TRANSCENDENTAL GENERATION, TAX COMPETITION AND REDISTRIBUTION
IN FEDERAL SYSTEMS

Naoto Aoyama and Emilson C. D. Silva

WP2011-004
November 15, 2011

http://www.econ.gatech.edu/research/workingpapers

School of Economics
Georgia Institute of Technology
221 Bobby Dodd Way
Atlanta, GA 30332-0615

© 2011 by Naoto Aoyama and Emilson C. D. Silva. All rights reserved. Short sections of
text, not to exceed two paragraphs, may be quoted without explicit permission provided
that full credit, including © notice, is given to the source.
Transboundary Pollution, Tax Competition and Redistribution in Federal Systems

Naoto Aoyama and Emilson C. D. Silva
WP2011-004
November 15, 2011
JEL: D62, D78, H41, H77

ABSTRACT

We examine the shape of federal policy making in three different policy scenarios, in which regional governments determine regional environmental policies to control correlated transboundary pollutants and the center implements interregional income transfers. We examine policy making under horizontal and hierarchical federal structures. In a horizontal structure, federal and regional governments make simultaneous policy choices. In hierarchical structures, federal and regional governments make sequential policy choices. Sequential choices may feature centralized or decentralized leadership. Our results indicate that hierarchical federal structures characterized by decentralized leadership may be socially superior to horizontal and hierarchical federal structures characterized by centralized leadership.

Naoto Aoyama
S.K.K. Information Business College,
1-3 Tokuda, Hirosaki,
Aomori 036-8032, Japan
naoyama7@hotmail.co.jp

Emilson C. D. Silva
School of Economics,
Georgia Institute of Technology
221 Bobby Dodd Way
Atlanta, GA 30332-0615, USA
1. Introduction

Most federal systems are characterized by interregional redistribution and policy making to address interregional spillovers. In the United States, there are many forms of federal grants that attempt to reduce regional (i.e., state) income disparities. Similarly, in the European Union, interregional redistribution schemes are in place to maintain cohesion and reduce income disparities amongst its objective regions. Perhaps, the clearest example of federal policy that attempts to address interregional spillovers is environmental policy making for the control of transboundary air pollutants, such as sulfur dioxide and nitrogen oxides, the main sources of acid rain. This form of federal policy is widely observable in the United States, the European Union and Canada, to cite just a few federal systems.

More recently, the European Union launched the second phase of the European Union Emissions Trading System (EU ETS) in order to account for some of the damages that are caused by emissions of greenhouse gases from significant European air polluting sources.\(^1\) A similar cap and trade program for the United States was part of President Barack Obama's agenda, but it stalled at the Senate in 2010 after passing in the House. Even though the United States has not yet implemented such an emissions trading scheme for greenhouse gases, it may become a reality in the future if President Obama is reelected.\(^2\)

Another key characteristic of federal systems is harmful tax competition. For example, as a clear demonstration that harmful tax competition has been a major concern in Europe, the European Union adopted in December 1, 1997 the Code of Conduct for

---

\(^1\) For a detailed description of the EU ETS, see http://ec.europa.eu/clima/policies/ets/index_en.htm.

\(^2\) See http://change.gov/agenda/energy_and_environment_agenda/.
business taxation to induce the member nations to reduce existing tax policies that produced harmful competition as well as to inhibit the implementation of such tax policies in the future.\textsuperscript{3} For the United States, Devereux et al. (2007) find evidence of harmful tax competition for cigarettes amongst neighboring states and for gasoline amongst state and federal governments. More recently, however, Chirinko and Wilson (2011) find evidence that, in the United States, the average state, in setting its capital tax policy, reacts negatively rather than positively to changes in capital taxes in neighboring states. This finding runs counter to the "race to the bottom" phenomenon typically associated with tax competition in federal systems.

In this paper, we consider the interplay of regional environmental policy designed to control net emissions of air transboundary pollutants, federal redistributive policy and horizontal tax competition between two regions that compete to attract capital. Our main settings are hierarchical federal systems in which two levels of government, federal and regional, have divided responsibility over implementation of socially desirable policies. The federal policy attempts to reduce regional inequality. The motivation for the regional environmental policy is to reduce pollution damages. Tax competition emerges from the facts that capital is mobile and the regional governments are obliged to utilize capital taxes to finance regional provision of air pollution abatement.

Silva and Caplan (1997) considered the potential pitfalls that divided responsibility between federal and regional governments over control of transboundary pollutants can have in hierarchical federal systems. They demonstrated that a federal system which features decentralized leadership, similar to the prevalent system in the

\textsuperscript{3} For a detailed history of measures undertaken by the European Union to reduce harmful tax competition, see http://ec.europa.eu/taxation_customs/taxation/company_tax/harmful_tax_practices/index_en.htm
European Union, may be socially superior to a federal system which features centralized leadership, similar to the prevalent system in the United States. Decentralized leadership dominates centralized leadership whenever regional welfare functions are quasilinear and the center, which cares about equity across regions, promotes interregional transfers. The subsequent paper by Caplan and Silva (1999) made the superiority of decentralized leadership even sharper, since in their setup decentralized leadership was socially efficient even when the center did not care about equity, and thus did not pursue redistribution across region. Decentralized leadership is shown to be superior whenever the regional governments choose pollution taxes and subsequently the center decides on the levels of pollution abatements the regions should produce.

The efficiency properties of decentralized leadership in federal-like institutional arrangements that attempt to control transboundary pollutants have been shown to be applicable to the Kyoto Protocol (see, e.g., Caplan et al (2003)) and to hybrid pollution control arrangements that may combine environmental policies for domestic and international pollution control (e.g., Aoyama and Silva (2008), Caplan and Silva (2005), Silva and Zhu (2009)) and for continental and global pollution control (e.g., Silva and Zhu (2011)). Of particular relevance to this paper is the framework introduced by Caplan and Silva (2005) in which pollution emission sources emit multiple correlated pollutants and the technology that is adopted to abate such emissions is coarse, in the sense that it is not finely designed to abate each type of pollution emission but to abate all pollutants (possibly at varying degrees).

The literature on interregional tax competition is vast (see, e.g., Wilson (1986, 1999) and Wildasin (1989)). Wilson (1986) states that “…tax competition exists if and
only if a rise in a single region’s public service output causes capital to flow out of the region.” (p.303). Wilson (1986) also shows that financing the provision of a local public good by levying a capital tax is inefficient. Wildasin provides an intuitive explanation for the inefficiency caused by capital tax competition: “The inefficiency associated with tax competition can be understood as a kind of externality. This externality occurs because an increase in the tax rate in one jurisdiction causes a flow of capital to other jurisdictions that increases their tax revenue.” (Wildasin 1989, p.194). Wilson (1999) surveys the large literature on tax competition and focuses on the tension there exists between Oates’ (1972) claim that tax competition is wasteful and Tiebout’s (1956) theory of local public good provision which calls for decentralization and competition at the local level to improve efficiency.4

As illustrated in studies of tax competition, regional governments in many federations adopt distortive taxation to finance the provision of public goods and services. In some papers, the regionally financed public goods may generate interregional consumption benefits. A particularly important example is Cremer and Gahvari (2004). These authors study tax competition in the presence of transboundary pollution. The regional governments control two policy instruments, an output tax and a pollution tax. In the absence of pollution, the regional governments would have incentives to lower their output taxes in order to attract capital, characterizing a race to the bottom. In the presence of pollution and without capital mobility, however, the regional governments may wish to set pollution taxes at levels that equate the marginal regional pollution damages. The

---

4 Hindricks et al. (2008) is related to this paper in that it considers settings in which tax competition occurs in the presence of equalization policies. The authors show that tax competition leads to underprovision of public investments and that equalization grants discourage public investments. This paper differs from theirs in many ways, including the fact that we consider settings with transboundary pollution.
authors demonstrate that uncoordinated government typically set inefficient environmental taxes, leading to too much pollution relative to the first-best level, and choose output taxes that support the "race to the bottom" argument.

In a recent paper, Hadjiyiannis et al. (2009) provide evidence that in most OECD countries over 40% of the costs of providing pollution abatement and control are financed by public expenditures. The authors utilize this fact in conjunction with the facts that transboundary pollution is ubiquitous and capital is internationally mobile to build a general equilibrium model in which regional provision of pollution abatement and tax competition are explicitly considered to analyze the efficiency of environmental policy designed by the governments of two politically independent regions (nations). They show that pollution taxes are efficiently chosen when countries are symmetric, but are inefficient when countries are asymmetric. Among other things, our paper differs from theirs in that we consider redistributive transfers and hierarchical federal structures.

The paper is organized as follows. Section 2 describes the basic model. Section 3 provides the center’s most preferable allocation. Section 4 examines the three federal policy scenarios. This section provides the main results of this study. Section 5 concludes the paper.

2. The Federal Economy

Consider a federation consisting of two regions, two autonomous regional governments and one central government. Each region is indexed by $j$, $j=1,2$. There are $n_j$ individuals in each region. Let $N > 0$ be the total population of individuals in the federation where $N = \sum_{j=1}^{2} n_j$. For simplicity, we consider a model in which there is production of a single composite (numeraire) good.
The production process for the numeraire good in region $j$ generates emissions of two types of pollutants: $S_j > 0$ units of sulfur dioxide, which causes acid rain damages, and $C_j > 0$ units of carbon dioxide, which causes climate change damages. Acid rain damages depend, among other things, on wind patterns and the (ground) resources (e.g., rivers, fish, fauna, infrastructure) available in a particular region. To distinguish from acid rain damages, we assume that climate change damages are solely associated with damages caused by deteriorating air quality. Air quality is equally shared by all residents of the federation.

In most of what follows, pollution abatement is provided by the regional governments. These governments may use a coarse abatement technology to simultaneously abate emissions of the two types of air pollutants. The technology produces pollution abatement utilizing the numeraire good as an input: it takes one unit of the numeraire good to produce one unit of pollution abatement.

In the absence of pollution abatement, the ground-level environmental quality in region $j$ is $L_j = Z_j - \alpha^j_S S_j - \alpha^m_S S_m$, where $Z_j > 0$ is the ground-level environmental quality that nature provides to residents of the region in the absence of acidic deposition and $\alpha^j_S S_j + \alpha^m_S S_m$ is the total amount of acidic deposition in the region, for $\alpha^j_S \in (0,1)$ and $\alpha^m_S \in (0,1)$, $j, m = 1, 2, \ j \neq m$. Thus, region $j$ suffers from acidic deposition which is in part caused by its own sulfur dioxide emission (i.e., the domestic component) and in part caused by the other region's sulfur dioxide emission (i.e., the transboundary or imported component). Note that $\alpha_{11} + \alpha_{12} = 1$ and $\alpha_{22} + \alpha_{21} = 1$ and that we allow for asymmetric transboundary sulfur dioxide depositions because we do not impose the
restriction that \(\alpha_{12} = \alpha_{21}\). In the absence of pollution abatement, the domestic and imported components of the acidic deposition in a region depend on regional sulfur emission levels and on exogenous wind patterns. The latter determine \(\alpha_{11}\) and \(\alpha_{22}\).

By employing the coarse abatement technology and producing \(g_j\) units of pollution abatement, regional government \(j\) can reduce its sulfur emission by \(\phi(g_j)\) units, where we assume that \(\phi(0) = 0\), \(\phi' > 0\) and \(\phi'' < 0\) for all \(g_j > 0\) and \(\lim_{g_j \to 0} \phi'(g_j) = \infty\). The Inada condition guarantees that \(g_j > 0\) in every allocation examined in this paper. Hence, region \(j\)'s ground-level environmental quality function is \(L_j(g_j, g_m) = Z_j - \alpha_{ij} \left[ S_j - \phi(g_j) \right] - \alpha_{mj} \left[ S_m - \phi(g_m) \right]\). Region \(j\)'s ground-level environmental quality produces a benefit of \(B_j = B(L_j(g_j, g_m))\) units to each resident of the region. We assume that that \(B' > 0\) and \(B'' < 0\) for all \(L_j \geq 0\) and that \(\lim_{L_j \to 0} B'(L_j) = \infty\). The Inada condition guarantees that \(L_j > 0\), \(j = 1, 2\), throughout.

Letting \(Y > 0\) denote federal air quality in the absence of carbon dioxide emissions and \(C = C_1 + C_2\) denote the total amount of carbon dioxide emitted in the federation, federal air quality is \(Q(G) = Y + G - C\), where \(G = g_1 + g_2\) denotes the total amount of carbon dioxide that is abated at the federal level. The benefit any individual in the federation derives from air quality is given by the function \(A(Q(G))\). We assume that for all \(Q \geq 0\), \(A' > 0\) and \(A'' < 0\) and that \(\lim_{Q \to 0} A'(Q) = \infty\). The Inada condition guarantees that \(Q > 0\).
For tractability, we assume that the utility of an individual who resides in region \(j\) is
\[
U_j(x_j, g_j, g_m) = x_j A(Q(g_j + g_m)) + B(L_j(g_j, g_m)),
\]
where \(x_j\) represents the quantity of the composite private good consumed by the individual. Given our modeling assumptions, one can interpret the first component of an individual's utility function as a Cobb-Douglas sub-utility function for the numeraire good and air quality provided that the transformation function, \(A(\cdot)\), takes the form \(A(Q) = Q^\beta, \quad \beta \in (0,1)\).

We shall assume throughout that each regional government is benevolent and has Benthamite preferences over residents’ welfare levels. Letting \(R_j\) denote the welfare level in region \(j\), this assumption implies that \(R_j = n_j U_j\). The central government is in charge of interregional redistribution. The center possesses a general Bergson-Samuelson social welfare function. The center’s utility is \(W(R_1, R_2)\). We assume that this function is strictly concave, twice continuously differentiable and satisfies the following conditions:
\[
W_j > 0, \quad W_{jj} < 0, \quad W_{jm} \geq 0, \quad W_j = \partial W_j/\partial R_j, \quad W_{jj} = \partial^2 W_j/\partial R_j \partial R_m \quad \text{and} \quad W_{jj} = \partial^2 W_j/\partial R_j^2, \quad j, m = 1, 2, \quad j \neq m.
\]
The center possesses an income tax instrument, which it can use to tax and redistribute income across regions. Let \(\tau_j\) denote the income transfer received (if positive) or paid (if negative) by a resident in region \(j\). The interregional income transfer constraint faced by the center is
\[
\sum_{j=1}^2 n_j \tau_j = 0.
\]
Let us now consider the supply side of the federal economy. In region \(j\), there are \(n_j\) units of labor, \(E_j\) units of an energy-generating resource (say, coal), and \(K_j\) units of capital. For simplicity, we assume that the regional supplies of labor and coal are fixed because labor and energy markets are regional and there is no mobility of either labor or
coal across regions. Each individual in region $j$ is endowed with one unit of labor, $E_j / n_j$ units of coal and $K_j / n_j$ units of capital. Each individual supplies his/her labor unit and her coal units inelastically in the region in which he or she resides. Each individual also supplies his or her capital units in the economy-wide capital market. The total amount of capital units supplied in the economy-wide capital market is $K = K_1 + K_2$.

The competitive industry in region $j$ rents $k_j$ units of capital and employs $n_j$ units of labor and $E_j$ units of coal to produce $F_j(\k_j; n_j, E_j) = f_j(\k_j)$ units of the numeraire good. We assume that the production function $f_j(\k_j)$ is increasing in its argument, strictly concave, twice continuously differentiable and satisfies the following Inada condition: $\lim_{k_j \to 0} f_j'(k_j) = \infty$. The production function exhibits decreasing returns to scale due to the regionally fixed labor and coal factors. The Inada condition implies that the capital input is essential in production. For simplicity, we assume that each unit of coal utilized in region $j$ generates one unit of sulfur dioxide and one unit of carbon dioxide, which are emitted in the atmosphere. Thus, $S_j = C_j = E_j$, $j = 1, 2$.

The profit of the industry in region $j$ is $\pi_j \equiv f_j(\k_j) - \left( r + t_j \right) k_j - w_j n_j - v_j E_j$, where $r > 0$ is the economy-wide rental rate, $t_j > 0$ is the regional capital tax rate levied by the regional government to finance provision of pollution abatement, and $w_j > 0$ and $v_j > 0$ are the labor wage and coal price in region $j$. In order to capture the effects of interregional capital tax competition on regional environmental policy making, we restrict our model by assuming that the regional governments do not have lump-sum taxes at their disposal.
Since the technology features decreasing returns to scale, profits are positive in equilibrium. Letting $\rho_j = r + t_j$ denote the rental rate after tax in region $j$, the first order condition for profit maximization is $f'_j(k_j) = \rho_j$, from which we derive the factor demand functions, $k_j(\rho_j)$, $j = 1, 2$. Observe that $k'_j(\rho_j) = 1/f''_j(k_j) < 0$.

The economy-wide capital market clears when $k_1(\rho_1) + k_2(\rho_2) = K$. Since $\rho_j = r + t_j$, $j = 1, 2$, this condition enables us to implicitly define the rental rate as function of the capital tax rates, $r(t_1, t_2)$. Plugging this function into the market-clearing condition and differentiating with respect to $t_j$ yields

$$\frac{\partial r}{\partial t_j} = -f''_m/(f''_1 + f''_2) \in (-1, 0), \ j, m = 1, 2, j \neq m.$$ (1a)

Conditions (1a) are standard in the tax competition literature; they are the marginal rental-rate functions (see, e.g., Wildasin (1989)).

Each individual in region $j$ obtains $r(t_1, t_2)K_j/n_j$ units of rental income from capital, $\nu_jE_j/n_j$ units of rental income from coal, $w_j$ units of labor income, and $\pi_j/n_j$ units of profit. Profits are not expatriated. The budget constraint for any resident of region $j$ is $x_j = \tau_j + [f_j(k_j(\rho_j(t_1, t_2)) + r(t_1, t_2)K_j - \rho_j(t_1, t_2)K_j(\rho_j(t_1, t_2))] / n_j$, where $\rho_j(t_1, t_2) = r(t_1, t_2) + t_j$, $j = 1, 2$. Adding up the individual budget constraints yields the regional resource constraints, for $j = 1, 2$,

$$n_jx_j + t_jk_j(\rho_j(t_1, t_2)) = f_j(k_j(\rho_j(t_1, t_2)) + r(t_1, t_2)(K_j - k_j(\rho_j(t_1, t_2))) + n_j\tau_j.$$ (1b)

Each regional government must balance its budget. The budget-balance conditions are
The left-hand side of each equation (1c) represents the governmental expenditure incurred in region \( j \) to provide pollution abatement. It takes one unit of numeraire to provide one unit of pollution abatement. Since the Inada conditions for the numeraire and sulfur-abatement production functions guarantee that in any equilibrium we must have \( k_j > 0 \) and \( g_j > 0 \) in equilibrium, equations (1c) inform us that we must also have \( t_j > 0, \ j = 1, 2 \). Equations (1c) enable us to express the regional pollution abatement levels as functions of the capital tax rates, \( g_j(t_1, t_2) = t_jk_j(\rho_j(t_1, t_2)), \ j = 1, 2 \). It follows that

\[
\frac{\partial g_j}{\partial t_j} = k_j(\cdot) + t_jk'_j(\cdot)[1 + \partial r/\partial t_j]
\]

(1d)

where \( t_jk'_j(\cdot)[1 + (\partial r/\partial t_j)] < 0, \ j = 1, 2 \).

\[
\frac{\partial g_j}{\partial t_m} = t_jk'_j(\cdot)(\partial r/\partial t_m) > 0, \ j, m = 1, 2, \ j \neq m.
\]

(1e)

These conditions demonstrate that any increase in a region’s capital tax rate causes outflow of capital to the other region. Therefore, the regions are tempted to set low capital tax rates in order to increase the total capital supply in the region. These results are standard in the literature (see, e.g., Wilson (1986) and Wildasin (1989)).

We use the budget constraint for the representative resident to define

\[
x_j(t_1, t_2, \tau_j) = \tau_j + \left[ f_j(k_j(\rho_j(t_1, t_2))) + r(t_1, t_2)K_j - \rho_j(t_1, t_2)k_j(\rho_j(t_1, t_2)) \right]/\pi_j,
\]

\( j, m = 1, 2, \ j \neq m \). It follows that

\[
\frac{\partial x_j}{\partial \tau_j} = 1 \quad \text{and} \quad \frac{\partial x_j}{\partial \tau_m} = 0, \quad j, m = 1, 2, \ j \neq m.
\]

(1f)
Equation (1f) tells us that the consumption of the numeraire good of the representative resident of region \( j \) varies at one-to-one rate with the interregional income transfer he or she receives or pays from the central government.

### 3. The Center’s Most Preferable Allocation

In this section, we consider the center’s most preferable allocation. This is the socially optimal allocation, which serves as our benchmark. For comparison purposes, we assume that the center must also utilize capital taxes to finance provision of pollution abatement in each region. The center chooses \( \{t_j, \tau_j\}_{j=1,2} \) to maximize \( W(R_1, R_2) \) subject to:

\[
\sum_{j=1}^{2} n_j \tau_j = 0. \tag{2a}
\]

To facilitate future comparisons, we solve the center’s maximization problem in two steps. First, we assume that the center chooses \( \{\tau_j\}_{j=1,2} \) to maximize \( W(R_1, R_2) \) subject to (2a) for fixed regional capital tax rates. This allows us to obtain the socially optimal interregional transfer levels as functions of the capital tax rates. We then plug the optimal income transfer functions into the social welfare function and optimize with respect to the regional capital tax rates.

Since \( R_j = n_j \left[ x_j(t_1, t_2, \tau_j) A(G(t_1, t_2)) + B(L_j(g_{1,2}(t_1, t_2), g_{2,2}(t_1, t_2))) \right] \), where \( G(t_1, t_2) \equiv g_1(t_1, t_2) + g_2(t_1, t_2) \), the first order conditions in the first step are the constraint (2a) and the following equations, for \( j = 1,2 \):
\[ W_j \left( \frac{\partial R_j}{\partial x_j} \right) = \lambda n_j \Rightarrow W_j A(G) = \lambda \Rightarrow W_1 = W_2 \] (2b)

Equation (2b) is implied by equation (1f). According to equation (2b), the center’s optimal redistribution scheme equalizes the social marginal utilities of income. Equations (2a) and (2b) define \( \tau_j(t_1, t_2), \ j = 1, 2 \). Let us \( x_j(t_1, t_2) \equiv x_j(t_1, t_2, \tau_j(t_1, t_2)) \) and \( R_j(t_1, t_2) \equiv n_j \left[ x_j(t_1, t_2) A(Q(G)) + B \left( L_j(g_j(t_1, t_2), g_m(t_1, t_2)) \right) \right], \ j, m = 1, 2, \ j \neq m. \)

Plugging \( \tau_j(t_1, t_2), \ j = 1, 2 \), into equation (2a) and differentiating the implied expression with respect to \( t_j \) yields

\[ \sum_{i=1}^{2} n_i \left( \frac{\partial \tau_i}{\partial t_j} \right) = 0. \] (2c)

The center now chooses \( \{t_j\}_{j=1,2} \) to maximize \( W(R_1(t_1, t_2), R_2(t_1, t_2)) \). The first order conditions are

\[ \sum_{i=1}^{2} W_i \left( \frac{\partial R_i}{\partial \tau_i} + \frac{\partial R_i}{\partial t_j} \right) = 0, \ j = 1, 2. \] (2d)

where

\[ \frac{\partial R_i}{\partial \tau_i} = n_i A(Q(G)) \frac{\partial \tau_i}{\partial t_j}, \ i, j = 1, 2, \] (2e)

\[ \frac{\partial R_i}{\partial t_j} = n_i A(Q(G)) \left( \frac{\partial x_i}{\partial t_j} \right) + n_i x_i A'(Q(G)) \left( \frac{\partial G}{\partial t_j} \right) + n_i B'(L_i) \sum_{m=1}^{2} \alpha_{m} \phi(g_m) \left( \frac{\partial g_m}{\partial t_j} \right), \ i, j = 1, 2. \] (2f)

Since \( \sum_{i=1}^{2} k_i (\rho_i(t_1, t_2)) = K \) and \( W_1 = W_2 > 0 \), substituting equations (1f)-(1h), (2e) and (2f) into equations (2d) yields

\[ \frac{1}{A(Q)} \left\{ \sum_{i=1}^{2} \sum_{h=1}^{2} n_h x_h A'(Q) + n_h B'(L_h) \alpha_{ih} \phi(g_i) \left( \frac{\partial g_i}{\partial t_j} \right) \right\} = k_j - \sum_{i=1}^{2} n_i \left( \frac{\partial \tau_i}{\partial t_j} \right), \ j = 1, 2. \] (2g)
Combining equations (2c) and (2g), we obtain:

$$\frac{1}{A(Q)} \left\{ \sum_{i=1}^{2} \sum_{h=1}^{2} [n_h x_h A'(Q) + n_h B'(L_h) \alpha_{ih} \phi'(g_i) \left( \frac{\partial g_i}{\partial t_j} \right)] \right\} = k_j, \ j = 1,2. \quad (2h)$$

Equations (2h) are modified Samuelson conditions. The condition shows the equalization of the sum of the marginal rates of substitution between pollution abatement and numeraire and the marginal social cost, which is the rate which capital is sacrificed for the provision of the regional pollution abatement products. The marginal rates of substitution between pollution abatement and numeraire contain two components, one which gives us the marginal social benefit of improving air quality and another which gives us the marginal social benefits of improving ground-level environmental quality in both regions. The socially optimal allocation is characterized by equations (1b), (1c), (2a), (2b) and (2h).

4. Federal Policy Game

We are now ready to consider federal policy making. There are three policy games: simultaneous policy making, decentralized leadership and centralized leadership. We compare each equilibrium allocation with the center’s most preferable allocation. Our analysis will enable us to clearly demonstrate the effects produced by the timing in federal policy making on efficiency and equity.

4.1 Simultaneous Policy Making

In the simultaneous game, the central government chooses \( \{\tau_j\}_{j=1,2} \) to maximize \( W(R_1, R_2) \) subject to constraint (2a) and \( x_j(\tau_j; t_1, t_2), \ j = 1,2, \) taking \( \{t_j\}_{j=1,2} \) as given. Each regional authority chooses \( \{t_j\} \) to maximize \( R_j \) subject to \( x_j(t_j; t_m, \tau_j) \) and
\( g_j(t_1, t_2) \), taking \( \{r_j, t_m\}_{j=1,2} \) as given, \( j, m = 1, 2 \) and \( j \neq m \). Since the center’s problem is identical to the problem it solved in section 3 except that here it does not control the regional policies, the solution for the center’s maximization problem is given by equations (2a) and (2b).

Let us now consider the equilibrium allocation implied by the regions’ maximization problems. The first order conditions are

\[
\frac{\partial R_j}{\partial t_j} = n_jA(Q)\left(\frac{\partial x_j}{\partial t_j}\right) + \sum_{i=1}^{2} \left[n_jx_jA'(Q) + n_jB'(L_j)\alpha_y\varphi'(g_i)\right]\left(\frac{\partial g_i}{\partial t_j}\right) = 0, \quad j = 1, 2. \tag{3a}
\]

Inserting equation (1g) into equations (3a), we obtain

\[
\frac{1}{A(Q)}\left[\sum_{i=1}^{2} \left[n_jx_jA'(Q) + n_jB'(L_j)\alpha_y\varphi'(g_i)\right]\left(\frac{\partial g_i}{\partial t_j}\right)\right] = k_j - (K_j - k_j)\frac{\partial r}{\partial t_j}, \quad j = 1, 2. \tag{3b}
\]

Pollution abatement products are determined according to conditions (3b). In each region, the rule equalizes the regional marginal rates of substitution between pollution abatement and numeraire and the regional marginal cost. Equations (3b) clearly demonstrate that the regional governments ignore interregional spillovers. This fact leads each region to under-provide pollution abatement.

In sum, the equilibrium allocation for the simultaneous game is characterized by equations, (1b), (1c), (2a), (2b) and (3b). Comparing this allocation with the socially optimal allocation leads us to the following conclusion:

**Proposition 1.** The equilibrium for the simultaneous game is characterized by socially optimal redistribution and fully decentralized capital tax policies. Therefore, the allocation is socially equitable but inefficient.
The center finds it desirable to intervene in the federal setting with simultaneous policy making because it has a strong taste for equity, as captured by the decreasing marginal social utilities of income. Thus, although the equilibrium for the simultaneous policy game examined here is not socially optimal, it represents an improvement in social welfare relative to a situation in which the center does not intervene. The latter would follow, for example, if the structure of the federal system was completely decentralized, with each regional government also being in charge of promoting intra-regional income transfers.

### 4.2 Decentralized Leadership

In this section, we show that federal policy making under decentralized leadership represents a social improvement relative to simultaneous federal policy making. The sequential game is as follows:

**Stage 1:** Regional government $j$ chooses nonnegative $\{t_j\}$ to maximize $R_j(t_1,t_2)$ taking the choices of all other regional government as given, $j=1,2$.

**Stage 2:** The center observes the regional policy makings and then chooses nonnegative $\{\tau_j\}_{j=1,2}$ to maximize $W(R_1(t_1,t_2),R_2(t_1,t_2))$ subject to the federation’s resource constraint (2a).

The equilibrium concept for the two-stage game is subgame perfection.

Applying backward induction, we first consider the second stage of the game. Since the center’s problem is identical to the problem it solved in section 3 except that here it does not control the regional policies, the solution in the second stage is given by equations (2a) and (2b). The center’s best response functions are $\tau_j(t_1,t_2)$, $j=1,2$.

Inserting the center’s best response functions into equations (2a) and (2b), we have
\[ \sum_{j=1}^{2} n_j \tau_j (t_1, t_2) = 0, \quad (4a) \]

\[ W_1(R_1(t_1, t_2), R_2(t_1, t_2)) = W_2(R_1(t_1, t_2), R_2(t_1, t_2)). \quad (4b) \]

We now consider the equilibrium allocation for the first stage. The first order conditions are as follows for \( j = 1, 2 \):

\[ \frac{\partial R_j}{\partial t_j} = n_j A(Q) \left( \frac{\partial x_j}{\partial t_j} + \frac{\partial x_j}{\partial \tau_j} \frac{\partial \tau_j}{\partial t_j} \right) + \sum_{i=1}^{2} \left[ n_j x_j A'(Q) + n_j B'(L_j) \alpha_j \phi'(g_i) \right] \left( \frac{\partial g_i}{\partial t_j} \right) = 0. \quad (4c) \]

Differentiating equations (4a) and (4b) with respect to \( \{t_j\}_{j=1,2} \) yields equations (2c) and

\[ (W_{11} - W_{12}) \left( \frac{\partial R_1}{\partial t_j} \right) = (W_{21} - W_{12}) \left( \frac{\partial R_2}{\partial t_j} \right), \quad j = 1, 2. \quad (4d) \]

Since \( W_{m m} - W_{j m} < 0 \) and \( \left( \frac{\partial R_j}{\partial t_j} \right) = 0, \quad j, m = 1, 2 \) and \( j \neq m \), equations (4d) imply

\[ \frac{\partial R_m}{\partial t_j} = n_m A(Q) \left( \frac{\partial x_m}{\partial t_j} + \frac{\partial x_m}{\partial \tau_m} \frac{\partial \tau_m}{\partial t_j} \right) + \sum_{i=1}^{2} \left[ n_m x_m A'(Q) + n_m B'(L_m) \alpha_{m \phi'(g_i)} \right] \left( \frac{\partial g_i}{\partial t_j} \right) = 0. \quad (4e) \]

Adding up equations (4c) and (4e) yields

\[ \frac{1}{A(Q)} \left( \sum_{i=1}^{2} \sum_{h=1}^{2} n_h x_h A'(Q) + n_h B'(L_h) \alpha_h \phi'(g_i) \right) \left( \frac{\partial g_i}{\partial t_j} \right) = k_j - \sum_{i=1}^{2} n_j \left( \frac{\partial \tau_j}{\partial t_j} \right), \quad j = 1, 2. \quad (4f) \]

Combining equations (4a) and (4f), we obtain:

\[ \frac{1}{A(Q)} \left( \sum_{i=1}^{2} \sum_{h=1}^{2} n_h x_h A'(Q) + n_h B'(L_h) \alpha_h \phi'(g_i) \right) \left( \frac{\partial g_i}{\partial t_j} \right) = k_j, \quad j = 1, 2. \quad (4g) \]

The equilibrium allocation for the decentralized leadership is characterized by equations (1b), (1c), (2a), (2b) and (4g).

**Proposition 2.** The subgame perfect equilibrium for the decentralized leadership game corresponds to the center’s most preferred allocation.
Proposition 2 follows from the fact that the redistributive policy implemented by the center provides the forward looking regional governments with incentives to internalize all externalities. To see this, consider the center's optimal redistribution rule (4b): \( W_1(R_1, R_2) = W_2(R_1, R_2) \). This condition can be used to express the welfare level of region 1 as an implicit function of the welfare level of region 2 (or vice-versa). Let \( R_i = \Psi(R_{\cdot i}) \), where \( \Psi(\cdot) \) is the implicit function. Then, the center's optimal redistribution rule can be rewritten as \( W_1(\Psi(R_2), R_2) = W_2(\Psi(R_2), R_2) \). Differentiating this equation with respect to \( R_2 \) yields \( \Psi'(R_2) = (W_{22} - W_{12})/(W_{11} - W_{21}) > 0 \), since \( W_{jj} < 0 \) and \( W_{jm} = W_{mj} \geq 0, \; j, m = 1, 2, \; j \neq m \). Thus, the center's optimal redistribution rule implies that one region's welfare level is an increasing function of the other region's welfare level. Knowing that the optimal redistributive rule will align regional welfare levels in the second stage, it is rational for each regional government in the first stage to make a choice with respect to the rate at which capital should be taxed in its region which internalizes all externalities.

4.3 Centralized Leadership

Suppose now that the center is the Stackelberg leader and the regions are the Stackelberg followers. Caplan and Silva (1999) considered a similar federal setting motivated by environmental policy making in the United States.

Applying backward induction, we first consider the second stage of the game. Since the region’s problem is identical to the problem it solved in section 4.1, the equilibrium conditions for regions’ maximization problems are characterized by equations (1b), (1c) and (3b) provided the solutions are interior.
Let \( t_j(\tau_1, \tau_2), \ j = 1,2, \) denote the regional optimal policy responses. Let us also define
\[ g_j(\tau_1, \tau_2) = g_j(t_1(\tau_1, \tau_2), t_2(\tau_1, \tau_2)), \quad k_j(\tau_1, \tau_2) = k_j(\rho_j(t_1(\tau_1, \tau_2), t_2(\tau_1, \tau_2))), \]
\[ U_j(\tau_1, \tau_2) = x_j(\tau_1, \tau_2)A(Q(G(\tau_1, \tau_2))) + B(L_j(g_j(\tau_1, \tau_2), g_m(\tau_1, \tau_2))), \]
\[ G(\tau_1, \tau_2) = g_1(\tau_1, \tau_2) + g_2(\tau_1, \tau_2) \text{ and } R_j(\tau_1, \tau_2) = n_j U_j(\tau_1, \tau_2), \ j = 1,2. \]

In the first stage of the game, the center chooses \( \{\tau_j\}_{j=1,2} \) to maximize
\[ W(R_1(\tau_1, \tau_2), R_2(\tau_1, \tau_2)) \text{ subject to the federation’s resource constraint (2a) and the regional tax policy responses.} \]

The first order condition of the first stage is
\[ \frac{\partial R_j}{\partial \tau_j} + \sum_{i=1}^2 \frac{\partial R_i}{\partial \tau_i} \frac{\partial R_j}{\partial \tau_j} + \sum_{h=1}^2 \frac{\partial R_h}{\partial \tau_h} \frac{\partial R_j}{\partial \tau_j} = \lambda n_j > 0, \ j = 1,2. \quad (5a) \]

Because \( \frac{\partial R_j}{\partial \tau_j} = n_j A(Q), \ j = 1,2, \) and \( \frac{\partial R_j}{\partial \tau_j} = 0 \quad i, j = 1,2, i \neq j, \) equations (5a) imply after some algebra
\[ W_1 A(Q) + \left( W_1 \frac{\partial R_1}{\partial \tau_1} + W_2 \frac{\partial R_2}{\partial \tau_1} \right) \left( \frac{1}{n_1} \frac{\partial t_1}{\partial \tau_1} - \frac{1}{n_2} \frac{\partial t_2}{\partial \tau_2} \right) \]
\[ = W_2 A(Q) + \left( W_1 \frac{\partial R_1}{\partial \tau_2} + W_2 \frac{\partial R_2}{\partial \tau_2} \right) \left( \frac{1}{n_2} \frac{\partial t_2}{\partial \tau_2} - \frac{1}{n_1} \frac{\partial t_1}{\partial \tau_1} \right) \quad (5b) \]

Substituting equations (3a) into equations (5b) yields,
\[ W_1 = W_2 + \left[ W_1 \frac{\partial R_1}{\partial \tau_1} \left( \frac{1}{n_2} \frac{\partial t_2}{\partial \tau_2} - \frac{1}{n_1} \frac{\partial t_1}{\partial \tau_1} \right) - W_2 \frac{\partial R_2}{\partial \tau_1} \left( \frac{1}{n_1} \frac{\partial t_1}{\partial \tau_1} - \frac{1}{n_2} \frac{\partial t_2}{\partial \tau_2} \right) \right]. \quad (5c) \]

Differentiating equations (3a) with respect to \( \{\tau_j\}_{j=1,2} \) yields
\[ \begin{bmatrix} \frac{\partial^2 R_j}{\partial t_j^2} & \frac{\partial^2 R_j}{\partial t_j \partial \tau_m} \\ \frac{\partial^2 R_m}{\partial \tau_j \partial t_m} & \frac{\partial^2 R_m}{\partial \tau_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial t_j}{\partial \tau_j} \\ \frac{\partial t_m}{\partial \tau_j} \end{bmatrix} = - \begin{bmatrix} \frac{\partial^2 R_j}{\partial t_j \partial \tau_j} \\ 0 \end{bmatrix}. \quad (5d) \]
By using Cramer’s Rule, we obtain

\[
\frac{\partial t_j}{\partial \tau_j} = -\frac{1}{\Delta} \left[ \frac{\partial^2 R_j}{\partial t_j \partial \tau_j} \frac{\partial^2 R_m}{\partial t_m \partial \tau_m} \right] \quad \text{and} \quad \frac{\partial t_m}{\partial \tau_j} = \frac{1}{\Delta} \left[ \frac{\partial^2 R_j}{\partial t_j \partial \tau_j} \frac{\partial^2 R_m}{\partial t_m \partial t_j} \right]
\]

where \( \Delta \equiv \frac{\partial^2 R_j}{\partial t_j^2} \frac{\partial^2 R_m}{\partial t_m^2} - \frac{\partial^2 R_j}{\partial t_j \partial t_m} \frac{\partial^2 R_m}{\partial t_j \partial t_m} \), \( j, m = 1, 2 \) and \( j \neq m \). Substituting equations (5e) into equations (5c), we have

\[
W_i = W_2 + \frac{1}{A(Q)\Delta} \left( W_2 \frac{\partial R_i}{\partial t} \left( \frac{1}{n_1} \frac{\partial^2 R_1}{\partial t \partial \tau_1} \frac{\partial^2 R_2}{\partial t_2} + \frac{1}{n_2} \frac{\partial^2 R_2}{\partial t_2 \partial \tau_2} \frac{\partial^2 R_2}{\partial t_2} \right) \right) - W_i \frac{\partial R_i}{\partial t_2} \left( \frac{1}{n_2} \frac{\partial^2 R_2}{\partial t_2 \partial \tau_2} \frac{\partial^2 R_2}{\partial t_2} + \frac{1}{n_1} \frac{\partial^2 R_1}{\partial t_1 \partial \tau_1} \frac{\partial^2 R_2}{\partial t_2} \right)
\]

Equation (5f) clearly demonstrates that the center’s interregional income transfer policy does not generally achieve the socially equitable goal of equalizing social marginal utilities of income. The policy rule implicit in equation (5f) balances the two objectives the center attempts to achieve; namely, equity and efficiency. Since the center knows that its redistribution policy influences the regions’ tax policies, it finds it optimal to deviate from the socially equitable goal of equalizing social marginal utilities of income. Had redistribution policy been unable to influence the regions’ tax policies, the center would have chosen to equalize social marginal utilities of income. The implied allocation would have been identical to the equilibrium allocation we obtained in the setting in which the center and the regions make simultaneous choices. Thus, the center’s redistributive policy would have been neutral.

We gather the results we obtained in the paper in the following proposition.

**Proposition 3.** The subgame perfect equilibrium for the centralized leadership game is generally socially inequitable and inefficient. The allocation differs from the subgame
perfect equilibrium for the decentralized leadership game and from the Nash equilibrium for the simultaneous game played by the center and the regions. The center’s redistributive policy is generally non-neutral.

By comparing the equilibrium for the decentralized leadership setting with the equilibrium for the centralized leadership setting, we are led to the same conclusion of Caplan and Silva (1999) that a federal system characterized by decentralized leadership may in many instances dominate a federal system characterized by centralized leadership. Our model, however, is much more general than the one presented by Caplan and Silva (1999). As pointed out by these authors and Caplan et al (2000), among others, the shape of federal policy making is sensitive to the timing of the game played by regional and central governments. In this paper, we have been able to clearly demonstrate that federal policy making under decentralized leadership is socially superior given our specifications of individual utilities and regional and federal welfare functions.

4. Conclusion

Following early and recent contributions to two important branches of the fiscal federalism literature, on transboundary pollution control and interregional tax competition, this paper makes contributions to each branch by demonstrating that efficient control of transboundary pollutants may be achieved even if regional governments do not have lump-sum taxes at their disposal and have thus to resort to tax schemes that sacrifice some critical resources, such as capital taxation. When capital taxes must be utilized, we show that tax competition is not necessarily wasteful, relative to what a fully coordinated system would accomplish, and that decentralized leadership may socially dominate alternative federal policy structures. A federal system
characterized by decentralized leadership may, therefore, be socially optimal. This implies that the prevalent system adopted in the European Union may be superior to its alternatives and that much more attention should be devoted to its efficiency and equity properties.

**References**


