ABSTRACT

Numerous non-profit organizations that contribute to collective goods also provide prestige to their members. Some of these institutions function as prestige clubs, with prestige levels and member contributions working as club goods and membership fees, respectively. We investigate the endogenous formation of prestige clubs. We show that the competitive equilibrium features prestige clubs and that competing club managers engage in a futile race for institutional aggrandizement. The competition, however, yields coordination benefits produced by internalization of positive and negative externalities within clubs. The competitive equilibrium is inefficient because clubs neglect external benefits and costs associated with their members’ contributions.
I. Introduction

Numerous non-profit organizations that contribute to one or more collective goods also provide prestige to their members or affiliates. Several charities group their members into multiple prestige categories and publicize their members’ contributions. Other good examples are orchestras, public radios, schools, universities and various religious organizations.

The prestige level enjoyed by a contributor may be a function of personal and institutional components. At the personal level, an individual may obtain prestige when her contribution is compared with other individual contributions; namely, it may depend on how important her contribution is relative to the contribution of others. At the institutional level, an individual may derive prestige by associating with a prestigious organization. Members of the same organization may obtain equivalent prestige levels if they are grouped in the same category of contributors or belong to the same team of contributors (e.g., a department in a given university) or may obtain different prestige levels if they are grouped in different categories or teams. The prestige level enjoyed by the institution may also depend on how important its overall contribution to the collective good relative to the contributions of peer organizations. A student enjoys prestige for his own accomplishments in exams and from the status enjoyed by his school, and the school’s ranking may in part be determined by its relative importance in producing collective goods such as knowledge and civil skills. A scholar earns prestige for her distinguished research output but also for her affiliation with a reputable university, and the latter’s reputation is in part a function of its relative contribution to the production and dissemination of scientific knowledge. A philanthropist enjoys prestige for the importance of his contribution relative to the contribution of others to a charitable cause, such as poverty alleviation, but also because of the relative standing of the charity of which he is a member, and the prestige enjoyed by the charity is partly due to its relative contribution to the charitable cause.

Organizations that promote prestige for their members seem to function as competitive prestige clubs. They compete for members who demand prestige services and they compete among themselves to advance their own prestige levels. The prestige levels offered to members of such institutions have, in many instances, the key characteristics of club goods, since members are often grouped into excludable prestige categories of limited size. The contributions of members who enjoy identical prestige levels also tend to be identical. Charities that supply multiple prestige categories provide a good example. Charity
members who belong to a particular prestige category often contribute identical amounts and enjoy the same prestige level.

In this paper, we build a simple model to investigate the incentives for the formation of competitive prestige clubs and examine their key behavioral features. In our economy, there are two types of consumers who enjoy the benefits of the provision of a collective good and the consumption of a composite private good, the economy’s numeraire good. One type of consumer also derives utility from prestige, and prestige can be produced from individual and institutional contributions to the collective good. The individual component of prestige produced by one’s contribution is modeled as the difference between the individual contribution amount and the economy’s average of individual contributions. Similarly, the institutional component of prestige produced by one’s club is modeled as the difference between the club’s contribution amount and the economy’s average of club contributions.

As in a standard public good’s model, an individual’s contribution to the collective good benefits every other individual in the economy because everyone derives positive utility from consumption of the collective good. Unlike a standard public good’s model, the contribution made by an individual or a club increases the individual or institutional average contribution level and this results in a loss of prestige for each prestige lover. In our model, therefore, there are positive and negative effects associated with expansions in one’s contribution to the collective good.

Prestige clubs are formed in an attempt to promote the collective good and prestige levels for their members by internalizing positive and negative externalities associated with expansions in contributions and by supplying prestige lovers with an institutional prestige component. Prestige clubs compete for members and among themselves. As a result, each club attempts to maximize the utility of its representative member.

We show that prestige clubs emerge in the competitive equilibrium. Each club internalizes positive and negative externalities associated with its members’ contributions to the collective good, but neglects the positive and negative effects of its members’ contributions on other individuals in the economy. The internalization of externalities within clubs generate coordination benefits and such benefits produce an increase in the overall provision level of the collective good relative to the level one would observe in an economy without prestige clubs. Each club’s membership size is limited by increasing crowding costs. The
optimal membership size is the level that maximizes a club’s contribution to the collective good, given the individual contribution made by each of its members. We also show that the race for institutional aggrandizement is futile because clubs are identical in equilibrium. They make identical contributions to the collective good and thus enjoy no institutional prestige.

We also show that the competitive equilibrium is inefficient because prestige clubs do not fully internalize positive and negative externalities associated with their members’ contributions to the collective good and because prestige clubs are of inefficient size. The efficient size is either zero, in which case the economy should not have any prestige club at all, or the entire population of prestige lovers, in which case the economy should feature one prestige club only.

This paper contributes to two strands of the literature, the literature on voluntary contributions to impure public goods and the literature on clubs. Starting with the seminal contribution by Cornes and Sandler (1984), the literature on voluntary contributions to impure public goods has grown enormously. Noteworthy contributions have examined the comparative static properties of such models (see, e.g., Cornes and Sandler (1994)), the roles played by warm glow in alleviating free riding incentives and the crowding out between public and private contributions to a collective good (see, e.g., Andreoni (1989), Andreoni (1990) and Andreoni and Payne (2003)), the effects of prestige incentives on contributions to charities or other prestigious organizations (see, e.g., Harbaugh (1998a), Harbaugh (1998b) and Glazer and Konrad (1996)) and the effects of leadership giving in charitable funding (see, e.g., Andreoni (2006a)).

Our paper contributes to this literature in that it models prestige production as a function of relative individual and group contributions to the collective good, prestigious organizations as clubs, and examines the endogenous formation of such prestige clubs and some of their key features.

The literature on clubs is also voluminous. The seminal contribution is Buchanan (1965). Of the many issues addressed by this literature, some noteworthy ones are those related to the benefits of forming small groups (see, e.g. Buchanan (1965) and Berglas (1976)), those related to the existence of a competitive equilibrium and the efficiency of such an equilibrium whenever it is shown to exist (see, e.g., Ellickson et al (1999), Ellickson et al (2001), Hammond, Kaneko and Wooders (1989), Page and Wooders (2008),

---

1 See Andreoni (2006b) for a survey of works related to philanthropy and Cornes and Sandler (1986) for references on the broader theme of voluntary contributions to impure public goods.
Scotchmer (1985a), Scotchmer (1985b), Scotchmer (2005), Scotchmer and Wooders (1987), Wooders (1978), Wooders (1980) and Wooders (1994)), those related to the efficiency of heterogeneous vis-à-vis homogeneous clubs (see, e.g., Brueckner and Lee (1989) and Sandler and Tschirhart (1984)) and those related to the degree of exclusion implemented by clubs (see, e.g. Hesley and Strange (1991), Oakland (1972), Silva and Kahn (1993) and Silva (1997)).

Our main contributions to this literature are that: (i) we model clubs as providers of both club goods and contributions to a nonexcludable and nonrivalrous collective good; (ii) we consider the potential interactions between clubs and individual contributors; and (iii) we examine the role that interclub externalities play on the efficiency of the competitive equilibrium.

This paper is organized as follows. The next section introduces the basic model. Section III derives the competitive equilibrium for the economy. Section IV describes the relevant Pareto efficient allocations and compares them with the competitive equilibrium in order to illustrate the sources of inefficiency of the latter. Section V concludes.

II. Basic Model

Consider an economy with two types of individuals, whom we shall refer as “purists” and “prestige lovers”. There are $N$ consumers of each type, where $N$ is large. Purists are consumers of type $a$ and prestige lovers are consumers of type $l$. The populations of purists and prestige lovers are indexed by $i$, $i = 1, ..., N$.

There are three goods, a numeraire good, a collective good and prestige. Each type of consumer is endowed with $w > 0$ units of a numeraire good. Purists do not derive utility from prestige. If a purist $i$ consumes $x^a_i$ units of the numeraire good and enjoys $Q$ units of the collective good, his utility is $U(x^a_i, Q)$. If a prestige lover $i$ consumes $x^l_i$ units of the numeraire good and enjoys $Q$ units of the collective good and $p^l_i$ units of prestige, his utility is $U(x^l_i, Q) + Bp^l_i$, where $B > 0$. We assume that $U$ is

\[2\] See Scotchmer (2008) for a recent survey of the literature on clubs.

\[3\] Note that these individuals have purist incentives to contribute to the collective good, but are called prestige lovers in order to distinguish them from the population of purists.
increasing in each argument, twice continuously differentiable, strictly quasiconcave and homothetic. The collective good is nonexcludable and nonrivalrous.

The prestige level enjoyed by prestige lover $i$ depends on whether he contributes to the collective good directly or channels his contribution through an intermediary institution, a prestige club. If he is an independent contributor, the prestige level he enjoys by contributing an amount $z'_i$ to the collective good is $p'_i = z'_i - Q/2N$. If he contributes the same amount through a prestige club $j$, his prestige level is $p'_i = z'_i + I_j - Q/2N$, where $I_j$ is the institutional component of the prestige production function. To keep things simple and focus our attention on the incentives promoted by the prestige function to the formation of clubs, we assume that there is common knowledge about individual and institutional contributions to the collective good and that the information transmission mechanism is without any cost.

An active prestige club $j$ contains $n_j > 1$ members. Let $Y_j = \sum_{i=1}^{n_j} z'_i - C(n_j)$ denote the contribution to the collective good made by club $j$, where $C(n_j)$ is the cost of running the club, which includes costs of operation and exclusion. We assume that $C$ satisfies the following properties: $C(1) = 0$, $C_s(n) > 0$ and $C_{ss}(n) > 0$ for $n_j \in [1,N]$. We also assume that the average cost function, $C(n)/n$, increases at a nondecreasing rate in $n_j$ for $n_j \in [1,N]$. We define the institutional prestige level enjoyed

---

4 These properties are sufficient for the existence and uniqueness of equilibrium in a voluntary contributions game (see, e.g., Bergstrom et al (1986), Cornes and Hartley (2005) and Cornes and Hartley (2007)). An implication of the general analysis of Cornes and Hartley (2005) is that the standard assumed properties of the component of preferences represented by the function $U$ are sufficient to ensure the existence of a unique equilibrium in a voluntary contributions game without prestige clubs, even though prestige lovers have the extra linear term in their utility function.

5 If we assumed that individuals and organization incurred costs of advertising their contributions, as it may seem natural, one could generate an additional motivation for the formation of clubs if one assumed that the advertising cost function were sub-additive on the number of individual contribution levels advertised. We decided to neglect advertising costs in order to demonstrate that the key characteristics of the prestige function, per se, provide incentives for the formation of clubs.

6 An example of a cost function that satisfies all the assumptions is $C(n_j) = n_j(n_j - 1)/2$. The requirement that the average cost function increases at a nondecreasing rate rules out functions such as $C(n_j) = (n_j^2 - 1)/2$. 

---
by every individual who joins club $j$ as $I_j = Y_j - \hat{Y}$, where $\hat{Y} = \left(\sum_{k=1}^{J} Y_k\right)/J$ is the average of contribution levels made by prestige clubs. Thus, in an economy with prestige clubs, the prestige level enjoyed by a club member is a function of how large his contribution is relative to the average of individual contributions and how large the contribution made by his club is relative to the average of institutional contributions.

III. Competitive Equilibrium

We start by considering a setting in which the economy does not feature prestige clubs. The equilibrium in this hypothetical setting is a candidate for the competitive equilibrium. Purists and prestige lovers play a standard voluntary contributions game, where $Q = \sum_{i=1}^{N} (z_i^a + z_i^l)$. A purist $i$ chooses nonnegative $\{x_i^a, z_i^a\}$ to maximize $U(x_i^a, z_i^a) + Bp_i^l$ subject to $x_i^a + z_i^a = w$, taking $Z_{i,j}^a = \sum_{a=1}^{N} z_{a,j}^a$ and $Z^l = \sum_{a=1}^{N} z_{a,l}^l$ as given. Letting $x_i^a = w - z_i^a$ and assuming that $z_i^a < w$, the Kuhn-Tucker conditions are

$$z_i^a \left(-U_i(\hat{w} - z_i^a, Q) + U_i(w - z_i^a, Q)\right) = 0, \quad z_i^a \geq 0, \quad -U_i(\hat{w} - z_i^a, Q) + U_i(w - z_i^a, Q) \leq 0. \tag{1}$$

Close inspection of these conditions reveals that $z_i^a = z_i^a, \forall i$ in the population of purists. This implies that $Z^a = Nz^a$.

A prestige lover $i$ chooses nonnegative $\{x_i^l, p_i^l, z_i^l\}$ to maximize $U(x_i^l, Q) + Bp_i^l$ subject to $x_i^l + z_i^l = w$ and $p_i^l = z_i^l - (z_i^l + Z_i^a + Z_i^l)/2N$, taking $Z^a$ and $Z_{i,j}^l = \sum_{a=1}^{N} z_{a,j}^l$ as given. Letting $x_i^l = w - z_i^l$ and assuming that $0 < z_i^l < w$ in the candidate equilibrium, the first order condition is

$$2N\left[U_i(\hat{w} - z_i^l, Q) - U_i(w - z_i^l, Q)\right] + (2N - 1)B = 0. \tag{2}$$

Since equation (2) holds for each $i$ in the population of prestige lovers, $z_i^l = z_i^l$ for each $i$ in this population. This implies that $Z^l = Nz^l$. Note that prestige lovers may be the sole providers of the collective good if the benefit from prestige, $B$, is sufficiently large.

In order to focus our attention on contribution incentives produced by the prestige function, we shall consider situations in which it is optimal for purists to free ride. Note that if
In the equilibrium candidate, we obtain $z^u = 0$ from conditions (1). Under these circumstances, the first order condition for each $i$ in the population of prestige lovers can be written as follows:

$$\frac{U_q(x', N^u)}{U_q(x', N^u') - U_q(x, N^u)} \cdot \frac{B}{2N} \cdot \frac{2N - 1}{2N} \cdot \frac{B}{U_q(x', N^u)} = 1.$$ (3)

Equation (3) informs us that each prestige lover accounts for the contribution’s pure and impure benefits when he decides how much to contribute. The first fraction on the left hand side is the pure marginal benefit. This benefit is the prestige lover’s marginal rate of substitution between the collective good and the numeraire good. The second fraction is the impure marginal benefit. It is his marginal rate of substitution between prestige and the numeraire good. Each prestige lover ignores the negative prestige externality that he imposes on every other prestige lover when he contributes to the collective good. Holding all other contributions constant, his contribution raises the average level of individual contributions, reducing prestige for each other individual at a rate of $1/2N$. In the equilibrium candidate, each prestige lover enjoys $z'/2$ units of prestige, the average level of individual contributions.

To illustrate that purists find it optimal to free ride if prestige lovers derive a sufficiently large benefit from prestige, we consider an example. Let $U\big(x^u_i, Q\big) = x^u_i Q$, $U\big(x^l_i, Q\big) + Bp_i = x^l_i Q + Bp_i$, $Nw \geq B \geq w$, $p_i' = z^l_i - \frac{Q}{2N}$, $Q = \sum_{i=1}^N z^l_i + z^l_i$, $N \geq 2$. Letting $x^u_i = w - z^u_i$ and assuming that $z^u_i < w$, the Kuhn-Tucker conditions for the problem faced by a purist $i$ are $z^u_i \left( w - Q - z^u_i \right) = 0$, $z^u_i \geq 0$, $w - Q - z^u_i \leq 0$, $i = 1, ..., N$. Note that $z^u_i = z^u_i \forall i$ in the population of purists in the candidate equilibrium. It also follows that if $Q \geq w$ in the candidate equilibrium, $z^u = 0$. We demonstrate below that this is the case.

---

It is straightforward to show that prestige lovers enjoy positive prestige levels in an equilibrium candidate in which each purist contributes a positive amount to the collective good because the extra prestige benefit derived by prestige lovers provide them with an incentive to make a larger contribution. The prestige level enjoyed by each prestige lover in such a case is $\left(z' - z^u\right)/2$. 

---

7 It is straightforward to show that prestige lovers enjoy positive prestige levels in an equilibrium candidate in which each purist contributes a positive amount to the collective good because the extra prestige benefit derived by prestige lovers provide them with an incentive to make a larger contribution. The prestige level enjoyed by each prestige lover in such a case is $\left(z' - z^u\right)/2$. 

---
Letting $x'_i = w - z'_i$ and assuming that $0 < z'_i < w$ in the candidate equilibrium, we obtain
\[
2Nz'_i = 2N(w - Q) + (2N - 1)B
\]
from the first order condition for the problem solved by prestige lover $i$, $i = 1,\ldots, N$. Since this first order condition holds for each prestige lover $i$, $z'_i = z'_i \ \forall i$ in the population of prestige lovers. Thus, $Nz'_i = Z'$. Since $Q = Z'' + Z'$, we have
\[
Z' = \left\{2N(w - Z'') + (2N - 1)B\right\}/2(N + 1).
\]
If $z'' = 0$, we have $Z'' = 0$ and 
\[
Q = Z' = \left\{2Nw + (2N - 1)B\right\}/2(N + 1).
\]
We need to show that $Q \geq w$ and that $w > z'$. If $Q \geq w$, we have $(2N - 1)B \geq 2w$. Now note that $(2N - 1)B \geq (2N - 1)w$ because $B \geq w$ and that $(2N - 1)w > 2w$ because $N \geq 2$. Hence, $Q > w$ and $z'' = 0$, as we have assumed. Note that $w > z'$ if $w > Q/N$ or $Nw > Q$. This holds because
\[
Nw > \frac{2Nw + (2N - 1)Nw}{2(N + 1)} \geq \frac{2Nw + (2N - 1)B}{2(N + 1)} = Q.
\]
Since the prestige function informs us that the amount of prestige received by any prestige lover may increase if this individual associates with others, prestige clubs may form to attract prestige lovers. We assume that the population of prestige lovers is sufficiently large and that marginal crowding costs rise sufficiently quickly so that in the equilibrium candidate the club industry features multiple clubs.

Let $J$ denote the number of potential prestige clubs. These clubs are indexed by $j$, $j = 1,\ldots, J$. Each club competes for members. Assuming potential entry in the club industry, every active club must attempt to maximize the utility of a representative member subject to feasibility and sustainability constraints. The feasibility constraint is the requirement that the total contribution amount raised by the club is at least as large as the total cost of running the club.

A club is sustainable if it induces individuals to join and provides individual and group incentives for club members to stay. Since an individual’s decision of whether or not to join a club is a participation decision and the decision of a club member of whether or not to exit the club is a defecting decision, the offer made by a feasible and sustainable club must satisfy both participation and no-defection constraints. Prestige clubs emerge in the competitive equilibrium if and only if they are feasible and sustainable.

In general, the participation and no-defection constraints differ because the utility level one gets from not joining a club is typically not the same as the utility level one obtains from defecting. The utility
of not joining is the payoff an independent contributor obtains in a voluntary contributions game in which the other players may be independent contributors or members of clubs. The independent contributor makes his contribution taking as given the other players’ choices of whether or not to join clubs and their subsequent contributions.

To formally derive the utility of not joining a club, let $Q_i \equiv Q - z_i'$ denote the total amount contributed to the collective good in the economy excluding the contribution made by an independent contributor $i$. Let $V_i(Q_i)$ be the (indirect) utility level obtained by an independent contributor $i$. This utility level is defined as

$$V_i(Q_i) = \max_{z' \in [0, z_i']} U( w - z_i', z_i' + Q_i) + B \left( z_i' - (z_i' + Q_i) / 2N \right).$$

We now examine an individual’s utility level associated with exiting a club. Let us suppose that each defector becomes an independent contributor. Among other things, the utility of defection depends on whether or not the individual is the first defector. For the first defector, the utility of defection is the payoff such an individual obtains from leaving his club and making an independent contribution, given the actions of his ex-club members and other contributors, who may be members of other clubs or independent contributors. For every other member of a club which has already experienced defection, the utility of defection is the payoff the individual obtains from leaving the club and making an independent contribution, knowing how many individuals have already defected, given the actions of his ex-club members, the defectors and other contributors, who may be members of other clubs or independent contributors. From the second defector on, the optimal defecting strategy is derived in an environment where the defecting player knows how many individuals have previously defected. The utility of defection for each subsequent defector may increase or decrease with the number of individuals who have already defected. There may be economies or diseconomies of scale in defection.

To formally derive a club member’s utility of defection, we must first consider the offer made by a particular club $j$. Each member of club $j$ pays the same “membership fee,” $z_j$. Hence, $z_i' = z_j$ for every individual $i$ who belongs to club $j$. This implies that $x_i' = x_j$ for every member of club $j$ because each individual has the same income. The club also offers the same amount of “club good,” $p_j$, to each of its members. The amount of club good supplied is
\[
p_j = \left( z_j - \frac{n_j z_j - C(n_j) + Q_{-j}}{2N} \right) + \left( n_j z_j - C(n_j) - \frac{n_j z_j - C(n_j) + Y_{-j}}{J} \right),
\]

where \( Q_{-j} = Q - Y_j \) and \( Y_{-j} = \sum_{x \neq j}^n [n_x z_x - C(n_x)] \). If there are no independent contributors, \( Q_{-j} = Y_{-j} \).

Finally, the club’s offer must be feasible. Feasibility requires that total collections are at least as large as the club’s total cost

\[
n_j z_j \geq C(n_j).
\]

Let \( e_j \in [0, n_j] \) denote the number of individuals who exit club \( j \). Define the number of individuals who stay in club \( j \) as \( s_j = n_j - e_j \). Thus, \( s_j \in [1, n_j] \). When \( s_j \) individuals stay, club \( j \) contributes \( Y(s_j, z_j) = s_j z_j - C(s_j) \) units to the collective good. Let \( Z^{\infty} = \sum_{j=1}^{e_j} z_{j,i}^t \) denote the total amount contributed to the collective good by the defectors. These definitions enable us to rewrite \( Q_{-j} \) as \( Q_{-j} = Q - Y(s_j, z_j) - Z^{\infty} \). Now, let \( Q_{(j,i)} = Q_{-j} - z_{j,i}^t \) be the amount of collective good provided in the economy excluding the contributions made by club \( j \) and by an independent contributor \( i \). The payoff a club member \( i \) obtains from exiting club \( j \) when \( e_j \) individuals have already exited the club is

\[
V_i(Q_{(j,i)}) = \left\{ \text{Max}_{z_{j,i} \in [0,u]} \left[ U\left( w - z_{j,i}^t + Q_{(j,i)} \right) + B\left( z_{j,i}^t - \left( z_{j,i}^t + Q_{(j,i)} \right) / 2N \right) \right] \right\}.
\]

Assuming an interior solution to the problem faced by a club member \( i \) who exits club \( j \) after \( e_j \) individuals have already exited, we have the following first order condition:

\[
4N^2 \left[ U_Q \left( w - z_{j,i}^t + Q_{(j,i)} \right) - U_s \left( w - z_{j,i}^t + Q_{(j,i)} \right) \right] + (2N - 1) B \left( 2N - 1 \right) z_{j,i}^t - Q_{(j,i)} = 0.
\]

Let \( z_{j,i}^t(e_j) \) be defined implicitly by equation (6). A straightforward exercise in comparative statics yields

\[
\frac{d z_{j,i}^t}{d e_j} = \frac{\left( z_j - C_s(s_j) \right) \left( U_{i,Q} - U_{i,s} \right)}{U_{i,s} + U_{i,Q} - 2 U_{i,Q}}.
\]
Equation (7) informs us that the optimal contribution of a defecting club member rises with the number of individuals who have previously defected if \( z_j > C_a(s_j) \). We show below that this inequality holds in the equilibrium candidate for \( e_j > 0 \). Now, note that

\[
\frac{dW_i(Q_{i(j,i)})}{de_j} = \frac{dW_i(Q_{i(j,i)})}{dQ_{i(j,i)}} \frac{dQ_{i(j,i)}}{dy(s_j, z_j)} \frac{dy(s_j, z_j)}{de_j} = - \left( W_{iQ} - \frac{B}{2N} \right) (z_j - C_a(s_j)) \tag{8a}
\]

and

\[
\frac{d^2W_i(Q_{i(j,i)})}{de_j^2} = \left( z_j - C_a(s_j) \right)^2 \left( \frac{U_{iQ}U_{iQQ} - \left( U_{iQQ} \right)^2}{U_{iQQ}^2 + U_{iQQ}^2 - 2U_{iQQ}^2} \right) - \left( \frac{U_{iQ} - B}{2N} \right) C_m(s_j). \tag{8b}
\]

Suppose that \( z_j > C_a(s_j) \) for \( e_j > 0 \) (i.e., \( s_j < n_j \)). If \( U_{iQ}(w, Q) > B/2N \) for \( z'_i \in [0, w] \) and \( Q \in [0, Nw] \), equation (8a) yields \( dW_i(Q_{i(j,i)})/de_j < 0 \) for \( e_j > 0 \). If \( U \) is strictly concave, equation (8b) implies that \( d^2W_i(Q_{i(j,i)})/de_j^2 < 0 \) for \( e_j > 0 \). When \( dW_i(Q_{i(j,i)})/de_j < 0 \) for \( e_j > 0 \), there are diseconomies of scale in defection. Defection becomes less attractive the larger it is the number of individuals who have previously defected. If, on the other hand, we have \( z_j > C_a(s_j) \) for \( e_j > 0 \) and \( U_{iQ}(w, Q) < B/2N \) for \( z'_i \in [0, w] \) and \( Q \in [0, Nw] \), we obtain \( dW_i(Q_{i(j,i)})/de_j > 0 \) for \( e_j > 0 \) from equation (8a). In this case, there are economies of scale in defection. Of course, it is also possible that for some interval of \( e_j \) values we observe diseconomies of scale while for some other interval of \( e_j \) values we observe economies of scale in defection.

If \( V_i(Q_{i(j,i)}) \) is monotonically decreasing (increasing) in \( e_j \), the payoff received by the first defector is the largest (lowest) and the payoff enjoyed by the last defector is the lowest (largest) in the set of defection payoffs. If \( V_i(Q_{i(j,i)}) \) is not monotonic in \( e_j \), there will be at least one value of \( e_j \) that maximizes \( V_i(Q_{i(j,i)}) \). In any event, letting \( V^M_i \equiv \left\{ \max_{s_j \in [0, s_j]} V_i(Q_{i(j,i)}) \right\} \), the no-defection constraints faced by any club can be expressed in terms of one inequality stating that the utility level offered by the club is at least as high as \( V^M_i \).
We are now ready to consider the problem faced by the manager of club \( j \). Noting that 
\[
Q = n_j z_j - C(n_j) + Q_j, \]
the manager of club \( j \) chooses \( \{n_j, p_j, x_j, z_j\} \) to maximize 
\[
U(x_j, n_j z_j - C(n_j) + Q_j) + Bp_j \quad \text{subject to} \quad U(x_j, n_j z_j - C(n_j) + Q_j) + Bp_j \geq \text{Max} \{V_i(Q_j), V^M_i\},
\]
equations (4) and (5), \( x_j + z_j = w, \ x_j \geq 0, \ z_j \geq 0, \) and \( n_j \in [1, N] \), taking \( Q_j, Y_j, V_i(Q_j), V^M_i \) and \( J \) as given.

Suppose that the offer made by club \( j \) satisfies the feasibility, participation and no-defection constraints. We will later show that these constraints are indeed satisfied in a feasible equilibrium candidate.

Assuming an interior solution, the first order conditions are equation (4), the budget constraint and the following two equations for \( j = 1, \ldots, J \):
\[
n_j \left(\frac{U'_j}{U'_s} + \left(\frac{J-1}{J}\right)B \frac{U'_s}{U'_j} + \left(\frac{2N-n_j}{2N}\right)B \right) = 1, \tag{9a}
\]
\[
\left[z_j - C_u(n_j)\right] \left[\frac{U'_j}{U'_s} + \left(\frac{J-1}{J} - \frac{1}{2N}\right)B\right] = 0. \tag{9b}
\]
Since \( J \leq N \), we have \((J-1)/J > 1/2N \) if \( J \geq 2 \). Assuming that \( J \geq 2 \) in the equilibrium candidate implies that \( z_j = C_u(n_j) \) from equation (9b).

Close inspection of equations (9a) and (9b) reveals that prestige clubs are identical in the equilibrium candidate. This implies that \( J = N/n_j \). Letting \( z_j = z' \) and \( n_j = n \) for \( \forall j \), we have \( Y_j = Y = nz' - C(n) = Y, \) \( Q = N[nz' - C(n)]/n \) and \( p_j = p = [z' + C(n)/n]/2 > 0 \). Note that \( nz' - C(n) = nC_u(n) - C(n) > 0 \), where the inequality follows from the fact that \( C \) is strictly convex.

Thus, the feasibility constraint is satisfied in the equilibrium candidate.

In sum, the equilibrium candidate is characterized by the following conditions:
\[
n \left[\frac{U'_j}{U'_s} + \left(\frac{N-n}{N}\right)B \frac{U'_s}{U'_j} + \left(\frac{2N-n}{2N}\right)B \right] = 1 \Rightarrow n \left[\frac{U'_j}{U'_s} + \left(\frac{N-n}{2N}\right)B \right] = 1, \tag{10a}
\]

\(^8\) The assumption that \( J \geq 2 \) is thus equivalent to \( N \geq 2n_j \). This assumption is harmless if \( N \) is sufficiently large or if marginal crowding costs rise sufficiently quickly in \( n_j \). To illustrate, we provide an example below in which \( N = 120, \ n_j = 30 \) and \( J = 4 \). The cost function is \( C(n_j) = n_j(n_j - 1)/2 \).
\[ z' = C_n(n) , \]  
\[ Y = nz' - C(n) = \hat{Y} , \]  
\[ Q = Z' - \frac{NC(n)}{n} , \]  
\[ p = z' - \frac{Q}{2N} = \frac{1}{2} \left( z' + \frac{C(n)}{n} \right) , \]  
\[ x' + z' = w , \]  
\[ U(x', Q) + Bp = \max \left\{ V_i(Q_i), V_i^M \right\} , \]  
\[ U(x^*, Q) = U(w, Q) . \]  

The first equation in (10a) states that the representative prestige club determines its contribution level by equating the sum of marginal group and individual contribution benefits, properly discounted by the marginal utility of income, to the marginal rate of transformation between contribution and the numeraire good. The marginal group benefits consist of two components, the marginal pure benefits from grouping and the marginal impure benefits from grouping. The marginal pure benefits from grouping are the sum of the group members’ marginal rates of substitution between collective and numeraire goods. The marginal impure benefits from grouping are the weighted sum of the group members’ marginal rates of substitution between prestige and numeraire goods. The rationale for the weight, \( (N-n)/N \), comes from the fact that a unit increase in the contribution made by the representative club member is perceived to produce a net increase of \( (N-n)/N \) units in the prestige level enjoyed by each club member stemming from the institutional component of the prestige production function. The marginal individual benefit is the net marginal impure benefit associated with an expansion of each club member’s prestige level stemming from the individual component of the prestige production function. The club manager knows that an expansion in a club member’s contribution produces positive and negative effects on the welfare of each club member. The manager internalizes the positive and negative prestige externalities associated with an expansion in each member’s contribution for all individuals who belong to the club. He neglects, however, the positive and negative prestige externalities that are imposed on members of other clubs.
The bracketed term in the second equation in (10a) represents the perceived net effect in prestige production for each unit of contribution made by a club member. Consistent with our assumption that either $N$ is sufficiently large or crowding costs rise sufficiently quickly so that multiple clubs emerge in the equilibrium candidate (i.e., $N \geq 2n$), it is straightforward to show that

$$
\frac{(N-n)(2n+1)}{2N} > \frac{1}{2}
$$

since $2(N-n) > 1$. In light of result (10i), one can clearly see that the bracketed term in the second equation in (10a) is larger than one. This implies that a club manager perceives a net increase of more than one unit in the prestige level enjoyed by every club member for each unit of contribution made by a club member.

Equation (10b) informs us that the representative prestige club selects its membership size by equating the club’s marginal benefit of expanding membership to its marginal crowding cost. The club’s marginal benefit of expanding membership is the contribution collected from each club member. Given the optimal member fee, the optimal club size maximizes the club’s contribution to the collective good.

Equation (10c) shows that each club’s attempt to advance its institutional prestige level is futile. In equilibrium, institutional prestige levels are identically zero since all clubs make identical contributions to the collective good. However, as illustrated by equation (10a), the race for maximizing institutional prestige yields coordination benefits, which induce each club to expand its contribution level. Equation (10d) makes it clear that the formation of prestige clubs has a social cost. This cost is measured in terms of units of the collective good that are given up in order to advance prestige. The total social cost is the club industry’s operating cost.

Equation (10e) is immediately implied by equations (4), (10c) and (10d). Each individual’s prestige level is the average of individual contributions, which together finance provision of the collective good and the club industry’s operating cost. The prestige level in this equilibrium candidate is higher than the prestige level in the previous equilibrium candidate without prestige clubs, since here each individual contributes a higher amount to the collective good.\(^9\)

\(^9\) It is straightforward to demonstrate this. For the sake of comparison, suppose that $n = 1$ and note that the left hand side of equation (10a) with $n = 1$ is larger than the left hand side of equation (3). Since the right
We now demonstrate that the offer made by a feasible representative club satisfies the participation and no-exit constraints (10g). Consider the problem faced by an individual \( i \) who gets the highest payoff from defecting a representative club \( j \). This problem is identical to the problem faced by the manager of club \( j \) if we restrict the manager’s set of feasible actions as follows: (i) except for the contribution made by individual \( i \), fix the contribution of every other member; (ii) set club membership size equal to \( s_j \); and (iii) set the club’s contribution equal to the average of contributions made by clubs. Since the maximum of such a problem cannot be larger than the maximum of the problem in which the additional restrictions are not imposed, we can affirm that the indirect utility level the club manager delivers to each club member is at least as large as \( V_i^{M} \).

Now consider the problem faced by an independent contributor \( i \) who refused the club’s offer. This problem is identical to the problem faced by the manager of a feasible representative club \( j \) with the additional restrictions (i), (ii) and (iii) described above, but for \( s_j = 1 \). Thus, the payoff obtained by this contributor cannot be larger than the utility level offered by the feasible representative club in the equilibrium candidate. In sum, a feasible representative club is sustainable because its offer satisfies both no-defection and participation constraints.

We gather the results of our analysis in the following proposition:

**Proposition 1.** The competitive equilibrium features identical prestige clubs. It is characterized by equations (10a) – (10h).

To illustrate our results, we provide a numerical example. Suppose that the utility functions are the same as we used in our previous example. Let \( B = w \) and \( C(n) = n(n - 1)/2 \). Equation (10b) implies that \( z' = n - 1/2 \). Since \( U_Q^{i'} = x' = w - z' = w - n + 1/2 \) and \( U_s^{i'} = Q = Nn/2 \), we can express equation (10a) as follows: \( Nn(2w + 1 - 2n) + 2n(N - n)w + (2N - n)w = N^2n \). Letting \( N = 120 \) and \( w = 50.3 \), the quadratic hand side of each equation is the same, we reach the conclusion that the individual contribution amount that satisfies equation (3) is “too small” to satisfy equation (10a) under the hypothesis that \( n = 1 \). Since \( N > n > 1 \) in the interior optimum, one can easily verify that the left hand side of equation (10a) is larger than the left hand side of equation (3). This implies that the individual contribution in this equilibrium candidate is higher than in the previous one.

\(^{10}\) Even though the institutional prestige level is zero in the equilibrium candidate, each club’s attempt to maximize it yields coordination benefits, as we discussed in the text.
equation yields $n \equiv 30$. Then, $z' = 29.5$, $Y = 450$, $J = 4$, $Q = 1800$, $p = 22$ and $x' = 20.5$. The utility level for each prestige lover is $U(20.5, 1800) + 50.3(22) = 38006.6$. The utility level for each altruist is $U(50, 1800) = 90000$.

For comparison purposes, the outcome for the voluntary contributions game is $Q = 99.56$, $z' = 0.83$, $p = 0.41$ and $x' = 49.17$. The utility levels reached by each prestige lover and each altruist in this game are respectively 4915.99 and 4978. The competitive equilibrium yields higher levels of prestige and collective goods and higher utility levels for both types of consumers than the equilibrium candidate with independent contributions.

IV. Pareto Efficient Allocation

We now derive the conditions for a Pareto efficient allocation with positive utility levels for both types of consumers and compare it with the competitive equilibrium in order to highlight the latter's sources of inefficiencies.

A Pareto efficient allocation in which purists and prestige lovers enjoy positive utilities is the solution to the following problem: Choose $\{x^e, x', z^e, z', Q, n\}$ to maximize $U(x', Q) + B\left(z' - \frac{Q}{2N}\right)$ subject to

\begin{align*}
U(x^e, Q) &\geq \bar{U}^e > 0, \quad (11a) \\
N(z^e + z') & = Q + \frac{NC(n)}{n}, \quad (11b) \\
x^e + x' + z^e + z' & = 2w, \quad (11c) \\
x^e \geq 0, x' \geq 0, z^e \geq 0, z' \geq 0, Q \geq 0, N \geq n \geq 1.
\end{align*}

According to (11c) an increase in $z^e$ must be accompanied by a reduction in at least one of $\{x^e, x', z'\}$. A reduction in either $x'$ or $z'$ lowers the objective function. A reduction in $x^e$ lowers the utility function in (11a). Since $z^e$ does not enter directly in either the objective or (11a), we must have $z^e = 0$ in the optimum. Equation (11b) now implies that
\[
z' = \frac{Q}{N} + \frac{C(n)}{n}, \quad (11d)
\]

Plugging (11d) into both the objective function and constraint (11c) implies that the problem simplifies to

the choice of \(\{x^*, x', Q, n\}\) to maximize

\[
N\left(x^* + x'\right) + Q + \frac{NC(n)}{n} = 2Nw, \quad (11e)
\]

subject to (11a) and

\[
x^* \geq 0, \quad x' \geq 0, \quad Q \geq 0, \quad N \geq n \geq 1.
\]

Since the average cost function, \(C(n)/n\), is increasing at a nondecreasing rate in \(n\), the objective function
is convex in \(n\) and the constraint set is convex. It follows that the solution involves either \(n = 1\) or \(n = N\)
deping, among other things, on the degree of convexity of \(C\) and on the value that \(B\) takes. We shall
assume that \(2Nw > C(N)\) and that the utility functions are such that \(x^* > 0, \quad x' > 0\) and \(Q > 0\) in any
optimum.11

Suppose first that \(n = 1\). The resource constraint (11e) becomes

\[
N\left(x^* + x'\right) + Q = 2Nw. \quad (11f)
\]

Let \(\lambda_i \geq 0\) and \(\mu_i \geq 0\) be the Lagrangian multipliers associated with (11a) and (11f), respectively. The first
order conditions are constraints (11a), (11f) and the following equations:

\[
x^* : \quad \lambda_i U_i^* = N \mu_i, \quad (11g)
\]

\[
x' : \quad U_i' = N \mu_i, \quad (11h)
\]

\[
Q : \quad U_0' + \frac{B}{2N} + \lambda_0 U_0^* = N \mu, \quad (11i)
\]

Combining equations (11g), (11h) and (11i), we obtain

\[
N\left(\frac{U_0'}{U_0^*} + \frac{U_i'}{U_i^*}\right) + \frac{1}{2} \frac{B}{U_i'} = 1. \quad (11j)
\]

11 A sufficient condition for this result is that \(U\) satisfies the Inada conditions with respect to both
arguments. Since the Inada conditions are not necessary, as the example below demonstrates, we shall not impose it.
Equation (11j) is the modified Samuelson condition. It equates the marginal social benefit of contributing to the collective good to the marginal social of doing so. The marginal social benefit is the sum of two components: the pure marginal social benefit and the impure marginal social benefit. The pure marginal social benefit is the sum of the marginal rates of substitution between collective good and the numeraire good. The impure marginal social benefit equals half the marginal rate of substitution between prestige and the numeraire good. This represents the increase in prestige faced by each prestige lover as the collective is expanded. The marginal social cost of provision is the marginal rate of transformation between contribution and the numeraire good.

Suppose now that \( n = N \). The resource constraint (11e) becomes

\[
N\left(x^a + x^l\right) + Q + C(N) = 2Nw, \tag{11k}
\]

Let \( \lambda_N \geq 0 \) and \( \mu_N \geq 0 \) be the Lagrangian multipliers associated with (11a) and (11k), respectively. The first order conditions are constraints (11a), (11k) and the following equations:

\[
\lambda_N U_a^a = N\mu_N, \tag{11l}
\]

\[
U^a = N\mu_N, \tag{11m}
\]

\[
U^l + \frac{B}{2N} + \lambda_N U^a = N\mu_N. \tag{11n}
\]

Combining equations (11l) – (11n) we obtain the modified Samuelson condition (11j).

Let \( \begin{pmatrix} x^a(B), x^l(B), z^l(B), Q_l(B) \end{pmatrix} \) denote the solution to the system of equations (11a), (11d), (11f) – (11i). This solution is a candidate for a Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods, positive utility levels for all consumers and featuring independent contributors. Let \( \begin{pmatrix} x^a_N(B), x^l_N(B), z^l_N(B), Q_N(B) \end{pmatrix} \) denote the solution to the system of equations (11a), (11d), (11k) – (11n). This solution is a candidate for a Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods, positive utility levels for all consumers and featuring one prestige club. Since both solutions satisfy the utility constraint (11a) and \( \bar{U}^a \) is the same in both situations, the solution featuring independent contributors is a Pareto efficient allocation if

\[
U \left( x^a_l(B), Q_l(B) \right) + \frac{BQ_l(B)}{2N} > U \left( x^a_N(B), Q_N(B) \right) + B \left( \frac{Q_N(B)}{2N} + \frac{C(N)}{N} \right). \tag{12}
\]
The solution featuring one prestige club is a Pareto efficient allocation if the inequality in expression (12) is reversed. If the left and right hand sides of expression (12) are equal, both solutions are Pareto efficient.

We gather the results in the following proposition:

**Proposition 2.** Whenever condition (12) holds, a Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods and positive utility levels for all consumers is given by equations \( n = 1 \), \( z^e = 0 \), (11a), (11d), (11f) – (11i). If the reverse of condition (12) holds, a Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods, positive utility levels for all consumers is given by \( n = N \), \( z^e = 0 \), (11a), (11d), (11k) – (11n).

Comparing the conditions that characterize the competitive equilibrium with those that characterize any type of Pareto efficient allocation, it is immediately clear that the competitive equilibrium is inefficient because it features clubs of inefficient size, it yields an inefficient amount of collective good and an inefficient prestige level to each prestige lover. Comparing equations (10a) and (11j) reveals that they differ with respect to the “weights” placed on both components of the marginal benefit of a contribution to the collective good. Consider the second equation in (10a). Its pure marginal benefit consists of the sum of the marginal rates of substitution between collective and numeraire goods for the members of a representative club only. The pure marginal social benefit in equation (11j) is the sum of the marginal rates of substitution between collective and numeraire goods for all members of society. Since the pure marginal benefit that the representative club takes into account in the competitive equilibrium neglects the sum of the marginal rates of substitution between collective and numeraire goods for all members of society who do not belong to the club, the representative club places a lower weight on the pure marginal benefit of a contribution than it is efficient.

As for the comparison between impure marginal benefits, we can clearly see from the bracketed term in the second equation in (10a) that the weight placed by the representative club manager on the impure marginal benefit is larger than the efficient weight. In fact, as we discussed before, the weight placed on the impure marginal benefit by a club manager in the competitive equilibrium is larger than one. This follows from the fact that the representative club internalizes prestige externalities for its own members only.
As the competitive club places a lower than efficient weight on the pure marginal benefit but a higher than efficient weight on the impure marginal benefit, it is unclear whether the competitive club industry undersupplies or oversupplies the collective good. The lower than efficient weight on the pure marginal benefit provides incentives for underprovision. The higher than efficient weight on the impure marginal benefit, on the other hand, provides incentives for overprovision. Since the prestige level enjoyed by a prestige lover in either competitive equilibrium or a relevant Pareto efficient allocation is an increasing function of the overall level of collective good and since the competitive equilibrium does deliver an efficient amount of collective good, the prestige level supplied by each competitive club is also inefficient.

To illustrate our results, we consider a numerical example. Suppose that the utility functions are identical to those we used in our previous examples. Let \( N = 120 \) and \( \bar{U}^a = 90000 \). It is straightforward to show that if \( B = w = 50.3 \), the Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods is \( \{ x^i(B), x'_B(B), z'_B(B), Q_i(B) \} = \{14, 88, 32, 22, 50.4, 6048.56\} \). The prestige level enjoyed by each prestige lover is 25.2. The utility level for a representative prestige lover is 214903.41. Remember that each competitive club offers a prestige level equal to 22 to each of its members, the competitive industry supplies 1800 units of the collective good and the utility level for a representative club member is 38006.6. Thus, the competitive equilibrium undersupplies the collective and prestige goods and yields a much lower utility level to each prestige lover than it is efficient.

Suppose now that \( \bar{U}^a = 9000 \) and \( B = 6000 < Nw = 6036 \). In this case, the Pareto efficient allocation with positive consumption of numeraire, prestige and collective goods is the solution featuring a single prestige club: \( \{ x^i(B), x'_B(B), z'_B(B), Q_i(B) \} = \{2.27, 5.78, 92.55, 3966\} \). The prestige level enjoyed by each prestige lover is 76.025. The utility level for a prestige lover is 479073.48. The solution featuring independent contributors is \( \{ x^i(B), x'_B(B), z'_B(B), Q_i(B) \} = \{1.19, 36.61, 62.8, 7536\} \). The prestige level enjoyed by each prestige lover is 31.4. The utility level for a prestige lover is 464260.56.

V. Conclusion

The incentives to promote the individual and institutional components of prestige motivate individuals (managers) to form prestige clubs. The race for advancement of both prestige components produces coordination benefits within clubs, even though the race per se is futile. The coordination benefits stem
from internalization of positive and negative prestige externalities within clubs. Such coordination benefits, in turn, motivate managers to expand membership sizes, but the expansions are limited by growing crowding costs. Net of administrative costs, the prestige club industry produces a greater amount of collective good than one would observe in a hypothetical economy without prestige clubs.

The competitive equilibrium, however, is inefficient because it features clubs of inefficient size and each club does not fully internalize the positive and negative externalities associated with its members’ contributions to the collective good. As a result, the competitive equilibrium produces inefficient amounts of prestige and collective goods.

References


