Date: April 5, 1979

Project Title: Classical and Quantum Theory of Turbulent Behavior In Nonlinear Systems

Project No: G-41-677

Project Director: Dr. Joseph B. Ford

Sponsor: National Science Foundation; Washington, D. C. 20550

Agreement Period: From 4/1/79 Until 9/30/79

Includes 6 month flexibility period

Type Agreement: Grant No. DMR-7827584

Amount: $25,400 NSF Funds (G-41-677)
8,470 GIT Contribution (G-41-321)
$33,870 Total

Reports Required: Annual Progress Reports; Final Project Report

Sponsor Contact Person(s):

Technical Matters

Dr. John W. D. Connolly, Associate Director—Solid State Physics Program
Division of Materials Research
National Science Foundation
1800 G Street, N.W.
Washington, D. C. 20550

Phone: (202) 632-7334

Contractual Matters (thru OCA)

Ms. Mary Frances O'Connell
Grants Specialist - Area 4
National Science Foundation
1800 G Street, N. W.
Washington, D. C. 20550

Phone: (202) 632-2858

Defense Priority Rating: n/a

Assigned to: Physics (School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
Dean/Director—EES
Accounting Office
Procurement Office
Security Coordinator (OCA)
Reports Coordinator (OCA)

Library, Technical Reports Section
EES Information Office
EES Reports & Procedures
Project File (OCA)
Project Code (GTRI)
Other
SPONSORED PROJECT TERMINATION SHEET

Date 8/16/83

Project Title: Classical & Quantum Theory of Turbulent Behavior in Nonlinear Systems

Project No: G-41-677

Project Director: Dr. J. B. Ford

Sponsor: National Science Foundation

Effective Termination Date: 9/30/82

Clearance of Accounting Charges: 9/30/82

Grant/Contract Closeout Actions Remaining:

☐ Final Invoice and Closing Documents
☒ Final Fiscal Report Accounting (FCTR)
☒ Final Report of Inventions (only if positive)
☐ Govt. Property Inventory & Related Certificate
☐ Classified Material Certificate
☐ Other ____________________________

Assigned to: Physics ____________________________ (School/Laboratory)

COPIES TO:

Administrative Coordinator
Research Property Management
Accounting
Procurement/EES Supply Services
Research Security Services
Reports Coordinator (OCA)
Legal Services (OCA)
Library
EES Public Relations (2)
Computer Input
Project File
Other Ford

FORM OCA 10-781
PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION
Cover Page

FOR CONSIDERATION BY NSF ORGANIZATIONAL UNIT
Condensed Matter Theory Program
Division of Materials Research

IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY? Yes X No

NAME OF SUBMITTING ORGANIZATION TO WHICH AWARD SHOULD BE MADE (INCLUDE BRANCH/CAMPUS/OTHER COMPONENTS)
Georgia Institute of Technology

ADDRESS OF ORGANIZATION (INCLUDE ZIP CODE)
Atlanta, Georgia 30332

TITLE OF PROPOSED PROJECT
Classical and Quantum Theory of Turbulent Behavior in Nonlinear Systems

REQUESTED AMOUNT
$28,000

PROPOSED DURATION
One Year

DESIRED STARTING DATE
April 1, 1981

PI/PD NAME
Joseph Ford

ADDITIONAL PI/PD

ADDITIONAL PI/PD

FOR RENEWAL OR CONTINUING AWARD REQUEST, LIST PREVIOUS AWARD NO.
DMR-7827504

IF SUBMITTING ORGANIZATION IS A SMALL BUSINESS CONCERN, CHECK HERE

CHECK APPROPRIATE BOXES IF THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW:

☐ Animal Welfare
☐ Human Subjects
☐ National Environmental Policy Act

☐ Endangered Species
☐ Marine Mammal Protection
☐ Research Involving Recombinant DNA Molecules

☐ Historical Sites
☐ Pollution Control
☐ Proprietary and Privileged Information

PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR
NAME
Joseph Ford

AUTHORIZED ORGANIZATIONAL REP.
NAME
Dwight L. Allen

DATE
December 11, 1980

TITLE
Regent's Professor

DATE
December 11, 1980
I. PERSONNEL

In addition to the Principal Investigator, this research involved the post-doctoral student, Dr. Franco Vivaldi, and the foreign research collaborators, Professor Giulio Casati (University of Milan, Italy) and Professor Boris V. Chirikov (Novosibirsk University, USSR).

II. ACTIVITIES AND PAPERS

The Principal Investigator presented invited talks at the University of Virginia and the University of Missouri-Rolla. He also presented a series of invited lectures during a two week period in April as part of the Ergodic Year Program held by the Mathematics Department, University of Maryland. He was scheduled to visit Professor Chirikov in Novosibirsk for the month of August under a DOE-Soviet AEC agreement, followed by a two week visit with Professor Casati in Milan; however, this whole trip was cancelled when Moscow refused visa clearance for unknown reasons. Fortunately, Dr. Vivaldi, after spending the month of August in Milan with Professor Casati, was permitted to enter the USSR under an Italy-USSR agreement and to spend September and October in Novosibirsk with Professor Chirikov. Finally, during the spring of 1981, the Principal Investigator will present invited lectures at the

The following papers either have appeared or will appear:


III. BRIEF DESCRIPTION OF THE RESEARCH

Since the inception of this Grant (1/1/79), we have sought a detailed understanding of chaos (turbulence) in the well-known Duffing equation

\[ \dddot{Y} + b\ddot{Y} + \omega_0^2 Y = -RY^3 + D \cos \omega t \quad (1) \]
in the belief that the chaotic behavior of this simple system is characteristic of almost all driven, nonlinear oscillators. Georg Haubs, whose M.S. thesis was mentioned in the previous Progress Report for the period 4/1/79-12/31/79, discovered the existence of a strange attractor for Eq. (1); but Huberman and Crutchfield, Phys. Rev. Lett. 43, 1743 (3 Dec 1979), published first. In seeking to reproduce the Huberman-Crutchfield calculations, Franco Vivaldi discovered the existence of two neighboring strange attractors rather than just one. All the above attractors arise as the endpoint of some period doubling sequence of periodic orbits which appear as some system parameter is varied and, moreover, these sequences have precisely the properties predicted by the Feigenbaum theory of universality for dissipative systems. As Vivaldi continued his investigation, he noted that period doubling to chaos continued to exist as he reduced the damping parameter $b$ in Eq. (1) toward zero. He then set $b = 0$ and, as anticipated, found period doubling to chaos to persist, although now the endpoint is a non-attracting, chaotic point set rather than a strange attractor. In addition, he found that the Feigenbaum theory appears to apply even to the conservative, undamped, $b = 0$ system except that the values of certain numerical ratios are different. As a consequence, one suspects that period doubling and universality play a significant role in the appearance of chaos (turbulence) in a broad spectrum of systems. These results will be published in Paper # 4 listed above.
In the midst of these calculations, we learned that a number of other investigators were obtaining similar results for various nonlinear systems. We then organized a special, informal meeting during the spring of 1980 to coordinate the efforts of all. Vivaldi has been especially active in this collaboration and some of the research results will appear in Paper # 5. Of these results, perhaps the most significant is extension and generalization of Feigenbaum's original theory to cover conservative systems. Additional results are being published by several investigators as individual papers.

At the December, 1979, meeting on Nonlinear Dynamics in New York, Jurgen Moser requested Franco Vivaldi to perform certain computer studies of stability for a special area-preserving mapping with the intent to publish a joint paper co-authored by Moser and Vivaldi. This computer study has now been completed and forwarded to Moser. The final work will appear as Paper # 6 above.

The Principal Investigator continues to collaborate with Professor Chirikov (Novosibirsk) on the stability of relativistic particle beams and with Professor Casati (Milano) on the study of quantum chaos in simple billiard models. In addition, he continues to investigate certain significant but quite "far-out" questions whose answers appear to lie tantalizingly close to the reach of contemporary nonlinear dynamics. A brief discussion of these questions follows.

The fact that most classical many-body systems can now be proven to obey statistical mechanics and thermodynamics only emphasizes the
well-known fact that integrable systems obey neither. Thus, can we not devise an integrable system which cyclically absorbs energy in the form of heat and delivers this energy in the form of work, thus violating Kelvin's statement of the second law? Certainly this possibility is a "long-shot," but, nonetheless, armed with the new tools supplied by nonlinear dynamics, for the first time we know how to attack the problem. The search for a suitable integrable system is being conducted in collaboration with the low temperature experimentalist, Professor Bascom Deaver of the University of Virginia.

Newtonian mechanics is generally regarded as deterministic, not random. But the orbits of a chaotic Newtonian system are rigorously known to exhibit observable random properties. Thus, there is a very real, if little recognized, crisis or paradox in Newtonian mechanics; namely, is it random or deterministic in character? In seeking to resolve this paradox, one discovers that chaotic Newtonian systems can be regarded as deterministic if and only if one possesses truly infinite observational and computational precision; finite precision no matter how great is not enough. Since truly infinite accuracy is impossible, one concludes that Newtonian mechanics is random in some sense. If the randomness is not to remain forever arbitrary (depending on the accuracy of currently available instruments), then there must exist a new "uncertainty principle" which provides the ultimate upper limit to observational and/or computational precision. We are currently seeking to uncover this new "uncertainty principle" and to explore the multitudinous avenues connected to the random-deterministic paradox. A fuller
IV. CURRENT SUPPORT

In addition to the research support provided by the present NSF Grant DMR-7828584, Dr. Ford's research group will soon also be supported in part by the Department of Energy Contract No. DE-AS05-81ER40003.A000 (final contract arrangements are now being negotiated). This is a one year contract with a starting date of November 1, 1980, with a total budget of $52,000, and with the title "Numerical Studies of Arnol’d Diffusion in the Beam-Beam Interaction of Intersecting Storage Rings." As stated in the proposal to D.O.E., Dr. Franco Vivaldi will assume primary and full time responsibility for the performance of the proposed research for D.O.E. Dr. Ford will serve primarily as overseer for this research; one month's salary for Dr. Ford is included in the D.O.E. contract budget.

No further research support is pending or being sought.
## Third Year - Continuing Grant

### SUMMARY

**PROPOSAL BUDGET**

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<th>DURATION (MONTHS)</th>
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**AWARD NO.**

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<tr>
<th>FUND REQUESTED BY PROPOSER</th>
<th>FUNDS GRANTED BY NSF (IF DIFFERENT)</th>
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<td>$8,787</td>
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**ORGANIZATION**

Georgia Institute of Technology, Atlanta, Georgia 30332

**PRINCIPAL INVESTIGATOR/PROJECT DIRECTOR**

Joseph Ford

**Seniors, Co-PI's, Faculty and Other Senior Associates**

<table>
<thead>
<tr>
<th>Name</th>
<th>Title</th>
<th>Number</th>
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<tbody>
<tr>
<td>Joseph Ford</td>
<td>Regents' Professor</td>
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</table>

**Project Budget**

- **Total Senior Personnel (1-5)**: $8,787
- **Fringe Benefits (if charged as direct costs)** at 11.11% of Item A1: $976
- **Total Salaries, Wages and Fringe Benefits (A+B+C)**: $15,763
- **Other Personnel**
  - **Post Doctoral Associates**: $878
  - **Other Professionals (Technician, Programmer, etc.)**: $878
  - **Graduate Students**: Academic year and Summer: $6,000

**PERMANENT EQUIPMENT**

- **Total Permanent Equipment**: $11,763

**TRAVEL**

- **Domestic (incl. Canada and U.S. possessions)**: $600
- **Foreign**: $1,100

**PARTICIPANT SUPPORT COSTS**

- **Stipends**: $2,500
- **Travel**: $620
- **Subsistence**: $1,200
- **Other**: $1,000

**TOTAL PARTICIPANT COSTS**: $6,320

**OTHER DIRECT COSTS**

- **Materials and Supplies**: $1,200
- **Publication Costs/Page Charges**: $500
- **Consultant Services**: Boris Chirikov (USSR) 31 days at $20/day: $620
- **Computer (ADPE) Services**: $878
- **Subcontracts**: $878
- **Other**: $1,000

**TOTAL OTHER DIRECT COSTS**: $4,096

**TOTAL DIRECT COSTS (A THROUGH G)**: $25,774

**INDIRECT COSTS**

- **73% of Total Salaries and Wages through 6/30/81**: $2,699
- **75% of Total Salaries and Wages from 7/1/81**: $8,318

**TOTAL INDIRECT COSTS**: $11,017

**RESIDUAL FUNDS**

None

**AMOUNT OF THIS REQUEST (J) OR (J MINUS K)**: $28,000

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**FOR NSF USE ONLY**

**INDIRECT COST RATE VERIFICATION**

**Rate Sheet**

Initials: UGC

**INST. REP. TYPED NAME & SIGNATURE**

Joseph Ford

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PART I—PROJECT IDENTIFICATION INFORMATION

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PART II—SUMMARY OF COMPLETED PROJECT (FOR PUBLIC USE)

Despite the determinism which pervades the currently accepted theories in science, almost all natural phenomena exhibit randomness at every level from the turbulent flow of a mountain stream to the Brownian motion of a microscopic particle. The research here outlined has sought, on a practical level, to understand, to explain, and to illustrate the mechanisms whereby deterministic physical systems can yield turbulently random behavior, and simultaneously, at a deeper philosophical level, it has sought to fully resolve the logical contradiction in the term "determinate randomness." Over the past three years, this research has shown that determinate chaos is the underlying cause of diffusive energy flow in the form of heat, and it has revealed that weak chaos is a dangerous instability for high energy, colliding beam machines (p-p or p-3) now under design or construction around the world. On the mathematical side, this research has exposed several new "routes to turbulence" in model systems. Finally, at the deepest theoretical level, this research has used algorithmic complexity theory to resolve the centuries old random-determinate paradox, and, in so doing, it has uncovered ideas which point toward the need for a third revolution in physics as pervasive and extensive as the earlier revolutions of relativity and quantum mechanics.

PART III—TECHNICAL INFORMATION (FOR PROGRAM MANAGEMENT USES)

<table>
<thead>
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<th>NONE</th>
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<th>PREVIOUSLY FURNISHED SEPARATELY TO PROGRAM</th>
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<tr>
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<tr>
<td>b. Publication Citations</td>
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<tr>
<td>c. Data on Scientific Collaborators</td>
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<tr>
<td>d. Information on Inventions</td>
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<td>e. Technical Description of Project and Results</td>
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<tr>
<td>f. Other (specify)</td>
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</tbody>
</table>

2. Principal Investigator/Project Director Name (Typed)

Joseph Ford

4. Date

11/21/83
I. PERSONNEL

In addition to the Principal Investigator, this research involved the graduate students David Vogel, Georg Haubs, Ulrich Steiger, and Bruno Eckhardt, the post-doctoral student, Franco Vivaldi, and the visiting professors Dr. G. H. Walker (University of Tennessee at Chattanooga), Dr. Giulio Casati (University of Milano), Dr. Boris V. Chirikov (Institute of Nuclear Physics, Novosibirsk), and Dr. Nobu Saito (Waseda University, Tokyo).

II. ACTIVITIES AND PAPERS

During this three year period, the Principal Investigator has on average participated as an invited speaker at five conferences/year and presented seminar talks at universities in the US and abroad at an average rate of about eight talks/year. He has served as co-organizer of two distinct conferences to be held in June of 1983. During this period he has served as an editor of Physica D: Nonlinear Phenomena and on the editorial board of the Journal of Mathematical Physics.

Copies of the following papers are supplied herewith:

Abstracts for the Master of Science Theses of Georg Haubs and Bruno Eckhardt.


The following papers are at various stages of the publication process. Copies will be forwarded to NSF at a later date.


III. TECHNICAL DESCRIPTION OF PROJECT AND RESULTS

Earlier Progress Reports in concert with the above summary of Part II (Form 98A) and the enclosed reprints describe and summarize our scientific results of the past three years quite adequately. Here we need only briefly outline the contents of the four papers yet to be published. Reference 11 will, at long delayed last, firmly establish that the Fourier heat law is valid for chaotic dynamical systems but not otherwise. Reference 12 establishes that the Feigenbaum period doubling to chaos observed for a mapping system in Ref. 8 also occurs in differential equation systems. Reference 13 studies the character of stability (or its lack) in the solar system using an exotic mapping system. Finally, as mentioned in Part II (form 98A)
above, we have resolved the random-determinate paradox. Since we now know what random is, we also know what not-random is. As a consequence, in Ref. 14 we expose a broad new class of dynamical systems which have meaningful analytic solutions but which are not integrable in the historically accepted sense.