Project #: E-25-639
Center #: 10/24-6-R6752-0A0
Contract#: DE-FG05-89ER53287
Prime #: 
Subprojects ?: N
Main project #: 

Project unit: ME
Project director(s):
THOMAS C & JR ME (404)894-3731

Sponsor/division names: US DEPT OF ENERGY / DOE OAK RIDGE - TN
Sponsor/division codes: 141 / 017

Award period: 890601 to 900531 (performance) 900831 (reports)

Sponsor amount New this change Total to date
Contract-value 36,523.00 36,523.00
Funded 36,523.00 36,523.00
Cost sharing amount 0.00

Does subcontracting plan apply ?: N
Title: USING CROSSED-SIGHTILINE CORRELATION OF ELECTRON CYCLOTRON EMISSION TO ......

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820
Sponsor technical contact DONALD H PRIESTER / ER-64 (301)353-3421
OFFICE OF FUSION ENERGY, USDOE H/Q WASHINGTON, DC 20545

Security class (U,C,S,TS) : U
Defense priority rating : N/A
Equipment title vests with: Sponsor GIT X
PRIOR WRITTEN APPROVAL REQUIRED IF OVER $500.

Administrative comments - INITIATION.
GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 09/19/90

Project No. E-25-639
Project Director THOMAS C E JR
School/Lab MECH ENGR
Sponsor US DEPT OF ENERGY/DOE OAK RIDGE - TN
Contract/Grant No. DE-FG05-88ER53287
Prime Contract No.

Title USING CROSSED-SIGHTLINE CORRELATION OF ELECTRON CYCLOTRON EMISSION TO

Effective Completion Date 900531 (Performance) 900831 (Reports)

Closeout Actions Required:
Final Invoice or Copy of Final Invoice
Final Report of Inventions and/or Subcontracts
Government Property Inventory & Related Certificates
Classified Material Certificate
Release and Assignment
Other

Comments

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director Y
Administrative Network Representative Y
GTRI Accounting/Grants and Contracts Y
Procurement/Supply Services Y
Research Property Management Y
Research Security Services N
Reports Coordinator (OCA) Y
GTRC Y
Project File Y
Other N

NOTE: Final Patent Questionnaire sent to PDPI.
June 13, 1990

Ms. Melissa Y. Johnson, Contract Specialist  
U. S. Department of Energy-Oak Ridge Operations  
Procurement and Contracts Division  
P. O. Box 2001  
Oak Ridge, TN 37831-8758  

REFERENCE: Grant # DE-FG05-89ER53287

Dear Ms. Johnson,

Enclosed in triplicate is the final Financial Status Report (SF-269) for Grant No. DE-FG05-89ER53287 covering the period June 1, 1989 through May 31, 1990.

If you should have questions or need additional information, please contact Geraldine Reese of this office at (404) 894-2629.

Sincerely,

David V. Welch  
Director  
DVW/GMR/djt

Enclosures

cc: Dr. W. O. Winer, Mech Eng 0405  
   Dr. C. E. Thomas, Mech Eng 0405  
   Ms. Mildred Heyser, OCA/CSD 0420  
   File E-25-639/R6752-0A0
**FINANCIAL STATUS REPORT**  
**Short Form**  
*(Follow instructions on the back)*

1. **Federal Agency and Organizational Element to Which Report is Submitted**  
   U. S. Department of Energy

2. **Federal Grant or Other Identifying Number Assigned By Federal Agency**  
   DE-FG05-89ER53287

3. **Recipient Organization (Name and complete address, including ZIP code)**  
   Georgia Tech Research Corporation  
   P. O. Box 100117  
   Atlanta, GA 30384

4. **Employer Identification Number**  
   58-0603146

5. **Recipient Account Number or Identifying Number**  
   E-25-639/R6752-0A0

8. **Funding/Grant Period (See Instructions)**  
   **From (Month, Day, Year):**  
   June 1, 1989  
   **To (Month, Day, Year):**  
   May 31, 1990

10. **Transactions:**
   a. **Total outlays**  
      | Previously Reported | This Period | Cumulative |
      |---------------------|-------------|------------|
      | 36,523.00           | -0-         | 36,523.00  |
   b. **Recipient share of outlays**  
      | -0-                 | -0-         | -0-        |
   c. **Federal share of outlays**  
      | 36,523.00           | -0-         | 36,523.00  |
   d. **Total unliquidated obligations**  
      | -0-                 | -0-         | -0-        |
   e. **Recipient share of unliquidated obligations**  
      | -0-                 | -0-         | -0-        |
   f. **Federal share of unliquidated obligations**  
      | -0-                 | -0-         | -0-        |
   g. **Total Federal share (Sum of lines c and f)**  
      | 36,523.00           | -0-         | 36,523.00  |
   h. **Total Federal funds authorized for this funding period**  
      | 36,523.00           | -0-         | 36,523.00  |
   i. **Unobligated balance of Federal funds (Line h minus line g)**  
      | -0-                 | -0-         | -0-        |

11. **Indirect Expenses:**
   a. **Type of Rate**  
      | Provisional | Predetermined | Final | Fixed |
   b. **Rate**    | See Below    |             |       |       |
   c. **Base**    | MTDC         |             |       |       |
   d. **Total Amount**  
      | $13,760.77   |             |       |       |
   e. **Federal Share**  
      | $13,760.77   |             |       |       |

12. **Remarks:**  
   "Questions pertaining to this report should be directed to:  
   Geraldine Reese (404) 894-2629"

13. **Certification:**  
   "I certify to the best of my knowledge and belief that this report is correct and complete and that all outlays and unliquidated obligations are for the purposes set forth in the award documents."

**Typed or Printed Name and Title**  
David V. Welch, Director  
Office of Grants and Contracts Accounting

**Signature of Authorized Certifying Official**

**Telephone (Area code, number and extension)**  
(404) 894-2629

**Date Report Submitted**  
June 13, 1990

Previous Editions not Usable

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$745.00 excluded from Indirect Costs

2. **CONTRACT or grant number**  DE-FO5-89ER53287
   2A. MASTER contract number (GOCO's)
   2B. Responsible PATENT office

3. Performing organization CONTROL number (internal)
   3A. Budget and Reporting code E25-639
   3B. Funding YEAR for this award 1989-1990

4. Original contract start date  June 1, 1989
   4A. Current contract start date  June 1, 1989

5. Work STATUS
   - Proposed
   - Renewal
   - New
   - Terminated
   5A. Manpower (FTE)  0.6

5B. CONGRESSIONAL district  Fifth
5C. STATE or Country where work is being performed  Georgia
5D. COUNTRY sponsoring research  USA

6. Name of PERFORMING organization  Georgia Institute of Technology

6A. DEPARTMENT or DIVISION  Fusion Research Center
   Nuclear Engineering

6B. Street Address  ESM Bldg.
   Cherry Street

6C. City, State, Zip Code  Atlanta, Georgia 30332

7. Circle only one code for TYPE of Organization Performing R&D:
   - CU - College, university, or trade school
   - FF - Federally funded RD&D centers or laboratory operated for an agency of the U.S.
     Government
   - IN - Private industry
   - NP - Foundation or laboratory not operated for profit
   - ST - Regional, state or local government facility
   - TA - Trade or professional organization
   - US - Federal agency
   - XX - Other
   - EG - Electric or gas utility

8A. Contractor's PRINCIPAL INVESTIGATOR/s or project manager
   Name/s (Last, First, MI)  Thomas, C.E.

8B. PHONE's (in order of PI names with commercial followed by FTS)
   Comm.  (404)894-3731  : FTS

8C. PI's address (if different from that of Performing Organization)
10. FUNDING in thousands of dollars (KS). Funds represent budget obligations for operating and capital equipment (FY runs October 1 – September 30).

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10D. Does the current FUNDING cover more than one year’s work? Yes XX No 
E. If yes, provide dates (from when to when). 

11. Descriptive SUMMARY of work. Enter a Project Summary using complete sentences limited to 200 words covering the following: Objective(s), state project objectives quantifying where possible (e.g., “The project objective is to demonstrate 95% recovery of sulphur from raw gas with molten salt recycling at a rate of one gallon per minute.”); approach, describe the technical approach used (how the work is to be done); expected product/results, describe the final products or results expected from the project and their importance and relevance.

Determine the feasibility of fluctuation measurements using electron cyclotron emission correlation in large fusion experiments. This work is potentially important since it may help determine the reason for anomalous energy confinement in the devi


13. KEYWORDS (Listed five terms describing the technical aspects of the project. List specific chemicals and CAS number, if applicable.)

cyclotron, fluctuations, fusion, transport

14. RESPONDENT. Name and address of person filling out the Form 538. Give telephone number, including extension (if you have FTS number, please include it) at which person can be reached. Record the date this form was completed or updated. The information in Item 14 will not be published.

Respondent's Name: Thomas, C.E.  Phone No.: (404)894-3731  Date: 7/11/89

Street: Nuclear Engineering

City: Atlanta,  State: Georgia  Zip: 30332
NOTICE: Return this form to the office indicated in the reporting requirements for your award agreement covering this project. If you have completed a similar programmatic office project description during the current Fiscal Year, complete only the new data elements on this form and send it and a copy of the description completed earlier to Department of Energy, Office of Scientific Information, P. O. Box 62, Oak Ridge, TN 37831.
Autocorrelation and Crossed-Sightline Correlation of ECE for Measurement of Electron Temperature and Density Fluctuations on ATF and TEXT

C. E. Thomas Jr.
Nuclear Engineering Program, Georgia Institute of Technology
Atlanta, Georgia 30332

R. F. Gandy
Department of Physics, Auburn University, Auburn, Alabama 36849

June 26, 1990

Abstract

The great stumbling block in the quest for fusion power using magnetic confinement devices is anomalous transport. It is conjectured that turbulent plasma fluctuations may be responsible for the degraded energy confinement observed in experiments. There exists a clear need for more detailed experimental studies of plasma microturbulence. A conceptual design is presented for a diagnostic to measure electron temperature and density fluctuations using electron cyclotron emission (ECE). The proposed ECE systems will employ auto-correlation and cross-correlation techniques to measure radiation from the Advanced Toroidal Facility (ATF) at Oak Ridge National Laboratory (ORNL) and also from the Texas Experimental Tokamak (TEXT) at the University of Texas. This set of experiments on a stellarator and a tokamak will allow a unique comparative study of the fluctuation physics in the two different magnetic configurations. The paper below presents the theoretical basis and conceptual design of the diagnostic.
I Introduction

I-A Motivation

In the quest for controlled thermonuclear fusion, the magnetic confinement approach has encountered a formidable obstacle: anomalous transport. The experimental values of electron thermal diffusivity exceed those expected from neoclassical theory by orders of magnitude. There clearly exists a lack of understanding of thermal transport in high temperature plasma devices. It is conjectured that fine-scale plasma microinstabilities may be the cause of the anomalous transport.\[1\]-[3] Currently no accepted theory exists which can fully explain the experimental results obtained. One of the causes of this lack of understanding is the past experimental emphasis on measuring bulk, time-averaged plasma quantities. In order for the plasma community to understand the basic physics of anomalous transport, experimentalists need to design dedicated plasma diagnostics to measure the fluctuating components of plasma quantities. This is currently being done at many sites.

Electron cyclotron emission (ECE) has the potential to provide information on fluctuations in electron temperature, electron density and magnetic field. Initially, our experiments will focus on the detection of optically thick, second harmonic ECE to look at fluctuations in electron temperature. Later it is intended to include additional sightlines to allow measurements of fluctuations in electron density and plasma pressure using optically thin emission. We plan a set of experiments to measure fluctuating ECE from the ATF stellarator located at ORNL and TEXT (a moderate sized tokamak) at the University of Texas. This use of a tokamak and a stellarator will lead to insights into the similarities and differences of transport in the two main classes of toroidal devices.

I-B Background

The history of the use of optically thick electron cyclotron emission to measure fluctuations in plasma electron temperature is somewhat of a dichotomy. In the first case, ECE has been used with great success to measure coherent, large amplitude, MHD-type fluctuations (e.g., sawteeth) that have significant spatial extent ($\geq 2\ \text{cm}$) and occur on a relatively slow
(≥ 10 μs) time scale. The MHD-related temperature fluctuations are usually localized to a flux surface and can be spatially resolved using the standard ECE diagnostic techniques. References [4]-[6] contain examples of the use of ECE to measure these types of coherent $T_e$ fluctuations. On the other hand, the application of ECE to measure incoherent or turbulent temperature fluctuations has not been at all successful. The only attempt to date did not work because of photon noise limitations.[7],[8] A summary of the present experimental situation would be that coherent, large-scale fluctuations can be easily measured but one should use extreme caution when looking for incoherent, small-scale fluctuations. In the present work we present advanced methods of data analysis using autocorrelation and cross-correlation of ECE to infer turbulent temperature fluctuations.

The clear need for a better understanding of transport in magnetic fusion devices leads one to ask the following question: Can useful information on plasma fluctuations be deduced from ECE? To answer this question one must carefully consider the parameter range of possible $T_e$ fluctuations and the physical limits inherent in a measurement of ECE from a magnetic fusion device. This will be done in Section II below. In Section III the use of cross-correlation and autocorrelation to overcome the traditional limits of ECE physics will be discussed. Then the conceptual design for the diagnostic will be discussed in Section IV. Finally, the implications of all of the above will be discussed.

II Fluctuation Physics and Previous ECE Limitations

The first step in evaluating the possibility of using ECE to measure $T_e$ fluctuations is to address the nature of these fluctuations. In particular, what level and type of $T_e$ fluctuations are detectable? The answer to this question depends on the spectrum of the fluctuations in $\omega - k$ space and the amplitude of these fluctuation. Obviously we do not have the exact answers, but some limits are set by looking at present and past ECE results. A typical ECE system[9] used to measure $T_e$ has a spatial resolution of approximately 5 cm and a typical time resolution of 10 microseconds. A typical system noise level is of order 5 ev. The lack of evidence of incoherent $T_e$ fluctuations at these levels implies that one needs to look with better spatial resolution, sample at higher frequencies, and reduce the system noise level.
Since no one has yet reported turbulent $T_e$ fluctuations from the interior of a high temperature plasma, we do not know how much improvement in ECE system performance is required. However we do know that significant improvement in ECE system performance is possible if one designs and dedicates a system to measure fluctuations. As a guide to the type of system performance desired, one can assume that any temperature fluctuation lie in the same part of $\omega - k$ space as the density fluctuations observed to date. With this assumption, one requires a frequency response up to 1 MHz and the ability to resolve wavenumbers in the range of $1 - 10 \text{ cm}^{-1}$ (or wavelengths in the range of 0.5-5 cm). Electron density fluctuation levels on the level of a few percent have been reported. What is required then is an ECE system with spot size on the order of 1 cm or less, time resolution of 1 microsecond or better, and the ability to measure temperature amplitude fluctuations on the order of 1% or less. Of particular interest, in regard to electron temperature fluctuation studies, is the area of trapped electron modes. Recent theoretical work, applied to the ATF device, predicts that the trapped electron population will vary greatly for two specific magnetic configurations obtained by varying the quadrupole field. The first configuration is practically shearless on the inner half of the plasma radius, and the second configuration has strong shear throughout. Theory predicts that in the first case the dissipative trapped electron (DTE) mode will be localized and that in the second case it will be extended along field lines. The DTE mode should be more dangerous in the first configuration because of the localization of the drift wave. If these trapped electron modes are the driving cause of anomalous transport, then the effects of varying the trapped electron fraction may be measurable using the fluctuating ECE diagnostic system described herein. It would be exciting to compare scaling laws derived from varying the trapped electron fraction on ATF with the experimental results of ECE measurements on TEXT.

Recent theoretical predictions for mode localization, wavelengths, and frequencies have been discussed by Carreras. For ECH discharges in ATF, it is expected that the maximum fluctuation level for dissipative trapped electron (DTE) modes will be peaked in the inner third (both sides of the plasma center if orbits are confined) of the plasma radius because of the stellarator low shear. The poloidal wave length is expected to be comparable to the radial width of the mode and is of the order of a few cm. For a tokamak like TEXT, the fluctuation
level probably peaks at the half minor radius on the outside (no trapped particles on the inside) with the poloidal wave length being several times longer than the radial width of the mode and of the order of a centimeter. Contrary to other types of instabilities and because these fluctuations involve trapped electrons, it is expected that the electron temperature fluctuation ($\dot{T}_e/T_e$) will be larger in comparison to density fluctuations. Frequencies (with no Doppler shift) would be less than 100 kHz. Theoretical predictions of the mode amplitude involve complex nonlinear saturation calculations and are a subject of considerable discussion among plasma theorists.

II-A Previous Limitations on ECE Measurements

Now that we have a guide for the type of ECE system performance required to explore for electron temperature fluctuations, we address the physics limitations of such a system. The critical question to be addressed when assessing the possibility of using ECE to measure electron temperature fluctuations is: What is the minimum achievable spatial resolution? One needs as small a spot size as possible in order to resolve the large wavenumber fluctuations associated with the plasma microturbulence. If we consider viewing perpendicularly to the magnetic field, then the spot size for viewing in the horizontal mid-plane is determined by two factors: (1) transverse spot size set by the diffraction limit, and (2) longitudinal spot size set by frequency linewidth considerations.

Let us first consider the case of viewing the plasma with a horn/lens system which allows focusing in order to minimize spot size in the plasma (see Fig. 1). The spatial extent of the spot width $d$, transverse to the viewing direction, is determined by the diffraction limit. For the circular aperture case the diameter of the spot is given by the relation $d = 2.44 \lambda f / D$, (where $\lambda =$wavelength, $f =$focal length of the lens and $D =$diameter of the lens). This value of $d$ represents the distance between the first two minima in the Airy diffraction pattern and therefore encompasses 84% of the radiated power. Practically the factor $f / D$, the f#, has a minimum set by the finite size of the vacuum viewing window and lens construction limitations. For the two devices under consideration, these limitations set an f# minimum of approximately 2. Therefore one finds that $d \geq 5\lambda$. For operation at $B = 2$ Tesla, and second harmonic ECE, this implies that $d_{\text{min}} \approx 1.5$ cm. This result holds for ATF and TEXT. Note
that for machines with a higher magnetic field or optically thick higher harmonics the spot size will be even smaller since it scales as $B^{-1}$.

The second consideration involves the longitudinal extent of the viewing spot-size, spatial dimension $w$ in Fig. 1. This dimension is related to the geometry of the surfaces of constant magnetic field magnitude. For the TEXT and ATF cases under consideration (horizontal mid-plane, viewing toward the center of the plasma), surfaces of constant $|B|$ are near vertical. For emission perpendicular to the magnetic field the dimension $w$ is set by the natural frequency linewidth convolved with the instrumental bandwidth, which in turn depends on the magnetic field spatial gradient. The lower limit is set by the natural linewidth since the instrumental bandwidth can be set to an arbitrarily low value (while noting that setting the instrumental bandwidth very much lower than the natural linewidth decreases signal without improving spatial resolution). Therefore, we will assume that the instrumental bandwidth is chosen such that the natural linewidth dominates in the determination of dimension $w$.

The natural line broadening is due to relativistic and/or Doppler broadening depending on the relative angle between the direction of emission and the magnetic field. The spread due to Doppler broadening is minimized by viewing perpendicularly to the magnetic field. Therefore, in our case, the spread is due to relativistic line broadening. For second harmonic ECE the relativistic linewidth is given by\textsuperscript{14}: $\Delta f/f = 4.1 \frac{qT_e}{m_e c^2}$, (where $f = 2f_c$, $f_c$ is the local electron cyclotron frequency, $q$ is the electron charge, $T_e$ is the electron temperature in eV, $m_e$ is the electron mass, and $c$ is the speed of light). The spatial variation of the magnetic field (and therefore of $f$) then defines $w$, if the instrumental bandwidth approximately equals the natural linewidth.

The preceding analysis of the determination of the longitudinal extent of the spot size $w$ must be modified when dealing with optically thick harmonics (as is the case for TEXT and ATF at second harmonic, X-mode). The reason for this has to do with the reabsorption of radiation that occurs with a black body. The effect is illustrated in Fig. 2. Referring to Fig. 2, consider an optically thick emitting layer having a spatial width of $w_0$ and a total optical depth of $\tau_0$, where $\tau_0 > 2$. For simplicity, also assume that the absorption coefficient $\alpha$ is constant across the layer. Recall that the optical depth $\tau$ is defined as $\int \alpha dx$. Consider radiation emanating from $x_0$, toward the receiving horn. As this radiation traverses the
layer it will constantly be absorbed and reemitted with an intensity characteristic of the local electron temperature level. This process will continue as the radiation moves to higher values of \( x \), until the radiation passes a critical value of \( x \), \( x_{cr} \). The point \( x_{cr} \) is defined by the characteristic that the accumulated optical depth between \( x_{cr} \) and \( x = w_0 \) is approximately 2 (\( \tau = 2 \) is chosen as this region will account for approximately 86% of the emission). Therefore for purposes of viewing the emission from the emitting region, one sees a reduced layer (cross-hatched region) characterized by a smaller, effective width, \( w = w_0 - x_{cr} \). The consequences of this effect on the spot size for TEXT and ATF have been calculated for ECE at second harmonic, extraordinary mode emission where relativistic broadening is the dominant line broadening mechanism. Typical results are shown in Figures 3 and 4 for ATF and TEXT respectively. Note that this effect limits \( w_{min} \) to a value less than 1 cm for both devices.

Once the spot size has been reduced as much as possible, the next area of concern is the sensitivity of the receiver. For a typical heterodyne receiver the lower limit of the sensitivity is set by the quantum noise limit\(^{[15]}\): \( \Delta T_e / T_e \geq (\Delta f_v / \Delta f_r)^{0.5} \). Here \( \Delta f_v \) is the video bandwidth (defined as the inverse of the microwave crystal detector integration time constant) and \( \Delta f_r \) is the bandwidth of the radiation being detected. For a typical radiation bandwidth of 400 MHz and a video bandwidth of 100 kHz this implies that fluctuations in the range of 1 – 2% should be detectable. If the observed emission is from plasma regions where \( T_e = 100 \) to 500 eV, then detection limits of 1 – 5 eV are possible. Also note that lower levels of sensitivity can be achieved by reducing the video bandwidth (at the expense of time and frequency resolution). One should note that the signal being viewed may depend on magnetic field, temperature, and density. This will depend on which harmonic is being used.

In summary, an ECE diagnostic system using present methods can be optimized to explore a high temperature plasma for \( T_e \) fluctuations. The minimum detectable amplitude for fluctuations in the frequency range of interest (order of 100 kHz) is \( \approx 2\% \). It will be shown below that this amplitude detection limit can be improved by an order of magnitude with autocorrelation techniques.
III ECE Autocorrelation and Correlation Measurements of Temperature and Density Fluctuations

III-A Introduction

It is our belief that present ECE methods do not give adequate amplitude resolution to measure fluctuations at the frequencies of interest (~ 100kHz), and give no information about wavelengths of the fluctuation being studied. It has been suggested\cite{16}-\cite{18} that crossed sightline correlation of ECE would give another method of measuring temperature and density fluctuations. We propose here a variant of that method, auto and cross correlation of a single sightline ECE system, to measure temperature fluctuations including amplitude, frequency dependence, correlation length in the plasma (width in k-space), and correlation time (width in ω-space). We would like to mention here that Dr. P. Efthimion of PPPL has previously suggested\cite{19} the use of autocorrelation for the measurement of correlation length in the plasma. We have extended our previous work to show that this and considerably more can be measured.

Autocorrelation of an optically thick harmonic must be used to measure temperature fluctuations, as discussed in detail below. If in addition to the sightline for the optically thick harmonic, a crossing sightline for an optically thin harmonic is constructed, then density fluctuations (amplitude, frequency dependence, possibly wavelength dependence) can be measured by comparing the autocorrelation functions of the two sightlines. Additionally, the PHASE (important for inferring transport) between the density and temperature fluctuations can be inferred by cross-correlation of the signal between the two sightlines. It is not presently planned to build the second sightline on either TEXT or ATF. This will be proposed at a later date after demonstration of the temperature fluctuation measurement.

It is demonstrated below that the resolution of the autocorrelation system is limited only by the integration time available. The maximum time for integration is limited by the amount of time that an experiment is in a quasi steady state. On TEXT and ATF this time period is from 10 to 100 ms. For this integration time we estimate that the proposed ECE autocorrelation diagnostic can resolve temperature fluctuations of $2 \times 10^{-3} T_e$, i.e., 0.2%,
at frequencies of 100 kHz. This is an order of magnitude better than the resolution of a traditional heterodyne ECE system. The frequency resolution would be on the order of 1 kHz, and the diagnostic will be responsive to wavelengths longer than 1 cm, and will give the spread in k-space of the wavelengths for each frequency band measured. For machines with higher magnetic fields or temperatures, it may be possible to achieve sensitivity to fluctuation wavelengths of the order of 1 mm, since higher magnetic field and higher harmonics means smaller spot sizes for the antenna pattern, and higher temperature and density implies greater optical depth and a corresponding decrease in the emission line width.

There is of course some price to pay for the improved resolution of the autocorrelation ECE diagnostic. In this case it is not money (the diagnostic is relatively cheap) so much as time resolution. It is necessary to assume that the average amplitude of the turbulent fluctuations is constant over the time period of the integration. Since the integration period is restricted to a length of time such that the experiment is "steady-state", we believe that this is a reasonable assumption. Note that this does not restrict the frequency resolution of the diagnostic, only the time resolution of the average turbulent amplitude at a particular frequency.

### III-B Correlation Functions Defined

Figure 5 shows the conceptual optical design of the diagnostic and Fig. 6 shows the electronics. The design of the sightline in this diagnostic is very similar to the design of the ECE diagnostic on ATF. The received signal at the antenna (as described in more detail below) will be bandpass filtered into two identical passbands \( \delta \omega \) about several frequencies \( \omega \) at time \( t \) and \( t + \tau \) and fed into microwave detectors. The two signals thus derived for each \( \omega \) are identical except that one is delayed with respect to the other by a time \( \tau \). The delay must be achieved in hardware (to be demonstrated in Section III-H), and the purpose of the delay will be described in detail below. Basically it makes the quantum fluctuations (thermal fluctuations) of the two signals incoherent with respect to one another, but leaves
the bulk plasma fluctuations still coherent (correlated). Let the signal out of these detectors due to emission along the sightline be described by \( I(\omega_i, t) \) and \( I(\omega_i, t + \tau) \) for detectors 1 and 2, respectively, where \( \omega_i \) is the center frequency of the \( i^{th} \) double-channel of the detector array. For a turbulent plasma, the signal \( I \) will have a steady-state part \( \overline{I} \) and a fluctuating part \( i \), in which case

\[
I(\omega_i, t) = \overline{I(\omega_i)} + i(\omega_i, t)
\]  

(1)

The steady-state part of \( I \) is given by the mean value,

\[
\overline{I(\omega_i)} = \frac{1}{T} \int_0^T I(\omega_i, t) dt
\]  

(2)

where \( T \) is the averaging time. Note that \( I \) can be electronically high pass filtered to remove \( \overline{I} \), or this can be done in software.

The correlation function is defined as\(^{[21]}\)

\[
R_{ik}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T i_l(t) i_k(t + \tau) dt
\]  

(3)

where \( R_{ik} \) is precisely the cross-correlation function (CCF) if \( i_l \) and \( i_k \) have mean values of zero, and again \( T \) is the averaging or correlation time. The autocorrelation function (ACF) is defined as the special case when \( l = k \). For the single sightline of this diagnostic, the cross-correlation function is estimated by

\[
R_{ij}(\omega_i, \omega_j, t, \tau) = \frac{1}{T} \int_{-T/2}^{T/2} i(\omega_i, t + \xi) i(\omega_j, t + \xi + \tau) d\xi
\]  

\[
= \langle i(\omega_i, t) i(\omega_j, t + \tau) \rangle
\]  

(4)

where \( \langle \rangle \) is defined as the time correlation function operator, \( T \) is a finite period, and we also allow for the possibility of cross-channel correlation; i.e., the ECE signal at frequency \( \omega_i \) is correlated with the ECE signal at frequency \( \omega_j \). This has many of the characteristics of cross-correlation since looking at different ECE frequencies corresponds to looking at different spatial locations. Therefore, for \( \omega_i \) not equal to \( \omega_j \), it will be described as cross-correlation.
If \( j \) is defined as the local value of the fluctuating part of the emission at positions \( s \) along the sightline, that is,

\[
j(\omega_i, s, t) = \frac{d\bar{i}}{ds}
\]

then Eq. (4) can be written as

\[
R_{ij}(\omega_i, \omega_j, t, \tau) = \langle \int ds_1 \int ds_2 j(\omega_i, s_1, t)j(\omega_j, s_2, t + \tau) \rangle
\]

The region in a plasma where ECE of frequency \( \omega_i \) is emitted is limited to regions where the cyclotron harmonic is approximately equal to \( \omega_i \). The width of this region is the smaller of the natural line width or the absorption length at that point in the plasma, so that

\[
R_{ij}(\omega_i, \omega_j, t, \tau) = \int_{r(\omega_i)-l_e}^{r(\omega_i)+l_e} ds_1 \int_{r(\omega_j)-l_e}^{r(\omega_j)+l_e} ds_2 \langle j(\omega_i, s_1, t)j(\omega_j, s_2, t + \tau) \rangle
\]

where \( r(\omega_i) \) is the point of peak emission of the received signal at frequency \( \omega_i \) (not the same as the point of peak emission for an optically thick harmonic), the distance \( l_e \) is the smaller of the absorption length (optical depth) or natural line width, and the time operator has been passed through the two spatial integrals. The above integral can be approximated as

\[
R_{ij}(\omega_i, \omega_j, t, \tau) = (2l_e)^2 \langle j(\omega_i, r(\omega_i), t) j(\omega_j, r(\omega_j), t + \tau) \rangle
\]

### III-C Autocorrelation Time of Thermal Emission

It can be shown\(^{[22]}\) that the autocorrelation time of a steady-state thermal source of radiation (i.e., one without macroscopic turbulent fluctuations) is given by

\[
\tau_c = \frac{2\pi}{\delta\omega_i}
\]
where $\delta \omega_i$ is the radian frequency passband width, Full Width at Half Maximum (FWHM), of the filter in the channel used to look at the source, and it is assumed that the source is broad-band compared to the filter. This means that the statistical (quantum) fluctuations of a thermal source are 100% correlated with themselves for times short compared to $\tau_\epsilon$. This would appear to prevent autocorrelation from being used as a method of measuring the turbulent fluctuations of a plasma, since in most cases of interest the quantum fluctuations are larger than the turbulent fluctuations of interest. However, it is straightforward to resolve this problem. For passband frequency widths $\delta \nu \geq 100MHz$, the quantum correlation time is less than $10^{-7}$ nanoseconds. However, the turbulent frequency fluctuations of interest have timescales of the order $1ms$ to $1\mu s$. Thus if the autocorrelation signal in one of the dual paths is delayed by $\sim 10$ ns, then the quantum fluctuations will be uncorrelated and their auto or cross-correlation will go to zero, but the turbulent fluctuations remain 100% correlated with themselves. This allows autocorrelation to separate the quantum (thermal) fluctuations from the turbulent fluctuations, and explains why we have used the unequal time correlation function $R_{ij}(\tau)$ above, and will continue to use it below. A slightly more sophisticated treatment of this problem including the effect of the microwave crystal detector time constant (which forces the required delay in hardware, rather than software) will be given in Section III-H below.

### III-D Uncertainty in the Correlation Function

The variance of the calculated cross-correlation function of $i(\omega_i, t)$ and $i(\omega_j, t+\tau)$ for identical bandwidths, $\Delta f$, is

$$
\sigma_{ij}^2 = \frac{1}{2 \Delta f T} \left[ R_{ii}(0)R_{jj}(0) + R_{ij}^2(\tau) \right]
$$

where $T$ is the correlation integration time of $i(\omega_i, t)$ and $i(\omega_j, t + \tau)$ in Eq. (4), $\sigma_{ij}^2$ is the variance of $R_{ij}(\tau)$, $\Delta f$ is the effective bandwidth, and $R_{ij}(\omega_i, \omega_j, t, \tau)$ has been previously defined. For convenience we will often shorten $R_{ij}(\omega_i, \omega_j, t, \tau)$ to $R_{ij}(\tau)$. The requirement that $\Delta f T \geq 5$ is used in obtaining this equation.

The uncertainty in $R_{ij}(\tau)$ is estimated from (also from Ref. [21])
\[ \epsilon^2 \equiv \frac{\sigma_{ij}^2}{R_{ij}^2} = \frac{1}{2 \Delta f T} \left[ 1 + \rho_{ij}^{-2} \right] \]  

where

\[ \rho_{ij}(\tau) = \frac{R_{ij}(\tau)}{\sqrt{R_{ii}(0)R_{jj}(0)}} \]  

is the cross-correlation coefficient function. Equation (11) can be rewritten as

\[ \sigma_{ij} = R_{ij}(\tau) \left[ \frac{1 + \rho_{ij}^{-2}}{N} \right]^{1/2} \]  

where \( N \) is the number of data points used in calculating \( R_{ij}(\tau) \). See Bendat and Piersol[21] for the derivation of Eq. (13). The uncertainty in the autocorrelation function is found by using the above equations with subscripts \( ij \) replaced by \( ii \).

The signals \( i(\omega_i) \) and \( i(\omega_j) \) can be written as

\[ i(\omega_i, t) = s(t) + m(\omega_i, t) \]  

and

\[ i(\omega_j, t) = s(t) + n(\omega_j, t) \]  

where \( s(t) \) is the correlated emission, and \( m(\omega_i, t) \) and \( n(\omega_j, t) \) are uncorrelated emission (noise for our purposes). For this diagnostic \( \langle m^2 \rangle \) and \( \langle n^2 \rangle \) are probably due to equivalent physical phenomena. In this case, they can be treated as equivalent but uncorrelated noise sources, i.e. \( \langle n^2 \rangle = \langle m^2 \rangle \). Substituting these relations into the equation for the cross-correlation coefficient produces the desired result[18],

\[ \rho_{ij} = \frac{1}{1 + \left( \frac{\langle n^2 \rangle}{\langle s^2 \rangle} \right)} \]  

\[ (16) \]
III-E Expected Line Shape

The expected line shape of the cross-correlation function $R_{ij}(\delta \omega_{ij}, t, r)$ versus frequency difference $\delta \omega_{ij} = |\omega_i - \omega_j|$ is difficult to calculate as it depends on the natural spatial line width ($l_{ns}$) of the ECE, the correlation length ($l_c$) and correlation function shape in the plasma, the spot size of the antenna sightline $w$, the absorption length (optical depth) $l_a$, and the wavelength of the fluctuation being measured. It is quite probable that the cross-correlation function will have a Gaussian distribution. This statement is supported by the central limit theorem\cite{21}, which asserts that the Gaussian distribution will result quite generally from the sum of a large number of independent random variables acting together. Assuming the turbulent modes in the plasma are independent, the Gaussian shape follows.

It is assumed that the sightline will be arranged perpendicular to the magnetic field so that the natural line width will be due to relativistic broadening of the emission. In general, the requirement of perpendicularity is relaxed to requiring that the Doppler broadening be less than the relativistic broadening. This allows for some deviation from true perpendicularity of the sightlines to the magnetic field. The natural line width $\delta \omega_n$ (full width at half maximum) due to relativistic broadening is given by\cite{14}

$$\frac{\delta \omega_n}{\omega_b} = c_s \left[ \frac{v_{te}}{c} \right]^2$$

(17)

where $\omega_b$ is the fundamental cyclotron frequency, and $v_{te}$ is the electron thermal velocity (but is defined as $v_{te} = (T/m)^{1/2}$ rather than the often used $(2T/m)^{1/2}$). Here $s$ is the harmonic number, and $c_s$ is a constant which changes with harmonic number, e.g. for $s = 3$, $c_s = 4.8$.

We have also calculated line-broadening due to the Doppler shift caused by the finite angular spread of a Gaussian beam, and the line-broadening due to finite transit-time effects. Our calculations show that both of these are much less than the relativistic broadening.

To calculate the variance in frequency space of $R_{ij}(\delta \omega_{ij}, \tau)$, it is assumed that the two dominant factors are the correlation length and the emission line width. If the natural ECE line width in frequency space is considered as its equivalent spatial line width,
\[ l_{ns} = \frac{B}{\nabla B} \frac{\delta \omega_n}{\omega_b} = R \, c_s \left[ \frac{v_{te}}{c} \right]^2 \] (18)
	hen the variance is approximated by

\[ \delta \omega_c^2 = \frac{\omega_i^2}{R^2} (l_c^2 + l_e^2) \] (19)

where again we take the emission line-width \( l_e \) as the smaller of \( l_{ns} \) or \( l_a \). Then \( \delta \omega_c \) is the width (FWHM) in frequency space of the cross-correlation function \( R_{ij}(\delta \omega_{ij}, t, \tau) \), \( \omega_i \) is the approximate cyclotron harmonic frequency under consideration, and \( R \) is the average major radius of \( r(\omega_i) \) and \( r(\omega_j) \). Note that this defines the correlation function in frequency space,

\[ R_{ij}(\delta \omega_{ij}, t, \tau) = R_{ii}(\omega_i, \tau) \exp \left\{ -\frac{1}{2} \left[ \frac{\omega_i - \omega_j}{\delta \omega_c} \right]^2 \right\} \] (20)

Once \( \delta \omega_c \) and \( R_{ii}(\omega_i, \tau) \) are calculated, a continuous Gaussian distribution for \( R_{ij}(\delta \omega_{ij}, t, \tau) \) is defined. For our experiments we will measure \( R_{ij}(\delta \omega_{ij}, t, \tau) \) versus \( \delta \omega_{ij} \) [including \( R_{ii}(\omega_i, \tau) \)].

III-F Estimating the Correlation Noise

The next step is to estimate the uncertainty in each value of \( R_{ij}(\delta \omega_{ij}, t, \tau) \) using Eq. (13),

\[ \sigma_{ij} = R_{ii}(\omega_i, t, \tau) \left[ \frac{1 + \rho_{ij}^2}{N} \right]^{1/2} \] (21)

The cross-correlation coefficient \( \rho_{ij} \) is calculated using Eq. (16). To do this, we must estimate the sources of the noise \( \langle n^2 \rangle \). The uncorrelated signal (noise) can be divided into two categories: noise produced in the plasma and noise produced in the microwave hardware. If we approximate the hardware noise (after the IF amp) as 0.01 eV black body emission, our ECE power calculations for TEXT and ATF show the hardware noise to be down by at least four orders of magnitude relative to the signal, and we find that plasma noise dominates.

The remaining noise can be divided into three sources: the quantum noise at the detectors, the uncorrelated signal incident on the antenna due to reflections off the vacuum vessel (optically thin harmonics only), and uncorrelated emission along the sightline.
Radiometer theory\cite{15} has shown that for thermal emission the fluctuation about the average signal value is given by

\[ n_q = \frac{S_T}{\sqrt{\tau_d \delta \omega_i/2\pi}} \]  (22)

where \( n_q \) is the quantum noise, \( S_T \) is the total average received signal, \( \delta \omega_i \) is the bandwidth of the channel (previously defined), and \( \tau_d \) is the microwave crystal detector integration time. The total average received signal is defined as the dc signal plus vacuum vessel reflections. This can be written as

\[ S_T = \bar{I} + Q\bar{I}(1 - \eta) \]  (23)

where \( \bar{I} \) is as before, \( Q \) is the cavity resonance term for the vacuum vessel, and \( \eta \) is the viewing dump efficiency (absorption coefficient). For optically thick harmonics, \( \eta = 1 \). The vacuum vessel cavity resonance is defined as

\[ Q = \frac{1}{1 - R_w} \]  (24)

where \( R_w \) is the wall reflectivity of the vacuum vessel.

To calculate the correlation coefficient \( \rho_{ij} \), the noise terms must be normalized to the correlated signal \( s(\delta \omega_i, t) \) [see Eq. (16)]. Thus the normalized quantum noise can be written as

\[ \frac{n_q}{s(\delta \omega_i, t)} = \frac{1 + Q(1 - \eta)}{A_f \sqrt{\tau \delta \omega_i/2\pi}} \]  (25)

where \( A_f \) is the emission fluctuation fraction due to correlated fluctuations in density, temperature, or magnetic field and is defined as

\[ A_f \equiv \frac{s(\delta \omega_i, t)}{\bar{I}} \]

\[ \simeq \frac{1}{\bar{I}} \left[ R_{ii}(\omega, \tau) \exp \left\{ -\frac{1}{2} \left[ \frac{\omega - \omega_j}{\delta \omega_c} \right]^2 \right\} \right]^{1/2} \]

\[ = A_{f0} \exp \left\{ -\frac{1}{4} \left[ \frac{\omega - \omega_j}{\delta \omega_c} \right]^2 \right\} \]  (26)
and $A_{fo}$ is defined as the emission fluctuation fraction at $\omega_i$. The second equality above follows by using Eq. (20) as $\langle s^2 \rangle$ and approximating $s^2(t) \simeq \langle s^2 \rangle$.

For optically thick sight-lines and assuming $l_e \geq l_c$ we neglect the other two sources of noise.

### III-G Autocorrelation Measurements of Temperature Fluctuations

Having done the heavy work, it is now easy to show how to use autocorrelation of ECE to measure temperature fluctuations, and calculate the expected resolution of the measurement. For optically thick plasmas, the ECE emission is given by the low frequency limit of the black body radiation law $I \propto \omega^2 T_e$. Thus Eq. (4) becomes

$$R_{ij}(\omega_i, \omega_j, t, \tau) = k^2 \omega_i^2 \omega_j^2 \langle \tilde{T}_e [r(\omega_i)] \tilde{T}_e [r(\omega_j)] \rangle$$

(27)

where $k^2$ is an appropriate constant (easily calculated), and all the other symbols have been previously defined. The reader is reminded that $r(\omega_i)$ is the spatial point of maximum received emission on the sightline of the detected ECE signal at frequency $\omega_i$. Then for $i = j$ this becomes the autocorrelation function

$$R_{ii}(\omega_i, t, \tau) = k^2 \omega_i^4 \langle (\tilde{T}_e [r(\omega_i)])^2 \rangle$$

(28)

and it is immediately obvious that $R_{ii}(\tau)$ is proportional only to the temperature fluctuations, since we choose $\tau$ so that the quantum fluctuations are uncorrelated, and the system is designed so that the correlated system noise is less than the expected turbulent fluctuation amplitude.

The next question of interest is the resolution of the measurement of $\tilde{T}_e$. Assuming that the emission is optically thick (so that the "viewing dump noise") can be ignored, and also assuming that the correlation length is longer than the effective spatial emission linewidth, then Eq. (22) and Eq. (26) can be used in Eq. (21) to give the expression for the uncertainty:


\[ \epsilon = \frac{\sigma_T}{T_e} = \frac{\sigma_{ii}}{2R_{ii}(\tau)} = \frac{1}{2} \left[ 2 + \frac{(\tau_d)^2 A_{f0}^2}{\nu_d T} \right]^{1/2} \]  

(29)

Where \( f = \delta \omega_i/(2\pi) \) is the ECE channel bandwidth (\( \sim 400 \, MHz \)), \( \tau_d \) is still the detector time constant, \( A_{f0} \) is the normalized electron temperature fluctuation amplitude \( \bar{T}_e/T_e \) (assumed \( \geq 2 \times 10^{-3} \)), \( \nu_d \) is the data acquisition rate (\( \sim 1 \, MHz \)), and \( T \) is the integration time. Setting \( \epsilon \leq 1 \) and solving for \( T \) gives \( T \geq 5 \, ms \), where we set \( \nu_d = 2/\tau = 200 \, kHz \) so that we can resolve fluctuations at 100 \( kHz \). Note that the data can be smoothed and bandpass filtered in software, before it is correlated, for frequencies less than the actual physical data acquisition rate (thus justifying using \( \nu_d = 200 \, kHz \)). Also, if the effective data rate is averaged to 200 kHz, then the effective detector time constant is increased by the smoothing, so that \( \tau_d = 10 \, \mu s \). This demonstrates the advertised amplitude resolution of the measurement. The resolution in frequency space is limited only by our ability to smooth and filter the data in software for frequencies up to the Nyquist frequency \( \nu_d/2 \).

The correlation length in the plasma is inferred from the cross-channel correlation function measurements discussed above [Eq. (20)], and the time correlation (frequency width) of the turbulence is estimated by calculating \( R_{ii}(\tau) \) from the data for a range of \( \tau \)'s greater than the quantum autocorrelation time and taking the measured \( \tau_c \) for the turbulence to be the FWHM of \( R_{ii}(\tau) \).

### III-H Effects of Finite Detector Time Constant

In Section III-B above it was mentioned that a finite detector time constant (i.e., the microwave crystal detector time constant for each frequency channel) affects the autocorrelation of signals with self-correlation times shorter than the detector time constant \( \tau_d \) (e.g., thermal correlation). The basic result is that the signals remain correlated at a low level for times considerably longer than would be inferred from the autocorrelation time of the signal before detection. The derivation of the required results is nicely presented by Loudon\(^{22}\), and only the final equations are given here. He shows that the intensity-intensity correlation function
of the fluctuations of an electromagnetic signal due to thermal emission (thermal linewidth with a Gaussian shape) is given by:

\[
\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\mathcal{I}_1 \mathcal{I}_2} = [g_2(\tau) - 1] = \exp(-\delta^2 \tau^2) \quad .
\] (30)

Where the incident beam is split into two channels, one which is instantaneously detected \(i_1\), and the other \(i_2\) which is delayed by increasing the detector distance from the splitter. The time delay is \(\tau\) as before, and \(\delta = \delta \omega_i/(1.18)\), where \(\delta \omega_i\) is the FWHM in frequency space of the channel, also as before. The variables \(\mathcal{I}_1\) and \(\mathcal{I}_2\) are previously defined as the average DC signal for each channel. The function \([g_2(\tau) - 1]\) is Loudon's notation for the correlation function and is provided to assist the interested reader in locating the derivation.

It can be similarly shown that for a Lorentzian line shape with FWHM \(\gamma\) that the intensity-intensity correlation function is given by:

\[
\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\mathcal{I}_1 \mathcal{I}_2} = [g_2(\tau) - 1] = \exp(-2\gamma \tau) \quad .
\] (31)

Where for convenience we assume that \(\tau\) is positive (i.e., \(t_2 \geq t_1\)).

Both of the above equations were derived under the assumption that the detector had infinitely fast time response \((\tau_d = 0)\). For the case of a finite detector integration time \(\tau_d\), the correlation function must be convolved over the detector's memory of past signals with the detector response function for each detector. Loudon derives the finite detector time constant form of the intensity-intensity correlation function for a delay time between the two detectors \(\tau = 0\). Unfortunately our problem is to make the thermal correlations disappear by using a finite delay \(\tau = t_2 - t_1\), where \(t_1\) is the signal correlation time at the first detector, \(t_2\) is the signal correlation time at the second detector, and for convenience we again assume \(t_2 \geq t_1\). Therefore, following Loudon's lead, the intensity-intensity correlation function allowing for a finite detector time constant \(\tau_d\) and for a delay \(\tau\) between the two
signals, is given by:

$$\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\bar{I}_1 \bar{I}_2} = \frac{1}{\tau_d^2} \int_{-\infty}^{t_1} du \left\{ \int_{-\infty}^{t_2} dv \exp\left(-2\gamma |v - u|\right) \right.$$} 

$$\times \exp\left[-\frac{(t_1 - u)}{\tau_d}\right]$$ 

$$\times \exp\left[-\frac{(t_2 - v)}{\tau_d}\right] \right\}. \quad (32)$$

Where we have convolved the exponentially decaying detector time responses with (for convenience) the Lorentzian form of the intensity-intensity correlation function since the final result depends little on the exact line shape. Evaluating this rather horrible (for an experimentalist) integral leads to:

$$\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\bar{I}_1 \bar{I}_2} = \frac{1}{2\gamma \tau_d^2} \left\{ \left[ \frac{\gamma \tau_d^2}{2\gamma \tau_d + 1} \right] \exp\left(-\frac{\tau}{\tau_d}\right) \right.$$} 

$$+ \left[ \frac{\gamma \tau_d^2}{2\gamma \tau_d - 1} \right] \exp\left(-\frac{\tau}{\tau_d}\right)$$ 

$$- \left[ \frac{2\gamma \tau_d^2}{4\gamma^2 \tau_d^2 - 1} \right] \exp(-2\gamma \tau) \right\}. \quad (33)$$

For the case of interest where $\tau \geq \tau_d \gg (1/\gamma)$, this simplifies to:

$$\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\bar{I}_1 \bar{I}_2} = \frac{1}{2\gamma \tau_d} \left\{ \exp\left(-\frac{\tau}{\tau_d}\right) \right\}. \quad (34)$$

The expected correlation due to turbulent temperature fluctuations is given by:

$$\frac{\langle i_1(t) i_2(t + \tau) \rangle}{\bar{I}_1 \bar{I}_2} = A_{f0}^2 \left\{ \exp\left(-\frac{2\tau}{\tau_c}\right) \right\}. \quad (35)$$

Since we expect the temperature fluctuation correlation time to be much longer than the detector time constant, $\tau_c \gg \tau_d$, and where for simplicity in evaluation of the next equation below a Lorentzian line-shape has been assumed rather than the Gaussian line-shape we actually expect. The next conclusion remains the same with either line-shape. Here $A_{f0}$ is the normalized electron temperature fluctuation fraction, and $\tau_c = 1/\delta \omega_c$ is the self-correlation time of the turbulent temperature fluctuations as previously defined.
Requiring that the ratio of the thermal correlation to the turbulent correlation be less than \( r_1 \), where \( r_1 \ll 1 \), leads to the following inequality for the required correlation delay time \( \tau \):

\[
\tau \geq \frac{\tau_d}{1 - (\tau_d/\tau_c)} \left[ -\ln(\gamma) - \ln(2\tau_d) - 2\ln(r_1A_{f0}) \right] \quad (36)
\]

Further requiring in inequality 36 that \((\tau/\tau_c) = r_2\) where \( r_2 \ll 1 \) is also required (so that the delay to decorrelate the thermal fluctuations does not decorrelate the turbulent fluctuations), leads to a constraint on the detector time response:

\[
\tau_d \leq \frac{r_2\tau_c}{[r_2 - \ln(\gamma) - \ln(2\tau_d) - 2\ln(r_1A_{f0})]} \quad (37)
\]

Inserting reasonable values for the parameters (\( r_2 = r_1 = 0.1, A_{f0} = 2 \times 10^{-3}, \gamma = 400 \times 10^6 \text{MHz}, \tau_c = 10^{-5} \text{s} \)), and iteratively solving for \( \tau_d \) quickly leads to the conclusion that the data acquisition rate \( \nu_d \) must be greater than 10 MHz. This follows because our derivation of the signal to noise ratio for the \( T_e \) measurement requires \( \nu_d \approx (1/\tau_d) \). This kind of data acquisition rate is not only expensive (for the ADC’s, fast detectors, and memory) but leads to unwieldy amounts of data when the acquisition time is stretched over tens to hundreds of milliseconds. It follows immediately then that the delay time between channels to decorrelate the thermal emissions should be introduced in the microwave hardware so that Eq. 31 applies, and the data acquisition rate can be in the more reasonable range of \( \nu_d \approx 1 \text{MHz} \).

### III-I Required Hardware Delay Time \( \tau_h \)

Since it is still required that the hardware delay time \( \tau_h \) be long enough that the correlation of thermal fluctuations be much less than the turbulent correlation, the ratio of Eq. 31 and Eq. 35 can again be set to \( r_1, r_1 \ll 1 \), and solved to give a value for the required hardware (waveguide) time delay:

\[
\tau = \frac{-\ln(r_1A_{f0})}{2\Delta f_r} \quad (38)
\]
Where all quantities are previously defined and $\Delta f_r$ is the FWHM of the microwave passband filter just in front of each microwave filter detector, as before. If different filter passbands are used then either different delays must be used or the delay must be calculated using the smallest $\Delta f_r$. Using reasonable values for the parameters ($r_1 = 0.1$, $A_f_0 = 2 \times 10^{-3}$, and $\Delta f_r = 400 \times 10^6$) gives $\tau = 20$ ns. This means a waveguide delay of 20 ns must be introduced after the spatial signal is split in hardware, and before the microwave crystal detectors.

**III-J Allowable Correlated Electronic Noise**

The noise temperature of any correlated electronic noise must be much less than the signal we hope to detect. If the minimum local electron temperature looked at is 100 eV, and $A_f_0 = 2 \times 10^{-3}$ is to be resolved, then the allowable correlated noise should be an order of magnitude smaller, $n = 0.02$ eV. This will define where the incoming signal must be split. One very convenient place to split the signal and delay one side would be after the LO/mixer and IF amps. In this case the total noise temperature of the system up to the splitter must be less than 0.02 eV.

**III-K Effect of Optical Thickness**

For a finite optical depth, we require that the signal from turbulent fluctuations reflected off the vessel wall into our viewing system be small compared to the local fluctuation signal. The net reflected signal into the viewing system is:

$$s = \frac{i}{1 - R_w e^{-\tau} e^{-\tau}}. \quad (39)$$

Where $i$ is the signal emitted along the sightline, as before, $R_w$ is the wall reflectivity, and $\tau$ is now the previously discussed optical depth (rather than the time delay). Since we require $s/i = r_1$ where $r_1 \ll 1$, the equation can be solved for $\tau$:

$$\tau = \ln \left[ \frac{1 + r_1 R_w}{r_1} \right]. \quad (40)$$

For typical parameters ($r_1 = 0.1$, $R_w = 0.90$), we require that the optical depth be $\tau \geq 2.4$. For optical depths less than this there must be a viewing dump on the far wall.

22
III-L Effect of Local Oscillator Frequency Stability

If the local oscillator (LO) used to mix the received ECE radiation down to an intermediate frequency is not absolutely constant in frequency, the frequency width $\delta f$ of the LO will appear to be a temperature fluctuation (this assumes that a single LO is used for the immediate frequency channel and the delayed frequency channel). It can be shown that the apparent temperature fluctuation is given by:

$$\tilde{T}_e = \frac{\delta f T_e}{f \epsilon}$$  \hspace{1cm} (41)

Where $\epsilon$ is the inverse toroidal aspect ratio $r/R$, and all other symbols are previously defined. This sets an easily calculable limit on the required frequency stability of the LO, if a single LO per spatial sightline design is used.

III-M Effect of True Magnetic Field Fluctuations

True ripple of the toroidal field or helical field of a toroidal device always exists at some level, since no power supply is perfect. Fortunately these ripples are at frequencies generally less than $1 \text{ kHz}$, so they are expected to be in the region high pass filtered out by the data acquisition system. Because the ECE emission is proportional to $B^2$, the apparent temperature fluctuation caused by any such ripple will render that portion of the fluctuation spectrum unusable.

IV Conceptual Design

We intend to perform a set of experiments on ATF and TEXT to measure second harmonic, extraordinary mode, optically thick, fluctuating ECE during 2 Tesla operation. Using as small a spot size as possible, we will look for fluctuations from a given spot in the plasma. We will cross-correlate two or more different emission frequencies from the same sightline to look for fluctuations. The use of two or more channels has an additional advantage because one can in principle deduce the radial wave-vector range $\delta k_r$ of the fluctuations by finding
the e-folding distance ($e^{-1}$ distance) of the correlation function by measuring the correlation function versus channel separation. The capability to look for time-correlation information from a single point will be provided by dividing the signal in hardware, time delaying part of it (in hardware) and then autocorrelating the two signals. The signal for a particular ECE frequency is split in hardware rather than software to provide for improved noise rejection (rejection of thermal correlation by hardware delay is discussed in Section III-H above). Random noise added at any hardware stage in the frequency range of interest would be 100% correlated if the signal splitting were done in software. Random noise added by separate well isolated hardware stages is not correlated. Also if the signal is split in hardware, then the system noise level can by checked by firing plasma shots with the receiver horn covered and cross-correlating/autocorrelating as appropriate. The measured correlated signal under these conditions represents the intrinsic noise correlation level of the system, and should not be (as discussed above) the measurement limit of the system. That is we require the correlated signal under these conditions to be $\ll 10^{-3}$ of the correlated signal with the system exposed to the plasma.

IV-A Viewing System

The selection of a horn to couple the free space radiation to the waveguide for transport to the heterodyne receiver requires a design which provides adequate spatial resolution inside the emitting plasma volume. The radiation pattern of a standard horn usually subtends a large solid angle making a typical spot size in the plasma of about 5 cm. Utilizing Gaussian optics\cite{23} a horn-lens system can be used to form a viewing beam with a waist in the plasma, defining the viewing geometry (Fig. 7). Only radiation originating within the beam contours enters the horn. We propose to view horizontally from the outside of the machines. The selection of the beam parameters will be governed by the desire to maintain Gaussian purity, maximize power flow through the vacuum port, and attain minimum spot size in the plasma. A corrugated conical horn and a lens or focussing mirror will be designed to couple efficiently to the viewing beam while providing broadband $80 - 110 \text{ GHz}$ performance by matching the frequency scaling of the horn with that of the beam. The horn/lens will view the plasma through a wedged quartz vacuum window located on a horizontal port. An oppositely-
oriented, wedged Teflon piece will be added in order to compensate for beam deflection due to the quartz vacuum window. Beam dumps will not be required since we will be detecting optically thick emission.

Delivery of the radiation from the machine to the receiver will be accomplished using C-band, over-moded waveguide. Fundamental waveguide sections immediately after the horn provide polarization selection prior to transmission by the oversized section. Then H-plane, quasioptical 90 degree C-band bends give the necessary direction changes with minimal loss. A waveguide high pass filter is provided to eliminate the lower mixer sideband. A schematic of the viewing system is shown in Fig. 5.

IV-B Receiver

The configuration for 2 Tesla operation consists of a mixer with an intermediate frequency (IF) output of 2 – 18 GHz. The local oscillator (LO) frequency will be in the range 80 – 90 GHz and the system will be in WR-10 waveguide. The coaxial IF system consists of two low noise IF amplifiers each with 30 db gain, providing input to an 8 way power division network (7db loss per channel). The filter bank system offers simultaneous information at all frequencies of interest, thus a spectrum is available with each sampling. Using a fixed frequency, phase-locked, Gunn local oscillator makes for a quiet receiver easily calibrated with LN2. Simplicity, reliability, and cost are also attractive features.

Frequency resolution will be provided by a filter bank consisting of 8 bandpass filters for each IF amplifier. We will have an array of fixed filters with different channel center frequencies and bandwidths, each one optimized for a particular plasma radius and range of expected electron temperatures at 2 Tesla. There will also be a set of variable center frequency, YIG-tuned, bandpass filters which will allow us to vary the separation between channels for cross-channel correlation experiments. Video crystal detectors with a sensitivity of 0.5 mV/µW and video amplifiers provide signals to the data acquisition. A standard 12-bit, 1 – 5 MHz, CAMAC based data acquisition system will be used (eg., Lecroy Model 6810 ADC). A schematic of the receiver is shown in Fig. 6.
V Discussion

From the considerations discussed above and our design efforts on this problem to date, we believe it is reasonably likely that turbulent temperature fluctuations $\dot{T_e}/T_e \approx 2 \times 10^{-3}$, with wavelengths greater than $1.5 \text{ cm}$ can be measured on ATF and TEXT for frequencies less than $\approx 100 \text{ kHz}$. On higher field, higher temperature, longer pulse devices it is likely that considerably better resolution can be achieved. We hope to demonstrate the likely resolution on ATF and TEXT in less than two years.

VI Acknowledgments

This work is in support of the United States Department of Energy Office of Fusion Energy Transport Initiative and involves a collaborative effort between Auburn University, the Georgia Institute of Technology, the University of Texas at Austin and Oak Ridge National Laboratory.
References


Figure Captions

Fig. 1. Definition of ECE spot size. The image spot d is defined by a lens having focal length, f, and diameter, D.

Fig. 2. Optically thick emitting layer having a total optical depth much greater than 2. The effective width of the emitting layer (cross-hatched region) is determined by that part of the volume facing the viewing system that has an optical depth \( \approx 2 \).

Fig. 3. Second harmonic ECE spatial linewidth dimension, \( w \), for ATF with \( B = 2 \ T \), \( T_{e0} = 1 \text{ kev} \), and \( n_{e0} = 3 \times 10^{13} \text{ cm}^{-3} \). This particular calculation is for vertical (port location) emission and assumes parabolic density and temperature profiles. The results are similar for horizontal (port location) emission. Note that the optical depth equals 2 at approximately 24 cm.

Fig. 4. Second harmonic ECE spatial linewidth dimension \( w \) for TEXT with \( B = 2 \ T \), \( T_{e0} = 1 \text{ kev} \), and \( n_{e0} = 3 \times 10^{13} \text{ cm}^{-3} \). This calculation is for horizontal emission and assumes parabolic density and temperature profiles. Note that the optical depth equals 2 at approximately \( r = 20 \text{ cm} \).

Fig. 5. Proposed set-up of the ECE viewing system on ATF showing vacuum window, lens, horn and transition to over-moded waveguide.

Fig. 6. Proposed set-up of the ECE receiver system on ATF showing mixer, local oscillator, IF amplifier, multiplexer, power dividers, filter network, and detectors.

Fig. 7. Gaussian beam optics showing horn waist, focusing lens and beam waist.
Fig. 1
Thomas, et. al.

Fig. 2
Thomas, et. al.

Fig. 3
Thomas, et. al.

Fig. 4
Thomas, et. al.

Fig. 5
LOCAL OSCILLATOR FREQUENCY = 85 GHZ

MIXER

IF OUT

RF IN FREQUENCY = 87-103 GHZ

IF AMPLIFIER (2-18 GHZ)

MULTIPLEXER

2 - 6 GHZ

6 - 10 GHZ

10 - 14 GHZ

14 - 18 GHZ

2-WAY POWER DIVIDERS

FILTER NETWORK

DETECTOR NETWORK

TO VIDEO AMPLIFIERS & DATA ACQUISITION SYSTEM

Thomas, et. al.

Fig. 6
DOE F 1332.16 (10-84)
(Formerly RA-427)

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I INTRODUCTION

Because of the difficulty of understanding particle and energy confinement in experimental devices aimed at the goal of producing magnetic fusion energy, new diagnostics to understand these processes are constantly being sought. Of particular interest are diagnostics to measure the local spatial value (as opposed to chord integrated) of fluctuating quantities in present experiments, and diagnostics to measure the local value of any of the important plasma parameters in a true fusion reactor environment (i.e., in a high radiation environment). As discussed below, we (my graduate students and myself) have shown that crossed-sightline ECE (electron-cyclotron-emission) can be used for both local measurement of fluctuating quantities (electron temperature and density), and also for inferring the local value of the absolute magnetic field in present experiments, or in a reactor environment. The absolute magnetic field measurement is a direct measure of the local plasma pressure ($\beta$) in currentless devices, and in combination with temperature and pressure measurements is a measurement of the plasma current density in devices with significant plasma current. The work on fluctuations (electron density and temperature) is being pursued (continued) under
a separate contract in collaboration with Auburn University, Oak Ridge National Laboratory, and the University of Texas. The work on absolute measurement of magnetic fields is unfunded at this writing, but follow-on work will be proposed this fall in collaboration with InterScience, Inc., and/or the C.I.T. project or Alcator-C-Mod projects. Our calculations show that absolute measurements of magnetic field can be made to the order of 0.1%, even in a reactor environment, and that a scanning ECE system could be used to measure the beta profile or current profile in a reactor, as appropriate. The electron temperature fluctuation measurements have a similar resolution, and can also show the phase between density and temperature fluctuations, an extremely important quantity for understanding transport caused by these fluctuations.

II REPORT

The results of our studies on measurement of absolute magnetic field, measurement of beta and/or poloidal magnetic fields, and measurement of parallel magnetic field fluctuations have been published in the Review of Scientific Instruments, volume 61, issue No. 2 (February), of 1990. The article by G.R. Hanson (a Georgia Tech graduate student partially supported under this contract) and C.E. Thomas is attached to this report as enclosure (1). The article discusses our research in considerable detail, but can be summarized rather briefly. It is demonstrated that absolute measurements of magnetic field and of parallel magnetic field fluctuations by using crossed-sightline ECE correlation can be made with an accuracy of 0.1%, with a spatial resolution the order of 1 cm or better and with a time resolution of the order of 100 microseconds on both the TEXT and CIT devices (similar resolution could be expected for the Doublet Big-D device at General Atomics, and for TFTR). It is further demonstrated that beta could be measured on a currentless device with the same accuracy (this follows immediately), and that the current profile could be measured with an accuracy of better than 90% over the outer 80% of the plasma radius (i.e., from r/a=0.2 to r/a=1.0). The diagnostic is predicted to be extremely useful for absolute magnetic field profile measurements in burning devices or reactors, because the only components near the reactor are metallic mirrors and/or waveguides, which are extremely resistant to radiation
damage. The diagnostic also has the advantage of being a passive device, no beams, lasers or physical penetration of the plasma are required.

The use of the ECE diagnostic to measure electron temperature and density fluctuations has been presented as a paper at the Conference on High Temperature Plasma Diagnostics at Cape Cod in May, 1990. The paper prepared for this meeting presents the research to date on this subject in great detail, and is attached as enclosure (2). The basic results of this study are that electron temperature fluctuations of 0.1% can be resolved with a spatial resolution of the order of 1 cm. The frequency resolution of the diagnostic is of the order of 1 kHz or better from 10 kHz to 500 kHz, but the plasma must be steady-state (the fluctuation amplitude must be approximately constant) for 10 milliseconds in order to achieve this resolution. Diagnostics to implement this concept are presently being built for both ATF and TEXT (under a separate contract in collaboration with Auburn, ORNL, and Texas). The resolution discussed above is more than an order of magnitude better than any previous measurement of temperature fluctuations in a high temperature plasma. Simultaneous measurement of the relative phase and amplitude of density fluctuation will be attempted in a later phase after the temperature fluctuation measurements are successful.

III SUMMARY

In summary, we believe that DOE has gotten excellent value for its investment under this contract, with the promise of outstanding new diagnostics for both present day fusion experiments, and for fusion reactors.