FINAL REPORT

Project A-874

BUBBLE MEASUREMENT IN SEA WATER

Arthur L. Bennett

Contract N600(24)-59885
Modifications Nos. 15 and 16

March 1970

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia

Prepared for
Navy Ship Research and Development Laboratories
Panama City, Florida
Georgia Institute of Technology  
Engineering Experiment Station  
Atlanta, Georgia  30332

FINAL REPORT  
Project A-874

Arthur L. Bennett

Contract N600(24)-59885  
MODS. Nos. 15 and 16

March 1970

Performed for  
Navy Ship Research and Development Laboratories  
Panama City, Florida
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. BACKGROUND</td>
<td>2</td>
</tr>
<tr>
<td>A. Bubble behavior</td>
<td>2</td>
</tr>
<tr>
<td>B. Bubble measurements</td>
<td>4</td>
</tr>
<tr>
<td>C. The Coulter Counter</td>
<td>5</td>
</tr>
<tr>
<td>1. The aperture</td>
<td>7</td>
</tr>
<tr>
<td>III. EXPERIMENTAL</td>
<td>13</td>
</tr>
<tr>
<td>A. Equipment</td>
<td>13</td>
</tr>
<tr>
<td>1. Bubble generator</td>
<td>13</td>
</tr>
<tr>
<td>2. Photographic technique</td>
<td>16</td>
</tr>
<tr>
<td>B. Bubble measurements</td>
<td>17</td>
</tr>
<tr>
<td>1. Coulter Counter measurements</td>
<td>17</td>
</tr>
<tr>
<td>a. Count analysis</td>
<td>18</td>
</tr>
<tr>
<td>2. Photographic bubble measurements</td>
<td>19</td>
</tr>
<tr>
<td>IV. Conclusions</td>
<td>21</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>22</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The purpose of this investigation is the evaluation of the Coulter Counter for determining the bubble population of sea water.

The acoustic properties of sea water are of continuing interest to the US Navy because of the generally high transparency of the water compared to electromagnetic radiation. There are areas, however, such as wakes, the upper layers in the presence of waves, and the surf zone, where bubbles may strongly influence propagation.
II. BACKGROUND

The influence of bubbles on the transmission of sound in sea water was first investigated by Willis. Not only is the target area many times the dimension of the bubbles, but the attenuation is also high. Additional data and analysis are provided in "Propagation of Sound Through a Liquid Containing Bubbles" by Carstensen and Foldy, and by Fox, Curley, and Larson. Acoustical measurements of tap water over the frequency range 10 to 10 kc/sec are given by Iyengar and Richardson.

The generation of bubbles by wave action has been investigated by Glotov, Kolobaev, and Neuimin. This paper describes a bubble catcher for the photographic measurement of bubbles. The contribution of bubbles near the surface to the propagation of sound at grazing incidence was investigated by Clay and Medwin.

A. Bubble behavior

Liebermann has described the evanescent character of bubbles. Two dissipative effects are operative, the rise of the bubble toward the surface because of its buoyancy, and its absorption by diffusion of the gas into solution in the water. Liebermann shows, for bubbles (100 microns radius (1 micron = 10^-6 meter), that the Stokes relation for the velocity of rise, \( v \),

\[
v = \frac{(2/9) g R^2 (\rho - \rho')}{\eta}
\]

is valid, where \( g \) is the acceleration of gravity, \( R \) the bubble radius,
\((\rho - \rho')\) the difference in densities of the bubble and the surrounding liquid, and \(\eta\) the viscosity. It should be noted that the gas density is dependent on the depth as well as temperature, even though it can usually be neglected entirely. For small bubbles, surface tension contributes substantially to the internal pressure, e.g., 1.4 atmosphere at a radius of one micron.

The dissipation of the gas of the bubble by solution in the surrounding liquid as controlled by the gas concentration in the liquid was formulated by Liebermann in the approximate relation,

\[ R^2 = R_0^2 - \frac{2\kappa}{\rho} (C_s - C_o)t \]

where \(R\) is the bubble radius at time \(t\), \(R_0\) the radius at \(t = 0\), \(\kappa\) the coefficient of diffusion, \(\rho\) the density of the liquid, \(C_s\) the gas concentration at saturation, and \(C_o\) the concentration in the liquid remote from the bubble.

The slight departure from linearity in the \(R\) dependence on the time in the experimental measures may be accounted for by the contamination of the bubble surface. Liebermann postulates a film of thickness \(\Delta R\) with another diffusion coefficient \(\kappa_1\). The relation then becomes

\[ R^2 = R_0^2 - \left(\frac{2\kappa}{\rho}\right) (C_s - C_o)t + \left(\frac{2\kappa\Delta R}{\kappa_1}\right) (R_0 - R). \]

The value of \(\kappa\Delta R/\kappa_1\) derived from the best fit is about 0.06. The coefficient of diffusion of a freely rising bubble (about twice that of a stationary bubble) was found to be about \(6.25 \times 10^{-5}\) at 25°C.

Turner uses the expression for the change in radius due to absorption
in terms of the partial pressure of the gas dissolved in the liquid, $P_p$, and the hydrostatic pressure, $P_o$:

$$\frac{\partial (R/R_0)}{\partial t} = -\frac{\chi R}{R_0^2} \cdot \frac{P_o + 2/R - P}{P_o + (4\sigma/3R_0)}$$

where $\chi$ is the diffusivity constant, $\alpha$ the solubility constant, and $\sigma$ the surface tension between the liquid and the gas. This expression is derived on the implicit assumption that there is no film or skin at the bubble surface; that is, the partial pressure of the gas in the water is maintained at the bubble surface. Stokes law is used for the rise velocity.

Turner has computed the behavior of bubbles in filtered tap water saturated with air. In his Fig. 2 the change in size and height of rise of bubbles from 10- to 100-microns initial radius released at a depth of 1.37 meters in water saturated with air is shown as a function of time after bubble formation. A critical initial bubble radius of 60 microns is evident for this depth of release. Bubbles of radius 60 microns or less will be absorbed before they reach the surface, the rise increasing with the initial radius. Bubbles larger than 60 microns initial radius will reach the surface; the loss in radius decreases with the size because of the higher velocity of rise and the lower ratio of surface to volume. In water less than saturated, solution would be more rapid.

B. Bubble measurements

Even though the current interest in bubbles centers on acoustic reflection and absorption, these effects must be interpreted by independent
measurements of the physical properties of the medium. Gross foreign bodies in the water, such as fish and other sizeable objects are not under consideration, but only solids or bubbles in the millimeter to micron range of the order of a tenth of an acoustic wave length.

Solid particles small relative to the wave length are generally assumed to have negligible effect on the transmission of sound except for a small amount of Rayleigh scattering. Preliminary results of measurement of volume back-scattering strength at 19.5 kHz and particle counts from 0.7 to 70 microns diameter in the vicinity of Key West have been reported by NRL. The log of the particle count measured at 100-foot intervals in depth follows the volume scattering strength (db), including a rough agreement of the maximum count at 750-foot depth with the deep scattering layer peak at 600-foot depth. The acoustic back-scattering, however, was found to follow a first power variation with depth in the shallower layers, rather than the fourth power expected for Rayleigh scattering. The particle counts may, therefore, indicate small organisms. The size of the acoustic effect implies either gas bubbles in the animals or attached to them.

Bubble measurement at Georgia Tech concerns the evaluation of the Coulter Counter as a means of measurement of bubbles in sea water. Other commercial devices, such as the HIAC Counter which depends on the absorption or scattering of light by suspended particles, appear to offer no advantage over the Coulter Counter and would pose a comparable problem of calibration for counting bubbles.

C. The Coulter Counter

The industrial Model A of this equipment measures an equivalent volume
of a particle which has an electrical resistivity which is high relative to the resistivity of the electrolyte serving as suspension. Most particles including finely divided metal show a resistance high enough to qualify.

A large range of equivalent particle diameters can be measured by the successive use of apertures of increasing size. Apertures are made ranging in diameter from 11 microns to 2000 microns. Reliable counts are obtained for a given aperture over the diameter range from 2% to 40% of the aperture diameter.

A DC potential of 300 volts with one or more of a set of resistors in series is placed across platinum electrodes in the electrolyte, one on each side of the aperture.

The I-setting (which selects the series resistance) determines the current through the aperture, hence the voltage (or height) of the pulse corresponding to a particle of a certain size passing through the aperture. The ten steps of the I-switch each increase the current in steps of \( \sqrt{2} \).

As particles are drawn through the aperture by the flow of electrolyte, pulses are produced in the current. The voltage drop across the series resistor is amplified and presented on a cathode-ray tube for monitoring the operation. The CR presentation shows the pulses brightened above the level corresponding to the threshold setting. A pulse amplifier delivers the pulses above threshold to a counter. The first three decades are high-speed neon lights; four higher decades are counted on a mechanical register. The gain of the amplifier is controlled by a Gain switch with six steps, each step increasing the gain by \( \sqrt{2} \). The amplified pulses are also fed to a linear voltage divider operated by the Threshold dial, calibrated in percentage, 0 to 100.
The Threshold setting, by eliminating pulses less than the corresponding height, provides an independent setting of sensitivity and establishes the lower limit of the pulse height counted.

In the use of the Coulter Counter bench-stand a measured quantity, 50, 500, or 2000 microliters is passed through the aperture for a count. The flow is drawn through the aperture by the release of mercury from a reservoir with about 15 cm head. Contacts in a horizontal section of tubing control the start and end of the counting period corresponding to the volume selected. The counter can also be used with an external timer with constant flow through the aperture; this alternative was used with the bubble generator described later.

Each aperture is calibrated by a dispersion of particles of fairly uniform size in the electrolyte in use. With sea water, the particulate matter was removed with a 0.45 micron filter and the calibrating particles added. The size distribution of the standard was determined by measurement with the microscope. The larger particles (in the range 20 to 100 microns) are usually pollen. Latex suspensions are available for small particle calibration.

1. The aperture

The flow pattern through the Coulter Counter aperture and the effect of the flow on entrained bubbles must be evaluated before the counter can be used with confidence for the measurement of bubbles. Examination of several apertures confirms the statement of the local representative that the thickness of the sapphire plate in which the round hole is drilled is three-fourths the diameter of the hole. The edges of
the orifice are slightly chipped. The volume coefficients for the hydraulic head in use, 55 cm of water, are 0.80 and 0.77 for the 400 and 280 micron apertures, respectively. For the velocity corresponding to the head, 330 cm/sec, the Reynolds numbers are 1350 and 940, respectively. It is unlikely that turbulence is appreciable within any of the apertures because of the short length.

A search has not established the details of the flow in a (nearly) sharp orifice entering a short flooded tube. Since the Coulter Counter is known to give reliable measurements for particles 40% to, at most, 50% of the diameter of the aperture, the flow pattern in the tube appears not to be restrictive on the use.

The operating pressure at the entrance of the aperture was a maximum of 90 cm of water and 35 cm at the exit in the present equipment. At Atlanta the average atmospheric pressure corresponds to 1000 cm of water. The static expansion of the bubble due to the pressure decrease from 1090 cm absolute head to 1035 cm is 5% in volume or 1.5% in radius.

The distortion of the bubble by the acceleration of the fluid may be important. Birkhoff and Zarantonello state that a bubble in an accelerated liquid accelerates two or three times as fast as the surrounding liquid. Under Helmholtz instability an equatorial bulge, transverse to the direction of acceleration, then occurs and later the bubble "dishes", that is, the following surface becomes concave. Although none of the circumstances of the above observation are stated, it would be appropriate for bubbles of centimeter size. As Birkhoff notes, surface tension becomes important in bubbles under 2 mm and provides an important stabilizing influence.
The change of resistance of the Coulter Counter aperture for a cylinder particle with the axis in the flow direction has been analyzed under the following assumptions:

a) the aperture contents form a cylindrical resistor in which current density is uniform,

b) multiplying the aperture length by an appropriate factor covers the electrically effective zones outside the aperture,

c) the passages of individual particles occur at random and are evenly distributed through the aperture cross-section,

d) the electrically effective volume of a particle in the aperture may be expressed as a cylinder having the same resistivity as the particle. Measurements indicate that most materials respond as insulators, including copper, aluminum, and silver.

Let the particle cylinder be $a \cdot d$ in length and $b \cdot d$ in diameter, where $d$ is the diameter of a sphere having the same volume as the cylinder. This $d$ is thus the particle dimension as measured electrically (not necessarily the same as the physical dimension).

Now consider the aperture as having a disc segment (containing a given particle) of a diameter $D$ equal to that of the aperture and a thickness $a \cdot d$ equal to the length of the particle-cylinder. Let $\rho_0$ and $\rho$ be the resistivities of the liquid and the particle, respectively. Then, the disc segment resistance without the particle is:

$$R_0 = \rho_0 \frac{a \cdot d}{(\pi/4)D^2}$$

and the resistance with the particle is that of two resistors in parallel, or:
Thus, the resistance change caused by the particle is:

\[ \Delta R = R - R_0 = \frac{1}{\left(\frac{\pi}{4}\right) (D^2 - b^2 d^2)} + \frac{1}{\left(\frac{\pi}{4}\right) b^2 d^2 a} - \frac{\rho a d}{\left(\frac{\pi}{4}\right) D^4} \]

simplified:

\[ = \frac{\rho a d}{\left(\frac{\pi}{4}\right) D^4} \cdot \frac{d^3 \left(1 - \frac{\rho_o}{\rho}\right)}{b^3 - \frac{d^3}{D^3} \left(1 - \frac{\rho_o}{\rho}\right)} \]

But, for an equivalent sphere and cylinder of equal volume:

\[ \left(\frac{1}{b^3}\right) = 1.5a \]

Thus,

\[ \Delta R = \frac{\rho a d}{\pi D^4} \cdot \frac{1.5}{1 - \frac{\rho_o}{\rho}} \cdot \frac{d^3}{a D^3} \]

Response is thus directly proportional to particle volume, except as modified by the last term in the denominator. This effect is limited because \( d/D \) should not exceed a maximum of about 0.5. In practice, deviation from volumetric response has proven to be negligible for aqueous electrolytes and relatively insulating particles, due to these probable
compensating factors:

a) a larger particle makes a greater increase in the current density and hence electrical heating in the rest of the aperture, thus momentarily lowering the resistivity of the electrolyte and the response to particle passage,

b) the factor a is probably somewhat greater than unity especially for larger particles which are the more likely to have irregular shapes and thus become "feathered" to align with the flow stream.

Thus, ignoring the last term in the denominator, the response equation becomes:

$$\Delta R = \frac{4d^3}{1.5\pi d^2} \rho_0 (1 - \rho_0/\rho) = A \rho_0 (1 - \rho_0/\rho).$$

Differentiating the setting equal to zero for maximum response:

$$\frac{d(\Delta R)}{d\rho_0} = A - 2A \rho_0/\rho = 0, \text{ and } \rho_0 = \rho/2.$$  

Usually, it will be impractical to achieve this "best" resistivity ratio of 2:1, as the required fluid resistivity for many particulate materials will be too high to permit the required current in the aperture without excessive resistive noise, heating of the aperture contents, or both. Adequate pulse height is readily achieved, however, so this is of little concern.

This analysis indicates that a bubble elongated in the direction of flow will give no appreciable error for an oblate spheroid with difference in radii as much as, say, 20%. The flattening in the transverse direction
(as indicated by Birkhoff et al.) occurs only when the pressure builds up beyond the orifice. Ivany et al. have examined vapor cavities in a two-dimensional venturi under much more severe conditions.

For the 400 micron aperture, the transit time at 330 cm/sec is 90 microseconds; because of the symmetry, the transit time is proportional to the diameter of the aperture at constant head.
III. EXPERIMENTAL

A. Equipment

1. Bubble generator

A bubble generator sketched in Figure 1 was constructed. The ultra-fine glass frit filter has a specified pore size of 1.2 microns. With 15 psi air pressure on the lower side of the frit, fairly large bubbles, up to 1 mm diameter, are generated. The 1-inch o.d. tube with flat bottom (riser) channels the flow of water across the surface of the frit, sweeping the bubbles away while the size is still small. The gap between the frit and the end of the riser is controlled by three narrow strips of 0.0015-inch mylar film cemented radially on the lower surface of the riser.

The bubble generator is attached to the bottom of the plenum chamber shown in Figure 2. The air passes upward through the frit and the bubbles are swept into the riser by the flow of water from outside to inside of the riser. The large bubbles rise to the top of the chamber and thence to the outlet. The slower-rising, smaller bubbles drift along to the vicinity of the counting aperture.

This bubble generator gave highly variable bubble counts because of turbulence in the plenum. A glass tube 0.79-inch O.D. and about 0.67-inch I.D. was bent to a 45° curve and inserted in the riser. The upper end was cut vertically to face the counter tube, with a clearance of about 0.2 inches. The flow of water was increased to 10 ml/second by increase of the head across the generator and by adjusting the clearance between the frit and the riser to produce a discharge speed of about 5 cm/sec from the riser extension. Relatively smooth flow around the counter aperture tube was produced. The larger bubbles rise to the upper
Figure 1. Bubble Generator.
Figure 2. Plenum For Bubble Measurement.
surface of the curved riser extension and are discharged about a centimeter above the counter aperture.

Early measurements were made by forcing synthetic sea water through the bubble generator from one reservoir under air pressure to a vented one. The second design introduced a constant level reservoir 300 cm above the frit. The flow was regulated as desired so that usually the actual head was substantially less. The minimum would be the static head to the overflow, 65 cm. From the overflow, vented to the air, the water entered a flow meter, thence to a 2-liter reservoir which supplied the circulating pump at constant head. A peristaltic pump with adjustable drive operated on the plastic tubing, avoiding contamination. From the pump the water was returned to the upper reservoir. This arrangement provided for continuous operation with two liters of water, important in view of the difficulty in providing a supply of Gulf water.

2. Photographic technique

The plenum, fabricated from 1/2-inch polymethyl methacrylate, was four inches high, six inches long and two inches wide inside. The counter aperture tube, about 7/16-inch in diameter, projected through the top of the plenum. The tube could be aligned so that the aperture plate was parallel to the front side of the plenum, facing the camera. The camera axis was perpendicular to the front of the plenum so that it could be focussed on the aperture plate.

Illumination was provided by two Strobotac 1531A light sources which were mounted to right and left of the plenum, facing each other. Each source was at 90° from the camera axis. The sources were set to single flash, both operated from a push button so that one or a counted number of flashes could be made for each exposure.
The camera was constructed with a Microtessar F/4.5 lens of 48 mm focal length, the shortest which would focus on the aperture, behind 1/2 inch of plastic and nearly 1 inch of water. An EYEMO 35 mm camera, lever-operated to open the shutter for one frame and then advance the film when the shutter was closed, was used. The dimensions were adjusted to give a magnification of eight.

The lighting was chosen to give a highlight on the surface of the bubble from each source so that each bubble would appear as two dots in the correct orientation. The sources should have been at a greater angle from the camera since total reflection from water to air in the bubble occurs at 41.4° and the angle should exceed twice this. Reflection at 90° is 3.8/25.3 or 15% of that at 97.2° between camera and lamp. At an angle greater than 90°, part of the field would have been in the shadow of the aperture plate and tube.

B. Bubble measurements

1. Coulter counter measurements

Each aperture used was calibrated with pollen of suitable size: ragweed, 19 microns; pecan, 48 microns; and corn, 98 microns. Since calibrated pollen was not available, laborious micrometer microscope counts were required. Since no effective means was found to keep the pollen in suspension in the circulating system, the calibration was done on the bench stand, where a stirrer could be used.

The electronics of the Model B counter are overloaded by an excess of particles with a wide range of sizes. With the bench stand, dilution of a suspension decreases the count to a manageable number. In counting bubbles, the count must be reduced to an acceptable level by control of the number of bubbles.
In the early measurements, the usable limit was exceeded. Progressive steps were taken in reducing the area of frit so that the number of bubbles was within acceptable limits with low differential air pressure on the frit. The standard procedure for correction for coincident particles, varying as the square of the count, was used. This correction becomes increasingly uncertain with the concentration of particles.

Measurements of bubbles were corrected for background particulate matter. The samples of Gulf water kindly supplied by the Laboratory varied greatly in the amount of coarse material in suspension. Usually runs could be made with the 100-micron aperture, but the 20-micron aperture clogged repeatedly. This difficulty was avoided by scalping with a 30-micron screen. Bubbles were removed by boiling under vacuum at room temperature.

a. Count analysis. The bubble counts above each diameter, \( d \), indicated by the setting of the counter were reduced to the count in one cm\(^2\), \( n \). The flow discharge from the aperture was measured with a graduate over a long enough time to give a reliable measurement and reduced to the counting time, usually 20.8 seconds. A plot of \( \log n \) vs \( \log d \) is linear for a log-normal distribution. The plots were generally linear over a range of diameters of about 3 to 1. Counts of large bubbles indicate too short a counting period for statistical reliability. The fall-off in counts for small bubbles is instrumental. The valid range was extended by exchange of the aperture for a smaller one.

From the plot the count of particles smaller than a desired diameter was read, then subtracted from the count for the next larger size, giving the number in that size interval. More than 100 runs were made during the evolution of the equipment. Since each size limit was counted in
sequence, a run required twenty minutes or more. Repeat readings indicated that the stability of the system was often less than desired.

2. Photographic bubble measurements

The camera described above was aligned and focussed on the Coulter aperture at F/4.5 then reduced to F/16 to increase the depth of focus. The image of the aperture provided a calibration of the scale of the photograph. The computed depth of focus, confirmed by measurements in air, is 0.1 cm with half this depth in front of the aperture. In water the depth of focus is reduced to 0.05/1.33 or 0.038 cm.

The area photographed, 0.27 x 0.19 cm, with the usable depth of field gives a volume of 1.9 x 10^{-3} cm^3. Twenty-nine photographs taken during a particular run were exposed to a total of 182 flashes of the strobe light. The volume searched, therefore, is 0.35 cm^3. The bubble diameters, estimated to 1 micron, were sorted in 10-micron intervals as shown in Table I.

Just before the photos were taken, a run with a counter aperture of 560 microns gave the size distribution indicated. When this count is reduced to the same effective volume as used for the photo, it is seen that the photo count confirms the trend, approaching the same count in the range of 50 to 90 microns. The rapidly decreasing photo count at small diameter indicates the difficulty in photographing the highlights since the brightness decreases rapidly with the diameter and the image is lost in the background.

In retrospect, the drawback of the photographic technique is recognized as the decrease in reflected intensity with the second power of the radius of the reflecting sphere, seriously limiting the size range which can be photographed.
**TABLE I**

Bubble Counts

**Photo vs Counter**

<table>
<thead>
<tr>
<th>Size Interval Microns</th>
<th>No. Photo in 0.33 cm³</th>
<th>No. Counter in 1 cm³</th>
<th>No. Counter in 0.35 cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 to 40</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40 to 50</td>
<td>12</td>
<td>73</td>
<td>26</td>
</tr>
<tr>
<td>50 to 60</td>
<td>10</td>
<td>43</td>
<td>15</td>
</tr>
<tr>
<td>60 to 70</td>
<td>0</td>
<td>22</td>
<td>7.7</td>
</tr>
<tr>
<td>70 to 80</td>
<td>4</td>
<td>12</td>
<td>4.2</td>
</tr>
<tr>
<td>80 to 90</td>
<td>1</td>
<td>7.0</td>
<td>2.4</td>
</tr>
<tr>
<td>90 to 100</td>
<td>0</td>
<td>4.0</td>
<td>1.4</td>
</tr>
<tr>
<td>100 to 110</td>
<td>0</td>
<td>2.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>
IV. CONCLUSIONS

The work with the Coulter Counter indicates no basic reason to doubt the capability of the instrument to measure bubbles as accurately as particles. Many of the difficulties encountered due to stability of the equipment would be relieved by use of the later models which count a number of size ranges simultaneously. Stoppage of the aperture by large particles can be largely avoided by screening with provision for back flushing.

There appears to be little likelihood that the flow through the aperture appreciably distorts the measured bubble volume. It was noted that millimeter bubbles, although broken up to a cluster of small bubbles on emergence, gave reliably only one count.

The photographic technique, while avoiding many problems of back-lighting, is applicable only over a small size range.
REFERENCES


