N-GRAM MODELING OF TABLA SEQUENCES USING VARIABLE-LENGTH HIDDEN MARKOV MODELS FOR IMPROVISATION AND COMPOSITION

A Thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Master of Science in
Music Technology

Georgia Tech Center for Music Technology
Georgia Institute of Technology
December 2011

N-GRAM MODELING OF TABLA SEQUENCES USING VARIABLE-LENGTH HIDDEN MARKOV MODELS FOR IMPROVISATION AND COMPOSITION

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To Appa and Amma,

for everything that they've done for me

ACKNOWLEDGEMENTS

I am deeply indebted to Dr. Parag Chordia, without whose guidance this thesis would not be here today. I would also like to thank Dr. Gil Weinberg and Prof. Jason Freeman for their invaluable comments and their feedback throughout the entire process. I'm extremely grateful to Dr. Rebecca Fiebrink, from the the Dept. of Computer Science at Princeton University for sparing the time and effort to review the thesis on very short notice. Also, I would be nowhere, assuming I am *somewhere* right now, without the contributions of my colleagues Trishul Mallikarjuna, Aaron Albin and Sertan Şentürk to this project. I would like to thank Trishul for laying down the basic architecture of the Prediction Suffix Tree, parts of which are still being used intact within the framework. Aaron's help in researching and designing the smoothing algorithms for the project is greatly appreciated, and Sertan's MATLAB code for audio segmentation and feature extraction ensured that the project could be completed in time. I would also like to thank Alex Rae, whose tabla sequencer proved to be very handy in creating the audio database. Finally, I would like to express my gratitude towards my family, all my friends, and my colleagues at the GTCMT for their undying moral support over the last two years. This project is supported by the National Science Foundation under NSF grant no. 0855758.

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SUMMARY

This work presents a novel approach for the design of a predictive model of music that can be used to analyze and generate musical material that is highly context dependent. The system is based on an approach known as n-gram modeling, often used in language processing and speech recognition algorithms, implemented initially upon a framework of Variable-Length Markov Models (VLMMs) and then extended to Variable-Length Hidden Markov Models (VLHMMs). The system brings together various principles like escape probabilities, smoothing schemes and uses multiple representations of the data stream to construct a multiple viewpoints system that enables it to draw complex relationships between the different input n-grams, and use this information to provide a stronger prediction scheme. It is implemented as a MAX/MSP external in C++ and is intended to be a predictive framework that can be used to create generative music systems and educational and compositional tools for music. A formal quantitative evaluation scheme based on entropy of the predictions is used to evaluate the model in sequence prediction tasks on a database of tabla compositions. The results show good model performance for both the VLMM and the VLHMM while highlighting the expensive computational cost of higher-order VLHMMs.

CHAPTER I

INTRODUCTION

The work presented here describes a context dependent predictive system for modeling rhythmic sequences from a set of tabla compositions. Its main task is to extract information from audio or symbols, use this information to build a database of musical sequences, and make predictions on possible continuations for a given sequence. To achieve this, it draws upon concepts from various fields - n-gram modeling and Markov models from Natural Language Processing(NLP), the concepts of entropy and smoothing from information theory, and Variable-Length Hidden Markov Models, or VLHMMs from speech processing. These concepts are built into a unified framework known as a *multiple viewpoints* system. Such a system makes use of multiple representations of the data to model and represent complex relationships between different streams of information, and uses a merging schema to combine this information into a single prediction for the next event. The result is a versatile model that is capable of using information across all orders of the model, providing very specific matches for sequences found within the database, and falling back on general lower-order predictions if it unable to find suitable matches. The system is evaluated on two fronts - a symbolic library and a synthesized audio database of tabla compositions. Note that it is not intended to be a generative work, but a predictive framework that can be used to implement generative systems and educational software.

The tabla is one of the main percussive instruments used in traditional North-Indian classical music (NICM). It consists of two drums capable of producing a wide range of timbres, from short clicks to sustained bass tones. Music on the tabla is generally rhythmic in nature, consisting of patterns constructed by the juxtaposition

of sounds of different timbres. Each of these sounds is produced by striking specific parts of the drum membranes. Tabla players make use of basic principles of rhythmic and timbral organization along with sophisticated techniques of composition and improvisation to create complex progressions of events. This is discussed in much greater detail in Section 2.3.

The project consists of two parts. The first is a system based on Variable-Length Markov Models (VLMMs) that is designed to work on symbolic notation. The second part extends this concept to audio data, using Variable-Length Hidden Markov Models(VLHMMs) instead of VLMMs, however the underlying processes for both parts are strongly related. Both systems make use of an ensemble of prediction models - a set of VLMMs or VLHMMs working together on different streams of data - to construct a set of Prediction Suffix Trees (PSTs). Each stream of data and its corresponding predictive model together form what is known as a *viewpoint*. The PSTs together keep count of every single sequence entered into the system, and this data can be used to calculate the probability of any given sequence. In addition, the PSTs also incorporate *escape probabilities*, thereby accounting for sequences that have not been encountered yet, and a *smoothing* scheme to decide the importance given to a term based on its position (or level) in the tree. Each viewpoint consists of two PSTs within itself - a Long Term Model (LTM) that is constructed during the training phase, and a Short Term Model (STM) that is built up during testing. The LTM provides a general overview of the training data, providing information for predictions when the context is unclear, while the STM is included to address the pattern and repetitions specific to each individual testing piece. Each model, LTM or STM, returns a probability distribution consisting of all possibilities for the next event. A merging scheme based on the *entropy* of these distributions, is used first to combine the distributions returned by the LTM and the STM, and then again to combine the predictions of each viewpoint into a single prediction. A formal evaluation scheme

based on the entropy of the predictions at every step is used to provide a quantitative description of the performance of the systems under different circumstances. Chapter 3 provides a theoretical overview of each of these concepts, and describes their roles within the context of this work.

Although the two systems use the same process for prediction and evaluation, there are significant differences in their contexts and their implementations. For the VLMM system, a symbolic database of 34 tabla compositions was first prepared. These compositions were taken from tabla maestro Aloke Dutta's tutorial [30] for novices, and were encoded into a Humdrum [37] format known as **bol (read "bol"). Because it operates on symbolic data, the VLMM system is able to fully utilize the power of high-order contexts, using the information provided by the last 20 events or less to make its decision. Also, since the number of calculations associated with symbolic data is much fewer than that for audio, the VLMM system runs in real-time and uses the full set of 42 symbols allowing the notation to be directly used as input to the model. The VLHMM, on the other hand uses audio data, created by synthesizing the compositions of the symbolic database on a sample-by-sample basis. This was done so that the symbolic versions of these compositions could be used to train the transition probabilities of the model to improve its training accuracy. As for the emission probabilities, the audio samples are segmented (automatically) into individual tabla strokes and used to construct a Multi-Variate Gaussian (MVG) distribution. The first 21 MFCCs (Mel-Frequency Cepstral Coefficients), used to describe the power-spectrum of a sound - are then extracted from these segments and written to files. The VLHMM uses the MFCCs as its input for the process of sequence prediction, but due to the number of calculations involved for each prediction, it is restricted to using the information from a maximum of 3 events. The VLHMM is also forced to use a reduced set of 9 symbols instead of the original 42 due to the similarities in the acoustic properties of the strokes. These similarities are largely due

to the system of nomenclature, and it is possible for the same stroke to be denoted by several different symbols based on its role within a composition. Since the strokes sound exactly the same, it would be impossible for the system in its present state to differentiate between them using their MFCCs. Chapters 4 and 5 describe the specific changes and design decisions for the VLMM and the VLHMM systems respectively.

Over the course of this work, a number of different experiments were conducted on each of the systems to draw conclusions on the basic aspects of the working of high-order context models on symbolic and audio data. The system of evaluation used for the study is a simple scheme that uses the entropy of a predicted event as a measure of its accuracy. A leave-one-out cross-validation method at the song level is used to measure the entropy for each prediction, and then average these entropies across all 34 trials. Results are typically expressed in perplexity instead of entropy to make them easier to interpret - the smaller the average perplexity, the more accurate the result. For the VLMM, the experiments focus on the relationships between the average perplexity and model order, perplexity and different smoothing schemes, and the relationships between different viewpoints and the information presented by each of them. For the VLHMM, the experiments focus on the performance of the model in terms of its average entropy, its classification accuracy, and its time cost with increasing order. The results show that high-order context models provide a significant gain over models with little or no context in terms of the average perplexity. For the VLMM, we see that average perplexity tends to decrease with increasing model order, down to a certain limit. Experiments with different smoothing schemes show that the relative weights between higher and lower-order predictions can cause subtle, but significant changes in the perplexity. Finally, as more viewpoints are included into the final prediction, average perplexity tends to decrease, meaning that predictions get better with the inclusion of more information. The experiments on the VLHMM show similar trends in the average perplexity with increasing order. In addition, they

also show an exponential increase in the time cost associated with increasing model order. While these results do suggest that high-order context models are well-suited for modeling musical sequences, they also highlight the fact that high-order VLHMMs are extremely expensive in their time cost and need to be optimized further before they can used effectively. Detailed results are given in Chapter 5.4, while Chapter 6 describes some possible enhancements to our current implementation.

1.1 Motivation

My motivation for this work stems from the basic problem of using computers to model music. Of course, this is by no means a trivial task; it is a point that lies somewhere between the realms of engineering, art and psychology, and till date no single process has presented itself as a potential solution, though there have been several methods of analysis that have proven to be partially successful. Early attempts to model music using computation, such as the works of Lidov et. al. [44], Hiller [34] and Cope [67] have largely been focused on the application of semantic methods, however, recent advancements in machine learning and language modeling, have led researchers towards using statistical methods [61, 1] instead. In particular, the use of n-gram modeling in conjunction with Markov models has proven to be a reliable approach for tasks such as algorithmic composition [1, 2], machine improvisation [50, 41, 4] and stylistic analysis [29, 20]. In fact, many of the concepts used as part of the VLMM framework for modeling symbolic tabla notation have largely been inspired by the work of Conklin and Witten [20]. Past research in the field of information theory, most notably by Shannon [63], Kneser and Ney [40], and works on data compression, by Cleary and Teahan [17], and Cleary and Witten [16] have played a crucial role in influencing some of the algorithms used within the framework.

The VLHMM framework began as an extension of the existing VLMM system in an effort to transfer the successes of this approach of modeling symbolic music

[13, 15] to audio data. It was inspired by recent research involving the application of HMMs in speech recognition and music information retrieval, such as the work of Thede and Harper [65], Lee and Slaney [43], Cao [11] and Chordia [14]. Since higher-order HMMs are incredibly expensive in terms of their computational cost, the implementation of a generalized variable length HMM is still very much a topic of discussion. Researchers are currently seeking alternate implementations like Mixed-Order Models [62] and equivalent first-order representations [54] to try and minimize the computation time and memory associated with such models. My main aim with the VLHMM system is to extend the concepts used in the VLMM wherever possible, and to create a unique approach to modeling the music of the tabla, while exploring the constraints surrounding this implementation of the model.

1.2 Hypothesis

To summarize my expectations from this work, I hope to thoroughly explore the capabilities of the two predictive systems and show, using a quantitative evaluation scheme, that VLMMs and VLHMMs have the potential to provide a better framework for the prediction of musical sequences over fixed-order Markov models. For the VLMM system, I will demonstrate, through a series of experiments, that the model perplexity decreases with the increase in order; that the addition of smoothing schemes improves model accuracy, and finally, the incorporation of information across multiple streams of data can be used to further helps to decrease model perplexity. For the VLHMM, I will show that such a system is not only feasible in terms of its time-complexity and memory usage given the limits of a modern desktop computer, but also presents a usable approach to modeling improvisation in music for real-time interactive systems in terms of time taken per prediction.

1.3 Contribution

This work represents the confluence of a number of ideas and concepts from various disciplines, and to the best of my knowledge, is the first attempt at an implementation of a predictive system for music that analyzes audio and symbols and utilizes information across all orders of the VLMM/VLHMM for prediction. It brings together concepts from language modeling, speech processing and information theory, all in an effort to model improvisation in tabla sequences. Although, each of these concepts have been well-explored in their respective fields, they have very rarely been applied in combination towards music modeling. I believe that by bringing these ideas together under one roof, this system occupies its own unique place among predictive frameworks.

One of the biggest lessons learnt over the course of this work is that some techniques adopted from other fields like language processing and information theory, have proved to be invaluable to the performance and evaluation of the VLHMM. The system demonstrates that N-gram modeling, coupled with high-order variable length Markov chains, makes for a robust approach to sequence prediction. In addition, results with smoothing systems indicate that information across different orders can be used to make better decisions, thereby resulting in better musical material. For evaluation, the concept of cross-entropy, a term borrowed from information theory, shows considerable promise in terms of a quantitative metric for model performance. The n-gram modeling approach reinforces the notion that there are significant similarities between music and language. Both are no more or no less than forms of human expression, and understandably, techniques that work in language processing could potentially work for music and vice versa. At the same time, it is important to remember that there are significant differences between the two. My experience with smoothing schemes shows that while it is a good idea to borrow techniques from speech and language processing, those techniques will need to be adapted for

applications in music.

The main contribution by this system to the MIR community is the implementation of the VLHMM. Few predictive systems make use of higher-order markov chains. Even fewer attempt to use hidden Markov models for audio analysis, let alone the analysis of tabla compositions. As mentioned before, formal theory on higher order HMMs is still not entirely in place, and our efforts look to throw some light in this direction. This work demonstrates that it is feasible to use VLHMMs of up to orders 2 and 3 for audio analysis, and that with a few optimizations they can be considered viable options for real-time systems. The PST is used to devise a branching system that eliminates all pathways that are impossible, allowing the model to work only on the relevant continuations to a given sequence. This enables the system to exploit the PST architecture to gain a significant advantage in speed and memory over conventional HMMs. Moreover, the concept of multiple viewpoints can be used to decide between several predictions for the same event, and this information can be further used to adapt each prediction depending on its context.

Finally, in terms of the music chosen for the model, the decision to model tabla sequences instead of tonal western music places this work in a region that is largely unexplored. Very few researchers have attempted to model music on the tabla till date [31, 8, 14, 55, 56]. Chordia's work [14] uses a first order HMM for the recognition of tabla strokes from audio, while Rae's thesis [55] involves a recombinant model for generating structured variations. The VLMM/VLHMM framework combines these two approaches within a single system, capable of performing both functions and working on either symbols or audio. Given the quantitative results and the **bol database, this work is definitely a significant addition to this domain. Apart from this though, this decision also reflects my philosophy and approach to modeling music. I strongly believe that to truly understand music, one must look to all forms of the art for inspiration. Tabla compositions offer a succinct overview of the basic principles of

rhythmic improvisation and timbral organization without the additional load of tonal harmony, providing a model that can be easily evaluated regardless of the musical complexity of its output. Since many ethnic styles of music, such as the tabla, focus on certain aspects of the music alone, a specific model, has the potential to pick up trends that are much harder to spot in conventional tonal music - information which can then be generalized and re-applied to Western music, to enhance our understanding of how it should be modeled. Through this work, I wish to encourage other researchers to consider alternate forms of music to further our notions about generative composition and improvisation.

CHAPTER II

BACKGROUND AND RELATED WORK

The inter-disciplinary nature of this work makes it necessary to look at several different areas of research before we can examine the core ideas and concepts involved.

2.1 Creativity and Computation

Creativity is a dangerous word. It is a part of who we are, of what we do, of how we live our lives. It is a fundamental aspect of human thinking, and yet it stubbornly eludes any standard definition, refusing to be constrained within the bounds of conventional language. Perhaps this is because creativity is often defined using the *circumstances* surrounding a thought, action, work or individual, and not on the agent, event or process that actually leads to creativity. Perhaps what we deem to be creative is dependent only on its value to society or to us as individuals. Perhaps creativity is not meant to be defined, in which case, we are better off trying to describe it instead. The reason for this philosophical opening is that if we are to understand creativity by *simulating* it using computers, we need to know what it is that we are trying to accomplish, and whether we are heading in the right direction. Although the focus of this work is purely technical, and not artistic, this section aims to do no more than provide a glimpse of the historical context involved and establish the relevance of this work to its initial motivation.

A rather well-known definition of creativity was put forward by David Cope [22] in response to criticism against his principal work, *Experiments in Music Intelligence*: "The initialization of connections between two or more multifaceted things, ideas, or phenomena hitherto not otherwise considered actively connected.". The controversy surrounding his project, EMI (or Emmy, as it is often called), is that it is capable

of analyzing a large corpus of work in a particular musical style, retain patterns and phrases that define the characteristics of that style, and use this information to generate new pieces emulating that style of music [21]. In fact, when faced with a Turing test, most listeners failed to distinguish between music generated by EMI against the works of Bach. This is the reason why his definition of creativity, unlike most of the earlier definitions, makes no mention of a person, or a consciousness agent. As you can imagine, this opens up a whole new debate about whether a conscious mind is a pre-requisite for creativity, which leads to more questions about how we define intelligence and consciousness! A discussion of this magnitude is well beyond the scope of this document, and I do not intend to light fires that I am not equipped to put out.

Harold Cohen's AARON is a robotic painter that is renowned for its still-life portraits. Indeed, some of AARON's art work has been featured at museums around the world [46], and some would say, it has received more fame than most human artists! AARON creates its drawings very much like a human would - using lines and closed shapes to create representations of people and objects, yet its creator does not hold the view that it is actually creative [18]. Cohen believes that AARON lacks knowledge and understanding about its work to qualify as a truly creative machine. On the other hand, Cope believes that Emmy is a creative program, often coming up with musical material that he would never have thought of. He attributes its creativity to a process of pattern-recognition, selection, and recombination [21]. Another, rather entertaining, example of "machine creativity" is JAPE. JAPE is a riddle generator, capable of generating sets of questions and answers, most of which are hilarious, using a set of structures and rules that define the relationships between a list of words, their meanings, usage and pronunciation [57]. In many cases, the argument is that creativity is a process, and once the basic attributes of this process have been defined, even a machine can be "programmed" to be creative, while its

counter argument is that machines can never be truly creative - it is the creativity of the programmer that is reflected in the operation of the machine. Regardless of whether it is the machine or its creator that is truly creative, it is a fact that the works produced by *computational models of creativity* like EMI, AARON and JAPE clearly demonstrate that works produced by computer programs can be deemed creative, even when judged by human standards.

This leads us to an examination of the current work. Are we, in fact, modeling creativity using this VLMM/VLHMM framework, and more importantly, is there a system to judge how creative our model is? How can we possibly *measure* the creativity of the output of such a model? The simple answer to the first question is that it is important to remember that all computational models of creativity are exactly what they claim to be - algorithmic or statistical models that mimic the operation of a creative process, without actually performing every single aspect of that process. In other words, they simulate human creativity, and in doing so they tend to produce output that could be called by some as creative works. In this sense, the answer is yes. This framework analyzes a database of encoded tabla sequences that are said to represent the basic aspects of tabla music, and given a typical rhythmic pattern attempts to mimic the progressions, substitutions and transformations that would be performed by a typical tabla player under those very same circumstances. It does this, not by going through years of rigorous training in rhythmic and timbral improvisation, but using a simple statistical process that assigns a finite probability to each of the options for valid continuations available to it. A thorough qualitative assessment of the musical material produced by the model, however, is not the focus of this work, and to make any comments on the creativity of the output at this stage would be based on premature assumptions. Based on the quantitative results however, it is possible to state that for a majority of musical contexts, the model does show a high likelihood of selecting continuations which would be deemed "correct"

by the theoretical techniques of improvisation on the tabla.

2.2 Generative Music

The term "generative music" used within the context of this document refers to the process of composing music using an algorithmic process. A generative music system, in this case, is an autonomous or semi-autonomous model that carries out this process of composition. While generative systems come in all shapes and sizes - from tiny applications, running on arduinos and mobile phones to giant musical robots [35], we will not consider the physical setup of a system as part of it. Given our context, what we are interested in is the software that controls the musical material generated by such a system. Although this thesis is only concerned with the predictive aspect of composition, it is presented here as a potential generative system for two reasons. Firstly, most predictive models, even if they use similar algorithms for prediction are usually concerned with the degree of data compression, or the efficiency of their performance. This makes comparisons between models almost impossible unless very similar data is used to train and test these models. The second reason, is that this work is not concerned with maximizing the predictive efficiency of the model - its focus is on investigating a relatively new technique to generate stylistically consistent variations on the given input. Conceptually, this puts this system much closer to a generative music system rather than a purely predictive model. Chapter 3 provides specific examples to related predictive models, while this section provides an overview of the different approaches to generative systems.

The concept of using algorithmic processes to generate music is, by no means a new concept, whether it be statistical, like Mozart's *Musikalisches Wurfelspiel*(dice game) or the works of Xenakis [70], or deterministic processes like the serialist compositions of Schafer. Yet modeling music using computation is a relatively recent aspect of this field. Broadly speaking, there are two approaches to the problem. The first is the

semantic approach, where music is modeled explicitly using rules and strcutures to build compositions. The second, is the stochastic approach, where a style of music is modeled using probabilistic algorithms. Early attempts at modeling music began with semantic models, most notably those by Hiller [34], Lidov [44], Rowe [58] and Cope [67]. Most of these early works use recombination as a compositional technique - various permutations and combinations of chunks of musical material are strung together with the intention that the resulting combination leads to the generation of creative outcomes. EMI [21] is probably one of the best examples of this technique, although it incorporates many other approaches as well. George Lewis's *Voyager* is another such system, employing a variety of recombinant algorithms along with statistical models to generate complex polyrhythmic structures. Rae's work [55] uses a recombinant model for generating variations on a theme of tabla music.

A more recent approach to a creative compositional process is used by Horner and Goldberg [36], Dahlstedt and Nordhal [24], and programs such as GenJam [10]: using genetic algorithms to promote certain characteristics of generated material. GenJam is based on the approach of a novice jass player learning to improvise. As it generates musical material, the user has the option of giving it instructional feedback, to either accept or reject a melodic line. If a phrase is accepted, certain characteristics of the phrase are retained and used to generate new generations of similar phrases. Genetic algorithms are very powerful for controlling the general output of a model within a parameter space that cannot be described precisely. Their biggest drawback, however, is the fitness function used to promote certain characteristics over others. This function and its corresponding system of mutation must be chosen carefully to prevent the model from being overly selective, thereby restricting itself to a few basic types of phrases. Cellular automata(CA), is another recent school of thought. It is also based on an evolutionary process, where cells on a grid progress in number and position based on certain criteria, like the number of neighbours around each cell, for

instance. The positions of these cells can be used to control a generative process like the creation of evolving MIDI sequences [48, 49]. Stephen Wolfram [69] provides an overview of these techniques in his book *A New Kind of Science*.

What we are concerned with however, are statistical models of music, where a program is trained on an existing corpus of music and then used to generate works that are statistically (and sometimes stylistically) similar. Sequence prediction techniques like Markov models have proven to be very effective tools for modeling music - Charles Ames describes the use of Markov models for composition [1]; Dubnov and Assayag have conducted numerous studies on musical structure [29, 27, 28] and musical sequences [26, 3]; and Conklin and Witten present their use of a multiple viewpoint system [20] for stylistic models. Francois Pachet's *Continuator* [50] is one of the best known works of this type, and has served as the inspiration for many other such systems, including this work. It is perhaps one of the best examples of an improvisatory system based on variable-length Markov chains. The Continuator uses a prefix tree, very similar in architecture to the VLMM system, to keep track of patterns played by a user. It continuously records symbolic MIDI data from the user, building up a database of phrases and gestures. It then uses this information to generate possible continuations to what the user has played so far. A certain level of creativity is achieved by selecting any one out of the many possible continuations, often leading to some surprising results. The raw output of the Markov chain is then modified by a set of reduction functions to handle polyphony, preserve meter, and maintain musical structure.

The VLMM framework represents a more general approach over that used by the Continuator, and is not suited to any single style of music. The different viewpoints can be used to form several different criteria for selecting predictions at any given time, and the entropies returned by the model help in evaluating these predictions

Figure 1: A picture showing the two drums of the tabla. The one on the left is called *bayan*, and it is used to produce resonant bass tones. The drum on the right is the *dayan*, capable of producing sharp clicks and ringing tones.

quantitatively. It is intended to be a purely predictive framework over which generative systems like the *Continuator* can be built.

2.3 Tabla

The tabla is a traditional Indian percussive instrument extensively used in both North-Indian classical and folk music. It consists of two wooden drums, the *bayan* and the *dayan*, one for each hand, of very different shapes and sizes (see Figure 1). The difference in their construction gives the two drums very different timbral qualitites and each is played with its own technique. Their construction opens up a wide range of timbres, and in the hands of a good performer, the tabla is capable of producing sounds ranging from sharp, short clicks to deep, resonating bass tones. Despite its rather recent origins (atleast compared to other Indian instruments), music on the tabla is built around surprisingly sophisticated techniques of performance, improvisation and composition. Though there are many different styles of performance and composition, all of them revolve around similar principles of rhythmic phrasing and timbral organization.

The tabla began as a simple percussive instrument designed for rhythmic accompaniment, yet its versatility and timbral variety inspired entirely new styles of

performance. Fixed compositions and improvisation techniques for solo performance soon emerged and the tabla took on a more prominent role in Hindustani(North Indian Classical) music. A typical tabla composition, a *qaida*, for example, starts with a simple theme. The performer starts by repeating a theme a few times to set a rhythmic cycle. Then, using established rules of syntactic development, the performer proceeds to create more challenging versions of the theme and with each cycle the theme evolves into something more complex than the last. The pattern is allowed to develop further, with the performer ensuring that every new cycle adds to the previous one to create a coherent progression demonstrating the artist's technical as well as improvisational skills until it reaches a certain speed and intensity at which point the composition is concluded with a dramatic *tihai*, typically a longer sequence that is based on the theme and repeated three times to signal the end of the piece.

The drum usually played with the left hand, the bayan, is the bigger of the two. It is generally played with the heel of the palm and is used to produce deep bass tones. The performer can modulate the pitch of these tones by sliding his/her palm across the membrane after striking it, and this tends to add a lot of expression into the performance. The dayan is the smaller drum, generally used to produce short clicks or longer, resonant strokes. The timbral quality of the sound depends on the manner and location of the strike on the membrane. By rapidly switching between these different locations, a tabla player can create an endless variety of rhythmic patterns. Each of these sounds is assigned a name based on its timbre - for instance, *na* is a sharp, short resonating sound produced by striking the very edge of the membrane of the dayan, whereas *ge* is a long bass tone produced using the bayan. However, most compositions make use of a slightly higher level of organization defined by the concept of *bols*. A *bol* is better understood as a compositional unit, rather than a single stroke or sound. Some bols like *te*, *na*, *ge*, refer to single hits, while others like *dha* refer to compound sounds where two strokes, in this case *ge* and *na*, are

played together. Still others refer to short sequences, like *tete*, or *terekete* that are often used as musical gestures within a composition. *Bols* are also broadly divided into two categories. *Open* strokes are long resonating strokes, and *closed* strokes are short clicks and strikes. Specific rules of composition dictate what substitutions between the two types are allowed, and compositions are often built by highlighting the contrast between the open and closed versions of the same pattern.

Note that a *bol*, as defined here, is a compositional unit, and not an absolute sound or timbre. The name of a bol often depends on the context of what has been played, and the style of the composition, so it is possible for the same stroke to be denoted by two or more *bols* simply because of its place within the piece. This is a very significant aspect of the music of the tabla. This system of *bols* can almost be considered a musical language where the *bols* are like words, and sentences can be formed by following grammatical rules of composition! It also means that the tabla can be treated as a monophonic instrument, greatly simplifying the task of modeling compound strokes. Table 2.3 shows a list of the most common tabla strokes and their corresponding categories.

2.3.1 Why the tabla?

My reasons for considering the tabla were, atleast initially, mostly a matter of convenience and aesthetics. Convenience, because I already knew where to look to find a good set of compositions that exemplified common improvisation techniques. As it happened, Dr. Chordia possessed a book of standard tabla compositions selected and compiled by tabla maestro Aloke Dutta[30]. It had been written as a tutorial to demonstrate the principles of improvisation on the tabla and the various rules and techniques that came along with it, and it was simply a matter of encoding these songs into text files to create a database. Previous work by Chordia [14] and Rae [55] had led to the use of the **bol notation for representing tabla(see Section 4.1)

Table 1: Tabla strokes used in the current work. The drum used is indicated, along with basic timbral information. "Ringing" strokes are resonant and pitched; "modulated pitch" means that the pitch of the stroke is altered by palm pressure on the drum; "closed" strokes are short, sharp, and unpitched.

Stroke name	drum used	timbre
dha	compound	ringing *bayan*
dhe	compound	ringing *bayan*
dhec	*dayan*	closed
dhen	*dayan*	ringing *bayan*
dhin	compound	ringing *bayan* and *dayan*
dun	compound	ringing *bayan* and *dayan*
ge	*bayan*	ringing *bayan*
geM	*bayan*	ringing *bayan*, modulated pitch
ke	*bayan*	closed
na	*dayan*	ringing *dayan*
nec	*dayan*	closed
nen	*dayan*	ringing *dayan*
rec	*dayan*	closed
te	*dayan*	closed
tin	*dayan*	ringing *dayan*
tun	*dayan*	ringin *dayan*
tunke	compound	closed *bayan*, ringing *dayan*

sequences in a text-based format.

It was also a matter of aesthetics because of my background and ethnicity. One could say my musical tastes are biased in favor of Indian music, however it is also true that I wanted to model non-western styles of music. A large number of predictive systems make use of a relatively small set of databases. For instance, most chord estimation algorthms are trained and evaluated on Chris Harte's Beatles database [32]. This has its advantages - the annotated data saves researchers a considerable amount of time and a common database allows them to compare the performance of one algorithm with the next. I feel, however, that there is much to be gained by modeling ethnic musical styles, and that this knowledge can help provide a general understanding of all musical cultures [66]. In most cases, a specific style of ethnic music is built around some core principles - for example, almost all forms of Indian music are concerned only with rhythm and melody, allowing me to examine the melodic structure of a piece without any fear of being influenced by its harmonic composition. A specialized ethnic music model has the potential to give valuable insight on such principles - insight that can then be re-applied to tonal music to create better predictive and generative systems [23].

As the technical aspects of this research came into focus, I realized that the music of the tabla offered a number of advantages that would never have been possible with tonal music. One very valuable feature is that the music of the tabla almost resembles a language of sorts. Fixed rules of substitution and improvisation define a large part of the structure of a piece. When considered along with the fact that it also involves a considerable amount of structured repetition, it becomes quite obvious that music on the tabla is very well suited for analysis using linguistic models like Markov chains. The research(Chapter 4) in this regard gave good results, which prompted the implementation of the VLHMM framework to allow us to move to the audio domain. Another aspect of tabla solos is their theme-variation structure. Since

music on the tabla is based on principles of rhythmic and timbral improvisation, it allows us to study these principles without interference from melodic and harmonic influences. This provides a better idea of how sequences are constructed, making it easier to model creativity in this context.

The final motivation for researching tabla compositions was that this was an area that few researchers and ventured into, and since we were already in a position to explore this field, our efforts would be perhaps more useful here than anywhere else. Not many researchers have tried to model the music of the tabla. One of the most significant works, by Bell and Kippen [8] applies linguistic modeling techniques to try and understand the grammatical rules surrounding the concept of *bols* in tabla solos. Chordia's work [14] involves the automatic transcription of tabla music from audio using HMMs, and this work can, in some ways, be considered an extension of this earlier project. Gillet and Richard carried out sequence modeling on the tabla, again with the goal of transcription [31]. Our work also draws quite heavily from Alex Rae's work [55], where techniques of analysis and recombination are used to produce coherent variations on existing tabla sequences. I believe that this research will contribute significantly to the existing repertoire of research on the tabla, and the addition of an annotated database of tabla compositions will be a welcome addition to this relatively sparse community.

CHAPTER III

CONCEPTS

This chapter explains the basic concepts involved in this work and offers a brief summary of their roles within the framework. Examples are presented so that the terms and definitions associated with each of these concepts are established and are easily understood in the subsequent chapters of the thesis.

3.1 N-Gram Modeling

N-Gram modeling is an approach of data analysis designed for sequential streams of data. The principle is elementary, quite literally. A stream of data is split into its constituent terms, or *n-grams*, as they are called, and the information extracted from the relationships between individual n-grams is used to construct an *n-gram model*, which can then be used for tasks like predicting the next term in a sequence, or constructing new sequences similar to an existing one. The concept is easily demonstrated using a simple example.

Let us consider a sequence of letters $S = \{ABAACAABAA\}$. Let us construct an n-gram model on this data using a simple histogram. Splitting the stream into its constituent letters, we see that there are 7 As, 2 Bs and 1 C. We can now estimate $P(A) = 7/10$, i.e. there is a 70% chance that the next element in the sequence will be an A. A more sophisticated approach would be to use bi-grams, or groups of 2 letters (see Table 2). To estimate the probability of the next term in the sequence being an A, we are looking for the probability of the bi-gram AA, since the last term in the sequence S is an A. $P(A|A) = P(AA)/(P(AA) + P(AB) + P(AC)) = 3/(3+2+1) =$ 50%. This means that the probability of A being the next element in the sequence is 50% and not 70% as we predicted earlier!

Table 2: Histogram showing the list of all possible bi-grams and their counts in a sequence $S = \{ABAACAABAA\}$.

Bi-gram	AA	AB	AC	BA	BB	BC	CA	CB	CC
Counts	3	2	1	2	0	0	1	0	0

A model that uses still more information can be constructed using sequences of tri-grams, or even 4-grams and so on. Of course, longer n-grams involve exponentially larger sets of possible observations which lead to more complex n-gram models which in turn lead to greater demands on memory and computational resources. Another point to keep in mind is that as n-grams get longer, they also become fewer in number until their count becomes too low to serve any purpose. In other words, the information becomes so specific, that after a certain length, information extracted from the observed n-grams is no longer applicable to sequences that might occur in the future. To consider an extreme case, only two 9-grams occur in the sequence S, $S_1 = \{ABAACAABA\}$ and $S_2 = \{BAACAABAA\}$, and each of them occurs only once, so it gives us no information about the 9-gram $S_3 = \{AACAABAAA\}$ which would have given us the probability of A being the next term in the sequence S.

Despite these shortcomings, n-gram modeling has proved itself to be a robust approach to sequential data analysis. It has been applied rather successfully in speech processing and statistical natural language processing [45], and is now being applied to music modeling as well. Downie's work in this regard was responsible for drawing a considerable amount of attention to studies relating the analysis of music and language [25]. More recent works, by Pearce [52] and Suyoto [64], investigate these relationships using context models to gain a better understanding of the structure of longer sequences. Perhaps the most important advantage of using n-gram modeling is that it has an inherent approach to addressing the hierarchy within a sequence. In music, as with language, a single stream of events can often be interpreted in multiple ways - as a stream of notes, as a set of gestures and short phrases, or as a coherent

melody - and considering n-grams of different lengths allows us to consider all options before making a prediction about the next event. More sophisticated n-gram models, Markov models, for instance, use statistical methods to handle the different levels of information and provide processes for sequence prediction and generation.

3.2 Markov Models

A Markov Model is a statistical model of a time dependent series of events. Each of these possible events is represented by a unique symbol, and the model is defined as a set of states, one for each symbol and the probability of moving from one state to the next. The model is trained by calculating the transition probabilities between the different states by observing a known sequence of events. If the model is accurately trained, then the next symbol in a sequence can be predicted simply by looking up the state having the highest transition probability from the current state of the model.

Developed in the early 1900s by Russian mathematician Andrei A. Markov, Markov models were intitally used to conduct a statistical analysis on the order of words in Russian texts [45], but over time they developed into more sophisticated tools and began to find applications for a wide variety of tasks ranging from data compression [16], language modeling [60] and speech processing [39] to biological sequence analysis [7] and financial models [9], and have found their place in music modeling as well. They form the basis for several generative music systems [50, 47] and classification algorithms [29]. Markov models have been used for timbre analysis [5] and also towards the development of a model of music cognition [51].

A classic example of a Markov Model is shown in Figure 2 [39]. Let us suppose that we need to train a model to predict the weather at a given place on a daily basis based on the weather of the previous day. Let us also assume, for simplicity, that the weather can only be rainy(R) or sunny(S) on a given day. On observing the weather for eleven consecutive days, we find the sequence of weather conditions

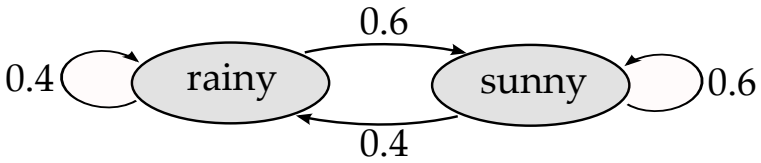


Figure 2: A simple first-order Markov model built on observed weather conditions on a daily basis [39]. The figure shows two states for the weather on a given day, rainy(R) and sunny(S). The arrows represent the transition from one state to the next.

Table 3: Histogram showing the counts for all possible bi-grams for the sequence $S = \{RSSRRSSRRSS\}$.

Bi-gram	RR	RS	SR	SS
Count	2	3	2	3

to be $S = \{RSSRRSSRRSS\}$. The simplest form of a Markov model is one that considers the next state to be dependent only on the current state. Such a model is termed a *first-order* Markov model because it looks back one time-step into the past. A first-order Markov model is built by observing the number of bi-grams in a given sequence, represented as a histogram in Table 3.

We can now re-order this histogram into a *transition matrix* a such that a_{ij} (see Table 4) denotes the probability of transition from the current state i at time t to the next state j at $t + 1$. Mathematically, a_{ij} is defined as $a_{ij} = P(\omega_{t+1} = j|\omega_t = i)$ where ω denotes the state of the model at time t.

Given that the 11th day is sunny, the transition on the 12th day is going to be either S→S, or S→R. Therefore, we can calculate the chance of the weather being sunny or rainy on the 12st day as: $P_S.P_{SS} = 1.a_{SS} = 0.6$ or $P_S.P_{SR} = 1.a_{SR} = 0.4$.

Table 4: Transition matrix of a first-order Markov model built on the sequence $S = \{RSSRRSSRRSS\}$.

$a_{ij} =$

	R	S
R	0.4	0.6
S	0.4	0.6

This means that there is a 60% chance that it will continue to be sunny on the 12th day. Note that the sum of the rows of the matrix a should be 1, since it represents all possible transitions from the previous state. There is however, one very important assumption in order for this framework to be successful - the series of events must obey the Markov property, which states that, for a Markov model of order n, the next event at time $t + 1$ is dependent only on the last n events. In other words, the system is strictly causal - events in the future have no influence on events in the past. Considering this assumption, the application of Markov models to language and, in our case, music, raises some questions which must be addressed. Firstly, both music and language are mostly non-causal - very often events in the present are almost completely dependent on events in the future. For instance, the notes chosen by a soloist are often dependent on the upcoming transition to the next chord, or to the next section of the piece. Secondly, creativity and innovation provide the motivation for almost all forms of composition: music is *designed* to be unpredictable. Composers and improvisers are always looking to create something new and unexpected, and more often than not, various compositional aspects are decided entirely by the intent of the performer or the composer, and there might be no significant dependence on the previous state. How, then, can Markov Models possibly take these factors into account, and how is it that they have worked reasonably well for applications in music and sequence prediction?

The short answer to both these questions is that music and language are "considerably more" causal than most of us would like to think. Although individual progressions are driven by hidden, non-causal factors like future events and personal intentions, on a much larger scale, the music itself is based on standard rules and techniques for creating these progressions, quite similar to how the order of words in a sentence is decided almost entirely by grammar. Over a large database of events,

these hidden factors manifest themselves as repeating patterns, expressions and gestures that can be predicted by the model without any knowledge of the actual circumstances. In fact, artists and composers spend years practicing these standard gestures and techniques to build up expectations among their listeners and hold their attention. It is the regularity of music that makes it alluring to most of us [38, Chapter 10], and it is this very same regularity that also makes it predictable. At the same time, the trademark of a good composer or artist is his/her creativity. On one hand, we do hope that given a large enough database of a genre of music, a markov model should be able to "capture" the style of music and the intent of the artist within its framework, yet on the other, a creative progression of events, by its very definition, is unpredictable. Any Markov model, however large and complex it might be, is a poor substitute for the human mind, and the failure to predict certain events does not always represent a bad prediction, rather it should be considered a testament to the creativity of the composer.

3.3 Prediction Suffix Trees

A logical extension to a first-order Markov model is an n-th order model, where the next state ω_j, is dependent on the last n states. Here, the probability of transition to ω_j is expressed as $P(\omega_{t+1} = j|\omega_t = i, \omega_{t-1} = k, \dots)$ so that the probability of the next event is dependent on the last n events, where n is the order of the Markov model. While this may seem like a reasonable approach at first glance, it soon becomes apparent that the extension is not without its flaws. The most obvious obstacle to the implementation of such a model is the curse of dimensionality. As the order of the model increases, the size of the transition matrix increases exponentially. For an n^{th} order model with N states, the transition matrix a has N^n values, making higher-order models extremely expensive.

A more practical implementation is to use a Prediction Suffix Tree(PST). A PST

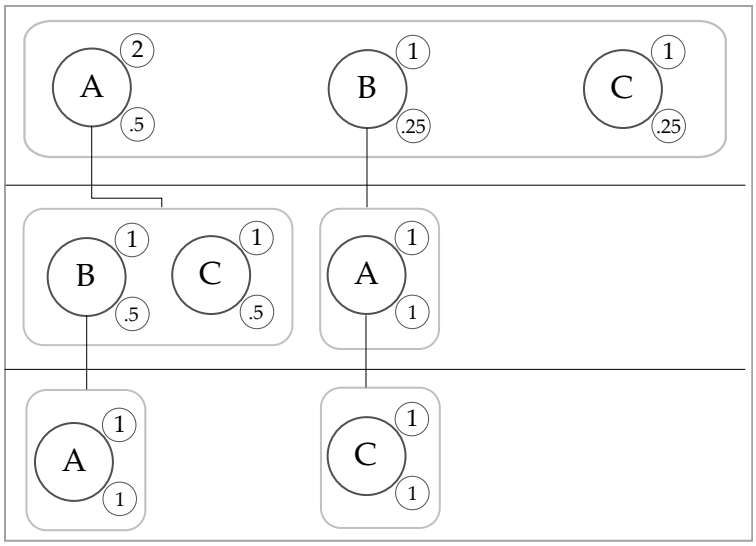


Figure 3: Diagram showing a PST trained on the sequence $S = \{ABAC\}$. The top level represents independent symbols 'A', 'B', 'C'. The children represent bi-grams that start with the parent, and their children represent tri-grams and so on. The cirle on the top right of every node is the number of times that a node has been seen so far, and the circle to the bottom right shows its probability.

is a data structure, similar to other n-ary trees. The root of the tree represents the state of the model at time $t = 0$. Below the root, the very first level of the tree contains nodes that represent all the symbols seen in the training process. The nodes at the next level, the children of the independent symbols, represent all known bi-grams in the database, with the connections from each parent to its children deciding the valid transitions. The "branches" of the tree are paths that represent all unique sequences present in the training data. As we move further down the tree, we move forward in time along the particular sequence denoted by that branch. Since the tree only contains branches representing sequences that have been seen so far, the number of calculations is far less than that of an n^{th} order Markov model and this greatly reduces the memory and computation time needed for such a model. Mathematically though, the PST is nothing but an alternate representation of the transition matrix for an n^{th}-order Markov model.

Figure 3 shows a PST built on a sequence of letters 'ABAC'. The top level of

the tree contains individual elements of the sequence and their respective counts and probabilities. There are 3 unique symbols, 'A' which occurs twice, 'B', and 'C' occur once in the sequence. The second level contains the nodes for the bigrams, so we have 'AB', 'AC' and 'BA', and each of them occurs only once. Note that the nodes in the second level of the PST contain the probability that the second symbol occurs *after* the first has already been seen, therefore the probability of a bi-gram, say 'AB', is given by $P(AB) = P(A) \cdot P(B|A) = 0.5 \cdot 0.5 = 0.25$, which is simply the product of the probabilities of the two terms in the sequence. This probability is identical to that returned by a first-order Markov model for the same sequence, with $P(B|A)$ taking over the role of the transition probability a_{ij}. The immediate advantage here is the gain in efficiency, since we no longer perform unnecessary calculations for the bi-grams that have not been seen. The bigger advantage though, is that the PST simplifies the extension of the Markov chain to any order. It eliminates the need for a large matrix of transition probabilities, and makes the generalization much simpler to implement.

3.3.1 The Zero Frequency Problem

Consider a sequence of letters, say $S = \{ABAC\}$, that has already been fed into a PST (see Figure 3). Imagine that we now request the probability of occurrence for a sequence $S_1 = \{ABCD\}$. This leads to a potential problem, because the probability for D is 0, since it has never been seen before. Theoretically, there is nothing wrong with this. In fact, that is exactly what is represented by the PST, and on an ideal dataset consisting of all possible symbols, this situation should never even occur at all. In practice however, almost all datasets are invariably incomplete, and at higher orders, the sheer number of possible outcomes make it very likely to encounter sequences that are not part of the training set. In music, where basic variables like notes and durations are typically seen as limited in number, an unusual melodic scale,

or a new rhythmic contour will almost certainly lead to sequences that are not part of the training database. In a practical situation then, it stands to reason that our PST needs to assume that its training is imperfect and stay open to the possibility of new material. In other words, a small amount of the total probability mass needs to be reserved for unseen events and new symbols. This is termed the zero-frequency problem. It is usually discussed in the context of arithmetic coding algorithms for data compression [17], but is equally relevant for context models.

Let ϵ be a number between 0 and 1 that represents the probability of encountering a new event, so $P(S1) = \epsilon$ and not 0. Conceptually, this has some very important implications. Since the PST now considers the possibility of an "unknown" event, this information can be used to our advantage. From a generative stand point, it allows us to model and generate new material by replacing this new event symbol with a random symbol from the tree. Mathematically though, the biggest impact of this system is that the PST will *always* return a number that is greater than zero and less than 1, allowing us to shift these probabilities from a linear scale to a logarithmic scale. This shift allows the PST to deal with the minute probabilities associated with higher-order terms in the tree.

3.3.2 Smoothing

The process of smoothing addresses another well-known problem with high-order context models. As the context gets longer, the information also gets more specific. For instance, for an alphabet containing 10 symbols, there are 10^2 or 100 possible bi-grams and 10^4 possible 4-grams. The chance for any one of the bi-grams to occur, considering a uniform distribution, is 1%. The chance for any one 4-gram to occur is 0.01%. For a sequence containing 10 terms, the chance for any one 10-gram is 10^{-10}! In a practical distribution, the data will be even more sparse, and most high-order n-grams will hardly be seen more than a few times. This means that beyond a certain

depth of the PST, most of the information present is only slightly better than random noise, and we would be wasting time and computational resources keeping track of all this data. On the other hand, the information at the lower orders, although significantly more consistent, is often too general to be very useful. Even if this information is useful, there is a good chance that good lower order predictions occur *too frequently*, which leads to very repetitive motifs and patterns being produced by the model. We are therefore faced with a balancing act - to utilize the specifity of the lower levels of the tree to generate novel material, yet retain the generality of the higher levels to preserve the character of the composition.

Smoothing is a process that blends the different predictions offered by different levels of the tree, attempting to strike this balance. It is equivalent to observing all predictions made by the different n-gram models in the PST and blending them together to obtain one prediction that incorporates all the information generated by the tree. It is a point-by-point weighted average of the probability distribution generated at each level of the tree for the next symbol. For a given symbol ω, found at multiple levels of the PST, this can be expressed as:

$$P(\omega) = \frac{\sum_{i=1}^{n} w_i P_i(\omega)}{\sum_{i=1}^{n} w_i} \tag{1}$$

where $P_i(\omega)$ represents the probability of state ω at i^{th} level of the PST and w_i is the weight assigned to that level. The core of the algorithm however, lies in how each of these weights is assigned. How do we decide which prediction deserves more weight than the other? Chapter 4 addresses this question by comparing three different methods for smoothing - the Kneser-Ney smoothing algorithm [40]; a simple fixed-weight scheme known as the $1/N$ method, and a third parametric approach which is essentially a generalized form of the $1/N$ scheme.

3.4 Multiple Viewpoints

The PST described in Section 3.3 represents the framework for a Variable-Length Markov Model(VLMM) that can make a combined prediction across all orders from 1 to N, and I can begin to introduce the concept of a Multiple Viewpoint Model (MVM). MVMs use several of these VLMMs in parallel to make a prediction on a given stream of data that is represented simultaneously in multiple ways. For instance, a melody comprising discrete pitches (MIDI numbers, perhaps) can be represented in many ways - absolute MIDI numbers, melodic interval, pitch class representation, melodic contour, etc. Each of these representations, or *viewpoints* as we shall now call them, has the potential to highlight certain aspects of a composition that might prove to be useful in making a better prediction. While absolute pitch numbers tend to work well for songs in the same key, a relative representation might give better results for transposed melodies. A pitch class representation, or a functional notation might emphasize common relationships between notes that might not be easy to see across all octaves, and contour information can help in reinforcing other predictions or detecting common musical gestures. In addition, MVMs also incorporate techniques to model mathematical relationships between these different representations and are capable of extracting very specific information across several streams of data. Consider, for example, a pattern that contains certain notes at regular rhythmic intervals. An MVM could be constructed to examine the melody in conjunction with the rhythmic position and use the link between the two to predict the occurrence of these notes. MVMs have the potential to use the information gained by using alternate representations and improve upon the prediction given by any single VLMM. Furthermore, MVMs can use several VLMMs of varying length and specificity to find matches for contexts that would otherwise be almost impossible.

The concept of multiple viewpoints was first introduced by Darrell Conklin and Ian Witten [20], and later carried forward by others such as Pearce and Wiggins [53].

Conklin and Witten provide a robust mathematical framework for the application of MVMs in [20], describing its capabilities and potential applications and then proceeded to evaluate their system on a database of Bach chorales. Their results proved that this approach is very well suited for musical sequence prediction. Experiments by Conklin and Anagnostopoulou [19] further explore this concept for musical patterns.

The smallest unit of an MVM is a *type*. A type is defined as an abstract property of events, like note numbers, note durations or scale degree, or something much more complex, like all accented notes that fall on the first beat of a measure. There are different kinds of types, the ones relevant to us being:

- Basic types: Basic types are the simplest properties of events. They do not depend on any of the other types. Eg. note numbers, inter-onset intervals.
- Cross types: A single type consisting of the cross-product of two or more basic types. Eg. Notes X IOI (Inter-Onset-Intervals)
- Derived types: Types derived from a basic type. Eg. Accented Notes

A *viewpoint* includes a type and its corresponding context model, but for simplicity, we shall refer to a viewpoint by its type alone. The power of a multiple viewpoint system stems from its ability to *decide* between the different viewpoints and apply either one or a combination of them, at every time step. We therefore require a system, or a quantity, that allows the system to evaluate or *judge* the relevance of its predictive distribution. For this, we use a property known as *entropy*.

3.4.1 Entropy

We define entropy to be the negative logarithm of the probability of an event e, given a context c.

$$E(e|c) = -log_2(P(e|c)) \tag{2}$$

The definition comes from Shannon's seminal paper on a predictive theory of the English language [63], where a context model is used to encode the letters in a stream of words. Although the main objective of the model is data compression, the paper lays down the fundamental rules for a general predictive coding model, and uses the dual principles of prediction and generation to compress and transmit data. Raw data is fed into a predictive model, which then encodes the letters into a stream of bits. The number of bits representing each letter is dependent on the context preceding the letter, and can be no less than the entropy of the letter in question. The encoded stream is then sent to an identical statistical model, only this time it is used to generate the actual sequence from the encoded bits. This scheme of working forms the basis of almost all forms of data compression systems.

The term entropy literally corresponds to its namesake, the physical quantity. Lower entropies correspond to predictions with high probabilities, i.e. lower randomness, and high-entropy predictions signify unpredictability. We can also define another quantity, the cross-entropy of a probability distribution, as the expectation of the entropy for each point in a discrete distribution, given by

$$H = -\frac{\sum_{i=1}^{n}(P_i)log_2(P_i)}{\sum_{i=1}^{n} P_i} \tag{3}$$

where n now represents the number of discrete points in the distribution, and i is any one of those points. This quantity effectively estimates the randomness of a given distribution. Distributions with low entropies are more predictive than distributions with higher entropies, but a lower entropy is invariably the result of an uneven distribution. The maximum possible entropy is seen when the distribution is uniform, since it has no predictive power at all. This result is quite significant, because it now allows us to consider the cross-entropy of a distribution akin to a measure of confidence. A distribution with a low cross-entropy is more uneven, and therefore *more confident* as compared to a distribution with higher entropy. The

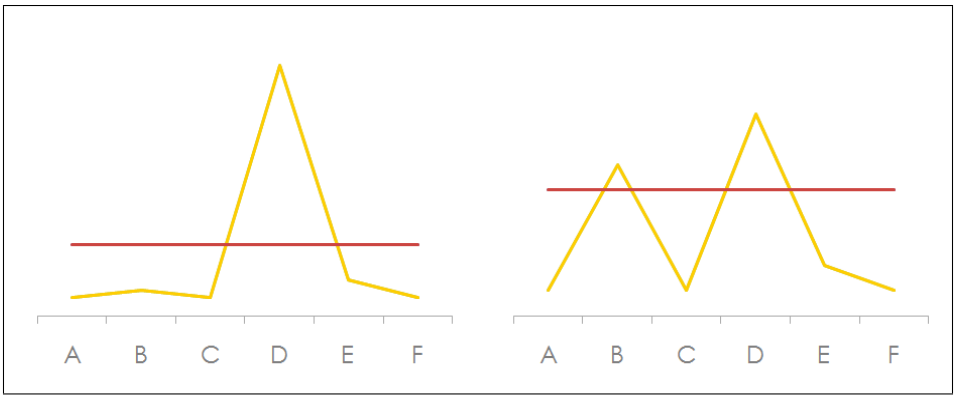


Figure 4: Sketch showing the relationship between the probabilities in a finite distribution (the yellow lines) and the entropy of the distribution (represented by the red lines). The distribution on the left has a lower entropy because it has one clear prediction.

inverse of the cross-entropy H of a distribution is used as a measure of confidence, to decide the which viewpoints deserve more weight in the merging process.

3.4.2 Merging Viewpoints

The process of combining the predictions of the different viewpoints is simply a weighted average of all the predictive distributions, where the weights for each distribution depend upon its entropy. The weight for each distribution is given by a normalized average across the entire distribution.

$$w_m = \frac{H(p_m)}{H_{max}(P_m)} \tag{4}$$

Higher average entropies result in lower weights for the distribution, this way in the final merging process, viewpoints that are less confident (having higher entropies) make much smaller contributions to the final prediction. This allows us to choose the best predictions depending on the context of the event, leading to better matches and lower entropies for the model as a whole.

3.4.3 Perplexity

For the purpose of evaluating the performance of a model, we also use another metric, *perplexity*, defined as 2 raised to the power of the entropy. While entropy represents the minimum bound of the number of bits used to encode the occurence of that event, the perplexity is simply another way of looking at the same measure. The perplexity of an event is an estimation of the number of uniform choices that the model is faced with in the prediction of that event. Since the perplexity of a model is mapped to an exponential scale, it increases very rapidly as the probability decreases, making it a good metric to amplify the differences in predictions with small probabilities. In fact, most of our results are expressed using perplexity since it makes them a little easier to read and understand.

CHAPTER IV

VLMM IMPLEMENTATION

The system is implemented as a MAX/MSP object and is written in C++. MAX/MSP was chosen as the platform for development because of its vast library of pre-built objects and libraries. It provided the flexibility to design and run a variety of tests over the entire database, making it easy and efficient to change parameters between test runs, with the patches being completely re-usable for the VLHMM as well. This chapter describes the details of this implementation of an integrated VLMM framework that makes use of all the concepts discussed until now to model a set of tabla compositions, leading to two conference publications [13, 15]. It involves designing an MVM, finding the relevant viewpoints, and designing an evaluation scheme that returns an accurate description of the performance of the model. The first step in this process is the creation of a training database.

*4.1 **bol Database*

The VLMM system is trained on a symbolic database, encoded from Aloke Dutta's book of tabla compositions [30]. The book is a tutorial for novice tabla players, outlining the basic principles of improvisation on the tabla using transcriptions of various forms of improvisations. The book also lists some renowned fixed compositions, which were also incorporated into the database. In total, it contains 34 songs of 8 different types - 13 Qaidas, 5 Relas, 4 Tukdas, 4 Gats, 4 Chakradars, 2 Keharva thekas, 1 Dadra and 1 Laggi. An important point to note here is that about 50% of the database is comprised of *qaidas* and *relas*, which are both improvisational forms, based around a central theme. The database comprises these 34 songs - a total of 27,189 strokes, using 42 unique symbols. These songs were encoded manually from

Table 5: Snippet of a typical **bol file.

```
!! Qaida 1 (8 beats)
**bol
!DATA>dhin--(3)dhagena(3) dha--(3)dhagena(3)
                    dhatete(3)dhatidha(3) genatu(3)nakena(3)
8dhin
24dha
24ge
24na/
.
.
*break
*-
```

the book and then converted using scripts into a special format called **bol, designed for notating music on the tabla, and follows the standards of the Humdrum music notation, developed by David Huron [37].

The **bol notation was devised by Craig Sapp [59] at Stanford University. A small snippet of a typical bol file is shown in Table 5. Lines that begin with "!" are comments and are used for metadata only. The data to the VLMM is contained in the terms below the DATA line. A typical **bol term, "8dhin", for example, consists of two parts - the current stroke *dhin* and the time interval between the current stroke and the next one, represented by the number 8. These durations are represented by the inverse of their fraction in a bar, i.e. larger numbers denote smaller durations.

4.2 VLMM

4.2.1 Viewpoints

Considering the information in the **bol database, 4 viewpoints are used in the VLMM system. The two basic types are *strokes*(S) and *durations*(D). The S viewpoint consists of a set of unique integers derived from the names of the each of the tabla strokes found in the database. The D viewpoint represents the inter-onset interval(IOI) between adjacent strokes. Both of these can be directly extracted directly

Table 6: Table summarizing the four viewpoints used in the VLMM system.

Viewpoint	Description	Symbol	Example
Strokes	Integer derived from stroke names	S	8 (*dha*)
Durations	Integer derived from IOI	D	16
Strokes x Dur	Cross-type between S and D	SxD	8,16
Strokes x PiC	Derived type between S and Position in Cycle	SxPiC	8,0.5

from the **bol notation (see Section 4.1). In addition to these basic types, there exists a cross-type between the S and D viewpoints. This viewpoint, SxD, proved to be very useful to differentiate between patterns that comprised the same strokes but different durations. For example, the pattern *8dha 16te 16te* sounds entirely different from *16dha 16te 16te*. The first stroke *dha* is twice as long in the first pattern as compared to the second, a difference that can only be realized when the strokes and durations are examined together. In many cases, themes and variations show very prominent contours in timbre along with their rhythm. This timbral contour is determined to a large extent by the relative position of some strokes in the rhythmic cycle. For example, a *dha*, being a heavier bass stroke, is usually played at the very beginning of the cycle to signify the beginning or the ending of a phrase. Such structure can only be retained if the position of the stroke is considered along with it. The Stroke x Position-In-Cycle viewpoint, or SxPiC, consists of tuples of the form (S, PiC) where S denotes an integer representing the stroke and PiC represents a floating-point number that denotes its position within the rhythmic cycle. The SxPiC viewpoint did not add much to the quantitative evaluation of the model, but it improved the generated sequences substantially in terms of their structure and quality.

4.2.2 Prediction Scheme

Figure 5 provides a simplified overview of the functioning of the entire VLMM system. The notation from the **bol files is read into the model, one term at a time, and for every term that is read in, the model attempts to predict the next event. The data

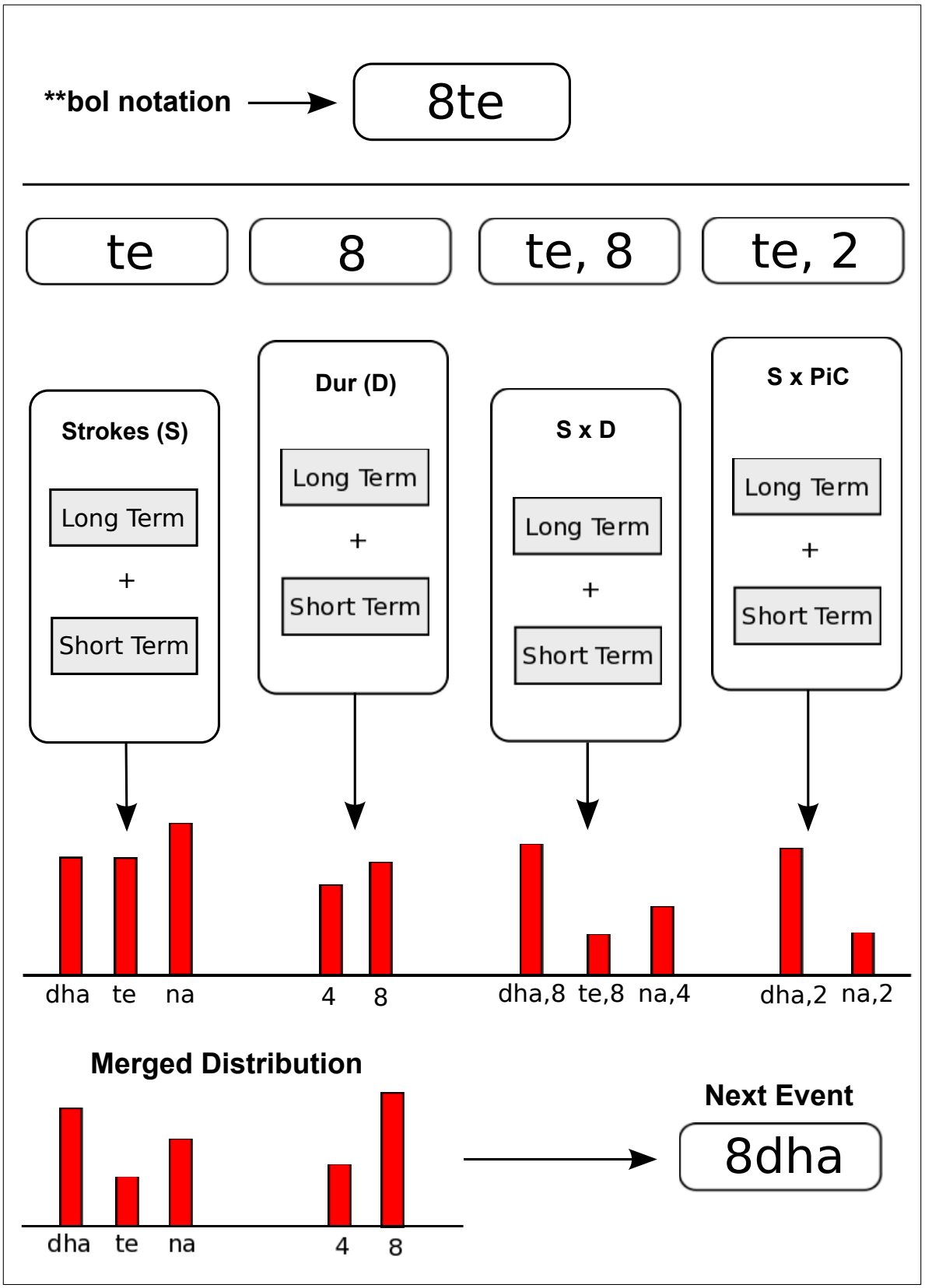


Figure 5: Figure showing an overview of the prediction scheme used in the VLMM.

is first split into the different viewpoints, and each viewpoint constructs two PSTs - a Long Term Model(LTM) and a Short Term Model(STM). The LTM is constructed during the *training* phase of the model, on 33 out of the 34 available compositions, while the STM is constructed on *testing* data, i.e. the one remaining composition. The reason for constructing two PSTs instead of a single one, is to balance the amount of repetition with the extent of variation. A typical tabla composition, like a *qaida*, is built around a theme variation structure, where each song consists of its own characteristic patterns and motifs. A large database, such as the LTM is unaware of such local details due to the sheer number of possible variations available to it. In other words, it is too general to be of any use to a particular song, and many obvious repetitions are easily missed by the LTM. The STM however, is constructed on this one song alone, and is able to identify these repeating motifs very easily. When a prediction is called for, each viewpoint, therefore, makes use of both models, merging their individual distributions into a single one. These distributions are merged yet again before a final prediction is made. Note how the SxD and SxPiC viewpoints can alter the distributions returned by the S or D viewpoints.

Let us take another look at the PST discussed in Section 3.3 in Figure 3. Let us assume that this PST is built on a sequence of strokes, $S = \{dha, ge, na, ge\}$. Now, assume we are given a sequence $S' = \{dha, ge\}$ and asked to predict the the probability of *na* being the next term in this sequence. We start by considering sequences of all orders less than the given sequence S', upto the maximum order N with the term *na* appended to the sequence. This gives us $S'_0 = \{na\}$; $S'_1 = \{ge, na\}$ and $S'_2 = \{dha, ge, na\}$. The probability of *na* following the sequence S' is given by

$$P(na) = \frac{P(S'_0) + P(S'_1) + P(S'_2)}{3} \tag{5}$$

which is simply the sum of the probabilities of all possible pathways for the PST to generate a *na*. We can now rewrite equation 5 as

$$P(na) = \frac{P(na|\{\}) + P(na|\{ge\}) + P(na|\{dha, ge\})}{3} \tag{6}$$

This is where the smoothing scheme is used. Applying the weights from the smoothing scheme to the PST, this now becomes

$$P(na) = \frac{w_0 P(na|\{\}) + w_1 P(na|\{ge\}) + w_2 P(na|\{dha, ge\})}{w_0 + w_1 + w_2} \tag{7}$$

so that the final probability is the weighted sum of the probability of the term *na* at different levels of the PST. We can now generalize this to derive a single equation that describes the process for each prediction with in the VLMM:

$$P(x_{n+1}|\{x_0 \dots x_n\}) = \frac{w_0 P(x_{n+1}|\{\}) + w_1 P(x_{n+1}|\{x_n\}) + \sum_{i=2}^{n} w_i P(x_{n+1}|\{x_{n-i+1} \dots x_n\})}{\sum_{i=0}^{N} w_i} \tag{8}$$

Thus a VLMM of order N is equivalent to combining the predictions of all fixed-order Markov models from order 0 to N. This process occurs for both the LTM and STM of every single viewpoint whenever a prediction is called. The basic architecture of the PST was designed by Trishul Mallikarjuna from the Georgia Tech Center for Music Technology. A large part of it has changed considerably over the course of this work, but some parts still remain as they were. His help in this regard is greatly appreciated. There are still two details about the PST that must be made clear before we can proceed with the merging scheme for the viewpoints - the escape probabilities and the smoothing scheme.

4.2.2.1 Escape probabilities

Witten and Bell, in their work on tackling the zero-frequency problem [68], provide a quantitative comparison between the different methods used to address the zero-frequency problem in modern data compression algorithms. Since each PST in the

VLMM system is a context dependent model, not unlike a predictive coding model, the approach deemed to be the most effective by Witten and Bell was chosen for this work as well. The Poisson process model, as it is called, is a method for estimating the escape probabilities of a model based on the number of terms in the training set and their frequency. In very simple terms, the escape probability for a given model (for a level of the tree, in our case) increases along with the ratio of new terms encountered to the total number of terms.

Let us assume that out of an unknown number q of unique tokens, we have encountered c_i of type i, in a sample set of size n. Assuming that these tokens of type i occur according to a Poisson process, the expected probability that the next encountered event will be novel is given by

$$\epsilon = \frac{t_1}{n} - \frac{t_2}{n^2} + \frac{t_3}{n^3} - \dots \tag{9}$$

where t_i is the number of unique terms that have occurred i times. Now assuming that we are dealing with a reasonable amount of repetition in our sample set, and given that n is sufficiently large, we can neglect the higher terms of the series shown in equation 9 and approximate this to $\epsilon = t_1/n$. Since n needs to be large enough, we apply these escape probabilities to all sequences of a specific order, like we would if we were maintaining separate Markov models for each order of the VLMM instead of a PST. We can now define the escape probability at each level of the tree as:

$$\epsilon_n = \frac{t_{1n}}{N_n} \tag{10}$$

for the n^{th} level of the tree. N is the total number of sequences of length n and t_{1n} represents the number of terms at the n^{th} level that have been seen only once.

4.2.2.2 Smoothing

The smoothing process used in a PST decides the ratio of weights $w_0 \ldots w_n$ in equation 8. Based on the results of a study of smoothing algorithms for language models by Chen and Goodman [12], two basic approaches were implemented for the VLMM - the Kneser-Ney and the 1/N smoothing schemes.

The Kneser-Ney [40] smoothing scheme is essentially a discounting scheme that sets the weight for a given level of the tree based on certain pre-assigned discount factors for each level. The weight assigned to each level of the tree depends on the number of elements that have occurred once or more, along with the weighted sum of the discount factors of all previous levels of the tree. The scheme assigns higher weights to those levels of the tree that show more repetition. A full explanation of the scheme is beyond the scope of this document, however, [12] and [40] both provide very accurate descriptions of the process.

On the other hand, the 1/N approach, assigns weights to each level of the tree using a fixed-weight scheme designed to give more priority to higher orders regardless of the amount of repetition at each level. The weight for the i^{th} level of a tree of maximum order N is given by

$$w_i = \frac{1}{N - i + 1} \tag{11}$$

Surprisingly, a formal comparsion between the 1/N and Kneser-Ney schemes (described in Section 4.4) showed that the 1/N approach performed much better than the Kneser-Ney approach in terms of the entropies of the predicted output, despite being much simpler. One reason for this could be that the Kneser-Ney approach is designed for words in sentences, which almost always will have fewer repetitions than musical phrases, thus overcompensating for the amount of repetition in a theme-variation composition. The results of these comparisons prompted a third smoothing scheme,

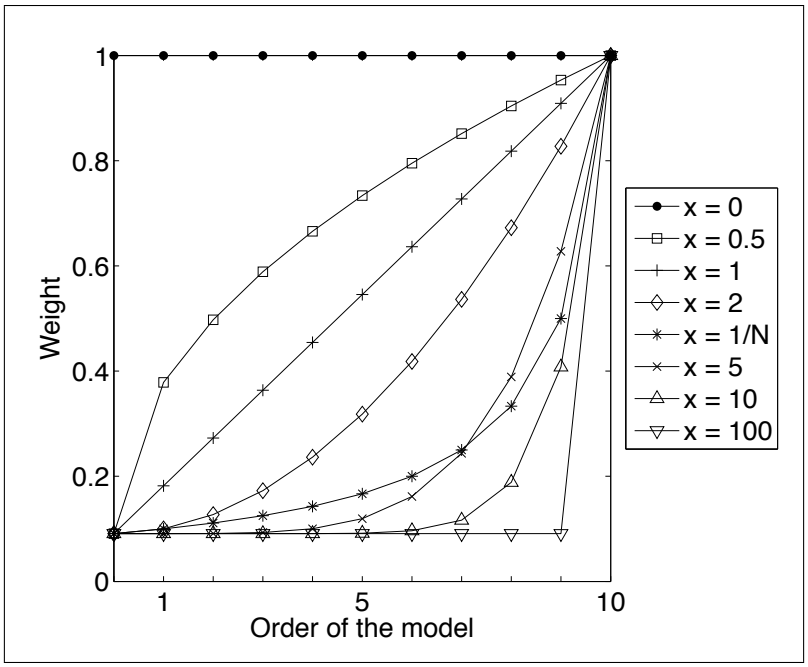


Figure 6: Figure showing the family of curves represented by the Parametric smoothing scheme.

intended to be a much broader look at how smoothing affects the resulting entropy distribution of a context model. A generalized form of the 1/N scheme was designed and implemented by Aaron Albin and myself. We call it the parametric weighting scheme (denoted by P), based on a family of curves of the form

$$w_i = a\left\{\left(1-\frac{c}{a}\right)\left(\frac{i}{N}\right)^x\right\}+c \tag{12}$$

By adjusting the values for the amplitude (a), the offset (c) and the exponent (x), the weights can be shifted, allowing the model to choose between more specific matches or general predictions. Figure 6 shows the range of curves that can be produced using this weighting scheme. We, Aaron and I, set out to explore two potential questions here. Firstly, how much of an effect did these parameters actually have on the system? And second, is there an optimum set of values for a, b and x that would give us the best results in terms of cross-perplexity?

To address these questions, we trained a VLMM model of maximum order 10

and compared the cross-entropy results between the 1/N system and the parametric system, with different values of the exponent value (x) ranging from 0 to 100. We also compared these models against a simple *back-off* (BO) model that would consider only the longest context match possible. In addition, we used the data from these trials to hone in on an optimum set of parameters.

4.2.3 Merging Predictions

When a prediction is called for, all the information gathered so far has to be integrated and presented to the user in the form of basic types, i.e. the next stroke and its corresponding duration. The first step is merging the LTM and the STM for each viewpoint to get a combined prediction. Once this is done, the distributions returned by individual viewpoints are merged again. The merging scheme for both these processes is identical, making use of the cross-entropy of the distributions to decide the weightage for each.

Suppose we have two viewpoints V_1 and V_2, whose respective distributions must be combined. The cross-entropy of each distribution is first calculated using the relation

$$H = -\sum_{i=1}^{N} P_i log_2(P_i) \tag{13}$$

The next step is the normalization of this measure by the maximum entropy of the distribution H_{max}, which is obtained by taking the negative logarithm of the minimum probability in the distribution, denoted by P_m. This returns the weight associated with that distribution.

$$w_m = \frac{H}{H_{max}(P_m)} \tag{14}$$

The actual process of merging is simply a point-by-point weighted average of the two distributions. For the basic types, strokes and durations, the combined prediction of the LTM and the STM is returned as the final distribution. For the SxD viewpoint,

the combined distribution has to be marginalized and split into two distributions - one corresponding to strokes, and another corresponding to durations. Each of these distributions is then merged with its corresponding basic type. The derived viewpoints SxPiC demands some explanation. We decided not to use this viewpoint in the LTM, mainly because different compositions used different time signatures and rhythmic accents, and incorporating this viewpoint in such a diverse database would cause more harm than good. This viewpoint was only used in the STM to preserve the rhythmic structure specific to each piece. At each prediction, the position in the cycle is checked and a distribution for all strokes occuring at this position is returned. This distribution is then merged with the strokes viewpoint. The S and D distributions are then normalized so that cross-entropy can be calculated.

4.3 Evaluation

Perplexity, which is in turn derived from entropy, is used as the metric to evaluate model accuracy and performance. A leave-one-out cross-validation scheme is used at the song level. For each run, 33 out of the 34 songs are used as training data to build up the VLMMs for the LTM. The remaining song is then fed in token by token and a prediction is called at every step before the token is actually fed to the STM. The model returns a probability distribution for that time-step and the entropy for the actual testing token at that step is checked and recorded. This is repeated for every song until the log contains a set of entropies corresponding to every token in the database. These entropies are then averaged and the average entropy is converted to perplexity. If the model has been trained well, the average perplexity across all 34 runs should be relatively low. Recall Shannon's original definition of the entropy E_L of a language L

$$E_L = -\frac{\sum_{i=1}^{n} log_2(P(e_i|c_i)}{n} \tag{15}$$

where e_i is the i^{th} event occurring after its corresponding context c_i and n is the total number of subsequences used in the estimation process. This is exactly what we use to estimate the performance of our model. A lower E_L means a better predictive theory, which means that we also have a better generative system. Note that the measure E_L represents the average of the entropy values for each prediction; it should not be confused with the cross-entropy of a distribution H (Eqn. 13. E_L is used to measure the performance of the model, whereas H is used to assign weights to each of the models before the merging process.

This system of evaluation was used to test the performance of the model over a variety of experiments. The first of these is a comparison of the average entropies of the LTM, STM and their combined prediction for orders ranging from 1 to 20. It conveys the relationship between entropy (or perplexity) and model order, and highlights the difference in predictions between the corpus-wide VLMM of the LTM, and the song-specific STM. The impact of different smoothing schemes for a VLMM of order 10 was also studied using a detailed study of the average entropy of each viewpoint under each smoothing scheme. The results establish the trend of decreasing entropy as the number of viewpoints are increased, and also show which smoothing scheme works best for each viewpoint. To add to this, a separate study on the parametric model alone shows that its exponent value can be tuned to minimize the average entropy for given system.

4.4 Results

Figure 7 shows the change in the average perplexity of the VLMM with the change in the maximum order. As you can see, there is a clear downward trend in the values, with the LTM Order 0 showing an entropy of 3.614 (perplexity of 12.24) and Order 20 shallowing out at an entropy of 1.584 (perplexity of 3.0). Results are statistically significant at the 0.01 level. Clearly, increase in the model order leads to better

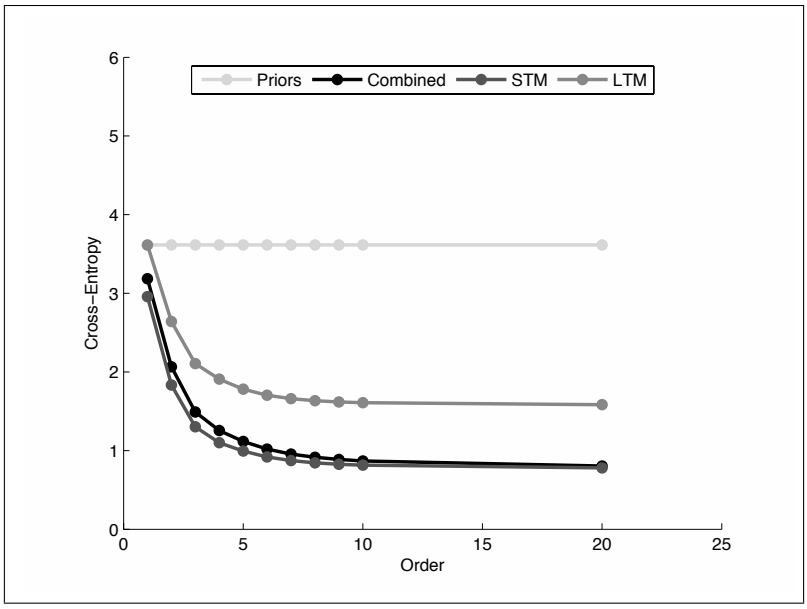


Figure 7: Entropy of predictions plotted against increasing model order

performance, although with diminishing returns. Beyond this limit (Order 20), even drastic increases in the model order caused only negligible changes in perplexity, and so we did not pursue this any further. The drop in perplexity can be easily explained by considering the nature of the task at hand. Over 42 symbols, the chance of picking any one of them randomly is only about 1/42 or 2.3%. As soon as some context information is provided to the system, most of these possibilities get filtered out, and number of valid paths that could continue the sequence gets considerably smaller. This leads to a sharp decrease in the average perplexity from Order 0 to Order 1. As we go deeper and deeper down the PST, however, we start reaching the limit of the sequences that have actually been seen by the model so far. In fact, a large number of songs in the database are built around sequences that are 16 beats long, leading to about 32 strokes for a many of the sequences. At around Order 30 and beyond, the PST is reaching the limits of its sequence length, and further increases to say, 40 or 50 make no difference to the model. Moreover, exact matches are extremely difficult to find for higher orders, therefore, the model ends up picking possibilities from lower orders instead, leading to only marginal improvements for these higher orders. The

increase in order also led to noticeable changes in the generative aspect of the model, and higher-order models tended to produce larger fragments of coherent material, as we would expect. Beyond order 20 though, differences in the musical material produced were no longer easily noticeable.

Along with the experiment discussed above, an informal qualitative evaluation of the musical material generated by the VLMM was also conducted. This evaluation was very brief and completely subjective, based only on the opinions of Dr. Chordia and myself, and therefore will not be described here. However, it did yield two useful results that influenced some technical decisions that deserve mention. The first is that although the STM seems to show lower entropies than the LTM and the combined model (see Figure 7), its generative output tends to be very repetitive. The combined model, on the other hand, may show a higher average entropy but tends to produce more interesting phrases and variations. This means that the LTM provides the variety for creating novel variations, while the STM provides the structure to help keep the composition coherent. A corollary of this is that since the STM is getting trained as each song progresses, the entropy per term tends to decrease over time, flattening out after approximately 50-100 tokens (a theme and one or two variations worth). This indicates the minimum amount of material required for the STM to incorporate the song-specific structure. The second result is that the SxPiC viewpoint caused a surprising amount of improvement in the coherence of the generated phrases by preserving the rhythmic structure within a theme or variation. Although this improvement is not visible in the quantitative results, from a practical standpoint, the SxPiC viewpoint is essential to the phrase generation process of the VLMM.

A variety of experiments were conducted to compare the relationship between smoothing algorithms and model performance. Table 7 summarizes the results for the Back-Off(BO) model, 1/N approach(1/N) and the Parametric(P) model using a VLMM of maximum order 10. The BO model is clearly the least effective of the

Table 7: Average and median of perplexity results for for back-off, 1/N, and parametric (with exponent coefficient equal to 1) smoothing methods. Results for combined models using a maximum order of 10. MV refers to the multiple-viewpoints model in which the SxD and SxPIC viewpoints have been incorporated.

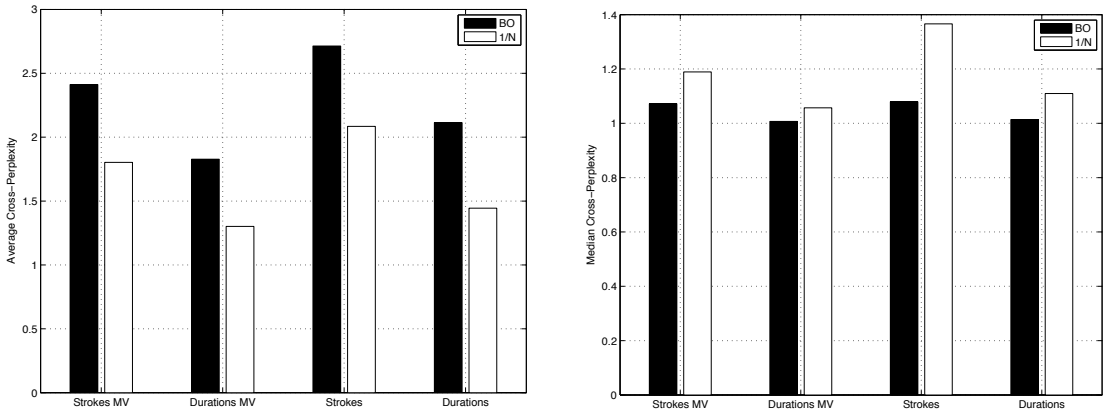
Model	**Strokes**		**Strokes MV**		**Durations**		**Durations MV**		**SxD**		**SxPIC**	
	Av.	Med.	Av.	Med.	Av.	Med.	Av.	Med.	Av.	Med.	Av.	Med.
BO	2.71	1.08	2.41	1.07	2.11	1.014	1.83	1.01	3.84	1.29	2.58	1.40
1/N	2.08	1.37	1.80	1.19	1.44	1.11	1.30	1.06	2.69	1.58	2.55	1.37
P (x=1)	2.00	1.33	2.03	1.27	1.55	1.09	1.42	1.07	2.60	1.43	2.57	1.37

Table 8: Summary of perplexity results for LTM for order 10.

Order	**Durations**	**Strokes**	**SxD**
10	1.76	3.05	4.66

three, however, it still gives the lowest values for median perplexity. This suggests that the BO model tends to either find very good matches, or not at all, while the other two models manage to strike a better compromise because of their weighting schemes. Figure 10 shows us a histogram of all perplexity values for the BO model. As expected, there is a large concentration of values around 0, however, the distribution continues till values of 30 or more. Between the other two models, the parametric model, with an exponential coefficient of 1 (denoted P(x=1)) seems to do marginally better than the 1/N model, though not always. Figure 4.4 gives us a much better look at the differences between the BO model and the 1/N model for the different viewpoints. All models however, compare very favorably to the baseline perplexity of 12.24 obtained using only the prior probabilities of the independent strokes.

Another trend that relates to Table 7 is regarding the drop in perplexity with the addition of viewpoints. Reading across the table, we see that the addition of viewpoints seems to improve model performance. For the BO model, perplexity drops from 2.71 to 2.41 between Strokes and Stroke MV, meaning that the addition of the SxD viewpoint to the Strokes viewpoint brought some useful information to the table. The same trend is also seen between Durations and Durations MV for the



(a) Comparison of average perplexity between BO and 1/N for single and multiple viewpoints.

(b) Comparison of median perplexity between BO and 1/N for single and multiple viewpoints.

Figure 8: Comparison of perplexity between BO and 1/N models.

BO model. Again, all the results discussed so far are statistically significant on the Tukey-Kramer test at the 0.01 level.

Figure 9 shows the variation of average perplexity for different values of the exponent for the parametric model. The curves points towards a minima between $x = 0.5$ and $x = 2$, an optimum value of the exponent within this range. The results show that the weighting scheme does make a small but significant difference to the prediction scheme of the model. Clearly, higher weights for higher orders leads to lower perplexities, though this rule does not hold beyond a certain limit. What is not seen here is the effect of changing the smoothing scheme on individual predictions. Though the average perplexities might not vary too much, it might lead to subtle changes in prediction and generation. It might be worthwhile to explore the effect of smoothing in a generative context rather than in a quantitative analysis.

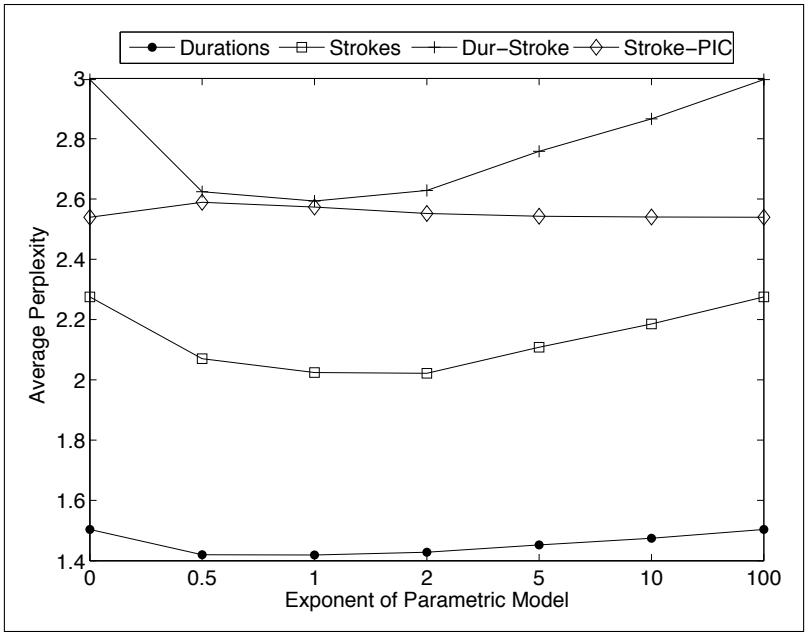


Figure 9: Perplexity as a function of exponent coefficient in parametric model. There seems to be an optimal range between 1 and 2.

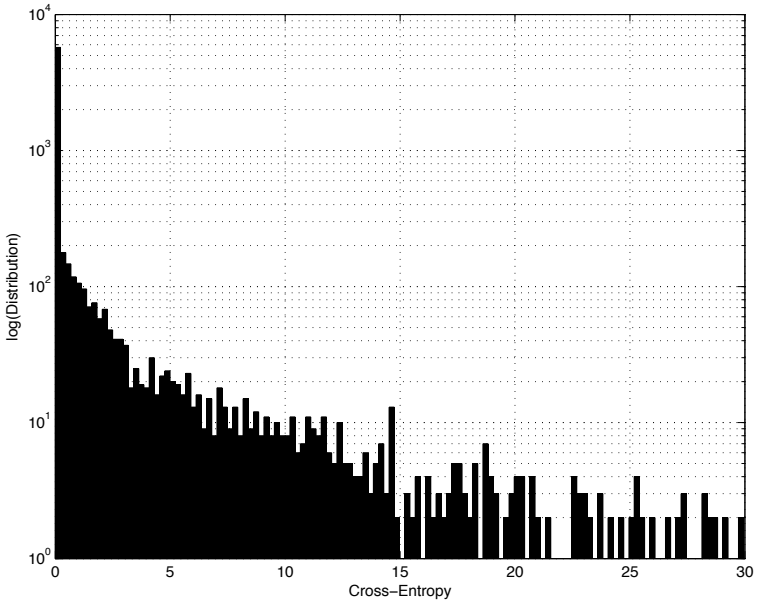


Figure 10: Histogram showing range of perplexity values when predicting strokes for the Back-Off model

CHAPTER V

VLHMM IMPLEMENTATION

The model described so far only handles symbolic data, and while there are plenty of applications for such a model, a more practical scenario would be to try and apply these principles to audio data. This involves using VLHMMs instead of VLMMs, applying the principles learnt using the VLMM to the audio domain. The transition from symbols to audio is not a very simple one has some major consequences on the system as well as its behaviour.

5.1 Hidden Markov Models

The extension from Markov models to Hidden Markov models is one that was born out of practical necessity. While symbolic information is a useful source of input, it is not always available. In most cases, what is available is real-world data such as sensor measurements like raw audio input from a microphone, or acceleration from accelerometers. HMMs provide the means to apply sequence modeling effectively to real data. They have proven to be very successful for applications such as speech recognition and music information retrieval. Juang and Rabiner explain the power behind HMMs and their success in speech processing in their seminal paper on first-order HMMs [39]. Thede and Harper describe the use of a second-order HMM in [65]. In MIR applications, HMMs have proved to be quite successful at chord estimation and key extraction [42] [43] and automatic accompaniment for vocal melodies [11].

Figure 11 gives a simplified explanation of the concept behind an HMM. Consider the Markov model described in Figure 2. Recall that it shows the transition probabilities to predict whether the next day is going to be sunny or rainy, depending on the

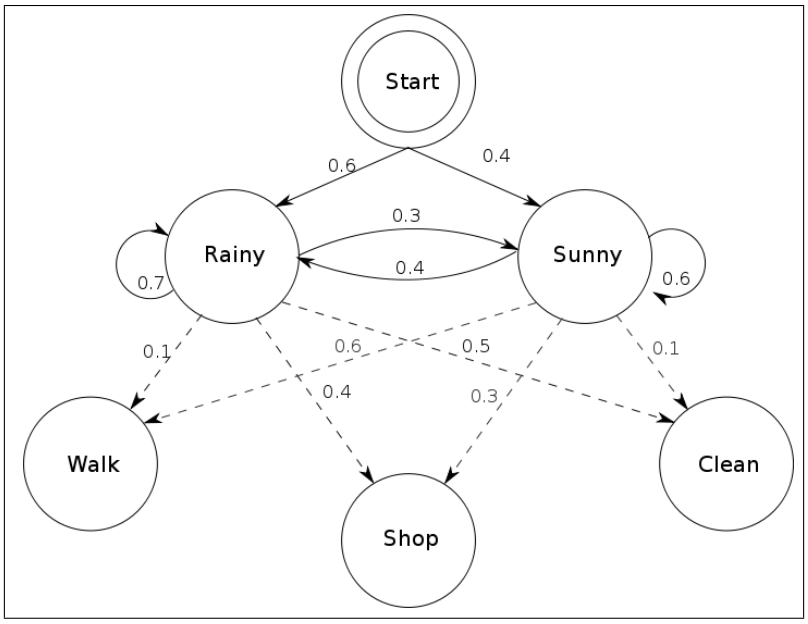


Figure 11: An example of a simple Hidden Markov Model with 2 hidden states(Rainy, Sunny) and 3 visible symbols(Walk, Shop, Clean).

current weather conditions. Now suppose we add a second process: a person experiencing these weather conditions can do one out of three things on a given day. He/she can either walk, shop, or clean. The task given to our new model is to predict the weather on a given day based on the weather of the previous day, only by observing whether this person is walking, shopping or cleaning on this day. We can formulate this as follows. The states of the weather, (R)ainy and (S)unny, cannot be directly observed, and are called the hidden states. Each hidden state is capable of *emitting* a visible symbol, any one of Walk(A), Shop(B) and Clean(C). Say our HMM is now fed with a sequence of the person's daily activities over the last few days. To predict our next state, the HMM has to calculate the best sequence of hidden states that led to this context, and based on that information calculate what the next hidden state is most likely going to be. When this operation is performed for one particular state, it is termed the *forward probability* of that state following the given sequence.

Let us re-formulate this problem mathematically. Given a sequence of observations $O_t = \{x_1, x_2, x_3, ..., x_t\}$, we need to calculate $P(\omega_{t+1}^j | O_t)$ for $j = 1, ..., c$, where c is

the number of hidden states, and $P(\omega_{t+1}^j)$ represents the probability of the jth state at time $t+1$. This is simply $P(\omega_{t+1}^j|O_t) = \sum_{i=1}^{c} \alpha_i^t a_{ij}$, where α_i^t is the probability of being in state i at time t having emitted O_t, i.e. the forward probability of state i. If a_{ij} is the probability of making a transition from i to j then the required forward probabilities can be calculated recursively as follows:

$$\alpha_j^t = \sum_{i=1}^{c} \alpha_i^{t-1} a_{ij} P(x_t|\omega_j) \tag{16}$$

giving us the probability of state j occurring after the sequence O_t has been emitted. This needs to be repeated for every possible state, to calculate a discrete distribution across all hidden states.

5.2 Database

A synthetic audio database based on the symbolic **bol files described in Section 4.1 was generated for the purpose of evaluating the VLHMM. Two sets of files were recorded, each using a different set of samples, for training and testing. The samples were taken from two previously recorded performances by renowned tabla players. These recordings were part of a previous experiment [14] and had already been segmented into strokes and labelled manually. The **bol files were then used to synthesize the compositions stroke by stroke. To achieve this, a tabla stroke sequencer, designed in PureData, by Alex Rae as part of his thesis [55], was used - stroke labels and IOIs were read from **bol files and the corresponding stroke samples were triggered in sequence. The resulting audio was recorded as wav file (16-bit, 44.1 kHz). A randomized selection process was used to choose between about 10 different samples for each stroke to increase the variability and difficulty of classification. For each song, the timestamps of each stroke were also noted. The audio, timestamps as well as the bol files were organized into separate folders and now form the JNMR Tabla Database. It is accessible at http://paragchordia.com/jnmrTabla.

Using the recorded time-stamps, the audio files were programmatically segmented into individual strokes and read into MATLAB. Sertan Şentürk's help with this part of the process proved to be invaluable. His code on segmentation and feature extraction was instrumental for the timely completion of this work. Although beat detection could have been employed to segment the files, which would allow us to model durations as well, in the interest of time, we decided to use the time-stamps for the segmentation process. It remains for future work to implement an onset detection system for the HMM. Based on [33], we found that the MFCCs were a good choice of features for percussive samples and the first 21 MFCCs were extracted for each of the strokes. The 0^{th} coefficient, representing signal energy was discarded. On average, the length of each sample was within about half a second, so the features were calculated on the entire stroke, without any division into frames. This resulted in a longer FFT length than a standard such as 4096 samples, therefore offering very high frequency resolution for the features. On the symbolic side, the 42 stroke names were re-mapped to 9 acoustic categories. Several strokes used different names depending on their context in the composition. To simplify the task of audio classification, similar sounding strokes were grouped into the same acoustic category and a new set of labels was prepared. The features were then aligned with their respective labels and written to another set of files which were then read in to the VLHMM system.

The reason for creating a synthetic database, instead of original recordings, is that we wanted a database that was annotated on a stroke-by-stroke basis. Since the symbolic database provided the names of the strokes for us, it was easier, both in terms of time and effort, to synthesize a new database instead of manually annotating a the recordings. The synthesized database also has the advantage of having a much larger number of strokes because the recordings were only a couple of hours in length. The use of a synthetic database usually produces optimistic results since the variations within a small synthesized set are much smaller than those encountered in a real

scenario. This is accounted for by the use of two completely different sets of samples for the training and testing database. This should simulate a a real-world case, where the model has no prior information about incoming audio.

5.3 VLHMM

The VLHMM framework is built upon the PST architecture developed for the VLMM, however, the complexity of a VLHMM system led to considerable differences between the two models. The first of these, is that the **bol input to the model is now replaced by feature vectors extracted from segmented audio samples. The audio features also add an additional layer into the predictive process - not only do we have to try and predict what stroke comes next, we also have to evaluate the probability of the input features representing that stroke. The PST itself is re-designed to accommodate two distributions instead of one - every node in the tree contains a transition probability for the state itself, and an emission distribution for the visible symbols. To add to this, the state space for the visible symbols is a continuous spectrum consisting of the entire range of MFCC values, and not a discrete space. Therefore, the emission distribution consists of a multi-variate gaussian(MVG) mapping all observed values of the MFCCs into all observed stroke types. The biggest consequence of these changes is processing time. There is a massive amount of computation required to calculate the forward probability of each sequence, and this computation increases exponentially as we go down the tree. At the same time, a significant amount of uncertainty is added to the system by the continuous input, causing a noticeable decrease in performance.

Another change in the VLHMM is the reduction in the state space - the number of strokes - from 42 to just 9. There are two reasons for this, the first being that the system of nomenclature of the tabla is based on *bols* and not the strokes themselves. Since a *bol* is a compositional unit, rather than a specific sound, the same stroke can be represented by several different *bols* depending on its position in the piece. Since

the model has no way of differentiating these *bols* by their acoustic features, all *bols* of similar acoustic properties were grouped together into a single stroke. This reduced state space also has the added benefit of reducing the computation time drastically. The final change, in a effort to reduce the calculation time taken per prediction, the VLHMM model does not use the STM. The inclusion of the STM would double the time taken per prediction, making the model too slow for any practical purpose, so it was decided that only the LTM would be used, and the order of the model would be restricted to a maximum of 3.

The remaining aspects of the VLMM structure are kept as they are. All low-level data structures and functions are retained except for minor changes to accommodate the new input types. Higher level functions such as those used for building the PST - smoothing, merging and prediction - remain largely unchanged, and all the concepts developed for the VLMM are adapted to the architecture of the HMM.

5.3.1 Training

The training procedure for the HMM is comprised of two independent processes. The first involves creating the MVG system for the set of strokes. We used a leave-one-out approach as before, creating 34 sets of MVGs, each one built without using the strokes of one of the songs. Each MVG was built using the MFCC coefficients of the strokes using a shared covariance matrix. The means and covariances were then written to files and read in directly into the VLHMM. The second process involves building the PST. This was done exactly as before, since we are only using stroke labels to estimate the transition probabilities between the nodes of the PST. Here, we use the same emission distribution for all nodes of the tree.

5.3.2 Prediction Scheme

Prediction for the VLHMM is almost identical to the process used for the VLMM. As discussed in Section 5.1, the probability of a state, or in our case, a stroke, ω_{t+1}^{j}

is given by

$$P(\omega_{t+1}^{j}|O_t) = \sum_{i=1}^{c} \alpha_i^t a_{ij} \tag{17}$$

Note that this probability is only for a single HMM of a fixed order. For a VLHMM system, all subsequences of the original sequence must also be considered and their weighted sum is then taken. This now gives us the equation

$$P(\omega_{t+1}^{j}) = \frac{w_0 P(\omega_{t+1}^{j}|\{\}) + w_1 P(\omega_{t+1}^{j}|\{x_t\}) + \sum_{i=2}^{N-1} w_i P(\omega_{t+1}^{j}|\{x_{t-i+1}, \ldots, x_t\})}{\sum_{i=0}^{N-1} w_i} \tag{18}$$

which returns the probability that the stroke ω_j is going to occur at time $t+1$. This process is then repeated for all possible symbols and the resulting distribution is then normalized to sum up to 1. The probabilities are then converted to entropies and averaged as part of the evaluation proces.

However, as noted earlier this quickly becomes unmanageable because the state space expands rapidly for high-order models. The PST architecture, provides a practical solution and allows us to consider a much smaller number of terms in the calculation of each forward probability α_t^i. To calculate $P(\omega_{t+1}^{j}|O_t)$, for an HMM of order m, we look for all nodes at level $m+1$ that correspond to category j. Let the set of such nodes be A. The total number of nodes that fit this definition will be the number of terms in our sum, which will in general be much less than the theoretical maximum. Since each node has only one parent, we multiply the forward probability of the parent node by the transition probability from the parent to child. These partial probabilities are then summed for all $i \in A$ to get $P(\omega_{t+1}^{j})$. In other words, we now have $\alpha_j^t = \sum_{i \in A} \alpha_i^{t-1} a_{ij} P(x_t|\omega_j)$. Figure 12 shows how the number of nodes grows with the depth of the tree for this data set compared with the theoretical maximum

given by c^m. Between order 2 and 3 the number of possible nodes explodes, however the actual number of nodes in the PST is approximately an order of magnitude less than this. Another caveat with high-order HMMs is that labeled symbolic data might not always be available and so the training process comes down to estimating the transition probabilities between the states using the Baum-Welsch algorithm [6]. In this case, since the hidden states correspond to the strokes, and that information was easily available, the training procedure for the HMM was greatly simplified.

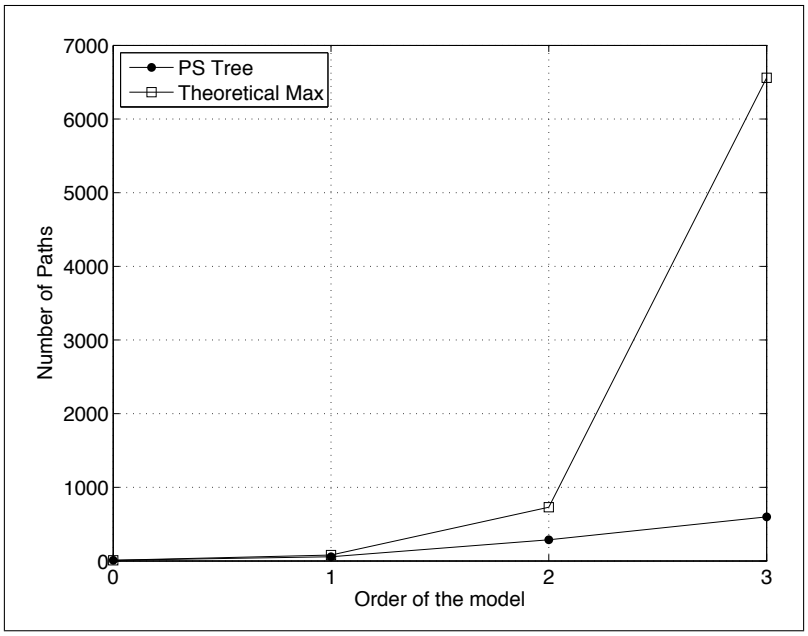


Figure 12: Comparison between number of nodes in the PST at a given level vs. the theoretical maximum

5.3.3 Evaluation

The evaluation process is notably different from that of the VLMM. Input data was first split into two different parts - the labels and the features for all the strokes. The features were saved in a context stream, and the forward probability of those features was then calculated. The forward probability for the stream is then multiplied with the emission distribution of the final set of symbols to give the next symbol distribution for that particular context. This process is repeated for every possible

Table 9: Table showing the change in perplexity with increasing order of the VLHMM

	Order 0	**Order 1**	**Order 2**	**Order 3**
Average	2.64	2.45	2.50	2.31
Median	1.23	1.20	1.20	1.16

context stream (from orders 1 to N) and then smoothed and merged together to form a single next-symbol distribution. The probability of the actual label corresponding to this feature vector is then checked in the next symbol distribution and recorded. A graphical representation of this process is shown in Figure 13. As before, this process is repeated for all 34 songs, and the average entropy and perplexity are calculated.

As with the VLMM, this evaluation scheme was used to conduct a few basic experiments to gauge the performance of the VLHMM. The most important of these is the relationship between model order and perplexity, providing a clear picture of how performance improves as the VLHMM is extended. Another important metric is the average time taken per prediction and its increase with model order. Its relationship with the order gives an idea of the efficiency of the system and its use in a practical generative system. A song-wise analysis of the data is also conducted, for a detailed picture of the performance of the VLHMM on the different types of songs, and explore possible shortcomings with the current approach.

5.4 Results and Discussion

Tables 9 and 5.4 summarize the performance of the VLHMM. Four evaluation runs of the VLHMM using different orders (see Table 9) were conducted. Order 0 represents the baseline perplexity, i.e. using only the MVG distribution to perform a classification task on each stroke - the context model was not used for this evaluation - and the average perplexity of 2.64 represents the minimum bound for the performance of the model. With increasing order of the VLHMM, the average perplexity shows

Table 10: Average perplexity for each test song, and the median perplexity across all test songs for each order.

	Order 0	**Order 1**	**Order 2**	**Order 3**	**Composition Type**
Song 1	1.82	1.71	1.75	1.72	qaida
Song 2	4.79	4.38	4.36	4.3	laggi
Song 3	2.06	1.95	1.89	1.83	qaida
Song 4	2.77	2.47	2.4	2.35	qaida
Song 5	2.9	2.67	2.53	2.44	qaida
Song 6	2.22	2.06	2.01	2.02	qaida
Song 7	4.3	4.08	3.95	3.78	qaida
Song 8	1.59	1.47	1.44	1.41	rela
Song 9	2.06	1.97	2.9	1.84	rela
Song 10	2.03	1.97	1.93	1.89	tukda
Song 11	2.33	2.26	2.26	2.28	gat
Song 12	2.34	2.34	2.33	2.29	chakradhar
Song 13	1.94	1.83	1.82	1.8	qaida
Song 14	2.61	2.41	2.35	2.27	qaida
Song 15	3.06	2.85	2.81	2.74	qaida
Song 16	1.66	1.53	1.45	1.4	rela
Song 17	1.92	1.87	1.83	1.79	tukda
Song 18	3.4	3.11	2.97	2.83	gat
Song 19	2.31	2.19	2.13	2.14	chakradhar
Song 20	2.54	2.23	2.13	2.1	qaida
Song 21	2.66	2.32	2.23	2.16	qaida
Song 22	1.63	1.55	1.68	1.39	rela
Song 23	2.66	2.53	2.48	2.4	tukda
Song 24	3.95	3.53	3.39	3.23	gat
Song 25	2.89	2.77	2.73	2.6	chakradhar
Song 26	2.24	2.07	2.07	2.03	qaida
Song 27	2.26	2.11	2.05	2.03	qaida
Song 28	2.19	1.98	1.88	1.79	rela
Song 29	3.05	2.9	2.82	2.7	tukda
Song 30	31.24	28.35	27.29	26.73	gat
Song 31	4.42	4.46	4.39	4.31	chakradhar
Song 32	3.81	3.74	3.65	3.68	keharva
Song 33	21.21	15.16	13.73	13.55	keharva
Song 34	9.19	8.15	8.07	7.86	dadra
Median	2.575	2.33	2.34	2.275	

a decreasing trend, going from 2.45 for a first-order VLHMM to 2.31 for the third order. On a Tukey-Kramer test, all results showed statistical significance at the 0.01 level except the average perplexities between Order 1 and Order 2.

Note the significant drop in perplexity between Order 1 and Order 0, showing that even the slightest bit of context information can lead to a considerable change in the perplexity values. On the other hand, the drop from Order 1 to Order 3 is a bit less than expected, though it is still a good improvement. In comparison with the earlier results of the Strokes(S) viewpoint of the VLMM model, the VLHMM seems to show perplexity results around a similar range of values, when it should clearly have suffered a more significant decrease in performance due to the shift to audio. The reason for this is that the VLMM uses 35 different symbols for its prediction task and has a lot more choices to choose from. In contrast, the VLHMM only has 9 choices to choose from due to the categorization of similar sounding strokes into the same acoustic category.

Table 5.4 offers a list of the average perplexities for each song. A closer look at the table shows that there is a considerable amount of variation in these values. Song 1 shows the best performance with a perplexity of 1.71 while Song 30 forms the other end of the spectrum with a perplexity of 26.73. Since the perplexity values are scaled along an exponential scale, the difference in performance is greatly amplified, yet this difference is a clear indication that certain songs, such as 30, 33 and 34 show very poor model performance. There is, however, good reason for this drop in performance. Song 33 is a *gat*, a complex fixed composition containing very different sequences as compared to most of the other songs (*qaidas* and *relas*) which are built on a theme variation structure. In addition, the song was set in a meter of 12 beats per cycle, while most other songs are set in a cycle of 16 beats. Examining Songs 33 and 34 also shows a similar trend. Song 33 is a *keharva theka* and Song 34 is a *dadra theka*, both of which are very different from the other songs in the database. These

Table 11: Table showing the average perplexity by composition type

	Qaidas	**Relas**	**Gats,Tukdas**	**Thekas**
Avg. Perplexity	2.25	1.57	4.97	7.34

two songs happen to be have the maximum amount of internal repetition, and the poor performance of the VLHMM in these two cases clearly shows that without an equivalent STM approach, the system is far too generic to adapt itself to individual songs.

Table 11 shows the perplexities averaged for each of the composition types. Since *qaidas* and *relas* comprise 50% out of the 34 compositions, the database is clearly biased towards them, with perplexities of 2.25 and 1.57. *Gats* and *tukdas*, which are both forms of fixed compositions show a much higher perplexity, while the *thekas* show the worst performance. This is quite consistent with the theoretical aspects of these compositions. *Relas* and *qaidas* are generally more predictable because of their theme variation structure. The fixed compositions are usually a lot less repetitive and contain sequences with unusual phrasing and juxtaposition. The *thekas*, as mentioned earlier, despite being repetitive, are outliers in terms of their structure within this database and therefore show high perplexities. Another major source of error is the lack of high-level information in the VLHMM. Since the context is never more than three events long, the model has no information about long term repetition or progression. This leads to some very obvious failures in both prediction and generation. This can easily be corrected by manually coding some higher level structural details into the model, and this would certainly be a necessary requisite for a generative model. We chose not to do this for the quantitative study to allow us to better understand the working of the VLHMM on different types of songs.

Table 12: Table showing the average computation time per stroke in seconds for VLHMMs of orders 1, 2, 3

	Order 1	**Order 2**	**Order 3**
Time(sec)	0.066	0.198	0.534

Table 12 shows the average computation time taken per stroke and its relationship with the maximum order of the VLHMM. These experiments reflect the time taken to run on Mac OS X with a CPU speed of 2x3 Ghz and 4GB of RAM. It is quite clear that only the first-order model can be considered a viable option for a real-time interactive system as of now, however it does show that with a few optimizations, second and third-order models are well within reach. The use of parallel processing algorithms and architecture-specific libraries for the calculations can significantly reduce computation time, while running the C++ libraries outside of MAX/MSP can further improve speed and memory management. These optimizations may justify the use of high-order VLHMMs in future systems given the limited increase in accuracy.

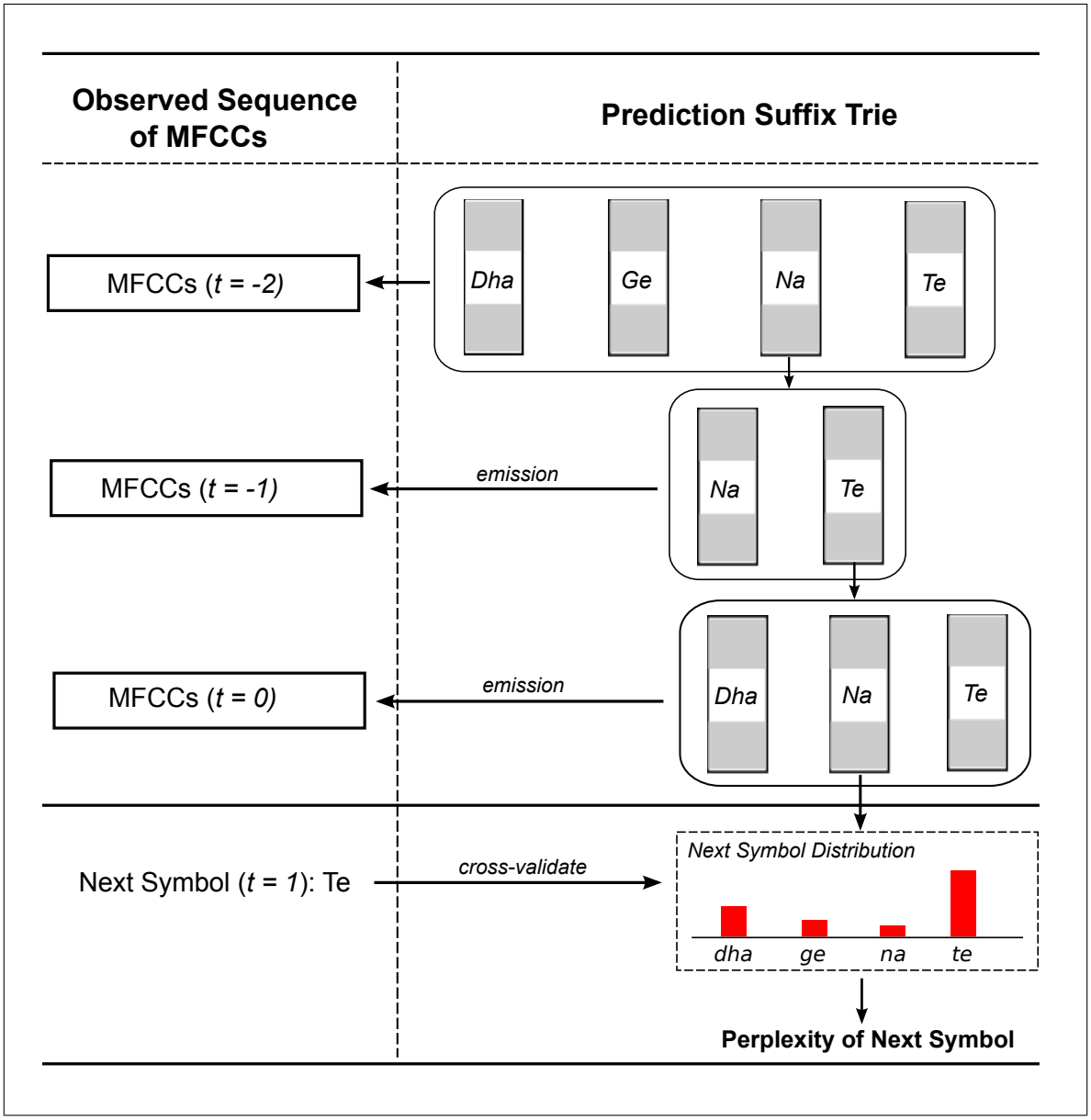


Figure 13: Figure describing the cross-validation scheme used for the VLHMM system

CHAPTER VI

FUTURE WORK

An overall assessment of the VLHMM system makes it quite obvious that the scope of this research goes far beyond that of a simple thesis. There are plenty of loose ends at this stage, and many possible options for improving the system from its current state. The first step should be to increase the speed and computational efficiency of the model, to help lessen the constraints on order and state space. Although established methods like the PST do provide a considerable advantage in this regard, further optimizations to the prediction algorithms are necessary to fully harness the power of higher-order HMMs. Other techniques for HMMs, such as Mixed-Order Modeling [62], will be required to improve the speed of the model.

Regarding classification and predictive accuracy, this is where the maximum amount of improvement in the system can take place. One strong possibility is to experiment with different audio features to improve the baseline classification accuracy of the system. This in turn will reinforce its predictive ability. Another modification that was considered was the use of emission distributions built using the set of bi-grams instead of individual strokes. Potentially, this could improve the classification of the VLHMM for strokes even when the audio contains the reverberation of the previous stroke. Sadly, the distribution of strokes in the database was too sparse to construct MVGs for all categories, and in the interest of time, we decided not to incorporate this into our work for now. A useful modification to the VLHMM was the implementation of the Baum-Welsch algorithm to modify the HMM during the testing phase. This would bias the probabilities in favor of the current composition, and allow the model to adapt to incoming input, much like the application of the short-term model in the

symbolic case. Given the size of the VLHMM though, and the number of different paths in the PST, our implementation did not yield promising results, and I prefer to leave it unpublished till a better approach is found.

From a generative perspective, one possible modification is to include adding duration modeling to create rhythmically coherent material. This would also require the incorporation of onset detection into the audio analysis of the model - a theoretically viable option, but one that brings its own share of problems to the table, especially when one considers the inconsistencies associated with current algorithms. Error handling mechanisms that could be used to compare sequences between training and testing sessions to potentially correct errors due to pitch tracking and onset detection will also need to be used to counter these shortcomings.

As far as long term goals for the project are concerned, generalizing this model to work in other domains like chord recognition is one objective, but to me, the implementation of a real-time improvisation system seems far more relevant. I believe that the best evaluation for any generative system is in the context of a real-world scenario, such as its interaction with a human performer. Not only does this impose restrictions on the speed and accuracy of operation, it is also perhaps the only way to judge qualitative aspects of the generated material like its coherence, its creativity and its formal structure. To me, this represents the ideal destination for my work.

CHAPTER VII

CONCLUSION

This thesis describes a computational model of tabla sequences based on VLMMs and VLHMMs. To the best of my knowledge this is the first use of VLHMMs for music prediction. When predicting the next stroke or duration of a tabla composition VLMMs are highly predictive, with a minimum perplexity of 1.80 using a median perplexity of 1.07 using Multiple Viewpoint modeling. Incorporating a short-term model substantially improves performance compared with only using a corpus-wide long-term model. The reflects the fact that patterns in musical pieces often differ substantially from corpus-wide patterns, while being internally quite consistent. The incorporation of additional rhythmic viewpoints leads to small, but significant improvements in the entropy of stroke predictions. These results open up several possibilities for applications in generative systems. The low perplexity values (¡ 2.0) indicate that on average the model has to choose between upto 2 options for the next step, indicating that it does generate variations very similar to those that are present in the training database. At the same time, the weight for the LTM can be increased to add more variety and generate unexpected material. Further study will be required to develop the system from an artistic perspective, but as a technical framework, the level of prediction and control seems satisfactory.

The work also shows that VLHMMs can be used to predict stroke continuations from audio. The approach is based on computing the forward probabilities for the high-order HMMs that constitute the VLHMM, efficiently using a PST that had been learned from stroke patterns using symbolic data. Increasing the maximum model order from 1 to 3 decreases perplexity by a small amount. Depending on the

application, the computational burden of computing the high-order HMM is probably not justifiable given this performance improvement.

However, currently there is no analog to the STM in the hidden framework. Since the strokes are not visible it is more difficult to learn the song-specific n-grams. One approach of solving this is to allow the transition probabilities to adapt within a song. Results from the symbolic domain suggest that incorporating such song-level information could lead to dramatic improvements.

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