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This research was directed at the application of Industrial and Systems Engineering methodologies normally used in quality control and network analysis to the investigation of the graceful degradation properties of an integrated air defense system when the components, i.e. operation centers and fire units are developed piece-meal.

Work under this task was originally to include more fire units, adjacent (over)
operations centers, and a computerized simulation using MICON data. When data was not furnished by the sponsor in the summer of 1981, a no-cost one year extension was proposed in September 1981. This request was turned down, and the research reported here reflects work during the period December 1980 through 30 September 1981.
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Conducted by
The School of Industrial and Systems Engineering
Georgia Institute of Technology

Leslie G. Callahan, Jr., Project Director
James Lovell, Investigator
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>I. INTRODUCTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Description of the Problem</td>
<td>1</td>
</tr>
<tr>
<td>B. Project Objective</td>
<td></td>
</tr>
</tbody>
</table>

| II. SYSTEMS MODELING AND ANALYSIS | 3 |
| III. MODELING SYSTEMS AS NETWORKS | 6 |
| IV. MODEL DEVELOPMENT | 10 |
| V. MODEL APPLICATIONS | 17 |
| VI. RESULTS AND CONCLUSIONS | 20 |

## APPENDIX

| A. Development of Micro-Weapon System Structure | 21 |
| B. System and Target Characteristics | 25 |
| C. Case 1: No System Degradation | 26 |
| D. Case 2: Battery A degraded (Only 11 Missiles Ready) | 27 |
| E. Case 3: Battery A Degraded (Only 11 Missiles Ready), Battery B Degraded (Communications Lost to AN/TSQ-73) | 28 |
| F. Case 4: Battery A Degraded (Only 11 Missiles Ready), Battery B Out of Action | 29 |

**BIBLIOGRAPHY** | 30 |
Modern air defense systems can be viewed as large interacting networks of man-machine configurations of sensors, communication links, operations centers, launchers and missiles. The general objective of this research was to investigate the application of Industrial and Systems Engineering methodologies normally used in quality control and network analysis to the investigation of graceful degradation in air defense systems whose components have been developed piece-meal.

This report includes a brief review of approaches to weapons systems modeling, a review of network methodologies, and the development of a model of a battalion level air defense network of one operation center and four fire units. Based on this model, sample cases are investigated using hypothetical data and analytical evaluations to the Minimum Cost Flow Algorithm to illustrate the effects of graceful degradation.

Work under this task was originally planned to include more fire units, adjacent operations centers, and a computerized simulation using MICOM data. When data was not furnished by the sponsor in the summer of 1981, a no-cost one year extension was proposed in September 1981. This request was turned down, and the research reported here reflects work during the period December 1980 through 30 September 1981.
Chapter I
INTRODUCTION

A. Description of the Problem

Modern weapon systems can be viewed as large interacting networks of man-machine configurations of sensors, communication links, operations centers, launchers, and missiles. The level of automation, or dependence on computer driven processes in lieu of human decision is highest for air defense systems, and lowest for forward area infantry systems. Consequently, air defense systems require a higher level of technical integration for effective field operations than infantry or field artillery. However, the military research and development and procurement system is geared toward piecemeal development of new components across the whole spectrum of weapon systems, and doesn't take into account the unique requirements of air defense systems for integrating successive generation of new missiles, radars, computers, and communication components.

Degredation denotes a reduction to a lower state of effectiveness due to normal failures of components due to reliability, availability, maintainability, or enemy action. Graceful degredation implies a non-catastrophic reduction in system effectiveness by linking alternative components in different combinations of modal connections still capable of field operations. Although a number of analytical tools have been developed for the application of graceful degredation principles to the design of communication networks and airborne avionics systems, and more recently to distributed data processing systems (1) relatively little research has been conducted on their use in the design of air defense weapon systems.
B. **Project Objective**

The general objective of this research is to investigate the application of Industrial and Systems Engineering methodologies normally used in quality control and network analysis to the investigation of graceful degradation in air defense systems whose components have been developed piece-meal.

C. **Scope**

This report includes a brief review of approaches to weapon systems modeling, a review of network methodologies, and the development of a model of a battalion level air defense network of one operations center and four fire units. Sample cases with hypothetical data and an analytical solution to the Minimum Cost Flow Algorithm have been solved to illustrate the effects of graceful degradation. Work under this task was originally planned to include more fire units, adjacent operation centers, and a computerized simulation run using MICOM data. When data was not furnished by the sponsor in the summer of 1981, a no-cost one year extension was proposed in September 1981. This request was turned down, and the research reported here reflects work during the period December 1980 through 30 September 1981.
Chapter II
SYSTEMS MODELING AND ANALYSIS

Systems modeling and analysis is a relatively new discipline and many of its early applications were military systems. It has since been applied to a wide spectrum of activities, such as hospitals, banks, assembly lines, etc. Systems modeling and analysis is now the primary approach to solving problems involving large, complex systems. Parallel development of sophisticated computers and specialized languages, such as GPSS, COBOL, GASP, etc., facilitated the solution of many problems that would otherwise have been mathematically forbidding. Even without computers, the systems approach presents a way of tackling problems that would otherwise be unsolvable for all practical purposes. Unfortunately, definition of the components of the system and their interrelationship is frequently a complex problem itself. A solution to an improperly defined or oversimplified problem is of no practical value. The key is to accurately model the system so it reflects the true state of nature. Some systems can be simply analyzed with straightforward operations research techniques; while other systems may not be readily recognized as problems and may require some "massaging" before being tackled with OR solution techniques. Because OR techniques are useful in almost all cases, properly defining the system using OR terminology is of prime importance. Regardless of the problem's simplicity or complexity, when properly applied, OR techniques can provide valuable information about system performance characteristics without actual operation of the system.

The value of this approach is quite obvious. Monetary savings can be considered when OR techniques are used instead of actual system testing. System configurations can be varied and measurements taken without any component of the system actually moving. Possible additions or deletions to the
system can be tested; and equipment that does not currently exist can be evaluated and its impact on system performance ascertained. Performance characteristics can be altered, the system reconfigured, and many other permutations can be tested in a matter of seconds, rather than months or years. The benefits are numerous; and, in a climate of increased cost consciousness, cost efficiency is perhaps the most significant benefit.

As indicated earlier in this chapter, the cornerstone of systems analysis, either by computer simulations or analytical methods is the system model developed in the problem definition phase. The concept of a "weapon system" had its genesis in the World War II era, and many attempts were made in the 1950's and 1960's to develop standard models and definitions. The Weapon System Communication (WESCOM) Project at the University of Pennsylvania in the mid 1950's is an example of one of the early multidisciplinary attempts directed at the semantical and taxanomical problems related to weapon system modeling, particularly for air defense systems (2). WESCOM developed a glossary of terms classifying weapons systems by modes of operation. That is, either a weapons system operated independently or it operated under coordinated conditions, either centralized or cooperative. Subsequent doctrinal considerations have changed the terminology to independent/autonomous operation, centralized control, and decentralized control. WESCON was also one of the first attempts and functional breakdowns of subsystems and components (3). In the same time frame a number of individual investigations attempted to complement the large scale multidisciplinary effort as exemplified in WESCOM by developing a Weapon System Philosophy (4), and at least 20 different definitions appeared in the literature related to defense research and development.
One of the last attempts at developing a generalized weapons system model was made by the Army staff in 1969 in "Force Structure Planning - Determination of Micro-Weapon Systems." (5) This effort was directed towards defining an "Elemental Destructive Weapon System" which could be utilized for the comparative analysis of alternative technologies against a hierarchy of enemy targets. It addressed the problem of the basic functional micro-structure of weapon systems, i.e. sensing, communications, movement, and delivery of warheads. The intended application would range from the individual soldier, to a whole air defense battery, and would offer a framework for an optimum cost/effective methodology for the design of organizational structures, particularly at the battalion and bigrade level. The basic symbology and an illustration of its application to modeling air defense systems is shown in Appendix A.

By the 1970's efforts at developing general purpose models of standard definition, and common functional structures of "weapon systems" for use with computer simulations, had almost ceased. Today there is no consensus, or general agreement with the Department of Defense on standard definitions or models. Consequently this research has been oriented towards the application of standard Industrial and Systems Engineering models and methodologies currently employed in quality control and network analysis.
Chapter III
Modeling Systems as Networks

The concept of modeling systems as networks has recently received much attention. Systems that can be modeled as networks possess certain properties which make them particularly attractive both to understanding and solution. The study of network flow problems has produced a variety of solution techniques and the development of numerous efficient algorithms with wide application (6).

In its simplest form, a network flow problem is a connecting of nodes with a system of links over which information, materials, or commodities are transmitted. It is obvious that all air defense system configurations, from the NORAD system to a Redeye section, meet that basic network definition. An air defense system can be modeled as a network and can be studied as a network flow problem. It is, therefore, constructive to survey some of the more useful solution techniques and to discuss how they might be applied in the context of an air defense network.

One of the most common network problems encountered is that of a simple assignment problem. A standard (M - jobs and M - machines) or a nonstandard (M - machines and N - jobs where M ≠ N) assignment problem has been efficiently solved using the transportation algorithm described in many texts, as well as Vogel's approximation techniques. The problem can be quickly solved with computers and feasible solutions can be easily computed by hand.

It is possible to pattern an air defense network as an assignment problem by considering hostile aircraft as jobs and air defense launchers as machines. A measure of effectiveness, combining factors such as reliability, kill-probability, site location, and other parameters, can be used as a substitute for costs in the original assignment problem. Although assignment
algorithms are possible solution techniques and are easily adaptable to subsequent analysis under degraded conditions, the dynamic nature of the aircraft/launcher problem constrains their value in a practical sense.

A related specialized form for networks is the transportation problem. In its most general application, a transportation problem involves shipping commodities from sources to sinks. The main considerations are least-cost and least-time problems. As with other common network problems, algorithms have been developed to efficiently solve transportation problems. The algorithms include the MODI (modified distribution) method and the Northwest Corner Method. With little difficulty, it can be seen that a model of an air defense system can be constructed as a transportation problem; and the model can then be solved using well-known algorithms. As with an assignment-type modeling format, a transportation model for an air defense network has serious operational shortcomings which limit its usefulness.

A special form of the transportation problem, the transshipment problem, possesses a structure which has an intuitive appeal when related to an air defense network. When transshipment nodes are viewed as control centers (ADCCP/TOC/SOC, etc.), an air defense network resembles a transshipment structure. Algorithms have been developed to solve the transshipment problem in an indirect manner by decomposing the transshipment problem into a series of transportation problems. A direct method, the minimal cost network flow method, has also been developed and can be efficiently applied to solve the transshipment problem. Again, the dynamic nature of the problem and parameter calculation present certain difficulties in determining a solution, but the structure of the problem enables degraded conditions to be efficiently solved.

A related area of network analysis is the construction of a minimum spanning tree (i.e., the connecting of all nodes in a tree at minimal cost).
An important characteristic is that any conclusion drawn about a minimum spanning tree (MST) is equally applicable to a maximum spanning tree. When the requirement of total information flow in a control system, such as an air defense network, is considered, the MST seems quite applicable. Kruskal and Dykstra have both studied the MST problem and proposed algorithms to solve it. These algorithms are easily computerized and are quite efficient. When modified so they are rooted at a given node, the MST structure is applicable to an air defense network under optimal conditions and increasingly degraded conditions. The MST algorithm, in either form, could be used to reconstruct the network as a node or arc is destroyed, but it does not give any indication of how targets should be allocated, so its usage is limited. By modifying arc costs and equating costs to some function weighted on such parameters as system reliability, target speed, proximity to defended area, etc., the MST algorithm has possible, although limited, application to the air defense problem.

Other solution techniques for specialized classes of networks have been proposed and proven to be viable in the solution of these problems. As with previously cited examples, an air defense network can be modeled to fit practically any network class; and, by imaginatively defining constraints and parameters, the appropriate algorithm can be used to obtain a solution, if one exists. Some of these classes of problems are the shortest path problem, multicommodity network flows, and minimum cost flow. Bradley provides an excellent survey of the research done and algorithms proposed to solve the above-mentioned problems.

Generally, network analysis is valuable in solving air defense problems. It has the advantages of being computationally efficient and more intuitive to the layman; and cost efficient solution techniques are readily available.
As previously cited, however, there are drawbacks to network analysis with reference to air defense systems.

Mallon researched and proposed a network flow approach to a military related problem in 1974 (7). The problem he considered was to develop a method for controlling telephone communication networks during periods when demand exceeds capacity. Mallon attempted to employ several techniques to solve this problem and described a procedure to configure a communications network. His chapter on network theoretics applies to the air defense problem, as well as it applies to the communication problem. Where Mallon was concerned with the routing of tactical communications, the air defense problem concerns the distribution of targets. Mallon's coverage of the problem of changing user requirements (i.e., network alteration) is directly comparable to the degraded air defense system problem.

Fault tree analysis is another related area of analysis which can be used to gain insight into the underlying structure of an air defense system. Fault tree analysis, which is primarily a technique used in reliability analysis, would provide some understanding of subsystems' strengths, weaknesses, and critical components. Gordon Rankin studied fault tree analysis as applied to operational testing (8). He cites the numerous advantages and disadvantages of the method and concludes that despite its shortcomings, fault tree analysis is an excellent way of gaining a greater understanding of the system. Rankin recommends areas for future research; and to his list, air defense configurations seem to be an appropriate and logical addition.
Chapter IV
MODEL DEVELOPMENT

The previous discussion highlighted some of the background research accomplished in the areas of system definition and representative models. The development of operations research techniques and their application to military systems has been an evolutionary process, sometimes rapid, other times slow, but constantly changing and evolving. It is not the purpose of this paper to add to the techniques available, but rather to show an application of operations research techniques to a particular problem. The problem to be addressed is to model a basic air defense system and to demonstrate the systems approach to this problem.

The specific problem to be addressed is the ability of a basic air defense system to detect, allocate, engage, and destroy hostile aircraft. A basic air defense system is considered to be a battalion size unit, which includes a centralized control element and a number of subordinate fire units. In particular, the system to be modeled is comprised of the AN/TSQ-73 Missile Minder, a command and control system for surface to air missiles and four subordinate fire units or batteries, either Hawk or Nike Hercules. Figure 1 shows typical operational interfaces.

It is important to note that as stated before, there are many ways to model the basic air defense system as defined. The model utilized herein is to view the system as a network, composed of nodes and interconnecting arcs. In particular, the system can be modeled as a variation of a transshipment problem. This can be done so long as each battery is considered as being able to engage only those targets allocated to it, without the capacity to acquire and engage targets independently of the AN/TSQ-73. In the interest of simplicity and ease of calculations, we will consider single commodity
FIGURE 1: AN/TSQ-73 (Missile Minder) Interfaces
operations (i.e., one type of aircraft with identical flight characteristics) and not a multicommodity problem. The multicommodity problem can be solved with multicommodity network algorithms, but such a solution although a more complex problem, is merely an extension of the application demonstration herein. Therefore, for the purposes of illustration, a single commodity network is sufficient.

The basic air defense system modeled as a transshipment problem would be as shown in Figure 2:

FIGURE 2: Basic Air Defense System Modeled as a Transshipment Problem

If this air defense system were to operate as part of a larger organization, the battalion AN/TSQ-73 would operate subordinate to a Group or Society Air Defense Operations Center. In addition, adjacent battalions would have the ability to provide command and control for this air defense system's fire units should its AN/TSQ-73 become non-operational/destroyed.

As shown, the problem, even for a basic air defense system, is complex and, when the possible connecting links with higher and adjacent AN/TSQ-73's are included (not shown in Fig. 2 for simplicity), the problem becomes even more formidable. With current computer resources and solution algorithms
available, solutions to problems of this magnitude are easily attainable and even very complex systems are solvable, although not with a great deal of difficulty.

Having specified the modeling technique to be used, the costs and capacities of the arcs in the network must be defined. The capacities can be described in a functional form to represent the variables which determine the system's performance and thereby enable the system to be evaluated under a variety of conditions.

These functional relationships can be derived using a number of techniques available to the researcher. An obvious and very powerful technique is multiple linear regression. Another means available is purely analytical curve fitting, seeing what relationships are required to have the data fit the results. In any event, the functional form developed can be as simple or complex as the modeler desires.

In this problem the arc capacities must be expressed as two separate functional relationships. One function defines the relationship between targets and the AN/TSQ-73; while the other characterizes the interrelationship of the AN/TSQ-73, fire units and targets.

There are many factors which determine the functional relationship between the AN/TSQ-73 and target aircraft. Some of the more obvious are the aircraft's speed, altitude, electronic warfare capability, raid size, etc. Other factors such as radar masking, acquisition range, target resolution, etc. are also factors in determining the ability of the AN/TSQ-73 to detect, identify and allocate targets to the fire units. Of these parameters, some are more easily quantified than others. Some of these parameters must be given subjective weightings based upon experience. Irregardless of the values assigned to the parameters, each factor present in the derived functional relationship will have an impact on system performance.
In this model, the only concern is the allocation of targets to the fire units, so it is assumed that either all targets are detected, that is the AN/TSQ-73 is fully operational, or no targets are detected, the AN/TSQ-73 is non-operational/destroyed. If no targets are detected, there is no flow in the network and in effect no problem. If all targets are detected, the arc capacities represent the relationship between the AN/TSQ-73 and the fire units. It is assumed that with all targets detected, capacity on the arcs from target to AN/TSQ-73 is infinite and cost is 1.

The ability of the fire units to successfully engage allocated targets is a function of many variables. Some of these variables are totally dependent upon the communications links between the AN/TSQ-73 and the fire units, while others are functions of the fire units' abilities to acquire, track and engage allocated targets.

Two major factors which affect the capacity of the arc from the AN/TSQ-73 to the fire unit are the communications/data link reliability and fire unit system availability. The same factors which impact on AN/TSQ-73 capability to detect and identify targets are primary considerations in a fire unit's ability to destroy allocated aircraft.

To completely describe the performance of a system would require the inclusion of all possible factors in the functional relationship. However, this is not necessarily desireable or required. For the purposes of illustration, the following hypothetical functional relationship is proposed.

\[ U_{ij}(t,w,a,u,m,r) = \frac{mwx}{t} \left( \frac{a}{v} \right)^{-w} \]

- \( U_{ij} \) = Upper bound arc capacity from node i to node j
- \( t \) = crew standard engagement time (min.)
- \( a \) = target altitude (in 1,000 ft.)
\[ v = \text{target velocity (in knots)} \]
\[ m = \text{missiles available (on launches)} \]
\[ r = \text{engagement range (in 10k yds)} \]
\[ W' = \text{electronic warfare capacity of target minus counter-electronic warfare capacity of fire unit (non-negative integer 0, 1, 2, 3)} \]

Assuming for illustration the cost function to be dependent only upon probabilities that a target will not be successfully engaged, the following cost function is presented as a type functional relationship:

\[
C_{ij} = \frac{1}{P_o \times P_m \times P_c}
\]

where:
\[ C_{ij} = \text{cost of arc from node i to node j} \]
\[ P_o = \text{probability a fire unit is operational} \]
\[ P_m = \text{probability a target engaged will be destroyed} \]
\[ P_c = \text{probability communications and software interfaces are operational (probability a target allocation from AN/TSQ-73 is correctly received at the proper fire unit).} \]

Utilizing the hypothetical functional relationships stated, it is possible to generate data of sample cases of gracefully degrading systems to be analyzed. The model as proposed, indicated commodity flows and labels is as shown in Figure 3.
\((c_{ij}, u_{ij})\)

- \(c_{ij}\) = cost on arc
- \(u_{ij}\) = capacity on arc

**FIGURE 3:** Air Defense System Modeled as Commodity Flow Network
To determine the utility and practicality of the model, it must be tested under varying conditions of successive degradation to insure its results are reasonable and usable. Although it is beyond the scope of this paper to enter into the model validation procedure, it is worthwhile to present some sample cases to illustrate the manner in which the model would function and to extend these sample results into some recommendations and conclusions.

The model, as stated previously, is four fire units under the operational control of an AN/TSQ-73 Missile Minder System (see Fig. 3). Using the hypothetical functional relationships specified in Chapter IV for "costs" and "capacities," arc costs and arc capacities can be calculated for each arc of the air defense network. The solution algorithm employed is the Minimum Cost Flow Algorithm, but any of many other techniques such as the Out-of-Kilter Algorithm are equally valid approaches to the problem.

In conducting the sample tests of the model, for purposes of illustration, all target information will remain constant, with the only changes being the changing parameters of fire unit capability and availability, to reflect the successive graceful degradation of the system being modeled.

To interpret the results for each set of conditions, the network cost, as computed by the Minimum Cost Flow Algorithm, is the cost incurred by the air defense system to engage, with a high probability of success, the total aircraft allocated by the AN/TSQ-73 or the total capacity of the system, whichever is smaller. Given that the total engagement capacity of the system is the sum of the capacities of the individual fire units in the system, when the total raid size is greater than the total capacity of the system,
there will be penetration of the air defense system, unless another system is available to engage these targets.

Therefore, it is reasonable to use as a measure of the system's effectiveness the total cost incurred to engage the hostile aircraft and the number of aircraft which were not engaged. Using these two parameters, one could determine the system's effectiveness by comparing the cost and number of penetrators to a predetermined fixed index, thereby gaining an appreciation of the effect of graceful degradation upon system effectiveness.

In each of the cases presented, the target characteristics are held constant, as stated previously. These target characteristics are presented in Appendix B, Table 1. The initial system characteristics are presented in Appendix A, Table 2. As these parameters are changed due to system degradation in the various cases considered, only those parameters which are changed will be listed. Those parameters not specifically stated for the individual cases are assumed to be the initial system characteristics. The network representations of these various cases are presented as appendices to this paper.

In Case 1, (see Appendix C) the system is presented as a fully operational system, with the only constraints on the system capability being those imposed by the initial system characteristics and target characteristics. Solving this system as a maximum flow, minimum cost network yields the solution as shown in Appendix B.

In case 2 (see Appendix D), the system has been degraded by decreasing the number of available missiles at Battery A from 18 to 11. Even with just this slight degradation in system capabilities, there has been a substantial change in overall system effectiveness, resulting in a failure to engage 3 aircraft.
In Case 3 (see Appendix E), the communications from Battery B to the AN/TSQ-73 is out of action, while Battery A is still degraded by having only 11 missiles available. Here, the system again has been reduced in efficiency by having 3 aircraft penetrate without being engaged and because of the requirement to allocate targets to Battery B by way of Battery A, the cost has increased significantly. This cost is occasioned by the additional cost of 5 units per target for each allocation to Battery B.

In Case 4 (see Appendix F), the system is further degraded by having Battery B become non-operational. This results in system effectiveness being further reduced as 14 aircraft are now able to penetrate the defended area without being engaged.
Chapter VI
RESULTS AND CONCLUSIONS

From the costs of engagements and the numbers of aircraft which penetrate the defended area without being engaged for each of the cases presented, it is easily seen that as system capabilities are degraded, these costs increase. In addition, as system capabilities are degraded, system effectiveness measured against aircraft engaged also decreases. In a mode as simple as this where the effects of specified parameters can be controlled, it becomes immediately evident the impact each level of degradation has upon system capabilities. From this, one can recognize the value of utilizing the techniques of operations research, systems analysis to examine real life systems in part and as a whole to gain a greater appreciation of how the parts of the system act and interact to create the system as it operates.

In this context, it can be concluded that operations research techniques can be successfully employed to analyze an air defense system and that from such an analysis significant information can be obtained. This information can then be utilized to give the commander on the ground a more detailed impact of the effects of graceful degradation upon his system effectiveness. In addition, information of this type is extremely valuable to the systems designer in that it enables him to focus more clearly upon the critical items of the system to ensure improved system availability through greater reliability, surviveability and redundancy.
APPENDIX A

Development of Micro-Weapon System Structure

I. Basic Functions of Weapon Systems
   a. In order to develop a comparison between elementary or micro-weapon systems, it is useful to consider certain basic functions with a commonality of functions across the several families of weapon systems. A unit weapons systems is a set of inter-dependent related man and equipment elements capable of performing a combination of four elementary functions with the objective of delivering a warhead and destroying a specified target array.

   b. The functions which are common, and their definitions, are:

      (1) SENSING. A systems has the ability to sense; to judge, distinguish, discriminate, or estimate external conditions of the target.

      (2) COMMUNICATE. A system has the ability to communicate; to receive, pass along, transmit, or make known information pertinent to the target.

      (3) MOVE. A system has the ability to move; to place itself in a position of advantage, change place or position, and respond to movements of the target.

      (4) LAUNCH. A system must be able to launch/shoot; to send forth or discharge a missile, projectile, warhead, or other means intended to accomplish the objective relative to the target.

II. Case for Employment of Weapon Systems

   It is convenient to discuss the micro-weapon system in relation to its movement or position and the relative movement or positions of the target. There are four cases of relativity between the micro-weapon systems and the target array.
III. Elemental Micro-Weapon Structures

Using the fundamental functions and the basic cases outlined, it is a simple matter to synthesize the basic micro-weapon structure. One way of relating the basic functions is as follows.

**LEGEND:**

- ○ Man function
- □ Equipment function
- S Sensing
- C Communication
- M Movement
- L Launcher
- T Target

--- Information flow
--- Logistics support
--- Command and control

Output

Denotes either man or machine function
AIR DEFENSE
(Fixed)

AIR DEFENSE
(Mobile)
APPENDIX B

System and Target Characteristics

TABLE 1: Target Characteristics

1. Raid Size: 40
2. Target Altitude: 30,000 ft.
3. Target Speed: 400 knots
4. Electronic Warfare Capability: 0

TABLE 2: System Characteristics

<table>
<thead>
<tr>
<th>Unit</th>
<th>Missiles</th>
<th>Eng. Rng.</th>
<th>Time to Eng.</th>
<th>$P_o$</th>
<th>$P_m$</th>
<th>$P_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18</td>
<td>200,000 yds</td>
<td>2.1 min</td>
<td>.80</td>
<td>.90</td>
<td>.80</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>150,000 yds</td>
<td>1.6 min</td>
<td>.75</td>
<td>.85</td>
<td>.85</td>
</tr>
<tr>
<td>C</td>
<td>24</td>
<td>120,000 yds</td>
<td>2. min</td>
<td>.85</td>
<td>.80</td>
<td>.75</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>90,000 yds</td>
<td>1.1 min</td>
<td>.70</td>
<td>.80</td>
<td>.65</td>
</tr>
</tbody>
</table>

$P_{AB} = .20$  $P_{BC} = .20$  $P_{CD} = .20$
APPENDIX C

CASE 1; No System Degradation

\[(C, u_{ij})\]
\[C_{ij} = \text{cost on arc}\]
\[u_{ij} = \text{capacity on arc}\]

COST OF ENGAGEMENT: 118.28 units
APPENDIX D

CASE 2; Battery A Degraded (Only 11 Missiles Ready)

\[(c_{ij}, u_{ij})\]
\(c_{ij}\) = cost on arc
\(u_{ij}\) = capacity on arc

COST OF ENGAGEMENT: 112.08 units + 3 aircraft not engaged
APPENDIX E

CASE 3; Battery A Degraded (Only 11 Missiles Ready), Battery B Degraded (Communications Lost to AN/TSQ-73)

\[(c_{ij}, u_{ij})\]
\[c_{ij} = \text{cost on arc}\]
\[u_{ij} = \text{capacity on arc}\]

COST OF ENGAGEMENT: 167.08 units + 3 aircraft not engaged
APPENDIX F

CASE 4; Battery A Degraded (Only 11 Missiles Ready), Battery B Out of Action

\[(C_{ij}, u_{ij})\]
\[C_{ij} = \text{cost on arc}\]
\[u_{ij} = \text{capacity on arc}\]

\[
\begin{array}{c}
\text{TGT} 40 \\
\text{TSQ 73} \\
\text{C} \\
\text{D} \\
\text{TGT}
\end{array}
\]

\[
\begin{array}{c}
(1.00) \\
(1.00) \\
(5.00) \\
(272.7)
\end{array}
\]

\[
\begin{array}{c}
(1.00) \\
(1.96.11) \\
(1.74.8)
\end{array}
\]

COST OF ENGAGEMENT: 801.73 units + 14 aircraft not engaged
BIBLIOGRAPHY


