A Robust Scheme for Direct Adaptive Control of Flexible Arms

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Abstract

In exchange for light weight in the design of arms, one must accept an increase in system flexibility and the associated difficulty in accurately controlling a flexible structure. Both rigid body motions and flexural vibrations are required to model the dynamic system. A robust feedback control which is constructed is based on the Lyapounv function for the analysis of uniform boundedness. The control signal is synthesized from estimated states. This is accomplished by separating the system into two parts: linear and nonlinear (uncertainty in the linear model). Performance of a one-link flexible arm resulting from this control algorithm is compared with that of purely linear feedback control.

I. Introduction

In recent years, programmable multi-functional manipulators, or industrial robots, have become increasingly important in automation. Robot control is one of the important ways to improve robot performance, especially robot motion. In the meantime, with the rapid development of computing ability, advanced strategies of robot control become feasible in a practical way. However, one of the major drawbacks of today's robots is that they tend to be slow. The robot motion speed is severely limited by the weight of the manipulator arm. Furthermore, the excessive arm weight not only hampers the rapid motion of the manipulator arm, but also increases the robot's consumption of energy and the size of the required actuators. The reduction of the component weight has been proposed as one way to reduce the cost of industrial manipulators while improving high speed performance. In exchange for light-weight, however, one must accept an increase in system flexibility along with the associated difficulty in accurately controlling a flexible structure. Meanwhile, increased manipulator performance (higher speeds) requires a controller which allows both nonlinear link dynamics as well as link flexibility.

The various algorithms for modeling and control of flexible structures are found in many works. Balas [1978] developed a feedback controller for a finite number of modes of the flexible system and the controllability and observability conditions of the system necessary for successful operation were determined. A truncated mode series was used for finite element modeling of flexible manipulator arms, [Usoro, Nadira, and Mahil] while linear equations were derived for control from linearizing with respect to a prescribed reference motion [Truckenbrodt]. Schmitz [1984] discussed characteristics of a very flexible manipulator with an open truss construction operating in the horizontal plane and the end-point position measured by an optical sensor. The linear optimal control was imposed for designing the feedback controller. Moreover, three types of linear feedback schemes, joint angle and velocity feedback with and without cross joint feedback, and feedback of flexible state variable, were first proposed to show some results for the flexible arms by Book et al [Book, Maizza-Neto, and Whitney].

A modeling approach based on Lagrange's equation with assumed flexible modes [Book 1984] is used here in developing the accurate dynamic model for the flexible manipulator. This leads to a nonlinear equation with coupling terms without assumptions of a linear decoupled system.[Cannon, Schmitz] [Sakawa, Matsuno, and Fukushima] [Meldrum, Balas] The aim of our study is the construction of an estimated, state feedback control which guarantees the neighborhood of the pre-defined nominal state after a finite interval of time, including the effect of gravity.

The implementation of a feedback composed of linear and nonlinear functions of the state, in a system with uncertainties can result in robust stabilization.[Gutman] [Leitmann] This approach is also the basis of the present work. The system first is separated into the linear nominal part and the uncertainty (nonlinear) part. The Lyapunov function for stability analysis results in the controller design. The theory of uniform boundedness, which permits more relaxed assumptions than that of asymptotic stability, is utilized [Yoshizawa]. The observer design will show a sufficiently fast convergence rate on the difference between the state and the estimator. Therefore, the estimated state can be utilized in determining the actuating control signal without the problem of the positive dynamic system existing in most output feedback controls [Siciliano, Calise, and Jonnalagadda] [Bossche, Dugard, and Landau]. Digital simulation results are presented to show that the guaranteed behavior of the system under this combined control is superior to that due to linear control in spite of nonlinearities or uncertainties.

The organization of the paper is as follows. The dynamic model is derived in Section II. Control properties of the system are presented in Section III. Sections IV and V show the controller and the observer designs respectively. In Section VI, we present a case of simulations for the one-link flexible arm and the final remarks are in Section VII.

II. Dynamic Model of Flexible Arms

In this section, the formulation of a state space model for flexible arms is derived and the output measurements form the output matrix. First of all, the position of every point along the flexible arm is described as a linear combination of vibratory modes and rigid body motion.(Figure 1.) The flexible deflection is assumed to be an infinite series of separable modes which are the product of admissible functions $\phi(\xi)$ and time-dependent generalized coordinates $q(t)$.

$$Z(\xi,t) = \sum_{i=1}^{\infty} \phi_i(\xi)\, q_i(t) \qquad (2.1)$$

where $Z(\xi,t)$ satisfies the Bernoulli-Euler beam equation [Meirovitch]

$$EI \frac{\partial^4 Z}{\partial \xi^4} + \frac{\partial^2 Z}{\partial t^2} = 0 \quad , \; EI = \text{Flexural rigidity} \qquad (2.2)$$

with clamped-mass boundary conditions.[Hastings] However, the first few modes will be accurate enough to describe the flexible deflection while the amplitudes of higher modes of the flexible link are small compared to them. [Hastings] [Hughes] The first two modes will be imposed here from a practical point of view.

Next, the potential and kinetic energies can be constructed to compute the Lagrangian for the entire system. Due to the distributed character of the flexible arms, the kinetic energy K.E. is taken into account by integrating over the entire system.

$$K.E. = 1/2 \int \dot{R}\cdot\dot{R}\, dm \qquad (2.5)$$

$\dot{R}$ is the absolute velocity vector of the point along the flexible arms. The potential energy P.E. includes the gravity and the "stored energy" which is attributed to the modal stiffness K_{ji}

$$K_{ji} = EI \int_0^L (\ddot{\phi}_{ji}(\xi))^2\, d\xi \quad , \text{ for } i = 1,2 \qquad (2.4)$$

where L is the length of the arm an j means the jth link.

Then, the differential equations of motion can be formed through Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial K.E.}{\partial \dot{z}_i}\right) - \frac{\partial K.E.}{\partial z_i} + \frac{\partial P.E.}{\partial z_i} = U_i \qquad (2.5)$$

where $z_i(t)$ are the generalized coordinates and U_i are the generalized forces. Note that the generalized coordinates are classified as rigid body variables $\theta(t)$ associated with rigid body modes and flexible variables $q(t)$ associated with flexible modes.

Finally, a state space form will be easily organized through the dynamic differential equations. Equation (2.5) can be written as

$$M(\theta,q)\begin{bmatrix}\ddot{\theta}\\ \ddot{q}\end{bmatrix} + \begin{bmatrix}f_1(\theta,q,\dot{\theta},\dot{q})\\ f_2(\theta,q,\dot{\theta},\dot{q})\end{bmatrix} + \begin{bmatrix}g & 0\\ 0 & Kq\end{bmatrix} + \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} = \begin{bmatrix}U\\ 0\end{bmatrix} \qquad (2.6)$$

where

M is the inertia matrix.

f_1 and f_2 are vectors containing the nonlinear coupling terms of rigid body variables and flexible variables with their derivatives.

g is the gravitational force matrix.

$K = \text{diag}(K_{11}, K_{12}, K_{21}, \ldots\ldots, K_{n2})$ is the modal stiffness matrix.

U is the vector of the generalized force.

η_1 and η_2 are uncertainty vectors caused by friction, backlash, unmodeled modes and etc.

The set of nonlinear equations (2.6) is formulated as the following state equation

$$\frac{d}{dt}\begin{bmatrix}x^p\\ x^v\end{bmatrix} = \begin{bmatrix}0 & I\\ A_1(x^p) & A_2(x^p,x^v)\end{bmatrix}\begin{bmatrix}x^p\\ x^v\end{bmatrix} + \begin{bmatrix}0\\ B_2(x^p)\end{bmatrix} U + \eta$$

$$\Delta \; A(X)\, X + B(X)\, U + \eta \qquad (2.7)$$

where $x^v = [\dot{\theta}^T , \dot{q}^T]^T = \dot{x}^p$ (2.7a)

$$A_1(x^p)x^p = M^{-1}\begin{bmatrix}g & 0\\ 0 & Kq\end{bmatrix} \qquad (2.7b)$$

$$A_2(x^p,x^v)x^v = M^{-1}\begin{bmatrix}f_1\\ f_2\end{bmatrix} \qquad (2.7c)$$

$$B_2(x^p) = M^{-1}\begin{bmatrix}I\\ 0\end{bmatrix} \qquad (2.7d)$$

$$\eta = M^{-1}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} \qquad (2.7e)$$

Furthermore, the system cannot be controlled without signal measurements of the output. Since not every state in this system is available, the output measurements which construct the output matrix are described in the following: 1. Joint rotational angle, 2. Joint rotational angle rate, 3. Flexible modes measured by strain gauges. [Hastings, Book] So, there is

$$Y = C\,X + w \qquad (2.8)$$

where

Y is the output measurement.

C is the constant matrix.

w is the uncertainty vector containing disturbances, unmeasurable modes and etc.

Due to the flexural character, the motion of flexible modes must be restricted within certain bounds around the motion of the rigid body modes.

Therefore, equation (2.7) can be separated into the nominal parts and the uncertainty parts, the nominal parts being dominant in this system. In other words, equation (2.7) is linearized about the equilibrium point to get the nominal parts and the higher order terms are attributed to the uncertainty parts.

III. Properties of the Dynamic Equations

Before applying the control algorithm, some control properties need to be verified for the dynamic equations of flexible arms. Without loss of generality, let us rewrite equations (2.7) and (2.8) as

$$\dot{X}(t) = [A_0 + \bar{A}(t) + \Delta A(\sigma(t))]\, X(t) + [B_0 + \Delta B(\sigma(t))]\, U(t) + \eta \qquad (3.1)$$

$$Y(t) = C_0\, X(t) + w \qquad (3.2)$$

where

$X(t) \in R^n$ is the state, $U(t) \in R^m$ is the control input, and $Y(t) \in R^l$ is the measured output.

A_0, B_0, and C_0 are the nominal matrices which are constant.

$\bar{A}(t) \in R^{nxn}$ is linear and time-varying.

$\Delta A(\sigma(t)) \in R^{nxn}$ and $\Delta B(\sigma(t)) \in R^{nxm}$ are the system uncertainties.

Note that there exists $\bar{A}(t)$ in the system in order to reduce the boundedness of ΔA. Therefore, the following properties have been proven:

i) $(A_0+\bar{A}(t)\ ,\ B_0)$ is differentially controllable. This implies that $(A_0\ ,\ B_0)$ is controllable.[Chen]

ii) $(A_0\ ,\ C_0)$ is observable.

iii) $\bar{A}(t) = B_0\, K_{IA}(t)$, which is called the "Parameter matching condition", is satisfied, where $K_{IA}(t)$ is a linear integral gain.[Siciliano, Yuan, and Book]

Now, let us impose more restrictive conditions for this system (3.1) and (3.2) with the following assumptions: [Leitmann]

iv) $\Delta A(\sigma)$ and $\Delta B(\sigma)$ depend on the parameters σ (t) $\in R^p$ and are prescribed functions which are continuous on R^p.

v) Uncertainty parameters:

$$\sigma(\bullet) : R^1 \to \Omega \subset R^p$$
$$\eta(\bullet) : R^1 \to V \subset R^n$$
$$w(\bullet) : R^1 \to W \subset R^l$$

are Lebesque measurable, where Ω, V, and W are prescribed compact subsets of the appropriate spaces.

vi) The "matching conditions" are met: There exist continuous matrix functions (of appropriate dimensions) $D(\bullet)$ and $E(\bullet)$ such that

$$\Delta A(\sigma) = B_0\, D(\sigma) \qquad (3.3a)$$
$$\Delta B(\sigma) = B_0\, E(\sigma) \qquad (3.3b)$$

vii) $\max\|E\| < 1$, where $\|\bullet\|$ is an induced matrix norm. (3.4)

Remark 1): Physically, the dynamic system of flexible arms is composed of rigid body modes and flexible modes, while the linear combination of flexible modes is used to specify the deflection of any point along the arm from the motion of assumed rigid body modes. If the dominant dynamics is related to the rigid body modes,[Siciliano, Book, and De Maria] then the flexible modes will contribute the most to the system uncertainties which are continuous and bounded. However, those properties meet requirements iv), v) and vii).

Remark 2): From the controls point of view, the signal-synthesis adaptation implemented here assumes the satisfaction of the matching conditions described in vi). These conditions guarantee that the uncertainty vector does not influence the dynamics more than the control input U does.[Gutman] Instead, the perfect model following condition is satisfied [Landau], condition vi) also indicating that the uncertainties can be compensated, while ΔA, ΔB belong to the range space of B_0.

Remark 3): Equation (2.7d) shows that the uncertainty ΔB is associated with the variation of the inverse of the inertia matrix. Consequently, condition vii), it is claimed, will be satisfied when the variation of the inertia matrix is comparatively small and the magnitude of the inertia matrix is comparatively large in the system.

IV. Robust Adaptive Controller

The goal of adaptive control is to eliminate the state error between the plant and the reference model so that the behaviour of the plant and the model follow each other. Therefore, consider the reference model first

$$\dot{X}_m = A_m\, X_m + B_m\, U_m \qquad (4.1)$$

and let

$$A_m = A_0 + B_0\, K_A \qquad (4.1a)$$
$$B_m = B_0\, K_B \qquad (4.1b)$$

where K_A and K_B are constant matrix gains and there exists a Lyapunov matrix function

$$A_m^T\, P + A_m\, P + Q = 0 \qquad (4.2)$$

where P, Q are positive definite and symmetric matrices.

Simply stated, the reference model is chosen as the controllable and stable system related to the plant in order to reduce the tracking error.[Siciliano, Yuan, and Book]

However, the signal-synthesis method is implemented here to control the system by adjusting input U which is described in the following equation

$$U = K_A\, X + K_B\, U_m + K_{IA}\, X + \gamma(e) \qquad (4.3)$$

where K_{IA} is an integral gain which will be defined later, and the function $\gamma(\bullet) : R^n \to R^m$ is such that

$$\gamma(e) = \frac{B_0^T Pe}{\|B_0^T Pe\|}\, \rho_v(e) \qquad (4.4)$$

here $e = X_m - X$ and function $\rho_v(\bullet) : R^n \to R_+^1$ will also be specified subsequently.

As usual, the error dynamics is derived from the difference between equations (4.1) and (3.1)

$$\dot{e} = \dot{X}_m - \dot{X} \qquad (4.5a)$$

$$= (A_m X_m + B_m U_m) - [(A_0 + \bar{A} + \Delta A)X + (B_0 + \Delta B)U]$$

Substituting (4.3), (3.3a,b) and (4.1a,b) into the above, yields

$$\dot{e} = A_m e - 2B_0 K_{IA} X - B_0(\psi + \upsilon) \quad (4.5b)$$

where $\upsilon \triangleq E(K_A X + K_B U_m + K_{IA} X + \psi)$ (4.5c)

(4.4) and (4.5c) give the following inequality

$$||\upsilon|| \leq$$

$$\max||E||(\max||K_A X||+\max||K_B U_m||+\max||K_{IA} X||+\rho_v)$$

$$\triangleq \rho_v \quad (4.6)$$

The definition of ρ_v in (4.6) is valid, i.e. (4.6) can be solved since (3.4) is satisfied. Therefore, we have

$$\rho_v = (1 - \max||E||)^{-1}(\max||E||(\max||K_A X||$$

$$+ \max||K_B U_m|| + \max||K_{IA} X||) \quad (4.7)$$

Now, the objective is to show that the error dynamics (4.5b) is stable by Lemma 1.[Yoshizawa]

Lemma 1 : Given the dynamic equations (3.1) and (4.1) with the feedback input (4.3), we have the error dynamics (4.5b). If the properties i) - vii) (Section III) are satisfied, then the error dynamics is uniformly bounded.

Proof : Choose a positive definite function V as the Lyapounv function

$$V = e^T p e + tr\ (\bar{A}^T S \bar{A}) \quad (4.8)$$

where $S = S^T > 0$ and $tr(\bullet)$ means the trace of the matrix.

Then, consider the Lyapunov derivative with equation (4.5b) and property iii), namely

$$\dot{V} = e^T P\dot{e} + \dot{e}^T p e + 2tr\ (\bar{A}^T S \dot{\bar{A}})$$

$$= e^T(PA_m + A_m^T P)e - 2tr\ [(B_0 K_{IA} X)^T Pe$$

$$- (B_0 K_{IA})^T S B_0 \dot{K}_{IA}] - 2\ e^T P B_0(\psi + \upsilon) \quad (4.9)$$

Now, let $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$ (4.10)

From equation (4.2)

$$\dot{V}_1 = e^T(PA_m + A_m^T P)e = - e^T Q e < 0 \quad (4.11)$$

$$\dot{V}_2 = -2\ tr\ [(B_0 K_{IA} X)^T Pe - (B_0 K_{IA})^T S B_0 \dot{K}_{IA}] \quad (4.12)$$

$$= -2\ tr\ [(B_0 K_{IA})^T (PeX^T - SB_0 \dot{K}_{IA})]$$

So, we have $\dot{V}_2 = 0$, if $\dot{K}_{IA} = S^{-1}(B_0^T B_0)^{-1} B_0^T PeX^T$

From equations (4.4) and (4.6)

$$\dot{V}_3 = - e^T P B_0(\psi + \upsilon) \quad (4.13)$$

$$= - B_0^T Pe(\frac{B_0^T Pe}{||B_0^T Pe||} \rho_v + \upsilon)$$

$$\leq - ||B_0^T Pe||\ \rho_v + ||B_0^T Pe||\ \upsilon$$

$$\leq 0$$

Therefore,

$$\dot{V} < 0 \qquad \text{Q.E.D.}$$

Remark 1): The convergence speed in this error dynamics can be compared by a positive value $-\dot{V}/V$. This implies that the state error approaches zero as time tends to infinity, i.e. the state tracks the reference model more closely as time continues.

Remark 2): Since the control input (4.3) is discontinuous at $||B_0^T Pe|| = 0$, it may excite the higher modes in a system of flexible links and equation (4.5b) may not have a solution in the usual sense.[Gutman] To overcome this problem, equation (4.4) can be modified as [Leitmann]

$$\psi(e) = \begin{cases} \dfrac{B_0^T Pe}{||B_0^T Pe||} \rho_v(e)\ , \text{ when } ||B_0^T Pe|| > \beta \\ \dfrac{B_0^T Pe}{\beta} \rho_v(e)\ , \text{ when } ||B_0^T Pe|| \leq \beta \end{cases} \quad (4.14)$$

where β is a prescribed positive constant.

Remark 3): If ΔA, $\Delta B \rightarrow 0$, the adaptive controller is derived in the same formula as the previous work.[Siciliano, Yuan, and Book]

IV. Robust Observer

From the last section, the state of the system converges to a certain bound by an appropriate choice of the Lyapunov function, that is , $||X|| < \delta$ for all $t \geq 0$, where δ is a positive constant. Moreover, the controller design is dependent on the full states which are physically measurable. These lead to the proposed observer design for the nonlinear dynamic system subjected to bounded nonlinearities or uncertainties.

Consider the dynamic equations (3.1) and (3.2) with property ii) (Section III) and the observer equation [Luenberger]

$$\dot{X}_0 = A_0 X_0 + B_0 U + L(Y - C_0 X_0) \quad (5.1)$$

where $X_0 \in R^n$ is the estimated state and $L \in R^{l \times n}$ is the constant matrix.

Again, the "observer" error dynamics is

$$\dot{e}_0 = \dot{X} - \dot{X}_0 \quad (5.2)$$

$$= (A_0 - LC_0)(X - X_0) + \bar{A}X + \Delta AX + \Delta BU$$

$$= A_e e_0 + \mu$$

where $A_e = A_0 - LC_0$

$$\mu = \bar{A}X + \Delta AX + \Delta BU$$

Note that μ is bounded, since $||X|| < \delta$, and ΔA, ΔB, ΔC, and $\bar{A}$ are bounded form properties iv) and v) (Section III), i.e. there exists a constant γ , such that $||\mu(t)|| < \gamma$ for all $t \geq 0$. Also, there is a transition function $\Phi(t)$ for the linear equation $\dot{e}_0 = A_e e_0$, then $\Phi(t) = \exp(A_e t)$. Considering that λ is the maximum eigenvalue of A_e, we have the following Lemma 2. [Vidyasagar]

Lemma 2 : μ is bounded. If λ has a negative part, then the observer error dynamics (5.2) is uniformly bounded.

Proof : First, there is an inequality for the transition function $\Phi(t)$, such that $||\Phi(t)|| < me^{-at}$ for $m > 0$ and $0 > -a >$ real part(λ). Then, we express equation (5.2) as the equivalent nonlinear integral equation

$$e_0(t) = \Phi(t)e_0(0) + \int_0^t \Phi(t - \tau)\ \mu(\tau)\ d\tau$$

$$||e_0(t)|| \leq ||\Phi(t)||\,||e_0(0)|| + \int_0^t ||\Phi(t-\tau)||\,||\mu(\tau)||d\tau$$

$$\leq me^{-at}\ ||e_0(0)|| + \int_0^t me^{-a(t-\tau)}\ \gamma\ d\tau$$

$$= me^{-at}\ ||e_0(0)|| + \frac{m\gamma}{a}\ (1 - e^{-at})$$

Since $a > 0$, $||e_0(t)|| < N$, for all $t \geq 0$ where N is a positive constant.

Q.E.D.

Remark 1): If $a \to \infty$, then $||e_0(t)|| \to 0$ as $t \to \infty$. Therefore, the state X approaches the estimator X_0 as in the case of a linear system. [Luenburger] From the physical point of view, we would like to have the convergence rate of the "observer" error faster than the rate of variation of the uncertainties in the dynamics so that the estimated state can be utilized as the actuating control signal.

Remark 2): The separation principle which exists in the linear time-invariant system may also appear in a dominantly linear system with uncertainties. Many works [Galimidi, Barmish] [Hollot, Galimidi] were attributed to the result, while Lemma 2 is another approach.

VI. Case Study

In the following, a case study is developed for a one-link flexible arm moving in the operating space of gravity, where a prototype exists in the Flexible Automation Laboratory at Georgia Tech. Specification of this system and the dynamic model (2.6) can be found in the previous work.[Sciliano, Yuan, and Book]

In equation (3.1), A_0 and B_0 are derived from linearizing the model (2.7) around the nominal point of zero, while the other terms represent the uncertainties. The measurements of joint angle, rate, and strains due to the deflection of the link form the output matrix (3.2). Computer simulation will show that the one-link flexible arm moves with high-speed motion and fast vibratory-setting time from 90° to 0° . (Fig. 1)

In order to have the reference model (A_m , B_m) stable (4.1), the constant gains K_A and K_B are chosen according to the optimal regulator with a prescribed degree of stability α, whose performance index is

$$I = \int_0^\infty \exp(-\alpha t)\ (X^T RX + rU^2)\ dt \qquad (6.1)$$

where R is an identity matrix and r is 1. The integral gain K_{IA} is zero here, due to the simplicity of the dynamic system and motion trajectory. In equation (4.14) ρ_V is taken as 1 and β is 0.1. Then, two different sets ($\alpha = 1$, $\alpha = 5$) of simulations have been carried out, while the maximum eigenvalue of the "observer" error dynamics (5.2) is chosen as -10, results in sufficient convergence rate for recovering the unmeasurable states.

With the feedback input (4.3), Runge Kutta-Verner method is implemented to solve the nonlinear differential equation (2.7) at a sampling rate of 1ms. Fig. 2 to Fig. 9 show that the controller indeed is capable of tolerating the uncertainties in the parameters with the given actual bound. Obviously, we notice that well damped responses with the combined linear and nonlinear controller are obtained but the responses with the linear controller are oscillatory. (Fig. 2 - Fig. 4) Due to the dominance of A_0 and B_0 in the system, the higher-speed motion is attained in the case of higher linear gain ($\alpha = 5$). (Fig. 5 - Fig. 8) Figure 9 shows the control input U.

VII. Conclusions

A dynamic model has been derived by Lagrange's equations and the assumed mode method and the measurements form the output matrix. Moreover, the required control properties of the signal synthesis method are satisfied by the dynamic model. A robust adaptive controller has been developed for flexible arms, and a full order state observer has been presented to recover the states which cannot be measured. However, the feedback input is a control signal composed of a proportional (linear) , an integral and a nonlinear (saturated) part, which tolerates a larger variation in uncertainties than a linear controller as shown in the example. A modified controller has been introduced to avoid the discontinuity when the state error reaches the region of the boundedness. Due to high gain of the observer, the error dynamics of the observer is guaranteed to be stable. But the separation principle of controller-observer in a system with nonlinearity or uncertainty is still an open question.

Simulations in the case of a one-link flexible arm have shown the robustness of the system control to the uncertainties. However, the bound of the uncertainty has not been addressed in this paper. For multi-link flexible arms, we should take into account that the nonlinear coupling terms between rigid modes and flexible modes are the uncertainties with boundedness; consequently, it is expected that this control approach is superior to most present algorithms .

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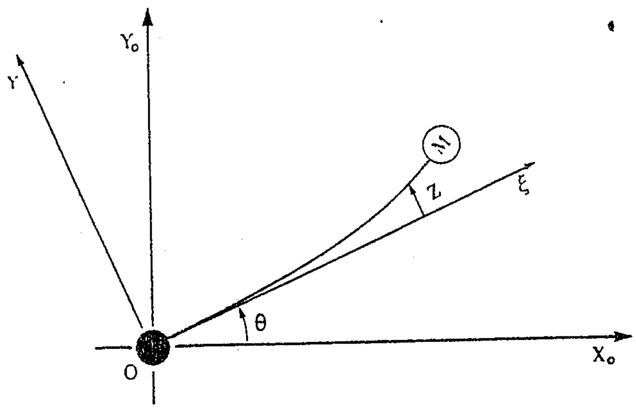

Fig. 1. Flexible manipulator.

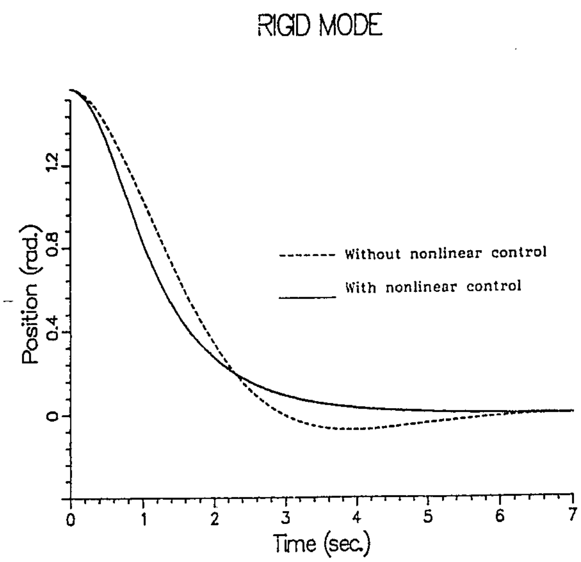

Fig. 2. Position response of rigid body mode (a=1).

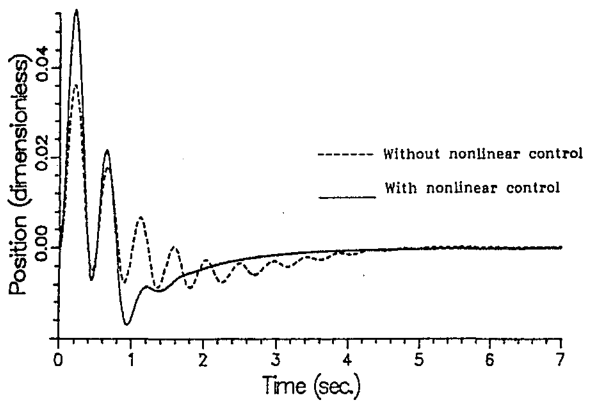

Fig. 3. Position response of first flexible mode (a =1).

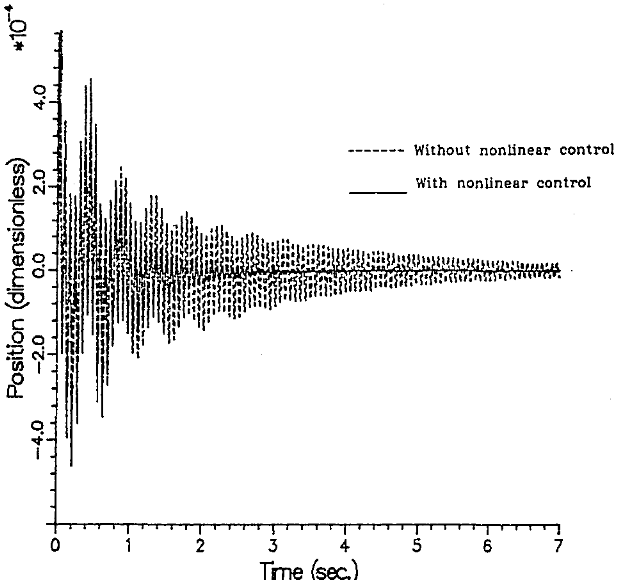

Fig. 4. Position response of second flexible mode (a =1).

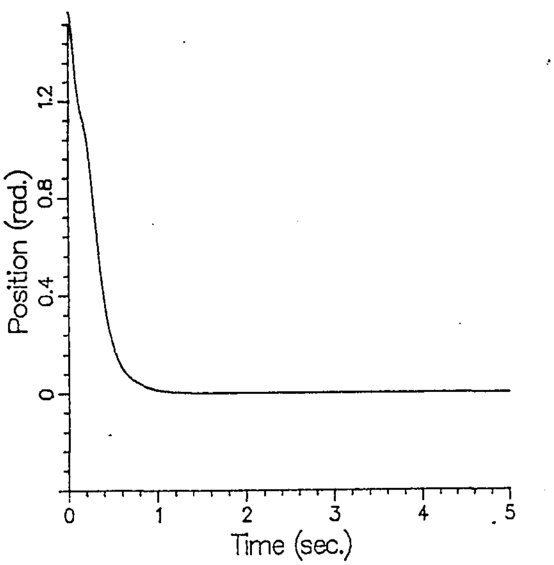

Fig. 5. Position response of rigid body mode (a =5).

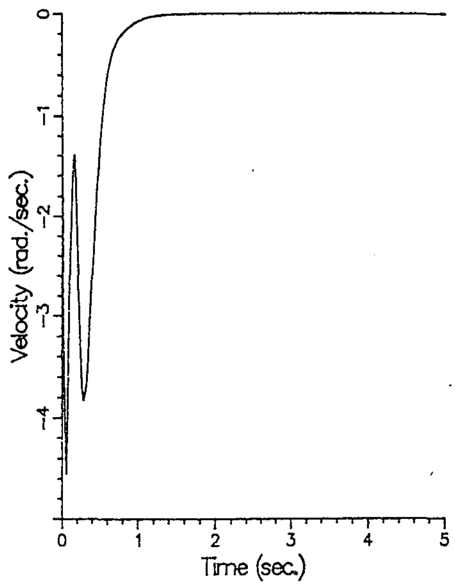

Fig. 6. Velocity (rotational rate) response of rigid body mode (a =5).

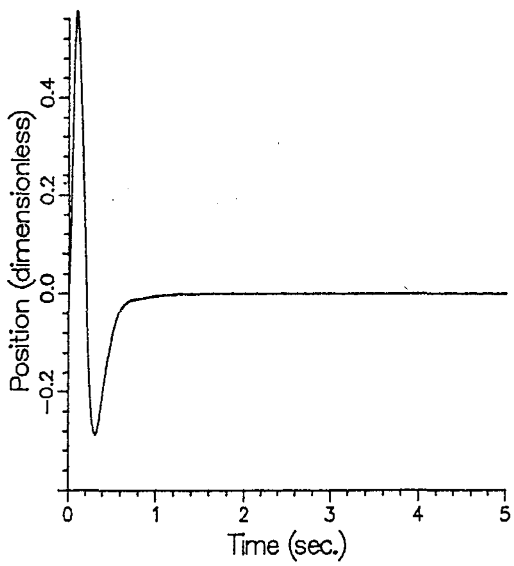

Fig. 7. Position response of first flexible mode (a =5).

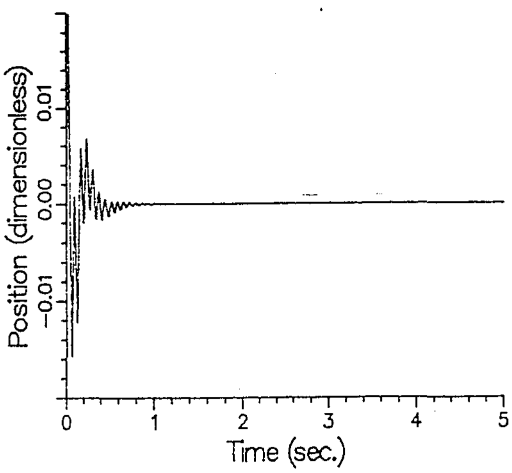

Fig. 8. Position response of second flexible mode (a =5).

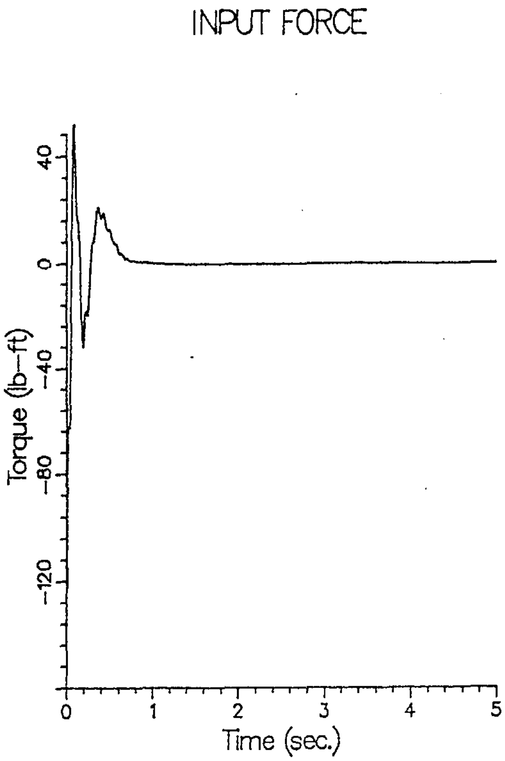

Fig. 9. Control input (α =5).