DESIGN AND IMPLEMENTATION OF A SOFTWARE TOOL
FOR DAY-AHEAD AND REAL TIME ELECTRICITY GRID OPTIMAL MANAGEMENT
AT THE RESIDENTIAL LEVEL FROM A CUSTOMER’S PERSPECTIVE

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by

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SUMMARY

This thesis focuses on the design and implementation of a software tool able to achieve electricity grid optimal management in a dynamic pricing environment, at the residential level, and from a customer's perspective.

The main drivers encouraging a development of energy management at the home level are analyzed, and a system architecture modeling power, thermodynamic and economic subsystems is proposed. The user behavior is also considered.

A mathematical formulation of the related energy management optimization problem is proposed based on the linear programming theory.

Several cases involving controllable and non-controllable domestic loads as well as renewable energy sources are presented and simulation scenarios illustrate the proposed optimization strategy in each case.

The performance of the controller and the changes in energy use are analyzed, and ideas for possible future work are discussed.
A home grid can be defined as a small-scale energy system operating at the home level.

Today, home grids are essentially made up of loads which are most of the time manually controlled: lights, TV, washing-machine, etc. Additionally, electricity tariffs for residential customers are in most cases constant, or piecewise constant.

The future penetration of distributed generation, energy storage, and controllable loads located downstream of the meter, along with the likely development of dynamic pricing policies, will make energy management a more complex optimization problem which may be very difficult for the common user to solve. Residential customers will need the assistance of management software to optimize their consumption and react to varying price and incentive signals in order to minimize their electricity bill.

This thesis proposes an optimization strategy to achieve electricity grid optimal management at the residential level, from a customer's perspective. This strategy involves the modeling of power, thermodynamic and economic systems, as well as user behavior modeling, and can operate in both day-ahead and real-time, acting as the brain of the home grid.
The benefits of the proposed optimization strategy are:

- to help the residential customer minimize his electricity costs by creating a tool which can achieve home grid optimal management.
- to help utilities reduce the peak demand which results in energy and cost savings by enabling the residential customers to respond to day-ahead price signals and real-time consumption reduction requests.
- to motivate the deployment of smart grid technologies at the residential level, including distributed generation and storage, intelligent appliances, and smart meters enabling dynamic-pricing, and to enable the optimal management of these technologies.
- to propose a model for residential customer behavior which could be included in future multi-agent based simulations.

The rest of the thesis is organized as follows. Chapter 2 gives an overview of the main drivers encouraging a development of home energy management in the coming years. Chapter 3 proposes a modeling for the home grid system. Chapter 4 and 5 discuss the mathematical formulation of the optimization problem related to home energy management in two particular cases, and chapter 6 gives the mathematical formulation in the general case. In chapter 7, we extend the general case to a scenario involving several utilities, and in chapter 8 we specifically discuss real-time response. Finally, chapter 9 provides the conclusions of the present work and describes possible future work.
CHAPTER 2
ORIGIN AND HISTORY OF THE PROBLEM

2.1 - All-powerful utility vs. unresponsive customer

Since the two first electrical networks opened simultaneously in New York, USA and Bellegarde, France, in the Fall of 1882, electricity has become universally used at the commercial, industrial and residential levels. Today, electricity is certainly the most important energy carrier for human activity. Electrical energy can be produced from a wide variety of primary sources, including fossil, nuclear and renewable, and can satisfy multiple energy needs thanks to a distribution network relatively dense, efficient and reliable.

The immediate availability of electricity to the customer is considered in developed countries as part of our lifestyle, along with water availability everywhere and at anytime. However, contrary to water for instance, electrical energy storage is still difficult to achieve. Consequently, total generation must equal total demand at all times.

To facilitate the fulfillment of the above constraint along with the other capability limitations of the network, and due to the economies of scale of the electricity industry, electrical power companies have historically owned the whole infrastructure from generating units to transmission and distribution infrastructure, leading to natural monopoly situations.

In the early 1990s, various deregulation processes started reshaping the electricity markets separating the generation and retail functions from the transmission and distribution functions. Competition was progressively introduced at the generation level,
establishing wholesale electricity markets where generation companies can sell their production to retailers. The transmission and distribution functions remained natural monopoly functions.

The ultimate form of competitive electricity market requires an additional market at the retail level where consumers can buy electricity from a pool of retailers. Retail markets have been established in some places, while the retail function remains a monopoly in some others.

Consumers are in theory able to consume electrical energy whenever they wish, the local utility being responsible to make sure that generation equals demand plus losses at all times, regardless of the amount of energy the customers are demanding. This is achieved through complex load forecasting estimations combined with frequency regulation using different types of resources:

- Online reactive reserves with automated controls that can respond immediately to maintain voltages within the required range.
- Online power sources with automated control that can respond instantly to minute-to-minute load fluctuations (regulation) and to major demand spikes (spinning reserves)
- Standby power sources with manual controls that can respond to demand spikes in 10 to 30 minutes.

In practice, situations where the demand starts exceeding the generation capacity may lead to power outage.
In return, utilities have a strong market power, especially when a retail market is not established, and are able to name their prices for electricity based on a regulatory process.

### 2.2 - Electricity pricing policies

Our human life cycle is divided into periods of activity and inactivity and this directly affects our energy needs. Consequently, the electricity demand curve as a function of time is not flat, but presents cyclic variations: the demand is low during the night, higher during the day, and very high in the evening.

These variations have an influence on the electricity prices which include the generation costs, and the utility margins. Indeed, the cost of generating electricity significantly increases during peak-demand periods because generating units used during such periods are more expensive to operate. Thus, the marginal cost varies during the day.

Along with the human life cycle, other factors may affect the electricity demand and prices: weather, season, public holiday, etc.

Today, most of the customers are not exposed to price variations because most of them have standard flat rate contracts with their utility. These contracts reflect the average price of electricity throughout the day. In some cases, electricity prices vary depending on the season.

Such static pricing policy does not prevent utilities from making profits, but make difficult to manage the demand during peak-demand periods, when energy-supply systems are constrained. Indeed, customers adjust their demand for a given type of good to the price
variations of this good. In the case of electricity, they have no reason to modify their demand of energy because the price variations of electricity do not affect them in the short term.

2.3 - Existing demand side management programs and their limitations

Following the 1973 oil crisis, governments of many countries have encouraged the development of energy demand management programs. Demand side management consists of a set of actions that influence the quantity and/or demand curve profiles of energy consumed by end-users. Such actions particularly target reduction of demand during peak periods.

Demand side management programs require implementing a communication layer on top of the electrical layer. This enables the utility to send signals in advance or in real time to the end-users in order to influence their energy consumption.

Most of the existing demand response programs have been oriented towards large industrial or commercial customers, and are essentially load management based. Two main types of programs are in use, time-based and incentive-based programs:

- In time-based programs such as Time-Of-Use (TOU), Real-Time Pricing (RTP) and Critical Peak Pricing (CPP), the electricity price changes for different periods according to the electricity supply cost.
- In incentive-based programs such as Direct Load Control (DLC), Interruptible/Curtailable (I/C) Service or Demand Bidding, customers may receive incentives if they are willing to provide load reductions.
Some utilities also run demand management programs oriented towards residential customers. *Electricité de France* for instance, one of the world’s largest utility companies, runs two programs based on TOU and CPP in addition to its standard flat rate contract to encourage its customers to shift loads to off-peak periods.

- The day/night TOU-based program [1] proposes to customers above 6kVA a differential pricing between the day and night periods, with a fixed rate during the day (6am to 10pm) above the standard tariff, and a fixed rate during the night (10pm to 6am) below the normal tariff.

- The *Tempo* program [2] combines the previous day/night pattern with a CPP-based pattern. The days of the year are divided into three groups: blue (300 days), white (43 days) and red (22 days). Each category corresponds to a given set of rates for the day and night periods, blue days corresponding to economical rates, white days to average rates and red days to surcharged rates. The color of the next day is released online each day at 5pm.

However, the response of residential customers to demand management programs is usually very limited. In some places, residential customers simply cannot respond because they are only offered standard flat rate contracts. Additionally, the communication layer is most of the time inadequate, or simply does not exist, which prevents the utility to broadcast price signals to its customers, and to record their response.

Finally, when a residential demand management program is implemented, the decision process on the customer side is rarely automated, except in some cases for the heating system, which limits the customer’s ability to respond.
2.4 - Consequences of the emerging smart grid technologies on the utility-customer relation

In its *Smart Grid System Report* published in July 2009 [3], the U.S. Department of Energy underlines that, in the smart grid environment, consumers will become an integral part of the electric power system:

« They help balance supply and demand and ensure reliability by modifying the way they use and purchase electricity. These modifications come as a result of consumers having choices that motivate different purchasing patterns and behavior. These choices involve new technologies, new information about their electricity use, and new forms of electricity pricing and incentives. »

Among the new smart grid technologies, several will have a direct impact at the residential level:

- *Distributed generation* - solar, wind, biofuel - enabling the customer to produce power. This will lead to bi-directional energy flows between the home grid and the utility grid.
- *Distributed storage*, enabling the customer to choose when to sell the energy he has in stock.
- *EV/PHEV vehicles* plugged-in to the home grid with power transferred back and forth.
- *Intelligent appliances*, able to communicate and coordinate with a local controller at the house level.
- *Advanced meters*, enabling bi-directional communication with the smart grid.
These technologies will change the utility-customer relation by enabling the residential customer to eventually become a *prosumer*, not only consuming but also producing power while interacting with the grid in a dynamic pricing environment. Prosumers will play an active role in the balancing process between supply and demand along with the utility, and more generally in adjusting operations based on grid conditions.

2.5 - **Necessity of a local controller at the residential level**

It has to be noted that some of the smart grid technologies have been around for a while. Differential pricing was first introduced by Landis & Gyr, a Swiss electricity meter manufacturer, which designed the first double tariff meter in 1903. Similarly, research has been conducted on electric vehicle or distributed generation since the 18th century: Gustave Trouvé demonstrated a working three-wheeled automobile in 1881, and Charles Fritts created the first working solar cell in 1883.

The challenge today, from the customer's point of view, is to be able to *optimize* the use of these existing technologies, as well as all the emerging technologies which are to be deployed in the coming years.

In a smart grid environment, energy management at the residential level involves making economic choices based not only on the variable cost of electricity and the ability to shift load, but also on the ability to store or sell energy. And these choices have to integrate the customer's preferences in terms of comfort.

In that context, the optimal control of the home grid cannot be achieved manually requiring the assistance of a local controller, possibly embedded in the advanced meter.
This controller should be able to:

- integrate the customer’s preferences, and the parameters describing the home grid
- analyze price signals broadcasted by utilities, and external factors such as weather forecasts
- make decisions to optimize the consumer’s energy use in order to minimize the energy-related costs
- report these decisions to the customer, and to the various elements plugged-in to the home grid
- be flexible to fit a wide range of home grid configurations, and easy to use for the customer

The objective of this thesis is to propose an optimization strategy to successfully implement such controller.
3.1 - Introduction
The proposed controller achieves home grid optimal management in a smart grid environment from a customer’s perspective. In this context, achieving optimal management means to minimize the customer’s electricity bill while meeting the customer’s needs and meeting the constraints of the system.

The originality of this thesis is to encompass all the smart grids elements mentioned above - distributed generation, storage and smart appliances - as well as the customer’s comfort preferences while being able to respond to dynamic price signals in both day-ahead and real-time situations.

3.2 - Day-Ahead Optimization
At the day-ahead level, this thesis recognizes the importance of the ability to shift the time when electric energy is transferred from the utility grid to the home grid. This allows the customer to respond to day-ahead price signals by planning to buy energy when electricity cost is low.

The time shifting of this energy transfer can be achieved in two ways: first, by hooking up a storage device to the home grid in order to decouple the timing of generation and consumption of electric energy. Over the last decades, electricity storage has made progress along with the development of renewable energies such as solar or wind which generate energy intermittently. Additionally, EV/PHEV vehicles may also be used in the
future as storage resources during their vehicle-to-grid operation periods. The average vehicle is idle for 96% of the time [4] and usually parked at night at the customer's home allowing energy to be exchanged back and forth with the home grid.

In the following, we will assume the presence of a storage device connected to the home grid. The optimized management of this resource will be one of the degrees of freedom the customer will have to optimize his energy consumption.

The second way to enable energy transfer time shifting is load rescheduling. While with a storage system, timing of transfer to the grid and consumption are decoupled, with load rescheduling the energy transferred is used immediately and the storage system is not needed anymore.

In the following, we will assume the presence of reschedulable loads at the residential level, such as washing machine, dryer, dishwasher, refrigerator-freezer defrost, water-heater, heating and A/C, swimming pool heater and pump, etc. The clever scheduling of these reschedulable loads will be the second way for the customer to optimize his electricity consumption.

In conclusion, the customer will be able to shift the time when electric energy is transferred from the utility grid to his home grid by using both his storage system and load rescheduling.
3.3 - Real-Time Response

In the future smart grid environment, utilities will be able to broadcast every hour, every 15 minutes, or even in real-time signals to their residential customers. These signals will be either real-time price signals, or requests for electricity consumption reductions.

The later type of requests can happen during peak hours when the electrical grid is operating almost at its maximum generation capacity. In return, customers receive an incentive from their utility which is proportional to their energy consumption reduction. Customers have two degrees of freedom to respond to real-time requests from utilities: curtailment/interruption of loads and augmentation of local generation. At the residential level, the heating and air-conditioning are the two loads which can be interrupted instantly. Generation units based on solar and wind are not controllable, but the use of biofuel-burning micro-turbines can give another degree of freedom to the customer. The use of energy previously stored in a storage device connected to the home grid is another option.

The choice to respond or not to a request is based on costs, and consumer's preferences. For heating and air-conditioning, preferences are expressed in terms of temperature over a given period of time. For biofuel-burning micro-turbines, the fuel cost is compared to the incentive proposed by the utility.

3.4 - System Modeling

The electrical system considered in this thesis consists of a set of solar photovoltaic modules, a small wind turbine, a biofuel-burning micro-turbine, an energy storage system, and a set of controllable and non-controllable loads (Fig. 1). In this section we
describe how the various elements of the grid are modeled and integrated in the controller.

3.4.1- Solar Photovoltaic Modules

The power $P_i^{(s)}$ delivered by the photovoltaic modules over an interval of time $d_i$ can be expressed as a linear function of the solar irradiance $I_i$ over that interval, the total surface of the modules $S^{(s)}$, and the overall efficiency of the photovoltaic system $\eta^{(s)}$:

$$P_i^{(s)} = \eta^{(s)}S^{(s)}I_i$$

We will assume in the following that weather forecasts allow a day-ahead approximate estimation of solar irradiance and are available to the residential customer.

3.4.2- Small Wind Turbine

The power $P_i^{(w)}$ delivered by the wind turbine over an interval of time $d_i$ can be expressed, in first approximation, as a function of the wind speed $V_i$, the air density $\rho$, the equivalent surface of the wind turbine $S^{(w)}$, and the efficiency of the overall system $\eta^{(w)}$:

$$P_i^{(w)} = \eta^{(w)}\frac{1}{2}\rho S^{(w)}V_i^3$$

In reality, as soon as the wind speed reaches a specific limit -corresponding to the Reynold number (dimensionless) reaching 2500- the flow becomes turbulent, and non-linearities appear.
FIGURE 1: Home Grid – Electrical Grid Components
Although it is still difficult today to forecast with precision the wind speed because of its stochastic nature, we will assume in the following that weather forecasts allow a day-ahead approximate estimation of the wind speed and are available to the residential customer.

3.4.3- Biofuel-burning micro-turbine

The biofuel-burning micro-turbine is assumed to be fully controllable by the residential customer, delivering a power $P_{i}^{(bt)}$ over the time interval $a_i$.

In the following, we will assume that $P_{i}^{(bt)} = P_{nom}^{(bt)}$ when the turbine is turned on, with $P_{nom}^{(bt)}$ the nominal power of the micro turbine, and $P_{i}^{(bt)} = 0$ when the turbine is turned off. Also assume that $p^f$ is the cost to operate in c/kWh.

3.4.4- Energy Storage System

Energy storage systems behave differently whether they are charging or discharging and their characteristics may also vary as a function of the number of charge/discharge cycles they have experienced.

In this thesis, we will consider a simplified model in which the storage system behaves like a water tank. Then, the behavior of the system can be described over a period of time $[t, t + 1]$ by the initial amount of energy $E^{init}$ stored in that system at $t$, the final amount of energy stored at $t + 1$, and the maximum capacity $E^{max}$ of the system. However, our methodology allows these to be extended to more detailed models in future studies.
3.4.5- Controllable and Non-Controllable Loads

From an optimization point of view, the residential loads can be separated into two groups, the controllable loads, and the non-controllable loads.

The non-controllable loads include all the domestic loads the customer wants to be able to turn on and off immediately: lights, microwave, TV, etc. It is desirable to inform the customer when he should reduce his use of these loads in order to minimize his electricity bill. But integrating these loads into a fully automated optimization process would be irrelevant as residential customers want to keep their ability to turn on certain loads such as lights, watch TV, take a shower or use their toaster whenever they need, including peak-pricing time periods.

One the other side, the controllable loads include the dishwasher, the defrost mode of a freezer, the AC, or the water heater tank. The customer is more flexible with the controllable loads as long as they fill their role. For instance, as soon as hot water is available at anytime, the customer does not mind if the water tank turns on at midnight or at noon in order to heat water.

Finally, the controllable loads can also be divided up into two subcategories, the interruptible loads, and the non-interruptible loads. The interruptible loads such as heating or air conditioning systems can be interrupted immediately whereas non-interruptible loads such as a dishwasher or a washing machine are not designed to allow unexpected interruptions when their cycle has started.
3.4.6- **Grid Connection**

The system considered is connected to the utility electrical grid, and power can be transferred back and forth to the grid. The electrical connection point of the home grid to the utility grid constitutes the point of common coupling (PCC).

3.4.7- **Price signal, incentive signal**

Day-ahead price signals are to be broadcast to the local optimal controller directly through the smart meter, or alternatively through a web portal. Utility companies charge the customer when power is transferred from the grid to his premises. We will assume that utilities have obligation to buy the power the customer is willing to send back to the grid at anytime. Buying and selling electricity prices may differ. Practically, a net metering system determines what the resulting cost is for the customer.

It is also assumed that incentives signals can be broadcast in real time to encourage customers to reduce their energy consumption.
FIGURE 2: Home Grid - System Architecture
3.5- Mathematical Formulation

The energy $E$ stored in the storage system is the state variable which will allow us to describe the state of the system throughout the next day.

$E$ is a positive and continuous function defined on a period of time $T_E \subseteq \mathbb{R}^+$. The values of $E$ belongs to the interval $[E^{min}, E^{max}] \subseteq \mathbb{R}^+$. The storage system capacity determines the maximum amount of energy $E^{max}$ which can be stored in the system. The depth of discharge of the storage system determines the fraction of energy that can be withdrawn, hence the minimal amount of energy $E^{min}$ which has to remain in the storage system at all times.

If $E^{min} \neq 0$, the behavior of the storage system can be modeled by a virtual storage system described by the energy function $E_0$, with $E_0^{min} = 0$ and $E_0^{max} = E^{max} - E^{min}$. Consequently, we can assume from now on without any simplification that $E$ takes its values in $[0, E^{max}] \subseteq \mathbb{R}^+$.

The variations of $E$ must remain finite at all times as it is not physically possible for the storage system to exchange an infinitely great quantity of energy over an infinitely small time interval with the external environment. The physical characteristics of the storage system allow the definition of a maximal charging rate $R^c > 0$ and a maximal discharging rate $R^d > 0$.

The age and past history of the system, the number and frequency of charging and discharging cycles and the temperature may affect the characteristics and performance of the storage system. For simplicity, we will not consider these effects in the following,
and assume that $E^{max}, R^c, R^d$ remain constant over the temporal horizon during which the energy consumption is to be optimized.

In conclusion, four physical constraints apply to $E$:

\begin{align*}
  \forall t \in T_E, & \quad 0 \leq E(t) \quad (1) \\
  \forall t \in T_E, & \quad E(t) \leq E^{max} \quad (2) \\
  \forall t_1, t_2 \in T_E, & \quad E(t_2) - E(t_1) \leq R^c \quad (3) \\
  \forall t_1, t_2 \in T_E, & \quad -R^d \leq E(t_2) - E(t_1) \quad (4)
\end{align*}

Let $D$ be the period of time over which we want to optimize the home grid electricity consumption. In day ahead optimization $D$ usually corresponds to the next 24 hour period. Let us divide $D$ into $N$ adjacent time intervals $d_i$ of length $\tau_i$. Assume that the intervals have the same length such that \( \forall i, \tau_i = \tau \), and call $d = \{d_1, d_2, ..., d_i, ..., d_N\}$ a subdivision of $D$.

Assume for instance that $D$ is 24 hour long. Then, if $N = 24$, $D$ is divided into time intervals of $\tau = 60 \text{ min}$ each. $N = 48$ corresponds to intervals of $\tau = 30 \text{ min}$, $N = 72$ corresponds to intervals of $\tau = 20 \text{ min}$, $N = 96$ corresponds to intervals of $\tau = 15 \text{ min}$, etc.
For each interval $d_i$:

- $E_i^0$ is the amount of energy stored at the beginning of the interval
- $P_i^{(g)}$ is the power exchanged between the home grid and the utility grid with $P_i^{(g)} > 0$ when power is transferred from the utility grid to the home grid
- $P_i^{(bt)}$ is the power generated by the biofuel-burning turbine
- $P_i^{(s)}$ is the power generated by the photovoltaic modules
- $P_i^{(w)}$ is the power generated by the wind turbine
- $P_i^{(ct)}$ is the power consumed by the controllable loads
- $P_i^{(ncct)}$ is the power consumed by the non-controllable loads

The recursive relation between $E_{i+1}^0$ and $E_i^0$ is:

$$\begin{align*}
E_{i+1}^0 &= E_i^{\text{init}} \\
E_{i+1}^0 &= E_i^0 + (P_i^{(g)} + P_i^{(bt)} + P_i^{(s)} + P_i^{(w)} - P_i^{(ct)} - P_i^{(ncct)}).\tau
\end{align*}$$

The four physical constraints $\{(1), (2), (3), (4)\}$ must be verified.

$\{(1), (2)\}$ gives:

\[
\forall i \in [1, N],
-(P_i^{(g)} + P_i^{(bt)} - P_i^{(ct)}).\tau \leq E_i^0 + (P_i^{(s)} + P_i^{(w)} - P_i^{(ncct)}).\tau
\]

\[
\forall i \in [1, N],
(P_i^{(g)} + P_i^{(bt)} - P_i^{(ct)}).\tau \leq E_i^{\text{max}} - E_i^0 - (P_i^{(s)} + P_i^{(w)} - P_i^{(ncct)}).\tau
\]
Our objective now is to find the optimal controllable loads schedule—and biofuel burning turbine schedule—for the period of time $D$ so that the electricity cost is minimized.
4.1 - Introduction

We have divided the controllable loads into two groups: the non-interruptible loads (washing machine, dryer, dish washer, etc), and the interruptible loads (heater, A/C). Let us focus first on the non-interruptible loads in the absence of interruptible loads.

4.2 - Mathematical formulation as a linear programming problem

4.2.1 - Equality and inequality conditions

Assume that the cycle of the non-interruptible load \( L \) is a multiple of \( \tau \), say \( k_L \tau \). We must have \( k_L \leq N \).

Then there are \( S_L = N - k_L + 1 \) positions possible to schedule the load \( L \) over \( D \). Each position \( j \) corresponds to a binary number \( \delta_j^L \in \{0,1\} \). If the load \( L \) is scheduled in the position \( j \), \( \delta_j^L = 1 \), otherwise \( \delta_j^L = 0 \).

Example: for \( N = 6 \) and \( k_L = 2 \):

\[
\begin{array}{ccccc}
\text{Position 1} & & \text{Position 3} & & \text{Position 5} \\
\hline
\delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 \\
\hline
\text{Position 2} & & \text{Position 4} \\
\end{array}
\]

Let us assume that the load \( L \) is to be scheduled \( \gamma^L \) times over the next day. Then we must have:

\[
\sum_{j=1}^{S_L} \delta_j^L = \gamma^L \quad (11a)
\]
Additionally, we need to make sure that two cycles do not overlap.

We must have:

\[
\forall i \in [[1, S_L - (k_L - 1)]], \\
\begin{align*}
\begin{cases}
\delta^L_{i+1} + \delta^L_{i+2} \leq 1 \\
\delta^L_{i+2} + \delta^L_{i+3} \leq 1 \\
\vdots \\
\delta^L_{i+(k_L-2)} + \delta^L_{i+(k_L-1)} \leq 1 
\end{cases}
\end{align*}
\]

If \( i = S_L - (k_L - 2) \),

\[
\begin{align*}
\begin{cases}
\delta^L_{i+1} + \delta^L_{i+2} \leq 1 \\
\delta^L_{i+2} + \delta^L_{i+3} \leq 1 \\
\vdots \\
\delta^L_{i+(k_L-3)} + \delta^L_{i+(k_L-2)} \leq 1 
\end{cases}
\end{align*}
\]

If \( i = S_L - (k_L - 3) \),

\[
\begin{align*}
\begin{cases}
\delta^L_{i+1} + \delta^L_{i+2} \leq 1 \\
\delta^L_{i+2} + \delta^L_{i+3} \leq 1 \\
\vdots \\
\delta^L_{i+(k_L-4)} + \delta^L_{i+(k_L-3)} \leq 1 
\end{cases}
\end{align*}
\]

If \( i = S_L - (k_L - 1) = S_L - 1 \),

\[
\begin{align*}
\begin{cases}
\delta^L_{i+1} \leq 1 
\end{cases}
\end{align*}
\]

Let us consider \( \ell \) non-interruptible loads \( L_1, L_2, ..., L_\ell \) which are connected to the home grid. The power consumption of these loads is \( P^L_1, P^L_2, ..., P^L_\ell \). The objective now is to determine the contribution of these non-interruptible loads to the total controllable loads consumption, and to find the optimal fashion to schedule these loads.
Let us take \( q \in \mathbb{N} \). The parameter \( q \) allows us to refine the subdivision by dividing each time interval into \( q \) new intervals of same length \( \frac{T}{q} \). In this chapter we will keep \( q = 1 \). We will discuss this parameter more in depth in chapter 6.

Define \( p_{\text{nom}}^{bt} \) the nominal power of the biofuel burning turbine.

Define \( \forall j \in [1, N] \), \( \delta_j^{bt} \in [0, 1] \), with \( \delta_j^{bt} = 0 \) if the turbine is off over \( d_i \), and \( \delta_j^{bt} = 1 \) if the turbine is on over \( d_i \).

In the rest of this section, for the system of \( \ell \) non-interruptible loads considered, we express the corresponding inequality conditions \( \{ (\alpha), (\beta), (11b) \} \) and equality conditions \( (11a) \) in matrix form.

### 4.2.2 - Inequality conditions in matrix form

For a non-interruptible load \( L \), let the matrices \( M_{N,q}^{\alpha L} \), \( M_{N,q}^{\beta L} \), \( M_{SL}^{k_L-j} \) and \( M_{SL} \) be:
$M_{N,q}^{_{RL}}$ is a $((N - k_L)q + k_L) \times S_L$ matrix and $M_{N,q}^{_{RL}}$ is a $Nq \times S_L$ matrix.

$M_{S_L}$ is a $(k_L - 1)(S_L - k_L) / 2 \times S_L$ matrix.
Let also $T_N$ and $I_N$ be:

$$
T_N = \begin{bmatrix}
1 & 0 & \ldots & \ldots & 0 \\
1 & 1 & \backslash & \ldots & 0 \\
\vdots & \vdots & \backslash & \backslash & \vdots \\
1 & 1 & \backslash & \ldots & 0 \\
1 & 1 & \ldots & \ldots & 1 \\
\end{bmatrix} \quad \quad I_N = \begin{bmatrix}
1 & 0 & \ldots & \ldots & 0 \\
0 & 1 & \ldots & \ldots & 0 \\
\vdots & \vdots & \ldots & \ldots & \vdots \\
0 & 0 & \ldots & \ldots & 1 \\
\end{bmatrix} 
$$

$T_N$ and $I_N$ are $N \times N$ matrices.

Finally, define $A^{(I)}$, $B^{(I)}$ and $X^{(I)}$:

$$
A^{(I)} = \tau \begin{bmatrix}
-T_N & -p_{\text{bet}}^N T_N & p_{L_1}^N M_{N,1}^{a_{L_1}} & p_{L_2}^N M_{N,1}^{a_{L_2}} & \cdots & p_{L_i}^N M_{N,1}^{a_{L_i}} \\
T_N & p_{\text{bet}}^N T_N & -p_{L_1}^N M_{N,1}^{a_{L_1}} & -p_{L_2}^N M_{N,1}^{a_{L_2}} & \cdots & -p_{L_i}^N M_{N,1}^{a_{L_i}} \\
I_N & p_{\text{bet}}^N I_N & -p_{L_1}^N M_{N,1}^{b_{L_1}} & -p_{L_2}^N M_{N,1}^{b_{L_2}} & \cdots & -p_{L_i}^N M_{N,1}^{b_{L_i}} \\
-I_N & -p_{\text{bet}}^N I_N & p_{L_1}^N M_{N,1}^{b_{L_1}} & p_{L_2}^N M_{N,1}^{b_{L_2}} & \cdots & p_{L_i}^N M_{N,1}^{b_{L_i}} \\
\frac{1}{\tau} M_{S_{L_1}} & \frac{1}{\tau} M_{S_{L_2}} & \cdots & \frac{1}{\tau} M_{S_{L_{i}}}
\end{bmatrix}
$$

$$
B^{(I)} = \begin{bmatrix}
b_{a_1}^{(1)} \\
b_{a_2}^{(1)} \\
b_{\beta_1}^{(1)} \\
b_{\beta_2}^{(1)} \\
1_{p^{(g)} \cdot (s_{L_1} - 1) \cdot (s_{L_2} - 1)}
\end{bmatrix} \quad \quad X^{(I)} = \begin{bmatrix}
p_{(g)}^1 \\
p_{(g)}^2 \\
p_{(g)}^N \\
\frac{1}{\tau} M_{S_{L_1}} \\
\frac{1}{\tau} M_{S_{L_2}} \\
a_{L_1}^1 \\
a_{L_1}^2 \\
\vdots \\
a_{L_{i}}^1 \\
\vdots \\
a_{L_{i}}^2 \\
\vdots \\
a_{L_{i}}^N \\
\delta_{S_{L_1}} \\
\delta_{S_{L_2}} \\
\vdots \\
\delta_{S_{L_{i}}}
\end{bmatrix}
$$
With:

\[
\begin{align*}
 b_{\alpha,1} &= \begin{bmatrix}
 E^{\text{init}} + (p_{1}^{(s)} + p_{1}^{(w)}) \tau - p_{1}^{(ncl)} \tau \\
 \vdots \\
 E^{\text{init}} + \sum_{i=1}^{1<n<N} (p_{i}^{(s)} + p_{i}^{(w)}) \tau - \sum_{i=1}^{1<n<N} p_{i}^{(ncl)} \tau \\
 \vdots \\
 E^{\text{init}} + \sum_{i=1}^{N} (p_{i}^{(s)} + p_{i}^{(w)}) \tau - \sum_{i=1}^{N} p_{i}^{(ncl)} \tau 
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
 b_{\alpha,2} &= \begin{bmatrix}
 E^{\text{max}} - E^{\text{init}} - (p_{1}^{(s)} + p_{1}^{(w)}) \tau + p_{1}^{(ncl)} \tau \\
 \vdots \\
 E^{\text{max}} - E^{\text{init}} - \sum_{i=1}^{1<n<N} (p_{i}^{(s)} + p_{i}^{(w)}) \tau + \sum_{i=1}^{1<n<N} p_{i}^{(ncl)} \tau \\
 \vdots \\
 E^{\text{max}} - E^{\text{init}} - \sum_{i=1}^{N} (p_{i}^{(s)} + p_{i}^{(w)}) \tau + \sum_{i=1}^{N} p_{i}^{(ncl)} \tau 
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
 b_{\beta,1} &= \begin{bmatrix}
 R^{c} - (p_{1}^{(s)} + p_{1}^{(w)}) \tau + p_{1}^{(ncl)} \tau \\
 \vdots \\
 R^{c} - (p_{i}^{(s)} + p_{i}^{(w)}) \tau + p_{i}^{(ncl)} \tau \\
 R^{c} - (p_{N}^{(s)} + p_{N}^{(w)}) \tau + p_{N}^{(ncl)} \tau 
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
 b_{\beta,2} &= \begin{bmatrix}
 R^{d} + (p_{1}^{(s)} + p_{1}^{(w)}) \tau - p_{1}^{(ncl)} \tau \\
 \vdots \\
 R^{d} + (p_{i}^{(s)} + p_{i}^{(w)}) \tau - p_{i}^{(ncl)} \tau \\
 R^{d} + (p_{N}^{(s)} + p_{N}^{(w)}) \tau - p_{N}^{(ncl)} \tau 
\end{bmatrix}
\end{align*}
\]

Then:

\[
\{(\alpha), (\beta), (11b)\} \text{ is equivalent to } A^{(I)}X^{(I)} \leq B^{(I)}
\]

4.2.3- Equality conditions in matrix form

Let now \(A_{eq}^{(I)}\) and \(B_{eq}^{(I)}\) be:

\[
A_{eq}^{(I)} = \begin{bmatrix}
0 & \ldots & 0 & 1 & \ldots & 1 & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 1 & \ldots & 1 & \ldots & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\
0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 1 \\
0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 1 \\
\end{bmatrix}
\]

\[\ell\]
\[ B_{eq}^{(I)} = \begin{bmatrix} y^{L_1} \\ y^{L_2} \\ \vdots \\ y^{L_{\ell}} \end{bmatrix} \]

Then:

(11a) is equivalent to \( A_{eq}^{(I)} X^{(I)} = B_{eq}^{(I)} \)

4.2.4- Equivalent system in matrix form

Define \( \mathcal{P}^u = [p_1^u \ p_2^u \ \ldots \ p_{N-1}^u \ p_N^u]^T \) the electricity prices corresponding to the subdivision \( d \).

Define \( \mathcal{P}^f = p^f \mathbf{1}_{\mathbb{R}^N}^T \) the biofuel prices corresponding to the subdivision \( d \). Biofuel prices are supposed constant over \( D \) and equal to \( p^f \).

Finally define \( \mathcal{P}^{(I)} = [p^{uT} \ p^{fT} \ 0 \ \ldots \ 0]^T \)

\[ \begin{array}{c}
S_{L_1} + \ldots + S_{L_{\ell}}
\end{array} \]

The non-interruptible loads optimal schedule problem can then be expressed as the following mixed-integer linear programming problem:

\[
\begin{align*}
\text{Minimize} & \quad C^{(I)} = \mathcal{P}^{(I)}^T \cdot X^{(I)} \\
\text{Subject to} & \quad A^{(I)} X^{(I)} \leq B^{(I)} \\
& \quad A_{eq}^{(I)} X^{(I)} = B_{eq}^{(I)} \\
& \quad \forall i \in [1, N], \ \delta_{i}^{bt} \in [0,1] \\
& \quad \forall i \in [1, \ell], \ \forall j \in [1, S_{L_i}], \ \delta_{ij}^{ti} \in [0,1]
\end{align*}
\]
4.3 - Example

Example: for \( N = 4 \), \( \ell = 2 \), \( k_{L_1} = 2 \), \( k_{L_2} = 3 \), \( \gamma_{L_1} = 1 \), \( \gamma_{L_2} = 1 \)

\( S_{L_1} = N - k_{L_1} + 1 = 3 \), \( S_{L_2} = N - k_{L_2} + 1 = 2 \)

\[
\begin{array}{cccc}
\text{Load } L_1 & \delta_1^{L_1} & \delta_2^{L_1} & \delta_3^{L_1} \\
 & d_1 & d_2 & d_3 & d_4 \\
\text{Position 1} & \text{Position 2} & \text{Position 3} \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Load } L_2 & \delta_1^{L_2} & \delta_2^{L_2} & \delta_3^{L_2} \\
 & d_1 & d_2 & d_3 & d_4 \\
\text{Position 1} & \text{Position 2} & \text{Position 3} \\
\end{array}
\]

\[
M_{\alpha,L_1}^{N,1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}, \quad M_{\beta,L_1}^{N,1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_{\alpha,L_2}^{N,1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \\ 3 & 3 \end{bmatrix}, \quad M_{\beta,L_2}^{N,1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}
\]

\[
M_{S_{L_1}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad M_{S_{L_2}} = \begin{bmatrix} 1 & 1 \end{bmatrix}
\]
\[
A^{(1)} = \tau, \\
B^{(1)} = \begin{bmatrix}
E^{\text{init}} + \left( p_1^{(s)} + p_1^{(w)} \right) \cdot \tau - p_1^{(\text{ncl})} \cdot \tau \\
E^{\text{init}} + \sum_{i=1}^{4} \left( p_i^{(s)} + p_i^{(w)} \right) \cdot \tau - \sum_{i=1}^{4} p_i^{(\text{ncl})} \cdot \tau \\
E^{\text{max}} - E^{\text{init}} - \left( p_1^{(s)} + p_1^{(w)} \right) \cdot \tau + p_1^{(\text{ncl})} \cdot \tau \\
R^c - \left( p_1^{(s)} + p_1^{(w)} \right) \cdot \tau + p_1^{(\text{ncl})} \cdot \tau \\
R^d + \left( p_1^{(s)} + p_1^{(w)} \right) \cdot \tau - p_1^{(\text{ncl})} \cdot \tau \\
E^{\text{init}} + \sum_{i=1}^{4} \left( p_4^{(s)} + p_4^{(w)} \right) \cdot \tau + \sum_{i=1}^{4} p_i^{(\text{ncl})} \cdot \tau \\
E^{\text{max}} - E^{\text{init}} - \sum_{i=1}^{4} \left( p_4^{(s)} + p_4^{(w)} \right) \cdot \tau + \sum_{i=1}^{4} p_i^{(\text{ncl})} \cdot \tau \\
R^c - \left( p_4^{(s)} + p_4^{(w)} \right) \cdot \tau + p_4^{(\text{ncl})} \cdot \tau \\
R^d + \left( p_4^{(s)} + p_4^{(w)} \right) \cdot \tau - p_4^{(\text{ncl})} \cdot \tau \\
\end{bmatrix}
\]

\[
X^{(1)} = \begin{bmatrix}
p_1^{(g)} \\
p_2^{(g)} \\
p_3^{(g)} \\
p_4^{(g)} \\
p_1^{(s)} \\
p_2^{(s)} \\
p_3^{(s)} \\
p_4^{(s)} \\
\end{bmatrix}
\]

\[
A_{eq}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ \end{bmatrix}
\]

\[
B_{eq}^{(1)} = [1]
\]
\[ A^{(l)} X^{(l)} \leq B^{(l)} \] gives:

\[
(-p_{1}^{(g)} - \delta_{1}^{bt} P_{1}^{bt} + \delta_{1}^{v1} p_{1}^{v1} + \delta_{1}^{v2} p_{2}^{v2}) \tau \leq E^{init} + (p_{1}^{(s)} + p_{1}^{(w)}) \tau - p_{1}^{(ncl)} \tau
\]

\[
(-p_{1}^{(g)} - \delta_{1}^{bt} P_{1}^{bt} + \delta_{1}^{v1} p_{1}^{v1} + \delta_{1}^{v2} p_{2}^{v2}) \tau
\]

\[
\leq E^{init} + \sum_{i=1}^{2} (p_{i}^{(s)} + p_{i}^{(w)}) \tau - \sum_{i=1}^{2} p_{i}^{(ncl)} \tau
\]

\[
(-p_{1}^{(g)} - p_{2}^{(g)} + p_{3}^{(g)}) + (\delta_{1}^{bt} + \delta_{1}^{v1} + \delta_{1}^{v2}) P_{1}^{bt} P_{nom} + 2\delta_{1}^{v1} p_{1}^{v1} + 2\delta_{1}^{v2} p_{2}^{v2} + \delta_{1}^{v3} p_{1}^{v3} + 3\delta_{1}^{v3} p_{2}^{v3} + 2\delta_{1}^{v4} p_{1}^{v4} + 2\delta_{1}^{v4} p_{2}^{v4}) \tau
\]

\[
\leq E^{init} + \sum_{i=1}^{3} (p_{i}^{(s)} + p_{i}^{(w)}) \tau - \sum_{i=1}^{3} p_{i}^{(ncl)} \tau
\]

\[
(-p_{1}^{(g)} - p_{2}^{(g)} + p_{3}^{(g)} + p_{4}^{(g)}) + (\delta_{1}^{bt} + \delta_{1}^{v1} + \delta_{1}^{v2} + \delta_{1}^{v3}) P_{1}^{bt} P_{nom} + 2\delta_{1}^{v1} p_{1}^{v1} + 2\delta_{1}^{v2} p_{2}^{v2} + \delta_{1}^{v3} p_{1}^{v3} + 3\delta_{1}^{v3} p_{2}^{v3} + 2\delta_{1}^{v4} p_{1}^{v4} + 2\delta_{1}^{v4} p_{2}^{v4}
\]

\[
+ 3\delta_{1}^{v5} p_{1}^{v5} + \delta_{1}^{v6} p_{1}^{v6}) \tau
\]

\[
\leq E^{init} + \sum_{i=1}^{4} (p_{i}^{(s)} + p_{i}^{(w)}) \tau - \sum_{i=1}^{4} p_{i}^{(ncl)} \tau
\]

\[
(p_{1}^{(g)} + \delta_{1}^{bt} P_{1}^{bt} - \delta_{1}^{v1} p_{1}^{v1} - \delta_{1}^{v2} p_{2}^{v2}) \tau \leq E^{max} - E^{init} - (p_{1}^{(s)} + p_{1}^{(w)}) \tau + p_{1}^{(ncl)} \tau
\]

\[
(p_{1}^{(g)} + \delta_{1}^{bt} P_{1}^{bt} - \delta_{1}^{v1} p_{1}^{v1} - \delta_{1}^{v2} p_{2}^{v2}) \tau
\]

\[
\leq E^{max} - E^{init} - \sum_{i=1}^{2} (p_{i}^{(s)} + p_{i}^{(w)}) \tau + \sum_{i=1}^{2} p_{i}^{(ncl)} \tau
\]

\[
(p_{1}^{(g)} + p_{2}^{(g)} + p_{3}^{(g)}) + (\delta_{1}^{bt} + \delta_{1}^{v1} + \delta_{1}^{v2}) P_{1}^{bt} P_{nom} - 2\delta_{1}^{v1} p_{1}^{v1} - 2\delta_{1}^{v2} p_{2}^{v2} - \delta_{1}^{v3} p_{1}^{v3} - 3\delta_{1}^{v3} p_{2}^{v3} - 2\delta_{1}^{v4} p_{1}^{v4} - 2\delta_{1}^{v4} p_{2}^{v4}) \tau
\]

\[
\leq E^{max} - E^{init} - \sum_{i=1}^{3} (p_{i}^{(s)} + p_{i}^{(w)}) \tau + \sum_{i=1}^{3} p_{i}^{(ncl)} \tau
\]

\[
(p_{1}^{(g)} + p_{2}^{(g)} + p_{3}^{(g)} + p_{4}^{(g)}) + (\delta_{1}^{bt} + \delta_{1}^{v1} + \delta_{1}^{v2} + \delta_{1}^{v3}) P_{1}^{bt} P_{nom} - 2\delta_{1}^{v1} p_{1}^{v1} - 2\delta_{1}^{v2} p_{2}^{v2} - 2\delta_{1}^{v3} p_{1}^{v3} - 3\delta_{1}^{v3} p_{2}^{v3} - 2\delta_{1}^{v4} p_{1}^{v4} - 2\delta_{1}^{v4} p_{2}^{v4}
\]

\[
- 3\delta_{1}^{v5} p_{1}^{v5} - 3\delta_{1}^{v6} p_{1}^{v6}) \tau
\]

\[
\leq E^{max} - E^{init} - \sum_{i=1}^{4} (p_{i}^{(s)} + p_{i}^{(w)}) \tau + \sum_{i=1}^{4} p_{i}^{(ncl)} \tau
\]
\[ (p_{1}^{(g)} + \delta_{1}^{h}p_{1}^{ht}p_{nom}^{bt} - \delta_{1}^{l}p_{1}^{l} - \delta_{2}^{l}p_{2}^{l}), \tau \leq R^{c} - \left( p_{1}^{(s)} + p_{1}^{(w)} \right), \tau + p_{1}^{(nc)} \cdot \tau \]

\[ (p_{2}^{(g)} + \delta_{2}^{h}p_{2}^{ht}p_{nom}^{bt} - \delta_{2}^{l}p_{2}^{l} - \delta_{1}^{l}p_{1}^{l} - \delta_{2}^{l}p_{2}^{l} - \delta_{3}^{l}p_{3}^{l}), \tau \leq R^{c} - \left( p_{2}^{(s)} + p_{2}^{(w)} \right), \tau + p_{2}^{(nc)} \cdot \tau \]

\[ (p_{3}^{(g)} + \delta_{3}^{h}p_{3}^{ht}p_{nom}^{bt} - \delta_{3}^{l}p_{3}^{l} - \delta_{2}^{l}p_{2}^{l} - \delta_{3}^{l}p_{3}^{l} - \delta_{3}^{l}p_{3}^{l} - \delta_{2}^{l}p_{2}^{l}), \tau \leq R^{c} - \left( p_{3}^{(s)} + p_{3}^{(w)} \right), \tau + p_{3}^{(nc)} \cdot \tau \]

\[ (p_{4}^{(g)} + \delta_{4}^{h}p_{4}^{ht}p_{nom}^{bt} - \delta_{3}^{l}p_{3}^{l} - \delta_{2}^{l}p_{2}^{l} - \delta_{3}^{l}p_{3}^{l}), \tau \leq R^{c} - \left( p_{4}^{(s)} + p_{4}^{(w)} \right), \tau + p_{4}^{(nc)} \cdot \tau \]

\[ (-p_{1}^{(g)} - \delta_{1}^{ht}p_{1}^{ht}p_{nom}^{bt} + \delta_{1}^{l}p_{1}^{l} + \delta_{1}^{l}p_{1}^{l}), \tau \leq R^{d} + \left( p_{1}^{(s)} + p_{1}^{(w)} \right), \tau - p_{1}^{(nc)} \cdot \tau \]

\[ (-p_{2}^{(g)} - \delta_{2}^{ht}p_{2}^{ht}p_{nom}^{bt} + \delta_{2}^{l}p_{2}^{l} + \delta_{2}^{l}p_{2}^{l} + \delta_{1}^{l}p_{1}^{l} + \delta_{2}^{l}p_{2}^{l}), \tau \leq R^{d} + \left( p_{2}^{(s)} + p_{2}^{(w)} \right), \tau - p_{2}^{(nc)} \cdot \tau \]

\[ (-p_{3}^{(g)} - \delta_{3}^{ht}p_{3}^{ht}p_{nom}^{bt} + \delta_{3}^{l}p_{3}^{l} + \delta_{3}^{l}p_{3}^{l} + \delta_{3}^{l}p_{3}^{l} + \delta_{2}^{l}p_{2}^{l}), \tau \leq R^{d} + \left( p_{3}^{(s)} + p_{3}^{(w)} \right), \tau - p_{3}^{(nc)} \cdot \tau \]

\[ (-p_{4}^{(g)} - \delta_{4}^{ht}p_{4}^{ht}p_{nom}^{bt} + \delta_{3}^{l}p_{3}^{l} + \delta_{2}^{l}p_{2}^{l}), \tau \leq R^{d} + \left( p_{4}^{(s)} + p_{4}^{(w)} \right), \tau - p_{4}^{(nc)} \cdot \tau \]

\[ \begin{align*}
\delta_{1}^{l} + \delta_{1}^{l} & \leq 1 \\
\delta_{2}^{l} + \delta_{2}^{l} & \leq 1 \\
\delta_{3}^{l} + \delta_{3}^{l} & \leq 1 \\
\delta_{4}^{l} + \delta_{4}^{l} & \leq 1
\end{align*} \]

\[ \text{cf (11b)} \]

\[ \text{Aeq}(f)X(f) = \text{Beq}(f) \text{ gives :} \]

\[ \begin{align*}
\delta_{1}^{l} + \delta_{2}^{l} + \delta_{3}^{l} & = 1 \\
\delta_{1}^{l} + \delta_{2}^{l} & = 1
\end{align*} \]

\[ \text{cf (11a)} \]
4.4 - Simulations

4.4.1 - Introduction

To conclude this chapter, we simulate successively five scenarios in order to illustrate the mathematical theory discussed above. For each scenario, the system (LP-I) is solved for the home grid defined in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Assumed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Time horizon</td>
<td>24h</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of time intervals</td>
<td>48</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Length of each time interval</td>
<td>30min</td>
</tr>
<tr>
<td>$p^f$</td>
<td>Biofuel price</td>
<td>7.4 c/kWh</td>
</tr>
<tr>
<td>$E_{\text{init}}^\text{max}$</td>
<td>Energy stored at $t = 0$</td>
<td>0 kWh</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>Maximum capacity - storage device</td>
<td>3 kWh</td>
</tr>
<tr>
<td>$R^c$</td>
<td>Maximal charging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$R^d$</td>
<td>Maximal discharging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$P_{\text{nom}}^{\text{biofuel}}$</td>
<td>Nominal power - biofuel-burning turbine</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$\eta^{(s)}$</td>
<td>Solar panel efficiency</td>
<td>0.15</td>
</tr>
<tr>
<td>$S^{(s)}$</td>
<td>Solar panel eq. surface</td>
<td>1m$^2$</td>
</tr>
<tr>
<td>$\eta^{(w)}$</td>
<td>Wind turbine efficiency</td>
<td>0.20</td>
</tr>
<tr>
<td>$S^{(w)}$</td>
<td>Wind turbine eq. surface</td>
<td>1m$^2$</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Number of non-interruptible loads</td>
<td>6</td>
</tr>
<tr>
<td>$P_{L_1}^L$</td>
<td>Washing machine consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$P_{L_2}^D$</td>
<td>Dryer consumption</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$P_{L_3}^D$</td>
<td>Dishwasher consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$P_{L_4}^D$</td>
<td>Defrost cycle consumption</td>
<td>1.3 kW</td>
</tr>
<tr>
<td>$P_{L_5}^D$</td>
<td>Water heater consumption</td>
<td>2 kW</td>
</tr>
<tr>
<td>$P_{L_6}^D$</td>
<td>PHEV battery consumption</td>
<td>3 kW</td>
</tr>
<tr>
<td>$k_{L_1}$</td>
<td>$k_{L_1}$ is the cycle of $L_1$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{L_2}$</td>
<td>$k_{L_2}$ is the cycle of $L_2$</td>
<td>2</td>
</tr>
<tr>
<td>$k_{L_3}$</td>
<td>$k_{L_3}$ is the cycle of $L_3$</td>
<td>2</td>
</tr>
<tr>
<td>$k_{L_4}$</td>
<td>$k_{L_4}$ is the cycle of $L_4$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{L_5}$</td>
<td>$k_{L_5}$ is the cycle of $L_5$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{L_6}$</td>
<td>$k_{L_6}$ is the cycle of $L_6$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_1}$</td>
<td>$L_1$ to be scheduled $\gamma^{L_1}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_2}$</td>
<td>$L_2$ to be scheduled $\gamma^{L_2}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_3}$</td>
<td>$L_3$ to be scheduled $\gamma^{L_3}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_4}$</td>
<td>$L_4$ to be scheduled $\gamma^{L_4}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_5}$</td>
<td>$L_5$ to be scheduled $\gamma^{L_5}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_6}$</td>
<td>$L_6$ to be scheduled $\gamma^{L_6}$ times</td>
<td>4</td>
</tr>
</tbody>
</table>

Note that there are $\prod_{i=1}^{\ell} \left( \binom{S_{L_i}}{\gamma^{L_i}} \right) = 4.75 \times 10^{13}$ potential solutions in the 6-dimensionnal solution space for the non-interruptible loads schedule.
The day-ahead price signal is broadcasted to the smart meter and the non-controllable consumption forecasts are determined by a corresponding forecasting module which is out of the scope of this thesis.

It is assumed for this simulation that a maximum of 7kW can be exchanged at any time between the utility grid and the home grid.

The price signal and the non-controllable loads consumption forecasts assumed in this chapter are shown in Figure 3.

In scenario 1 to 5, we progressively enable the devices connected to the home grid and examine their impact on the electricity bill. The successive results are summarized at the end of this chapter in Table 2.

**FIGURE 3:** Price forecasts for scenarios 1 to 5

**FIGURE 4:** Non-controllable loads forecasts for scenarios 1 to 5
4.4.2 - **Scenario 1**

In this scenario, we assume that the renewable energy units (solar, wind, biofuel-burning turbine) and the storage system are all off-line. No time constraints apply to the controllable loads.

![Graph](image1)

**FIGURE 5:** Optimal loads schedule - scenario 1

![Graph](image2)

**FIGURE 6:** Power exchanged with the grid - scenario 1

The optimal schedule for this scenario corresponds to a forecasted cost of $1.51 for the next day. The controller obviously schedules all the loads between 1am and 4am since this time period corresponds to the lowest electricity prices for the next 24 hours. Notice that the 7 kW limit is enforced.
4.4.3- **Scenario 2**

In this scenario, time constraints apply to several controllable loads, and are transposed into constraints on the $\delta_i^{L,j}$:

- Loads 1 and 2 must run between 10am and 5pm
- Load 3 must run between 3pm and 7pm
- Load 6 must run between 0am and 6am

The purple areas in the loads schedule (Fig. 7) correspond to the non-scheduling periods.

The renewable energy units and the storage systems are still off-line.

![Graph showing optimal loads schedule for scenario 2](image)

**FIGURE 7**: Optimal loads schedule - scenario 2

![Graph showing power exchanged with the grid for scenario 2](image)

**FIGURE 8**: Power exchanged with the grid - scenario 2
The optimal schedule corresponds to a forecasted cost of $1.57 for the next day, which is obviously higher than the cost of a scenario not enforcing time constraints.

4.4.4- Scenario 3

In this scenario, the previous conditions are maintained, except that the solar and wind units are now online. Their production can be forecasted based on the irradiance and wind speed forecasts shown in Figures 9 and 10.

FIGURE 9: Irradiance forecasts - scenarios 3 to 5

FIGURE 10: Wind speed forecasts - scenarios 3 to 5
The optimal schedule corresponds to a forecasted cost of $0.75 for the next day. The cost savings are due to the local power production. Notice that part of the local production is sold back to the utility grid, creating time periods where the resulting power flow exchanged between the utility grid and the home grid is negative.

4.4.5- **Scenario 4**

In this scenario, the previous conditions are maintained, except that the biofuel-burning turbine can now be turn on or off as needed.
FIGURE 13: Price forecasts - scenario 4

FIGURE 14: Biofuel-burning turbine schedule - scenario 4

FIGURE 15: Optimal loads schedule - scenario 4
The optimal schedule corresponds to a forecasted cost of $0.70 for the next day. Notice that the biofuel-burning turbine starts as soon as the utility price signal goes above the biofuel price.

4.4.6- Scenario 5

In this scenario, the previous conditions are maintained, except that the storage system is now active.
The optimal schedule corresponds to a forecasted cost of $0.57 for the next day.

Notice that the storage device charges when prices are relatively low, and discharges when prices are relatively high.

Also notice that the storage device is completely discharged at the end of the 24 hour period as having energy stored in the device at the end would mean that the energy consumption is not optimized.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Comfort preferences</th>
<th>Solar and wind units status</th>
<th>Biofuel-burning turbine status</th>
<th>Storage status</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No</td>
<td>Off-line</td>
<td>Off-line</td>
<td>Off-line</td>
<td>1.51</td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
<td>Off-line</td>
<td>Off-line</td>
<td>Off-line</td>
<td>1.57</td>
</tr>
<tr>
<td>3</td>
<td>Yes</td>
<td>Online</td>
<td>Off-line</td>
<td>Off-line</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>Yes</td>
<td>Online</td>
<td>Online</td>
<td>Off-line</td>
<td>0.70</td>
</tr>
<tr>
<td>5</td>
<td>Yes</td>
<td>Online</td>
<td>Online</td>
<td>Online</td>
<td>0.57</td>
</tr>
</tbody>
</table>
CHAPTER 5
THE INTERRUPTIBLE LOADS CASE

5.1 - Introduction
In this chapter, we focus on the heating and A/C systems which are interruptible loads, and assume that the non-interruptible loads are off-line. We also assume for now that the storage system and the generation capabilities are turned off. The non-controllable loads are not considered in order to simplify the derivation. Then, the utility grid is the only source providing power to the interruptible loads.

5.2 - Mathematical formulation as a linear programming problem
Let us consider the thermodynamic system modeling the house. This system exchanges heat with two other thermodynamic systems, the heating/AC, and the outside.

In winter, the heating system transfers heat to the house. As a result of this some heat is transferred to the outside world (thermal losses). In summer the heating system is replaced by the AC system and the heat flow is reversed.

In the following, we will use the term heating to refer to the system capable of producing heat in winter time, and cold in summer time.
The absolute temperature of the room $T^r$ is the state variable which will allow us to describe the temporal evolution of the thermodynamic system throughout the next day.

$T^r$ is a continuous function defined on a period of time $\mathcal{T}_{T^r} \subseteq \mathbb{R}^+$. The user sets comfort preferences on $T^r$ by defining two limit functions $T^{r,\min}$ and $T^{r,\max}$ defined on $\mathcal{T}_{T^r}$.

$T^r$ must verify the following constraints:

$$\forall t \in \mathcal{T}_{T^r}, \quad T^{r,\min}(t) \leq T^r(t) \leq T^{r,\max}(t)$$  \hspace{1cm} (12)

The evolution of the outside temperature $T^{\text{out}}$ leads to variations in the quantity of heat exchanged between the outside and the room. This leads to variations in $T^r$. Thanks to the heating, heat is transferred to the room so that $T^r$ can meet the constraints stated in (12) despite of external temperature variations.

For a given value of $T^r$, the user can control the value of $\left(\frac{dQ}{dt}\right)_{\text{losses}}$ through the temperature of the heater $T^h$, and then control the variations of $T^r$.

$T^{\text{out}}$ and $T^h$ are defined on $\mathcal{T}_{T^r}$. Additionally, physical constraints apply on the temperature of the heater: $T^h$ must stay in $[T^{h,\min}, T^{h,\max}] \subseteq \mathbb{R}$, and the variations of $T^h$ are bounded by $R^{\max} > 0$ and $R^{\min} > 0$. 
The first law of thermodynamics applied to the thermodynamic system modeling the room gives $dU = \delta Q + \delta W$.

Since the process is isochoric, $dV = 0$, then $\delta W = -pdV = 0$. Hence $dU = \delta Q$.

The internal energy $U$ is a state function, then $dU = \left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial P}\right)_T dP$, and $dP \approx 0$ (the pressure does not vary significantly in the room). Hence $dU = \left(\frac{\partial U}{\partial T}\right)_p dT$.

Then, $dU = \delta Q = \left(\frac{\partial U}{\partial T}\right)_v dT = \zeta C dT$, with $\zeta$ the heat capacity of air. And $\zeta = M_{air}.c$ with $M_{air}$ the mass of air inside the house and $c$ the specific heat capacity of air at constant pressure (heat capacity per unit mass).

Finally $\delta Q = M_{air}.c dT$ leads to:

$$\forall \ t \in \mathcal{T}_r, \quad \frac{dT_r(t)}{dt} = \frac{1}{M_{air}.c} \left\{ \left(\frac{dQ(t)}{dt}\right)_{heater} - \left(\frac{dQ(t)}{dt}\right)_{losses} \right\}$$

(15)

From $\delta Q = M_{air}.c dT$ we can also derivate:

$$\forall \ t \in \mathcal{T}_r, \quad \left(\frac{dQ(t)}{dt}\right)_{heater} = \dot{M}.c \times \{T^h(t) - T^r(t)\}$$

(16)

$\dot{M}$ is the air flow rate through the heater and is assumed to be constant.
Let $R_{eq}$ be the equivalent thermal resistance of the house.

Refer to Appendix A for the geometry of the house, and Appendix B for the detailed calculation of $R_{eq}$.

We have:

$$\forall t \in T^r, \quad \left(\frac{dQ(t)}{dt}\right)_{\text{losses}} = \frac{T^r(t) - T^{\text{out}}(t)}{R_{eq}}$$

Let now $d = \{d_1, d_2, \ldots, d_i, \ldots, d_N\}$ be a subdivision of $D$.

Define $T^r_{1i} \ldots$ the temperature of the room at the beginning of the time interval $d_i$ and $T^r_i$ the function defined on $d_i$ such that $T^r_i$ and $T^r$ are identically equal on $d_i$. The initial condition is given by $T^{\text{init}} = T^r_{1i}$.

Assume that $N$ is large enough so that $\tau \ll \tau^\text{out}$, with $\tau^\text{out}$ the characteristic time of $T^\text{out}$. Then we can assume that $T^\text{out}$ is constant on $d_i$ and equal to $T^\text{out}_i$.

Assume that $T^h$ will be maintained constant on $d_i$ and equal to $T^h_i$.

Assume that $N$ is large enough so that $\tau \ll \tau^r_i$, with $\tau^r_i$ the characteristic time of $T^r_i$. 

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Then we can assume that:

\[
\forall t \in d_i, \quad \left( \frac{dQ(t)}{dt} \right)_{heater} = \dot{M} \cdot c \times \{ T^h(t) - T^r(t) \} \approx \dot{M} \cdot c \times \{ T^h_i - T^r_i \} \]  \quad (18)

Define:

\[
\forall i \in [1, N], \quad P^h_i = \dot{M} \cdot c \times \{ T^h_i - T^r_i \} \]  \quad (19)

When heat is transferred to the room (winter time), \( P^h_i > 0 \). When the heat flow is reversed (summer time, the “heater” behaves like an AC), \( P^h_i < 0 \).

Define \( P^{h,+}_i > 0 \) and \( P^{h,-}_i > 0 \) such that \( P^h_i = P^{h,+}_i - P^{h,-}_i \).

Let us call \( \eta^h > 0 \) and \( \eta^{ac} > 0 \) the energy conversion efficiency of the heating and the A/C. Then, providing \( P^{h,+}_i \) to the room requires \( \eta^h \times P^{h,+}_i \) electric energy, and extracting \( P^{h,-}_i \) from the room requires \( \eta^{ac} \times P^{h,-}_i \) electric energy.

Let us now express \( T^r_i \) as a function of the time, \( P^h_i \) and \( T^{out}_i \).

With (15), (16), (17), (18), (19):

\[
\forall t \in d_i, \quad \frac{dT^r_i(t)}{dt} = \frac{1}{M_{air \cdot c}} \left\{ P^h_i - \frac{T^r_i(t) - T^{out}_i}{R_{eq}} \right\}
\]

Then:

\[
\forall t \in d_i, \quad \frac{dT^r_i(t)}{dt} + \frac{T^r_i(t)}{M_{air \cdot c} R_{eq}} = \frac{P^h_i}{M_{air \cdot c}} + \frac{T^{out}_i}{M_{air \cdot c} R_{eq}}
\]  \quad (20)
Solving the differential equation for $T_i^r$ with the origin of time being taken at the beginning of $d_i$:

$$\forall \ t \in d_i, \ T_i^r(t) = (T_i^{r,0} - R_{eq}p_i^h - J_{out}) \exp\left(\frac{-t}{M_{air,c,eq}}\right) + R_{eq}p_i^h + J_{out} \tag{21}$$

$T^r$ is continuous, then $T_i^r$ is continuous and the continuity at the boundary between $d_i$ and $d_{i+1}$ gives:

$$T_i^r(\tau) = T_{i+1}^{r,0} \tag{22}$$

Combining (21) and (22):

$$(T_i^{r,0} - R_{eq}p_i^h - J_{out}) \exp\left(\frac{-\tau}{M_{air,c,eq}}\right) + R_{eq}p_i^h + J_{out} = T_{i+1}^{r,0}$$

Hence:

$$T_i^{r,0} \exp\left(\frac{-\tau}{M_{air,c,eq}}\right) - T_{i+1}^{r,0} + R_{eq}\left(1 - \exp\left(\frac{-\tau}{M_{air,c,eq}}\right)\right) p_i^h = \left(\exp\left(\frac{-\tau}{M_{air,c,eq}}\right) - 1\right) J_{out}$$

Define $K_\tau = \exp\left(\frac{-\tau}{M_{air,c,eq}}\right)$

Finally:

$$\forall \ i \in [1,N], \ T_i^{r,0}K_\tau - T_{i+1}^{r,0} + R_{eq}\left(1 - K_\tau\right)p_i^h = (K_\tau - 1) J_i^{out} \tag{23}$$
Let now $A_{eq}^{(II)}$, $B_{eq}^{(II)}$ and $X^{(II)}$ be:

\[
A_{eq}^{(II)} =
\begin{bmatrix}
K_r & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{eq}(1 - K_r) & -R_{eq}(1 - K_r) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_r & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{eq}(1 - K_r) & -R_{eq}(1 - K_r) & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B_{eq}^{(II)} = (K_r - 1) \begin{bmatrix}
T^{\text{out}}_1 \\
\vdots \\
T^{\text{out}}_N \\
\end{bmatrix}
\]

\[
X^{(II)} = \begin{bmatrix}
T^{r,0}_1 \\
\vdots \\
T^{r,0}_{N+1} \\
p^{r,+}_1 \\
\vdots \\
p^{r,+}_N \\
p^{r,-}_1 \\
\vdots \\
p^{r,-}_N \\
\end{bmatrix}
\]

Then:

(23) is equivalent to $A_{eq}^{(II)} X^{(II)} = B_{eq}^{(II)}$

(13) and (14) gives:

\[
\forall i \in [1, N], \quad T^{h, \text{min}} \leq T^h_i \leq T^{h, \text{max}} \tag{24}
\]

\[
\forall i \in [1, N], \quad R^{\text{min}} \leq T^r_{i+1} - T^r_i \leq R^{\text{max}} \tag{25}
\]

then, with (19):

\[
\forall i \in [1, N], \quad T^{h, \text{min}} \leq \frac{p^h_i}{M_c + T^r_i} \leq T^{h, \text{max}}
\]

\[
\forall i \in [1, N - 1], \quad -R^{\text{min}} \leq \left(\frac{p^h_i}{M_c + T^r_{i+1}}\right) - \left(\frac{p^h_i}{M_c + T^r_i}\right) \leq R^{\text{max}}
\]
And finally:

\[ T_{i}^{r,0} + \frac{p_{i}^{h+,c}}{M,c} - \frac{p_{i}^{h-,c}}{M,c} \leq T_{i}^{h,max} \]  
\[ \forall \ i \in [1, N], \tag{26} \]

\[ -T_{i}^{r,0} - \frac{p_{i}^{h+,c}}{M,c} + \frac{p_{i}^{h-,c}}{M,c} \leq -T_{i}^{h,min} \]  
\[ \forall \ i \in [1, N], \tag{27} \]

\[ -T_{i}^{r,0} + T_{i+1}^{r,0} - \frac{p_{i}^{h+,c}}{M,c} + \frac{p_{i+1}^{h+,c}}{M,c} - \frac{p_{i+1}^{h-,c}}{M,c} \leq R^{max} \]  
\[ \forall \ i \in [1, N - 1], \tag{28} \]

\[ +T_{i}^{r,0} - T_{i+1}^{r,0} + \frac{p_{i}^{h+,c}}{M,c} - \frac{p_{i+1}^{h+,c}}{M,c} - \frac{p_{i+1}^{h-,c}}{M,c} \leq R^{min} \]  
\[ \forall \ i \in [1, N - 1], \tag{29} \]

Let now \( A^{(II)} \) and \( B^{(II)} \) be:

\[
A^{(II)} = \begin{bmatrix}
I_{N} & 0_{R^{N}} & \frac{1}{M,c}F_{N} \\
-I_{N} & 0_{R^{N}} & -\frac{1}{M,c}F_{N} \\
G_{N-1} & 0_{R^{N}} & H_{N-1} \\
-G_{N-1} & 0_{R^{N}} & H_{N-1}
\end{bmatrix}
\]

\[
B^{(II)} = \begin{bmatrix}
T_{i}^{h,max} & 1_{R^{N}} \\
-T_{i}^{h,min} & 1_{R^{N}} \\
R_{N}^{max} & 1_{R^{N-1}} \\
R_{N}^{min} & 1_{R^{N-1}}
\end{bmatrix}
\]

With:

\[
F_{N} = \begin{bmatrix}
1 & -1 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & & & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \cdots & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 1 & -1
\end{bmatrix}
\]

\[
G_{N-1} = \begin{bmatrix}
-1 & 1 & 0 & \cdots & \cdots & 0 \\
0 & -1 & 1 & \ddots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \vdots & \ddots & -1 & 1 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & -1 & 1
\end{bmatrix}
\]
Then:

\[(26),(27),(28),(29) \text{ are equivalent to } A^{(II)} X^{(II)} \leq B^{(II)}\]

\[\mathcal{P}^u = [p_1^u \quad p_2^u \quad \ldots \quad p_{N-1}^u \quad p_N^u]^T\]

is still defined as the electricity prices corresponding to the subdivision \(d\).

Define \(\bar{\mathcal{P}}^{(II)} = [0_{\mathbb{R}^{N+1}} \quad \eta^h p_1^u \quad \eta^a p_1^u \quad \ldots \quad \eta^h p_N^u \quad \eta^a p_N^u]^T\)

Note that we have assumed that the power consumed by the heating/AC was directly provided by the utility grid.

The interruptible loads optimal schedule problem can finally be expressed as the following linear programming problem:

Minimize

\[\mathcal{C}^{(II)} = \mathcal{P}^{(II)}^T X^{(II)}\]

Subject to

\[A^{(II)} X^{(II)} \leq B^{(II)}\]

\[A_{eq}^{(II)} X^{(II)} = B_{eq}^{(II)}\]

\[\forall \quad i \in [1, N], \quad T_{i, min}^r \leq T_{i, 0}^r \leq T_{i, max}^r\]

(LP-II)
Note about the operation of the heating/AC system:

Equation (19) shows that if \( T_i^{h} = T_i^{r,0} \) then \( P_i^{h} = 0 \), which means physically that the heating/AC does not exchange heat with the room over \( d_i \).

Then, in situations where we will have \( T_i^{h} - T_i^{r,0} < \varepsilon \) with \( \varepsilon \) a “small” quantity to be defined when implementing the algorithm, we will conclude that the heating/AC is turned off over \( d_i \).

5.3 - Simulations

5.3.1 - Introduction

To conclude this chapter, we simulate successively a winter scenario and a summer scenario in order to illustrate the mathematical theory discussed above. For each scenario, the system (LP-II) is solved for the home grid defined in Table 3.

| TABLE 3 | HOMEGRID PARAMETERS — CHAPTER 5 |
|-----------------|-----------------|-----------------|
| **Symbol**      | **Quantity**    | **Assumed value** |
| \( D \)         | Time horizon    | 24h              |
| \( N \)         | Number of time intervals | 288             |
| \( \tau \)      | Length of each time interval | 5min            |
| \( \eta^h \)    | Heating efficiency | 1                |
| \( \eta^{ac} \) | AC efficiency    | 1                |
| \( M \)         | Air flow rate    | 1 kg/s           |
| \( T^{h, \text{min}} \) | Minimum heater-AC temperature | 5°C             |
| \( T^{\text{h, max}} \) | Maximum heater-AC temperature | 32°C            |
| \( R^{\text{min}} \) | Slew rate, temperature decrease | 6°C            |
| \( R^{\text{max}} \) | Slew rate, temperature increase | 6°C           |
| \( T_i^{\text{init}} \) | Initial room temperature | 20°C          |
| \( \varepsilon \) | ON/OFF status threshold | 1°C            |

Recall that the storage system and the generation capabilities are off-line in this chapter.

The utility grid is then the only source providing power to the heating/AC.
5.3.2- Winter scenario

FIGURE 20: External temperature forecasts—winter scenario

FIGURE 21: Price forecasts—winter scenario
Notice how the controller lets the room temperature decreasing when the user’s comfort preferences are relaxed in order to save energy (for instance from 9am to 12am).

The forecasted cost for the next day is $1.1.
5.3.3- Summer scenario

FIGURE 26: External temperature forecasts – summer scenario
Similarly to what happened in the winter scenario, the controller lets the room temperature increasing when the user’s comfort preferences are relaxed (for instance from 9am to 12am). Notice that from 2pm to 5pm, because the electricity price is lower on average than around 6pm, the room temperature is maintained close to 24°C.
For the summer scenario, the forecasted cost for the next day is $1.86.
CHAPTER 6
THE GENERAL CASE

6.1 - Introduction

In this chapter, we now combine the two controllable loads groups, the non-interruptible loads and the interruptible loads, into one single system, similarly to what we did in (LP-I) for the non-interruptible loads, and (LP-II) for the interruptible loads. The storage system and the distributed generation units are now connected to the home grid again.

6.2 - Mathematical formulation as a linear programming problem

First, notice that a subdivision adapted to the non-interruptible loads (washing machine, dryer, dish washer, etc), is not necessarily adapted to the interruptible loads (heating, A/C).

For instance, with \( D = 24 \) h, choosing \( N = 48 \) allows to divide \( D \) into intervals of length \( \tau = 30 \) min. This means approaching the cycles of the non-interruptible loads by intervals of the type \( n\tau, n \in \mathbb{N} \) ie 30min, 60min, 90min, etc. This subdivision seems adapted to the non-interruptible loads, but may not be adapted to the heating and A/C. Indeed one of the assumptions we made is that \( \tau \ll \tau_{\text{out}} \) and \( \tau \ll \tau_{\text{in}} \), but with \( \tau = 30 \) min, \( \tau_{\text{out}} \) may vary significantly over that period of time, leading to significant variations on \( T_{\text{in}} \) as well.

Then, we see the impossibility to combine directly (LP-I) and (LP-II), as we need first to find a subdivision which is adapted to both the interruptible and non-interruptible loads.
A possible solution is to divide each interval of the initial subdivision into a new subdivision of \( q \) intervals of equal length: the initial subdivision remains adapted to the non-interruptible loads, and we choose the new subdivision such that it is consistent to the interruptible loads.

We have the following relations:

\[
\tau' = q\tau \\
N'q = N
\]

The variables in \( X^{(I)} \) and \( X^{(II)} \) remain the same, except that each \( p_i^{(g)} \) in \( X^{(I)} \) is replaced by \( q \) variables \( p_{i,1}^{(g)}, \ldots, p_{i,q}^{(g)} \) to fit the new subdivision \( D \). All the \( p_i^{(g)} \) are then re-indexed as \( p_1^{(g)}, \ldots, p_N^{(g)} \).

Define:

\[
X = \begin{bmatrix}
p_1^{(g)} & \cdots & p_N^{(g)} & \delta_1^b & \cdots & \delta_N^b & \delta_1^l & \cdots & \delta_N^l & \delta_1^u & \cdots & \delta_N^u & \tau_1^r & \cdots & \tau_N^r & p_1^{h,+} & \cdots & p_N^{h,+} \end{bmatrix}^T
\]
\[
A = \begin{bmatrix}
-\tau T_N & -\tau P_{\text{nom}}^I T_N & \tau P_1^I \cdot M_{N,q}^{a,l_1} & \tau P_2^I \cdot M_{N,q}^{a,l_2} & \ldots & \tau P_{N-1}^I \cdot M_{N,q}^{a,l_{N-1}} & 0_{\mathbb{R}^{N \times N}} & 0_{\mathbb{R}^{N \times N}} & \tau \eta^a \\
\tau T_N & \tau P_{\text{nom}}^I T_N & -\tau P_1^I \cdot M_{N,q}^{a,l_1} & -\tau P_2^I \cdot M_{N,q}^{a,l_2} & \ldots & -\tau P_{N-1}^I \cdot M_{N,q}^{a,l_{N-1}} & 0_{\mathbb{R}^{N \times N}} & 0_{\mathbb{R}^{N \times N}} & -\tau \eta^a \\
\tau I_N & \tau P_{\text{nom}}^I I_N & -\tau P_1^I \cdot M_{N,q}^{a,l_1} & -\tau P_2^I \cdot M_{N,q}^{a,l_2} & \ldots & -\tau P_{N-1}^I \cdot M_{N,q}^{a,l_{N-1}} & 0_{\mathbb{R}^{N \times N}} & 0_{\mathbb{R}^{N \times N}} & -\tau \eta^\beta \\
-\tau I_N & -\tau P_{\text{nom}}^I I_N & \tau P_1^I \cdot M_{N,q}^{a,l_1} & \tau P_2^I \cdot M_{N,q}^{a,l_2} & \ldots & \tau P_{N-1}^I \cdot M_{N,q}^{a,l_{N-1}} & 0_{\mathbb{R}^{N \times N}} & 0_{\mathbb{R}^{N \times N}} & \tau \eta^\beta \\
M_{S_1} & & & & \cdots & & M_{S_{11}} & & \\
& & & & M_{S_{12}} & & \vdots & & \vdots \\
& & & & & & & M_{S_{1N}} & & \\
& & & & & & & \vdots & & \vdots \\
& & & & & & & & M_{S_{1N}} & & \\
& & & & & & & & \vdots & & \vdots \\
& & & & & & & & & & \vdots \\
& & & & & & & & & & & \vdots \\
& & & & & & & & & & & \vdots \\
& & & & & & & & & & & \vdots \\
& & & & & & & & & & & \vdots \\
& & & & & & & & & & & \vdots \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
B^{(I)} \\
B^{(II)}
\end{bmatrix}
\]

With:

\[
\eta^a = \begin{bmatrix}
\eta^h & \eta^{ac} & 0 & 0 & \ldots & \ldots & 0 & 0 \\
\eta^h & \eta^{ac} & \eta^h & \eta^{ac} & \ldots & \ldots & \ldots & \ldots \\
\vdots & \vdots & \ddots & \vdots & \ldots & \ldots & \ldots & \ldots \\
\eta^h & \eta^{ac} & \ldots & \ldots & \eta^h & \eta^{ac}
\end{bmatrix}
\]

\[
\eta^\beta = \begin{bmatrix}
\eta^h & \eta^{ac} & 0 & 0 & \ldots & \ldots & 0 & 0 \\
0 & 0 & \eta^h & \eta^{ac} & \ldots & \ldots & \ldots & \ldots \\
\vdots & \vdots & \ddots & \vdots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \ldots & \eta^h & \eta^{ac}
\end{bmatrix}
\]

\[
A_{eq} = \begin{bmatrix}
A^{(I)} & 0_{\mathbb{R}^{(2N+1) \times (2N+1)}} \\
0_{\mathbb{R}^{(2N+1) \times (2N+1)}} & A^{(II)}
\end{bmatrix}
\]

\[
B_{eq} = \begin{bmatrix}
B^{(I)} \\
B^{(II)}
\end{bmatrix}
\]

62
Define $\mathbf{p}^u = [p_1^u \ p_2^u \ \ldots \ p_{N-1}^u \ p_N^u]^T$ the electricity prices corresponding to the subdivision $d$.

Define $\mathbf{p}^f = p^f 1_{\mathbb{R}^N}$ as the biofuel prices corresponding to the subdivision $d$. Biofuel prices are supposed constant over $D$ and equal to $p^f$.

Finally, define $\mathcal{P} = \begin{bmatrix} \mathbf{p}^u & \mathbf{p}^{bt}_{\text{nom}} & \mathbf{p}^f & 0 & \ldots & 0 \end{bmatrix}^T$

$(\Sigma_{i=1}^{\ell} S_i) + 3N + 1$

Note that $\mathbf{p}^{bt}_{\text{nom}}$ has to be included in $\mathcal{P}$ since $X$ only contains the logic numbers $\delta_1^{bt}, \ldots, \delta_N^{bt}$ defining if the biofuel-burning turbine is on or off for each time interval $d_i$.

The optimization problem combining the non-interruptible loads and the interruptible loads can finally be expressed as the following mixed-integer linear programming problem:

\begin{align*}
\text{Minimize} & \quad \mathcal{C} = \mathcal{P}^T \cdot X \\
\text{Subject to} & \quad AX \leq B \\
& \quad A_{eq}X = B_{eq} \\
& \quad \forall i \in \llbracket 1, N \rrbracket, \ \delta_i^{bt} \in \llbracket 0, 1 \rrbracket \\
& \quad \forall i \in \llbracket 1, \ell \rrbracket, \forall j \in \llbracket 1, S_i \rrbracket, \ \delta_j^{Li} \in \llbracket 0, 1 \rrbracket \\
& \quad \forall i \in \llbracket 1, N \rrbracket, \ T_{i,\text{min}}^{r} \leq T_i^{r,0} \leq T_i^{r,\text{max}}
\end{align*}

(LP)
6.3 – Simulations

6.3.1- Introduction

To conclude this chapter, we simulate a scenario involving both interruptible and non-interruptible loads in order to illustrate the mathematical theory discussed above. The system (LP) is solved for the home grid defined in Table 4.

6.3.2- Scenario simulation

![Price forecasts – general case](image1)

**FIGURE 32:** Price forecasts – general case

![Wind speed forecasts – general case](image2)

**FIGURE 33:** Wind speed forecasts – general case
### HOMEGRID PARAMETERS – CHAPTER 6

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Assumed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Time horizon</td>
<td>6h</td>
</tr>
<tr>
<td>$N'$</td>
<td>Number of time intervals – subdivision $d'$</td>
<td>18</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>Length of each time interval – subdivision $d$</td>
<td>20min</td>
</tr>
<tr>
<td>$q$</td>
<td>Subdivision factor</td>
<td>4</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of time intervals</td>
<td>72</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Length of each time interval</td>
<td>5min</td>
</tr>
<tr>
<td>$p_f$</td>
<td>Biofuel price</td>
<td>5.8 ¢/kWh</td>
</tr>
<tr>
<td>$E_{\text{init}}$</td>
<td>Energy stored at $t = 0$</td>
<td>0 kWh</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>Maximum capacity - storage device</td>
<td>3 kWh</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Maximal charging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Maximal discharging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$p_{\text{bt}}^{\text{nom}}$</td>
<td>Nominal power - biofuel-burning turbine</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$\eta^{(s)}$</td>
<td>Solar panel efficiency</td>
<td>0.15</td>
</tr>
<tr>
<td>$S^{(s)}$</td>
<td>Solar panel eq. surface</td>
<td>1m$^2$</td>
</tr>
<tr>
<td>$\eta^{(w)}$</td>
<td>Wind turbine efficiency</td>
<td>0.20</td>
</tr>
<tr>
<td>$S^{(w)}$</td>
<td>Wind turbine eq. surface</td>
<td>1m$^2$</td>
</tr>
<tr>
<td>$l$</td>
<td>Number of non-interruptible loads</td>
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</tr>
<tr>
<td>$P_m^L$</td>
<td>Washing machine consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$P_d^L$</td>
<td>Dryer consumption</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$P_d^D$</td>
<td>Dishwasher consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$P_{fc}^L$</td>
<td>Defrost cycle consumption</td>
<td>1.3 kW</td>
</tr>
<tr>
<td>$P_{he}^L$</td>
<td>Water heater consumption</td>
<td>2 kW</td>
</tr>
<tr>
<td>$P_{PHEV}^L$</td>
<td>PHEV battery consumption</td>
<td>3 kW</td>
</tr>
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<td>$k_{L_1}$</td>
<td>$k_{L_1}$ is the cycle of $L_1$</td>
<td>2</td>
</tr>
<tr>
<td>$k_{L_2}$</td>
<td>$k_{L_2}$ is the cycle of $L_2$</td>
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<tr>
<td>$k_{L_3}$</td>
<td>$k_{L_3}$ is the cycle of $L_3$</td>
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<tr>
<td>$k_{L_4}$</td>
<td>$k_{L_4}$ is the cycle of $L_4$</td>
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</tr>
<tr>
<td>$k_{L_5}$</td>
<td>$k_{L_5}$ is the cycle of $L_5$</td>
<td>1</td>
</tr>
<tr>
<td>$k_{L_6}$</td>
<td>$k_{L_6}$ is the cycle of $L_6$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_1}$</td>
<td>$L_1$ to be scheduled $\gamma^{L_1}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_2}$</td>
<td>$L_2$ to be scheduled $\gamma^{L_2}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_3}$</td>
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<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_4}$</td>
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</tr>
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<td>$\gamma^{L_5}$</td>
<td>$L_5$ to be scheduled $\gamma^{L_5}$ times</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma^{L_6}$</td>
<td>$L_6$ to be scheduled $\gamma^{L_6}$ times</td>
<td>4</td>
</tr>
<tr>
<td>$\eta^h$</td>
<td>Heating efficiency</td>
<td>1</td>
</tr>
<tr>
<td>$\eta^{AC}$</td>
<td>AC efficiency</td>
<td>1</td>
</tr>
<tr>
<td>$M$</td>
<td>Air flow rate</td>
<td>1 kg/s</td>
</tr>
<tr>
<td>$T_{\text{h, min}}$</td>
<td>Minimum heater-AC temperature</td>
<td>5°C</td>
</tr>
<tr>
<td>$T_{\text{h, max}}$</td>
<td>Maximum heater-AC temperature</td>
<td>32°C</td>
</tr>
<tr>
<td>$R_{\text{min}}$</td>
<td>Slew rate, temperature decrease</td>
<td>5°C</td>
</tr>
<tr>
<td>$R_{\text{max}}$</td>
<td>Slew rate, temperature increase</td>
<td>5°C</td>
</tr>
<tr>
<td>$T_{\text{init}}$</td>
<td>Initial room temperature</td>
<td>22°C</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>ON/OFF status threshold</td>
<td>1°C</td>
</tr>
</tbody>
</table>
Similarly to the simulation results for the summer scenario discussed in chapter 5, notice how the controller lets the room temperature increasing when the user’s comfort preferences are relaxed (for instance from hour 2.5 to hour 3). Notice than from hour 0 to hour 1, because the price is relatively low compared to the next period, the controller
keeps the room temperature fairly low compared to the maximum limit allowed by the user for that time period.

From hour 4 to hour 5, the room temperature is maintained fairly constant in prevision of the sudden increase in external temperature occurring at hour 5.

FIGURE 37: Heater temperature – general case

FIGURE 38: Heater power consumption – general case
Similarly to what we observed in chapter 4, the biofuel-burning turbine starts as soon as the utility price signal goes above the biofuel price.
FIGURE 42: Storage schedule – general case

FIGURE 43: Power exchanged with the grid – general case
CHAPTER 7
EXTENSION TO THE CASE OF SEVERAL ENERGY PROVIDERS

7.1 - Introduction
In this chapter, we assume that \( u \) energy providers \( U_1, U_2, \ldots, U_u \) are now serving the residential customer. Then, for each \( d_i \), the residential customer has now the ability to deal with one of these providers based on their price signals.

The software now not only has to find the optimal load schedule, but has also to decide which utilities to deal with and when.

7.2 - Mathematical formulation as a linear programming problem
The \( N \) variables \( P_1^{(g)}, \ldots, P_N^{(g)} \) are now replaced by \( u \times N \) variables \( P_1^{(g,1)}, \ldots, P_N^{(g,1)}, \ldots, P_1^{(g,u)}, \ldots, P_N^{(g,u)} \).

Define: 
\[ \forall i \in [1, u], \forall j \in [1, N], \quad P_j^{(g,i),+} > 0 \text{ and } P_j^{(g,i),-} > 0 \text{ such that } P_j^{(g,i)} = P_j^{(g,i),+} - P_j^{(g,i),-}. \]

Let us call \( \textbf{X}^u \) the updated vector \( \textbf{X} \) in the case of \( u \) energy providers:
\[ \textbf{X}^u = \left[ P_1^{(g,1)} \ldots P_j^{(g,a)} \ldots P_N^{(g,a)} \delta_1^{\text{g-}} \ldots \delta_R^{\text{g-}} \delta_1^{\text{g+}} \ldots \delta_R^{\text{g+}} \delta_2^{\text{g+}} \ldots \delta_R^{\text{g+}} \delta_2^{\text{g+}} \ldots \delta_R^{\text{g+}} T_2^{\text{g+}} \ldots T_N^{\text{g+}} P_1^{b, +} \ldots P_u^{b, +} P_1^{b, -} \ldots P_u^{b, -} \right]^T. \]

Recall that by definition:
\[ F_N = \begin{bmatrix} 1 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \]
Define $K_N$:

Let now $A^u, B^u, A_{eq}^u, B_{eq}^u$ be:

$$A^u = \tau \times N$$

$$B^u = B$$

$$A_{eq}^u = [0_{\mathbb{R}((4N+\ell)\times(n-1))} A_{eq}]$$

$$B_{eq}^u = B_{eq}$$
Also define ∀ i ∈ [1, u], \( \mathcal{P}^i = [p_{i}^{l^+} - p_{i}^{l^-} \ldots p_{N}^{l^+} - p_{N}^{l^-}]^T \)

- \( p_{i}^{l^+} \) is the price provider \( i \) is willing to sell electricity to the customer over \( d_j \)
- \( p_{i}^{l^-} \) is the price provider \( i \) is willing to buy electricity from the customer over \( d_j \)

We will assume in the following that:

∀ i ∈ [1, u], ∀ j ∈ [1, N], \( \exists k ∈ [1, u]\setminus\{i\} \mid p_{j}^{k^-} > p_{j}^{i^+} \)

With this condition, we assume that energy providers make contracts at the provider level to exchange power between them, and that their price signals cannot be in such a way that residential customers act as third parties which could buy large quantities of power from provider \( i \) over \( d_j \) to sell this power immediately to provider \( k \) which would pay \( p_{j}^{k^-} > p_{j}^{i^+} \) for it (arbitrage).

Define \( \bar{\mathcal{P}}^u = [\mathcal{P}_1^T \ldots \mathcal{P}_u^T \mathcal{P}_{nom}^{bt} \mathcal{P}_f^T 0 \ldots 0]^T \)

\( (\Sigma_{i=1}^N S_i) + 3N+1 \)

The optimization problem combining the non-interruptible loads and the interruptible loads can finally be expressed as the following mixed-integer linear programming problem:

\[
\begin{align*}
\text{Minimize} & \quad \mathcal{C} = \bar{\mathcal{P}}^u \cdot X^u \\
\text{Subject to} & \quad A^u X^u \leq B^u \\
& \quad A_{eq}^u X^u = B_{eq}^u \\
& \quad \forall i \in [1, N], \quad \delta_{i}^{bt} \in [0,1] \\
& \quad \forall i \in [1, \ell], \forall j \in [1, S_{i}], \quad \delta_{j}^{r,i} \in [0,1] \\
& \quad \forall i \in [1, N], \quad T_{i}^{r,min} \leq T_{i}^{r,0} \leq T_{i}^{r,max}
\end{align*}
\]
7.3 – Simulations

7.3.1- Introduction

To conclude this chapter, we simulate a scenario involving three energy providers in order to illustrate the mathematical theory discussed above. The system (LP-u) is solved for the home grid defined in Table 5.

Assume the following time constraints:

- Load 1 must run between 1:20 and 5
- Load 2 must run between 0 and 2
- Load 3 must run between 4 and 6
- Load 4 must run between 0 and 3:20

Also assume solar and wind modules are off-line for this simulation.
### TABLE 5
**HOMEGRID PARAMETERS – CHAPTER 7**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Assumed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Time horizon</td>
<td>6h</td>
</tr>
<tr>
<td>$N'$</td>
<td>Number of time intervals – subdivision $d'$</td>
<td>18</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>Length of each time interval – subdivision $d$</td>
<td>20min</td>
</tr>
<tr>
<td>$q$</td>
<td>Subdivision factor</td>
<td>4</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of time intervals</td>
<td>72</td>
</tr>
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<td>$\tau$</td>
<td>Length of each time interval</td>
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</tr>
<tr>
<td>$\mu$</td>
<td>Number of energy providers</td>
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</tr>
<tr>
<td>$p_f^t$</td>
<td>Biofuel price</td>
<td>5 c/kWh</td>
</tr>
<tr>
<td>$E_{\text{init}}$</td>
<td>Energy stored at $t = 0$</td>
<td>0 kWh</td>
</tr>
<tr>
<td>$E_{\max}^\text{max}$</td>
<td>Maximum capacity - storage device</td>
<td>3 kWh</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Maximal charging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Maximal discharging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$p_{\text{nom}}$</td>
<td>Nominal power - biofuel-burning turbine</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Number of non-interruptible loads</td>
<td>6</td>
</tr>
<tr>
<td>$p_{L_1}^1$</td>
<td>Washing machine consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$p_{L_2}^1$</td>
<td>Dryer consumption</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$p_{L_3}^1$</td>
<td>Dishwasher consumption</td>
<td>1 kW</td>
</tr>
<tr>
<td>$p_{L_4}^1$</td>
<td>Defrost cycle consumption</td>
<td>1.3 kW</td>
</tr>
<tr>
<td>$p_{L_5}^1$</td>
<td>Water heater consumption</td>
<td>2 kW</td>
</tr>
<tr>
<td>$p_{L_6}^1$</td>
<td>PHEV battery consumption</td>
<td>3 kW</td>
</tr>
<tr>
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<td>$k_{L_1}$ is the cycle of $L_1$</td>
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</tr>
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</tr>
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<td>$y_{L_2}^1$</td>
<td>$L_2$ to be scheduled $y_{L_2}^1$ times</td>
<td>1</td>
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<td>$y_{L_3}^1$</td>
<td>$L_3$ to be scheduled $y_{L_3}^1$ times</td>
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</tr>
<tr>
<td>$y_{L_4}^1$</td>
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<td>$L_5$ to be scheduled $y_{L_5}^1$ times</td>
<td>2</td>
</tr>
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<td>$L_6$ to be scheduled $y_{L_6}^1$ times</td>
<td>4</td>
</tr>
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<td>Heating efficiency</td>
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</tr>
<tr>
<td>$\eta^{ac}$</td>
<td>AC efficiency</td>
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</tr>
<tr>
<td>$\dot{M}$</td>
<td>Air flow rate</td>
<td>1 kg/s</td>
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<tr>
<td>$\tau_{h,\text{min}}$</td>
<td>Minimum heater-AC temperature</td>
<td>5°C</td>
</tr>
<tr>
<td>$\tau_{h,\text{max}}$</td>
<td>Maximum heater-AC temperature</td>
<td>32°C</td>
</tr>
<tr>
<td>$R_{\text{min}}$</td>
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<tr>
<td>$R_{\text{max}}$</td>
<td>Slew rate, temperature increase</td>
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<tr>
<td>$\varepsilon$</td>
<td>ON/OFF status threshold</td>
<td>1°C</td>
</tr>
</tbody>
</table>
As we mentioned in the previous section, we assume in this chapter that the selling price signal and the purchasing price signal are different. Price forecasts for this simulation are given in Figures 44 and 45.

FIGURE 44: Price forecasts – sale – extension to several energy providers

FIGURE 45: Price forecasts – purchase – extension to several energy providers
FIGURE 46: Non-controllable loads forecasts – extension to several energy providers

FIGURE 47: External temperature forecasts – extension to several energy providers

FIGURE 48: Room temperature – extension to several energy providers
FIGURE 49: Heater temperature – extension to several energy providers

FIGURE 50: Heater power consumption – extension to several energy providers

FIGURE 51: Heater status – extension to several energy providers
Notice in Figure 53 and 54 that the biofuel-burning turbine schedule and the storage schedule are not as predictable as they were in the previous simulations because we now have differentiated prices for selling and purchasing electricity.
Notice in Figure 55 that, logically, the energy provider chosen at a given time is always the cheapest one. The only exception is for an interval where providers 1 and 3 both sell electricity at exactly the same price (*).
Figure 56 shows that in this scenario, electricity is never sold back to any energy providers because the purchasing prices are always lower than the selling prices. Then, there is no reason for the customer to buy and store electricity in order to sell it back later as he is certain to lose money in the process. It is also in the interest of the user to store his local production in excess, which is free, in order to use it later instead of selling it to the grid.
8.1 - Introduction
In this chapter, we consider a real-time scenario where the energy provider sends an incentive signal at \( t = t_{u,0} \) to encourage residential customers to reduce their energy consumption for a given period of time. This situation can result of excessive demand or emergency conditions for example. The controller analyses the incentive signal and updates the optimum load schedule accordingly.

8.2 - Mathematical formulation as a linear programming problem
Assume that the optimization algorithm was run prior to the beginning of the period \( D \). A subdivision \( d = \{ d_1, d_2, ..., d_i, ..., d_N \} \) was chosen, and there is an existing optimal schedule for the controllable loads.

The monitoring systems has turned on and off the non-interruptible loads accordingly since the beginning of \( D \), and should keep doing so until its end. The heating/AC is run in real time by a separate control system which bases its decisions on both the optimal scenario forecast and the actual temperatures of the room and the outside. If the temperature forecasts and the thermal model of the house are reliable, the actual schedule of the heating/AC should be very close to the optimal schedule determined beforehand and based on the temperature forecasts.

At \( t = t_{u,0} \), the provider sends a signal to the customer proposing an incentive to encourage the customer to reduce his consumption over a period \( \tau_u = t_{u,f} - t_{u,0} \).
At time $t_{u,0}$, assume that we are somewhere in the time interval $d_m$. Then, after receiving this signal, we are going to re-evaluate our controllable load schedule from the beginning of $d_{m+1}$ to the end of $d_N$. Call this interval $D_{new}$.

Any non-interruptible load which has been already turned on before receiving the provider signal has to complete its cycle and cannot be interrupted.

If non-interruptible loads were scheduled over $D_{new}$, this schedule is going to be re-examined to check if, with the new offer received from the provider, a better schedule can be found.

The signal received from the provider allows the customer to define a vector $[i_1 \ i_2 \ ... \ i_n]^T$, with $i_j$ the incentive the provider is willing to pay per power unit reduced over $d_j$. Thus, if the customer reduces his power consumption by $\Delta P_j^g$ over $d_j$, the provider will pay him $i_j \times \Delta P_j^g$ for this time interval.
Knowing this, the user is going to solve again the mixed-integer linear programming problem (LP) over $D_{new}$ conserving the same $\tau$.

Note that if $m \not\equiv 0 \pmod{q}$, meaning that $m$ and 0 are not congruent modulo $q$, then $\sigma$ intervals $d_{N-\sigma+1} \ldots d_N$ are removed at the end of $D_{new}$ so that $N - \sigma - m \equiv 0 \pmod{q}$. If $r$ is the remainder of the division algorithm of $m$ by $q$, we have $\sigma = q - r$.

First, knowing the value of the parameter $m$, we need to detect which non-interruptible loads could potentially be rescheduled. Non-interruptible loads which were run before the beginning of $d_{m+1}$ are now off picture; non-interruptible loads which started before the beginning of $d_{m+1}$ cannot be interrupted; non-interruptible loads which were scheduled to start at or after the beginning of $d_{m+1}$ can be rescheduled.

The initial optimization provided for each load $L$ optimal values of the position parameters $\delta_L^1 \ldots \delta_L^L$. Given the value of $m$, we can determine for each load $L$ what is the status of the position $j$ with respect to $d_{m+1}$:

**Case a**: the period corresponding to the position $j$ is to start at or after the beginning of $d_{m+1}$. This case occurs when $m + 1 \leq (j - 1)q + 1$

**Case b**: the period corresponding to the position $j$ started before the beginning of $d_{m+1}$, and ends after the beginning of $d_{m+1}$.
This case occurs when $(j - 1)q + 2 \leq m + 1 \leq (j - 1)q + 1 + (k_L - 1)q + (q - 1)$
**Case c:** the period corresponding to the position \( j \) ended before the beginning of \( d_{m+1} \).

This case occurs when \((j - 1)q + 1 + (k_L - 1)q + (q - 1) + 1 \leq m + 1\)

This allows us, for each load \( L \), to scan all the possible position statuses, and to come up with the positions corresponding to the time slots starting at or after the beginning of \( d_{m+1} \).

Once these positions are identified, the corresponding values of the position parameters \( \delta^L_i \) are scanned in order to determine how many times the load \( L \) was scheduled in \( D_{new} \).

This allows us to determine how many times this load is to be rescheduled. This number is called \( y_{new}^L \).

Finally, the number of new possible positions between \( d_{m+1} \) and \( d_{N-\sigma} \) for the load \( L \) is given by:

\[
S_{L,new} = (N - \sigma - m) - k_L + 1
\]

**Example:** for \( N' = 6 \), \( q = 3 \), \( \ell = 1 \), \( k_{L_1} = 2 \), \( y'^{L_1} = 1 \)

From this we can derivate \( N = 18 \), \( S_{L_1} = N' - k_{L_1} + 1 = 5 \)
Assume that before the beginning of $d_1$ the optimization problem was solved, and that the software determined that the best position to run $L_1$ was position 5.

Then $\delta_{1}^{L_1} = \delta_{2}^{L_1} = \delta_{3}^{L_1} = \delta_{4}^{L_1} = 0$, $\delta_{5}^{L_1} = 1$.

Assume that at some point over $d_8$ we receive an incentive signal from the provider.

Then we have $m = 8$.

Let us determine the status of each position $j$:

\[
j = 1 : \quad (j - 1)q + 1 + (k_L - 1)q + (q - 1) + 1 = 7 \leq m + 1 = 9 \rightarrow \text{case c}
\]

\[
j = 2 : \quad (j - 1)q + 2 = 5 \leq m + 1 = 9 \leq (j - 1)q + 1 + (k_L - 1)q + (q - 1) = 9 \rightarrow \text{case b}
\]

\[
j = 3 : \quad (j - 1)q + 2 = 8 \leq m + 1 = 9 \leq (j - 1)q + 1 + (k_L - 1)q + (q - 1) = 12 \rightarrow \text{case b}
\]

\[
j = 4 : \quad m + 1 = 9 \leq (j - 1)q + 1 = 10 \rightarrow \text{case a}
\]

\[
j = 5 : \quad m + 1 = 9 \leq (j - 1)q + 1 = 13 \rightarrow \text{case a}
\]

From the status analysis we see that only positions 4 and 5 are to be considered for rescheduling. Given that $\delta_{4}^{L_1} = 0$, $\delta_{5}^{L_1} = 1$, we know that the load $L_1$ is to be rescheduled only once over $D_{new} \cdot y_{new}^{L_1} = 1$.

Also, $m \not\equiv 0 \pmod{q}$, and $8 = 2 \times 3 + 2$, then $\sigma = q - r = 3 - 2 = 1$.

Finally, we are to reschedule $L_1$ once between $d_9$ and $d_{17}$.
Let us now come back to the general case, and call $X$ the previous optimal solution to (LP). With the previous notations, we had:

$$C = \tilde{P}^T \cdot X = p^u_1 r_1^{(g)} + \cdots + p^u_N r_N^{(g)} + p^{bt}_{nom} p f \times (\delta^b_1 + \cdots + \delta^b_N)$$

Re-indexing the $p^{(g)}_{i,j}$ we have:

$$C = \tilde{P}^T \cdot X = p^u_1 r_1^{(g)} + \cdots + p^u_N r_N^{(g)} + p^{bt}_{nom} p f \times (\delta^b_1 + \cdots + \delta^b_N)$$

Let us call $c'$ the cost corresponding to the power exchanged with the utility grid over $D_{new}$ according to the pre-existing loads schedule:

$$c' = p^u_{m+1} r_{m+1}^{(g)} + \cdots + p^u_{N-\sigma} r_{N-\sigma}^{(g)} + p^{bt}_{nom} p f \times (\delta^b_{m+1} + \cdots + \delta^b_{N-\sigma})$$

Define $\tilde{P}'$ and $X'$ such that $c' = \tilde{P}'^T \cdot X'^T$.

$$\tilde{P}' = \begin{bmatrix} p^u_{m+1} & \cdots & p^u_{N-\sigma} & p^{bt}_{nom} p f \mathbf{1}_{\mathbb{R}^{N-\sigma}} T \mathbf{0}_{\mathbb{R}^{3(N-\sigma)+1+\Sigma_{i=1}^{m} s_{i,new}}} \end{bmatrix}$$

$$X' = \begin{bmatrix} r_{m+1}^{(g)} & \cdots & r_{N-\sigma}^{(g)} & \delta^b_{m+1} & \cdots & \delta^b_{N-\sigma} & \mathbf{0}_{\mathbb{R}^{3(N-\sigma)+1+\Sigma_{i=1}^{m} s_{i,new}}} \end{bmatrix}$$

Let us now call $p^{(g)}_{m+1,new}, \ldots, p^{(g)}_{N-\sigma,new}$ the power exchanged over $D_{new}$ after the re-evaluation of the customer’s consumption following the reception of the provider signal.

The new cost $c_{new}$ can be expressed as:

$$c_{new} = p^u_{m+1} r_{m+1,new} + \cdots + p^u_{N-\sigma} r_{N-\sigma,new} + p^{bt}_{nom} p f \times (\delta^b_{m+1,new} + \cdots + \delta^b_{N-\sigma,new})$$

$$- i_1 (p^{(g)}_{m+1,new} - p^{(g)}_{m+1,new}) \cdots - i_n (p^{(g)}_{m+n,new} - p^{(g)}_{m+n,new})$$
Then,

\[ C_{\text{new}} = (p_{m+1}^u + i_1)p_{m+1,\text{new}}^{(g)} + \cdots + (p_{m+n}^u + i_n)p_{m+n,\text{new}}^{(g)} - i_1 p_{m+1}^{(g)} - \cdots - i_n p_{m+n}^{(g)} \]

\[ + p_{m+n+1}^u p_{m+n+1,\text{new}}^{(g)} + \cdots + p_{N-\sigma}^u p_{N-\sigma,\text{new}}^{(g)} + p_{\text{nom}}^t p^f \]

\[ \times (s_{m+1,\text{new}}^{bt} + \cdots + s_{N-\sigma,\text{new}}^{bt}) \]

Let us define \( J = [i_1 \ i_2 \ \ldots \ i_n \ 0_{(N-\sigma)+1}\ \gamma_{l=1}^{s_{L_i,\text{new}}} \] \( J^T \)

Then we have:

\[ C_{\text{new}} = (J + \bar{\mathcal{P}})^T X_{\text{new}} - J^T X' \]

After receiving the provider signal, we then solve the following mixed-integer linear programming problem:

\[
\begin{align*}
\text{Minimize} & \quad C_{\text{new}} = (J + \bar{\mathcal{P}})^T X_{\text{new}} - J^T X' \\
\text{Subject to} & \quad A_{\text{new}} X_{\text{new}} \leq B_{\text{new}} \\
& \quad A_{eq,\text{new}} X_{\text{new}} = A_{eq,\text{new}} \\
& \quad \forall i \in \{m+1, N-\sigma\}, \ \delta_i^{bt} \in [0,1] \\
& \quad \forall i, j, \ \delta_i^{ij} \in [0,1] \\
& \quad \forall i \in \{m+1, N-\sigma\}, \ T_{r,m}^{r,\text{min}} \leq T_i^r \leq T_{r,max}^{r,\text{max}}
\end{align*}
\]

\( X_{\text{new}}, A_{\text{new}}, B_{\text{new}}, A_{eq,\text{new}}, A_{eq,\text{new}} \) are defined similarly to \( X, A, B, A_{eq}, B_{eq} \) substituting \( N - \sigma - m \) for \( N \) and \( S_{L_i,\text{new}} \) for \( S_{Li} \).

(LP-new) gives us a new optimal schedule. If \( C_{\text{new}} < C' \), this new schedule is adopted.

At the same time, the system displays a message inviting the customer to reduce the consumption of his non-controllable loads until \( t = t_{u,f} \).
8.3 - Simulation

8.3.1- Introduction

To conclude this chapter, we simulate a scenario involving both interruptible and non-interruptible loads in order to illustrate the mathematical theory discussed above. The system (LP-new) is solved for the home grid defined in Table 6.

Assume the following time constraints apply:

- Load 1 must run between 1:20 and 6
- Load 2 must run between 0 and 2
- Load 3 must run between 3:40 and 6
- Load 6 must run between 0 and 1:40

Assume solar module is off-line for this simulation.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Assumed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D'$</td>
<td>Time horizon</td>
<td>6h</td>
</tr>
<tr>
<td>$N'$</td>
<td>Number of time intervals – subdivision $d'$</td>
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</tr>
<tr>
<td>$\tau'$</td>
<td>Length of each time interval – subdivision $d$</td>
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<tr>
<td>$q$</td>
<td>Subdivision factor</td>
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<tr>
<td>$N$</td>
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<td>Energy stored at $t = 0$</td>
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<tr>
<td>$E_{\text{max}}$</td>
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</tr>
<tr>
<td>$R^c$</td>
<td>Maximal charging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$R^d$</td>
<td>Maximal discharging rate - storage device</td>
<td>0.5 kWh</td>
</tr>
<tr>
<td>$p_{\text{nom}}$</td>
<td>Nominal power - biofuel-burning turbine</td>
<td>1.5 kW</td>
</tr>
<tr>
<td>$\eta^{(w)}$</td>
<td>Wind turbine efficiency</td>
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</tr>
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<td>$S^{(w)}$</td>
<td>Wind turbine eq. surface</td>
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<td>$I$</td>
<td>Number of non-interruptible loads</td>
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<td>Washing machine consumption</td>
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<tr>
<td>$P^L_2$</td>
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</tr>
<tr>
<td>$P^L_3$</td>
<td>Dishwasher consumption</td>
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<td>$P^L_4$</td>
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<td>PHEV battery consumption</td>
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<td>$\gamma^{L_2}$</td>
<td>$L_2$ to be scheduled $\gamma^{L_2}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_3}$</td>
<td>$L_3$ to be scheduled $\gamma^{L_3}$ times</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma^{L_4}$</td>
<td>$L_4$ to be scheduled $\gamma^{L_4}$ times</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma^{L_5}$</td>
<td>$L_5$ to be scheduled $\gamma^{L_5}$ times</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma^{L_6}$</td>
<td>$L_6$ to be scheduled $\gamma^{L_6}$ times</td>
<td>4</td>
</tr>
<tr>
<td>$\eta^h$</td>
<td>Heating efficiency</td>
<td>1</td>
</tr>
<tr>
<td>$\eta^{ac}$</td>
<td>AC efficiency</td>
<td>1</td>
</tr>
<tr>
<td>$M$</td>
<td>Air flow rate</td>
<td>1 kg/s</td>
</tr>
<tr>
<td>$\tau^{h,\text{min}}$</td>
<td>Minimum heater-AC temperature</td>
<td>5°C</td>
</tr>
<tr>
<td>$\tau^{h,\text{max}}$</td>
<td>Maximum heater-AC temperature</td>
<td>32°C</td>
</tr>
<tr>
<td>$R^{\text{min}}$</td>
<td>Slew rate, temperature decrease</td>
<td>5°C</td>
</tr>
<tr>
<td>$R^{\text{max}}$</td>
<td>Slew rate, temperature increase</td>
<td>5°C</td>
</tr>
<tr>
<td>$\tau^{\text{init}}$</td>
<td>Initial room temperature</td>
<td>22°C</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>ON/OFF status threshold</td>
<td>1°C</td>
</tr>
</tbody>
</table>
8.3.2- Initial optimization

Similarly to what we did in chapter 6, let us determine the best schedule over the time period $D$.

**FIGURE 57:** Price forecasts – Initial optimization

**FIGURE 58:** Wind speed forecasts – Initial optimization

**FIGURE 59:** Non-controllable loads forecasts – Initial optimization
FIGURE 60: External temperature forecasts – Initial optimization

FIGURE 61: Room temperature – Initial optimization

FIGURE 62: Heater temperature – Initial optimization
FIGURE 63: Heater power consumption – Initial optimization

FIGURE 64: Heater status – Initial optimization

FIGURE 65: Biofuel-burning turbine schedule – Initial optimization
The price forecasted for this initial schedule is $8.31.
8.3.2- Real-time response

Assume now that at \( t = 2:58 \), the provider sends an incentive signal with \( t_{u,f} = 4 \).

Then \( m = 36, \sigma = 0 \).

Assume that \( \forall j, \ i_j = 3 \text{ } \$/kW \) (constant incentive signal).

![Incentive Signal](image)

FIGURE 69: Incentive signal – Real time response

![Non-Controlable Loads Forecasts](image)

Load 1 is added (already ON at \( t = 2:58 \))

FIGURE 70: Non-controllable loads forecasts – Real time response

At \( t = 2:58 \), load 1 is already running and is expected to end at \( t = 3:20 \). Load 1 cannot be interrupted, then its remaining consumption is added to the non-controllable loads forecasts (Figure 70).
The controller allows the room temperature to go higher to save energy.

FIGURE 71a: Room temperature – Real time response

Compare with original schedule

FIGURE 71b: Room temperature – Original schedule

The biofuel burning turbine starts earlier.

FIGURE 72: Biofuel-burning turbine schedule – Real time response
The original schedule corresponded to a cost of $8.31. The new schedule corresponds to a cost of $6.54. Then, the new scheduled is accepted.
CHAPTER 9
CONCLUSIONS

The objective of the present thesis was to formulate the optimization problem faced by a home energy manager controller, to implement the optimization controller, and to perform various simulations to evaluate the performance of the controller and the changes in energy use.

Several conclusions can be drawn based on this work:

Regarding system modeling, most of the relations between the unknown variables describing the system at a given time were shown to be linear. Equation (16) is the only equation we had to approximate so that the final system remains a linear function of the room temperature. But provided that $\tau$ is small enough, this approximation is fairly reasonable.

In this thesis we have assumed that the parameters of the model were constant. In future implementations, additional equations will be included to take into account the effects of the age and past history of the system, the number and frequency of charging and discharging cycles, and the external temperature, on parameters such as the efficiency $\eta^{(s)}$ of the solar panels, the maximum capacity $E^{\text{max}}$ of the storage system, or the charging and discharging rates $R^c$ and $R^d$. 
Concerning the thermal model of the house, some state-estimation algorithm will be implemented in parallel to refine the empirical value of $R_{eq}$ throughout time.

In future implementations, parameter values will still be maintained constant for a given optimization, but their values will be updated between each execution of the optimization algorithm. Note that the linearity, or non-linearity, of the potential effects of these external factors on the numerical values of the equation parameters will not affect the mathematical formulation of the optimization problem.

Based on the system modeled, we demonstrated that the optimization problem faced by a home energy manager controller could be formulated as a mixed-integer linear programming problem. This mathematical formulation encompasses both the physical constraints and the customer’s comfort preferences. It is general enough to be used in both day ahead and real time response scenarios, and has been extended to the case of several energy providers.

Linear programming is a very important field of optimization as many practical problems can be expressed as linear programming problems in a wide range of fields (planning, production, transportation, etc.). This will allow us to use existing algorithms and numerical analysis methods applying to this class of problems in future implementations in order to improve the computational performance.

In this thesis, the controller was implemented in the Matlab environment and the algorithm used was based on the function *linprog*. Depending on the scenario, the algorithm was very fast (few seconds), or took up to several minutes to return a solution.
Scenarios limited to the heating/AC system presented in chapter 5 are the fastest to solve since they only involve continuous variables. In scenarios involving integer variables modeling non-interruptible loads, the computational time increases with the dimension of the solution space for the non-controllable loads schedule given by $\prod_{i=1}^{F} \left( \frac{\sum_{i}^{y} l_i}{y} \right)$. At some point, a multistart approach was necessary as the algorithm was sometimes unable to find a feasible point to begin with, or made an initial guess too far from the optimal point.

Future implementations will include pretreatment algorithms on the matrices $A$, and $A_{eq}$ to improve the condition numbers $\kappa(A)$ and $\kappa(A_{eq})$ in order to ensure a well-conditioned problem. Additional numerical simulations will be conducted to develop heuristic methods in order to formulate an educated guess for the initial solution point. This will increase the robustness and the resolution speed of the controller.

The proposed control architecture at the home level presents several advantages in terms of efficiency and privacy. This distributed control architecture avoids data transmission to controllers higher in the hierarchy, hence potential congestions on the communication network. With this architecture, the end-user keeps full control on all the devices connected to his home grid, and eventually decides what to turn on and off, and when. This architecture also protects the customer’s ownership on his consumption data since all the calculations are handled locally.

This distributed intelligence could be embedded directly into the smart meter, with communication capacities in order to receive the price and weather forecast signals, and broadcast the optimal schedule to the devices connected to the home grid. However,
because the meter is usually localized outside in order to facilitate access for the utility employees, this solution might present security and privacy weaknesses. Another solution would be to physically separate the metering function from the other functions. An electrical device would be maintained outside to take care of the metering function. At the same time, a local server would handle all the calculations. This server would be able to communicate with the home grid devices, including the energy meter, the loads and the various sensors, and could get the price and weather forecast signals from a dedicated web portal. The user could communicate with this server via a dedicated touch screen, or via other communication devices (phone, tablet computer, personal computer, etc.) in order to enter his comfort preferences or check his daily consumption.

The proposed pricing policy was shown to be very efficient in successfully achieving demand response. The use of day-ahead price signals allows the customer to optimally schedule his consumption in order to minimize his bill. Of course, the controller avoids scheduling loads during peak pricing period. In a dynamic pricing environment, energy providers are able to design price signals in such a way that they reflect the actual marginal prices. Our simulations showed that this eventually influences the consumption patterns of the customer. Future simulations will explore the implications of this result on demand peak shaving and loss reduction at the micro-grid and distribution level.

In chapter 8, we simulated a real-time scenario involving responding to an incentive signal. Our results show that the controller was successfully able to reschedule the controllable loads to respond to this signal, and increase the amount of power produced locally by the controllable generation unit. Other real-time scenarios will be considered in
future implementations, involving updated weather forecasts signals, or signals resulting of emergency situations affecting the utility grid.

Notice that in both day ahead and real time, the energy consumption was optimized over a given temporal window based on price variations rather than price absolute values. This makes the proposed pricing policy fully compatible with government regulations imposing minimum and/or maximum price limits to energy providers.

Finally, the extension of the general case to a case with several energy providers showed that the controller was able to optimally determine, for a given pool of energy providers, when to buy electricity from each company based on its price signal.

As described in chapter 2, energy providers today have a strong market power on the retail market for residential customers. The implementation of this feature at the home level would allow deciding almost in real time from which energy provider to buy electricity from, and re-equilibrate the relation between the end-user and the energy provider.

In the simulations illustrating the multi-providers scenario, we assumed that the providers had made contracts at the provider level to exchange power between them, and that customers could not act as third parties in that process. However, our mathematical formulation is general enough to allow future simulations where the customer will not only be able to consume, produce and store energy, but also to buy power from provider \( i \) in order to sell this power immediately to provider \( k \), or to another consumer, and make profits.
In order to minimize the customer's electricity bill, this thesis recognized the importance of the ability to shift the time when electric energy is transferred from the utility grid to the home grid. This was achieved through loads rescheduling, and using an energy storage system. Because the electricity time of use is directly related to the life habits of the customer and his comfort preferences, future work will develop this relation between the storage function and the ability to enforce the customers' consumption preferences. In particular, the economical value of storage at the residential level will be studied, and the consumer comfort preferences will be priced based on the costs of energy storage technologies.

Concerning the distributed energy resources, the proposed mathematical formulation will allow future work to simulate the financial viability of a deployment of such technologies at the residential level. Our simulations show that distributed generation capabilities allow the customer to reduce his electricity bill. Additionally, the use of controllable units such as the biofuel-burning turbine featured in that thesis allows the user to respond to real-time request from the energy provider by increasing the local production.

In future work, the mathematical theory will be extended to take into account feed-in tariff policies, and simulations using estimations of price signal profiles for the coming years will allow conducting cost-benefit analysis and rate of return calculations of such technologies at the residential level.
Finally, the mathematical formulation based on the linear programming theory will be used to conduct future multi-agent based simulations at the micro-grid and utility grid levels. These simulations will explore the consequences of the proposed architecture on topics such as the resulting demand profile of a given network made of several independent agents, the distribution losses and their potential reduction, and the price elasticity of electrical energy.
Assume for simulations:

\[ L = 30 \text{ m} \]
\[ W = 10 \text{ m} \]
\[ H = 4 \text{ m} \]
\[ \theta = 40^\circ \]

The total wall area is
\[ S_{\text{wall}} = 2HL + 2WH + \frac{WL}{\cos(\theta)} + 2W^2\tan(\theta) \]

Also assume the presence of 6 windows of a total area \( S_{\text{window}} = 6 \text{ m}^2 \)

Let us call \( L_{\text{wall}} \) the thickness of the walls, \( L_{\text{window}} \) the thickness of the windows.

Assume for simulation \( L_{\text{wall}} = 0.2 \text{ m} \) and \( L_{\text{window}} = 0.01 \text{ m} \)
APPENDIX B
CALCULATION OF THE EQUIVALENT THERMAL RESISTANCE

In the following, convective heat transfers between the walls and the air, and the windows and the air, are neglected.

Consider a section $S$ of a material of thermal conductivity $\lambda$, with a thickness $e$. The thermal resistance $R_{th}$ of this material is defined by:

$$R_{th} \equiv \frac{e}{\lambda S}$$

Define $\lambda_{wall}$ the thermal conductivity of the walls (including the roof), and $\lambda_{window}$ the thermal conductivity of the windows.

Assume for simulations $\lambda_{wall} = 0.038 \, J/s\cdot m/C$ and $\lambda_{window} = 0.78 \, J/s\cdot m/C$.

We have:

$$R_{wall} = \frac{L_{wall}}{\lambda_{wall} S_{wall}} \quad R_{window} = \frac{L_{window}}{\lambda_{window} S_{window}}$$

Let call $R_{eq}$ the equivalent thermal resistance of the house. By analogy with the equivalent resistance of two electrical resistances in parallel:

$$R_{th} = \frac{R_{wall} R_{window}}{R_{wall} + R_{window}}$$
REFERENCES


