Controlled Fluid Recirculation for Efficient Hydraulic Actuation of Conservative Loads

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List of Symbols

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<th>Symbol</th>
<th>Description</th>
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<td>A</td>
<td>Piston area</td>
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<tr>
<td>B</td>
<td>Damping effect</td>
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<tr>
<td>B_e</td>
<td>Bulk modulus</td>
</tr>
<tr>
<td>C</td>
<td>Valve Coefficient</td>
</tr>
<tr>
<td>C_{Ci}(U)</td>
<td>Control constraint</td>
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<tr>
<td>g(x,u)</td>
<td>Function to be minimized</td>
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<td>H(x,p,u)</td>
<td>Hamiltonian</td>
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<tr>
<td>h(x(t_f))</td>
<td>Final condition</td>
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<tr>
<td>h(SC_4(X))</td>
<td>Sign Function</td>
</tr>
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<td>i</td>
<td>Current input to electrohydraulic valve</td>
</tr>
<tr>
<td>J(x,u)</td>
<td>Performance measure</td>
</tr>
<tr>
<td>K</td>
<td>Valve sizing constant</td>
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<tr>
<td>K_1</td>
<td>Gain constant</td>
</tr>
<tr>
<td>k_l</td>
<td>Leakage coefficient</td>
</tr>
<tr>
<td>L</td>
<td>Piston stroke</td>
</tr>
<tr>
<td>M</td>
<td>Mass of load</td>
</tr>
<tr>
<td>P</td>
<td>Lagrange multiplier or costate</td>
</tr>
<tr>
<td>P_i</td>
<td>Pressure at i side</td>
</tr>
<tr>
<td>P_s</td>
<td>Supply pressure</td>
</tr>
<tr>
<td>P_r</td>
<td>Return pressure</td>
</tr>
<tr>
<td>P_v</td>
<td>Valve pressure drop</td>
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<tr>
<td>Q_o</td>
<td>Flow caused by actuator movement</td>
</tr>
<tr>
<td>Q_i</td>
<td>Control flow</td>
</tr>
<tr>
<td>Q_k</td>
<td>Leakage flow</td>
</tr>
<tr>
<td>Q_c</td>
<td>Compressible flow</td>
</tr>
<tr>
<td>R</td>
<td>Weighting factor</td>
</tr>
<tr>
<td>SC_4(X)</td>
<td>State constraint</td>
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<tr>
<td>SGN</td>
<td>Sign function</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>t_f</td>
<td>Final Time</td>
</tr>
<tr>
<td>U</td>
<td>Control vector</td>
</tr>
<tr>
<td>U*</td>
<td>Optimal control vector</td>
</tr>
<tr>
<td>U_m</td>
<td>Upper bound for control vector</td>
</tr>
<tr>
<td>y</td>
<td>Displacement</td>
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<tr>
<td>V_0</td>
<td>Half of the total volume of actuator</td>
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</table>

Introduction

Hydraulic actuators have traditionally found application in servo systems requiring high bandwidth, high power per unit mass, and permitting low overall energy efficiency, the presence of hydraulic oil, and a rather bulky power supply. This paper presents a theoretical, analytical and experimental study of controlled fluid recirculation around an actuator to provide a more energy efficient hydraulic servo system for motion control while retaining the traditional advantages.

The four way servovalve conventionally used in high performance hydraulic applications is a throttling device composed of four variable orifices arranged to form a Wheatstone bridge. (See Figure 1.) Spool valves and jet pipe valves are its common implementations. The orifices are mechanically connected as shown by the dashed lines in Figure 1. Two of the orifices are open at any time, depending on the direction of fluid flow. The fluid supply is usually a pump, an accumulator, and a means of regulation to maintain the supply fluid at almost constant pressure. All flow to the actuator comes from the fluid supply through a valve orifice. All flow out of the actuator goes through another valve orifice and returns to a sump at much lower constant pressure, often atmospheric. The power required is dependent only on the pressures and the volumetric flow rate of fluid from the supply. Integrating this power over time gives the energy consumed which is equal to the supply pressure and fluid volume taken from the supply (proportional to the travel of the actuator, assuming negligible leakage). Notice that the power consumed has no direct relationship to the power required by the load. Much of the power is dissipated to heat across the valve orifices. Even when power is being taken out of the load, power must still be supplied from the pump and accumulator.

Power consumption may be reduced by keeping the supply pressure as low as possible. The minimum supply pressure permitted is determined by the maximum pressure required and perhaps by flow demands to be supplied from the accumulator. Further reduction of
power consumption requires modification of the hydraulic circuit. A variable supply pressure would reduce power consumption during all phases of operation. Regenerative braking of the load would reduce the power required even further by returning energy taken from the load to the supply. Practical means for implementing these strategies that retain the high performance of the hydraulic servo do not exist. The variable jet pump valve [1] is a variable fluid transformer which potentially matches the supply and load impedances for better overall efficiency. The method explored here uses controlled fluid recirculation from the actuator, through a two way valve, and back to the actuator to reduce energy consumption.

Controlled fluid recirculation is appropriate for loads which conserve much of the energy provided to them. Examples are inertial loads, gravity loads, and spring loads. Inertial loads are considered exclusively throughout the remainder of this paper, although they represent a more demanding application of the strategy. These types of loads occur in robot arms, antenna, hoists, and machine tools. If the movement end point is known, the motion and the control of fluid recirculation can be optimized. This is the problem considered in detail below. If the motion end point is not known in detail or if the motion is completely specified at each point in time, controlled fluid recirculation is still potentially valuable, but this case is not considered below.

This paper describes the hydraulic circuit for fluid recirculation and the mathematical model of it used for analysis and controller design. The optimal control analysis of the circuit for moving an inertial load from one position to another in specified time with inequality constraints on states and control is then described. Implementation of a modification of the optimal control on a microcomputer and breadboard hydraulic system is presented with experimental results. All phases of the paper are oriented to producing an advanced, operating, motion control system with realistic component dynamics. Additional details are found in Punyapras [2].

A Circuit for Controlled Fluid Recirculation

The objective of energy saving by recirculation of fluid around the actuator can be achieved by several circuits. One alternative considered was four independently controlled orifices. This and other alternatives were rejected due to problems with cavitation or due to the expense of constructing the circuit. The circuit used consists of a 4-way valve and a two way valve as shown in Figure 2. It provides the following advantages when compared to the conventional four way valve circuit or alternatives.

1. Reduced energy consumption
2. Avoid pressure surge and cavitation
3. Stability enhancement through controlled leakage

The breadboard circuit was constructed from off-the-shelf components. The two way valve was constructed from a four way valve with two ports blocked.

Mathematical Model of the Circuit

To model the circuit, tests were performed to justify the assumptions regarding valve dynamics. With these assumptions and the fundamentals of conservation of mass and momentum, equations of motion were written. Simulation using these equations were compared to experiment. The assumptions used were:

1. The orifice edges are assumed to be perfectly sharp. This is the characteristic of servovalve (MOOG 73-102) used.
2. The flow through each orifice is assumed to be simple orifice-type flow with negligible viscous effects, and the flow rate is assumed to change instantaneously with both pressure drop across orifice and orifice area. Time delay has been neglected so that a simple mathematical model will be obtained. The effects of neglecting time delay are expected when analytical and experimental studies are compared.
3. All of the connecting passes are short enough and wide enough to eliminate the effects of fluid mass on the flows through them.
4. Friction losses in the line and passes are included in the orifice losses, and leakage flow across the ram is assumed to be laminar.
5. Supply pressure is constant, and the bulk modulus of the fluid is taken to be constant within the range of pressure employed.
6. Exhaust pressure is assumed to be zero since it is very small compared to the operating pressure.

With these assumptions the conventional models for orifice flow, leakage flow, fluid compressibility, and conservation of mass and momentum can be used to obtain the equations summarized below. The schematic in Figure 3 and the list of symbols shows the nomenclature for the equations.

Equations for Orifice Flow

\[ Q_1 = C_{L1} \sqrt{P_1 - P_1} \]

\[ Q_2 = C_{L2} \sqrt{P_1 - P_2} , \quad P_1 \geq P_2 \]

\[ = C_{L2} \sqrt{P_1 - P_2} , \quad P_2 \geq P_1 \]

\[ Q_3 = C_{L1} \sqrt{P_2} \]

where \( SGN = 1.0, \quad P_1 > P_2 \)

\( SGN = -1.0, \quad P_2 > P_1 \)

Leakage Flow

\[ Q_4 = k_1 (P_1 - P_2) \]
Compressible Flow

\[ Q_{c1} = \frac{\partial p_1}{\partial t} \]
\[ Q_{c2} = \frac{\partial p_2}{\partial t} \]

Conservation of Mass

For \( P_1 > P_2 \)

\[ CV_1 : Q_1 - Q_4 - Q_2 = Q_{c1} + Q_a \]
\[ CV_2 : Q_2 + Q_4 - Q_3 = Q_{c2} - Q_a \]

For \( P_2 > P_1 \)

\[ CV_1 : Q_1 + Q_4 + Q_2 = Q_{c1} + Q_a \]
\[ CV_2 : -Q_2 - Q_4 - Q_3 = Q_{c2} - Q_a \]

Conservation of Momentum

\[ \Delta (P_1 - P_2) = \frac{\partial^2 y}{\partial t^2} M \]

The equations are then combined into four state equations in four state variables

Let \( x_1 = y \). Then

\[ x_2 = \frac{dy}{dt} \]
\[ x_3 = P_1 \]
\[ x_4 = P_2 \]
\[ U_1 = \dot{x}_1 \]
\[ U_2 = \dot{x}_2 \]
\[ CV_1 = V_0 + AX_1 \]
\[ CV_2 = V_0 - AX_1 \]

Then state equations take the form:

\[ \frac{dx_1}{dt} = x_2 \]  
(1)
\[ \frac{dx_2}{dt} = \frac{A}{M} (x_3 - x_4) \]  
(2)
\[ \frac{dx_3}{dt} = \frac{Be}{V_0 + AX_1} [CU_1 \sqrt{P_0 - x_3} - k_1 (x_3 - x_4) - SGN \cdot CU_1 \sqrt{\mid x_3 - x_4 \mid - AX_2}] \]  
(3)
\[ \frac{dx_4}{dt} = \frac{Be}{V_0 - AX_1} [k_1 (x_3 - x_4) + SGN \cdot CU_2 \sqrt{\mid x_3 - x_4 \mid}] \]

Optimal Control Analysis

Nonlinear time invariant system state equations with nonquadratic performance measures will lead to problems for which many basic approaches cannot be used. Linearization of the system state equations is prohibited by large variations of system states. The performance measure used combines the final state errors and the energy consumed. The weighting functions are applied to every term in the performance measure. The energy consumed is proportional to the flow in from the fluid supply. The flow in is linearly dependent on the control input \( U_1 \). This performance measure leads to a singular optimal control problem. For the problem to be well posed, constraints must be included in the problem. The inclusion of inequality constraints to a singular problem of a multivariable system will make the problem difficult if not impossible to solve analytically. A large computer with high speed must be used to solve the problem numerically off line. One of the indirect approaches generally used is the gradient method. This method applies the concepts of the calculus of variations to solve for the optimal path. To cope with the inequality and/or equality constraints, the method has to be modified. Then penalty function approach will be used to take care of the constraints in this paper.

In this particular application, the performance measure used is linear in control inputs. By using the variational approach, the local minimum is not guaranteed because the second derivative of the Hamiltonian is zero (instead of positive definite). Several trials for the initial guess of control inputs must be used. A global minimum is assumed if every trial converges to the same result.

The linear performance measure also leads to a singular problem when the first derivative of Hamiltonian is zero. Several references demonstrate the method of solving this kind of problem analytically. A singular problem usually leads to a bang-bang or bang-singular-bang control when the controls used are restricted by constraints. Simple problems of low order (< 3) linear systems have been solved successfully by many authors using Pontryagin's
Maximum Principle. For problems of higher order with nonlinear state equations, with constraints on controls and states, and with non-quadratic performance measures, analytical solutions are very difficult.

### Table 1

<table>
<thead>
<tr>
<th>Parameter and Units</th>
<th>Value</th>
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<tbody>
<tr>
<td>$P_s$ supply pressure</td>
<td>1200. psi</td>
</tr>
<tr>
<td>$B_e$ bulk modulus</td>
<td>50,000. psi</td>
</tr>
<tr>
<td>$V_0$ half cylinder volume</td>
<td>1.434 in$^3$</td>
</tr>
<tr>
<td>$A$ cylinder net area</td>
<td>0.478 in$^2$</td>
</tr>
<tr>
<td>$M$ load mass</td>
<td>0.259 lb·sec$^{-2}$/in</td>
</tr>
<tr>
<td>$k_1$ leakage coefficient</td>
<td>0.003 in$^3$/sec·psi</td>
</tr>
<tr>
<td>$C$ valve sizing constant</td>
<td>35.12 in$^4$/amp·sec·lb</td>
</tr>
<tr>
<td>$U_m$ maximum input</td>
<td>20. milliamp</td>
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</tbody>
</table>

The gradient method or method of steepest descent consists of searching numerically for an optimum by making a new iteration move in the direction of the gradient, the direction of the maximum decrease (or increase) of the cost function. The method was developed and applied to the optimal control problem by Kelly [3] and Bryson and Denham [4]. Various modifications to include penalty functions, and other methods of treating equality and inequality constraints have been presented [5,6,7]. The gradient method has the characteristic that it converges very fast for a few iterations and tends to converge very slowly as the final value is approached. To improve this aspect, modifications of the gradient method have been made to include the second-order terms. These methods are known as methods based on second variations. Kelly [9], Breakwell, Speyer, and Bryson [9], Kelly, Kopp, and Moyer [10], and Merriam [11] have contributed to the development of this second-order method. Although the second-order method does accelerate convergence of the first-order method, it requires good initial estimates of the initial control and trajectory. Methods that have characteristics of both the first-order and second-order methods have been developed such as the conjugate gradient method by Fletcher-Reeves [12], and Powell [13], and the variable metric method by Davidon-Fletcher-Powell [14]. Conjugate gradient methods are much better than first-order methods and approximate the performance of the second-order methods. A final alternative indirect method is the variational metric method which is usually good but is not always practical to implement.

### Formulation of Optimization Problem

The system state equations 1, 2, 3 and 4 are a set of four first-order, non-linear equations. The control inputs to the system are the current or voltage applied to the electrohydraulic valve.
Inequality Constraints

The penalty function approach will be used to incorporate inequality constraints on states and controls into the previously defined optimization problem.

Let $SC_i(X)$ and $CC_i(U)$ be constraint functions required to satisfy

- $SC_i(X) < 0$, $i = 1, 2$
- $CC_i(U) < 0$, $i = 1, \ldots, 4$

The state constraints applied here are to prevent cavitation and excessive pressure:

- $SC_1(X) = -X_3(t)$
- $SC_2(X) = X_4(t) - X_3(t)$

The constraints on control limit the range of control values:

- $CC_1(u) = -U_1(t)$
- $CC_2(u) = U_1 - U_2$
- $CC_3(u) = -U_2(t)$
- $CC_4(u) = U_2 - U_1$

The approximate inequality constraints on state and controls are incorporated into a new Hamiltonian as

$$H'(X, p, U) = H(X, p, U) - \sum_{i=1}^{4} K_i h(\text{SC}_i)[SC_i(X)]^2 + \sum_{i=1}^{4} \mu_i CC_i(U)$$

where $K_i$ = positive penalty weight number ..., $i = 1, 2, \ldots, 4$
The Experimental Results

The modified control was implemented on a TI 990/4 microcomputer to control the breadboard system. In addition to the optimal control law determined by a switching point based on position, a proportional feedback control law was used after the final position was reached. This prevented drift away from the position due to the inexact null position of the valve. Only the four-way valve was used for the proportional control.

The computer output went to a 12-bit digital to analog converter, then to amplifiers, then to the two valves. Position was measured by a linear variable differential transformer (LVDT), converted to a 12 bit digital signal and used to determine the switching point. The measurements were also recorded and plotted.

The modified control history and the mass position are shown in Figure 7. The successful completion of the desired move was observed in all cases. The analysis did not incorporate the 10 ms delay known from previous experiments to exist. Accounting for this delay the agreement between simulation and experiment were quite good.

Summary and Comments

A hydraulic circuit and optimal control law for controlled fluid recirculation have been devised to reduce energy consumption from that of a conventional four-way servo valve. The simulations for moving a 100 lb. mass horizontally 5 inches in 0.210 sec. show an energy savings of 28%. The control history was simplified and implemented on a breadboard system and the successful operation verified experimentally.

The optimization penalized energy consumed and the final position and velocity errors. The controlled fluid recirculation concept could also be used to follow a specified trajectory. This problem could be simpler to implement than the one considered. Other types of loads which store energy in potential and/or kinetic form could also benefit from this technique.

Further study should consider the sizing of the two valves and the use of a less expensive on-off two way valve. Further attention should also be given to the possibility that the global optimum has not been found. Multiple switching points may exist for move-ments greater than a certain distance. The results would be a pulsed operation alternating between accelerating and coasting.

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References


Fig. 1. The Four-way valve as a Wheatstone bridge.

Fig. 2. The Experimental Circuit as Constructed.

Fig. 3. Schematic with Notations for Motion in One Direction.

Fig. 4. Optimal Control Result for R = 100, \( R_1 = 20,000, R_2 = 20 \). (Conjugate Gradient Method).

Fig. 5. Simulation of Controls in Figure 4.

Fig. 6. Approximation of Optimal Control for R = 100, \( R_1 = 20,000, R_2 = 20 \).

Fig. 7. Plots of Displacement and Control History from Experiment.