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Security class (U,C,S,TS) : U
Defense priority rating : 
Equipment title vests with: Sponsor

ONR resident rep. is ACO (Y/N): N
supplemental sheet

GIT
GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 07/08/91

Project No. G-37-641 Center No. R6563-0A0

Project Director MEYER G H School/Lab MATH

Sponsor MCDONNELL DOUGLAS CORP/ST LOUIS, MO

Contract/Grant No. Z81156 Contract Entity GTRC

Prime Contract No.

Title ANALYSIS OF EM SCATTERING FROM CURVED SURFACES

Effective Completion Date 881231 (Performance) 881231 (Reports)

Closeout Actions Required: 

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<td>Final Invoice or Copy of Final Invoice</td>
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<td>Final Report of Inventions and/or Subcontracts</td>
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Comments

Subproject Under Main Project No.

Continues Project No.

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To: Ms. F. Gleason
From: G. Meyer, Mathematics

November 14, 1988

Enclosed please find a copy of my reports which I have sent to Ron Pearlman at McDonnell Douglas Aircraft Company. They are for your records.

In a telephone conversation on Thursday last week, Ron has asked that Georgia Tech bill McAir for the balance of the contract at its earliest convenience to avoid getting caught in some accounting backlog at McAir.

Thanks.

G-37 641
The Fock theory as presented in [1, 2] is designed to provide the electromagnetic fields near the vertex of a conducting scatterer $S$ of the form

$$z = -\frac{1}{2} (ax^2 + 2bxy + cy^2).$$

In this theory it is assumed that the scatterer is a paraboloid ($ac-b > 0$) and that the incident wave is a time harmonic plane wave traveling along the $x$-axis. The total field satisfies the Helmholtz equation

$$\psi_{xx} + \psi_{yy} + \psi_{zz} + k^2 \psi = 0$$

with appropriate boundary conditions on $S$. The key assumption is made that

$$\psi = \psi^* (x, y, z) e^{ikx}$$

where $\psi^*_{xx}$ and $\psi^*_{yy}$ are small compared to $k\psi^*_x$ and $\psi^*_{zz}$. A formalism is then presented which allows an approximate solution $\psi^*$ in terms of Airy functions.

The work carried out so far is based on the observation that after the order of magnitude approximations the resulting equations, including the reduced Leontovich boundary condition, contain the $y$-coordinate only as a parameter. Specifically, it is shown that the Fock transformations remain valid essentially without change if the scatterer $S$ is of the form

$$z = -\frac{1}{2} (a(y)x^2 + 2b(y)x + c(y))$$

where $a(y)c(y)-b^2(y) > 0$. In contrast to the old theory the cross section through $S$ at constant $z$ no longer need to be elliptic. For example, $z=x^2 - y^6$ now is permissible. In general, the new formulation allows somewhat more freedom in approximating a given scatterer $S$ with a shape with parabolic cross sections in $x$ only. For this more general setting the fields near the vertex are again expressible in terms of Airy functions.
Given an arbitrary scatterer and a point $x_0$ near the shadow boundary the question now arises of where to place the origin for the Fock theory in order to find the fields near $x_0$. Ray theory suggests that it be determined by the geodesic from $x_0$ to the shadow boundary which has the direction of the incident wave there. A computer program based on a piecewise polynomial representation of the geodesic is currently in the debugging stage.

Concurrent work carried out in collaboration with the EM signatures group of MDRL suggests that the whole Fock theory can be based on a local near-geodesic coordinate system. If successful, this approach would free the theory from any specific representation of the scatterer but would place a premium on geodesic calculations. This serves as an added incentive for the code development.

November 14, 1988

Dr. Ron Pearlman
Dept. 313, Building 61
McDonnell Douglas Aircraft Company
P. O. Box 516
St. Louis, Missouri 63166

Dear Ron:

Enclosed please find a listing and a disk for a Fortran program to compute geodesics. The program was written and checked out on the Cyber 855 of Georgia Tech. It was transferred with the CDC communications package Connect to my Macintosh II and copied to the enclosed floppy disk. The listing was taken from the disk and printed with Microsoft Word. Please let me know if this program can be easily transferred to the VAX. If not I shall try to get a VAX tape made here at Georgia Tech.

I shall attach a brief description of the program and a representative calculation. Please let me know if the present program meets your needs or whether modifications are required.

Sincerely yours,

Gunter H. Meyer

GHM: cw
Program GEODESY is a Fortran 77 program to compute a geodesic curve \((x(t),y(t),z(t))\) on a surface \(z = f(x,y)\).

One end of the geodesic is a specified target point \((x_1,y_1,z_1)\). The other end is either a specified point \((x_0,y_0,z_0)\) on the surface \(z = f(z,y)\), or to be computed such that the geodesic ends on a prescribed curve \(x = sb(y)\) on the surface with a specified direction \((k_1,k_2,k_3)\). (The idea is that the curve \(x = sb(y)\) is the shadow boundary on the surface for an incoming plane wave traveling in the direction \((k_1,k_2,k_3)\). The geodesic then is the ray path to the target point.)

The program requires the following modifications before use.

The user must provide in SUBROUTINE SURFACE the function \(f(x,y)\) and its first and second partial derivatives. The user must provide in FUNCTION SB the equation \(x = sb(y)\) for the shadow boundary. It is obtained by solving 
\[
\begin{pmatrix}
-\frac{df}{dx} & -\frac{df}{dy} & 1
\end{pmatrix}
\begin{pmatrix}
k_1 \\
k_2 \\
k_3
\end{pmatrix}
= 0
\]

On execution the program asks for the following input:

1. Number of nodes for the approximation of the geodesic. 3 to 5 nodes suffice for a short geodesic. Up to 20 nodes are allowed. The execution time increases drastically with the number of nodes.
2. Base coordinate \((x_1,y_1)\) of the target point \((x_1,y_1,z_1)\).
3. Option which determines whether (1) the geodesic from point to point or (2) from curve to point is to be computed.
4. Base coordinates \((x_0,y_0)\) of the starting point \((x_0,y_0,z_0)\) in the case of option 1.
5. Direction of the incident wave in case of option 2.
6. Guess for the coordinate \(y_0\) of the endpoint on the shadow boundary in case of option 2.

Output:

1. After completion of each secant step the program prints out the number of conjugate gradient iterations and the length of the computed curve. These lengths must converge if the program is successful.
2. After convergence the program prints the endpoints of the geodesic, its direction at the endpoint \((x_0,y_0,z_0)\) and the length of the geodesic.
3. If the last step of the conjugate gradient calculation terminates with an error message (e.g. \(IER = 129\)) the final result is of doubtful value.
Restrictions:

1. The program uses the conjugate gradient subroutine ZXCGR of the IML Program Library which must be attached before execution.

2. The function \( f(x,y) \) and its derivatives must be defined for all \( x \) and \( y \) in order for the conjugate gradient method to work. This limits use of the program for surfaces like \( z = \sqrt{1 - x^2 - y^2} \).

3. The program will likely experience difficulties if the geodesic is not uniquely defined.
Explanations for the sample output:

The program is provided is set up for the surface

$$z = 0.5 \left( A x^2 + 2 B x y + C y^2 \right)$$

where

$A = 4, B = 1, C = 3$. The plot shows the surface over the rectangle $[-1, 1] \times [-2, 2]$.

Run 1: 5 nodes are requested.

The incident vector is $(1, 1, -1)$.

The program did not converge within the error tolerance of $1 \times 10^{-5}$ in the allowable 20 secant iterations. On the other hand, the length of the geodesic remains nicely stationary.

Run 2: 10 nodes are requested and the final point from run 1 is given as initial guess. We have convergence after 9 secant iterations.

Run 3: 20 nodes are requested. The final point from run 2 is entered as the endpoint under option 1. The answers are the same as in run 2.
The Fock theory as presented in [1, 2] is designed to provide the electromagnetic fields near the vertex of a conducting scatterer $S$ of the form

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In this theory it is assumed that the scatterer is a paraboloid ($ac - b^2 > 0$) and that the incident wave is a time harmonic plane wave traveling along the $x$-axis. The total field satisfies the Helmholtz equation

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Concurrent work carried out in collaboration with the EM signatures group of MDRL suggests that the whole Fock theory can be based on a local near–geodesic coordinate system. If successful, this approach would free the theory from any specific representation of the scatterer but would place a premium on geodesic calculations. This serves as an added incentive for the code development.
