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GEORGIA INSTITUTE OF TECHNOLOGY
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RESEARCH PROJECT INITIATION

Date: June 27, 1975

Project Title: Expectationa & Equilibrium Over Time

Project No: M-50-611 (Co-project G-37-602)

Principal Investigator: Dr. Frederic B. Shipley & Dr. David Nachman

Sponsor: National Science Foundation

Agreement Period: From 6/1/75 Until 11/30/77*
~~*24 months budget period plus 6 months for submission of required reports, etc.~~

Type Agreement: Grant SOC75-14663

Amount:

	NSF		GIT	
IM:	M-50-611	$39,009		
MATH:	G-37-602	11,191	G-37-313	$3,919
		$50,200		

Reports Required:

Annual Letter Technical; Final Report

Sponsor Contact Person (s):

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Assigned to: Industrial Management

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Date: 3/3/78

Project Title: Expectations & Equilibrium over Time

Project No: M-50-611 and G-37-602 (Co-Projects)

Project Director: Dr. F.B. Shipley & Dr. D. Nachman (IM)
Dr. R.P. Kertz (Mathematics)

Sponsor: National Science Foundation

Effective Termination Date: 11/30/77

Clearance of Accounting Charges: 2/28/78

Grant/Contract Closeout Actions Remaining:

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SUMMARY OF COMPLETED PROJECT

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1. INSTITUTION AND ADDRESS		2. NSF PROGRAM	3. GRANT PERIOD
ollege of Industrial Management eorgia Institute of Technology tlanta, GA 30332		Economics	June 1, 1975 to September 30, 1977 *from* *to*
4. GRANT NUMBER	**5. BUDGET DUR. (MOS)**	**6. PRINCIPAL INVESTIGATOR(S)**	**7. GRANTEE ACCOUNT NUMBER**
0C75-14663	24	Kertz, Nachman, Shipley	

SUMMARY (Attach list of publications to form)

The inquiry made in this research concerns the role of expectations in questions f existence of long run equilibrium in market economies that evolve in time under ncertainty. The vehicle for this inquiry is a mathematical model of an economy here markets are open at each of a sequence of dates for spot trading and limited utures contracting and where agents at each date are uncertain about prices that ill prevail on markets at future dates. No institutional arrangements for handling ankruptcy and in particular no a priori bounds on size of futures contracts are ssumed.

The primary objectives in this context are the study of existence of solutions o an individual agent's optimal choice problem and the study of conditions on the xpectations of agents of the model economy that ensure the existence of a sequence f temporary competitive price equilibria, an equilibrium path, for this economy. revious work in partial snapshots of sequential models indicate that some similarity f agents' expectations is required for existence of temporary price equilibrium at given date, the date of the snapshot. This research extends this finding to the oving picture, i.e., the full sequential model. In addition, the moving picture eveals the opposing need for some disparity in agents expectations. The consistency f these opposing requirements, which lies at the heart of viability of a competitive ricing mechanism in an environment where bankruptcy is possible, remains an nresolved issue. The results obtained in the study of existence of solutions to ndividual agent choice problems and the resulting precise mathematical characterization f the opposing requirements, provide the needed framework within which to resolve nis important issue and pursue the economic consequences of this resolution.

SIGNATURE OF PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR	TYPED OR PRINTED NAME	DATE
	David C. Nachman	2/24/78

Form 98A (5-76)

Publications and Working Papers

The following three working papers were generated by the research done under NSF Grant No. SOC75-14663. Papers 1 and 2 have been submitted for publication to the Annals of Probability. Paper 1 has been excepted for publication and is now being revised. Paper 2 has been sent back for specific reply to a referee's comments before a publication decision is made. This reply as well as a revision of Paper 2 is under preparation. Paper 3 is subject to revision given the resolution of the central issue alluded to in the preceeding summary.

1. Robert P. Kertz and David C. Nachman, Optimal plans for discrete-time non-stationary dynamic programming with general total reward function I: the topology of weak convergence case, Working Paper MS-77-1, College of Industrial Management, Georgia Institute of Technology, (April, 1977).
2. Robert P. Kertz and David C. Nachman, Optimal plans for discrete-time non-stationary dynamic programming with general total reward function II: the ws^{∞}-topology case, Working Paper MS-77-6, College of Industrial Management, Georgia Institute of Technology, (June, 1977).
3. Robert P. Kertz and David C. Nachman, Temporary competitive equilibrium in a sequence of spot and futures markets, Working Paper E-77-22, College of Industrial Management, Georgia Institute of Technology, (November, 1977).

Final Technical Report

on

National Science Foundation

Grant No. SOC75-14663

"Expectations and Equilibrium over Time"

Granted to

Georgia Institute of Technology

Starting Date

June 1, 1975

Completion Date

November 30, 1977

Principal Investigators

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I. GENERAL OBJECTIVE OF RESEARCH

The general inquiry of this research concerns the role of expectations in questions of existence of various concepts of long run economic equilibrium in sequential trading models with some mechanism for inter-temporal transfers of wealth or for hedging against an uncertain future. It is in this context that the problem of bankruptcy is encountered and it is through a study of equilibrium existence questions in this context that one discovers the need for and kind of alternative (to competitive pricing mechanisms) institutional arrangements for handling bankruptcy. Indeed, it is clear that the antithetical nature of the concepts of temporary competitive price equioibrium and the existence of bankruptcy entails that a competitive price mechanism is viable only in economic environments, particular specifications of agents and resources, in which bankruptcy is only an unrealized potential. It is equally as clear that the key elements of the economic environment in this regard are agents' probabilistic expectations.

The first task of this research then becomes the specification of environments, in the context of particular models of market economies that evolve in time (sequential trading models), in which the existence of a sequence of temporary competitive price equilibria can be demonstrated. The choice of the particular model or models for this investigation is dictated in part by the essential character of the bankruptcy problem and in part by the need to avoid <u>a priori</u> institutional arrangements that affect the potential for bankruptcy.

The essential characteristic of the problem of bankruptcy is the existence of actions, such as investment in capital goods or inventories, the holding of money balances, or the making of futures contracts, available to economic agents that affect in an unpredictable way their ability to choose a feasible consumption plan usually at some later date. In the context of a competitive market economy under uncertainty regarding prices at future market dates, the future value of such actions is what is unpredictable. The wealth of an agent at any date is a function of the value, determined by market prices at that date, of such actions taken at previous dates. If this wealth is insufficient to allow an agent to choose a feasible consumption plan at that date, the agent is declared bankrupt, otherwise the agent is solvent.

To make clear the burden placed on the probabilistic expectations of economic agents by the need to avoid bankruptcy, modeling assumptions involving market or other institutional arrangements that affect the potential for bankruptcy must also be avoided. Such assumptions include the implicit or explicit assumption of bounds on the size of agents' indebtedness [11], assumptions regarding the supply of money, e.g., a fixed or inelastic supply of money [3], [4], and assumptions setting out bankruptcy laws or economic and extraeconomic penalties for bankruptcy [2], [6].

In following these dictates, we chose to study an infinite horizon generalization of a two period model of an exchange economy with spot and futures markets developed by Green [5]. In this work, Green's emphasis is on the existence of a temporary competitive price equilibrium

for markets open at the initial date. The important feature of this model is the deliberate absence of any institutional arrangements for bounding the size of futures contracts made by individual agents.

In the following section, the research conducted in generalizing Green's model and the results obtained in the analysis of the generalized model are discussed. In the final section of the report, some comments on the remaining tasks of this research program are made.

II. SCIENTIFIC DESCRIPTION OF RESEARCH AND RESULTS

There are two parts to a general equilibrium study of the kind, for example, that Debreu has done in [1]. The first is the modeling of the choice behavior of individual economic agents, and the second is the study of conditions under which individual choice can be aggregated in a consistent manner. The institutional market structure investigated places restrictions on both parts of such a study.

The structure of sequential trading entails a dynamic programming model of individual choice. An abbreviated version of such a model based on the expected utility hypothesis has become the corner stone of temporary equilibrium analysis [4: Section 2.1]. For finite horizon problems, the standard method of backward induction can be used to establish existence of optimal plans and the sequential analysis of such plans needed to derive demand correspondences for each market date, even when the problem is inherently non-stationary and non-Markovian.

For studying long run concepts of economic equilibrium, however, the individual agent choice behavior must be viewed as an infinite horizon

(non-stationary, non-Markovian) dynamic programming problem with expected utility criterion. When we began our research there was no literature on this class of problems save one unpublished manuscript by James S. Jordan (this work has since appeared in Econometrica as [9]), and it was clear that we would have to develop the theory for this class of problem ourselves.

This theory is essential not only to our particular generalization of Green's model, where except for a significant problem of relativization and a more traditional view of the concept of strategy Jordan's analysis is adequate, but also to the further development of temporary equilibrium theory in general. Indeed, the analysis of sequential trading models in which uncertainty exogeneous to economic activity exists, agents have varying amounts of information regarding both the economic and exogenous environment, and in which an agent's own action provides information regarding these environments requires a rather general model of agent expectations and consequently a general dynamic programming model of agent choice. Studies involving such general models, some of which fall within the scope of our intended research program, cannot proceed without a theory of existence of solutions to such problems.

The essential issue in establishing existence of solutions to a general infinite horizon dynamic programming problem with expected utility criterion is matching the right topology on the space of probability measures on posterities (possible futures) with given measurability and topological assumptions on the data of the problem including the laws of motion (probabilistic expectations), the feasible action correspondences, and

the utility function. Of particular interest from the point of view of economic theory are problems in which all uncertainty is endogeneous as in our generalization of Green's model discussed below, and problems in which exogenous uncertainty is present and utility is state dependent. In the first case, endogeneously determined states such as prices are likely to have a natural topological structure with respect to which agents' probabilistic expectations can be assumed to be continuous, and continuity and boundedness assumptions on agents' utility functions are also standard. In the second case, there may be no such natural topological structure of exogeneously determined states. In this case, certain primitive data such as agents' utility functions may be assumed to be regular (continuous) with respect to endogeneously determined states and actions, but only measurable with respect to exogeneously determined states.

Our research in these two cases are contained in the papers

1. Robert P. Kertz and David C. Nachman, Optimal plans for discrete-time non-stationary dynamic programming with general total reward function I: the topology of weak convergence case, Working Paper MS-77-1, College of Industrial Management, Georgia Institute of Technology (April, 1977).
2. Robert P. Kertz and David C. Nachman, Optimal plans for discrete-time non-stationary dynamic programming with general total reward function II: the ws^{∞}-topology case, Working Paper MS-77-6, College of Industrial Management, Georgia Institute of Technology (June, 1977).

Paper 1 presents an existence theorem for the dynamic programming problem with expected utility criterion when the state and action spaces are standard Borel spaces, the feasible action correspondence is compact-valued and upper hemi-continuous, the probabilistic laws of motion are continuous functions of histories in the topology of weak convergence on the space of probability measures on states, and where utility is bounded above and upper semi-continuous in both states and actions. This result covers the first case mentioned above.

In Paper 2, a similar existence theorem is obtained with weakening of the hypotheses regarding regularity of the utility function with respect to states but some needed strengthening of the regularity hypotheses on other data with respect to actions. While this result encompasses the second case mentioned above, it does so in the context where the measurability structure of the state spaces arises from an assumed topological structure. In particular, state spaces are assumed to be locally compact as well as standard Borel.

Both papers have been submitted for publication in the Annals of Probability. Paper 1 has been accepted for publication and at this time we are awaiting a decision on Paper 2. Both papers are currently under revision, but we have attached copies of the working papers as submitted for publication. When revisions are finished, we will send along a revised copy of each paper. Paper 1 was presented at the SIAM 1976 Fall Meeting held at Georgia Tech, October 18-20, 1976, and Paper 2 was presented at the 83rd Annual Meeting of the American Mathematical Society, St. Louis, Missouri, January 27-31, 1977. These presentations were made

by Professor Robert P. Kertz. In addition, Professor Kertz presented the results of these two papers together with some research of his own at the International Conference on Dynamic Programming, University of British Columbia, April 14-16, 1977.

The research on these two dynamic programming papers proceeded simultaneously with work on generalizing Green's [5] model of an exchange economy with spot and futures markets. Green's [5] model is really a one and a half period model where markets are open at the initial date for spot trading in commodities and for trading in unconditional futures contracts for delivery at the second market date. At that date only markets for spot exchange are open. Again, Green's interest here is in showing existence of a temporary competitive price equilibrium for the markets open at the initial market date. This interest does not directly address the problem of bankruptcy. There are no preexisting contractual obligations at the initial market date and there is no consideration of equilibrium at the second market date when futures contracts made at the initial date come due.

The most straight forward generalization of Green's model consistent with the objective of long run equilibrium analysis in which bankruptcy is a potential problem is of an economy where markets are open at each of an infinite sequence of dates for spot trading and limited (one period) unconditional futures contracting in elementary commodities, and where agents at each market date are uncertain about prices that will prevail at future market dates, both spot and futures prices. Elementary commodities are those distinguishable by their physical characteristics

and perhaps location, but not by date or event of nature. This generalization can be viewed as an attempt to make a moving picture out of Green's partial snapshot of an economy with spot and futures markets. With some modification to allow for trading known future endowments on current futures markets, Green's model appears as the first frame of this moving picture. Successive frames are Green type models with preexisting contracts.

The full model together with the complete analysis of individual choice and the analysis of equilibrium is presented in some detail in the paper

3. Robert P. Kertz and David C. Nachman, Temporary competitive equilibrium in a sequence of spot and futures markets, Working Paper E-77-22, College of Industrial Management, Georgia Institute of Technology (November, 1977).

Under a set of seven assumptions on the economic environment, the infinite sequence of frames, i.e., the movie, is shown to indeed move in the sense that an equilibrium in the markets open at the initial date exists, as Green has demonstrated [5: Theorem 4.11], and the existence of a finite history of equilibria through any date gives rise to initial conditions at the following date that ensure existence of an equilibrium in markets open at that date [10: Theorem 5.2.2].

The sequence economy is thus shown to have paths of evolution of trajectories where the state at each date is a temporary competitive price equilibrium for markets open at that date. In short, the economy has equilibrium paths. In addition, these equilibrium paths exhibit equilibrium over time [7], [8] in a weak sense: every neighborhood of

an equilibrium price system for markets at date n+1 is assigned positive probability by every agent at date n.

The seven assumptions mentioned above actually specify the class of economic environments for which the existence theorem is established. These assumptions are stated precisely in the context of the mathematical model developed in [10] and the economic significance and interpretation of these assumptions is given there is some detail. We mention here only the most significant aspects of these assumptions with regard to the role of expectations in the existence result.

The seminal insight of Green's work is two-fold. First he shows in the context of his model [5: Theorem 2.1] that an agent's expected utility maximizing behavior is determinate only for initial date price systems in a particular open set of prices. With appropriate mathematical qualifications, these price systems can be interpreted as those whose relative futures price vectors are forecasted by the agent as possible future (second market date) spot prices. Green then demonstrates the existence of a temporary competitive price equilibrium for the initial market date under an assumption [5: (4.2)] that requires some agreement by agents regarding possible future spot prices. In addition he shows [5: Example 5.2] that such an equilibrium need not exist without this assumption of agreement. The problem that arises is that if agents' expectations of possible date two spot prices is sufficiently diverse, aggregate excess demand may be unbounded.

The counterpart of Green's result on the determinateness of individual agents' choice behavior in our model requires an additional

assumption on agents' expectations since such choice behavior extends over a sequence of successive dates with uncertainty present at each date. For each date and each price history up to that date, there is a particular open set of current price systems, having the same interpretation as in Green's model, such that the agent's current set of feasible actions is non-empty and compact if and only if the current price system determining this set of feasible actions is a member of this particular open set [10: Lemma (2.3.1) and Theorem (2.3.3)]. It follows that an agent's choice behavior at any market date is determinate if and only if at every date this particular open set of price systems (for the subsequent market date) has subjective probability one. This required new assumption is embodied in an indirect but technically convenient way in Assumption (A.3) (v) [10: p. 11] and (A.4) (ii) [10: p. 20].

Also as in Green's model, in order to ensure boundedness of aggregate excess demand at any market date, some agreement among agents regarding possible price systems at the next market date is required. The actual assumption, (A.7) [10: p. 88], is somewhat technical in form but has the same effect as Green's assumption [5: (4.2)].

"Sufficient agreement" regarding possible future market prices, however, is not sufficient to avoid the problem of bankruptcy. Ironically, rather than agreement of expectations, avoidance of bankruptcy requires a sufficient amount of diversity of agents' expectations. The potential for bankruptcy arises say at date n if there exists a price system and an agent such that the agent's action at date n is determined at this price system, but where the value at this price system of the agent's date n

endowment augmented by futures contracts made at date n-1 is zero. It follows that every neighborhood of such a price system contains price systems at which this agent would be bankrupt.

It also follows from the definition of the open set of prices at which this agent's date n set of feasible actions is non-empty and compact [10: (2.3.1)] that this agent is of the opinion that the relative prices expressed by the vector of futures prices of the given price system are possible date n+1 relative spot prices. In order to avoid such a price system materializing as a candidate for an equilibrium at date n, it is necessary to assume that there is some other agent with the opposite opinion, i.e., there is another agent who does not view these relative prices as possible date n+1 relative spot prices. It then follows from (A.4) (ii) and [10: Lemma (4.3.5)] that this second agent's demand for futures contracts is unbounded at this price system.

The formal assumption needed is stated as (A.6) (ii) [10: p. 86]. This precise characterization of the problem of bankruptcy in terms of the diversity or lack thereof of agents' expectations together with the formal extension of Green's insight to a sequential trading model constitute the major contribution of the research completed to date. With respect to the initial task of this research outlined in Section I above, there remains an important unresolved issue in Paper 3 that is discussed in the following section. Paper 3 was presented at the recent Winter Meeting of the Econometric Society, New York City, December 28-30, 1977, by Professor David C. Nachman. A copy of Paper 3 is included with this report though it is subject to revision and is not for general circulation.

III. COMMENTS

Despite the precise mathematical characterization of the problem of bankruptcy given in Paper 3, the model developed there does not completely accomplish the first task of the research program undertaken with this grant. This task is to specify economic environments, or economies for short, in the context of the model of Paper 3 in which the existence of a sequence of temporary competitive price equilibria can be demonstrated. In this regard, the existence theorem [10: Theorem (5.2.2)] should be read as follows: if there exist specifications of agents' endowments, preferences, and opinions (probabilistic expectations) that satisfy (A.1) - (A.7), then the resulting economy has an equilibrium path. The assumptions (A.1) - (A.7) give mathematical properties, with economic interpretation or justification, of the mathematical objects that are the models for agents' characteristics (endowments, preferences, opinions) that are sufficient to ensure the existence of an equilibrium path for an economy whose agents have these characteristics. There is no guarantee, however, that there exists a collection of such mathematical objects that have all the required mathematical properties. Indeed, no example has been constructed to date. This raises the issue that the domain of the existence theorem may in fact be empty.

The heart of the problem does not lie in the requirements made on agents' endowments and preferences. Although as we indicate below the assumption (A.1) on agents' endowments is unrealistic, the assumptions (A.1), (A.2), and (A.5) are more or less standard kinds of assumptions

in general equilibrium theory and pose no real problem with regard to specification of an example. The heart of the problem lies in the assumptions (A.3), (A.4), (A.6), and (A.7) made on individual agents' opinions and on the collection of all agents' opinions. Each of these assumptions have both technical justification and economic interpretation as indicated in Section II above. Owing to the sequential nature of the model and the consequent sequential nature of these assumptions, however, they are extremely complicated and their relationships complex and therefore difficult to verify. In addition, there is some reason to believe, because of the opposite demands made by assumptions (A.6) and (A.7), that these assumptions may prove to be logically inconsistent.

Indeed, this issue of logical consistency of the assumptions of the model in Paper 3 is at the very root of the incompatibility of the concepts of competitive price equilibrium and bankruptcy. It is clear that this issue must be resolved before further research can be done. A proposal by Professor Nachman is being made to the National Science Foundation for renewed support in resolving this issue and in pursuing the economic implications of the resolution.

If the resolution is positive, i.e., the assumptions of Paper 3 prove to be logically consistent, the remaining tasks of the research program include a study of the necessity of merging of opinions as a long run economic equilibrium concept both in the context of the model in Paper 3 and a slightly more realistic version in which certainty of endowment sequences (assumption (A.1)) is replaced with an exogeneously given stochastic process of endowments with predictable semipositive lower bounds.

This later adjustment in the basic model also permits the study of fulfilled expectations or rational expectations equilibrium, a more realistic representation of Hichs' additional requirement for equilibrium over time than the appropriate but weak sense of fulfilled expectations in Paper 3. This work should lead ultimately to a study of expectation formation mechanisms which yield, in an adaptive sense, equilibrium or fulfilled expectations. A crucial element in the modeling of any such mechanism is the institutional arrangements that condition this adaptive behavior.

If the assumptions are logically inconsistent, the necessity of these assumptions with regard to the existence of equilibrium paths will have to be explored and the extent of the inconsistency in similar sequential trading models where the problem of bankruptcy exists, e.g., in models with money and personalized debt and in the adjusted model of Paper 3 mentioned above, will have to be determined. After the extent of the inconsistency is determined, work should begin directly on the modeling and study of economies with institutional arrangements for handling bankruptcies.

David C. Nachman
Principal Investigator

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