAL PART} \\
\text{OPTIMUM ADMITTANCE DISTRIBUTION}
\end{align*} \]

Constant Velocity on the Driver

Figure 21d
GCSEE INLET LESS CENTERBODY, KA=2.0, VEL. SPECIFIED
(Absolute Power)

Figure 21e
QCSEE INLET LESS CENTERBODY, KA=2.0, VEL. SPECIFIED
(RELATIVE POWER)

Figure 21f
IMAGINARY PART

OUTER

INNER

REAL PART

$\text{OPTIMUM ADMITTANCE DISTRIBUTION}$

Constant Phi on the Driver

Figure 22a
Figure 22b
QCSEE INLET LESS CENTERBODY, KA=3.0, PHI SPECIFIED
(RELATIVE POWER)

Figure 22c
OPTIMUM ADMITTANCE DISTRIBUTION
Constant Velocity on the Driver

Figure 22d
OCSEE INLET LESS CENTERBODY, KA=3.0, VEL. SPECIFIED (ABSOLUTE POWER)

Figure 22e
QCSEE INLET LESS CENTERBODY, KA=3.0, VEL. SPECIFIED (RELATIVE POWER)

Figure 22f
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 23a
QCSEE INLET LESS CENTERBODY, KA=5.0, PHI SPECIFIED
(ABSOLUTE POWER)

Figure 23b
OCSEE INLET LESS CENTERBODY, KA=5.0, PHI SPECIFIED
(RELATIVE POWER)

Figure 23c
OPTIMUM ADMITTANCE DISTRIBUTION
Constant Velocity on the Driver

Figure 23d
QCSEE INLET LESS CENTERBODY, KA=5.0, VEL. SPECIFIED
(Absolute Power)

Figure 23e

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QCSEE INLET LESS CENTERBODY, KA=5.0, VEL. SPECIFIED (RELATIVE POWER)

Figure 23f
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 24a
QCSEE INLET LESS CENTERBODY, KA=7.0, PHI SPECIFIED (ABSOLUTE POWER)

Figure 24b
QCSEE INLET LESS CENTERBODY, KA=7.0, PHI SPECIFIED (RELATIVE POWER)

Figure 24c
INLET LESS CENTERBODY

OPTIMUM ADMITTANCE DISTRIBUTION
Constant Velocity on the Driver

Figure 24d
QCSEE INLET LESS CENTERBODY, KA=7.0, VEL. SPECIFIED
(Absolute Power)

Figure 24e
QCSEE INLET LESS CENTERBODY, KA=7.0, VEL. SPECIFIED
(RELATIVE POWER)

Figure 24f
OPTIMUM ADMITTANCE DISTRIBUTION

Constant Phi on the Driver

Figure 25a
QCSEE INLET LESS CENTERBODY, KA=10.0, PHI SPECIFIED
(Absolute Power)

Figure 25b
OCSEE INLET LESS CENTERBODY, KA=10.0, PHI SPECIFIED
(RELATIVE POWER)

Figure 25c
INLET LESS CENTERBODY

REAL PART

OPTIMUM ADMITTANCE DISTRIBUTION

Constant Velocity on the Driver

Figure 25d
QCSEE INLET LESS CENTERBODY, KA=10.0, VEL. SPECIFIED (ABSOLUTE POWER)

Figure 25e
QCSEE INLET LESS CENTERBODY, KA=10.0, VEL. SPECIFIED (RELATIVE POWER)

Figure 25f