Final Report for Period: 06/2010 - 08/2010

Principal Investigator: Baker, Matthew H.
Organization: GA Tech Res Corp - GIT

Submitted By: Baker, Matthew - Principal Investigator

Title: Analysis on Berkovich spaces and applications

Project Participants

Senior Personnel
- Name: Baker, Matthew
- Worked for more than 160 Hours: Yes
- Contribution to Project: Post-doc

Graduate Student

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts
I collaborated with Serguei Norine on two research papers, and with Robert Rumely on a joint research monograph.

Activities and Findings

Research and Education Activities:
In addition to writing research papers and a research monograph, I also wrote online lecture notes for a graduate algebraic number theory course, gave a number of invited lectures on my research in the U.S. and Europe, supervised two REU students in summer 2006, organized an undergraduate research conference in summer 2006 at Georgia Tech, supervised two undergraduate senior projects in 2006-2007, and began supervising the Ph.D. work of David Krumm. In March 2007 I gave a course at the Arizona Winter School on potential theory on Berkovich curves, and wrote a set of lecture notes on this material which has been published in an AMS book on p-adic Geometry. I am currently supervising graduate research on graphs and their Jacobians by Farbod Shokrieh, a Ph.D. student, and by Ye Luo, a master's student. I co-organized two workshops/conferences in 2007-2008.

Findings:
I extended some of my results with Rumely concerning potential theory on the Berkovich projective line from $\mathbb{C}_p$ to an arbitrary complete and algebraically closed valued field. We also discovered a number of new results concerning dynamics on the Berkovich projective line.
I used Berkovich spaces to prove a conjecture of Szpiro and Tucker concerning preperiodic points of non-isotrivial rational maps over function fields.

I found a simple characterization of affinoid Berkovich spaces as the completion of a canonically defined uniform structure.

With Serguei Norine, I discovered and proved a graph-theoretic analogue of the classical Riemann-Roch theorem, as well as some new results about the Abel-Jacobi map from a graph to its Jacobian. The computational aspect of this work was assisted by my REU student Dragos Ilas.

My undergraduate REU student George Ander Steele proved some new results about Carmichael numbers in abelian extensions of Q and wrote a paper on the subject which has been accepted for publication by the journal of number theory.

With Serguei Norine, I investigated the notion of harmonic morphisms between graphs, proving several foundational results and giving several equivalent characterizations of hyperelliptic graphs.

I investigated the specialization of linear series from curves to graphs, and proved a basic inequality (the 'Specialization Lemma') which allowed me to deduce some new results in arithmetic and tropical geometry. With the help of my undergraduate student Adam Tart, I formulated some interesting combinatorial conjectures about graphs which are motivated by the connection with divisors on curves.

With Xander Faber, I proved some new results about metric properties of tropical curves and their Jacobians.

Training and Development:
My two REU students from summer 2006, Dragos Ilas and Ander Steele, gained valuable research experience. Ander presented his work at a regional undergraduate research conference called 'Meeting of the Minds', and he was awarded the 2007 Undergraduate Research Award by the Georgia Tech chapter of Sigma Xi. Ander's paper, written under my supervision, was subsequently published in the Journal of Number Theory.

A Ph.D. student, David Krumm, gained valuable research experience and learned many useful things through an NSF-sponsored research assistantship with me. In particular, we studied Arakelov Class Groups and Elliptic Curves.

My former postdoc, Clay Petsche, assisted my graduate student project at the Arizona Winter School, which involved approximately 10 graduate students during the course of one week in March 2007.

My 2008 REU students Stefan Froehlich and Daniel Connelly worked on harmonic morphisms of graphs and preperiodic points of dynamical systems, respectively.

I am currently mentoring a Ph.D. student, Farbod Shokrieh, a Master's student, Ye Luo, and a first-year graduate student, Spencer Backmann, on research projects involving graphs, metric graphs, and their Jacobians.

Outreach Activities:
I organized a two-day REU mini-conference at Georgia Tech in summer 2006 which was attended by undergraduates, graduate students, postdocs, and research faculty. This was just one aspect of my organization of the entire Georgia Tech School of Math REU program in summer 2006. I also organized the summer 2008 and summer 2009 REU programs at Georgia Tech.

My lectures at the Arizona Winter School in March 2007 were attended by over 100 graduate students, and also by postdocs and research mathematicians. My research project at the Arizona Winter School involved about 10 graduate students, who gave a presentation on their work at the end of the week.

I performed mathematical magic for the Georgia Tech High School Math Competition in March 2006, and for a Georgia Tech teacher training class in June 2006. In 2006-2007 I was a member of Project Magic, which gives monthly magic performances at local hospitals. In 2009, I performed a mathematical magic show for over 100 high school students at the Georgia Tech High School Mathematics Competition.

I was a faculty representative at the Georgia Tech Presidential Scholar welcoming lunch in August 2006.

I coordinated Georgia Tech's participation in the Putnam Exam from 2006 through 2009.

Journal Publications


Books or Other One-time Publications

Bibliography: Mathematical Surveys and Monographs Volume 159, American Mathematical Society

Editor(s): David Savitt, Dinesh Thakur
Collection: p-adic Geometry (Lectures from the 2007 Arizona Winter School)
Bibliography: AMS University Lecture Series

Web/Internet Site

URL(s):
www.math.gatech.edu/~mbaker/papers.html
Description:
The papers that are listed in this report are posted at the URL and all the papers acknowledge NSF support. In addition, many of the papers are posted on other preprint servers, such as the ArXiv, further enhancing the dissemination of research supported by this grant.

Other Specific Products

Contributions within Discipline:
My work on canonical heights over function fields has introduced a powerful new tool (the theory of Berkovich spaces) into the subject.

My work with Norine clarifies and presents new aspects of the interplay between algebraic curves and graphs, and has provided unexpected links between combinatorics, number theory, algebraic geometry, and tropical geometry.

My research monograph with Rumely promises to make the theory of Berkovich spaces accessible to a wider audience, and introduces a number of new ideas and constructions.

My work on specialization of linear systems from curves to graphs introduces a new method for proving theorems in tropical geometry.
Contributions to Other Disciplines:
My work with Norine promises to be influential in combinatorics and graph theory as well as in number theory, and should foster new interactions between experts in these different fields.

My work on specialization of linear systems promises to foster new interactions between arithmetic geometry, graph theory, and tropical geometry.

Contributions to Human Resource Development:
I supervised four undergraduate research projects; two of those students have already gone on to graduate school (one in mathematics, one in computer science).

I organized an REU mini-conference which provided valuable presentation experience to approximately 15 undergraduate participants.

I am currently training two graduate students, Ye Luo (Master's) and Farbod Shokrieh (Ph.D.). I also supervised research or reading courses with several graduate students, including David Krumm, Yi Huang, and Spencer Backmann.

I gave lectures on mathematics and magic and performed magic to high school students and middle school teachers.

My lectures and research project at the 2007 Arizona Winter School reached a large audience of graduate students in mathematics from around the world.

Contributions to Resources for Research and Education:
My senior project student Adam Tart wrote two useful java applets (available online) illustrating my work with Norine on Riemann-Roch for graphs. My REU student Dragos Ilas wrote Mathematica software for calculating the Abel-Jacobi map from a graph to its Jacobian.

I wrote a set of online lecture notes for an algebraic number theory graduate course.

I wrote a set of lecture notes for the 2007 Arizona Winter School aimed at introducing graduate students to the theory of Berkovich spaces.

Contributions Beyond Science and Engineering:

Conference Proceedings

Categories for which nothing is reported:
Organizational Partners
Any Product
Contributions: To Any Beyond Science and Engineering
Any Conference