Vibration Cancellation in Complex Support Structures for Precision Robots

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Abstract
This paper examined the effectiveness of the Optimal Arbitrary Time-delay filter applied to cancel the undesired elastic modes of vibration in complex supporting structures for precision robot which is under development. To achieve the desired positioning precision, suppressing the structure vibration is essential. For this to be done OAT filter method was chosen. Modeling of the system has been performed and using this model, simulations were done without and with OAT filter. The ability of OAT filter in canceling the vibration of the structure was demonstrated in the simulation results. And to verify this results, experiment involving OAT filter within the robot control system has been done on the current version of the precision robot. Even though the dynamics response of supporting structure in the precision robot system has an inherent characteristics of small magnitude and high value of natural frequencies, response due to the filtered command showed that the filter successfully reduced vibration in magnitude and got rid of the residual vibrations faster than the response with unfiltered command.

Introduction
Reduction of cost capital equipment cost is deemed critical to the widespread application of robotic technologies in assembly. A project underway at Georgia Institute of Technology seeks to capitalize on the improving price/performance of semiconductor technology to overcome the cost of precision construction of a massive structure and drive system. The project seeks a symbiosis of three technologies: active vibration control, learning control, and machine vision to achieve these ends. When combined with, a consistent mechanical design philosophy, state of the art drive motors, processor and sensor technology, a significant reduction of cost becomes possible. This paper concentrates on one aspect of this project, the use of command filtering to reduce the vibration settling time of the end effector. Singer and Seering[1] showed that time-delaying filtering of the command inputs can reduce or eliminate the residual vibration. This input shaping filter is restricted to have time-delay that is a function of the system parameters. Magee[3] developed the general filter resulting from optimization of the cost function that involves not only error signal but also time rate change in error signal too. The solution of this general filter approach called Optimal Arbitrary Time-delay Filter
has an arbitrary time-delay value which allows users to choose time-delay as an integer multiple of the sampling period and therefore can be represented perfectly in discrete-time control system. Magee and Book[2] applied this filtering method for positioning an micro manipulator that is attached to the end of long reach flexible manipulator where the base manipulator has a fairly big flexibility at low frequency around 5 Hz. And experiment result showed that the OAT filter successfully prevented the vibration.

In this paper the authors present results on the modeling and implementation of the OAT filter on a prismatic joint exciting the support structure of a high-speed, light-weight assembly arm. This joint is the first joint of a multi-joint arm under construction. The support structure is initially modeled as a simple spring and mass system. Even though the structure is designed to be fairly rigid, a number of resonances occur between the frequencies of 15 Hz and 90 Hz. The low amplitudes are not detected on the joint encoder but are obvious from accelerometer measurements. Even though the amplitudes at this stage of design are low, the addition of the remaining degrees of freedom threaten the demanding requirements set for settling time.

The paper proceeds by describing the overall concept for the arm, and producing a dynamic model for that concept including the flexibility of the supporting structure, the moving structure and the bearings. The order of the model is reduced. An OAT filter is designed and applied to the model of the concept. The first joint of the fabricated arm was then tested, and the realities of the more complex structure are illustrated with experimental results. The use of the OAT filter is illustrated in the time and frequency domain and shown to be effective on the lowest modes. The continuing work on the arm and the use of other advanced techniques are described.

A Light-Weight Arm for Assembly

The performance target of this light weight assembly robot under development is as followings. The precision robot handles a 1kg assembly part within the range of 0.5m x 0.2m x 0.2m (x-y-z) and it has placement within ±20 μm positional repeatability. One part placement takes one second and this performance is based on 0.5m motion of the main head to part feeder in the x-direction, 0.2 m vertical motion of probe to part feeder in z-direction and 0.2 m horizontal motion of tip between parts in y-direction. Two guides are constructed. One is the x-guide on which the main head rides. The other guide on which a vertical probe rides is the z-guide. The horizontal x-axis track consists of two anti-friction tracks and main head is constrained on this track by 2 lower anti-friction bearings and one recirculating bearing on the top. The vertical probe is restricted to slide relative to the main head in z-direction and this constraint is achieved by the 2 anti-friction bearings on the right side of the probe and on the other side it rides on a pair of recirculating ball bearings. The supporting structure is bolted down on the floor. As actuators, a linear motor and a screw motor are used for main head and the probe each. The magnetic rack for linear motor is implemented on the bottom of the x-track and the moving coil amateur is fixed on main head. The screw motor is placed on the main head. Encoders with 1 μm feedback resolution are used for both main head motion and probe motion. The structure supporting the first joint of the arm is designed without concern for moving its mass during operation. Precision is achieved by advanced control
concepts and innovative sensory system, not by dependence on open loop accuracy of the mechanical system.

Figure 1. Drawings of Manufacturing Assembly Precision Robot
Model of X and Z Axes

The dynamic behavior of the manufacturing precision robot in x-z plane is modeled as follows. The supporting structure motion is modeled as a rigid mass vibration ($x_b$) relative to the ground in a horizontal direction. The x track is assumed to have 2 degrees of freedom, which are vertical deflection ($z_b$) and the rotation ($\phi$) relative to the supporting structure. Those two degrees of freedom are allowed by compliance of the x-track which is assumed to be a beam. The main head sliding on the x-track is modeled as a rigid mass with a translating center of gravity at $L_1$ along the x-track. It is coupled to the x-track through a torsional stiffness resulting from the bearing compliance. That allows the main head to rotate in the x-z plane with pitch $\theta$. The torsional rotation center is assumed to be at the midpoint of two lower bearings. The mass center of z-probe slides up and down ($L_2$) relative to main head by a distance $z$ and also rotates ($\psi$) about the midpoint of two right side bearings with a coupling torsional stiffness which is coming from the bearing compliance to the main head. The linear motor driving main head is presented as force $f_1$ and the screw motor driving vertical probe is modeled as force $f_2$. The complete x-z plane model of the system has 7 generalized coordinates, 2 rigid and 5 elastic, and two control forces.

![Figure 2. Modeling of Manufacturing Assembly Precision Robot](image)
With the established model, non-linear equations of motion are derived using the Lagrange’s equation.

\[
 [M(q)]\ddot{q} + [C(q, \dot{q})]\dot{q} + [K(q)]q = \{F\}
\]

The damping matrix above contains the centrifugal effect caused by the simultaneous rotational motions and translational motions. Non-linear equations are linearized around the operating point. At this stage, not only the small displacement assumption but also the small velocity and acceleration assumptions are made.

\[
 [M(q)] \approx [M(q_0)] + \left[ \frac{\partial M(q)}{\partial q} \right]_{q = q_0} \{\Delta q\}
\]

\[
 [K(q)] \approx [K(q_0)] + \left[ \frac{\partial K(q)}{\partial q} \right]_{q = q_0} \{\Delta q\}
\]

\[
 [C(q, \dot{q})] \approx [C(q_0, \dot{q}_0)] + \left[ \frac{\partial C(q, \dot{q})}{\partial q} \right]_{q = q_0, \dot{q} = \dot{q}_0} \{\Delta q\} + \left[ \frac{\partial C(q, \dot{q})}{\partial \dot{q}} \right]_{q = q_0, \dot{q} = \dot{q}_0} \{\Delta \dot{q}\}
\]

Finally linearized equations of motion are obtained as following

\[
 [M(q_0)]\ddot{q} + [C(q_0, \dot{q}_0)]\dot{q} + [K(q_0)]q = \{F\}
\]

**Reduction of Degrees of Freedom**

For the linearized system, mode vectors and corresponding eigenvalues are found to analyze the effect of each modes to the overall motion of the system. In each plot of following Figure 3, mode vector and corresponding eigenvalue are shown with the horizontal axis indicating the index of each generalized coordinates and vertical values representing magnitude of the coordinate. The first and second mode vectors describe that two rigid motions related to \(L_1\) and \(L_2\). The third graph shows the lowest frequency elastic mode of the system. The natural frequency of this mode is 73.33Hz. This out of phase mode caused by the applied control force \(f_1\) consists of only the horizontal motion of \(x_b\) and \(L_1\). Examining the mode vectors, we find out that non-zero values of vertical motion of x-track, \(z_b\) and rotational motion of x-track relative to the structure, \(\phi\), appear only in those two highest modes. So, if we are going to focus our concerns onto the first 5 lowest modes including two rigid modes, two generalized coordinates \(z_b\) and \(\phi\) can be eliminated.
Design and Application of OAT Filter to Simulation

Optimal Arbitrary Time-delay Filter which has been developed by Magee and Book can be applied to the flexible manipulator system to reduce the undesirable single or multi modes of elastic vibration. The following equation shows the OAT filter coefficient for the single elastic vibration mode.

\[ f_{\text{1opt}}(t) = \delta(t) - 2 \cos(\omega_d T_1) e^{-\zeta \omega_n T_1 \delta(t - T_1)} + e^{-2\zeta \omega_n T_1 \delta(t - 2T_1)} \]

\[ \omega_n = \text{natural frequency} \]
\[ \zeta = \text{damping ratio} \]
\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

\( T_1 \) is the delay time which can be arbitrary. And \( \omega_n, \omega_d \) and \( \zeta \) are the physical properties of the elastic vibration mode whose amplitude of vibration is desired to be zero. The OAT filter has an infinite number of zeros in s-plane at

\[ s = -\zeta \omega_n \pm \frac{2n\pi}{T_1} \]

for \( n = 0, \pm 1, \pm 2, \ldots \)

This ensures that any time-delay cancels the oscillatory poles of the system and allows arbitrariness in choosing delay time. And also, as a consequence of the infinite number of zeros in filter, frequency response of the filter gives infinite number of local minima in magnitude response. Compared to the previous input shaping methods, the most practical
advantage of this OAT filter method is the arbitrary time-delay value which makes adapting the filtering algorithm to changes in the system parameters almost trivial. Using the linearized 5 D.O.F. modeling, first, a pseudo-step simulation was performed without filter where the pseudo-step force input \( f_t \) is applied to the main head. The response of \( X_b \) shows the vibration having an amplitude of 6 \( \mu m \) about 10 \( \mu m \) displacement position. \( L_1 \) shows a parabolic response which is rigid motion of main head caused by step force input.

![Figure 4. Pseudo-step response simulation with non-filter input](image)

Then, in order to reduce the elastic vibration mode of the lowest frequency at 73.33 Hz which is exclusively related to the \( x_b \) and \( L_1 \), an OAT filter is applied to the pseudo-step force command. The time-delay is set to be one half of the vibration period \( T_n \). In the first plot of following Figure 5, the filtered command is delayed by the amount of \( (1/2)T_n \) from the original input command. The simulation results indicate the possibility that, in control of precision robot with complex supporting structure, applying the OAT filter can suppress the vibration of \( x_b \) successfully.
Experimental Identification of Flexible Dynamics

Since the vibration of the supporting structure which deteriorates the accuracy of robot performance cannot be detected by the joint encoder, it is desired for the supporting structure vibration to be suppressed as small as possible. To achieve this goal in experiment by applying an OAT filter, the physical characteristics of the vibratory mode of structure such as $\omega_n$, $\omega_d$, $\zeta$ need to be identified. To find out those parameters, an identification experiment was done on the initial version of robot system which consists of the supporting structure, x-track, main head, linear motor and amplifier driving main head and encoder to feedback the main head position. The frequency response function of the supporting structure was obtained using the acceleration data from two accelerometers attached on the main head and C-channel of the x-track. As an excitation band-limited white noise was used. Initially, the structure showed a lot of resonant peaks between 15 Hz and 90 Hz. In trying to remove those complex peaks, structure has undergone stiffening process of bolting up neighboring two beams in x-z plane together with wood panel. And high peak at 15 Hz was reduced in magnitude after the right side connection between x-track and supporting structure which had been initially connected tightly by bolts and rubber damper was loosen. Figure 6 shows the frequency response of the current system where the input is main head acceleration and output is supporting structure acceleration. From curve fitting, the poles of the first two dominant elastic modes were found at $-0.4073 \pm j38.29$(Hz) and $-0.8634 \pm j55.26$ (Hz) in s-plane. These values represent lower natural frequencies than the analysis showed. The discrepancies are caused by the difference between the initial system design properties which analysis is based on and the actual system built up.
Application of OAT Filter to First Joint of Prototype

Experiment were performed on the current version of the system to verify the effectiveness of the OAT filter in improving the response of the structure. Filtered and non filtered command responses are compared in time and frequency-domains. The following figure shows the block diagram used in experiment. The OAT filter is placed outside the closed PID feedback loop to avoid deteriorating the system stability.

Figure 7. Control system block diagram
With sampling rate of 1 kHz, natural frequencies and damping ratios of the modes ($\omega_n=240.6\text{rad/sec}$, $\zeta_1=1.693\times10^{-3}$ for 1st mode and $\omega_n=347.2\text{rad/sec}$, $\zeta_2=2.486\times10^{-3}$ for 2nd mode) estimated in the preceding identification experiment were used as filter parameters to cancel those first two dominant modes. Delay times were set to be an integer multiple of sampling time which are the closest possible value to the actual half the period of each mode. This amount of time delay gives the most robustness to the control system and a global minimum magnitude in frequency response at the target frequency that is the resonant peak of undesired mode. An actuating force was applied on the main head by a linear motor on the x-track. First, results are compared in the time domain. Plot (a) of Figure 8. is comparing the filtered and non-filtered reference input signal, both has final step magnitude of 6,000 encoder counts which is corresponding to 6 cm translational motion of main head relative to the x-track. Plot (b) is the response of structure without filter. Plot (c) is representing the response with OAT filter canceling the 1st mode and showing that initial peak amplitude of the filtered is reduced by 40% and residual vibration vanishes faster than the non-filtered response. The response to the command where the first and second modes are filtered out by OAT filter is on plot (d). The two filters are connected in series. In this plot, even smaller initial vibration amplitude and a faster disappearance of the residual vibration than the response with one filter is observed.
(b) Non-filtered command time response

(c) Filtered command time response (with one OAT filter for 1st peak)

Figure 8. Time domain responses of supporting structure acceleration due to step input (continued)
Now, frequency domain responses are compared. The power spectrum of the acceleration of the supporting structure and main head is presented in Figure 9. In each plot of Figure 9, top one corresponds to the main head response and bottom one is the structure response. The response to a non-filtered pseudo-step command is presented in plot (a). This shows the strong effect of the first two dominant modes. According to the plot (b) that corresponds to the response with the command where the first mode is filtered out, both peaks get narrower and reduced in magnitude by 2.5 dB for the first and 13.3 dB for the second. Reduced magnitude at the second peak with OAT filter aiming the first peak only is contributed to the infinite number of minimal magnitude response of the OAT filter frequency response characteristics. The other narrow peak at the filtering target frequency can be explained by seeing the free run response of the structure on plot (c), where there is no actuating input. It is found that the environment of system is exciting the structure constantly and the filtered input frequency response is superimposed on this response caused by those unfiltered ambient exciting sources. Considering this fact, it can be said that the OAT filter achieves the goal of canceling the vibration around undesired frequency.

(d) Filtered command time response (with two OAT filters for 1st and 2nd peaks)

Figure 8. Time domain responses of supporting structure acceleration due to step input (continued)
Figure 9. Power spectrum responses of supporting structure acceleration due to step input (plot (a) and (b)) and for free-run experiment (plot (c)).
Conclusion and Future Work

Optimal Arbitrary Time delay filtering method has been applied to reduce the complex supporting structure vibration in precision manufacturing robot. The x-z plane modeling of the precision robot system was performed and simulation results using this model showed that undesired elastic mode of the robot system can be reduced by applying OAT filter. Arbitrariness of the time delay value of OAT filter was advantageous in actual implementation of filter in discrete-time control system. Even though the complex supporting structure dynamics has the inherent response characteristics of small magnitude and high natural frequencies, experiment result shows that the OAT filter canceled the undesired vibration effectively. The effectiveness of OAT filter was demonstrated by comparing the non-filtered and filtered command responses in both time and frequency domains. In the future, 2 more joints including vertically sliding probe will be added to this precision robot system. With these additional joints, dynamics of the system will get more complex and the total structure is expected to be excited more. Then, OAT will be applied to the control system again with the verified effectiveness on current version of configuration. As an overall control strategy of precision robot positioning, integration with learning control and vision sensing is planned as a future work.
References


