Submitted on: 12/01/2010
Principal Investigator: Gangbo, Wilfrid
Organization: GA Tech Res Corp - GIT
Submitted By: Gangbo, Wilfrid - Principal Investigator
Title: Geometry on the Set of Probability Measures

Project Participants

Senior Personnel

Name: Gangbo, Wilfrid
Worked for more than 160 Hours: Yes
Contribution to Project:

Post-doc

Graduate Student

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts
My collaborators include Luigi Ambrosio, Alessio Figalli, Adrian Tudorascu, Tommaso Pacini, Hwakil Kim, Lincoln Chayes, Helen Lei, G. Savare.

Activities and Findings

Research and Education Activities:
During 2005-07, W. Gangbo has completed a research project with his postdocs A. Tudorascu and T. Nguyen. He worked with his graduate students, T. Yolcu, HK Kim. He is also supervising Sedjro who is in the PhD program at GTech, for a reading course. This supervision has lasted two semesters. Gangbo is taught a special topic class on gradient flows in metric spaces. The class is attended by 7 faculty members and 7 students. Gangbo give a ten hours lecture in Benin during summer 2006 and the same lecture in Dakar, Senegal that summer.

During 2007-08, W. Gangbo's previous works with his collaborators have been accepted for publication. This includes findings with his postdocs A. Tudorascu. W. Gangbo is completing two research project with his postdocs A. Tudorascu. This is a continuation of their previous works. With his PhD student Hwakil Kim and his former postdoc T. Pacini, W. Gangbo is completing a paper on symplectic structures on the Wasserstein space.

During 2008-09, W. Gangbo's works with his collaborators have been accepted for publication. This includes findings with G. Bouchitte, P. Seppecher and L. Ambrosio. W. Gangbo has completed two projects his former postdocs T. Nguyen and A. Tudorascu. W. Gangbo has
completed a paper with M. Westdickenberg on Euler equations of compressible fluids. A project which started with his former postdoc T. Pacini and his current student H.K. Kim was completed.

In the past decade, PDEs methods have been used to understand classical problems in dynamical systems such as the search of invariant measures. This has served as an inspiration for W. Gangbo and his postdoc A. Tudorascu, to start extending the Weak-KAM theory, originally developed by Mane/Mather, to infinite dimensional Hamiltonian systems such as the Vlasov systems and their monokinetic counterpart. This is reminiscent (not a duplication) of the extension of the KAM theory to PDEs as done by many people such as S. Bolotin, J. Moser and P. Rabinowitz.

During 2008-2010, W. Gangbo completed a project with T. Yolcu and A. Figalli. This has been accepted for publication as indicated below. The recent PhD student Helen Lei, currently a NSF postdoc at Caltech is working on a joint project with W. Gangbo and her former advisor L. Chayes.

**Findings:**

In a joint work with his postdocs A. Tudorascu and T. Nguyen, Gangbo obtained existence of a new class of solutions in the 1d Euler-Poisson system. These solutions minimize an action and are unique when they are prescribed at time $0$ and time $T<\pi$. They conserve the Hamiltonian unlike the entropy solutions whose existence were established by other authors in prior works. With HK Kim, Gangbo is investigating perturbation of evolutive conservative system. The idea is to approximate an original nonsmooth Lagrangian by a class of smooth ones and obtain existence of solution. These solutions will lie on an orbit. Passing to the limit, one would obtain a solution for the original Lagrangian. Approximation of Lagrangians with good properties in the infinite dimensional set of probability measures is poorly understood. With his student T. Yolcu, Gangbo is studying smoothness of SL$^+$ convex sets, where SL$^+$ is a Lagrangian on a manifold. On needs to prove that the boundary of such set is of zero measure. This will make it possible to study a larger class of parabolic PDEs without imposing that the manifold is without boundary.

In a joint work with his postdoc A. Tudorascu, Gangbo study invariant measures, homogenization and effective Hamiltonian on the Wasserstein space. With his student M. Sedjro and his collaborator M. Cullen, W. Gangbo is studying axisymmetric flows which arise in atmospheric sciences. This is a model derived by C.G. Craig from Hamiltonian principles.

In a joint work with his postdoc A. Tudorascu, Gangbo consider the Vlasov systems with periodic potential. In general, global existence results are not known for such systems. In that project, they identified a class of initial conditions for which they established a global existence result. They also described asymptotic behavior of their solutions. With M. Westdickenberg, Gangbo introduced a variational scheme for the Euler equation of compressible fluids. They established new energy estimates and hope to prove convergence of the scheme in a future project.

Existence of solutions for first order differential equations is not known in infinite dimensional setting such as Hilbert spaces when the entropy functional fails to have a Lipschitz gradient. Motivated by applications in Fluids mechanics, with T. Yolcu and A. Figalli, W. gangbo consider infinite dimensional differential equations on the Wasserstein space. They extend a powerful method introduced by De Giorgi, the so-called De Giorgi interpolation to prove existence of solution when a Lagrangian exists.

Under the supervision of the PI, his student Marc Sedjro has established a minimax formula in connection with the study of almost axisymmetric flow. That work is being completed with Mike Cullen, a meteorologist from the UK.

**Training and Development:**

T. Nguyen was a postdoc mentored by Gangbo. They have completed a long paper on Euler-Poisson system with another postdoc, A. Tudorascu. Truyen has been offered several position including two postdoc in Canada and a tenure-track position at the University of Ohio in Akron. He chose the latter.

The former postdoc of W. Gangbo, T. Nguyen, took a tenure track position at the University of Ackron which is trying to establish a PhD program. His recent postdoc A. Tudorascu has accepted a three-years visiting position at the University of Madison Wisconsin and has now moving to a tenure-track position in West Virginia. W. Gangbo is currently supervising one PhD students. Two of his students graduated in 2009.

Hwakil Kim who has been under the supervision of W. Gangbo defended his PhD dissertation on April 08 2009. He has accepted a position as a Courant Instructor at NYU. Turkay Yolcu, a PhD student of W. Gangbo has defended his PhD dissertation in Fall 2009. He accepted a visiting position in a local college in August 2008. He is currently holding a visiting position at Purdue University. Marc Sedjro, a current PhD student of Gangbo is making steady progress on his research. He is expected to complete his dissertation in 2010 or early 2011.
Gangbo gave several talks in international conferences. This includes talks in international conferences in Chili, Haifa. He gave several talks at IPAM--UCLA. W. Gangbo was one of the organizer of the Spring 2008 program at IPAM-UCLA. This program was very successful in bringing together people from various areas of sciences and engineering and interact on problems of common interest. Many collaborations started there including a collaboration involving A. Figalli, T. Yolcu and W. Gangbo.

in 2009-2010, W. Gangbo gave several talks to disseminate his findings under the current contract. W. Gangbo gave several talks in international conferences. He was a plenary speaker at the Pacific NorthWest Seminar in Vancouver in January 2009. His international talks includes a one-hour talk in Nice in February 2009, in Kyoto, Japan in June 2009, in Lisbon, Portugal in July 2009, in Oberwolfach, Germany in February 2010. He also gave a five hours mini-course in Diablerets, Switzerland in March 2009. He gave a talk in a workshop organized by IPAM at Lake ArrowHead in December 2009 and gave a seminar talk at Brown University in October 2009 and at the University of Madison, Wisconsin in May 2009. W. Gangbo gave a colloquium talk at UC Berkeley in April 2009.

W. Gangbo organized an international summer school followed by a conference in Benin in summer 2010. A careful selection of the best African experts participated in the activities. The summer program hosted representants from over 10 African countries. This includes graduate students, postdocs and full time faculty members. Over a total of 16 countries in the world participated in the activities. This includes Cedric Villani who gave five lectures on Landau-Damping. Three months later, as anticipated, Cedric Villani was awarded the Field Medal for the work he described in that summer program. N. Ghoussoub was the moderator for a public debate which involved a selection of potential donors who will be willing to support the African mathematics community to help them get involved in more international activities. Promises have been made by various individuals or professional associations. A group of scientists including C. Villani, N. Ghoussoub, JP. Bourguignon, I. Gamba, E. Carlen and W. Gangbo together with the African mathematics community are making plan for a long term scientific exchanges between the USA, Europe and Africa. For that reason, C. Villani, N. Ghoussoub and W. Gangbo travelled to Waterloo, CA in November 2010 to discuss with Neil Turok who is one of the founder of the African Institute of Mathematical Sciences.

W. Gangbo is supervising the research work of Cheryl Outing, a senior student of the mathematics department of Spelman College. This is a local women and minority college in Atlanta. Cheryl was recommended to W. Gangbo as one of the best students Spelman College has had in the past few years. The work of Cheryl consists in proving some approximation result of domain by smooth domain. That result which is extremely useful in partial differential equations seems to be well accepted by the community while one can hardly find the proof somewhere. Our goal is to have Cheryl complete a project which may turn out to be a useful reference for many people including senior researchers.

W. Gangbo serves on several editorial board: SIMA, CMS, NHM, ESAIM. He is currently serving on the AMS Committee of Human Rights. He is serving on the Ramanujan Prize Selection Committee as his term has been extended from 2008--2010 to 2010--2012. W. Gangbo has served as a referee on various journals.

Journal Publications


Ambrosio, L; Gangbo, W, "Hamiltonian ODEs in the wasserstein space of probability measures", COMMUNICATIONS ON PURE AND APPLIED MATHEMATICS, p. 18, vol. 61, (2008). Published,

Cullen, M; Gangbo, W; Pisante, G, "The semigeostrophic equations discretized in reference and dual variables", ARCHIVE FOR RATIONAL MECHANICS AND ANALYSIS, p. 341, vol. 185, (2007). Published, 10.1007/s00205-006-0040-


W. Gangbo, A. Tudorascu, "Lagrangian Dynamics on an infinite-dimensional torus; a Weak KAM theorem", Advances in Mathematics, p. 260, vol. 1, (2010). Published,


Books or Other One-time Publications

Web/Internet Site

URL(s):
www.math.gatech.edu/~gangbo

Description:
One can obtain a listing of all my NSF-sponsored works. A complete reference to the NSF-DMS projects that supported these works can be found in "Grants and NSF Sponsored Works.

Other Specific Products

Contributions within Discipline:
In 2006--2008, Gangbo was running the PDE seminar at Georgia Tech. This involves many students as speakers. He is giving a special topic class to 7 faculty members and 7 students at his host institution.

Bouchitte, Gangbo, Seppecher consider the new concept of Michell trusses they introduced as a first concrete step which support Michell theory of existence of Michell trusses. They anticipate that the probability approach in that paper would be useful in the study of hyperbolic equation. W. Gangbo hopes that his current work on invariant measures will generate interest in PDEs and the calculus of variations.

In his work with Adrian Tudorascu, Gangbo layed down arguments to introduce the concept of rotation numbers associated to PDEs whose physical spaces are one-dimensionals. This can be extended to PDEs whose physical spaces are d-dimensionals.
The extension of De Giorgi interpolation method extend a powerful tool to attack PDEs which are not homogeneous. It can be readily adapted to include PDEs on manifolds.

**Contributions to Other Disciplines:**
Gangbo anticipate that his work on Michell trusses will be of great interest for engineers in elasticity theory

W. Gangbo anticipate that his work on invariant measures and infinite dimensional Hamiltonian systems will be of interest to geometers.

**Contributions to Human Resource Development:**
Nothing yet to report

**Contributions to Resources for Research and Education:**
Nothing yet to report

**Contributions Beyond Science and Engineering:**
Nothing yet to report

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**Conference Proceedings**

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