Modes of vibration of piezoelectric quartz crystals

By

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The purpose of this research is the measurement, analysis, and prediction of the modes of vibration of quartz piezoelectric crystals.

Background

The program for the two-year period covered by this NSF grant is a continuation of the work started in 1958 under the sponsorship of the U.S. Army Signal Research and Development Laboratories. Support was provided during 1960-61 under Grant NSF-G13647.

A series of theoretical papers by Mindlin and associates are available on the prediction of the frequencies of vibration of the various modes which can be excited by a radio frequency voltage applied to surface electrodes on the crystal. That work, as well as the early approximations by Koga, was based on the differential equations governing the motion of the crystal. The latest paper by Mindlin\(^1\) provides an excellent representation of the measurements by Fukuyo\(^2\). The theory does not include, however, the effect of

\(^*\)Superscript numbers refer to references listed in Bibliography.
piezoelectric stiffness, and arbitrary constants are introduced to compensate in part for the omission of higher-order terms dependent on the thickness. The elastic constants used in the computation were derived from the experimental data to be fitted. These constants differ as much as one per cent from those reported by Koga.

Research Progress

Theoretical. The paper by Koga, was completed with substantial participation by the Georgia Tech group. This theory applies to a plate of limited size and in addition elastopiezoelectric stiffness coefficients are introduced to include the piezoelectric effect. Although this contribution to the stiffness has an effect on the computed frequencies of less than one per cent, its inclusion in the theory is desirable.

The procedure in the development of the theory is given in some detail in Interim Technical Report No. 1. This report is a most useful compilation of notation and of the constants of quartz, both in the orthogonal axes and in the rotation corresponding to the AT-cut.

A critical examination of the mathematical procedures employed by Koga has led to some doubt as to the validity of the results. Until this suspicion can be validated or removed, we are reluctant to base further interpretation on the computed results. This uncertainty is most distressing because the theory provided a basis both for comparison with the polarization distribution as noted by Koga and with the strain gradient measured with the x-ray diffraction topography technique.
The theoretical development by Tiersten and Mindlin\textsuperscript{6}, including the piezoelectric relations and the electric field equations, is being programmed for the calculation of resonance eigen values for comparison with the numerical results of the computation by Mindlin and Gazis\textsuperscript{1} and with the experimental data. Our computations are based on the crystal constants of Bechmann\textsuperscript{7} whereas Mindlin and Gazis derived constants from experimental data\textsuperscript{2} of lower reliability than that acquired under this project.

The feasibility is being examined of extending the theory of Tiersten and Mindlin\textsuperscript{6} to include the displacements of higher order than $U_j^{(0)}$, $U_j^{(1)}$, and $U_j^{(2)}$ neglected by the authors. This extension is needed to obtain the strain gradient in the $y'$ direction for comparison with the x-ray measurements.

**Experimental.** During the first year the resonant frequencies over the range 100 Kc to 1100 Kc and 2700 Kc to 3300 Kc were measured to an x-dimension of 16.2 mm, completing the series from the initial value of 25.56 mm. Plots of the responses are given in the Annual Report for 1961-62. Polarization records of strong responses were made at intervals of about 0.5 mm. A more complete record of responses for a similar crystal 25.56 mm in x was taken.

A group of 10 polished crystals about 25 mm square, fundamental frequency just over 1 Mc, was purchased. The orientation was checked by x-ray and the edges carefully finished to close tolerance. Measurements of these crystals have confirmed the repeatability of the detailed measurements with the first crystal.

Extensive measurements by the Lang\textsuperscript{8} technique have served to develop and perfect our application of the technique to the measurement of crystals in vibration. Initial results were published\textsuperscript{9}. 
The effect of crystal drive level on the "rocking curve", that is the apparent angular width of the source, has been investigated. It is found that there is a pronounced widening of the skirts of the curve; also the width of the response at half-height is doubled at a drive level of 6 mA/cm$^2$ for the fundamental near 1 Mc. Data from the rocking curve is essential for interpretation of intensity measurements of the diffracted beam as the amplitude of oscillation is increased by increase of the driving current. Measurements completed indicate an inflection at about 2 mA/cm$^2$ for the crystal noted above; the rate of increase diminishes at higher drive levels as predicted.

The physical interpretation of the interaction of the thickness-shear strain with the corresponding shear of the flexure wave propagated in the x-direction is more consistent with both the observed polarization and diffraction patterns than alternate hypotheses.

The spectrum recording equipment has been improved by the addition of new equipment. A stable signal generator is used to drive the crystal in a 50-ohm series impedance circuit. The voltage across the output is recorded on a logarithmic scale through the use of a commercial log amplifier and an oscillograph with 3 Kc response. A special transformer was constructed to balance out the constant capacity of the crystal and permit a direct measurement of the motional resistance.

The polarization measurement equipment has been improved to record phase information thereby removing the possible ambiguity in interpretation. The table and translation mechanism have been redesigned to take the larger crystals to be used in the continuation of the work.
In general, the second year has reflected the transition from the survey of the detailed behavior of a single crystal to the development of better equipment for the wider field of investigation planned for the future. The full evaluation of the measuring techniques and elimination of spurious effects, more troublesome than at first anticipated, has been of great value in the interpretation of the results.

Personnel

Dr. Koga spent July and part of August 1961 at Georgia Tech. Mr. Masahiro Toki, graduate student at Yokohama National University, started work on July 1, 1961. Mr. Toki continued the measurements of crystal D-1 with diligence and efficiency until July 1962 when he went to the University of Minnesota for graduate work.

Dr. G. C. Knollman left Georgia Tech for industrial employment. During the fall of 1962, Dr. Dar Veig Ho, Assistant Professor of Mathematics, started part-time analytical work and theoretical investigations. He was joined full-time during the summer by Mr. W. F. Martens, second year graduate student in Math.

During the spring of 1962, Mr. Norval K. Hearn, Jr., Assistant Research Physicist under the guidance of Dr. Young, Research Associate Professor of Physics, assisted substantially in developing the Ilang technique of x-ray diffraction topography. He assisted occasionally until the fall of 1963 when he entered the graduate school of Kansas State University. During the spring of 1963, Mr. K. R. Allen, graduate student in Physics assisted with the x-ray work.
Mr. J. I. Cochran, graduate student in Electrical Engineering worked part-time from April 1962 to December when he left for industrial employment. He was succeeded by Mr. H. G. Henderson, graduate student in Electrical Engineering, who has continued the work on the electronics equipment.

Mr. M. T. Spahr, Physics graduate student, completed his undergraduate work in March 1963 at which time he joined the project. He has participated in the analysis of data and in programming for the P-220 and B-5000 computers.
BIBLIOGRAPHY


