George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 1, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 September to 12 October 1964

Gentlemen:

In accordance with Contract Modification No. 5, Monthly Progress Letters will be submitted in lieu of Quarterly Program Reports, commencing 12 September 1964 and continuing through the remainder of the present contract period.

It is desired that the contract program be extended for an additional twelve months, beginning 12 December 1964, at approximately the same rate of effort and with essentially the same work objectives as are now in effect. Monthly letter reports would presumably be called for under such an extension, and interim technical results would continue to be made available to the sponsor in the form of Project Technical Notes.

Group 1 Progress Summary

Primary efforts during the past month have been applied to the testing of the Six-Diode Bridge. As mentioned in the last quarterly report, Fairchild Type 1N3595 diodes are being used in place of the General Electric 1N4443's which were originally suggested by Dr. Polstorff. (Further inquiry revealed that E.E. had encountered production difficulties with the 1N4443, but delivery is now promised for the week of 9 November.)

Results of tests on the various types of gates considered thus far may be summarized as follows. (The quantities tabulated are defined in the last Quarterly Program Report.)
### Gate Type

<table>
<thead>
<tr>
<th>Gate Type</th>
<th>$A_v$(on)</th>
<th>$A_v$(off)</th>
<th>$V_o$</th>
<th>$I_D$</th>
<th>$Z$(calc.)</th>
<th>Freq. Limit</th>
<th>$V_{in}$</th>
</tr>
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<tr>
<td>Modified Guennou Chopper Switch</td>
<td>0.69</td>
<td>0.06</td>
<td>0.2mv</td>
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<td>300Ω</td>
<td>60 kc</td>
<td>±1v</td>
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<tr>
<td>2-Transistor Nonsaturating Sw.</td>
<td>0.72</td>
<td>0.001</td>
<td>0.1</td>
<td>17</td>
<td>12</td>
<td>50</td>
<td>±5*</td>
</tr>
<tr>
<td>4-Transistor Nonsaturating Sw.</td>
<td>0.81</td>
<td>0.0008</td>
<td>0.2</td>
<td>0.2</td>
<td>5</td>
<td>50</td>
<td>±5*</td>
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<tr>
<td>Complementary Microvolt Bridge</td>
<td>0.9</td>
<td>&lt;0.0005</td>
<td>3.0</td>
<td>&lt;0.1</td>
<td>~250</td>
<td>500</td>
<td>±30**</td>
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<tr>
<td>6-Diode Bridge</td>
<td>0.99</td>
<td>0.0007</td>
<td>3.2</td>
<td>0.4</td>
<td>~90</td>
<td>#</td>
<td>±20***</td>
</tr>
</tbody>
</table>

* Switching frequency limited by drive circuitry.
* Test level set by supply voltage.
** Test level set by transistor breakdown.
*** Test level set by choice of drive circuitry.

During the next month, tests of the six-diode bridge will be continued. In addition, some investigation of field-effect transistor switches may be started.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During this period the major emphasis has been on the hybrid problem in the following three areas:

1. A comprehensive literature survey
2. Examination of several elementary hybrid problems.
3. Planning of a general attack on the error analysis problem.

Some effort has also been made in the area of generation and description of nonstationary stochastic processes. This work has concentrated on a careful mathematical description of the type of random process which is required for simulation of a missile flight.

The literature survey has been completed, and Technical Note No. 4 discussing the existing literature and listing the major references is being prepared. This Note will be completed in the next monthly report period.

Several elementary hybrid problems have been studied. This work has placed in evidence various aspects of the error analysis problem. Some of this work may be included in later Technical Notes, but it does not warrant immediate publication.
Planning of a general attack on the error analysis problem has not been completed and work will continue in this area during the next monthly report period.

One aspect of the investigation on describing nonstationary stochastic processes has been completed, and a Technical Note covering this material will be prepared during the next report period.

Respectfully submitted,

Fred Dixon
Project Director
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 2, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 October to 12 November 1964

Gentlemen:

On 16 October 1964, the undersigned and Dr. J. L. Hammond, Jr. attended the local Southeastern Regional Meeting of the Association for Computing Machinery and the Southeastern Simulation Council as representatives of the subject research project. During portions of the afternoons of 15 and 16 October, conferences were held with Dr. W. K. Polstorff and Mr. Fred Shaver from MSFC to review the current program efforts and discuss future work. The NASA Purchasing Office was subsequently notified (by Georgia Tech Research Institute letter dated 23 October 1964) of our interest in submitting a proposal to extend the present contract.

Group 1 Progress Summary

The experimental data on electronic switch performance presented in the last monthly report was mostly obtained using the arrangement of analog computer integrators and summers shown on pages 5 and 6 of Quarterly Program Report No. 64-3 (12 June to 12 September 1964). Since it is hoped that leakage effects in the better types of switch will be below the drift of the amplifiers themselves, a different measuring technique has been tried.

Brief consideration was given to Dr. Polstorff's suggestion of employing a capacitor discharge into a ballistic galvanometer for the low-level leakage measurements. However, a Keithley Model 610A Electrometer (with ranges down to 10⁻¹³ amperes or 10 millivolts full-scale) has been made available to the
project and appears to be adequate for present purposes. The Electrometer is connected from gate output to ground and either reads the voltage across a standard load resistor (usually 1 megohm) or else itself serves as the load and reads gate output current directly.

Work has been continued on the 6-diode bridge incorporating Fairchild 1N3595 units. Measurements obtained with the Electrometer are in general accord with those previously reported. The G.E. 1N4443 diodes have still not been received, but hopefully will be available for similar tests in the near future.

A series-shunt gate using the recently announced 2N3380 and 2N3386 analog-switching unipolar field-effect transistors has been constructed and measurements are presently being made on this circuit. Early indications are that the "unifet" gate compares favorably with the 6-diode type and may offer significant advantages over it for certain applications. An attempt will be made to obtain comparative performance data on these two electronic switches by the end of the next report period.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

Technical Note No. 4, "Bibliography and Summary of Methods Related to Error Analysis of Hybrid Computers" by T. M. White, is in process of publication and should be available to the sponsor during the next week or two.

Other activity in the hybrid computation area has been concentrated on planning a general attack on the error analysis problem. A consideration of the possible uses of hybrid computers in simulation reveals that identification of several modes of operation is possible. The simplest mode of operation from the point of view of error analysis is that for which the analog portion of the computer performs linear operations such as integration and multiplication by a constant, while the digital portion is used essentially as a function generator. This case is appropriate for the solution of any ordinary differential equation, including linear, nonlinear, and time-varying cases, as well as any system of such equations. Other typical modes of operation include programs using the digital portion of the hybrid computer to perform logical operations, and programs for which the analog computer performs nonlinear as well as linear operations.
A logical approach is to start with the simplest case, and thus attention has so far been restricted to that use of the hybrid computer for which the analog components perform linear operations and the digital components perform generalized function generation. Considerable progress has been made for this mode of operation on an approach which will estimate computing errors by solving a linear equation with a random forcing function. The forcing function has a statistical structure which is determined by the nature of the sampling error for the class of solutions to the given problem. Plans are being made for discussing the details of this approach along with current progress in a Technical Note in the next several months.

Technical Note No. 5, "Notes on a Class of Stochastic Processes for Use in Analog Simulation Studies" by D. L. Finn, has been completed and is being readied for publication. This report discusses the formulation and characteristics of a general mathematical model for random disturbances of a missile flight. It should be available to the Sponsor by the end of the present contract period (12 December 1964) or shortly thereafter.

Other activities in this area of the work have been devoted to preparation for further experiments in the generation of nonstationary random processes. Plans call for using the synthesis procedure described in Technical Note No. 3 to simulate several additional noise processes of increasing complexity. Work during the current report period was largely directed toward improving the measuring system which is used to determine the characteristics of the physical process produced by the computer. In that connection it may be noted that an Electronic Associates, Inc., Model 44.200 Low-Frequency Gaussian Noise Generator has just been acquired by the Analog Computer Laboratory and will be used on all future tests for this project.

Respectfully submitted,

F. Dixon
Project Director
17 December 1964

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter No. 3, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period 12 November to 12 December 1964

Gentlemen:

This letter report covers the final month of the current contract period. However, it is expected that the program will be extended for another year, in accordance with the detailed proposal submitted to the MSFC Purchasing Office on 30 November 1964.

Group I Progress Summary

Further measurements have been made on the 6-diode switch utilizing Fairchild type 1N3595 units in place of the G. E. 1N4443's originally suggested by Dr. Polstorff. The General Electric Co. recently advised that production problems (and/or difficulties in obtaining raw materials) now make it impossible to quote a firm delivery date on 1N4443 diodes. We have therefore cancelled our order of last June, and will attempt to locate suitable alternative diode types. If an acceptable substitute cannot be found, a new order may be placed with G.E. later on.

As mentioned in the preceding progress letter, a Keithley Model 610A Electrometer is now being used to obtain low-level leakage measurements. The offset voltage determined for the 6-diode switch by means of the Electrometer was 10.5 millivolts across a load resistance of 1 megohm, corresponding to an essentially constant-current source output of 10.5 nanoamperes. The earlier result of 3.2 millivolts for $V_o$ which was listed in Monthly Progress Letter No. 1 (12 September to 12 October 1964)
represented a leakage current of some 6.4 nanoamperes through a load resistance of about 0.5 megohm. The remaining discrepancy between the two measurements may be attributable to a difference in operating temperatures: in the second case the diodes were enclosed in an aluminum box at close to room temperature, whereas for the first tests they were less well isolated from the drive circuitry but had their temperature somewhat lowered by evaporative cooling.

In order to allow determination of the switching frequency limits for the 6-diode bridge, a high-speed transistorized driving circuit was designed and constructed. A frequency limitation of about 2 kc was found. This limit is due to the time constant of the switching circuit in the "off" or open mode. The switch may be used at higher sampling rates by connecting a second 6-diode unit so as to ground the output of the first when in the open mode.

Measurements have also been made on a series-shunt "unifet" switch, incorporating two unipolar field-effect transistors (one type 2N3380 and one type 2N3386) which were obtained as part of a trial kit from Siliconix, Inc. The circuit configuration is shown below, together with measured characteristics corresponding to those tabulated in Monthly Progress Letter No. 1 for the various other switches investigated.

\[
\begin{align*}
A_{V(on)} &= 0.95 & V_o &= 0.04 \text{ mv} & Z_{out} &\approx 150 \Omega \\
A_{V(off)} &= \angle 0.00005 & L_D &= 0.2-0.4 \text{ mv} & f_{\text{lim}} &= 500 \text{ kc} \\
V_{in} &= \pm 2 \text{ volts (limited by unifet characteristics)}
\end{align*}
\]
Information has been requested from several manufacturers on multilayer semiconductor devices suitable for the construction of a bridge-type switch. At the present time, the Texas Instruments type 2N3004 silicon-controlled switch appears to be a good possible selection. During the next report period an attempt will be made to arrive at a firm choice and obtain a set of four such units for evaluation.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

Technical Note No. 4, "Bibliography and Summary of Methods Related to Error Analysis of Hybrid Computers," by T. M. White was published during the past month and forwarded to the sponsor. Technical Note No. 5, "Notes on a Class of Stochastic Processes for Use in Analog Simulation Studies," by D. L. Finn is in process of publication and should be available to the sponsor within the next two to three weeks. Group 2 efforts during the current report period were fairly evenly divided between the two assigned areas of interest--i.e., (1) Error Analysis for Hybrid Computation, and (2) Nonstationary Noise Studies.

In Area 1, further progress was made in formulating an approach to error analysis for problems using a hybrid computer in such a way that the analog elements perform linear operations and the digital elements perform generalized function generation. Types of problems which can be solved in this manner were noted briefly in the last progress letter, and a detailed discussion is planned for a later technical report. As mentioned in the preceding progress letter, an approach which produces an estimate of computing errors by solving a linear equation with a random forcing function is presently being studied. Several specific examples have been examined in evaluating this approach, and the results so far are encouraging. Techniques for specifying the random forcing function in a simple manner are currently being investigated. An ideal procedure would make it possible to examine the differential equation being solved by the hybrid computer, and characterize the random forcing function for the error equation directly from the parameters of the equation.

Work in Area 2 has been concerned with using the synthesis procedure of Technical Note No. 3 to generate several specific types of nonstationary noise. This work is part of an effort to further evaluate and refine the
techniques discussed in Technical Note No. 3 by examining several specific examples.

The Electronic Associates Model 44.200 Low-Frequency Gaussian Noise Generator recently acquired by the Analog Computer Laboratory for use in the above studies is currently undergoing tests to verify its assumed stationary, Gaussian, band-limited white-noise properties. These tests involve sampling the analog output signal through an A/D converter (some 400 samples per second for stretches of 20 minutes' duration) and processing the taped digital data on the Burroughs B-5000 computer.

Respectfully submitted,

F. Dixon
Project Director A-588

FD:sh
GEORGIA INSTITUTE OF TECHNOLOGY
ENGINEERING EXPERIMENT STATION
ATLANTA 13, GEORGIA

22 January 1965

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, FA-EC

Subject: Monthly Progress Letter 4, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NA88-2473
Covering the Period from 12 December 1964 to 12 January 1965

Gentlemen:

Although the recently proposed one-year extension of the subject contract did not arrive until 18 January, work on the program had been continued throughout most of the past month, along the lines previously discussed. These efforts are briefly outlined herein.

Group 1 Progress Summary

Contrary to information given last month, the General Electric Co. was finally able to make delivery of the 1N443 diodes which were ordered last June. (Delivery was accepted although cancellation proceedings had already been initiated.) The attached Data Sheet No. 1 compares various measured characteristics of twelve G.E. 1N443 and twelve Fairchild 1N3595 diodes received on this project and also shows the manufacturers' published specifications. The slight excesses in forward voltage drop at 100 ma seen for four of each diode type are not considered significant. However, the reverse-current characteristics of all eight units from the "B" production lot of 1N3595's are so badly out of tolerance that they should be replaced. Unfortunately, units from this production lot had been used in the first 6-diode gate tested, which accounts for the unexpectedly high gate-leakage current of 10.5 nanoamperes reported in the last progress letter.* In any

*Since Fairchild units #9 and 10 formed the two output legs of the bridge, a gate-leakage current of some 7 nanoamperes was to have been expected—i.e., the difference between the individual diode leakages of 25 and 18 nanoamperes. The apparent discrepancy of 3.5 nanoamperes may be attributed to reading errors, nonidentical measuring conditions, surface leakages in the circuitry mounting, etc.
case, a new set of measurements is now being made with the good 1N3595 units appropriately paired to minimize gate offset voltage. Tests will also be made on a 6-diode gate using the most closely matched group of new 1N4443's. The laboratory setup has been modified so that tests with the experimental circuitry can be performed both at room temperature (nominally 25°C) and in a small oven controlled at 50°C.

In addition, it is planned that two other promising types of semiconductor switches will be investigated during the next few months. Orders are being placed for two pairs each of Crystalonics CM602 field-effect transistors and Siliconix 2N3631 metal-oxide-silicon (MOS) field-effect transistors, and for eight Texas Instruments 2N3008 PNP planar silicon-controlled switches. Preliminary measurements on a series-shunt unifet switch, as reported in the last progress letter, indicate that very low leakage currents and relatively high switching rates are characteristic of this type of gate. The four-layer semiconductor devices also have desirable properties which should be investigated for potential applicability to analog and hybrid switching problems.

Data sheet No. 2 attached hereto presents manufacturers' performance ratings on all commercially available electronic switches known to the project at this time. In general, the manufacturer's brochure gave no statement as to the type of device or circuit configuration represented by a given model. On the basis of results achieved in project experiments to date, it is believed that one or more gates offering significantly better performance for computer applications should be possible with relatively modest effort at component matching and leakage compensation. A compilation of earlier test results and laboratory findings on various gate circuits considered under this program is currently being prepared for inclusion in a project technical note.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

Time commitments of the personnel of Group 2 have changed as of the first of the year. Present commitments, which are expected to hold for the next quarter, are the following:

R. E. Bryan - 1/2 time
Dr. D. L. Finn - 1/2 time
J. W. Petway - 1/2 time
Dr. J. L. Hammond, Jr. - coordination only
Group 2 efforts during the current period have continued in the two areas of interest—namely, (1) Error Analysis for Hybrid Computation and (2) Nonstationary Noise Studies.

In Area 1, attention is still being given to problems for which the analog portion of the hybrid computer performs linear operations and the digital portion performs generalized integration. An approach which produces an estimate of computing errors by solving a linear equation with a random forcing function is being studied as reported earlier. During the past period attention has been given to describing the random forcing function. Some general results have been obtained and several examples have been worked out, but the work is not yet at a stage which will justify a technical note. Studies in this area by Mr. Petway will continue under Dr. Hammond's supervision for the next report period.

In Area 2, Dr. Finn is investigating the basic structure of a mathematical model for use in describing random disturbances affecting a missile flight, along the lines discussed in Technical Note No. 5. During this report period an investigation of the detailed properties of this model has been initiated. The ultimate objective of this investigation is to develop the model in sufficient detail that it can be used to predict the statistical properties of a random disturbance encountered on a single missile flight selected from a large class of possible flights. Also, an investigation will be made of the possibility of obtaining a direct mechanization of this model for use in simulation studies.

Other activity in Area 2 is directed toward working several examples of increasing complexity using the synthesis procedure described in Technical Note No. 3.

Respectfully submitted,

F. Dixon
Project Director A-588

Attachments:
- Data Sheet #1 - Sample Diode Characteristics
- Data Sheet #2 - Commercially-available Electronic Switch Ratings
The schematic for the circuit is shown in the image. The values provided in the table correspond to different load conditions and are as follows:

<table>
<thead>
<tr>
<th>Load (mA)</th>
<th>Voltage (V)</th>
</tr>
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<tr>
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<tr>
<td>20</td>
<td>1.8</td>
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</table>

The table entries are listed for various load currents ranging from 0 to 20 mA, with corresponding voltage drops measured across the circuit components. The schematic diagram provides a visual representation of the circuit configuration.
## Data Sheet No. 2

### Commercially Available Electronic Switch Ratings

<table>
<thead>
<tr>
<th></th>
<th></th>
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EES 507 (5-32)
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 5, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-21473  
Covering the Period from 12 January to 12 February 1965

Gentlemen:

In accordance with the telephone request by Dr. W. K. Polstorff on 4 February, a visit to Marshall Space Flight Center will be scheduled sometime in March to discuss plans for completing the current work on electronic switches. Additional project conferences will probably be held at the time of the next Southeastern Simulation Council meeting to be held in Huntsville during April.

Group 1 Progress Summary

A series of tests was performed on a 6-diode gate incorporating G.E. IN4443 units selected from the lot of twelve received in January (see characteristics on Data Sheet No. 1 attached to the last progress letter). A similar series of tests was performed on a 6-diode gate utilizing the four good Fairchild 1N3595 units from lot "A" in place of the defective units from lot "B" previously tried. Results of these measurements are tabulated below, along with calculated values based on individual diode properties.

A more complete description of the laboratory procedures and model analysis involved will be included in the summary technical note currently being prepared for discussion at the forthcoming project conference in Huntsville during March.
<table>
<thead>
<tr>
<th>Performance Figure</th>
<th>1N4148 Gate Data</th>
<th>1N3595 Gate Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>Offset Voltage ($V_o$) - $\mu V$</td>
<td>0</td>
<td>0 $\pm$ 10</td>
</tr>
<tr>
<td>Drive Leakage ($L_D$) - $\mu A$</td>
<td>(100)*</td>
<td>10</td>
</tr>
<tr>
<td>On-Voltage Gain ($A_{v(on)}$)</td>
<td>.994</td>
<td>.989</td>
</tr>
<tr>
<td>Off-Voltage Gain ($A_{v(off)}$) (Signal Leakage)</td>
<td>$2\times10^{-13}$</td>
<td>$10^{-7}$</td>
</tr>
</tbody>
</table>

* Based on nominal difference of 0.10 nanoamp in reverse currents of Diodes 5 and 10 as measured at 50 volts. Since potential difference in actual bridge circuit was only some 20 volts, one would expect individual diode leakages and their difference to be lower, as observed.

** Based on nominal difference of 0.005 nanoamp in reverse currents of Diodes 1 and 2 as measured at 125 volts. Subsequent measurement of individual diode leakages at 20 volts gave a difference of $0.248 - 0.190 = 0.056$ nanoamp, or a revised theoretical $L_D$ value of 56 microvolts. The remaining discrepancy with the measured $L_D$ of 120 microvolts has not been accounted for.

The low-level voltage readings involved in the above tests were taken with a John Fluke model 823A differential voltmeter, having a resolution of $\pm 10$ microvolts.

Efforts during the next month will be directed towards the determination of switching-frequency limitations of the 6-diode gates and to initial tests on the new FET's and other semiconductor switch components which have been ordered.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

Group 2 activities during the current period have been divided between Area 1 - Error Analysis for Hybrid Computation, and Area 2 - Nonstationary Noise Studies. Area 2 received slightly more attention than Area 1.

Mr. Petway is continuing his studies in Area 1 under Dr. Hammond's supervision. The problem being considered has been described in earlier reports. Briefly this problem has to do with error analysis for a class of problems such that the analog portion of the hybrid computer performs linear operations and the digital portion performs generalized function generation. At the present time a study is being made of the utility of a difference equation formulation of the computation error. The possibility of obtaining tractable approximations and bounds for the error using this technique are being explored.
Dr. Finn, Richard Bryan, and Jay Schlag were active in Area 2 during the report period. Dr. Finn is continuing a theoretical investigation of a general mathematical model for use in describing random disturbances affecting a missile flight. As was mentioned in last month's report, the possibility of obtaining a direct mechanization of this model will also be explored. During the current report period the mechanization of a model for a restricted case has been studied. For the restricted case, it is supposed that a random disturbance, \( n_s(x) \), is described as a stationary gaussian process in a position parameter \( x \). The position parameter in turn is a function of time so that the resulting random process \( n_s(x(t)) \) is in general nonstationary. A method for simulating this process for slowly varying functions of time has been proposed.

This special case, although of interest for the subject contract, is only one step toward the final objective. Dr. Finn has therefore assigned this particular investigation to a Ph.D. student under his supervision, but not employed by the contract, and will devote his own efforts to the more general problem.

Mr. Bryan and Mr. Schlag have been active in a further exploration of the utility of the synthesis procedure presented in Technical Note 3. Several mathematical descriptions for covariance functions have been examined and one has been chosen for implementation on the computer. When the results of the actual simulation of this covariance function have been analyzed, a decision will be made as to the necessity for further work with other assumed covariance functions.

Respectfully submitted,

F. Dixon
Project Director A-588

FD:pc
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 6, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 February to 12 March 1965

Gentlemen:

A project conference is to be held with Dr. W. K. Polstorff at Marshall Space Flight Center on 22 March, at the time of the Southeastern Simulation Council Meeting (incorrectly mentioned in the last progress letter as being in April). This conference will be attended by Drs. D. L. Finn, J. L. Hammond, Jr., and T. M. White, with the primary aim of discussing future work on "non-stationary random processes" and new problems in "combined digital/analog computation." In view of Mr. J. W. Robertson's scheduled attendance of the IEEE National Convention in New York City during that week, a discussion of project investigations on "electronic switches" will be delayed until April.

Group 1 Progress Summary

Testing of the 6-diode gates was continued during the past month to determine their switching-frequency limitations. An upper limit of approximately 2,500 cps was found for this type of gate, whether incorporating the Fairchild 1N3595 or the G.E. 1N4143 diodes.

As suggested by Dr. Polstorff in a telephone conversation on 26 February, drive-leakage and offset-voltage measurements on the 6-diode gates were repeated using a 10-megohm load resistance instead of the 1-megohm value previously chosen as standard. The larger load resistance permitted readings to be made within the resolution capabilities of the Keithley model 610A electrometer.
The new drive-leakage data, when referred to the standard 1-megohm load, appear as follows:

- 1N3595 Gate - 120µV
- 1N4443 Gate "A" - 1.00µV
- 1N4443 Gate "B" - 1.35µV

All of these readings are considered accurate to about ± 1%. The 1N4443 Gate designated "A" above was assembled with diode units Nos. 6 and 2 in the input legs of the bridge and with diode units Nos. 10 and 5 in the output legs (see Data Sheet No. 1 in Progress Letter No. 4, 22 January 1965). The 1N4443 Gate designated "B" above had Nos. 6 and 10 as the input pair and Nos. 2 and 5 as the output pair. This latter configuration would be rated the best possible choice on the basis of matching the leakage data taken at 50 volts, at which level the expected gate drive leakage would have been 5 microvolts. The observed value of 1.35 microvolts is consistent with the fact that the actual gates were operated at a 20 volt level.

In all of the above tests, the input potentiometer setting was adjusted so as to bring the gate offset voltage \( V_0 \) to zero at the beginning of each run. However, drifts in the gate output of up to 50 microvolts (1-megohm load) occurred during the several-minutes course of the runs, these drifts being associated with variations in the balance of the plus and minus power supplies. The Lambda model 71 supplies which were used could be set to within 0.01 volt of each other and fluctuated by no more than this amount during a run. Unfortunately, however, the diode-bridge type of switch is sensitive to such power supply imbalance; a figure of 5 microvolts shift in gate offset per 1 millivolt variation between power supplies is probably typical.

As previously noted, the present 6-diode gate circuitry was designed to operate at ± 20 volts. Following a suggestion by Dr. Polstorff, consideration was given to the possibility of working in the range of ± 1 volt, as might be appropriate for certain low-level computer operations. Initial calculations showed that the 20:1 reduction in voltage would produce about a 14:1 decrease in gate drive leakage \( L_D \). On the other hand, the relative magnitude of changes in gate offset voltage \( V_0 \) due to power supply variations would be effectively 20 times greater at the lower operating level. Also, the off-voltage gain \( A_{V(\text{off})} \) would increase by a factor of \( 10^4 \) in going to the 1-volt gate. For these reasons, actual construction of a low-level 6-diode gate is not planned at this time.
A summary technical report covering our literature survey and laboratory investigations on electronic switches is still in process of preparation, but should be completed shortly after Mr. Robertson's return from the IEEE Convention in New York (22-26 March). An advance copy of the material will be forwarded to Dr. Polstorff for review prior to the proposed project meeting in April and before committing the document to actual publication.

Shipment of three Crystalonics type CM602 field-effect transistors has recently been received. These will be used to make up a high-quality series-shunt unifet gate for definitive tests during April.

### Group 2 Progress Summary

Current Group 2 activities are to be discussed in detail at the forthcoming project conference with Dr. Polstorff. The present report is intended only as an outline of Group 2 activities for the past month.

Dr. Finn has continued investigations of a mathematical model for use in describing random disturbances affecting a missile flight. In Progress Letter 5 it was reported that a procedure has been proposed for simulating a stationary, gaussian, random process \( n_s(x) \) in a position parameter \( x \). The position parameter \( x \) is allowed to be a slowly varying function of time. During the past month an alternate approach has been found for developing the simulation procedure for the resulting nonstationary random process \( n_x(x(t)) \). This alternate approach shows promise of being applicable to the general problem of simulating a random process having both time and position parameters.

As mentioned in the last progress letter, Mr. Bryan and Mr. Schlag have been active in further exploration of the utility of the synthesis procedure presented in Project Technical Note 3. During the present report period their efforts have been directed primarily toward synthesizing and analyzing the time-varying filter for a second-order covariance function. Certain small errors in the resulting nonstationary process indicate a possible malfunction in some stage of the filter implementation or measuring procedure. Attempts are being made to pinpoint the source of the errors. It is hoped that during the next report period a positive evaluation of this phase of the program can be arrived at, for purposes of planning future effort under the contract.
Mr. Petway has continued his investigations on error analysis in hybrid computation under Dr. Hammond's supervision. For a certain class of hybrid systems, an equation has been derived which gives the error of the hybrid system (as compared with an ideal analog system) in terms of the sampling period. Present effort is being directed toward solution of this equation on the B-5000 computer.

Respectfully submitted,

F. Dixon
Project Director A-588

FD:pc
15 April 1965

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 7, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 March to 12 April 1965

Gentlemen:

A project conference was held on 22-23 March at Marshall Space Flight Center between Dr. W. K. Polstorff and Drs. D. L. Finn, J. L. Hammond, Jr., and T. M. White of Georgia Tech to discuss the present and proposed work objectives of Group 1. A similar meeting between Dr. Polstorff and Messrs. F. Dixon, J. W. Robertson, and F. R. Williamson, Jr., to discuss the Group 2 program has been tentatively scheduled for the week of 19 April.

Georgia Tech project personnel wish to extend sincere sympathies to the staff at Marshall Space Flight Center over the untimely death of Mr. Fred T. Shaver. Mr. Shaver was responsible for initiating the research work in analog computation at Georgia Tech under the ABMA Computation Laboratory in early 1960. Our continuing affiliation with him under the Computation Division of Marshall Space Flight Center has been most pleasant, and his interest in Georgia Tech over the years is greatly appreciated.

Group 1 Progress Summary

Most of the available effort during this period has been applied to the preparation of Technical Note No. 6, "Investigation of Electronic Switches for Analog and Hybrid Computation," by Mr. J. W. Robertson. A clean draft copy of the document was forwarded to Dr. Polstorff on 9 April for review prior to the proposed project meeting in Huntsville later this month.
The laboratory program has been continued with the construction of a new series-shunt unifet switch using the Crystalonics CM602 N-channel field effect transistors recently received. (The previous unifet switch incorporated Siliconix 2N3380 and 2N3386 P-channel FETs which had been obtained in an experimental kit.) The construction of the new switch differs from preceding units in that polymethyl methacrylate sheet is used instead of phenol-formaldehyde paper laminate for supporting and insulating the circuitry. It is hoped that the higher volume resistivity of the methacrylate, and the greater independence of its resistivity to humidity changes, will eliminate the possibility of significant signal leakage occurring across the circuit board.

Use of the N-channel FETs requires that the drive circuitry be modified to provide complements of the driving signals used with the P-channel units. In addition, the drive circuitry voltages must be raised to permit the switch to accept higher input signal levels. These modifications are presently underway.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

As previously noted, Drs. Finn, Hammond, and White visited MSFC on 22-23 March to discuss the progress and plans of Group 2. At this meeting Dr. Polstorff requested that our group make a preliminary appraisal of the problem of error analysis for a digital sampling system involving a time-compression device. Dr. Finn, assisted by Mr. Petway, has undertaken this problem. A report on this appraisal has been prepared and will be sent to Dr. Polstorff as soon as it has been processed. It is our understanding that after the preliminary appraisal has been studied by the appropriate persons at MSFC, we will be informed as to what, if any, portion of the work on this problem is to be undertaken as a part of our Project A-588.

At the present time, efforts in the two assigned areas, Area 1 - Error Analysis for Hybrid Computation, Area 2 - Nonstationary Noise Studies, are being directed as follows.

Area 1 - Mr. Petway is continuing a digital computer study of the solution to a difference equation expressing the error in a hybrid computer solution of a class of linear differential equations. The purpose of the study is to relate error to the sampling period of the hybrid computer, and to the parameters of the differential equation. The results will be in the form of normalized tables and curves showing the appropriate relationships for the special class
of equations being studied. It is expected that these curves and tables will be of some direct interest, but the central objective is to lay the foundation for a subsequent study of solution errors for more general types of equations.

**Area 2** - Dr. Finn and Mr. Schlag have made progress in implementing on the analog computer a simulation of an elementary noise process whose parameters depend on a variable which will ultimately be the state of a space vehicle. Specifically, they are presently simulating a Gaussian random process which depends on a single parameter that varies slowly with time.

Mr. Bryan, working with Dr. Webb, is making some preliminary studies relative to using Lagrange interpolation polynomials to arrive at a procedure for studies of missile flights.

During this report period a random process with a second-order covariance function has been simulated on the computer as a further experimental test of the procedures for synthesizing nonstationary random processes with prescribed first and second moments discussed in Technical Note 3.

Respectfully submitted,

Fred Dixon  
Project Director A-588
Subject: Monthly Progress Letter 8, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 April to 12 May 1965

Gentlemen:

A project conference was held on 27 April at Marshall Space Flight Center between Dr. W. K. Polstorff and J. Spear (Flight Simulation Branch) and F. Dixon, J. W. Robertson, and F. R. Williamson, Jr. of Georgia Tech to discuss the present and proposed work objectives of Group 1. An additional visit to MSFC by Drs. D. L. Finn, J. L. Hammond, Jr., and R. P. Webb, to discuss the Group 2 program, has been scheduled for 21 May.

Group 1 Progress Summary

During the project meeting with Dr. Polstorff on 27 April, a review was made of the material to be published as Technical Note No. 6, "Investigation of Electronic Switches for Analog and Hybrid Computation" by John W. Robertson. Some minor modifications based on this review were subsequently incorporated in the report, which is now in process of being issued.

Also discussed at the above meeting were specific work objectives to be pursued during the remainder of the current contract period. It was agreed that primary consideration should be given to the six-diode bridge gate and the series-shunt field-effect transistor gate, the former being most promising for switching-integrator applications at the ±100 volt level and the latter for similar applications in the range of perhaps ±10 volts. The ultimate performance capabilities of these two switch types should be determined under conditions which closely approximate those that would be encountered in actual analog or hybrid computer systems. Critical design
factors or operating requirements should be identified, and methods of compensating for voltage offset and drive leakage should be worked out so as to maintain switching accuracy over appropriate ranges of signal amplitude and frequency. The following steps have since been taken.

Driving circuitry for the series-shunt unifet switch was completed and tests were begun on a new model of this switch using the Crystalonics CM602 N-channel field-effect transistors. Initial difficulties were experienced in getting the switch to operate, due to changing of the gate-to-channel capacitance to a value causing pinch off. Earlier models of the switch had not demonstrated this characteristic, apparently because the previously used FETs (Siliconix 2N3380 and 2N3386 P-channel types) had a much lower gate-to-channel capacitance, and could be adequately discharged through the leakage currents of the gate-to-channel diode and drive diode. Several methods of providing the necessary discharge path were tried, and found effective for the shunt transistor. For the series transistor, however, these methods did not work well due to the difficulty in providing a signal equal to the input signal without introducing error into the signal path. A method of compensating for the charge on the gate-to-channel capacitance by dumping an opposite charge onto the gate circuit was used successfully. The modified circuit is now under test.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

Drs. Finn, Hammond, and Webb have a project meeting with Dr. Polstorff scheduled for 21 May 1965. We understand that the primary purpose of this meeting is to discuss progress and plans relative to the work on nonstationary noise studies (Area 2). Other matters which may be discussed include a brief budget statement from Group 2, and our appraisal of the problem of error analysis for a special purpose spectrum analyzer. The report on this appraisal (Technical Note No. 7, "Preliminary Error Analysis of a Spectrum Measurement System," by David L. Finn) will be completed in time to have copies at the May 21 meeting.

During the present progress period, most of the group’s effort went into Area 2 (Nonstationary Noise Studies). Dr. Finn, Mr. Bryan and Mr. Schlag were active in this area on a regular basis, with Dr. Webb contributing a small amount of time. Mr. Petway was active in Area 1 (Error Analysis for Hybrid Computation) on a regular basis, with Dr. Hammond contributing in a
Subject: Monthly Progress Letter 9, Project A-586
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 May to 12 June 1965

Gentlemen:

The project conference was held on 21 May at Marshall Space Flight Center between Dr. W. K. Polstorff and members of his staff (Flight Simulation Branch) and Drs. D. L. Finn, J. L. Hammond, Jr., and R. P. Webb of Georgia Tech to discuss progress and plans for Group 2.

**Group 1 Progress Summary**

Measurements on the series-shunt unifet switch were continued this month. A series of measurements, designed to detect any variation in the parameters of the switch due to heating from the input signal, was completed with no indication that the gate parameters are dependent on the input signal. Calculations on the amount of heat dissipated in the switch, with maximum signal applied, yield a worst case dissipation of less than 2 microwatts (over a temperature range of 25°C to 75°C). Such a dissipation would cause a temperature rise of less than 0.001°C.

A system for measuring linearity was also devised. A triangular wave was generated on the analog computer with an amplitude equal to the maximum allowable signal excursion, and a slope of 0.1 volt/second. This signal was fed to the switch input and also to an inverter and the X axis of an X-Y plotter. The output of the switch, which was held in the On position, was fed to the input of a summing amplifier, together with the attenuated output of the inverter. The summing amplifier was given a gain of 100, by using...
a 1-megohm feedback resistor and a 10-kilohm input resistor. In addition, some noise rejection was provided by a ¼-microfarad capacitor in the feedback. The output of the summing amplifier was fed to the Y axis of the X-Y plotter. By careful adjustment of the attenuator and of the input resistors to the summer, and by forming the attenuator of custom-wound resistors to avoid potentiometer noise, it was possible to balance the output for a given input voltage while in the Reset mode on the computer. Then, switching to Compute gave a sweep input to the switch, with the error introduced by the switch displayed on the X-Y plotter. A linearity of 0.0008% was demonstrated by several trials at 25°C. At 50°C a linearity of 0.0016% was observed.

As a check on the above method, and to check distortion at higher frequencies, a second method was tried using a type D high-gain differential pre-amp in a Tektronix 545 oscilloscope. A frequency-compensated attenuator was constructed and placed in one input line to the type D units. The switch was placed in the other line, and both inputs were connected to a sinewave oscillator. By adjustments on the attenuator, the output components of the switch which were contained in the input signal could be balanced out, leaving only the contribution of the switch. Maximum allowable input amplitude was used and measurements were taken at several frequencies, both at 25°C and at 50°C. At 25°C, distortion was less than 0.001% (limit of measuring instruments) at 10 c/s, and rose to 1.1% at 640 Kc/s. At 50°C, distortion was less than 0.001% at 10 c/s, and rose to 1.44% at 640 Kc/s.

Laboratory work was limited to the foregoing tests due to relocation of the Special Problems Branch in the new Chemical Engineering Building (across the street from the Electrical Engineering Building, containing the Analog Computer Laboratory).

Group 2 Progress Summary

Discussions held during the project conference at MSFC on 21 May centered primarily on three topics, as briefly reviewed below.

Item 1 - Preliminary error analysis of a spectrum measurement system. Preprint copies of Technical Note No. 7, "Preliminary Error Analysis of a Spectrum Measurement System" by David L. Finn, were made available and this work was discussed. It is our understanding that the results presented in Technical Note 7 are adequate for the present, and no additional work in this area will be done at Georgia Tech until further notice from the Sponsor.
Item 2 - Budget. An informal summary of project expenditures to date and of anticipated expenditures through the contract renewal date was presented. It was agreed that the bulk of the remaining effort for the current contract period would be applied during the summer quarter, when the key personnel would be most available.

Item 3 - Nonstationary noise studies. Progress and plans in area 2 were reviewed in detail with Drs. W. K. Polstorff and M. H. Reinfurth and members of their respective staffs. Dr. Reinfurth indicated that there are several applications for techniques of generating nonstationary random processes.

Dr. Reinfurth provided measured data on atmospheric wind disturbances. This data will be used at Georgia Tech in an effort to generate signals whose statistical properties approximate those of the actual winds.

Group 2 activity during the current report period in the two assigned areas may be summarized as follows:

Area 1 - Error Analysis for Hybrid Computation: This area, which has received less attention than Area 2 during the last two quarters, will receive the majority of attention during the next quarter. Previous efforts have resulted in the identification of a class of hybrid problems for which the analog elements perform integration and the digital elements perform generalized function generation. For this class of problems, progress has been made in developing an error analysis procedure which is relatively simple to implement and which will hopefully yield useful results. During the past month, analysis work on several specific examples was completed. Preparation of a Technical Note detailing the results to date, including these examples, will begin during the next report period.

Area 2 - Nonstationary Noise Studies: The major activity in this area during the past month was concerned with preparation of a Technical Note which will be entitled "The Generation of a Gaussian Random Process in a Position Parameter." A paper with this same title (coauthored by Mr. William Yates, doctoral fellow) was presented by Dr. Finn at the Southeastern Simulation Council meeting in Atlanta on June 7, 1965.

Dr. Finn will not be employed on the contract this summer, so completion of the above Technical Note is planned for early fall.

Respectfully submitted,

Fred Dixon
Project Director
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 10, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 June to 12 July 1965

Gentlemen:

The material presented below summarizes work performed on the subject contract program during the past month by the two project groups involved. Group 1, concerned with the experimental investigation of electronic switches for analog and hybrid computation, utilizes Instrumentation Laboratory facilities of the Special Problems Branch, which for the past year and a half has occupied temporary space in the Nuclear Research Center. During June the Branch moved to larger quarters on the fourth floor of the new Chemical Engineering Building (Atlantic Drive and 4th Street). The Group 2 effort centers around the Analog Computer Laboratory, which remains on the second floor of the Electrical Engineering Building (just across the street).

Group 1 Progress Summary

Only a limited amount of experimentation could be performed during June because of the laboratory relocation noted above. Efforts were further curtailed by personnel vacations during the first two weeks in July. However, it was possible to complete the series of tests being made on the series-shunt unifet switch model incorporating Crystalonics type CM602 N-channel field-effect transistors. The tests conducted on this particular switch have been somewhat more extensive than those on previous models—including, for example, observations with the switch operating in a 75°C environment and at frequencies up to 640 Kc/s. Results obtained using the Crystalonics units are tabulated below, the methods of measuring "linearity" and "distortion" having been described in the preceding progress letter. REVIEW
Parameter Measured | at 25°C | at 50°C | at 75°C
--- | --- | --- | ---
$A_v$(on) | 0.9993 | 0.9993 | 0.9997
$V_o$ | +5 μV | +30 μV | +40 μV
$A_v$(off) | 0 | $2.5 \times 10^{-5}$ | ≤$3.1 \times 10^{-5}$
$L_D$ | ≤5 μV | ≤5 μV | ≤5 μV
Linearity (+8V range) | 0.0008% | 0.0016% | 0.003%
Distortion at 10 c/s | <0.001% | <0.001% | <0.0025%
Distortion at 640 Kc/s | 1.1% | 1.4% | 0.3%

The switch performance parameter values listed above for the paired Crystalonics CM602 FET's may be compared with those obtained for the similar switch using Siliconix 2N3380 and 2N3386 P-channel FET's as reported in Technical Note No. 6 (Table 1 on page 14). It will be noted, for example, that the On voltage gain, $A_v$(on), here is considerably better than the value of 0.95 obtained with the earlier series-shunt unifet switch. Also of interest is the reduction in drive leakage, $L_D$, from 40 microvolts for the Siliconix version to less than 5 microvolts (the limit of measuring resolution) for the Crystalonics version.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period June 12–July 12, 1965, Drs. Hammond and Webb and Mr. Bryan have been active on the subject contract on a part-time basis. Hammond is working in Area 1 (Error Analysis for Hybrid Computation) and Webb and Bryan are working in Area 2 (Nonstationary Noise Studies). Progress in each of the two assigned areas is as follows.

**Area 1** - The major effort in this area has gone into the preparation of certain sections of a Technical Note which will deal with error analysis for a class of hybrid computer problems. The following is a tentative topical outline of this Note:

1. Introduction
2. Derivation of a Linear Error Equation
3. Forcing Functions of the Linear Error Equation
4. Discussion of the Error Equation
   a. Stability
   b. Steady-State Solutions
   c. Transient Solutions
5. Examples
In Sections 2 and 3 of this Technical Note the derivation of a linear error equation is presented and the nature of the forcing functions for this equation is discussed. These two sections have been completed in draft form. Section 4 presents several approaches which result in approximate solutions of the error equation or a determination of some of its properties. This section is partially complete. Some examples for inclusion in Section 5 have been worked and others are planned.

It would be useful to include in this Technical Note several examples of problems solved on a hybrid computer along with theoretical and actual error analyses of these problems. Since Georgia Tech does not have the hybrid computer facilities needed, we plan to ask the assistance of Marshall Space Flight Center personnel in preparing the examples. A more specific request for this assistance will be made in the near future.

Area 2 - Work in this area has been directed along two lines, namely (1) a continuation of the study of the properties of the synthesis procedure for nonstationary noise processes described in Technical Note 3, and (2) an investigation of more sophisticated techniques for directly simulating the trajectories of missiles subjected to random disturbances. In the first case, efforts are being made to synthesize processes which will approximate the data provided by Dr. Reinfurth. Along the second line, a method which treats the random disturbance as a perturbation about a nominal trajectory is being investigated. This approach leads to a stochastic differential equation as the perturbation equation. Attempts are being made to solve this equation by an interpolation scheme using actual measured data.

Respectfully submitted,

Fred Dixon
Project Director

FD:pc
The material presented herein summarizes work performed on the subject contract program during the past monthly period by the two project groups involved, as identified in the preceding progress letter.

Group 1 Progress Summary

Design and construction of a Metal-Oxide-Semiconductor (MOS) field-effect transistor switch was attempted during the past month. A series-shunt switch using Siliconix 2N3631 units was first designed and constructed, along with the necessary driving circuitry. An attempt at operation resulted in the destruction of one of the units and demonstrated that the characteristics of the 2N3631 are unilateral; that is, the device is capable of operation as a field-effect transistor only with one polarity of source-to-drain voltage. Application of a positive source-to-drain voltage results in characteristics only slightly different from those of a diode and showing only a very small change with a change in gate drive. Apparently this unilateral characteristic is caused by the channel-to-substrate junction becoming forward biased. A thin-film type transistor on an insulating substrate should not demonstrate this unilateral characteristic. Isolation techniques (such as have been developed by Radiation, Inc.) could also be used to produce an MOS transistor with the channel isolated by a silicon monoxide layer from the substrate. The unilateral characteristics would be typical, however, of any unit using junction-type isolation techniques for the channel-to-substrate isolation, and having
the substrate connected to the source or drain, as most of the commercial units presently available seem to do.

A further attempt towards an MOS type transistor switch was made in the hopes of being able to utilize the higher gate-to-drain voltage ratings of these units. Series back-to-back units were used with good results for the Off (non-conducting) mode. The On mode characteristic was highly nonlinear, however, and unusable beyond 1 volt.

Neither of the above MOS switch circuits performed well enough to indicate that further measurements would be desirable. A modified series circuit might perform somewhat better and will be explored next.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 July-12 August 1965, Drs. Hammond and Webb and Mr. Bryan were active on the subject contract on a regular part-time basis. It is anticipated that Mr. Bryan will phase out of work on this contract during the current period. The division of effort between the two assigned areas was essentially the same as for last month; namely, Hammond working in Area 1 (Error Analysis for Hybrid Computation) and Webb and Bryan working in Area 2 (Nonstationary Noise Studies). Progress in these two areas is as follows.

Area 1 - A rough draft of the Technical Note outlined in the last progress letter is essentially complete. This Note discusses the problem of error analysis for a class of hybrid computer problems, and makes the contribution of describing a procedure for developing a set of error equations which can be adjoined to the original equations and solved with them on the hybrid computer. A partial evaluation of the method has been made by simulating the sample and hold effect of the hybrid computer with an analog computer. The results of this elementary evaluation are good.

For completeness it would be very desirable for the Technical Note to contain an analysis of several problems worked on a hybrid computer. Thus as mentioned in the previous progress letter, it is planned to provide the sponsor with a rough draft of the Note and a detailed outline of several problems to be worked on the hybrid computer. It is hoped that arrangements can be made for these problems to be run at Marshall Space Flight Center. Plans call for having the rough draft and suggested problems completely prepared by late August or early September.
Area 2 - As in the previous month, work in this area has been directed along two lines—namely, (a) a continuation of the study of the properties of the synthesis procedure for nonstationary noise processes described in Technical Note 3, and (b) an investigation of more sophisticated techniques for directly simulating the trajectories of missiles subjected to random disturbances.

In subarea (a) work is continuing on simulation of the data provided by Dr. Rheinfurth. The data have been reduced to the required form for implementation of a covariance-function fit program on the digital computer. This program, which will provide an analytic function representation of the data required by the synthesis procedure, is under development.

In subarea (b) a study is being made of the relationship of the stochastic-process synthesis procedure to the overall simulation problem. In addition, other possible approaches to the simulation problem are being investigated. The work is not yet at a stage which would justify a technical note, but plans call for documenting this phase of the work in the not too distant future.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 12, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 August to 12 September 1965

Gentlemen:

The material presented herein summarizes work performed on the subject contract program during the past monthly period by the two project groups involved.

**Group 1 Progress Summary**

A modified series two-transistor switch using metal-oxide-semiconductor (MOS) field-effect transistors was constructed and tested this past month. Series back-to-back units with both drains common and sources common were tried. The major modification attempted was the provision of a diode clamp to prevent a voltage larger than 0.5-0.7 volt being applied to the MOS transistor that was back-biased. Operation to levels of about 2 volts peak seemed fairly linear, but above that voltage switch leakage increased rapidly. No noticeable change occurred when the switch temperature was raised to 50°C. Due to the much better performance of the junction-type FET switch, further modifications to improve the action of the MOS-FET switch were not attempted, and no further measurements were taken.

Design and construction of a bridge-type gate using silicon-controlled-switch (SCS) units was also accomplished this month. Some difficulties were originally experienced in getting all of the units to switch on and off at the switching pulse. Use of a constant-current power supply solved this problem, however. The switch was found to be quite sensitive to the input
voltage and frequency. From dc to approximately 60 c/s, a 20-volt peak input could be switched. Above 60 c/s, the input voltage that could be successfully switched decreased with increasing frequency until 1.9 volts at 1.28 Kc/s was reached. Above this frequency the maximum input voltage was a constant 1.9 volts. The frequency-sensitive area between 60 c/s and 1.28 Kc/s is due to rate effect in the SCS's—i.e., when the input signal varies rapidly, the anode-to-gate capacitance delivers a charge into the gate that is proportional to the rate of change and the amplitude of the input signal. This charge may be sufficient to cause the units to switch on. The gate has a minimum charge sensitivity, below which a charge will not cause conduction. Thus, above 1.28 Kc/s no reduction in input voltage is necessary, due to the decreasing time allowed for the delivery of charge. Because of its marked frequency sensitivity, the SCS gate was not considered a practical possibility for computer applications, and no further measurements were made on it.

The foregoing experiments essentially complete the planned study of electronic switches. A summary of results not previously covered in Technical Note No. 6 is now being prepared. Recommendations for continuing effort include (a) redesign of driving circuitry for the six-diode bridge gate so as to provide offset-voltage and leakage compensation, and (b) construction of an improved experimental generalized analog integrator (utilizing the previously developed analog-to-incremental-digital conversion scheme) which would incorporate the optimum electronic switch designs as determined from the present program. Since currently available funds will not permit further experimental effort at this time, the recommended work must await renewal of the contract.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 August-12 September 1965, Drs. Hammond and Webb and Mr. Bryan were active on a regular part-time basis. Mr. Schlag conducted portions of an analog computer study. Mr. Bryan terminates his activity on the subject contract as of the end of this report period to assume a position of more responsibility in another area at Georgia Tech.

Progress in the two assigned areas is as follows.
Area 1 - Error Analysis for Hybrid Computation

Dr. Hammond provided the principal effort in this area, assisted for a short time by Mr. Schlag in solving a problem on the analog computer. Final processing of the rough draft of Technical Note No. 8 was completed.

A test problem for solution on the hybrid computer was chosen and evaluated on the analog computer. The problem chosen is a second-order nonlinear equation sometimes referred to as the pendulum equation. This equation seems to be a desirable choice because it can display either linear or nonlinear behavior depending on the initial conditions. This problem has been solved for several choices of initial conditions and parameters on an analog computer with sample-and-hold circuits added to simulate hybrid effects. The approximate error equation was solved on the computer at the same time, and the results show that the solution of the approximate error equation compares favorably to the true error for both linear and nonlinear behavior.

The rough draft of Technical Note No. 8 and a description of the test problem were sent to the sponsor.

Area 2 - Nonstationary Noise Studies

Dr. Webb and Mr. Bryan were active in this area on a regular part-time basis. Mr. Bryan has completed numerical and experimental work (identified as Subarea (a) in the last progress letter) having to do with use of the synthesis procedure of Technical Note No. 3 to generate a stochastic process approximating static wind data. Dr. Webb will evaluate this material and report the results in a Technical Note to be issued this Fall. Since Bryan's work was completed, we have received a report from the sponsor containing more accurate wind data than was previously available to us. It is not likely that the new data can be used in Webb's report.

Dr. Webb has continued his investigations in what was identified in the last progress letter as Subarea (b). He is conducting a preliminary study of the use of perturbation techniques in determining the trajectories of rockets subjected to random disturbance. The procedure involves separating a trajectory into a deterministic part and a random perturbation about the deterministic part. Equations for both parts of the trajectory have been established, with interest centering on the equation for the random perturbation.
Two approaches to the solution of the stochastic perturbation equation are being considered—namely (1) a machine solution which yields on each run a single sample function from the solution process, and (2) a direct solution for either certain moments of the random perturbation or the probability that the perturbation variable is in a certain range.

Plans for the Remainder of the Contract Period

As indicated in previous discussions with the sponsor, project funds were expended at an accelerated rate in the early part of this year when academic personnel were most readily available. During the next three months, activity will be curtailed to a considerable extent. Dr. Hammond will complete Technical Note No. 8, Dr. Finn will prepare a technical note to be entitled "The Generation of a Gaussian Random Process in a Position Parameter," and Dr. Webb will prepare a technical note to be entitled "On the Simulation of Trajectory Equations with Random Disturbance" covering his work and that of Bryan. No further generation of research results can be accomplished until further funding is received.

Respectfully submitted:

F. Dixon
Project Director, A-588
Gentlemen:

The material presented herein summarizes work performed on the subject contract program during the past monthly period by the two project groups involved.

Group 1 Progress Summary

As mentioned in the preceding progress letter, the experimental study of electronic switches which was planned under this program has been carried to completion. A technical report summarizing results not previously covered in Technical Note No. 6 has been prepared and should be issued during the next monthly period. Currently available funds will not permit further experimental work to be performed under the present contract.

A proposal is being submitted to extend the contract for an additional year—until 12 December 1966. The primary objective of Group 1 under this extension would be to construct and evaluate an improved experimental generalized analog integrator, based on the previously developed analog-to-incremental-digital (AID) converter but incorporating the optimum electronic switch designs as determined from the present program.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 September - 12 October 1965, Drs. Hammond, Finn, and Webb were active on a part-time basis, with minor assistance from Mr. Schlag.
For reasons previously discussed, relatively little effort is being applied to this program during the final quarter of the current contract period. The principal activity for the past month has been (and will continue to be) the preparation of project technical notes. The status of work in the two assigned areas of research is as follows.

**Area 1 - Error Analysis for Hybrid Computation**

Plans call for incorporating the results of a test problem, currently being processed by Marshall Space Flight Center, with the present content of the rough draft of Technical Note No. 8 before issuing the Note in final form. It is hoped that MSFC results can be obtained in time to complete the report by the end of the present contract period.

**Area 2 - Nonstationary Noise Studies**

Dr. Finn has started preparation of a technical note to be entitled "The Generation of a Gaussian Random Process in a Position Parameter." This report will describe a procedure that may be used to synthesize an analog computer circuit which produces a Gaussian process having a prescribed expected value and autocorrelation function. The random process that is generated depends on a single position parameter. The position parameter, in turn, is allowed to vary with time. The synthesis procedure to be described in the report represents a preliminary step in research directed toward the development of a method for the generation of a random process that simulates the random wind disturbances encountered by a rocket in flight. Plans call for finishing this technical note by the end of the present contract period.

Dr. Webb, assisted by Mr. Schlag, has been concerned with the use of the synthesis procedure of Technical Note No. 3 to generate a stochastic process approximating static wind data. Mr. Bryan has completed the first stage of reducing the then available experimental wind data to a form which is compatible with the synthesis procedure. Mr. Schlag is now using some of this reduced data in the procedure of Technical Note No. 3 to generate a random process which has first and second moments approximating that of the static data.

Dr. Webb plans to prepare a technical note covering this work. It was indicated in the last progress letter that Webb would issue this report in late
fall. However, since new wind data have since been received from Marshall Space Flight Center, it may prove desirable to delay the report until the new data can be included.

Respectfully submitted:

F. Dixon
Project Director, A-588
GEORGIA INSTITUTE OF TECHNOLOGY
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ATLANTA, GEORGIA 30332

November 19, 1965

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 14, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from October 12 to November 12, 1965

Gentlemen:

The material presented herein summarizes work performed on the subject contract program during the past monthly period by the two project groups involved. A proposal was submitted on 29 October to extend the contract for an additional year from the present termination date of 12 December 1965.

Group 1 Progress Summary

Technical Note No. 9, "Supplemental Investigations on Electronic Switches for Analog and Hybrid Computation" by John W. Robertson, has been completed but not yet reproduced for distribution to the sponsor. This will be accomplished before the end of the present contract period. No further experimental work is contemplated until the contract renewal takes effect.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 October--12 November 1965, Drs. Finn and Webb each contributed a small amount of time to the project. Dr. Finn has completed the rough draft of Technical Note No. 10, entitled "The Generation of a Gaussian Random Process in a Position Parameter," and will begin preparation of the final draft immediately.
Currently available funds are not sufficient for Dr. Webb to complete his report on the generation of specified random processes using the procedure described in Technical Note No. 3. It is therefore planned to delay and subsequently expand this report so as to include new results based on the wind data received from MSFC earlier this fall.

Further work on Technical Note No. 8, "A Study of Error Approximations for Hybrid Computers" by J. L. Hammond, Jr., has been postponed until results of the test problem being processed by MSFC are received.

Respectfully submitted:

F. Dixon  
Project Director, A-588

-2-
April 18, 1966

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letters 15, 16, and 17; Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473, Request No. DCN 1-6-70-00118
Covering the Period from 13 December 1965 to 12 March 1966

Gentlemen:

Pending negotiation of the 12-month extension to Contract NAS8-2473
as requested in our proposal of 29 October 1965 and now authorized by Contract
Modification No. 7 (executed on 11 March 1966), there were no progress letters
submitted for this project following the previous termination date of 12
December 1965. However, a limited amount of effort was maintained on certain
of the established technical objectives in anticipation of continuing the
program. The present report therefore covers the status of work performed
within the two project groups during the three monthly periods from 13
December 1965 to 12 March 1966.

Group 1 Progress Summary

Project A-588 Group 1 comprises personnel assigned to the Special
Problems Branch Instrumentation Laboratory, a unit of the Engineering Experiment
Station's Physical Sciences Division. This group is responsible for
item 3 of the current contractual Work Statement:

"Laboratory efforts by the contractor will be devoted to the
construction and evaluation of an improved generalized analog
integrator based on the previous experimental EGI design but
incorporating optimum series-shunt field-effect-transistor
switches or six-diode bridge gates as determined from the study
program just completed."

The bulk of the effort on this task will continue to be provided by Mr. John
W. Robertson (Assistant Research Engineer), with minor consulting and technical
supervision from Mr. Frank R. Williamson, Jr. (Research Engineer).
During the period covered by this progress letter, the basic design of the Electronic Generalized Integrator (EGI) was examined in detail to determine where improvements might be made over the original experimental version (described in the 4th Quarterly Program Report and Annual Technical Summary, by F. R. Williamson, Jr. and F. Dixon, 12 November 1962). Several alternative methods of providing the basic timing for the unit were considered, but as none of these seemed to offer any significant advantage, it was decided to continue using a clock oscillator as in the previous unit.

A check was performed on the available Pace 6.282 operational amplifiers to determine if the original EGI had utilized this amplifier to the fullest extent. It was found that the amplifiers could respond to 5 millisecond pulses. Available equipment did not allow tests with faster pulses. Using the maximum output current of the amplifier it was found that a pulse 15 milliseconds long was required to reset the amplifier output to zero. By employing a non-symmetrical pulse with an "on" time of 15 milliseconds and an "off" time of 5 milliseconds, it should be possible to use a 2 microfarad feedback capacitor and thus reduce the input impedance to provide lower drift.

Further improvement should be realized by employing FET switches to provide the reset pulses and sample pulses. The FET switching circuit allows a single input resistor to be used, thereby lowering the drift gain by a factor of two.

Some small effort was exerted in an attempt to find better amplifiers, however, the best potential alternative uncovered was only about an order of two better with respect to drift, and its lower bandwidth made this advantage dubious. The Pace 6.282 amplifiers are therefore being retained.

The changes to a larger feedback capacitor, a non-symmetrical reset pulse, and a single input resistor for each amplifier should provide an overall performance improvement factor of eight. (This assumes the performance not to be otherwise limited by the switches used.) Any such overall improvement may be utilized as increased bandwidth or decreased drift.

In addition to the improvements in basic design mentioned above, it is expected that significant reductions in gate offset and temperature effects will be realized by the use of FET-type switches (Amelco 2N4091 or equivalent).

- 2 -
Integrated circuits have been chosen for the necessary digital logic functions in the new EGI unit, in order to reduce the design time required and also to provide increased reliability, high operating speed, and low cost. Several types were investigated before settling on Fairchild industrial planar epitaxial microcircuits.

As a temperature-controlled environment is needed for the FET switches and the $\Delta X$ detectors, and since the leakage of the FETs would be increased by a high temperature, it was decided to operate them in a low-temperature enclosure. Frigistors Ltd. type 1FB-32-015-G1 solid-state Peltier-effect cooling modules have been chosen to provide the necessary cooling in conjunction with a proportional control circuit. The proportional controller will be of basically the same design as used in the preceding EGI oven, except for the inversion in gain. Also, silicon transistors will be used in the control circuit in order to reduce the size of the heat sink required.

In an attempt to provide some improvement in the $\Delta X$ detectors, Fairchild type U5B771039X differential comparators have been ordered for this application. These are integrated circuit units which will be temperature-controlled.

The time-base oscillator to be used is a commercial tuning-fork unit (Accutronics JJ series) with a frequency of 200 kHz.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period from 13 December 1965 to 12 March 1966, Group 2 research activities were conducted primarily by Dr. Hammond, who was active for most of this period on a half-time basis. During the Spring Quarter (April-June), it is planned that both Dr. Finn and Dr. Hammond will be active one-half time. Progress and plans in the two assigned areas for Group 2 are as follows.

Area 1 - Error Analysis for Hybrid Computation

Technical Note No. 8 presents a detailed discussion of the theoretical aspects of a method of error analysis based on adjoining a set of error equations to the problem equation and solving the complete set of equations on the computer. Dr. Hammond's efforts during the past three-month period have been devoted to formulating and implementing plans for evaluating utility of
the theoretical error analysis procedure in solving practical problems with a hybrid computer. Several methods of evaluating the error analysis procedure have been given preliminary consideration and the area which will receive the major emphasis during the next report period has been identified.

The ultimate evaluation of the error analysis procedure must be made with hybrid computers in actual practice. As a first step in this type of evaluation, an elementary test problem has been formulated and submitted to MSFC for solution. The results obtained from this test problem will determine the next step in this type of evaluation.

A second method of evaluation using a digitally simulated hybrid computer has been formulated and this method offers several advantages. General purpose programs for simulating a hybrid computer with a general purpose digital computer have been developed by several computing centers primarily for use in obtaining check solutions for hybrid work. For solving production problems of practical magnitude, such programs have two serious limitations—namely, excessive computing time, and limited man-machine communication. However, for evaluating the error analysis method using small- and medium-scale problems, these limitations are relatively unimportant.

A general-purpose simulation program identified as SLASH has been developed in the Algol language by Captain James E. Funk of the USAF Academy. Captain Funk has been kind enough to supply Georgia Tech with the magnetic tape necessary to implement SLASH, and during the past report period this program has been successfully checked out on the Burroughs B5500 computer at Georgia Tech using an elementary problem. By using a Runge-Kutta integration procedure and a step-size adjustment, SLASH makes it possible to program a wide class of problems as for an analog computer, and to solve these problems with negligible errors due to sampling rate and execution time.

Efforts are now under way to effect modifications to the program so as to make it possible to simulate directly the effect of sampling rate and execution time in a true hybrid computer. A general simulation program has been planned to accomplish the following:

(1) Solve a differential equation of the form

\[ \dot{x}(t) = f(x,y,t) \]

with negligible error due to sampling rate and execution time.
(2) Solve the same differential equation with a specified sampling rate for the simulated hybrid computer and specified execution times for each operation performed digitally on the simulated hybrid computer.

(3) Solve the approximate error equation along with the equation of item (2).

(4) Compute the actual error as the difference between the solution of items (1) and (2).

(5) Compute the difference between the approximate and actual errors as a function of time.

Plans call for using this general simulation program with a variety of typical examples to study approximate error as a function of actual error for various sampling rates and execution times. The simulation program can also be used to evaluate the effect of various allocations of the computing operations between the analog and digital portions of the simulated hybrid computer, and a study of this sort is proposed for later work.

To make the evaluation studies of maximum value, it would be helpful if the sponsor could provide several typical problems.

Some attention has been given to a third, theoretical method of evaluating the error analysis procedure; however, results from this approach are as yet inconclusive.

Area 2 - Nonstationary Noise Studies

As described in Technical Note No. 10, a synthesis procedure for determining an analog computer program to generate a prescribed Gaussian random process depending on a single position parameter has now been developed and tested. This work is a preliminary part of the problem of practical interest to the contract, namely that of generating a prescribed Gaussian random process depending on a single position parameter and time. Using the results of the preliminary study, emphasis will now be shifted to the practical problem. Indications are that much of the preliminary work will be applicable to the more general case.

It seems desirable to examine and use the experimental wind data given in NASA Technical Memorandum NASA TMX-53290, dated 9 July 1965, early in the study of methods for generating random processes depending on a position parameter and time. Thus, Dr. Finn will give some attention to this data during the next quarter.
The report under preparation by Dr. Webb which was referred to in Monthly Progress Letter No. 14 (19 November 1965) will be delayed until the results of Dr. Finn's study of the new wind data are available. At that time (possibly early summer) a technical note covering generation of random processes conforming to the wind data will be issued. Since our work has now resulted in at least two different methods of generating the required random processes, it seems desirable that this technical note contain results and a comparison of both procedures.

Respectfully submitted,

F. Dixon
Project Director, A-588
22 April 1966

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-DG

Subject: Monthly Progress Letter 18, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NASA-2473, Request No. DGN 1-5-70-00118
Covering the Period from 13 March to 12 April 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project groups involved.

Group 1 Progress Summary

Orders have been placed for various commercial components and parts to be included in the new Electronic Generalized Integrator (EGI) as noted in the previous progress summary. Pending delivery of these items, effort has been applied to the detailed design of the temperature controlled enclosure for the PET switches and ΔX detectors. A value for the thermal leakage of an enclosure insulated with polystyrene foam was needed, and this value was derived from the oven data of the original EGI. On the basis that two frigistors would be used to cool the entire enclosure, a worst-case frigistor current of 10 amperes at 3 volts was calculated—calling for a total input power of up to 60 watts. Wakefield type MN-2131-6.0-K heatsinks have been chosen, together with type MN-2159-6.0 auxiliary radiator inserts, to provide the very low thermal resistance and fairly large mounting surface.
required for each frigistor. The heatsinks will be cooled by natural convection.

The two frigistors, mounted on their heatsinks, are to be located at opposite ends of the temperature controlled enclosure, which will be some six inches long so as to ensure clearance between the back-turned fins of the heatsinks. The core of the enclosure will consist of a solid copper bar, the central portion being a 3-inch cube into which the FETs and differential comparators are sunk, and the ends tapering down as truncated four-sided pyramidal sections to the frigistor cooling surfaces. Surrounding the bar on all sides will be styrofoam with a minimum thickness of one inch. The components to be operated in the temperature controlled environment will be inserted with heat-conducting paste in close-fitting holes symmetrically located in the central core, so as to minimize temperature differences and achieve as low a thermal resistance as possible. The use of copper (whose thermal conductivity is within six percent of that of silver and is almost twice that of aluminum) should result in quite small temperature gradients through the interior of the enclosure. The nominal core temperature will be sensed by means of a single thermistor probe inserted at the center of the bar, whose changes in resistance will serve to control the frigistor cooling rate.

Drive current is to be supplied to the two frigistors in series by an RCA type 40251 npn silicon industrial transistor. (Several alternative transistors could have been chosen equally well.) The maximum operating junction temperature of this transistor is 200°C—which figure, coupled with a low junction-to-case thermal resistivity, allows use of a relatively small heatsink. A Wakefield type NC-423-A, with mounting adapters to permit orienting the sink in a vertical position, has been placed on order for this purpose. The use of mounting insulators will permit the transistor to be placed
directly in contact with the heatsink, giving minimal thermal resistance.

The output of the 200-kilohertz clock oscillator will be fed to a microcircuit binary counter, and the outputs of the various stages will gate the nonsymmetrical reset pulses described in the last progress letter. By using the high frequency oscillator and counting down to provide the timing, the capability of adjusting the reset pulse frequency and shape is obtained, allowing easy modification if necessary.

**Group 2 Progress Summary (reported by J. L. Hammond, Jr.)**

During the period 13 March to 12 April 1966, the Group 2 project staff consisted of Drs. Finn and Hammond, each devoting approximately one-half time (Dr. Hammond 5/12 time.) Progress in the two assigned areas is as follows.

**Area 1 - Error Analysis for Hybrid Computation**

During the past report period primary attention in this area has been given to the digital program for simulating a hybrid computer. The specific problem receiving the most attention has been that of adding to the basic SLASH program the capability of simulating digital delay and execution times. The task of developing and checking out this program is not yet completed but progress and checks of preliminary work have been satisfactory.

As discussed briefly in the last progress letter, the digital simulation program being developed will make possible studies of the effect of digital delay and execution time for various allocations of the computing operations between the analog and digital portions of the simulated hybrid computer. The ability of the digital simulation program to generate a "true solution", in the sense of negligible error from digital delay and execution time, can give this method an advantage over studies made on an actual hybrid computer. Thus, the program being developed is expected to provide a useful general tool to supplement other methods of studying hybrid computer errors.
On another aspect of the problem, some attention was given to surveying problems which represent typical applications of hybrid computation. Among others, the restricted three-body problem would seem to be an appropriate one for use in evaluating error analysis procedures.

**Area 2 - Nonstationary Noise Studies**

In this area, Dr. Finn has initiated a study directed toward his development of a procedure for synthesis of an analog computer circuit to simulate random wind disturbances depending on time and altitude. Two possible approaches are being investigated—namely, (1) an extension of the procedure for approximate realization of a one-parameter random process as described in Technical Note No. 10, and (2) a possible generalization of the synthesis procedure described in Technical Note No. 3. Both procedures would simulate the random wind disturbance process without the necessity for an a priori assumption as to the trajectory of the vehicle being simulated.

Respectfully submitted,

F. Dixon  
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama  35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 19, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473, Request No. DCN 1-6-70-00118  
Covering the Period from 13 April to 12 May 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project groups involved.

Group 1 Progress Summary

Design efforts on a driver for the 2N4091 Field Effect Transistor switches have occupied a major portion of this report period. A single driver is to drive three FETs functioning as a single-pole three-position switch. These switches will be used both to provide reset pulses and to sample the Y input. Inputs to the switch drivers will be the + ΔX and - ΔX signals from the integrated-circuit flip-flops. A nand gate internal to the driver will provide the signal to drive the FET connecting the amplifier input to ground into conduction during the absence of any input signal. A voltage swing of + 16 to - 26 volts is to be provided to the FETs on the Y input, allowing a ± 14 volt swing on the input. Transistor types 2N697, 2N2102, and 2N4037 are to be used in the driver.

An integrated-circuit (IC) 4-stage binary counter was also constructed and tested during this report period. The construction utilized a Triad Distributor Division printed-circuit card (No. 13540), as will all further IC assemblies, wherever practical. The card will be plugged into a socket mounted on the main chassis of the EGI unit. This counter is to be driven from the clock oscillator (a 200 kHz Accutronics series JJ) which has a 15-volt
square wave output with a 10 kilohm load. A capacitively compensated voltage divider was used to attenuate the drive voltage to the proper amplitude (approximately 1 volt) to drive the integrated circuits. Considerable sensitivity to both the square wave rise time and amplitude was observed. Probably the divider will have to be readjusted for proper operation with the actual clock oscillator when available. The straight binary counter was implemented by connecting the "1" outputs of the flip-flops to the "Toggle" input of the following flip-flop. By changing the connections between stages and adding feedback, decade, duodecimal and other counters can easily be made from the flip-flops used, in case the need should arise for changing the pulse repetition rate.

By using the "0" outputs of the flip-flops in the counter described above as inputs to a four-input gate, a timing signal and its complement can be derived. This signal is high for five microseconds and low for seventy-five microseconds. These timing signals are brought off the card and will be buffered and used to provide timing to all other parts of the unit. Changes in the timing-signal pulsewidth can easily be accomplished by changing the inputs to the gate. All interconnections are made with wire jumpers, so that modifications are readily accommodated.

**Group 2 Progress Summary** (reported by J. L. Hammond, Jr.)

During the period 13 April 1966 to 12 May 1966, the Group 2 project staff consisted of Drs. Finn and Hammond, each devoting approximately one-half time. This same staff and time allotment is planned for the next report period.

A conference with the sponsor is planned for May 25th in Huntsville. Recent progress in the two assigned areas and plans for the remainder of this contract year will be discussed at this meeting.

Progress in the two assigned areas during the present report period is as follows.

**Area 1 - Error Analysis for Hybrid Computation**

Work on modifying and checking out the digital computer program for studying a hybrid computer on a digital computer has continued and is now almost completed. The essential features of this program were summarized in Monthly Progress Letter 18 for the last report period.
Some attention has been given to certain other aspects of the problem. For example, the theoretical error analysis procedure has been adapted so that it applies directly to cases for which the logical computing operations are divided between the analog and digital portions of the hybrid computer in large sections, rather than having digital elements in the feedback loop of each integrator. This case is felt to occur frequently in practice.

Plans have also been made for studying various allocations of the computing operations between the analog and digital portions of the hybrid computer in simple problems. These studies will be carried out using the digital simulation program when it has been perfected.

In addition, some preliminary work has been done on the problem of approximating the effect of round-off errors in such a way that these errors can be included in the error analysis at some later date.

**Area 2 - Nonstationary Noise Studies**

In the last progress letter it was noted that studies are being made of two approaches to the problem of simulating random wind disturbances on a space vehicle without an a priori assumption as to the trajectory of the vehicle. Preliminary results seem to indicate that both methods are feasible. Efforts are now being made to synthesize circuits for simulating an elementary random process using both techniques. It is intended to implement one, or if time permits, both of these circuits on the analog computer.

Respectfully submitted,

F. Dixon
Project Director, A-588
Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project groups involved.

Group 1 Progress Summary

Construction of the FET switch driver was initiated during the current report period, and has been carried to completion except for installation of the pnp silicon transistors. After further consideration of the requirements on these transistors, it was decided that the 2N4037s (reported last month as the choice for this unit) should be replaced by 2N4036s, which have a somewhat higher breakdown voltage rating. These transistors are now on order, and checkout of the driver unit awaits only their delivery.

Additional progress was made in constructing the portion of the frigistor control amplifier external to the temperature-controlled enclosure. Due to the higher power requirements of the frigistors, increased gain was required in the amplifier. This was supplied by an additional emitter-follower stage using an RCA 40250, inserted before the 40251 output stage. The extra stage is mounted on the same heat sink with the 40251. Since suitable resistors are not on hand to serve as a dummy load, checkout of this amplifier will be postponed until the copper interior of the temperature-controlled...
enclosure is available to provide the mechanical attachment of the frigistors to their heat sinks. The finished interior will also allow completion of the first two stages of the amplifier as well as installation of the thermistor probe sensor.

The A and B relay logic in the original EGI unit was designed to function with the slave outputs from the Berkeley EASE computer then available. This logic will be replaced with new logic designed to operate with the Donner 3100 analog computer now in use, without the necessity for a converter.

Mechanical work on the chassis of the main unit is proceeding well. However, the final selection and arrangement of front-panel controls has not yet been settled.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 May 1966 to 12 June 1966, the Group 2 project staff consisted of Drs. Finn and Hammond, each devoting approximately one-half time. During the next report period and for the duration of the Georgia Tech Summer Quarter, the Group 2 staff will consist of Dr. Hammond one-half time and Mr. J. H. Schlager (Graduate Research Assistant) approximately one-quarter time.

A conference was held with the project officer in Huntsville on 25 May 1966 to review progress and plans in both of the Group 2 areas of investigations. Several possible directions for the error analysis study were suggested by Dr. Polstorff at this meeting and have been largely incorporated into the planned activities.

Progress for the month in the two assigned areas has been as follows:

Area 1 - Error Analysis for Hybrid Computation

Activity in this area during the report period continued to be centered around solving specific problems using a digital computer to simulate the hybrid computer, as discussed in previous reports. Most of the work done to date pertains to the equations

\[
\begin{align*}
    x_1 & = x_2 \\
    x_2 & = - \sin x_1 - x_2
\end{align*}
\]
which was chosen as a first test case.

For this problem, work done at Georgia Tech and hybrid computer solutions prepared by MSFC both indicate that under most conditions the approximate error equations discussed in Technical Note 8 provide an adequate approximation to the true error.

Some further work is planned with these equations to evaluate the effect on true and approximate error of various allocations of the computing operations between the analog and digital equipment.

Following Dr. Polstorff's suggestion, the next test example to be chosen will be one involving coordinate transformations and Euler Angles. Emphasis will be placed on the use of the digital computer as a generalized function generator.

Some preliminary attention has been given to sensitivity studies which may be helpful in solving problems with almost periodic solutions and light damping.

Area 2 - Nonstationary Noise Studies

During this report period the two methods previously discussed for synthesizing an analog computer network to simulate random wind disturbances of a booster trajectory continued to receive the majority of attention. The investigation has now progressed to the stage of preparing a rough draft of a technical note covering the work to date. It is expected that the rough draft will be completed during the next report period and the final draft shortly thereafter.

Plans call for the utilization of experimental wind data in future simulation of random wind disturbances. Although a study of the available data has been started, it has not progressed far enough to be included in the report now being prepared. This study will be continued by Dr. Finn when he resumes work on the contract during the Fall quarter.

Respectfully submitted,

F. Dixon
Project Director, A-588
Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved.

Group 1 Progress Summary

Due to illness and summer vacation, Mr. John W. Robertson, the engineer responsible for design and construction of the new experimental EGI system, was able to spend only about 60 percent time on this development during the past month. However, the effort as a whole seems to be progressing satisfactorily and within the original time and budget estimates.

During the present report period, the desired copper stock was procured for making the "cold plate" interior of the temperature-controlled enclosure in which various critical semiconductor components will be housed. Machining and drilling operations to provide mounting facilities for the components and suitable surfaces for connection with the frigistor units are currently underway.

The modified relay logic mentioned in the last report has now been installed and checked out. Some modification of the switching which provides the "reset" and "hold" functions on the operational amplifiers will be necessary. The required modification appears elementary at this point.

Selection of the front-panel controls has been completed, and the controls themselves have now been mounted. Mechanical work on the chassis is virtually complete, and component wiring has been started.
Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 June to 12 July 1966, the Group 2 staff consisted of Dr. Hammond at 1/2 time, Dr. Finn on a small part-time basis, and Mr. Schlag at approximately 1/4 time. Progress for the month in the two assigned study areas has been as follows.

Area 1 - Error Analysis for Hybrid Computation

The present phase of the study in Area 1 is concerned with hybrid computer error due to execution time and nonzero sampling intervals. The objective of this phase of the study is a theoretical formulation and an experimental evaluation of several error-analysis techniques, all designed for implementation on a computer.

The basic procedure involved was presented theoretically in Technical Note No. 8, and has been successfully applied in one test example. During the past month, several possible modifications and extensions of the basic procedure were investigated and the theoretical development of one modification was completed. This modification makes possible a calculation of hybrid system error using an all-analog or all-digital computer. The result is presented as an approximation to the error as a function of the sampling interval. The method is particularly adapted to a digital-analog simulation such as the SLASH program being used at Georgia Tech, or one of the IBM programs such as MIMIC or the new 1130 CSMP or CSMP/360. It should be noted that this particular method differs from another technique being actively investigated which requires a simulation of the hybrid computer by the digital equipment.

The all-analog or all-digital error analysis technique is currently being evaluated with the first test and will be applied in other test cases as the work progresses.

During the present report period a second general test problem was developed. This problem involves expressing the angular velocity of a vehicle about its body axes in terms of Euler angles using the equations

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix} = \frac{1}{\sin \theta} \begin{bmatrix}
\sin \phi & \cos \phi & 0 \\
-\sin \phi \sin \theta & -\cos \phi \cos \theta & \sin \theta \\
\cos \phi \sin \theta & -\sin \phi \sin \theta & 0
\end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}
\]

Current plans are to employ several different types of input velocities in evaluating the error analysis techniques with this example.

A method is also being developed for utilizing the results of an error analysis of the above equations to estimate the error in solving certain more complicated equations. These more complicated equations would be programmed on a hybrid computer so that a portion of the program (that containing the above equations) is solved digitally while other parts of the program are solved with analog equipment.

Area 2 - Nonstationary Noise Studies

During the past month, a rough draft of a technical note presenting two methods of synthesizing an analog computer network to simulate random wind disturbances has been completed. Preparation of the final draft of this technical note is now being undertaken.

Respectfully submitted

F. Dixon
Project Director, A-588
22 August 1966

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 22, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from July 12 to August 12, 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved.

Group 1 Progress Summary

Machining of the "cold bar" interior of the EGI temperature-controlled enclosure has been completed and all components requiring temperature stabilization have been mounted on the completed bar. Wakefield Engineering heat conductive compound was used between the various pieces of the cold bar, and between the semiconductors, their mountings, and the cold bar, in an effort to keep the thermal resistance as low as possible. Some interconnections of the components mounted on the cold bar have been possible within the insulated enclosure, by wiring directly between components with solid wire and sliding insulating tubing over the wire to avoid shorts to the cold bar. Connections which must be made outside the insulated enclosure utilize stranded wires attached to the components and led through the thermal insulation and chassis base to their various destinations. All interconnections on the bar itself are completed and leads have been attached to the points requiring outside connections. Only the installation of the cold bar in its insulated enclosure, along with the frigistors, and the connection of internal and external portions of the temperature control amplifier are necessary before checkout of the cold bar.
Arrival of the necessary transistors has allowed the gate driver card to be completed and checked out. Some problems with intermittent connections were encountered but have now been solved, and the card seems to be in good working order.

Two neon drivers were designed and built to operate the front panel indicators from the gate driver. The front panel lights will indicate the presence or absence of a pulse and its polarity, just as in the earlier EGI.

It was determined that the output of the 200 kHz clock oscillator was not a true square wave as had been stated in the specifications for the unit. Since a square wave had been assumed in the design and testing of the counter which is to operate from this oscillator, a squaring circuit was designed and constructed to improve the wave shape. A Schmitt trigger and a transistor switch were used, mounted on the same card as the temperature control amplifier. Checkout has been completed on this supplementary circuit.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 July to 12 August 1966, the Group 2 staff consisted of Dr. Hammond at 1/2 time and Mr. Schlag at approximately 1/4 time. Dr. Hammond was away from the campus on a two-week vacation during the report period. Progress for the month in the two assigned areas is as follows.

Area 1 - Error Analysis for Hybrid Computation

A project technical note discussing theoretical aspects of the error analysis study is being prepared. This note will discuss error analysis techniques which are extensions of the work reported in Technical Note No. 8. Specifically, the following topics will be discussed:

1. A computational error equation requiring as inputs the approximate values of the problem variables as computed with a hybrid computer.
2. A computational error equation requiring as inputs the "true" values of the problem variables. This equation makes it possible to estimate hybrid error by solving the problem and error equations on an all-analog or all-digital computer.
3. An error equation applying to computational procedures using digital feedback around some, but not all, analog integrators.
4. An error equation expressing error directly as a function of the sampling interval. This equation can be solved by any appropriate method--analog, digital, or hybrid.
Plans call for evaluating each of these four computational procedures with at least the two test examples discussed in previous reports. Numerical results have already been obtained for the equations in items (1) through (3) above using the pendulum-equation problem. The results so far indicate that the methods are all valid.

Work now in progress will produce numerical results for the equation of item (4) using the pendulum equation and will begin similar computations for the more complicated Euler angle problem given in the last monthly progress letter.

Preliminary work with problem equations of the complexity of the Euler angle equations indicates that the SLASH program uses an excessive amount of computing time. During the current report period, therefore, a new digital program possessing all of the desirable features of the SLASH program has been devised by Mr. Schlag. This new program will require on the order of one-fifth the running time of SLASH on a typical problem, and will also have certain programming advantages.

A technical note covering work with the pendulum equation and Euler angle examples will be issued as soon as the work is completed.

Area 2 - Nonstationary Noise Studies

Work in this area was limited to final processing of the technical note described in the last progress letter.

Respectfully submitted,

F. Dixon
Project Director, A-588

FD/gj
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Officer, PR-EC

Subject: Monthly Progress Letter 23, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 13 August to 12 September 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved.

Group 1 Progress Summary

Efforts have been continued on the construction of the new experimental EGI system, which is now nearing completion. Insulation sections for the cold-bar enclosure were cut from styrofoam, and the cold bar was positioned between the heat sinks, with a frigistor on each end between the sink and the cold bar. Heat conductive compound was used on the interfaces between sinks, frigistors, and cold bar in order to minimize the thermal resistance in these areas. The heat sinks were then bolted to the main chassis and drawn together by means of wire loops around pins inserted in the mounting holes on top of the heat sinks. This placed the cold bar and frigistors in compression between the heat sinks. The remainder of the styrofoam insulation was installed, and the unit was checked out by running power directly to the frigistors. A temperature of about 10°C was reached after a considerable period with some 10 amperes current through each frigistor. After the temperature-control amplifier was wired up, the unit was again tested--this time in closed-loop operation. A temperature of 8°C was reached, a quiescent current of about 4 amperes being drawn at this temperature.

Wiring was continued by mounting and connecting the tuning-fork oscillator, shaper, and integrated-circuit countdown circuitry. After locating and correcting a short in one of the components, check-out showed the units to produce the desired timing pulse.
A logic card containing the flip-flops for the positive and negative reset pulses was constructed in accordance with earlier design. However, check-out revealed an error in the logic used for setting and resetting the flip-flops. A revised set and reset logic has been designed which ensures synchronous operation, and modification of the circuit card is now under way.

**Group 2 Progress Summary (reported by J. L. Hammond, Jr.)**

During the period 13 August to 12 September 1966, the Group 2 staff consisted of Dr. Hammond at 1/2 time and Mr. Schlag at approximately 1/4 time. In the next report period (which begins the Fall Quarter at Georgia Tech), Dr. Finn will be active 1/3 time and Dr. Hammond 1/2 time. Mr. Schlag will not be available to work on this program during the Fall Quarter, but may be replaced by another graduate research assistant.

Progress for the past month in the two assigned areas is as follows.

**Area 1 - Error Analysis for Hybrid Computation**

The technical note discussing error analysis techniques mentioned in the preceding progress letter has been completed and is designated Technical Note No. 12, "Approximations for Hybrid Computer Error Due to Sampling and Execution Time," by Joseph L. Hammond, Jr., 30 August 1966. This report is being processed and should be forwarded to MSFC early in the next monthly period.

Efforts during the past month were largely devoted to developing the equations presented in Technical Note No. 12 to their final form and to completing preparation of the report. Some time was spent in evaluating the equations, using the Pendulum Problem (discussed earlier) as a test example.

Computer studies using the Euler Angle example will begin during the next report period.

**Area 2 - Nonstationary Noise Studies**

Although no new work was done in this area, final processing was completed on Technical Note No. 11, "Generation of Nonstationary Random Processes Dependent on Time and Position Parameters," by David L. Finn, 15 August 1966. This report has been forwarded to MSFC.

Respectfully submitted,

F. Dixon  
Project Director, A-588
GEORGIA INSTITUTE OF TECHNOLOGY
ENGINEERING EXPERIMENT STATION
ATLANTA, GEORGIA 30332

24 October 1966

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 24, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period from September 12 to October 12, 1966

Gentlemen:

The material presented herein covers work performed on the subject con-
tact program during the past monthly period by the two project research
groups involved.

Group 1 Progress Summary

Construction and debugging of the new experimental EGI system should be
completed soon. Necessary modification of the logic card containing the flip-
flops for the set and reset pulses has been accomplished. The design revision
called for an extra four gates, which were placed on order, received, and
incorporated into the unit, all within the past few weeks. The card will be
checked out as soon as possible.

Another equipment modification proved to be necessary after attempts were
made to operate the differential comparators and associated dual gate. The cans
of these units were found to be connected to certain of the pins. When the cans
were clamped together in the copper cooling block and the pins connected as
required, one power supply was shorted out. Nothing is said on the manufacturer's
data sheets of such internal connection to the cans. It was necessary to modify
the cold plate by first removing all wiring on one face of the block and then
removing the associated cover plate and drilling a larger hole in it. The gate
can was mounted in an insulating heat sink, which was fitted with a copper plug.
The copper plug was pressed into the original mounting hole and the block rewired.
Changes in the reset pulse logic required some additional wiring on the block. A check of the modification was performed by operating the two differential comparators, as well as the dual gate, successfully. The block was then re-installed between the frigistors and the units were retested, again successfully.

The gate driver card was mounted and wired to the power supplies. Interconnecting wiring will be started as soon as the integrated-circuit logic card mentioned above is checked out. This integrated-circuit card supplies the control signals for the gate driver card.

Wiring is also proceeding on the analog signal paths. The X-input is partly wired and should be completely wired soon.

Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

During the period 13 September to 12 October 1966, the Group 2 staff consisted of Dr. Finn at 1/3 time, and Dr. Hammond at 1/2 time. A new graduate assistant, Karol Zakrzewski, was added to the Group 2 staff at the end of September. He will work approximately 1/3 time, primarily providing computer programming assistance. The same staffing is planned for the remainder of the Fall Quarter.

On September 27th, Dr. Finn and Dr. Hammond attended the Southeastern Simulation Council meeting in Huntsville, Alabama and also held a project conference with Dr. Polstorff at Marshall Space Flight Center. At this meeting it was agreed that in planning our research studies the area of Nonstationary Noise Studies would be phased out, and work in a new area, Hybrid Computer Methods for Solving Partial Differential Equations, would be initiated. A formal proposal reflecting this change and requesting a 12-month extension of the present contract (through 12 December 1967) was submitted to the NASA Purchasing Office under letter dated 11 October 1966.

Progress during the past month in the two currently assigned study areas is as follows.

Area 1 - Error Analyses for Hybrid Computation

Technical Note No. 12, "Approximations for Hybrid Computer Error Due to Sampling and Execution Time," by Joseph L. Hammond, Jr., 30 August 1966, has been completed and forwarded to MSFC.

During the past month, activities in Area 1 have been directed toward
application of the error analysis methods described in Technical Note No. 12 in certain computational problems. Work with the nonlinear pendulum equation, as a simple test case, has been largely completed and will be described in a project technical note in the near future. Preliminary error-analysis studies have been started on two somewhat more complicated, and hence more typical, problems--namely, the Euler Angle problem described in Monthly Progress Letter 21 of 21 July 1966, and hybrid computer equations for a nonlinear heat transfer problem. The latter problem is described in a paper by J. S. Spear, "Hybrid Solution of a Nonlinear Heat Transfer Problem," presented to the Southeastern Simulation Council meeting on 27 September 1966.

An effort is being made to keep abreast of the current literature pertaining to error analysis for hybrid computation. In that connection, preprint summaries have been received of papers in this area to be presented at the 1966 Fall Joint Computer Conference, 8-10 November 1966. Six of the papers are pertinent, namely:


The bibliographies of the above six papers list several additional references which are pertinent. These items and other current literature are being studied so that the project research efforts will be based on the present state of the art with as little duplication as possible.

Area 2 - Nonstationary Noise Studies

In this area, Dr. Finn is resuming the research effort directed toward the utilization of experimental wind data in a previously developed synthesis technique for the mechanization of nonstationary random processes. The current
objective is to select a suitably tractable analytical expression that approximates the covariance of the experimental wind process.

Respectfully submitted,

F. Dixon
Project Director, A-588
21 November 1966

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 25, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from October 12 to November 12, 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved.

Group 1 Progress Summary

Wiring of the new experimental EGI system is nearly complete. The attenuators, computing resistors, and other circuit components have all been hooked up and checked satisfactorily except for the reset resistor network and the computing capacitors. Trouble-shooting revealed an error in the wiring to the control relays, which will necessitate changing the wiring on the reset resistor network. A partial order of precision computing capacitors has been received, but delays in shipment of the balance may require that lower-grade units be substituted for purposes of conducting final checkout and tests.

Final debugging of the overall EGI system will hopefully take only about a week, in view of the checkout given each section of the unit before assembly. Performance tests will necessarily be rather brief due to the short time remaining before completion of the present contract period. An attempt will nevertheless be made to apply most of the tests performed on the original EGI unit, although not in as much detail.
Group 2 Progress Summary (reported by J. L. Hammond, Jr.)

There was no change in the staff of Group 2 during the period 13 October to 12 November 1966 from that stated in the last progress letter (Monthly Progress Letter 24). As noted earlier, the present staffing is planned for the remainder of the Fall Quarter.

Area 1 - Error Analysis for Hybrid Computation

The current emphasis in this area is on the application of the error analysis methods described in Technical Note No. 12 to certain specific problems. The objectives of this work are twofold: (1) to further evaluate and enhance, if possible, the utility of the error analysis methods, and (2) to begin to build up information as to the nature of errors due to sampling rate in typical problems. The two major problems being considered have been described in Monthly Progress Letters 21 and 24. Emphasis during the past period has been on the nonlinear heat transfer problem.

In considering the heat transfer problem, the tediousness of developing analytically any one of the error equations immediately became apparent. The computational problem is essentially one of determining the elements of two 7-by-7 matrixes where each element is defined as the partial derivative of a reasonably complicated function with respect to one of the solution variables. The computation in this problem, which is considered typical, is by no means prohibitive; but it does affect the ultimate utility of the method.

To circumvent the analytical calculations just noted, a computer program has been developed which operates from a general statement of the error equation. The program requires as input only a statement of the problem equations being solved and their true solution, or their solution on a hybrid computer. The program automatically develops the required error equations and computes the approximate sampling error.

The computer program has been tested using the pendulum equation and was found to work satisfactorily for this case.

The next step, that of using the computer program in studying the nonlinear heat transfer problem, is currently under way.
The preparation of a technical note on the results of studies with the nonlinear pendulum equation has begun, and with the possible exception of final processing, should be completed before the end of the present contract period.

Area 2 - Nonstationary Noise Studies

In this area a mathematical model has been developed for the characterization of the salient features of the experimental random wind data made available by the sponsor. Efforts will now be directed toward the utilization of this model in conjunction with a previously developed synthesis technique for the mechanization of a nonstationary random process simulating the random wind variations.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 26, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from November 12 to December 12, 1966

Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved. This letter report is submitted on the basis that the contract period will be extended for an additional twelve months, in general accordance with the proposal forwarded to Marshall Space Flight Center on 11 October 1966.

Group 1 Progress Summary

Efforts during the past month have been directed toward completing Task 3 of the present contract program—the construction and evaluation of an improved generalized analog integrator, based upon the previous experimental EGI design but incorporating recently developed field-effect-transistor switches, along with other new features. The special computing capacitors which were overdue at the time of last month's progress letter have since been received and incorporated into the new EGI unit, thereby permitting completion of all wiring and construction. During the subsequent system checkout attempts, however, malfunctions were encountered in several sections of the unit—principally in the digital section of the Delta-X detectors. These problems were compounded by the accidental runaway of a laboratory power supply being used for the tests. Although most of the difficulties have now been ironed out, it appears unlikely that performance evaluation runs can be made until after the Christmas and New Year holidays. Circuit diagrams have been prepared in essentially...
final form for inclusion in a technical note describing the new EGI unit, but very little of the text material has been written as yet.

Among the problems encountered in last month's checkout efforts was the fact that the reference voltage for the differential comparators in the Delta-X detectors required more adjustment range than had originally been considered necessary. Noise and pickup caused the comparators to appear to be oscillating until additional range was provided for the reference voltage.

The digital microcircuit logic in the Delta-X detectors gave a considerable amount of trouble, which was eventually traced to the particular J-K flip-flops used. Although the toggle input performed as it should, neither the set nor reset inputs had a detectable effect on the output in any of the units available. The manufacturer is being contacted about this difficulty, but because of the shortage of time a temporary fix was devised utilizing a TO-5 can containing two three-input gates, with the gates interconnected so as to form an R-S flip-flop. This single can could be mounted on the circuit card in place of the J-K flip-flop, and hence rewiring of the entire card was not necessary.

It was also found that the FET gate driver circuitry was loading some of the integrated-circuit logic to the point of inoperability. Addition of an IC buffer amplifier to unload the circuitry took care of this problem.

The temperature control system for the "cold plate," which houses critical semiconductor components, has been functioning for some time and appears quite satisfactory. Only the analog section of the EGI unit remains to be fully checked out. A minor problem exists in this area in that the X summer amplifier exhibits excessive drift in the Operate mode. It is anticipated that this problem will be solved by holiday time, so that performance tests can be started immediately after New Year's.

**Group 2 Progress Summary (reported by J. L. Hammond, Jr.)**

The project staffing listed in Monthly Progress Letter 24 has been continued through the current report period. Assuming that the contract is extended for an additional twelve months, Drs. Finn and Hammond propose to work approximately one-half time on the project during the Winter academic quarter. Mr. Zakrzewski is scheduled to continue at one-third time, providing assistance in computer programming and related tasks.

Progress in the two assigned areas during the past month has been as follows.
Gentlemen:

The material presented herein covers work performed on the subject contract program during the past monthly period by the two project research groups involved. Contract Modification No. 9, dated 10 January 1967, provides for a continuation of the program until 11 December 1967 but with the scope of work modified to include a new study of techniques for solving partial differential equations using hybrid computers. This study will effectively replace the previous Task 3—involving laboratory construction and evaluation of an improved generalized analog integrator—which is in process of completion.

Group 1 Progress Summary

A number of different problems encountered in the process of debugging the completed EGI system have delayed final performance tests. Most of the required corrective measures were of a minor nature, such as replacing a faulty potentiometer, relocating ground connections, etc. One of the more serious difficulties, however, involved isolating an intermittent high-frequency oscillation in the comparator circuits that was causing spurious signals at the output of the operational amplifiers. These comparator circuits are housed in transistor-type cans and seem to generate a high-frequency interference signal on power supply leads and on input and output leads when the input signal is in a region near the threshold level. It was found necessary to introduce rf filters at the power inputs to the operational amplifiers and to include a low-pass filter between the output of the Storage Integrator and the input to the comparators. (It was not practical to introduce sufficient filtering at the comparator circuits, due to space limitations, inasmuch as these two circuits are located on the cold plate.)
Another item that had to be taken care of was the provision of sufficient drive current to the field-effect-transistor gates in the Reset Pulse Generator. The previous technique for obtaining the biasing current for the FET gates was through a back-biased diode in series with the drive signal. However, the leakage current of these diodes was insufficient for the purpose, so resistors were placed in parallel with the diodes.

Except for some additional steps to improve baseline stability, the EGI unit appears to be in satisfactory functional condition. The basic test runs made thus far indicate that it should be capable of improved performance over the previous EGI, and every effort will be made to complete the planned evaluation tests within the next report period.

**Group 2 Progress Summary** (reported by J. L. Hammond, Jr.)

The staffing for Group 2 during the Winter academic quarter (January through the middle of March) will be unchanged from the Fall quarter—namely, Drs. Finn and Hammond one-half time and Mr. Zakrzewski one-third time.

Beginning in the period covered by the present report, Group 2 has initiated and will continue to build up activity in a new area to be identified as "Area 3 - Hybrid Techniques for Partial Differential Equations." Following the wishes of the sponsor, emphasis is being shifted from the present Area 2 (Nonstationary Noise Studies) to the new area.

Progress in the three areas of interest is as follows.

**Area 1 - Error Analysis for Hybrid Computation**

The present general objectives of Area 1 were stated in Monthly Letter 26 as: (1) to further evaluate and enhance, if possible, the utility of the error analysis methods, and (2) to build up information as to the nature of errors due to sampling in typical problems.

A major part of the first of these objectives has been accomplished and is documented in the forthcoming Technical Note No. 13. This note discusses several techniques for implementing the error equations and illustrates the use of the four different forms of these equations with several examples. The material presented indicates that (1) the methods for implementing the error equations are practical at least for small-scale problems, (2) all forms of the error equations are technically correct, and (3) the error
equations yield accurate results in the examples studied. Technical Note No. 13 has been prepared for reproduction and should be issued within the next two weeks.

Now that the error equations have been shown to be accurate, emphasis in the study will shift to studies of typical problems using the equations, and to formulating general conclusions as to the nature of sampling and execution time errors. Work in both of these directions is already in progress.

Area 2 - Nonstationary Noise Studies

The main effort in this area continues to be in the preparation of a technical note summarizing the results of Area 2 research over the past several years. Every effort is being made to complete this report in as little time as possible, consistent with maintaining quality and presenting a complete picture of the research accomplished. Present plans call for completing most of the rough draft of this note during the next monthly period.

Area 3 - Studies of Partial Differential Equations

Some work in this area was initiated during the period covered by this report, and a substantial effort will be built up as emphasis shifts from Area 2 to Area 3.

Plans call for preparing a bibliography on the utilization of hybrid computers in the solution of partial differential equations, as the first task in Area 3, and this work has begun.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812  

Attention: Purchasing Office, PR-EC  

Subject: Monthly Progress Letter 28, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 January to 12 February 1967  

Gentlemen:  

As discussed with Dr. Polstorff by telephone on February 6th, completion of the Project A-588 Group 1 laboratory program on development and evaluation of an improved generalized analog integrator was temporarily postponed in order that the engineer in charge might take time off to be married. Work has since been resumed on the integrator performance tests, and documentation of results should be available within the next four to six weeks. Future progress reports on this project will be concerned only with the topics of investigation assigned to personnel formerly identified with Group 2.

Project staffing from the School of Electrical Engineering remains the same as for the previous report period—namely, Drs. D. L. Finn and J. L. Hammond, Jr. one-half time, and Mr. K. Zakrzewski one-third time. Present plans call for this same staffing arrangement to be maintained through the remainder of the current academic year (ending the first part of June). Progress in the three technical areas of interest is as follows.

Area 1 - Error Analysis for Hybrid Computation (reported by J. L. Hammond, Jr.)

Publication of Technical Note No. 13, "Implementation and Evaluation of Equations for Hybrid Computer Error," was somewhat delayed by minor difficulties encountered in the final processing and proofing stages. How-
ever, the report should be released and forwarded to the sponsor before the end of this month.

With the publication of Technical Note No. 13, the first phase of the Error Analysis Study—namely, the formulation, evaluation, and implementation of a tractable set of error equations—would seem to be essentially completed. Thus, as noted in a recent telephone conversation with Dr. Polstorff, the program has now reached a point where mutual benefit might be derived from a meeting in Huntsville to review and discuss in detail the results documented in Technical Notes Nos. 8, 12, and 13. We understand that such a meeting can probably be arranged in late March or early April, allowing for a preliminary review of Technical Note No. 13 by MSFC Laboratory personnel.

The present emphasis in the continuation of the study is on an error analysis for a problem of practical magnitude and on formulating general conclusions as to the nature of sampling and execution time errors. With respect to the former problem, we are currently experiencing what we hope are routine difficulties arising from the greatly increased complexity of the practical problem as compared to the test example.

With respect to the latter problem, results for errors in the test examples of Technical Note No. 13 and for other tractable cases are being studied with the aim of establishing typical characteristics for sampling and execution time errors.

Area 2 - Nonstationary Noise Studies

Attention in this area is now devoted entirely to work associated with the preparation of a summary technical note covering the results obtained in this area over the past several years. For this report to accurately reflect the work which has been done, it is considered very desirable to include several illustrative wind profiles generated with an elementary analog computer network designed using methods evolved during the latter part of the study. This work with the analog computer has been formulated and is now being performed. It is expected that the desired results can be obtained routinely and in a relatively short time.

Satisfactory progress is being made in drafting the parts of the
technical note covering the theoretical aspects of the work. As the summary report nears completion more and more personnel time will be shifted to investigations called for under the newly assigned Area 3.

Area 3 - Studies of Partial Differential Equations

Efforts in this area are building up slowly as Area 2 is being phased out. As noted earlier, the first task will be a comprehensive review of the literature in this area, and satisfactory progress is being made in preparing a bibliography and reviewing the pertinent papers.

Respectfully submitted,

F. Dixon
Project Director A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 29, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 February to 12 March 1967

Gentlemen:

As indicated in the last monthly progress letter, project staffing from the School of Electrical Engineering will continue unchanged for the remainder of this academic year. Drs. Finn and Hammond plan to come to Marshall Space Flight Center some time in March or April to discuss the status of work primarily in Area 1 (Error Analysis for Hybrid Computation). Definite arrangements for this visit will be set up by telephone with Dr. Polstorff. Progress in the three currently assigned areas of interest is as follows.

Area 1 - Error Analysis for Hybrid Computation (reported by J. L. Hammond, Jr.)

The emphasis in this area continues to be directed toward two problems—namely, a detailed error analysis for the heat flow problem described in the paper by Mr. Spear, and a consideration of the nature of computational errors due to sampling and execution time in tractable cases.

The initial part of the study of sampling errors in the heat flow problem is being conducted using the equation for error versus sampling interval (i.e., equation 4.5 of Technical Note No. 12). To conserve computer time, the true solutions are taken from the curves in Mr. Spear’s report and read into the computer as numerical data.
After experiencing routine difficulties in reading in and interpolating the numerical data and in the choice of step size in the numerical integration procedure, what appear to be satisfactory preliminary results have been obtained. Further work with these equations is planned to determine maximum error versus sampling interval and to investigate the error in other methods for programming the hybrid computer to solve the problem equation.

During this report period, work in establishing the general nature of sampling errors was devoted to an investigation of such errors when the problem equations are linear with constant coefficients. Such cases are not of central practical interest in themselves but serve to shed light on the nature of errors in the more general cases. Some interesting results pertaining to methods of normalizing the error equations have been obtained, as well as certain geometrical interpretations which may help in understanding sampling errors.

Results for both of the problems mentioned above will be discussed in technical notes when the work has been completed.

Area 2 - Nonstationary Noise Studies (reported by D. L. Finn)

During the past report period, work in this area has continued to be devoted entirely to the preparation of a summary technical note covering the results obtained over the past several years. During the preceding report period an effort was initiated toward the utilization of an elementary analog computer network for the generation of several illustrative wind profiles. This work has been completed during the present report period, and it has been found that the profiles generated by the analog computer network have a satisfactory qualitative agreement with profiles plotted by use of experimental wind data.

Area 3 - Studies of Partial Differential Equations (reported by D. L. Finn)

Work in this area has continued to be of a preliminary and limited nature because of the concentration of effort on the preparation of the summary report concerned with nonstationary noise studies. Attention has
continued to be concentrated on the study of the literature concerning the use of hybrid computers in the solution of partial differential equations.

Respectfully submitted:

F. Dixon
Project Director, A-588
19 April 1967

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 30, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 12 March to 12 April 1967

Gentlemen:

Project staffing for the remainder of this academic year has been noted in previous progress letters. Plans for Drs. Finn and Hammond to summarize progress in Area 1 (Error Analysis for Hybrid Computation) at Marshall Space Flight Center have been discussed by telephone with Dr. Polstaff. Early May now seems to be the most convenient time for this discussion. Final arrangements will be made by telephone at a later date.

Progress in the three currently assigned areas of study is as follows.

Area 1 - Error Analysis for Hybrid Computation (reported by J. L. Hammond, Jr.)

Emphasis in this area is essentially unchanged from that noted in the last progress letter. Efforts are being directed toward a detailed error analysis for the heat flow problem and toward obtaining tractable expressions for errors due to sampling in simplified problems.

With respect to the heat flow problem, approximate error curves have been computed over the first ten percent of the desired time interval. During this interval the errors decrease to a relatively small size.

To obtain a reasonable accuracy in solving the error equation, the digital step size (using a Runge-Kutta method) has to be quite small, on the order of $10^{-3}$ seconds. With this step size 5 minutes of computer time is
required to obtain the solution to the error equations for 0.3 seconds. Solutions to the error equation over an interval of 3 or 4 seconds is desired, and such a solution using the present program would require 50 minutes of computer time. Before making a computer run of this length, additional thought is being given to scaling and possible modifications in the numerical method which might reduce the required computing time. If it proves to be impossible to reduce the required computing time, a long run can be made.

The study of errors due to sampling in solving linear differential equations with constant coefficients has been continued. Some interesting results have been obtained by expressing the true problem solution and the approximate error equation solution in terms of the modes of the problem equation. (That is, the solutions to the equations are expressed in terms of their components along the eigenvectors associated with the problem equation.) When this is done, it turns out that the modal components of the error are related to the modal components of the true solution by a simple multiplicative factor, which is the product of the sampling interval, time, and the square of the eigenvalue (natural frequency) corresponding to the particular mode.

Area 2 - Nonstationary Noise Studies (reported by D. L. Finn)

The preparation of the rough draft of a technical note summarizing the results obtained over the past several years in this area of investigation has been completed and work has been started toward the reviewing and processing of a final draft of this technical note. This technical note presents a mathematical model for random wind variations, discusses a synthesis technique previously presented for synthesizing an analog computer network that simulates random wind disturbances, and includes some wind profiles generated by an analog computer for comparison with wind profiles determined experimentally.

Area 3 - Studies of Partial Differential Equations (reported by D. L. Finn)

Work in this area is progressively being increased as the preparation
of the summary technical note in the area of nonstationary noise studies is being completed. As noted earlier, a thorough literature survey is the first objective, and efforts during the past month have continued to be directed toward this goal.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 31, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 12 April to 12 May 1967

Gentlemen:

Project staffing will remain as previously noted through the report period ending 12 June 1967. Drs. Finn and Hammond have both been awarded faculty fellowships for participation in simulation research and study away from the Georgia Tech campus during the summer academic quarter. Dr. Finn will be at Marshall Space Flight Center and the University of Alabama; while Dr. Hammond will be at the NASA Ames facility and Stanford University. Because of the absence of Drs. Finn and Hammond, work on the contract during the latter half of June, July, August, and part of September will be at a reduced level. Project staffing from the School of Electrical Engineering will consist primarily of Mr. K. Zakrzewski working one-third time on several problems for the error analysis study.

Reduction of the research effort during the summer will be compensated for during other parts of the year so that the effort for the full year will be of the same magnitude as originally planned.

On 10 May 1967 Drs. Finn and Hammond visited Marshall Space Flight Center to present a seminar summarizing the progress and current status of the work in Area 1--Error Analysis for Hybrid Computation. The discussions at the seminar with MSFC personnel and later with Dr. Polsteroff and Mr. Spear were found to be very helpful.
Progress in the three currently assigned areas of study is as follows.

Area 1 - Error Analysis for Hybrid Computation (reported by J. L. Hammond, Jr.)

During the past report period satisfactory progress has been made on the two specific problems currently under study—the heat flow problem and the class of linear differential equations with constant coefficients. Plans still call for documenting these two problems with technical notes, although it is taking longer to complete the work than was originally planned. An immediate result of work on these specific problems has been insight resulting in an approach to a more general formulation for the error equations.

The first objective of the error analysis study was to formulate approximate error equations applying for a hybrid computer used in such a way that integration is performed with the analog equipment and function generation is performed with the digital equipment. Both types of components are coupled together in a closed loop by A/D and D/A converters. This type of operation and several more general types are indicated in the table below.

<table>
<thead>
<tr>
<th>Case</th>
<th>Integration</th>
<th>Function generation for analog subsystem</th>
<th>Function generation for digital subsystem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>analog</td>
<td>digital</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>analog</td>
<td>digital, analog</td>
<td>digital</td>
</tr>
<tr>
<td>Case 3</td>
<td>analog, digital</td>
<td>analog</td>
<td>digital</td>
</tr>
<tr>
<td>Case 4</td>
<td>analog, digital</td>
<td>analog, digital</td>
<td>analog, digital</td>
</tr>
</tbody>
</table>

Work completed to date makes possible a rather general approach to cases 1 and 2. Recent progress makes it seem feasible to extend the treatment to cases 3 and 4, and apparently this can be done without extensive modification of the basic approach. It seems reasonable to pursue this generalization of the basic approach before completing and reporting on the study of sampling errors for differential equations with constant coefficients.
The more general approach does not affect the work on the heat flow problem, which will be documented as soon as the work is completed.

Area 2 - Nonstationary Noise Studies (reported by D. L. Finn)

Dr. Finn has completed the final draft of a technical note summarizing the results obtained over the past several years in the area of nonstationary noise studies. As was indicated last month, this technical note presents a mathematical model for random wind variations, discusses a synthesis technique previously presented for synthesizing an analog computer network that simulates random wind disturbances, and includes some wind profiles generated by an analog computer for comparison with wind profiles determined experimentally. Preparation of this technical note completes the present research in the area of nonstationary noise studies. The typing and other final processing of this technical note will be done as soon as reviewing and proofreading by other concerned project personnel has been completed.

Area 3 - Studies of Partial Differential Equations (reported by D. L. Finn)

Research effort that was previously directed toward Area 2 (nonstationary noise studies) has now all been shifted to Area 3 (studies of partial differential equations). As was noted last month, the initial effort in this area is being directed toward the completion of a thorough literature survey.

Respectfully submitted

F. Dixon
Project Director, A-588
Project staffing will be considerably reduced for the next three report periods since Drs. Finn and Hammond have both been awarded NASA-ASEE summer faculty fellowships and will be away from the Georgia Tech campus during the summer quarter. Dr. Finn will spend this time at Marshall Space Flight Center and the University of Alabama, while Dr. Hammond will be at the NASA Ames facility and Stanford University.

Mr. K. Zakrzewski is scheduled to continue working approximately one-third time during the summer quarter on several specific problems for the error analysis study, as noted in the progress summary below.

**Area 1 - Error Analysis for Hybrid Computation** (reported by J. L. Hammond, Jr.)

As indicated in the last progress letter, work with specific examples has suggested a method of extending the general error analysis so that it will include all four types of closed-loop hybrid programs. Work during the past period has been devoted almost entirely to this extension of the basic method. Results have been very encouraging and it now seems possible to derive equations for sampling error in any closed-loop hybrid program. These extensions will be documented as soon as the work has been completed.

During the next several months Mr. Zakrzewski will continue to work on several specific applications of the error equations, and this seems an
appropriate time to summarize the past and planned work in this direction. As is frequently the case in research studies, we have used specific examples to evaluate and provide guidance for the general theoretical work. Thus, over the past year, work on several examples has been initiated and then from time to time set aside in favor of the more general studies.

The nonlinear pendulum equation first mentioned in Progress Letter No. 20, June 1966, is an example which has proven its usefulness. Many results with this equation are given in Technical Note No. 13, "Implementation and Evaluation of Equations for Hybrid Computer Error," 15 January 1967.

The Euler angle example first mentioned in Progress Letter No. 21, July 1966, has not yet received the attention it deserves, and Mr. Zakrzewski is scheduled to concentrate on it this summer.

The nonlinear heat transfer problem, first mentioned in Progress Letter No. 24, October 1966, has received a reasonable amount of attention, with results which may be summarized as follows. The seven simultaneous error equations for this problem have been programed and solved over a short time interval. However, since the step size required to solve the error equations accurately is very small, the amount of computation for a solution of 3 seconds' problem time appears excessive and has not been carried out.

Following a theoretical study of linear equations with constant coefficients, the nonlinear heat transfer equations were linearized. The general results for linear equations were applied to this case and a considerable insight into the nature of the computing problem was obtained. (This result was discussed briefly in the recent seminar at MSFC.) A method has suggested itself for solving the error equations with much less computing time (which, incidentally, has an analog in solving the original problem equations). Mr. Zakrzewski will attempt to implement the method during the summer.

After a reasonable body of information on this problem has been accumulated, a technical note summarizing the work will be written.

Studies of the class of linear differential equations with constant coefficients, first mentioned in Progress Letter No. 29, March 1967, have
yielded some interesting results. Some of these were mentioned at the
MSFC seminar referred to above. Some more work is envisioned before
this part of the study is completed. When it is completed it will be
documented by a technical note.

Two additional examples, not mentioned explicitly in the progress
letters, have been used in the study, namely the nonlinear Duffin equa-
tion and an optimum servo problem. Both are discussed in Technical Note

Area 2 - Nonstationary Noise Studies (reported by D. L. Finn)

As was indicated last month, the present research investigation
in this area has been completed and is now documented in Project A-588
Technical Note No. 15, entitled "Analog Simulation of Wind Turbulence,"
and dated 1 June 1967. This report is currently in process of reproduction
and should be issued before the end of this month.

Area 3 - Studies of Partial Differential Equations (reported by D. L. Finn)

Effort in this area is now being devoted to a literature survey on
the use of hybrid computers in the solution of partial differential equa-
tions. This investigation will not be pursued at Georgia Tech during
the summer quarter in view of Dr. Finn's absence from the campus while on
assignment to the summer faculty fellowship at Marshall Space Flight
Center. However, this phase of the project will be resumed during the
fall quarter, when it is planned to publish a bibliography documenting
the literature survey.

Respectfully submitted,

F. Dixon
Project Director, A-588
31 July 1967

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 33, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 June to 12 July 1967

Gentlemen:

As noted in the previous progress letter, Drs. D. L. Finn and J. L. Hammond, Jr. are both away from the Georgia Tech campus on NASA-ASEE summer faculty fellowships. Project activity for the present and next two report periods is therefore limited primarily to part-time studies by Mr. K. Zakrzewski on specific problems in Area 1--Error Analysis for Hybrid Computation. Past and planned efforts in this direction were reviewed in the preceding progress letter. The following investigations have since been performed.

Heat Problem

The nonlinear heat transfer problem (first mentioned in Progress Letter 24, October 1966) has been reformulated as a new system of seven linearized equations in new decoupled variables. The solution involves two 7-by-7 matrix transformations \( P \) and \( P^{-1} \) as indicated in the general relationship

\[
\dot{\mathbf{y}} = P^{-1} f(P \mathbf{y}, b)
\]

The transformations are presently being written for computer solution.
However, there is some question about the ultimate efficiency of the method, since it may require excessive time to transfer variables from one system to the other and back again during each computer step.

**Euler-Angle Transformation**

Computer runs have been made for several cases of the Euler-angle problem (first mentioned in Progress Letter 21, July 1966), with rather good results. For runs involving constant forcing functions, the computed hybrid error lies within ±1 percent of the true hybrid error. The computational procedures are now being extended to cover cases which involve time-varying forcing functions.

**Literature Coverage**

References to other work on Hybrid Computer Error Analysis are being sought concurrently with the more direct project investigations noted above. The literature covered to date includes (1) IEEE Transactions on Electronic Computers, 1966-67, (2) IEEE Transactions on Automatic Control, 1966-67, and (3) the Soviet journal "Automation and Telemecanics," 1966.

**Project Technical Notes**

Project A-588 Technical Note No. 15, "Analog Simulation of Wind Turbulence," by David L. Finn, 1 June 1967, was processed in Dr. Finn's absence and forwarded to the sponsor on 27 June. This report represents the completion of work performed over a several-year period on Nonstationary Noise Studies (Area 2).

Material being generated in the current effort on Error Analysis for Hybrid Computation is to be documented sometime after the summer quarter ends.

Respectfully submitted,

F. Dixon
Project Director, A-588
Gentlemen:

As noted in the preceding two progress letters, Drs. D. L. Finn and J. L. Hammond, Jr. are both away from the Georgia Tech campus on NASA-ASEE summer faculty fellowships. Project activity until the beginning of the fall quarter will therefore be limited primarily to part-time studies by Mr. K. Zakrzewski on specific problems in Area 1--Error Analysis for Hybrid Computation. Past and planned efforts in this direction were reviewed in Progress Letter No. 32, dated 20 June, 1967. Developments during the past month are briefly noted below.

Heat Problem

The computer program for solving the nonlinear heat transfer problem by means of a set of linearized equations in new decoupled variables has been satisfactorily checked out. Solutions will now be tried using a changeable step size--very small at the beginning, and larger at the end.

Euler-Angle Problem

Several computer runs have been made for the Euler-angle problem with a time-varying forcing function (first-order polynomial). Results are far less accurate than for the previous case of a constant forcing...
function, compared at the same step size. Also, there is significant instability around the point where the transformation matrix becomes nearly singular, and it is necessary to stop the solution when such a point is reached.

Bibliography

The literature search on Hybrid Computer Error Analysis has been continued with a review of the 1964 and 1965 issues of all three journals cited in last month's progress letter. The periodicals covered to date now include: (1) IEEE Transactions on Electronic Computers, 1964-67, (2) IEEE Transactions on Automatic Control, 1964-67, and (3) the Soviet journal "Automation and Telemechanics," 1964-66.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 35, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 August to 12 September 1967

Gentlemen:

As noted in the preceding three progress letters, Drs. D. L. Finn and Joseph L. Hammond, Jr. have both been away from the Georgia Tech campus on NASA-ASEE summer faculty fellowships. Project activity during the past quarter has therefore been limited primarily to part-time studies by Mr. K. Zakrzewski on specific problems in Area 1--Error Analysis for Hybrid Computation. As of September 25th, the beginning of the next academic quarter, Drs. Finn and Hammond will resume their work under this program.

The status of the studies conducted by Mr. Zakrzewski remains largely unchanged since the last progress letter, in part because of his involvement with academic requirements to complete the degree of M.S. in Electrical Engineering. There is little new to report on the nonlinear heat transfer problem. Solutions of the linearized equations using modified coordinates are due to be run with variable step size (small at the beginning and larger at the end). However, an apparent error in the program has delayed this work. As previously noted, computer solutions for the Euler-angle problem with time-varying forcing functions are much less accurate than for the case of a constant forcing function, compared at the same step size. The procedure of taking derivatives by
the computer must be improved in order to achieve a useful level of error approximation. These difficulties will be investigated more thoroughly during the fall quarter.

Respectfully submitted,

F. Dixon
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 36, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 September to 12 October 1967

Gentlemen:

At the end of September, which is the beginning of the Georgia Tech Fall Quarter, project staffing returned to the level of effort of the Winter and Spring Quarters earlier this year. Drs. Finn and Hammond will each devote one-half time for the remainder of the present contract period, and Mr. Zakrzewski will continue at one-third time.

Area 1 - Error Analysis for Hybrid Computation (reported by J.L. Hammond, Jr.)

With the beginning of the Fall Quarter, research efforts in this area were increased to the previous level of the Winter and Spring Quarters. Thus it seems to be an appropriate time to review the results of the limited effort during the last three months and to indicate plans for the remainder of the current contract year.

Our limited efforts over the summer (one man at a one-third time rate) had the following nominal objectives: (1) to study the effect of sampling error in specific examples involving Euler angle transformations, (2) to investigate a technique for reducing the computation time in determining sampling error for the Heat Flow problem, and (3) to make a contribution to our continuing effort to keep abreast of the current literature on hybrid error analysis.

The accomplishments for the quarter may be summarized as follows.
In one example using an Euler angle transformation satisfactory calculations of error were made. A second example was formulated for conditions which caused one variable to go through a singularity of the transformation. For this case difficulties arose which have just recently been clarified.

On the Heat Flow problem, progress was made in programming a method expected to reduce computing time of the error equations. (Incidentally, the method can also potentially be applied to the given problem equations.) Work on this method was not completed during the summer and is being continued.

The literature survey was satisfactorily continued during the Summer Quarter.

Efforts during the current period have uncovered the fact that there is some, though a very limited amount, of literature available on the numerical solution of sets of differential equations which have widely differing time constants. As has been observed both at MSFC and in solving the error equations at Georgia Tech, the effect of the widely differing time constants is that one extreme time constant forces the step size of a numerical method to be quite small, while the other extreme forces the time interval of the solution to be quite long; the net result is excessive computing time. The most interesting reference is C. E. Treanor, "A Method for the Numerical Integration of Coupled First-Order Constants," Math of Computation (20), 1966, pp. 39-45. Efforts are now underway to digest this work and evaluate it on the Heat Flow problem along with our own method referred to above.

Our plans for the next several months in Area 1 are as follows: (1) to complete the work of generalizing the error equations so that they apply to essentially any closed-loop hybrid program, (2) to formulate a complete analytical error analysis applying to the hybrid solution of linear constant-coefficient equations, and (3) to complete the study of the several examples which are currently being examined. We will continue our practice of issuing a technical note as soon as possible after completing a given phase of the work.
Area 2 - Nonstationary Noise Studies

Work in this area has been completed and the results summarized in Technical Note No. 15, "Analog Simulation of Wind Turbulence," D. L. Finn, 1 June 1967, as previously noted.

Area 3 - Studies of Partial Differential Equations (reported by D.L. Finn)

Work in this area has been resumed during the September 12-October 12 report period with Dr. Finn's return to the campus. (Dr. Finn was a NASA-ASEE Faculty Fellow at Marshall Space Flight Center during the summer quarter.) The immediate objective in this area is the completion of a literature survey on the use of hybrid computers in the solution of partial differential equations. The major portion of this survey has been completed and work has started on the organization of a technical note enumerating and discussing the pertinent papers found in the literature survey. It is expected to include a commentary on a dozen or so papers dealing specifically with the use of hybrid computers in solving partial differential equations, and a bibliography of one hundred or so additional papers dealing with the numerical solution of such equations.

Respectfully submitted,

F. Dixon
Project Director, A-588
17 November 1967

George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 37, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the period from 12 October to 12 November 1967

Gentlemen:

Project staffing will continue as previously noted until the end  
of the present contract period. A proposal for a continuation of the  
research program past the current expiration date of 12 December 1967  
has been submitted for consideration by MSFC.

Progress in the assigned areas has been as follows:

Area 1 - Error Analysis for Hybrid Computation (reported by J.L. Hammond, Jr.)

Work is continuing in this area along the lines outlined in  
Progress Letter 36, 20 October 1967. The work in generalizing the error  
equations so that they account for sampling errors in any closed-loop  
hybrid system has been essentially completed and the results are being  
prepared for publication.

During the last month, emphasis in the application of the error  
equations in specific examples has been placed on the Euler angle trans-  
formation. The transformation has been regarded as a set of differential  
equations to be programmed on the hybrid computer in such a way that  
the nonlinear trigonometric functions are generated digitally and inte-  
grations are performed on the analog equipment. Several simple forms  
(constants and linear time variations) have been assumed for the inputs  
to the transformation, and true and approximate sampling errors have been
computed. The objectives of the study are to determine: (a) how sampling errors vary with the sampling period and with time after a reference time for which the error is zero, (b) the accuracy of the approximate error equation, and (c) the degree of compensation for sampling error that can be obtained by adjoining the error equations to the problem equations.

Preparation of a technical note detailing work to date on this problem will begin during the next report period. Also during the next period attention will be given to preparing for publication the results pertaining to a complete analytical analysis of sampling errors in linear constant-coefficient systems.

Area 2 - Nonstationary Noise Studies (reported by D. L. Finn)

As has been previously noted, work in this area has been completed and the results summarized in Technical Note No. 15, "Analog Simulation of Wind Turbulence," D. L. Finn, 1 June 1967.

Area 3 - Studies of Partial Differential Equations (reported by D.L.Finn)

The information survey referred to in earlier progress letters continues to receive primary attention. The literature portion of this survey has been completed during the present report period, and a substantial amount of work has been done toward the completion of a technical note documenting the results.

In addition to the literature survey, during the present report period a study of certain problems arising in the use of hybrid computers for solving partial differential equations has been initiated using the techniques of error analysis developed at Georgia Tech. It is hoped that the use of these techniques may provide information about satisfactory rates of operation of digital-to-analog and analog-to-digital converters in the hybrid system, and also provide information about the complexity or order of the digital numerical methods that are appropriate to use with the selected sampling rates. Such information is needed in
order to avoid unnecessarily high accuracy, and thus low computation speed, in the digital numerical methods when this high accuracy does not make a substantial contribution to the overall problem solution accuracy.

Respectfully submitted,

F. Dixon
Project Director, A-588
19 December 1967

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 38, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 12 November to 12 December 1967

Gentlemen:

Under our research proposal of 9 October 1967 to Marshall Space Flight Center, it was requested that the present contract period be extended for an additional 12 months to cover revised work objectives in the areas of (A) Analysis and Control of Error in Hybrid Computation and (B) Hybrid Computer Methods for Solving Partial Differential Equations. Pending notification of action thereon, this monthly progress letter is submitted in lieu of a Final Report on the project.

The status of efforts in the three previously assigned areas of study is as follows.

Area 1 - Error Analysis for Hybrid Computation (reported by J.L. Hammond, Jr.)

Several aspects of the problem in this area have been developed to the stage that publication of the results would seem desirable, as was noted in the preceding progress letter. A rough draft of the work pertaining to a generalization of the error equations has been completed, so that processing of the final technical note can begin during the next period. Preparation of the draft took somewhat longer than had been anticipated, for the reasons that (1) it seemed desirable to involve Dr. Finn in this aspect of the problem because of its potential application to Area 3 (see below), and (2) verification of the final developed equations proved to be relatively time consuming.
Preparation of report material covering the analytical analysis of sampling errors in linear constant-coefficient systems and the examples involving the Euler angle transformation was delayed by the work on the generalization of the error equation. Mr. Zakrzewski, however, continued his efforts on the latter problem. With the draft of the work pertaining to generalization of the error equations completed, attention can now be given to preparing technical notes on these additional topics.

Area 2 - Nonstationary Noise Studies

As previously noted, work in this area was completed during the past year and the results are summarized in Technical Note No. 15, "Analog Simulation of Wind Turbulence," by D. L. Finn, 1 June 1967.

Area 3 - Studies of Partial Differential Equations (reported by D.L. Finn)

The literature survey in this area has made it clear that most practical methods of solving partial differential equations depend on approximating such equations by potentially large sets of ordinary differential equations. Thus, considerations of sampling error and stability in solving systems of ordinary differential equations on the hybrid computer become of central importance in this area. In an effort to efficiently expedite the overlap between Areas 1 and 3, Dr. Finn will coauthor the report generalizing the sampling error study.

Some work was continued on documentation of the results of the literature survey, but at a reduced rate because of the effort noted above.

Project staffing during the proposed contract extension period will continue to be primarily by School of Electrical Engineering personnel. Drs. Finn and Hammond both plan major time commitments to the program, and Mr. Zakrzewski is expected to provide assistance on a one-third time basis. In keeping with this arrangement, Dr. Hammond is scheduled to serve as project director under the proposed contract extension.

Respectfully submitted,

F. Dixon
Project Director, A-588
19 February 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 39, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NASA-2473
Covering the period from 13 December 1967 to 12 February 1968

Gentlemen:

Under our research proposal of 9 October 1967 to Marshall Space
Flight Center, it was requested that the present contract period be ex-
tended for an additional 12 months to cover revised work objectives in
the areas of (1) Analysis and Control of Error in Hybrid Computation and
Pending notification of action thereon, we have requested by letter dated
January 30, 1968 that the subject contract be extended without additional
funds for approximately ninety days. Effort will be maintained on certain
of the established technical objectives in anticipation of continuing the
program.

Dr. J. L. Hammond, Jr. was named as project director in both re-
quested extensions, replacing Mr. Fred Dixon for the reason that project
staffing will be primarily from the School of Electrical Engineering.

This letter reports progress for the two-month period beginning 13
December 1967, when the present contract expired.

Planned project staffing for the Georgia Tech winter quarter, which
ends 15 March 1968, is as follows: Dr. Finn - 1/2 time, Dr. Hammond - 1/4
time, and Mr. Zakrezewski - 1/3 time.

The status of efforts in the assigned areas of study is as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

Final processing of Technical Note No. 17--to be entitled "General
Equations for Sampling Error in Hybrid Computer Systems," by J. L. Hammond and D. L. Finn--is being delayed pending availability of funding. As noted in Monthly Progress Letter No. 38, the rough draft of this technical note has been completed.

Current objectives for Area 1 are:

(1) To prepare for publication material previously developed in two problem areas, namely (a) sampling errors for linear constant coefficient systems and (b) studies concerning the Euler angle example.

(2) To continue the investigation of numerical and hybrid solution of differential equations with widely separated natural frequencies. This problem came to light in working the Heat Flow example and is felt to be of general importance in solving partial differential equations using finite difference approximations.

During the current report period some progress was made toward objectives 1(a) and (2).

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

The preliminary draft of Technical Note No. 16, entitled "Bibliography on Techniques for Solving Partial Differential Equations by Hybrid Computation and Other Methods" by D. L. Finn, has been completed. The bibliography lists on the order of four hundred references under the headings: Hybrid Computer Methods, Analog Computer Methods, and General Methods. About one hundred of the references describe methods for hybrid computers, and for most of the references in this category a brief summary of contents is given.

Final processing of this technical note is being delayed until additional funding is received.

The current objective of Area 2 is to classify existing methods for the numerical solution of partial differential equations. When this has been done those methods adaptable to hybrid computation will be identified and investigated in detail.

With respect to the overall program (i.e., both areas), a conference
with the sponsor would be desirable in the near future to establish priorities for the various parts of the work. Present staffing is not adequate to make rapid progress concurrently on all of the various aspects of the work presently identified.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
20 March 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 40, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 13 February 1968 to 12 March 1968

Gentlemen:

Pending notification of action on our proposal to extend the present contract, we are maintaining effort on certain of the established technical objectives. If funding is received, project staffing for the Georgia Tech spring quarter will be: Dr. Finn - 1/2 time, Dr. Hammond - 1/2 time, and Mr. Zakrzewski - 1/3 time.

As noted in Progress Letter No. 39, final processing of Technical Notes Nos. 16 and 17 is being delayed pending availability of funding.

The status of efforts in the assigned areas of study is as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

The immediate objectives for Area 1 were noted in Progress Letter No. 39. During the current report period, progress was made toward objective 1(a)--preparation of the rough draft of a new technical note concerning sampling errors for linear constant coefficient systems.

With respect to objective (2)--the solution of differential equations with widely separated natural frequencies--a technique described by Treanor ("A Method for the Numerical Integration of Coupled First-Order Differential Equations with Greatly Different Time Constants," Math. of Computation (20), 1966) is being investigated both theoretically and by application to a test problem. An understanding of techniques such as presented by Treanor is expected to be useful in hybrid computation where similar problems arise.
As was noted in Progress Letter No. 39, the preliminary draft of Technical Note No. 16 entitled "Bibliography on Techniques for Solving Partial Differential Equations by Hybrid Computation and Other Methods" has been completed. Attention is called to an error in the description of this bibliography in last month's progress letter, which states: "About one hundred of the references describe methods for hybrid computers, and for most of the references in this category a brief summary of contents is given." This sentence should be corrected to read "About one hundred of the references describe methods for analog and hybrid computers, and for most of the references in the latter category a brief summary of contents is given."

During the present report period, efforts to classify methods for the numerical solution of partial differential equations have been continued. After this is accomplished, three methods that appear adaptable to hybrid computation will be identified and investigated.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
Subject: Monthly Progress Letter 41, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 13 March to 12 April 1968

Gentlemen:

Notification of funding for the new contract year was received by Georgia Tech during the week of April 7, 1968. Receipt of funds makes it possible to begin final processing of Technical Notes Nos. 16 and 17. These reports will be processed in the order listed.

With respect to project staffing Dr. Hammond will work 1/2 time and Mr. Zakrzewski 1/3 time for the Spring Quarter as noted in Progress Letter 40. Unfortunately because of a health problem Dr. Finn will not be able to work 1/2 time during the Spring Quarter as planned. Instead, if it is possible for him to do so, Dr. Finn will work for the contract during the summer. This will have the effect of reducing the planned project staffing during the Spring Quarter and increasing it by a like amount during the Summer Quarter with no net change in the total commitment.

Now that the contract has been officially renewed a conference with the Sponsor for planning the research program for the current contract year would be very desirable.

The status of efforts in the assigned areas of study is as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

Current objectives in this area are unchanged from those stated in Progress Letter 39, namely:

(1) To prepare for publication material previously developed in two problem areas, namely (a) sampling errors for linear constant coefficient systems and (b) studies concerning the Euler Angle example
(2) To continue the investigation of numerical and hybrid solutions of differential equations with widely separated natural frequencies. As noted earlier the latter problem arises in approximate solutions of partial as well as ordinary differential equations.

Part (a) of objective (1) is nearing completion and considerable progress has been made in the investigation of numerical and hybrid solutions of differential equations with widely separated time constants (objective (2)). With respect to the latter, a bibliography of current literature is given as an Appendix to this progress letter. The method due to Treanor (reference 1) was chosen for initial investigation. It has been programmed and applied to a test case consisting of two first order linear differential equations having a solution of the form

\[ c_1 e^{-\alpha t} + c_2 e^{-\beta t} \]

with \( \alpha = 1000 \) and \( \beta = 0.1 \). As noted previously, in problems of this sort the step size of common numerical methods is restrained to be on the order of \( \frac{1}{\beta} \) to insure stability, while the solution time must be on the order of \( \frac{1}{\alpha} \). For the standard Runge-Kutta fourth order procedure a step size of 0.003 was stable with acceptable accuracy while a step size of 0.004 was unstable.

In this test example Treanor's method made possible a ten-fold reduction in step size with comparable accuracy -- the method was stable with a step size of 0.025 and unstable with a step size of 0.03.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

The current objectives in this area are to first classify existing methods for the numerical solution of partial differential equations and then investigate in detail those methods adaptable to hybrid computation.

A detailed, refined classification of an area as complex as the numerical solution of partial differential equations is a difficult task, (on the order of 400 references will appear in the bibliography of Technical Note No. 16). Such a detailed classification is probably unnecessary for present purposes and hence the immediate objective is a coarse breakdown which will place in evidence the central problem areas and the major sources of error.
Reasonable progress has been made in studying the literature and the following is a preliminary survey of the area of interest.

Two steps seem to be significant in most procedures which can be implemented on hybrid computers, namely:

(1) The use of some sort of difference approximation in either the time or the space variables to reduce the partial differential equation to a set of ordinary differential equations and

(2) The solution of the set of ordinary differential equations by hybrid techniques.

Each of these general steps can be carried out in many different ways and each has its own characteristics. The problem that arises in step (1) is the "truncation" error of the finite difference approximation with the attendant possibility that the resulting difference equations may not be stable. In step (2) the usual problems of stability and accuracy in solving large sets of ordinary differential equations arise and in addition the typical boundary conditions imposed on the ordinary differential equations are multipoint, a fact that in itself poses a difficult problem.

Many existing procedures have been tailored to take advantage of the special properties of particular types of equations. Hyperbolic, parabolic, and elliptic are common classifications of partial differential equations and many methods can be found designed specifically for each of these classes.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588


Gentlemen:

Project staffing for the Spring Quarter ending in June remains as previously noted - Dr. Hammond, 1/2 time; Mr. Zakrzewski, 1/3 time.

It now appears that Dr. Finn will not work for the contract during the Summer Quarter, and thus the following new personnel will be added during the Summer Quarter with the time commitments noted:

- Mr. W. M. O'Dowd, Graduate Assistant, E.E., approximately 1/3 time.
- Mr. P. B. Bergstrom, Instructor, E.E., 1/2 time.

A biographical sketch for Mr. Bergstrom has been mailed to Dr. Polstorff.

In addition to the new personnel, Dr. Hammond plans to continue a 1/2 time commitment during the Summer Quarter; and Mr. Zakrzewski will tentatively continue at 1/3 time although his plans for the summer are not yet definite.

Notification of funding for Modification No. 15 to the existing contract was received by the Project Director on May 16, 1968. The additional funds are for the purpose of adding a research area which will be identified as "Area 3 - Studies of Hybrid Software."

A conference with the Sponsor discussing directions for the research effort was held at MSFC on May 14, 1968. Detailed plans, which reflect the conclusions reached in this discussion, will be prepared for each research area and presented in the next progress letter. The discussion will include research objectives and staffing plans for the newly funded Area 3.
Typing of the multilith plates for Technical Note No. 16 has been completed and this note is now in the last stage of processing. Final processing of Technical Note No. 17 will begin during the next report period.

The status of efforts in the two assigned areas of study is as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

Current objectives in this area are unchanged from those stated in Progress Letter 39.

With respect to sampling errors in programming linear constant coefficient systems, certain refinements in bounds previously established for sampling error are being worked out concurrently with preparing the results of previous studies for publication. The bounds are expected to be a useful guide to the choice of sampling periods for programming general, not necessarily linear constant coefficient, mathematical models.

In the area of numerical solution of differential equations with widely separated natural frequencies, Treanor's method, evaluated previously for a simple second order example, is now being applied to the heat flow problem which has received attention in previous work.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

The objectives in this area are unchanged from those stated in Progress Letter 41, namely: To first classify existing methods for the numerical solution of partial differential equations and then to investigate in detail those methods adaptable to hybrid computation. A preliminary survey of characteristics of existing methods for numerical solution of partial differential equations was given in Progress Letter 41. During the report period a more detailed examination of existing methods was begun. The study is not yet at a point where a definite summary can be made.

As a part of the work, a weekly academic seminar meeting to study numerical methods in solving partial differential equations has been set up. Eight persons, including a representative of the School of Mathematics, are participating in the seminar.

Respectively submitted,

Joseph L. Hammond Jr.
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812  

Attention: Purchasing Office, PR-EC  

Subject: Monthly Progress Letter 43, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 12 May to 12 June 1968

Gentlemen:

Project staffing through the current May 13, 1968 - June 12, 1968 report period, as previously noted, has been Dr. Hammond - 1/2 time and Mr. Zakrzewski - 1/3 time. For the Summer Quarter beginning June 24, 1968 staffing will be as follows:

- Mr. P. D. Bergstrom, Instructor EE, 1/2 time
- Dr. J. L. Hammond, Professor EE, 1/2 time
- Mr. W. M. O'Dowd, Graduate Assistant, approximately 1/3 time
- Mr. K. Zakrzewski, Graduate Assistant, 1/3 time

Technical Note No. 16 has been completed and mailed to the Sponsor. Technical Note No. 17 should be completed and ready for distribution in the near future.

Detailed plans for each research area have been prepared following the May 14, 1968 conference with the Sponsor. These plans are summarized below along with the statement of current progress.

It was decided during the conference with the Sponsor that an effort should be made to organize a symposium, probably to be held at Georgia Tech, on the general subject of Numerical Techniques in Solving Partial Differential Equations. Because of the time required to set up the symposium and the difficulty in obtaining the desired speakers during the Summer, early Fall would seem to be the earliest practical time for such a meeting. Plans are to seek funds at Georgia Tech for the travel
expense and honoraria of the invited speakers. In addition, suggested
lists of technical areas to be covered and possible speakers will be
compiled along with suggestions as to the organization of the meeting.
When this material has been prepared it will be forwarded to the Sponsor
for comments before taking further action.

Progress and plans for the assigned research areas are as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

Concisely stated, the objectives of this research area are to
obtain general mathematical relations for various types of error in
hybrid computation and develop means for reducing error through compensa-
tion or improved programming.

General equations for sampling error have been derived and are reported
in Technical Note No. 17. The current objective of this area is to complete
preparation of a Technical Note analyzing sampling error in solving linear
constant coefficient equations with a hybrid computer. An important
contribution of this work will be the definition of sampling error bounds
which can be used as a guide in general cases.

After completing this technical note, attention will be given to
preparing for publication work on sampling error in the Euler angle
transformation which has been in progress for some time. Attention will
be given to sampling error in particular transformations and to possible
means for compensating against such error.

Dr. Hammond and Mr. Zakrzewski are expected to work in this area
during the Summer Quarter.

Area 2 - Hybrid Computer Methods for Solving Partial Differential
Equations

The general objectives of this area are to study efficiency and
error in the solution of partial differential equations using hybrid
computers, with the objective of developing means for possibly improving
hybrid programs with respect to these criteria. Technical Note No. 16
is an extensive survey of the literature in this area.

During the current report period attention has been given to studying
the literature in an effort to determine existing methods for numerical
solution of partial differential equations, and to review the possible
ways of implementing these methods on a hybrid computer. Briefly the
more important methods seem to be the following:

(1) Monte Carlo methods
(2) "Brute force" parallel methods
(3) Iterative methods
   (a) discrete space - continuous time
   (b) discrete time - continuous space
   (c) method of characteristics

Monte Carlo methods have not been investigated in detail in the study of the literature. The other methods (and Monte Carlo also) require that the partial differential equation be reduced to a set of ordinary differential equations by an appropriate differencing technique on all but one of the variables. It seems to be generally conceded that the parallel methods require a prohibitive amount of equipment for most practical scale problems. The iterative methods all seem to be practical and the choice of a particular iterative method is dictated by the form of the equation and its boundary values.

All iterative methods require function storage in the digital part of the hybrid computer and playback at appropriate times for use in the analog computer. The function storage and playback leads to sampling errors. Other performance considerations are stability and rate of convergence to the true solution.

Plans in this area call for investigating sampling error, stability and rate of convergence of several hybrid computer methods for solving a simple diffusion equation. As work on this simple problem progresses more general investigations will be planned.

The work begun previously concerned with widely separated natural frequencies will be continued and listed in Area 2 rather than Area 1 since the equations in question come from an ordinary differential equation approximation to a particular partial differential equation.

Dr. Hammond and Mr. O'Dowd are expected to work in this area during the Summer Quarter with the possible assistance of Mr. Zakrzewski in programming example problems.

Area 3 - Studies of Hybrid Software

Concisely stated the objective of this area is to develop efficient hybrid programs for the class of equations which can be expressed in state variable form. The work will lead to software development for the digital part of the hybrid computer.
Suggested plans divide the work into two phases of approximately six months duration, namely: Phase (A) theoretical formulation and Phase (B) hybrid implementations.

In the Phase (A) previously developed error equations and general studies will be used to formulate, if possible, practical rules for determining: (a) which equations are integrated on the analog and which on the digital computer, (b) which functions should be generated on the digital computer, (c) the type and order of the numerical method used in the digital computer, (d) the "best" sequencing for the A/D and D/A converters and (e) the practicability of compensation using previously developed methods. The first phase of the work should result in flow charts for efficient hybrid programs. A report on the first phase of the work will be published and discussed with the Sponsor before proceeding to Phase (B).

Phase (B) will have the objective of implementing the flow charted programs on particular hybrid computers with attention to the required software. Further discussions with the Sponsor are planned before undertaking Phase (B).

Mr. Bergstrom with general supervision from Dr. Hammond is expected to work in this area during the Summer Quarter. Assistance from Computer Center programmers can be obtained when needed. At the present time it would seem that the majority of such assistance would be required in Phase (B).

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
16 July 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 44, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 June to 12 July, 1968

Gentlemen:

There has been one change in the project staffing for the Summer Quarter from that noted in Progress Letter 43. Mr. W. M. O'Dowd has found that any employment on a government supported contract violates the conditions of a NASA Training Grant which he currently holds. Thus he will not be able to work 1/3 time as previously stated. There is a strong possibility that Mr. O'Dowd's Ph.D. thesis will be in an area related to the contract research, and if this is the case, we will provide the Sponsor with a copy of this thesis.

An effort is now being made to replace Mr. O'Dowd with a qualified graduate assistant, but in all likelihood it will be the Fall Quarter before this can be done. Tentative arrangements have been made for Dr. Cecil Alford, who will join the Georgia Tech staff in the Fall as an Associate Professor of Electrical Engineering, to participate on the project 1/2 time. A biographical sketch of Dr. Alford will be forwarded to the Sponsor as soon as one is available.

The present project staffing for the Summer Quarter is the following:

Mr. P. D. Bergstrom, Instructor, EE, 1/2 time
Dr. J. L. Hammond, Professor, EE, 1/2 time
Mr. K. Zakrzewski, Graduate Assistant, 1/3 time
Technical Note No. 17 has been completed and mailed to the Sponsor.

With respect to planning the symposium on Numerical Techniques in Solving Partial Differential Equations, a request has been made to the Dean's Office for help in supporting and planning this activity. Because of vacations, an answer to this request has not yet been received.

Dr. Hammond will be on vacation the first two weeks in August (August 3 - August 17), and thus the next Progress Letter will be approximately one week late.

With respect to the specific research areas, in all three cases work is underway to implement the plans outlined in detail in Progress Letter 43. Progress in each area is, briefly, as follows:

Area 1 - Analysis and Control of Error in Hybrid Computation

A technical note to be entitled Technical Note No. 18, "Sampling Errors in the Hybrid Solution of Linear Constant Coefficient Equations", by Dr. J. L. Hammond, is now in preparation. The first sections of this Note have been drafted, and although some technical work in refining certain error bounds is still in progress, plans are to finish the draft during the next report period.

As noted earlier, attention will be given to the Euler Angle Transformation after completing Technical Note No. 18.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

Efforts in this area are proceeding in two directions: (1) The three iterative methods noted in Progress Letter 43 are being applied to a one dimensional diffusion equation using a digital simulation of the hybrid computer. This work will aid in establishing the characteristics of the basic methods. (2) Stability and convergence of numerical methods for solving partial differential equations are being studied in general in order to evaluate such properties for hybrid computer solutions.

The weekly seminar concerned with numerical methods in solving partial differential equations, which was begun last quarter, has been continued.
Area 3 - Studies of Hybrid Software

General plans for this area were outlined in Progress Letter 43. Work is now underway in what was identified in that Letter as Phase (A). Specifically, hybrid programs for certain examples are being studied theoretically, through the general error equations, and experimentally, using a digital simulation of the hybrid computer, to establish, if possible, rules for dealing with the programming considerations noted in Progress Letter 43.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
August 23, 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 45, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2437
Covering the Period from 12 July to 12 August 1968

Gentlemen:

Project staffing for the Summer Quarter remains as given in Progress Letter 44, namely

Mr. P. D. Bergstrom, Instructor, EE, 1/2 time
Dr. J. L. Hammond, Professor, EE, 1/2 time
Mr. K. Zakrzewski, Graduate Assistant, 1/3 time

Vacations and a trip out of the country by the Dean of Engineering have delayed an evaluation of the request for support of a symposium on Numerical Techniques in Solving Partial Differential Equations. A meeting with the Dean is now scheduled for August 27, 1968, and thus it should be possible to present definite plans for the symposium in the next few weeks.

Two Ph.D. theses on problems related to the contract research program (but not supported by contract funds), were mailed to the Sponsor during the report period. A limited number of additional copies are available if they should be required.

The general goals of the three specific research areas are unchanged from those stated and discussed in detail in Progress Letter 43. Progress during the report period toward achieving these goals is given below for each area.
Area 1 - Analysis and Control of Error in Hybrid Computation

The typed draft of Technical Note No. 18 "Sampling Errors in the Hybrid Solution of Linear Constant Coefficient Equations" was completed during the report period and final processing of the report will be undertaken immediately.

Technical efforts in this area will now shift to a consideration of the Euler Angle Transformation as discussed in earlier progress letters.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

As previously noted, the current objective in this area is to study the characteristics of the three basic iterative methods for solving partial differential equations given in Progress Letter No. 43. To begin the study the simple diffusion equation in one and two dimensions is being solved using a digital simulation of the hybrid computer.

A successful solution has been made for the one dimensional diffusion equation using the discrete space - continuous time method. Work is now in progress on the two dimensional case, and after this is completed the discrete time - continuous space method will be programmed. When all three methods have been applied to the problem a comparison of methods will be made.

In considering the various possible methods for programming partial differential equations on digital or hybrid computers, attention should be called to the paper by McCann ("Simulation Techniques for Distributed Parameter Systems," Proc. Int. Assoc. for Analog Comp., Jan. 1967, p. 30; listed in Technical Note No. 18, p. 15, as Ref. 48) which gives a very complete summary of the methods. A Xerox copy of this paper can be provided if the reference cannot be easily obtained.

The study of the theoretical aspects of stability and convergence of numerical methods for solving partial differential equations has continued during the report period in conjunction with the Seminar mentioned in earlier progress letters.

Area 3 - Studies of Hybrid Software

As noted in Progress Letter 43 the first phase of this work is an examination of five general aspects of programming a hybrid computer. To date, two aspects have been examined in detail - guides for determining the allocation of (a) integration operations and (b) function generation operations between the analog and digital computers.
The approach to these questions is based on the use of the general error equations presented in Technical Note No. 17 and the specific error equations for linear constant coefficient problem equations to be contained in Technical Note No. 18.

For linear constant coefficient problem equations complete expressions for the root shift caused by sampling in the hybrid computer have been worked out and will be reported in Technical Note No. 18. The root shifts are a function of the allocation of integration and function generation operations as well as sampling period and digital execution time.

Bounds on root shift, and efficient computational procedures for approximating root shift for practical scale (but linearized) problems are being worked out. Using these techniques the root shift, or bounds on the root shift, for various allocations of operations can be determined. The allocation resulting in the least root shift can then be identified for use in the hybrid program.

It should be possible at a later stage in the study to develop techniques for linearizing general problem equations so that the results of the current work can be used as a guide to allocation of operations in the more general cases.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
September 17, 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 46, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 August to 12 September 1968

Gentlemen:

Project staffing for the Fall Academic Quarter is planned as follows:

Dr. C. O. Alford, Associate Professor EE, 1/2 time
Mr. P. D. Bergstrom, Instructor EE, 1/2 time
Dr. J. L. Hammond, Jr., Professor EE, 1/2 time

Two changes can be noted in project staffing. Mr. K. Zakrzewski who
has participated on the contract for approximately two years as a Graduate
Assistant has left Georgia Tech to enroll in a business program at Stanford
University. Dr. Cecil Alford, who has recently joined the Georgia Tech staff,
will participate on the contract one-half time. Dr. Alford has had extensive
experience in analog and digital computation and aerospace studies and is ex-
pected to make a significant contribution to our work. Efforts are now underway
to hire one or possibly two Graduate Assistants to replace Mr. Zakrzewski.

Discussions with the Dean of Engineering at Georgia Tech have raised some
questions relative to the technical content of the planned symposium on Numerical
Techniques in Solving Partial Differential Equations which need to be answered be-
fore continuing our efforts to obtain funds for this activity. It seems best to
discuss these questions at the next conference with the Sponsor.
Progress in the three current research areas is as follows:

**Area 1 - Analysis and Control of Error in Hybrid Computation**

Final processing of Technical Note No. 18, "Sampling Errors in the Hybrid Solution of Linear Constant Coefficient Equations" is almost completed and this document should be available to the Sponsor in the next few weeks.

During the past report period the major project effort has gone into Areas 2 and 3. Work on sampling errors in the Euler angle transformation will begin again during the next report period following the plans discussed in previous progress letters.

**Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations**

The major activity in this area is still concerned with the study of the characteristics of the three basic iterative methods for solving partial differential equations. These methods were discussed in Progress Letter No. 43.

The planned approach involves a study of typical elementary, but not trivial, examples to be programmed using the three basic methods. The work is being done on the Univac 1108 digital computer which on small scale problems can provide a very good approximation to the hybrid computer.

The objective of the first phase of the work is a comparison of the three methods on the basis of computed results for several examples. It is expected that several more months will be required in this phase of the activity.

In addition to the computer study, some effort has gone into surveying methods for solving partial differential equations other than the three currently being studied. In particular a method based on transformation of the partial differential equation into a set of ordinary differential equations has been examined.

The Seminar for studying stability and convergence of numerical methods for solving partial differential equations will continue during the Fall Quarter.

**Area 3 - Studies of Hybrid Software**

The current activities in this area were discussed in Progress Letter No. 45 where it was noted that the first two of five general aspects of programming a
Faculty staffing for the Fall Academic Quarter remains as noted in Progress Letter 46, namely:

Dr. C. O. Alford, Associate Professor EE, 1/2 time
Mr. P. D. Bergstrom, Instructor EE, 1/2 time
Dr. J. L. Hammond, Jr., Professor EE, 1/2 time

Three Student Assistants have been hired on an hourly basis to aid in computer programming. These Student Assistants, namely:

Mr. S. Purandare
Mr. C. Stoner
Mr. C. Mundie

are expected to work quarter time on the average and all three plan to work for the project the full Academic year.

A conference with the Sponsor in Huntsville is planned for Thursday, October 17, 1968. A general review of progress will be given at this meeting.
Progress in the three current research areas is as follows:

Area 1 - Analysis and Control of Error in Hybrid Computation

Technical Note No. 18, "Sampling Errors in the Hybrid Solution of Linear Constant Coefficient Equations" has been completed and mailed to the Sponsor. Research in this area is now being directed toward using the general sampling error equations to study such error in hybrid computer programs which perform an Euler angle transformation digitally while solving the remainder of the equations with the Analog Computer.

The general error equations are directly applicable to the Euler angle problem and when applied yield a set of error equations which have the same order as the complete set of equations to be solved on the hybrid computer. Thus, for any specific hybrid computer problem, the error equations can be formulated in a straightforward manner. If a doubling in the order of the machine equations can be tolerated, the error equations can be programmed directly to provide compensation for sampling error. A hybrid program compensated with the error equations would be expected to have sampling error dependent on the sampling period squared rather than the first power of this quantity. This would represent a sizeable reduction in sampling error for most problems.

The objective of the present research is to attempt to reduce the number of error equations required in analysis or compensation of sampling error by exploiting the fact that the Euler angle transformation is the same for a wide class of problems and consists of only three (or possibly four) equations. A study of methods for decoupling the major error equations from the complete set of error equations is now being made. If the work is successful, on the order of six equations will be required for compensation or analysis regardless of the order of the complete hybrid problem.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

The first phase of the study is proceeding via a numerical solution of the one dimensional diffusion equation using the three basic methods.
The study is currently investigating solutions using the continuous time-discrete space (CTDS) technique. This part of the study is considering the effects of quantization, interpolation and time shifting applied to the stored data when using central differencing techniques. The study will also consider error compensation techniques, using the work reported in TN 17 and TN 18.

Another part of the study has been devoted to solutions of the diffusion equation using the discrete time-continuous space (DTCS) method. This method has been reduced to a flow diagram and is currently being programmed for solution. This particular method leads to the solution of an ordinary differential equation with split boundary conditions.

A later aspect of this study will consider solutions by the method of characteristics. At present, the applicability of this technique is still being studied and no specific numerical example has been implemented.

It is anticipated that this study will yield comparative data, for the three methods, pertaining to storage requirements, computing time, convergence and errors.

Area 3 - Studies of Hybrid Software

The major effort in this area is still directed toward formulating rules for (a) allocating the function generation and (b) allocating the integration operations between the analog and digital equipment in a general hybrid program. These two considerations seem to represent the most serious of the five listed in Progress Letter 43.

The problem is being approached through a consideration of the root shift caused by sampling as discussed in detail in Technical Note No. 18. A digital program to calculate root shift, for a general linear constant coefficient problem equation, has been written and is now in the final stage of verification. The program will be used to gain insight into the relation between error and allocation of computer operations.
Therefore the emphasis on work in this area has been on the first two of the five programming considerations listed in Progress Letter 43. An approach to handling programming considerations (c), (d), and (e) will be presented in the near future.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
November 18, 1968

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 48, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 October to 12 November 1968

Gentlemen:

Project staffing for the report period has remained as stated in Progress Letter 47.

A conference with the Sponsor was held at MSFC on October 17, 1968. At this conference a review of progress in the three assigned areas was given and Dr. Polstorff indicated aspects of the work which should be emphasized. In particular Dr. Polstorff requested that the theoretical studies pertaining to the programming of hybrid computers be brought to bear on a particular hybrid programming problem of immediate interest to the Sponsor. In response to this request, work in Areas 1 and 3 was directed toward an analysis of the problem of programming the equations for Saturn 5 on a hybrid computer.

A conference with the sponsor to discuss this study is planned for Thursday, November 21, 1968 at MSFC.

Detailed progress in the current research areas is as follows:

Areas 1 and 3

The techniques being studied for analysis of error in hybrid computation and for programming hybrid computers are being used on the problem of programming the equations for Saturn 5.
The equations for Saturn 5, as furnished by the Sponsor, have been put in state variable form. The sampling error equations of Technical Note No. 18 have been used, both in the general form and specifically for the equations given, to study the effects of sampling error. An attempt is being made to allocate the function generation and integration operations between the analog and digital computers so as to minimize the sampling error subject to requirements on accuracy and speed of problem solution.

Several different methods of compensating for sampling error are also being considered.

**Area 2- Hybrid Computer Methods for Solving Partial Differential Equations**

Dr. Alford attended a conference on Mathematics, Computation and Computers at the University of Illinois during the period October 28 - November 1. The conference dealt with several topics of interest in the solution of partial differential equations. Of particular interest was a paper by R. S. Varga of Case Western Reserve University titled, "Spline Approximations for Nonlinear Differential Equations". The method typically leads to a significant reduction of computing time (digital) for an associated increase in programming. The question of applicability to hybrid systems is still not answered. It is expected that some effort will be devoted to an investigation of this method in future work.

In addition to attending the conference, progress was made toward the general objectives of this area as given in Progress Letter No. 43.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 49, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NA88-2473
Covering the Period from 12 November to 12 December 1968

Gentlemen:

Project staffing for the report period is unchanged from that stated in Progress Letter 47.

A conference with the sponsor to discuss plans for the hybrid simulation of the Saturn 5 space vehicle was held November 21, 1968 at MSFC. It was agreed that Georgia Tech personnel would continue to study this problem and attempt to contribute, principally in the areas of sampling error, methods of compensation and allocation of operations between the analog and digital computers. The study of the Saturn 5 equations is a logical application of the work assigned to Area 1 - "Analysis and Control of Error in Hybrid Computation".

Progress in the three current research areas is as follows:

Area 1 - Analysis and Control of Error in Hybrid Computation

As noted above, the work in this area has been directed toward a study of the Saturn 5 simulation problem. The following aspects of the simulation are receiving attention.

The problem of allocating operations between the analog and digital computers is being studied through the technique of developing a matrix which displays the interactions between the problem variables. The matrix, which relates the roughly two hundred variables identified along the rows and columns, will contain ones where the variables interact and zeros where there is
no interaction. It is hoped that a study of the matrix will make it possible
to suggest allocations which minimize the coupling between analog and digital
variables and hence will minimize the sampling error.

Another approach to the allocation problem is being studied as a part of
the work in Area 3.

The sampling error caused by the discrete nature of the wind data is being
investigated using a simplified model of the space vehicle obtained from the
paper by R. S. Ryan et. al., "Use of Wind Shears in the Design of Aerospace
Vehicles". The model which uses only four state variables, is sufficiently
tractable to make it possible to formulate and study the solution of the error
equations. In spite of its simplicity, the model should accurately describe
at least the gross structure of the error caused by the discrete wind data.

Studies of several types of compensation are planned for the next report
period.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

Work in this area is continuing using the one diimensional diffusion equa-
tion as a vehicle for study. Particular attention during the report period has
been given to function storage which is the central problem in most hybrid
methods for solving partial differential equations.

Typical results for the one dimensional example, which can be extrapolated
to two and three dimensions, serve to indicate the nature of the problem. In
the one dimensional case which has been implemented on the computer, reasonable
results have been obtained using 6 space points. For each of these points,
one function must be stored and each such function requires on the order of
200 points for reasonable definition. Thus the total storage requirement is
200 times 6 or 1200 points for the one dimensional case. In two dimensions
the corresponding figure is 200 times 6 squared or 7200 points and in three
dimensions the value is 200 times 6 cubed or 43,200 points. The latter figure
is of course prohibitively large for most computers and even this figure has
been obtained for a very simple equation.

A study is being made of methods of reducing function storage. The effect
on computational error of a coarser quantization of the points defining each
function has been investigated during the report period. Improvements resulting
from linear interpolation between the stored points is now being studied and
plans call for a later investigation of the possibility of defining each func-
tion by an appropriate subroutine rather than by direct storage.
Area 3 - Studies of Hybrid Software

As a part of this work a linear constant coefficient approximation to a portion of the Saturn 5 system has been formulated and the root shift due to sampling is being studied for these equations. The problem provides an example for checking theoretical work and may provide an order of magnitude approximation to root shifts for the actual nonlinear time varying equations.

A study of the sequencing of the digital operations to minimize sampling error, which is another aspect of the hybrid software study, was initiated during the report period. To illustrate one direction that this work will take, a study of the error equations indicates that execution time has a more serious effect on sampling error than does the sampling rate. Thus, it is important in sequencing the digital operations to output information to the analog computer as rapidly as it can be made available.

Respectfully submitted,

Joseph L. Ham mond, Jr.
Project Director, A-588
January 22, 1969

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 50, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 December 1968 to 12 January 1969

Gentlemen:

Project staffing for the report period is unchanged from that stated in Progress Letter 47 and no change is expected during the winter academic quarter.

Progress in the three current research areas is as follows:

Area 1 - Analysis and Control of Error in Hybrid Computation

As noted in Progress Letter 49, at the suggestion of the sponsor, work in this area is currently being devoted to certain aspects of the Saturn V simulation problem. A detailed summary of progress to date on this problem is contained in a memo to Dr. Polstorff dated January 22, 1969 and further comments here seem unnecessary.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations

As pointed out in Progress Letter No. 49, a major emphasis is being placed on methods for reducing storage requirements. In all of these methods, a central question is the determination of the error induced when a reduced set of data is used in computation.

The one dimensional heat flow equation has been solved using six space points and 200 time points. This data has been quantized in units of 2, 4,
8, 16, and 32 and the solutions obtained for each of these cases. As would be expected, the errors induced, increase as the quantization is made more coarse. The problem now being studied concerns the induced error and methods by which it can be reduced. The first method of error reduction is a data shift. It has generally been concluded that a shift equal to $1/2$ the quantization provides the minimum error. Results to date show this to be untrue, however, and substantial improvements can be made over this generally accepted rule.

Other methods for reducing storage requirements which will be studied include interpolation techniques, both linear and quadratic, and function approximation techniques of the type discussed in Progress Letter 49.

Area 3 - Studies of Hybrid Software

Plans for work in this area were outlined initially in Progress Letter 43. Specifically, the first phase of the work was to be devoted to formulating, if possible, practical rules for determining for a hybrid program:

(a) which equations should be integrated on the analog and which on the digital computers, (b) which functions should be generated on the digital computer, (c) the type and order of the numerical method to be used on the digital computer, (d) the "best" sequencing for the A/D and D/A converters and (e) the practicability of compensation using various methods.

To date items (a) (b) and (d) have received explicit attention as a part of this phase of the work. Item (e) has received some attention as a part of related activities including the Saturn V work.

Plans now call for attention to item (c) during the next report period and in addition, beginning preliminary preparation of a technical note covering Phase (A) of this work.

Respectfully submitted.

Joseph L. Hammond, Jr.
Project Director, A-588
Project staffing for the report period is unchanged from that stated in Progress Letter 47 and no change will be made during the February 12 - March 12, 1969 report period which ends the Winter academic quarter. It is expected that a reduction in the project staff will be made for the Spring academic quarter because of the decrease in funds available for that period.

The following information is provided as required by Modification 16 to the existing contract. All items are for the calendar month of January 1969 and are based on the monthly appropriation statement of the Georgia Tech Engineering Experiment Station.

1. Man-hours expended (estimate) .................. 404 hr.
2. Material dollars expended .................. $  9,42
3. Total dollars expended .................. $  4,380.48
4. Accumulated total dollars expended to date .. $267,261.10

Progress in the three current research areas is as follows.

Area 1 - Analysis and Control of Error in Hybrid Computation

A detailed summary of progress in studying sampling errors and means for compensating against such errors in the Saturn V simulation problem was
contained in a memo to Dr. Polstorff dated January 22, 1969. It is understood that a conference with the Sponsor concerning this simulation problem will be set up at MSFC in the near future.

In the memo referred to above two methods for compensation against sampling error were discussed. These methods were identified as (1) a hybrid scheme based on digital data as described in the reference by Chapelle and (2) an all digital scheme which depends on the use of high speed D-to-A converters transferring data which is updated by a simple calculation requiring little computation time.

During the subject report period a third method of compensation based on use of the error equation of Technical Note No. 17 has been studied. This method would use the software of the digital computer to compute correction signals to be added to the normal zero order D-to-A converter outputs. The type of calculation required to compensate each digital signal is a function evaluation and not the solution of a large set of equations.

All three methods of compensation which have been studied have attractive theoretical features. A choice between the methods should be made on the basis of hardware-software "trade offs" evaluated on both a practical and theoretical basis.

During the next report period some additional attention will be given to all three methods pending the conference with the Sponsor.

**Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations**

The error curves for the one dimensional heat flow equation have been obtained for the quantization values given in Progress Letter No. 50. A number of error curves have also been obtained for varying units of data shift corresponding to each of the quantization values. It is anticipated that this work will be followed by studies using linear interpolation techniques and then possibly higher order interpolation methods. The goal of this phase of the study will be to establish the utility of interpolation methods as a substitute for finely spaced data points. One result of the study will be a set of error curves which indicates the interplay of the techniques of data quantization, data shift and data interpolation.
Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 52, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 12 February to 12 March 1969

Gentlemen:

Project staffing for the current report period is unchanged from that given in Progress Letter No. 47. In keeping with the decrease in funds available after March 12, 1969, the project staff for the Spring academic quarter will be reduced to:

Dr. C. O. Alford, Associate Professor E.E., 1/3 time
Dr. J. L. Hammond, Jr., Professor E.E., 1/3 time

The Student Assistants, Mr. S. Purandare, Mr. C. Stoner, and Mr. C. Mundie, will continue to work on an hourly basis for approximately one quarter time each.

The following expenditures for the calendar month of February are based on the monthly appropriation statement of the Georgia Tech Engineering Experiment Station.

1. Man-hours expended (estimate) ..................... 376 hours
2. Material dollars expended ........................... $ 4,44
3. Total dollars expended ............................... $ 4,225.89
4. Accumulated total dollars expended to date ...... $271,486.99

Progress in the three current research areas is as follows.
Area 1 - Analysis and Control of Error in Hybrid Computation

As noted previously, work in this area has been directed toward a study of sampling errors in the Saturn V simulation problem. A memo detailing progress on this problem has been provided the Sponsor and further activity in this area will be delayed until after a conference with the Sponsor to evaluate several possible directions for continued work.

Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations
(reported by C. O. Alford)

The work in this area continues along the lines outlined in Progress Letter 51. It has definitely been established that the most accurate hybrid solution of a partial differential equation, when using quantized digital data, is not obtained by shifting the data forward in time by an amount equal to one-half the quantization interval. More accurate results can be obtained by using other values of data shift, which are, in general, difficult to determine a priori. It is believed that the interpolation methods, however, will prove far superior to the simple quantized data even when used with the optimum shift. The effects of these methods on the solution error will be investigated during the coming weeks.

The collection of papers on the hybrid solution of partial differential equations received from the Sponsor last month has been reviewed. It seems that the most pertinent work is that outlined by Hara and Karplus in the paper "Application of Functional Optimization Techniques for the Serial Hybrid Computer Solution of Partial Differential Equations." This method is of sufficient interest to be compared with the more conventional finite difference approach. This comparison will be included in the future, but will probably not be undertaken before June 1, 1969.

Area 3 - Studies of Hybrid Software

A technical note covering Phase A of this study is now being prepared. A tentative title and outline of major topics for this note are given below.

Title: Software Techniques for Reducing Hybrid Computer Error
Major Topics:

1. Introduction and Statement of Problem
2. Choice of Numerical Methods
3. Compensation against Execution Time
4. Compensation against the Effects of Zero-Order Hold Converters
5. Allocation of Computing Operations for Linear Constant Coefficient Problem Equations

Some further study is being made with respect to topics (2) and (4).

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 53, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 13 March to 12 April 1969

Gentlemen:

As noted in Progress Letter No. 52 project staffing for the Spring academic quarter will be:

Dr. C. O. Alford, Associate Professor of Electrical Engineering, 1/3 time  
Dr. J. L. Hammond, Jr., Professor of Electrical Engineering, 1/3 time.

The change from that of the Winter quarter took place April 1, 1969. The Student Assistants Mr. S. Purandare, Mr. C. Stoner, and Mr. C. Mundie will continue to work on an hourly basis for approximately one quarter time each.

The following expenditures for the calendar month of March are based on the monthly appropriation statement of the Georgia Tech Engineering Experiment Station.

1. Man-hours expended (estimate) .................. 371 hours
2. Material dollars expended ....................... $12.75
3. Total dollars expended ......................... $4,225.89
4. Accumulated total dollars expended to date .... $275,617.58

Progress in the three current research areas is as follows:
AREA 1--Analysis and Control of Error in Hybrid Computation

As noted in Progress Letter 52, further work in this area and specifically work on the Saturn V simulation problem will be delayed until after a conference with the Sponsor.

AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations

(Reported by C. O. Alford)

At this time it seems to be appropriate to review some recent results in this area. Of several numerical solution techniques which have been evaluated the most general method is still believed to be a finite difference approach. When using this method there are problems related to stability, convergence to the solution and function storage. In general, it has been established that techniques which improve stability and convergence place increased demands on the function generation and storage. Work has therefore been concentrated on aspects of the problem dealing with function generation and storage.

The problem of function generation from point to point data storage has been investigated for several data storage rates. Solution errors in each case have been plotted and they enforce the intuitive feeling that more data points give better solutions.

The data has also been stored at various rates and shifted in various amounts. Solution errors have been plotted and indicate certain improved solutions can be had by the simple algorithm of data shifting.

Linear polynomial function generation is now being studied, but significant results have not been obtained. The goal, as stated in previous letters, is to arrive at function generation and storage routines which; (a) minimize storage requirements, (b) give decided benefits in solution accuracy, and (c) require modest computer execution time.

AREA 3--Studies of Hybrid Software

As noted in Progress Letter No. 52 a technical note covering the work in this area since June 1968 is currently being prepared. The document, which will be entitled Technical Note No. 19, "Software Techniques for Reducing Hybrid Computer
Error," will cover four topics namely: (1) choice of numerical methods for use in the digital portion of the hybrid computer, (2) compensation against effects of zero order hold operation of digital-to-analog converters, (3) compensation against execution time of digital operations and (4) allocation of computing operations for linear constant coefficient problem equations.

During the current report period work continued on the draft of the technical note and on further research with respect to topics one and two.

Respectfully submitted,

\[ \downarrow\text{Joseph L. Hammond, Jr.} \]
Project Director A-588
Gentlemen:

Project staffing remains as noted in Progress Letter 52: Dr. Alford - 1/3 time, Dr. Hammond - 1/3 time and the student assistants Mr. S. Purandare, Mr. C. Stoner and Mr. C. Mundie on an hourly basis.

The following expenditures for the calendar month of April are based on the monthly appropriation statement of the Georgia Tech Engineering Experiment Station:

1. Man-hours expended (estimate) ................. 265 hours
2. Material dollars expended ......................... $2.46
3. Total dollars expended.......................... $2,045.28
4. Accumulated total dollars expended to date........ $277,662.86

Progress in the three current research areas is as follows:

AREA 1--Analysis and Control of Error in Hybrid Computation

As noted in Progress Letter 52, this area including work on the Saturn V simulation problem, is currently inactive pending a conference with the Sponsor.
Some of the results of work in this area are, however, currently being incorporated in Technical Note No. 19 (see the summary of Area 3 activity below).

**AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations**

During the current report period, satisfactory progress has been made in carrying out the program of studies outlined in Progress Letters 51 and 53. A major aspect of this program is the accumulation of data concerning the solution of typical partial differential equations by a variety of methods.

Plans call for completing the computer work concerned with this study of typical examples during the next report period. A technical note covering recent progress in the area will then be prepared.

**AREA 3--Studies of Hybrid Software**

Most of the effort on the project during the current report period has been concentrated in this area. The immediate objective is the preparation of a technical note to be entitled Technical Note No. 19, "Software Techniques for Reducing Hybrid Computer Error."

Much of the work done in both Areas 1 and 3 over the past year is included in the scope of this technical note. All of this work is therefore being reviewed and the results put together and presented in a general and unified fashion. The material being covered includes, among other things, the Memo of January 22, 1969 concerning the simulation of the Saturn V equations.

Satisfactory progress is being made on the technical note and plans call for completing the rough draft during the next report period.

Respectively submitted,

Joseph L. Hammond, Jr.
Project Director A-588
June 17, 1969

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 55, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the period from 13 May to 12 June 1969

Gentlemen:

Planned project staffing for the Summer Quarter (June 16 through September 29, 1969) will be: Dr. Alford--2/3 time, Dr. Hammond--1/2 time and Mr. Barry Folsom, Student Assistant--full time. Mr. Folsom replaces Mr. Stoner and Mr. Purandare who will no longer be employed by the contract.

Since project funds have been reduced to $30,000 for the June 12, 1969--June 12, 1970 year, the two senior staff members cannot be supported for a substantial time commitment throughout the full year. Thus plans now call for Drs. Alford and Hammond to work a significant fraction of time during the Summer Quarter and to participate a much smaller fraction of time during the remainder of the year.

A meeting with the sponsor (represented by Mr. George Prince) took place May 27, 1969 at MSFC. The primary topic of discussion was the Saturn V simulation problem which will be discussed under Area 1 below.
The following dollar expenditures for the calendar month of May are taken
from the monthly appropriation statement of the Georgia Tech Engineering Experi-
ment Station. Man-hours expended are estimated by the project director.

1. Man-hours expended (estimate) .............................. 227 hours
2. Material dollars expended .................................... 00.00
3. Total dollars expended .......................................... $2,366.26
4. Accumulated total dollars expended to date ................. $280,029.12

Progress in the three current research areas is as follows:

AREA 1--Analysis and Control of Error in Hybrid Computation

In the conference with the sponsor noted above, the Saturn V simulation prob-
lem was discussed. Preliminary studies at MSFC seem to indicate a minimum exe-
cution time of 5 ms for the digital portion of the hybrid computer on a reduced scale
problem. Using this figure, the error equations give a sampling error estimate of
0.25% for real time operation and 25% for 100 times real time operation. Since the
latter is the desired type of operation it would seem that some sort of compensation
will be required for the Saturn V simulation.

Some attention will be given to means for accomplishing this compensation
during the next report period.

AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations
(reported by Dr. C. O. Alford)

Solution of typical partial differential equations using interpolation methods
was begun during the current report period. Initial solutions have been made using
the one dimensional diffusion equation with linear interpolation. The goal of this investigation is to determine the trade-off between data storage and polynomial interpolation algorithms. A preliminary technical note covering results in this area is planned following the completion of this phase of the study.

AREA 3--Studies of Hybrid Software

The work on Technical Note No. 19 is progressing more or less on schedule. The final document should be completed by midsummer.

Respectfully submitted.

Joseph L. Hammond, Jr.
Project Director A-588
INTERIM PROGRESS REPORT NO. 1
and
MONTHLY PROGRESS LETTER NO. 55-56

Research Project A-588

DEVELOPMENT OF NEW METHODS AND
APPLICATIONS OF ANALOG COMPUTATION

By J. L. Hammond

Prepared for
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama

Contract No. NAS8-2473

12 March 1968 to 12 July 1969

School of Electrical Engineering
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
INTERIM PROGRESS REPORT NO. 1

and

MONTHLY PROGRESS LETTER NO. 55

PROJECT A-588

DEVELOPMENT OF NEW METHODS AND
APPLICATIONS OF ANALOG COMPUTATION

By J. L. Hammond

Contract NAS8-2473

12 March 1968 to 12 July 1969

Prepared for

George C. Marshall Space Flight Center

National Aeronautics and Space Administration

Huntsville, Alabama
INTRODUCTION


Detailed technical progress made on the subject contract is reported in technical notes issued at irregular intervals timed to coincide with the completion of assigned studies. Eighteen technical notes have been issued on the subject contract. Three notes, Technical Notes Nos. 16, 17 and 18, have been issued in the period of this interim report.

Project activity for the subject period will be summarized under four headings; namely, Project Personnel, Progress on Assigned Tasks, General Activities and Financial Summary.

PROJECT PERSONNEL

During the interim report period Dr. J. L. Hammond, Professor of Electrical Engineering, has served as project director and has devoted one-half time to project activities with the exception of the Spring 1969 academic quarter when his time commitment was one-third. Two other professional staff members were active during the report period, namely Dr. C. O. Alford, Associate Professor EE, and Mr. P. D. Bergstrom, Instructor EE. Mr. Bergstrom worked at a one-half time rate for the period March 12, 1968 through March 12, 1969. Dr. Alford began work on the project in the Fall 1968 academic quarter. His time commitments have been: Fall Quarter 1968 - 1/2 time, Winter Quarter 1969 - 1/2 time, Spring Quarter 1969 - 1/3 time, Summer Quarter 1969 - 2/3 time.
The project employs student assistants on an hourly basis to aid in routine tasks and computer programming. During the report period the following students have been employed during the periods indicated:

Mr. K. Zakrzewski - Spring and Summer Quarter 1968
Mr. S. Purandare - Fall, Winter and Spring Quarter 1968-69
Mr. C. Stoner - Fall, Winter and Spring Quarter 1968-69
Mr. C. Mundie - Fall, Winter and Spring Quarter 1968-69
Mr. B. Folsom - Summer Quarter 1969

With the exception of Mr. Folsom, who is full time for the current Summer Quarter, the student time commitments have ranged from $\frac{1}{4}$ to $\frac{1}{2}$ time.

**PROGRESS ON ASSIGNED TASKS**

During the report period three general tasks have been assigned, namely: Analysis and Control of Error in Hybrid Computation, Hybrid Computer Methods for Solving Partial Differential Equations and Studies of Hybrid Software. The latter task was added in June 1968. Progress in each of these three areas is now discussed.

**Analysis and Control of Error in Hybrid Computation**

The general objectives in this area during the report period have been: (1) to document and refine general equations relating sampling error to basic hybrid computer parameters and (2) to use the basic equations in the study of representative problems. A third objective concerned with programming techniques for reducing sampling error is
discussed with the Area, Studies of Hybrid Software.

Objective (1) was accomplished and the technical details are given in Technical Note No. 17 "Sampling Errors in Closed Loop Hybrid Computer Programs" by J. L. Hammond, May 15, 1968 and Technical Note No. 18, "Sampling Errors in the Hybrid Solution of Linear Constant Coefficient Equations" by J. L. Hammond and G. C. Caldwell, August 30, 1968.

With respect to objective (2), some attention has been given to sampling errors in hybrid implementations for the following problems: a simple Euler angle transformation, the solution of the partial differential equations for a heat flow problem, and the trajectory of a simplified model of the Saturn V. Results of the study of the Saturn V problem have been reported in a Memo to Dr. Polstorff dated January 22, 1969.

**Hybrid Computer Methods for Solving Partial Differential Equations**

The objective of this area during the report period has been to examine existing techniques for solving partial differential equations and to compare these methods with respect to accuracy, stability and ease of implementation when applied to test problems. Improvements in existing methods have also been sought.

Methods for the hybrid solution of partial differential equations fall principally into the following classes

(1) Monte Carlo methods
(2) "Brute force" parallel methods
(3) Iterative methods
   (a) discrete space - continuous time
   (b) discrete time - continuous space
   (c) method of characteristics
(4) Methods for Special Classes of Problems

A preliminary study of each of these classes indicated that iterative methods (a) and (b), which use a finite difference approach, are the most general techniques. Thus, emphasis has been given to an investigation of these methods although some attention is being given to other techniques and to keeping abreast of the current literature.

The iterative methods have been examined for a simple test problem. Results of this work show that the major problems in the use of such methods are stability, convergence to the solution and function storage. It has been established that techniques which improve stability and convergence place increased demands on function storage. Because of the key role of function storage this problem has been isolated for detailed consideration. At the present time characteristics of various methods of function storage are being investigated with the objective of finding storage routines which minimize storage requirements consistent with adequate accuracy and acceptable execution time.

Technical Note No. 16 "Bibliography on Techniques for Solving Partial Differential Equations by Hybrid Computation and Other Methods," by D. L. Finn April 22, 1968, covers work in this area immediately prior to and in the early part of the present report period. Plans call for a technical note summarizing the work with iterative methods of solving partial differential equations and with function storage in the near future when this phase of the study is completed.

Studies of Hybrid Software

This task was initiated in June 1968. The objectives for study included two phases, namely; Phase (A) - a theoretical formulation of
programming rules and Phase (B) - development of hybrid software to implement the rules.

Phase (A) was intended to explore the possibility of establishing rules for determining:

(a) which equations are integrated on the analog and which on the digital computer,
(b) which functions should be generated on the digital computer,
(c) the type and order of numerical methods to be used in the digital computer,
(d) the "best" sequencing for the A/D and D/A converters and
(e) the practicability of compensation for errors.

Phase (A) has been essentially completed. The work has provided considerable insight into questions (a) and (b) and what are hoped to be practical answers for items (c), (d), and (e). A technical note summarizing the results is currently being prepared. Some preliminary aspects of the work are reported in the Memo to Dr. Polstorff referenced above.

GENERAL ACTIVITIES

During the report period conferences with the Sponsor at MSFC took place on the following dates: May 14, 1968, October 17, 1968, November 21, 1968 and May 27, 1969.

Dr. Alford attended a conference on "Mathematics, Computation and Computers" at the University of Illinois during the period October 12 - November 1, 1968. The conference dealt with several topics of interest in the solution of partial differential equations.
Dr. Hammond attended a Systems Science and Cybernetics workshop on "Modeling and Simulation of Distributed Systems" in conjunction with the March 1968 IEEE convention.

Dr. Hammond is Secretary-Treasurer of the Southeastern Simulation Society for the 1968-69 term. In this capacity he has attended the quarterly meetings of this society.


In conjunction with the subject contract, a short course on "Hybrid Computer Solution of Partial Differential Equations" is being planned for October 30-31, 1969, at Georgia Tech. Several authorities in the field will conduct the course.
FINANCIAL SUMMARY

The following figures compiled from the monthly financial reports of the Ga. Tech Engineering Experiment Station give a financial summary of the contract for the period March 1968 through June 1969:

**PROJECT EXPENDITURES**

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George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 57*, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 13 July to 12 August 1969

Gentlemen:

Project staffing continues to be as indicated in Monthly Progress  
Letter 55, namely Dr. Alford, Associate Professor - 2/3 time,  
Dr. Hammond, Professor - 1/2 time and Mr. Folsom, Student Assistant -  
full time. The same time commitments are planned through September  
1969. Beginning in the Fall academic quarter the senior staff members  
will reduce their participation to a much smaller fraction of time in  
keeping with the reduced budget for the 1969-70 project year. A meeting  
with the Sponsor in September or October to plan the directions for the  
reduced effort would be very desirable.

Drs. Alford and Hammond were both on vacation during the 13 July -  
12 August report period and this, of course, reduced the project  
activity.

*The last progress letter, combined with the Interim Report No. 1, was  
incorrectly numbered 55. The number should be changed to 56.
The following expenditures for the calendar month of July are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.

1. Man-hours expended ............... 400 hours
2. Material dollars expended ........ $48.42
3. Total dollars expended ........... $8,113.34
   (Including $4,469.18 cumulative computer charges)
4. Accumulated total dollars expended to date .... $289,857.22

Concerning technical progress, tentative plans are to concentrate efforts in Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations for the remainder of the summer and early Fall. A concentrated effort in this area seems reasonable at this time from the point of view of technical progress and also because of the short course in this area planned for the latter part of October.

Specific comments concerning the three assigned areas are as follows:

AREA 1--Analysis and Control of Error in Hybrid Computation

A discussion of several methods of compensating for sampling error has been prepared for Technical Note No. 19. It would seem that one or more of these methods could be used in the Saturn V simulation.

AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations

During the report period emphasis continued to be given to the study of function storage and to methods for minimizing storage requirements. Data has been obtained in test cases for linear interpolation and quadratic interpolation is now being investigated.
Additional test cases are being studied and in particular work with a two dimensional problem has begun.

**AREA 3--Studies of Hybrid Software**

In the course of preparing Technical Note No. 19 some additional attention has been given to problems associated with the choice of numerical methods for hybrid computation.

In addition to the work which will be reported in Technical Note No. 19, some work has been done on the problem of relative root shift caused by sampling for various allocations of operations between the analog and digital computers. This work extends that reported in Technical Note No. 18.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director A-588
September 26, 1969

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 58, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 13 August to 12 September 1969

Gentlemen:

At the beginning of the Fall Academic quarter Drs. Alford and Hammond will reduce their time commitments to 1/8 time each in keeping with the smaller amount of money available for the 1969-70 contract year. It is expected that one student assistant, Mr. C. Mundie, will work at an average rate of approximately 1/3 time. Time commitments for the present report period have been at the rate projected in Progress Letter 57.

Because of the reduced rate of effort beginning in the next contract period it would seem to be necessary from the point of view of efficiency to concentrate activity in only one of the three assigned areas. A conference with the Sponsor in the near future to discuss plans for the project during the remainder of the contract year would thus seem to be desirable. Tentative plans for the next report period are to give the majority of attention to Area 2—Hybrid Computer Methods for Solving Partial Differential Equations.
The following expenditures for the calendar month of August are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.

1. Man-hours expended ........................................ 360 hours
2. Material dollars expended ................................. $25.15
3. Total dollars expended ................................. $2,066.90
4. Accumulated dollars expended to date ............. $291,924.12

Progress in the three assigned Areas is as follows:

AREA 1--Analysis and Control of Error in Hybrid Computation

During this report period, a detailed study was made of methods for compensating against execution time in the digital generation of functions. The problem arises because digital calculations based on values of arguments at say $t_i$ are not completed until a later time $t_i + \delta$. Methods for compensation against the errors caused by execution time have been developed using the technique of extrapolating the arguments to a time, $t_i + \delta$, greater than $t_i + \delta$. The digital calculation is then made using the extrapolated arguments and the result can be made available at the time $t_i + \delta$, thus effectively avoiding an execution time. Several methods of implementing the compensation procedure have been investigated and there seem to be no apparent practical difficulties.

A significant reduction in execution time errors has an effect on the practicality and effectiveness of methods for compensating against sampling error. The interrelations between these two sources of error have also been investigated during the report period.
The results of the studies noted above were felt to be sufficiently pertinent to warrant delaying the publication of Technical Note No. 19 to permit their inclusion. A rough draft of Technical Note No. 19, including the new material on compensation against execution time errors, has now been completed and final processing of the Note will begin immediately.

**AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations**

Much of the activity in this area has been directed toward setting up the short course in hybrid computer methods for solving partial differential equations. The course will be held at Ga. Tech, October 30 and 31, 1969. In addition to Ga. Tech personnel, lecturers will include T. E. Bullock, Associate Professor of Electrical Engineering, University of Florida; R. M. Howe, Chairman, Department of Aerospace Engineering, University of Michigan; and R. Vichnevetsky, E. A. I. Inc. A formal announcement of the short course was mailed out in the middle of September. It is expected that the short course will provide opportunity for review and exchange of ideas in this area with several well known specialists.

Processing of data pertaining to the study of test cases for investigating the use of linear and quadratic interpolation is continuing. Conclusions from this part of the study should be forthcoming.

**AREA 3--Studies of Hybrid Software**

Studies in this area for the report period were combined with those reported for Area 1.

Respectfully submitted

Joseph L. Hammond, Jr.
Project Director A-588
October 23, 1969

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 59, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 13 September to 12 October 1969

Gentlemen:

Project staffing at the senior level continues to be as indicated in Monthly Progress Letter 58, namely: Dr. Alford, Associate Professor - 1/8 time, Dr. Hammond, Professor - 1/8 time. Mr. W. M. O'Dowd will be added to the contract personnel effective October 24, 1969, as a Graduate Assistant. He will work at a 1/4 time rate through the Fall quarter and at an increased rate beginning in 1970. Mr. O'Dowd is a third year doctoral student. Mr. C. Munie will continue as a Student Assistant on an hourly basis approximately 1/3 time.

The short course on Hybrid Computer Solution of Partial Differential Equations will take place October 30-31, 1969 at Georgia Tech. Registration indicates an attendance of 35-40 from various sections of the country.

As noted in Monthly Progress Letter 58, emphasis in the research activity will be given to Area 2 - Hybrid Computer Methods for Solving Partial Differential Equations for the next several report periods.

The following expenditures for the calendar month of September are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man hours are estimated by the project director.
1. Man-hours expended ............... 280 hours
2. Material dollars expended ........ $21.29
3. Total dollars expended .......... $3,395.22
4. Accumulated dollars expended to date ....... $295,319.34

AREA 2--Hybrid Computer Methods for Solving Partial Differential Equations

Since emphasis has been shifted to this area for the next several months a brief review of progress and plans seems appropriate.

A general survey of hybrid methods for solving partial differential equations indicated that finite difference methods are the most versatile and the most frequently used for general work. Thus the decision was made to investigate such methods first.

Finite difference methods when implemented on hybrid computers have several significant problem areas worthy of attention, namely: stability, convergence, function storage and computation time. All of these areas are interrelated and the function storage area was chosen as having the most potential impact on the field if improved methods could be found.

Preliminary study suggested several alternative approaches to the function storage problem. These are:

(1) omission of certain data points
(2) omission of data points and shifting of stored points
(3) linear interpolation of data
(4) quadratic interpolation of data
(5) improved methods of storage using differential equation approximations to functions

Methods for implementing the first four techniques in elementary cases were developed and a computer study was undertaken to compare the four methods in two test cases, specifically a one and a two dimensional diffusion equation. The approach is to run through a sufficient number of cases with different parameter
values to establish the characteristics of each method and permit a comparison of the methods. It is felt that although the examples are elementary they contain all the attributes pertaining to function storage that are required.

The necessary computer data has been obtained for the one dimensional problem and much of the work has been done for the two dimensional problem.

Short range plans are to process the data for the one dimensional case to establish the characteristics of the methods and compare them. At the same time the computer study for the two dimensional case will be continued. The results from both cases should provide quantitative insight into the characteristics of each method and their comparative merits.

Method (5), above, will be investigated after the completion of the present phase of the study.

AREA 1--Analysis and Control of Error in Hybrid Computation
AREA 3--Studies of Hybrid Software

Technical Note No. 19 which pertains to both Areas 1 and 3 has progressed to the final draft stage and should be completed soon.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director A-588

JLH:brj
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 60, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period from 13 October 1969 to 12 November 1969

Gentlemen:

Project staffing is unchanged from that given in Progress Letter 59, namely: Dr. Alford, Associate Professor - 1/8 time, Dr. Hammond, Professor - 1/8 time, Mr. W. M. O'Dowd, Graduate Assistant - 1/4 time, and Mr. C. Mundie, Student Assistant - approximately 1/3 time.

The following expenditures for the calendar month of October are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man hours are estimated by the project director.
1. Man-hours expended          146 hours
2. Material dollars expended    $21.29
3. Total dollars expended       $2,035.00
4. Accumulated dollars expended to date  $297,354.34

Research activity continues to be concentrated in AREA 2—Hybrid Computer Solution of Partial Differential Equations. Progress and plans in this area are now discussed in detail.

The short course held October 30-31, 1969 culminated an effort to summarize the current state-of-the-art in the hybrid computer solution of partial differential equations. The Sponsor has been given several hundred pages of text for the lectures. A summary of the content of the short course is now given. The first lecture, given by Dr. C. O. Alford of Georgia Tech, gave an introduction to partial differential equations and an overview of the course. The objective of this lecture and the one to follow was to present background theory in several areas as considered necessary for the specialized applications. The lecture gave an introduction to partial derivatives, classifications of partial differential equations, and a look at the problem of obtaining complete solutions to partial differential equations. The latter topic pointed out the need for computational solutions and led to a survey of hybrid computer solution methods.

The second lecture, delivered by Dr. J. L. Hammond of Georgia Tech, was oriented around an introduction to analog and digital computers, definitions, finite differences, elements of matrices, and programming fundamentals.
Lectures three and four were given by Dr. R. M. Howe of the University of Michigan. These lectures were devoted to solution methods based on finite differences. The methods surveyed introduced the CSCT or all analog method, the CSDT or Serial method and the DSCT method. Most of these methods were illustrated by the one dimensional diffusion equation. The DSCT method received the majority of the lecture time and Dr. Howe pointed out that this method appeared to offer the most promising approach for a general procedure. Problems with the method result from excessive storage and data transfer.

An open forum in the evening following Lecture Four produced a multitude of questions relating to education, computers, particular problems, and solution methods. Perhaps the most far-reaching question pertained to an assessment of the future of hybrid computation. To this Dr. Vichnevetsky of Electronic Associates, Inc. replied that most people have looked on hybrid computers in much the same way as they have approached analog computers with the consequence that hardware has far outstripped software. As a result, many people have become disenchanted with hybrid computers without proper reason. Dr. Vichnevetsky did feel, however, that required software would be developed and the true usefulness of the hybrid computer would be realized.

Lecture five was given by Dr. T. E. Bullock of the University of Florida. The lecture was devoted to solution by functional approximation methods. The technique has appeal due to its rather thorough mathematical basis and classical foundation but at the present time the method does not appear to hold wide applicability as a hybrid solution technique. Some work with the techniques
has, however, been done by Dr. Vichnevetsky.

The final two lectures were given by Dr. Vichnevetsky. He concentrated on tying together the various solution techniques and presented considerable detail on the method of characteristics and the CSDT method. Dr. Vichnevetsky presented the stability problems of the CSDT method and a solution which he has implemented. In addition he commented rather extensively on several other methods in an attempt to show there is no one best method. In particular, he emphasized that the current state-of-the-art is an amorphous collection of methods for specialized sets of problems.

In conjunction with the short course, the bibliography of Technical Note No. 16 was updated to the present time by the addition of one hundred and forty six new references. The updated bibliography is included in the material given the Sponsor.

The approach now being taken on the contract is consistent with the overall picture of the area presented by the course. Dr. Howe, in particular, feels that finite difference methods are of central importance for general methods and he specifically cited problems of storage and data transfer which are being examined in our present studies. Our work on the underlying theory necessary for developing hybrid software is along the lines seen to be necessary by Dr. Vichnevetsky.
Our present work is with a two dimensional heat flow equation with the objective of investigating several techniques for minimizing storage. General plans for this work were discussed in the last progress letter.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director A-588
December 29, 1969

George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama  35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 61, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period from 13 November 1969 to 12 December 1969

Gentlemen:

Project staffing for the report period has been unchanged from that  
given in Progress Letter 60. For the academic quarter beginning January 5,  
1970, the time commitments are expected to be Dr. Alford, Associate Professor -  
1/4 time, Dr. Hammond, Professor - 1/8 time, Mr. O'Dowd, Graduate Assistant -  
approximately 1/4 time and Mr. Murdie, Student Assistant - approximately 1/3  
time.

The following expenditures for the calendar month of November are taken  
from the monthly appropriation statement of the Georgia Tech Engineering Experiment  
Station. Man hours are estimated by the project director.
1. Man-hours expended ................................. 74 hours
2. Material dollars expended ......................... 23.99
3. Total dollars expended .............................. $\=-1,461.37$
   (This figure results from a credit of $2,502.12 to correct a previous
   computer charge. Without this correction, total expenditures for the
   month are $1040.75).
4. Accumulated dollars expended to date ............ $295,892.97

A meeting with the Sponsor to discuss recent progress and plans for the
project has been tentatively planned for January 15, 1970 at MSFC.

Research activity continues to be concentrated in Area 2 although some
work was done during the report period in Area 3.

Details of work in the two areas are as follows:

Area 2 -- Hybrid Computer Methods for Solving Partial Differential Equations

General objectives in this area are unchanged from those given in previous
progress letters. Progress is being made on two aspects of the work.

A profile of the various techniques for hybrid solutions of partial differen-
tial equations and a comparative evaluation of these techniques has now been
essentially developed. The short course held at Georgia Tech October 30 and 31
1969 and described in the last progress letter was a major step in accomplishing
this objective. Dr. Alford attended an Eastern Simulation Society Symposium on
This served as a further input in firming up our opinions concerning the
directions of present work in this area and the techniques which are of potential
value in applications such as those encountered at MSFC.
A verbal report on this part of the study will be given at the meeting with the Sponsor planned for January 15, 1970.

The other activity in this area, namely investigating techniques for minimizing function storage in the class of methods depending on finite difference approximations, is continuing along the lines previously discussed. Specifically a two dimensional example is being studied with a polynomial being used to extrapolate data and reduce storage.

**Area 3 -- Studies of Hybrid Software**

Some attention was given during the report period to a comparison of numerical methods based on z-transforms and classical methods such as the Runge-Kutta method. Quantitative results seem difficult to obtain for non-linear equations of a practical nature. Fowler's method, which has received some recent attention, is based on z-transforms with the property that subjective judgments are required as a part of the procedure. This latter property would seem to made the use of Fowler's method risky for equations whose solutions are not known approximately a priori. Reduction in computing time is of course an advantage claimed for the method.

Further work on numerical integration methods is planned but priority will be given to the partial differential equation area.

Respectfully submitted,

Joseph L. Hammond  
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 62, Project A-588
"Development of New Methods and Applications
of Analog Computation"
Contract No. NAS8-2473
Covering the Period 13 December 1969 to 12 January 1970

Gentlemen:

Project staffing for the period December 13, 1969 to January 4, 1970
continued as given in Progress Letter No. 60. As noted in Progress Letter
No. 61, time commitments for the Winter academic quarter, beginning January
5, 1970, are the following: Dr. Alford, Associate Professor - 1/4 time,
Dr. Hammond, Professor - 1/8 time, Mr. O'Dowd, Graduate Assistant - hourly,
approximately 1/4 time and Mr. Mundie, Student Assistant - hourly, approxi-
mately 1/3 time.

The following expenditures for the calendar month of December are taken
from the monthly appropriation statement of the Georgia Tech Engineering
Experiment Station. Man hours are estimated by the project director.
1. Man-hours expended ........................................ 53 hours
2. Material dollars expended ............................... $39.29
3. Total dollars expended ................................... $491.52
4. Accumulated dollars expended to date ............ $296,384.49

A meeting with the Sponsor to discuss recent progress and plans for the project took place January 20, 1970 at MSFC. The project is presently funded to June 12, 1970 with a free balance (at the end of December 1969) of approximately $9500 for personal services. In view of the limited funds available and the fact that a final report is required at the end of this contract year, it was agreed that efforts to wind up existing work should begin immediately.

Comments concerning the work in Areas 2 and 3 are presented below:

**Area 2 -- Hybrid Computer Methods for Solving Partial Differential Equations**

Work in this area has continued along the lines specified in Progress Letter No. 59. Results are not yet in a form to justify detailed comment.

**Area 3 -- Studies of Hybrid Software**

Technical Note No. 19 "Hybrid Computer Software for Solution Error Reduction", by C. O. Alford and J. L. Hammond, Jr. has now been completed and is currently being reproduced. The Note should be mailed to the Sponsor in the near future.

Some current work is being done in comparing numerical integration methods from the point of view of accuracy and stability.

Respectfully submitted.

Joseph L. Hammond, Jr.
Project Director, A-588
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 63, Project A-588
"Development of New Methods and Applications of Analog Computation"
Contract No. NAS8-2473
Covering the Period 13 January 1970 to 12 February 1970

Gentlemen:

Project staffing for the report period was unchanged from that noted in Progress Letter 62.

The following expenditures for the calendar month of January are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man hours are estimated by the project director.

1. Man-hours expended ........................................ 96 hours
2. Material dollars expended .............................. $29.42
3. Total dollars expended ................................. $1,355.25
4. Accumulated dollars expended to date ........... $297,739.73
Efforts on the project are now devoted exclusively to winding up the studies currently underway. In this connection, in order to obtain the maximum value from the work of two students (one a graduate research assistant and one undergraduate research assistant) who are employed through the spring academic quarter, an extension of time without additional funds has been requested. If granted, the extension of time to September 1970 will make it possible to complete work in progress between now and June 1970 and then prepare the final report during the summer.

It would seem desirable to give additional attention to three aspects of work in progress before the conclusion of the project. This will involve the following: (1) documentation of an error equation for systems using a first order (rather than a zero order) hold, (2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19, and (3) some additional work with methods for solving partial differential equations.

During the report period attention has been given to items (1) and (3). It is expected that item (1) will be completed soon and that work will continue on items (2) and (3) through the academic spring quarter.

Respectfully submitted,

"Joseph L. Hammond, Jr.
Project Director, A-588

JLH:emy
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 64, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 February 1970 to 12 March 1970

Gentlemen:

Project staffing for the report period was unchanged from that noted  
in Progress Letter No. 63.

The following expenditures for the calendar month of February are  
taken from the monthly appropriation statement of the Georgia Tech Engineering  
Experiment Station. Man hours are estimated by the project director.

1. Man-hours expended .......................... 60

2. Material dollars expended .................... $20.08

3. Total dollars expended ....................... $1289.90

4. Accumulated dollars expended to date ....... $299,029.63
In Progress Letter No. 63 a summary of plans for winding up studies currently under way before expiration of the contract was given. Three items requiring additional attention were noted, namely: (1) documentation of an error equation for systems using a first order (rather than a zero order) hold, (2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19, and (3) some additional work to conclude the survey of hybrid methods for solving partial differential equations.

During the report period development of the error equation referred to in item (1) was completed and plans are being made to summarize the development in a technical note.

With respect to item (2) an example will be chosen and several methods of compensation will be investigated using the digital hybrid computer simulation program developed earlier on the contract. If time permits, the compensation techniques will also be investigated with a small scale hybrid facility presently being set up in the School of Electrical Engineering at Georgia Tech.

The work pertaining to item (3) involves completion of the evaluation of several hybrid computer methods for solving partial differential equations through a digital simulation study of one and two dimensional test samples.
In addition to completing the development of the first order error equation, some progress was made during the report period with respect to items (2) and (3). It is expected that the work on all three items will be completed by June 1970 and that the extended period of the contract (June through September) can be used for preparing the final report.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588

JLH:emy
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 65, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 March 1970 to 13 April 1970

Gentlemen:

Project staffing for the report period was unchanged from that noted in Progress Letter 64.

The following expenditures for the calendar month of March are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.

1. Man-hours expended ................................... 72 hours
2. Material dollars expended ............................... $56.42
3. Total dollars expended ................................. $1260.92
4. Accumulated dollars expended to date ............... $300,290.55
As stated in the previous Progress Letter, plans for winding up current work before expiration of the contract call for the following: (1) documentation of an error equation for systems using a first order (rather than a zero order) hold, (2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19, and (3) some additional work to conclude the survey of hybrid methods for solving partial differential equations.

During the report period attention was given to items (1) and (2). With respect to the former, some progress was made in preparing a technical note covering this equation. With respect to the latter, modifications to the digital hybrid computer simulation program were carried out to make it possible to truncate word lengths at various points in the program in order to more accurately simulate A/D converters.

In addition to the three tasks noted above which will wind up work in progress, every effort will be made to document completed studies both for the benefit of the sponsor and in the open literature. In this connection a write up is being prepared on numerical integration techniques. The work, which extends portions of Technical Note No. 19, will be prepared probably as a Technical Note and then for publication in the open literature.

A paper covering earlier work on the digital hybrid computer simulation program, along with recent modifications, has been accepted for presentation at the 1970 Summer Computer Simulation Conference, June 10-12, at Denver, Colorado.

Respectfully submitted,

Joseph L. Hammond, Jr.
Project Director, A-588

JLH:emy
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama  35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 66, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 April 1970 to 13 May 1970

Gentlemen:

Project staffing for the report period consisted of Dr. Alford,  
Associate Professor - 1/4 time and Dr. Hammond, Professor - 1/8 time.  
The two student assistants assigned to the project have been unable to commit a significant amount of time to the project. Thus plans are now being made to engage one of these students, Mr. O'Dowd - Graduate Assistant, and possibly two other graduate assistants during the summer quarter.

The following expenditures for the calendar month of April are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.
1. Man-hours expended ...................... 68 hours
2. Material dollars expended .............. $17.29
3. Total dollars expended .................. $1,137.15
4. Accumulated dollars expended to date .. $301,427.70

Plans for winding up current work before expiration of the contract were stated in Progress Letter No. 64. The plans consist of the following:
(1) documentation of an error equation for systems using a first order hold,
(2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19 and (3) some additional work to conclude the survey of hybrid methods for solving partial differential equations.

In addition, as noted in Progress Letter No. 65, a paper is being prepared on the digital hybrid computer simulation program for presentation at the 1970 Summer Computer Simulation Conference June 10-12 at Denver, Colorado. Material on numerical integration techniques which extends portions of Technical Note No. 19 is also being prepared for publication.

During the report period the paper for the Summer Computer Simulation Conference and preparation of the material on numerical integration techniques received the most attention. Item (1) in the first listing above is essentially complete. Progress on items (2) and (3) has been slower than expected due to lack of time on the part of the two student assistants assigned to the project. Since the students are on an hourly basis, the funds not used now will be carried over to the summer quarter. Plans now call for the incompleted work
on items (2) and (3) to be done during the summer quarter in time to be included in the final report.

Respectfully submitted.

Joseph L. Hammond, Jr.
Project Director, A-588
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama  35812  

Attention:  Purchasing Office, PR-EC  

Subject:  Monthly Progress Letter 67, Project A-588  
"Development of New Methods and Applications  
of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 May 1970 to 12 June 1970  

Gentlemen:  

Project staffing for the report period remained as noted in previous  
letters, namely, Dr. Alford, Associate Professor - 1/4 time and Dr. Hammond,  
Professor - 1/8 time. Three Graduate Assistants have been employed on the  
contract for the summer quarter. These assistants are Mr. S. T. Li - 25%  
time, Mr. J. Leon - 83% time and Mr. W. M. O'Dowd - 50% time.  

The following expenditures for the calendar month of May are taken  
from the monthly appropriation statement of the Georgia Tech Engineering  
Experiment Station. Man-hours are estimated by the project director.
1. Man-hours expended .......................... 64 hours
2. Material dollars expended ................... $ 19.78
3. Total dollars expended ....................... $ 2,549.87
4. Accumulated dollars expended to date ....... $303,977.57

A paper entitled "DIHYSYS - A Hybrid Systems Simulator" was delivered at the Summer Computer Simulation Conference June 10-12, 1970 at Denver, Colorado. This paper describes work done on the contract over a period of several years pertaining to simulation of a hybrid computer; (including A/D and D/A conversion, execution time, word length, etc.) on a digital computer.

The three tasks receiving attention in winding up work in progress before the contract expiration in September are: (1) documentation of an error equation for systems using a first order hold, (2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19 and (3) some additional work to conclude the survey of hybrid methods for solving partial differential equations. A final report will, of course, be prepared before the conclusion of the contract.

At the present time task (2) is receiving the most attention. Task (1) is essentially completed and task (3) will be undertaken during July and August.

With respect to task (2), several of the methods for compensating hybrid computers to reduce error due to execution time and first order hold will be implemented for a typical small scale example using the digital hybrid computer simulator. The example will be used to study the extent of error
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812

Attention: Purchasing Office, PR-EC

Subject: Monthly Progress Letter 68, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 June 1970 to 12 July 1970

Gentlemen:

Project staffing for the report period and until the end of the contract is: Dr. Hammond, Professor - 1/4 time, Mr. S. T. Li, Graduate Assistant - 1/4 time, Mr. J. Leon, Graduate Assistant - 83% time, and Mr. W. M. O'Dowd - 1/2 time.

Dr. Hammond was on vacation the last two weeks in July.

The following expenditures for the calendar month of June are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.
1. Man-hours expended ......................... 66 hours
2. Material dollars expended .................... $ 17.38
3. Total dollars expended ....................... $ 1,563.84
4. Accumulated dollars expended to date ..... $ 305,541.41

For the past several months efforts have been directed toward documenting and completing work in progress before the contract expiration date, September 30, 1970. As has been noted in previous progress letters, the principle tasks receiving attention are: (1) documentation of an error equation for systems using a first order hold, (2) some practical experimentation with the methods of compensation suggested in Technical Note No. 19 and (3) some additional work to conclude the survey of hybrid methods for solving partial differential equations.

During the report period satisfactory progress was made on tasks (2) and (3). With respect to task (2), it was noted in Progress Letter No. 67 that several methods of compensation are being evaluated with digital hybrid computer simulation. This work is progressing well so that it may also be possible to program a simple compensation example on a true hybrid system consisting of a Honeywell 316 digital computer and a small Donner analog computer.

Previous work in the area of task (3) has resulted in numerical data from the digital simulation of several hybrid computer methods for solving one and two dimensional forms of the diffusion equation. Present activity in this area is directed toward writing up the results of this work.
Plans call for documenting as much as possible of the work now being completed in one or more technical notes. The final report will then consist of a summary of accomplishments during the complete period of the contract with appropriate references to the technical notes documenting the technical work.

Respectfully submitted.

Joseph L. Hammond, Jr.
Project Director, A-588

JLH:emy
George C. Marshall Space Flight Center  
National Aeronautics and Space Administration  
Huntsville, Alabama 35812  

Attention: Purchasing Office, PR-EC  

Subject: Monthly Progress Letter 69, Project A-588  
"Development of New Methods and Applications of Analog Computation"  
Contract No. NAS8-2473  
Covering the Period 13 July 1970 to 12 August 1970  

Gentlemen:  

Project staffing for the report period and until the end of the contract is: Dr. Hammond, Professor - 1/4 time, Mr. S. T. Li, Graduate Assistant - 1/4 time, Mr. J. Leon, Graduate Assistant - 83% time, and Mr. W. M. O'Dowd - 1/2 time.  

The following expenditures for the calendar month of July are taken from the monthly appropriation statement of the Georgia Tech Engineering Experiment Station. Man-hours are estimated by the project director.
1. Man-hours expended .................. 340 hours
2. Material dollars expended .......... $ 3.38
3. Total dollars expended ............. $ 2,932.79
4. Accumulated dollars expended to date .... $308,474.20

Project activities are now all directed toward final documentation of completed work. Plans call for publishing the final report during the month of September.

One or more technical notes covering progress during the latter phases of the contract will be published but plans are not yet final with respect to these notes.

Respectfully submitted,

/\ Joseph L. Hammond, Jr.
Project Director, A-588

JLH:emy
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION
I - An Experimental Electronic Generalized Integrator
II - Nonstationary Noise for Monte Carlo Studies

by
R. S. Johnson
F. R. Williamson
and
R. D. Loftin

QUARTERLY PROGRAM REPORT
on
Contract No. NAS8-2473

12 September 1961 - 12 December 1961

For
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

I - An Experimental Electronic Generalized Integrator
II - Nonstationary Noise for Monte Carlo Studies

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R. S. Johnson
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QUARTERLY PROGRAM REPORT
on
Contract No. NAS8-2473

12 September 1961 - 12 December 1961

For
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted:
Robert S. Johnson
Project Director A-588

Approved:
P. Dixon, Head
Special Problems Group
Physical Sciences Division
# CONTENTS

<table>
<thead>
<tr>
<th>I</th>
<th>BACKGROUND INFORMATION</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1-2. Project Objectives</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>ELECTRONIC GENERALIZED INTEGRATOR</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2-1. Introduction</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2-2. Generalized Integration</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2-3. The Electronic Generalized Integrator</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2-4. The Incremental Detector</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>2-5. The Electronic Sampling Gate</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2-6. The Analog Program</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2-7. Work Completed</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2-8. Future Work</td>
<td>13</td>
</tr>
<tr>
<td>III</td>
<td>GENERATION OF NONSTATIONARY NOISE FOR ANALOG-COMPUTER MONTE CARLO STUDIES</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>3-1. Analysis of Certain Stochastic Processes</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>3-2. Functionals Useful in Specifying Nonstationary Noise</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>3-3. Linear Networks for Nonstationary Shaping</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>3-4. Planned Work for the Coming Quarter</td>
<td>27</td>
</tr>
</tbody>
</table>

## List of Figures

1. Geometrical Interpretation of a Definite Integral  3
2. Block Diagram of Analog Program Used for Generalized Integration  3
3. Basic Block Diagram of the Electronic Generalized Integrator  6
4. Typical Waveforms Found in the Electronic Generalized Integrator  8
5. Block Diagram of the Threshold Detectors and Logic Section  10
6. Circuit Diagram of the Threshold Detectors  11
7. Circuit Diagram of the Electronic Sampling Gates  11
8. Analog Program of the Electronic Generalized Integrator  12
Chapter I
BACKGROUND INFORMATION

1-1. Introduction

Georgia Tech Research Project No. A-588 was established on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at the George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project has been assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates under the Special Problems Group of the Engineering Experiment Station's Physical Sciences Division.

1-2. Project Objectives

In accordance with decisions reached during the project organizational meeting with the Contracting Officer's Representative (Dr. W.P. Krause of MSFC), primary emphasis has been placed on areas A and B mentioned above. It was agreed that the following specific assignments would be undertaken initially:

Task I. Investigation of electronic techniques for integrating analog voltage signals with respect to arbitrary variables. The task includes construction of a working model of a generalized integrator suitable for use with existing analog equipment.

Task II. Investigation of analog techniques for the generation of non-stationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian, band-limited, white noise.
Chapter II
ELECTRONIC GENERALIZED INTEGRATOR

2-1. Introduction

The ability to perform integration with respect to an arbitrary variable can increase the flexibility of the general-purpose analog computer. We may distinguish the process of integration with respect to variables other than time as "generalized integration." An all-electronic device is being developed under this project to perform such integration. It will be referred to herein as the Electronic Generalized Integrator. Electromechanical devices designed for the same purpose, but with more limited bandwidth, have been previously described in the literature.

2-2. Generalized Integration

In order to understand better the operation of the Electronic Generalized Integrator, we should briefly examine the definition of a definite integral. This is expressed in Equation (2-1), where the function \( f(X) \) is defined to be continuous over the interval between \( a \) and \( b \).

\[
\int_a^b f(X) \, dX = \lim_{n \to \infty} \sum_{i=1}^{n} f(X_i^') \Delta X_i
\]

where

\[
\Delta X_i = X_i - X_{i-1}
\]

and

\[
X_{i-1} < X_i^' < X_i
\]

The limit is evaluated as \( n \) is increased and as the maximum of the numbers \( \Delta X_1, \ldots, \Delta X_n \) is made to approach zero. A geometrical interpretation of Equation (2-1) is illustrated in Figure 1. If we now define \( \Delta X \) to be a constant and make this value very small, Equation (2-1) may be replaced by the following approximation:

\[
\int_a^b f(X) \, dX = \Delta X \sum_{i=1}^{n} f(X_i^')
\]

This approximation describes the integral as the summation of a number of
AX MAGNITUDE DEI_ECTOR AX SIGN DETECTOR.

At PULSE WIDTH GENERATOR

SAMPLE AND DIFFERENCE HOLD CIRCUIT AMPLIFIER

POLARITY SWITCH

Figure 2: Geometrical Interpretation of a Definite Integral

Figure 2: Block Diagram of Analog Program Used for Generalized Integration
samples of \( f(X) \) times the constant \( \Delta X \). Since \( X \) and \( f(X) \) will both be functions of time, we may make the substitution of a sampling width of \( \Delta t \) seconds for the \( \Delta X \) appearing in Equation (2-2). This substitution will require that the length of the time \( \Delta t \) for which the sample is taken be less than the shortest time required for a change in the variable \( X \) of amount \( \Delta X \).

The instrumentation of Equation (2-2) was described by Bekey in 1958\(^x\). A simplified block diagram of the analog program he used to perform this instrumentation is illustrated in Figure 2. In this program, the variable \( X \) is examined for changes of a preset amount which is defined to be \( \Delta X \). These changes are detected by storing the value of the \( X \) input at some time \( t_o \) in the Sample and Hold Circuit and comparing this value with the value of \( X \) at some later time \( t_1 \). The comparison is made by the Difference Amplifier and the output of this section may be described as in Equation (2-3):

\[
(2-3) \quad E_1 = K[X(t_1) - X(t_o)] .
\]

When the magnitude of this value reaches the predetermined level, an output signal is generated in the \( \Delta X \) Detector. This signal causes a sample of \( f(X) \) that is \( \Delta t \) seconds long to appear at the input of the Accumulator. The time, \( t_o \), at which the Sample and Hold Circuit is reset to the value of the variable \( X \) is the instant just after that when \( f(X) \) is sampled. Since the change in the \( X \) input may be either positive or negative, the output of the Difference Amplifier is simultaneously examined for its polarity by the Sign Detector. The Sign Detector controls the polarity of the \( f(X) \) samples that appear at the input of the Accumulator. If the incremental change is negative, the negative of \( f(X) \) is sampled. The output of the Accumulator then contains the summation of all the input samples for \( X \) on the interval between \( a \) and \( b \). If the value of \( \Delta X \) that is preset into the \( \Delta X \) Detector is sufficiently small when compared to the difference between the two limits of integration, then the output of the Accumulator is a good approximation to the integral as expressed in Equation (2-2).

2-3 The Electronic Generalized Integrator

The Electronic Generalized Integrator constitutes an instrumentation of Equation (2-2) which is similar in many respects to that used by Bekey. It

differs primarily in the method whereby the incremental changes in the X input are detected and in the use of electronic switches for the sampling gates. The basic block diagram is shown in Figure 3.

The integrator may be divided into two separate parts, the first being the section used for the detection of incremental changes in the X input. This Incremental Detector section is a simplified version of the A/D Converter*. The information derived from this section may be considered an incremental digital representation of the X input. When the input to this section changes by a prechosen amount, the output signal relays this event and information on the sign of this change to the sampling circuits contained in the other section of the integrator.

The Incremental Detector is a closed-loop circuit that uses an operational amplifier with capacitive feedback for the analog storage element. The inputs to this amplifier are the Reset Pulses and the analog variable X. The analog input to the amplifier is coupled through a series capacitor which results in the analog signal appearing at the output of this amplifier with a sign inversion and a suitable scale factor. The Reset Pulses are introduced to the grid of this amplifier through a series resistor which results in the integration of these pulses. The volt-second area of the Reset Pulses and the integrating gain of the operational amplifier circuit are so chosen that the change in amplifier output voltage resulting from one Reset Pulse is equal to that produced by a change in the input analog signal of an amount equal to ΔX. The output voltage of this amplifier is the error voltage of the closed loop circuit. When it exceeds an amount equal to ΔX, this event is sensed by the Threshold Detector. The output of the Threshold Detector is used to generate a Reset Pulse and to close the sampling gate when this event occurs. The polarity of the Reset Pulse is selected to reduce the magnitude of the voltage at the output of the operational amplifier providing negative feedback in the closed-loop circuit of the Incremental Detector section. An illustration of typical waveforms is given in Figure 4.

The second portion of the Electronic Generalized Integrator has circuitry similar to that described in Section 2-2. Electronic switches are used for the

Figure 3: Basic Block Diagram of the Electronic Generalized Integrator
sampling gates in order to allow the sampling time of the variable \( f(X) \) to be decreased. This decrease is necessary in order to increase the bandwidth of the Generalized Integrator. The accumulation of the samples of \( f(X) \) is accomplished with an operational amplifier with capacitive feedback and resistive input. The output voltage of this amplifier changes in a staircase manner, with each voltage step being proportional to the amplitude of the corresponding sample of \( f(X) \). This effect is illustrated in Figure 4. For the output of the Electronic Generalized Integrator to be a useful representation of the integral of \( f(X) \), the size of \( \Delta X \) must be made small so that the curve pictured in Figure 4 approaches a smooth curve.

2-4. The Incremental Detector

It was pointed out in the previous section that the Incremental Detector of the Electronic Generalized Integrator is a simplified version of the Aid Converter. The detailed block diagram of this section is shown in Figure 5. Since the need for digital readout circuits has been eliminated, a simplification in the logic circuit may be made to reduce the total number of stages required. The other simplification is based on the elimination of the absolute-value circuit from the analog program. This was done in an effort to avoid some of the difficulties experienced in the frequency response in this section of the AID Converter. The absolute value circuit has been replaced by separate positive and negative threshold detectors. The threshold detector circuit is identical to that used in the AID Converter. It is basically a NOR circuit in which the condition at the output is determined by both of the input signals. The circuit is shown in Figure 6. One input signal is a square wave that is supplied as an interrogation pulse to both detectors. If and only if the error voltage is above the threshold level of a detector during the interval of the interrogation pulse, a pulse appears at the output of this detector. The Positive Threshold Detector is a complementary circuit of the Negative Threshold Detector. That is, the NPN transistor is replaced by a PNP transistor and all voltages are reversed in sign. It then performs identically to the Negative Threshold Detector for error voltages of the opposite polarity. A pulse from the output of one detector indicates that the \( X \) variable has changed by an amount equal to \( \Delta X \) in the positive direction, while a pulse from the output of the other detector indicates that the \( X \) variable has changed by an amount equal to \( \Delta X \) in the negative direc-
Figure 4: Typical Waveforms Found in the Electronic Generalized Integrator
tion. The reset pulse then reduces the magnitude of the error voltage below the threshold value.

A pulse occurring at the output of a given Threshold Detector is amplified and used to set a flip-flop circuit that is associated with that detector. A pulse from a timing source contained in the Electronic Generalized Integrator is applied simultaneously to the reset inputs of both flip-flop circuits. This pulse follows the Interrogation Pulse after an accurately controlled interval. This arrangement allows the output pulses of the flip-flops to be used to control the length of the Reset Pulses in the Incremental Detector and the length of the \( f(X) \) sampling pulses appearing at the input of the Accumulating Integrator.

2-5. The Electronic Sampling Gate

The Sampling Gate is a transformer-driven switching circuit using two transistors as shown in the circuit diagram of Figure 7. The transformer connections to the transistors are so phased that an input signal into the primary of the transformer turns on both transistors and causes them to saturate. In the absence of this input signal, no base current is supplied to the transistors and they may be considered in a cutoff state. This condition is equivalent to having a very large resistance between the input and the output terminals of the gate circuit. When the pulse is applied to the input of the transformer the two transistors saturate, reducing the equivalent resistance of the gate circuit by a factor of approximately a million. Since the two transistors are connected with their emitters together, they form a bidirectional switch and will accept signals of both polarities. The time required for the gate to change states may be made small compared to the time the gate is closed. Errors due to the finite resistance in the gate circuit in the open condition are somewhat compensated by the second gate circuit, which is connected to a voltage that is equal in magnitude but opposite in sign to that connected to the first gate. If this compensation is not complete, the remainder of the analog signal may be nulled out. The resistance in the gate circuit remaining in the on period is in series with the resistor to the grid of the Accumulating Amplifier and is accounted for in adjusting the gain of this circuit.

2-6. The Analog Program

The analog program included in the Electronic Generalized Integrator is illustrated in Figure 8. Two operational amplifiers are being included in the
Figure 5: Block Diagram of the Threshold Detectors and Logic Section
Figure 6: Circuit Diagram of the Threshold Detectors

Figure 7: Circuit Diagram of the Electronic Sampling Gates
Figure 8: Analog Program of the Electronic Generalized Integrator
initial design. If drift problems in the Storage Integrator contained in the Incremental Detector section become significant, it may be necessary to share the amplification in this stage over two amplifiers. Two relays are included in the circuits to provide a means of slaving the Electronic Generalized Integrator to the controls of an analog computer. The relay in the circuit of the Accumulator Integrator is used to bring the output of this amplifier to zero, which resets the Electronic Generalized Integrator. This relay is intended to be controlled by the RESET control on the analog computer. The relay in the circuit of the Storage Integrator of the Incremental Detector is used to place the Electronic Generalized Integrator in a HOLD condition. This relay resets the Storage Integrator, holding the output voltage of this amplifier at zero, but not disturbing the voltage stored in the Accumulator Integrator.

2-7. Work Completed

The major portion of the design of the Electronic Generalized Integrator has been completed and the parts necessary for construction of a working model have been placed on order. It is intended that this working model be a complete, self-contained unit capable of being used with a standard general-purpose analog computer.

Construction has begun on a minor portion of the circuit, but this work has not progressed very far due to the lack of parts. A breadboard of the circuit used in the Sampling Gate has been built and preliminary evaluation of this circuit has shown that it will operate with a linearity of better than 0.2 percent over the intended full-scale range. Better performance is expected by adjusting the operating point of the gate transistors. If still further improvement is needed, matched transistors will be used.

2-8. Future Work

The work during the next quarter should include the completion of a working unit of the Electronic Generalized Integrator and a preliminary evaluation of its performance.
Chapter III

GENERATION OF NONSTATIONARY NOISE FOR ANALOG-COMPUTER MONTE CARLO STUDIES

Task II of GIT/EES Project A-588 concerns the investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian, band-limited, white noise. As discussed in the Sections below, efforts during the past quarter have centered about three principal topics—analysis of certain specific stochastic processes, selection of statistical functionals useful in specifying nonstationary noise, and investigation of linear networks for nonstationary shaping.

Current plans are to issue separately a project technical note covering the fundamentals of stochastic process theory as it pertains to Task II. This note will serve as a project reference manual and will define several functionals which are inconsistently defined in the open literature.

3-1. Analysis of Certain Stochastic Processes

It is not possible to exhibit a convenient representation of the arbitrary stochastic process. It is instructive, however, to examine the statistics of a few "closed-form" processes which are expressible as time functions with random parameters.

a. Sinewave of Random Phase and Amplitude

The stochastic process represented by

\[ X(t) = x \sin(t+y) , \]

where \( x \) and \( y \) are independent random variates, might arise through the use of a sinewave generator whose output amplitude varies slowly in a random way so that for each sample function the amplitude may be considered a constant. The random variate \( y \) takes into account the random phase of the signal with respect to the beginning of the sample. (To make this process more nearly an exact representation of the experiment, the interval between sample functions should be sufficiently long to provide independence.)

\[ (3-1) \quad E[X(t)] = E[x] E[\sin(t+y)] = (\sin t) E[\cos y] + (\cos t) E[\sin y] , \]

which, in general, is a function of \( t \).

The covariance function is given by

\[ (3-2) \quad \text{Cov}(X) = C_X(T,t) = E[X(t)X(t+T)] = E[X^2 \sin(t+y) \sin(t+T+y)] \]

\[ = \frac{1}{2} E[X^2 \{ \cos T - \cos(2t+T) E\cos 2y + \sin(2t+T) E\sin 2y \}] , \]
which, in general, is also a function of time, \( t \).

The time average of \( X(t) \) is given by

\[
(3-3) \quad AX(t) = \lim_{a \to \infty} \frac{1}{2a} \int_{-a}^{a} X(t) \, dt = \lim_{a \to \infty} \frac{1}{2a} \int_{-a}^{a} x \sin(t+y) \, dt = 0,
\]

invariant with respect to the sample function chosen.\(^*\)

The time-statistic equivalent of the covariance function, the correlation function, is

\[
R_x(T,x,y) = AX(t)X(t+T) = \lim_{a \to \infty} \frac{1}{2a} \int_{-a}^{a} x^2 \sin(t+y) \sin(t+y+T) \, dt,
\]

which reduces to a form invariant with respect to \( y \):

\[
(3-4) \quad R_x(T,x) = \frac{1}{2} x^2 \cos T.
\]

If we assume that the random phase, \( y \), is uniformly distributed over the interval \((-\pi, \pi)\) with probability density function

\[
(3-5) \quad p_y(y) = \begin{cases} \frac{1}{2\pi}, & |y| < \pi \\ 0, & |y| \geq \pi \end{cases}
\]

then the process mean, as given by Equation (3-1), reduces to zero identically in \( t \). That is,

\[
EX(t) = AX(t) = 0, \text{ all } t.
\]

Also, under the assumption of (3-5), the covariance becomes invariant in \( t \) and Equation (3-2) reduces to

\[
(3-6) \quad C_x(T) = \frac{1}{2} \cos T \, \text{EX}^2,
\]

which is characteristic of the "wide-sense stationary" process.

\(^*\)The symbol "\( A \)" is used herein to denote the time-average operator, as contrasted with the ensemble-average operator "\( E \)".
If the power spectral density function, $f_x$, is defined as the Fourier transform (with respect to $T$) of the covariance function,

$$f_x(\omega, t) = \int_{-\infty}^{\infty} C_x(T, t) \exp(-j\omega T) \, dT,$$

then Equation (3-6) yields

$$f_x(\omega) = \pi \frac{\text{E}x^2}{2} \left[ \delta(\omega+1) + \delta(\omega-1) \right].$$

Note that the definition of (3-7) applied to the nonstationary process of Equation (3-2) yields a complex power spectral density function which is of little physical significance. Other definitions in common use, such as those suggested by Page** and Middleton***, have other drawbacks which limit their usefulness. Page's "instantaneous power spectrum" yields a random variate and Middleton's "intensity density" is identically zero for all finite-energy processes. Considerable attention is given to this problem in the forthcoming technical note.

b. Sinewave of Random Phase and Frequency

Let $x$ and $y$ be independent random variates with the probability density function of $y$ given by

$$p_y(y) = \frac{1}{2\pi}, \quad |y| \leq \pi$$

$$p_y(y) = 0, \quad |y| > \pi$$

and that of $x$ satisfying

$$p_x(x) = p_x(-x).$$

*There does not appear to be a consistent definition of the power spectral density function for nonstationary processes. The definition used here is essentially that of Kharkevich: A.A. Kharkevich, Spectra and Analysis, Consultants Bureau, New York, 1960 (Translated from Russian), p. 149.


Construct the stochastic process

\[ X(t) = 2\sin(xt + y) , \]

where the factor "2" is chosen (without loss of generality) to make the average power unity. This process would arise, for example, through the use of a sine-wave generator whose frequency varies slowly in a random way so that for each sample function the frequency may be considered a constant. As in Section 3-1a, variate \( y \) accounts for the random phase of the signal with respect to the origin.

The expected value of \( X \) is zero for all \( t \) and the covariance function is invariant in \( t \):

\[ C_x(T) = E(\cos xT) = \int_{-\infty}^{\infty} p_x(x) \cos xT \, dx . \]

The process is therefore wide-sense stationary and the definition of power spectral density given by Equation (3-7) is applicable. Applying this definition, we obtain

\[ f_x(\omega) = \int_{-\infty}^{\infty} C_x(T) \exp(-j\omega T) \, dT = \int_{-\infty}^{\infty} \exp(-j\omega T) \int_{-\infty}^{\infty} p_x(x) \cos xT \, dx \, dT . \]

Interchanging the order of integration,

\[ f_x(\omega) = \int_{-\infty}^{\infty} p_x(x) \int_{-\infty}^{\infty} \exp(-j\omega T) \cos xT \, dT \, dx \]

\[ = \pi \int_{-\infty}^{\infty} p_x(x) [\delta(\omega - x) + \delta(\omega + x)] \, dx \]

\[ = \pi [p_x(\omega) + p_x(-\omega)] . \]

From Equation (3-9) we may write, finally,

\[ f_x(\omega) = 2\pi p_x(\omega) . \]
Thus, this process has the unusual property that the power spectral density (in the sense of Equation (3-7)) is proportional to the probability density of the principal primitive variate.

c. Gaussian, Band-Limited, White Noise

Most commercially available noise generators deliver an output voltage which approximates the Gaussian, band-limited, white noise process. It is useful, therefore, to express this process in closed form. Such a representation can be developed as shown below.

It is convenient to discuss first the following lemma:

Lemma I: Let \( S = \sum_{n=-\infty}^{\infty} \frac{\sin(u-nn)}{u-nn} \cdot \frac{\sin(u+v-nn)}{u+v-nn} \)

then \( S = \frac{\sin v}{v} \).

Proof: Write \( S \) in the form

\[
S = \sum_{n=-\infty}^{\infty} \frac{\sin u}{u-nn} \cdot \frac{\sin(u+v)}{u+v-nn} \cdot \frac{1}{n(n-u-nn)(u+v-nn)}
\]

Expand the summand by partial fractions:

\[
S = \sin u \cdot \sin(u+v) \sum_{n} \frac{\frac{1}{v}}{n} \left[ \frac{1}{n (u-nn)} - \frac{1}{(u+v-nn)} \right].
\]

Now

\[
\sum_{n \neq 0} \frac{1}{u-nn} = \frac{1}{u} + \sum_{n \neq 0} \frac{1}{u-nn} = \text{ctn} u - \sum_{n \neq 0} \frac{1}{nn}
\]

and

\[
\sum_{n \neq 0} \frac{1}{u+v-nn} = \text{ctn}(u+v) - \sum_{n \neq 0} \frac{1}{nn}.
\]

*See, for example, D.H. Menzel, Fundamental Formulas of Physics, Section 14.8, Dover Publications, New York, 1960.
Then $S$ may be expressed as

$$S = \frac{\sin u \sin(u+v)}{v} \left[ \cot u - \sum_{n \neq 0} \frac{1}{n \sin n \pi} \cot(n+1)u + \sum_{n \neq 0} \frac{1}{n \sin n \pi} \cot n \pi \right].$$

And, finally,

$$S = \frac{\sin v}{v}, \text{QED}.$$

For the construction of the Gaussian, band-limited, white noise process, let $x_n$ be the generic symbol for a set of independent, zero-mean, Gaussian random variates with common variance, $\sigma^2$. Form the stochastic process, $X_N$,

$$X_N = \sum_{n=-N}^{N} x_n \frac{\sin(at-n\pi)}{at-n\pi}.$$ 

Then $X_N(t_1),...,X_N(t_i),...,X_N(t_k)$ is a $k$-variate Gaussian variable for every $N$. Since

$$\text{Ex}_x x_j = 0, \text{ if } j,$$

and

$$\text{Ex}_x^2 = \sigma^2, \text{ all } i,$$

and

$$\sum_{n=-\infty}^{\infty} \left[ \frac{\sin(at-n\pi)}{at-n\pi} \right]^2 = 1 \quad \text{(a special case of Lemma I)},$$

then there exists a limiting random variable, $X(t)$, given by

$$X(t) = \lim_{N \to \infty} X_N(t) = \sum_{n=-\infty}^{\infty} x_n \frac{\sin(at-n\pi)}{at-n\pi}.$$

Further, $X(t)$ is Gaussian with zero mean and variance $\sigma^2$. The covariance of $X$ is given by

$$C_X(T) = EX(t)X(t+T)$$

$$= E\left\{ \sum_{n} x_n \frac{\sin(at-n\pi)}{at-n\pi} \right\} \left\{ \sum_{n} x_n \frac{\sin(at+T-n\pi)}{at+T-n\pi} \right\}.$$
Because of the independence of the $x_n$'s, this may be written

$$ C_X(T) = \mathbb{E} \sum_n x_n^2 \frac{\sin(at-n\pi)}{at-n\pi} \cdot \frac{\sin(at+aT-n\pi)}{at+aT-n\pi} = \sigma^2 \sum_n \frac{\sin(at-n\pi)}{at-n\pi} \cdot \frac{\sin(at+aT-n\pi)}{at+aT-n\pi}. $$

By Lemma I, this reduces to

$$ C_X(T) = \sigma^2 \frac{\sin aT}{aT}, $$

which is invariant in $t$ and, therefore, represents a wide-sense stationary process. But $X(t)$ has been shown to be a Gaussian process and is then strictly stationary.

The power spectral density may now be computed as the Fourier transform of $C_X$:

$$ f_X(\omega) = \frac{\sigma^2}{a}, \quad |\omega| < a $$

$$ f_X(\omega) = 0, \quad |\omega| \geq a. $$

Hence, the stochastic process represented by

$$ X(t) = \sum_{n=\infty}^{\infty} x_n \frac{\sin(at-n\pi)}{at-n\pi}, $$

where the $x_n$'s are independent Gaussian variates with zero mean and common variance, is a stationary Gaussian band-limited white noise process with band-width $a$.

3-2. Functionals Useful in Specifying Nonstationary Noise

The specification of statistical properties in stationary processes is not a particularly difficult proposition. In a given Monte Carlo study, for example, we might require a noise source which delivers a stationary process having a specified power spectral density (or, equivalently, a certain covariance function) and a specified univariate probability distribution. It may be difficult, in cases, to generate the desired process, but it is generally not hard to determine what is required.

---

The selection of appropriate functionals to describe a nonstationary process, on the other hand, may require much effort. In the first place, several of the functionals commonly used are not universally defined in the same way. Secondly, the nonstationarity rules out the possibility of ergodicity and requires that meaningful specifications be made on the ensemble basis.

Considerable effort has been devoted to this problem during the past quarter, with the following tentative conclusions:

(a) The covariance function, as defined by Equation (3-2), appears to be a more useful and meaningful function than the power spectral density. As mentioned in Section 3-1, this latter quantity is not well defined and may lead to a complex functional or a random variate.

(b) Generally speaking, the complete specification of a nonstationary process is impractical. If \( X(t) \) represents the process, a complete description is provided only by citing the \( k \)-variate distribution of \( [X(t_1), \ldots, X(t_k)] \) for every integral \( k \) (or the equivalent information). Thus it is necessary to select carefully the statistical parameters of principal interest. (This is true of many stationary processes, also, but appears to be a more vital question in the nonstationary case.)

With regard to the input random process, two decisions must be made when setting up a Monte Carlo study: (1) Which functionals or parameters will be used to describe the random process, and (2) What is the detailed form of the chosen functionals. Decision (1) is usually made on the basis of the response variables of interest in the system under study. Decision (2) is dictated by the physical nature of the stochastic process being simulated. To illustrate the way in which these latter specifications are derived, we consider the following very general example.

A rocket is to be fired in a vertical attitude and assumed to rise at a constant rate. As it moves through the atmosphere, it is subject to buffeting winds which are random in nature. It is desired to simulate the rocket system and observe its response to the simulated random wind. A number of computational runs will be performed to gather statistical data on the rocket performance.

Conceptually, at least, the required data on the statistical behavior of the wind could be obtained by placing sensing elements at various altitudes and recording sample functions of wind velocity. If we assume that the speed
and direction of the wind are independent scalar quantities, then these may be simulated separately and combined in the proper relationship by a resolver. For simplicity we will consider only the scalar wind speed.

Viewed as a whole, the wind speed may be thought of as a stochastic process of two parameters—the altitude, \( h \), and time, \( t \). Let the process be denoted by \( X(h,t) \) and symbolize the mean and covariance by \( \mu_X(h,t) \) and \( C_X(h,H,t,T) \):

\[
\mu_X(h,t) = \text{EX}(h,t)
\]
\[
C_X(h,H,t,T) = \text{EX}(h,t)X(h+H,t+T)
\]

If it is understood that the rocket will be launched only under good weather conditions and at a particular time of day, then it may be assumed that the wind at a given altitude may be represented by a stationary stochastic process. Under these conditions, the mean and covariance are invariant in \( t \) and may be written as \( \mu_X(h) \) and \( C_X(h,H,T) \).

Since the rocket is to rise at a constant rate, \( v \), we have for the altitude

\[
h = vt
\]

and

\[
H = vT.
\]

The process may now be written in terms of a single parameter, \( t \): \( X(vt,t) \). This is expressed more concisely as a new process \( Y(t) \) having mean and covariance given by

\[
\mu_Y(t) = \text{EY}(t) = \mu_X(vt) = \text{EX}(vt,t)
\]
\[
C_Y(T,t) = \text{EY}(t)Y(t+T) = C_X(vt,vT,T) = \text{EX}(vt,t)X[v(t+T),t+T]
\]

If these quantities seem appropriate to test that portion of the rocket in which we have an interest, then efforts may be made to generate a nonstationary process having these properties. Otherwise, more appropriate functionals may be derived in the same way and attempts made to generate a suitable process.

*However, the rocket will be changing altitude constantly and this consideration will lead to a nonstationary process.*
3-3. Linear Networks for Nonstationary Shaping

There appear to be at least three techniques which may be employed (separately or in combination) to derive nonstationary stochastic processes having prescribed properties: nonlinear networks (possibly time-varying), time varying linear networks, and utilization of transient effects in time-invariant networks. During the past quarter attention has been given primarily to the use of linear networks. The results are summarized below.

a. Time-Varying Linear Networks

The analysis of time-varying networks has been extensively studied by several authorities, notably Zadeh and Darlington. Zadeh in particular has developed techniques for obtaining the system function and impulsive response of linear variable networks from the so-called fundamental equation of the network--i.e., the differential equation which relates the input and output. If the fundamental equation of the network is of the form

\[ L(p,t) y(t) = K(p,t) x(t) \]

where \( x(t) \) and \( y(t) \) are the input and output of the network and \( L \) and \( K \) are linear differential operators, then the system function, \( H(j\omega,t) \), for the network must satisfy the following differential equation:

\[ \frac{1}{n!} \frac{\partial^n L}{\partial (j\omega)^n} \frac{d^n H}{dt^n} + \ldots + \frac{1}{L} \frac{\partial L}{\partial (j\omega)} \frac{dH}{dt} + H = \frac{K}{L}. \]

The boundary conditions for the above equation must either be given or be derivable from the network. In practice, even for relatively simple networks, the equation for \( H(j\omega,t) \) is so complex as to preclude the possibility of obtaining a closed-form solution and requires the use of some rather sophisticated

---

*One rather interesting nonlinear technique has been discovered. Let a diode function generator be driven by a noise source having probability distribution \( P(x) \). Adjust the function generator to the functional form of \( P(x) \). Then the function generator output will have probability density uniform on the interval \((0,1)\) and zero elsewhere. This technique follows from the theorem:

"Any density for a continuous variate \( x \) may be transformed to the uniform density \( f(y) = 1 \ (0 < y < 1) \) by letting \( y = G(x) \), where \( G(x) \) is the cumulative distribution of \( x \)." (A.F. Mood, Introduction to the Theory of Statistics, McGraw-Hill Book Company, New York, 1950, page 107.)

approximation techniques which are, at best, applicable only over a limited range of the independent variable.

As an example of the techniques developed by Zadeh, consider the following circuit.

\[ \begin{array}{c}
\text{x(t)} \\
\downarrow & \text{R(t)} \\
\downarrow & \text{C} \\
\rightarrow & \text{y(t)} \\
\end{array} \]

It is a simple matter to write the fundamental equation for this circuit:

\[ CR(t) \frac{dy(t)}{dt} + y(t) = x(t) , \text{ or} \]

\[ [CR(t)p + l] y(t) = x(t) , \quad p = \frac{d}{dt} . \]

The system function is then obtainable from the derived equation

\[ \frac{dH}{dt} + \frac{j\omega CR(t) + l}{CR(t)} H = \frac{1}{CR(t)} , \]

which yields

\[ H(j\omega,t) = \exp \left[ -\int_{\infty}^{t} \frac{j\omega CR(a) + 1}{CR(a)} da \right] \int_{0}^{t} \frac{1}{CR(p)} \exp \left[ \int_{0}^{p} \frac{j\omega CR(\gamma) + 1}{CR(\gamma)} d\gamma \right] dp . \]

Having obtained the system function for the network, it is now possible to derive the impulsive response—i.e., the response of the network at time \( t \) to an impulse applied at time \( t_o \). Zadeh shows that the impulsive response \( W(t,t_o) \) and the system function are related by the following integral equation:

\[ W(t,t_o) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega,t) e^{j\omega(t-t_o)} d\omega , \quad t > t_o , \]

which yields for \( W(t,t_o) \)

\[ W(t,t_o) = \frac{1}{CR(t_o)} \exp \left[ -\int_{t_o}^{t} \frac{da}{CR(a)} \right] , \quad t > t_o . \]

Notice that when \( R(t) \) is constant, the results obtained above reduce to the
system function and impulsive response for the simple R-C section low-pass filter as expected:

$$H(j\omega, t) = H(j\omega) = \frac{1}{jCR\omega + 1}$$

$$W(t, t_o) = W(t-t_o) = \frac{1}{RC} e^{-(t-t_o)/RC}, \ t > t_o.$$  

Suppose the network in the example is driven by band-limited Gaussian white noise such that the input power spectral density function is given by

$$G_1(\omega) = \begin{cases} b, & |\omega| \leq a \\ 0, & \text{otherwise} \end{cases}$$

and the corresponding covariance function is given by

$$C_1(T) = E[x(t)x(t+T)] = \frac{ab}{\pi} \frac{\sin(aT)}{aT}.$$  

Since the input has zero mean, the output also has zero mean; further, the covariance function for the output process is given by

$$C_2(t,T) = E[y(t)y(t+T)]$$

$$= \frac{ab}{\pi} \exp \left[-2\int \int \frac{da}{CR(a)} \frac{da}{CR(a)} \right] \int \int \frac{\sin(a(\beta-\gamma))}{a(\beta-\gamma)} \exp \left[ \int \frac{da}{CR(a)} + \int \frac{da}{CR(a)} \right] d\beta d\gamma.$$  

No attempt will be made here to derive an output power spectral density function. It may be noted in this connection, however, that Fourier transformation of the output covariance function yields a complex expression with little physical significance.

Consider now a particular form for $R(t)$:

$$R(t) = \frac{t}{C}.$$  

Substituting this expression for $R(t)$ back into the previous results yields

$$H(j\omega, t) = \frac{1}{j\omega t} \left[ 1 - e^{-j\omega t} \right],$$

$$W(t, t_o) = \frac{1}{t},$$

$$C_2(t,T) = \frac{ab}{\pi t(t+T)} \int \int \frac{\sin(a(\beta-\gamma))}{a(\beta-\gamma)} d\beta d\gamma.$$  

-25-
It is clear from the example and from the general equation for the system function of a variable network that even relatively simple networks having only one variable element lead to expressions for network parameters which are so cumbersome as to be all but unmanageable. Since the analysis of known networks lead to such unwieldy expressions, it is easy to see that the synthesis of such networks borders on the impossible. Because the synthesis problem is of primary importance, the class of variable networks under consideration has been restricted to the so-called separated networks. These are variable networks which can be represented by the cascaded combination of a linear invariant network and a linear variable network that contains no reactive components. The block diagram for a typical separated network is shown below.

\[ X(t) \xrightarrow{N(j\omega)} Y(t) \xrightarrow{K(t)} Z(t) \]

The behavior of the network \( N \) is well known; further, it is easy to show that since \( Z(t) = K(t)Y(t) \), then \( E[Z(t)] = K(t)E[Y(t)] \) and the covariance function for \( Z(t) \) is given by

\[ C_Z(t,T) = E[Z(t)Z(t+T)] = K(t)K(t+T) C_Y(T) \]

It is clear that this class of variable networks yields more tractable results than does the general variable network, while not seriously compromising either flexibility or utility. The properties of this general class and applicable synthesis techniques will be more extensively investigated during the coming report period.

b. Invariant Linear Networks

The technique of utilizing the transient response of time-invariant networks to derive nonstationary stochastic processes has been investigated by Lampard.\(^\text{a}\) The covariance function and probability distribution function of the output process are derived when the input is stationary, white, and Gaussian, but no attempt is made to develop synthesis techniques.

It is felt that this technique and the results derived by Lampard hold a great deal of promise for that restricted class of problems where it is sufficient to specify only a single parameter of the output process. As time permits beyond the work outlined in the preceding subsection, this technique will be further investigated in light of its application to the problem of interest and an attempt will be made to develop procedures for synthesis.

3-4. Planned Work for the Coming Quarter

During the coming report period, efforts on Task II will be directed toward completion of the technical note discussed in the introductory paragraph of Chapter III and toward further development of the linear-network shaping techniques. If time permits, a limited amount of experimental investigation will be conducted using the Georgia Tech Analog Computer Laboratory facilities. The principal aim of this empirical work will be verification of the shaping techniques to be developed, particularly those resulting from the "separated networks" discussed in Subsection 3-3a.
DEVELOPMENT OF
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I - An Experimental Electronic Generalized Integrator
II - Nonstationary Noise for Monte Carlo Studies
by
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Chapter I
BACKGROUND INFORMATION

1-1. Introduction

Georgia Tech Research Project No. A-588 was established on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at the George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project has been assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates under the Special Problems Group of the Engineering Experiment Station's Physical Sciences Division.

1-2. Project Objectives

In accordance with decisions reached during the project organizational meeting with the Contracting Officer's Representative (Dr. Walter Krause of MSFC), primary emphasis has been placed on areas A and B mentioned above. It was agreed that the following specific assignments would be undertaken initially:

Task I. Investigation of electronic techniques for integrating analog voltage signals with respect to arbitrary variables. The task includes construction of a working model of a generalised integrator suitable for use with existing analog equipment.

Task II. Investigation of analog techniques for the generation of non-stationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian band-limited, white noise.
2-1. Outline of Work Completed to Date

At the end of the current quarter, construction of the Electronic Generalized Integrator model has proceeded through the assembly of approximately 75 percent of the circuitry and hardware. The major system—the Incremental Detector Section including the Positive and Negative Error Detectors (See Figure 1), both Flip-Flops and Pulse Indicators, the Positive and Negative Reset Pulse Generators, and all associated pulse amplifiers—has been completed. The analog amplifier to be used in the Storage Integrator is mounted, but stabilized power and relay control for its operation is not yet provided. Partial tests of the Incremental Detector Section have been carried out, however, using an external Berkeley 1048 amplifier.

The 100-Kc Oscillator, the Frequency Divider, and the associated multivibrator and pulse amplifiers are completed and tested. The temperature-controlling oven, in which all critical components are housed, is complete and long-term checks indicate very satisfactory performance.

Detailed information on the completed sections is given in the paragraphs below and in the schematic diagrams of Figures 1 through 4.

2-2. The Incremental Detector Section

The Incremental Detector Section (IDS) is a closed-loop system used for detecting incremental changes in the \( x \) input. Referring to the block diagram of Figure 1 and the schematic of Figure 2, the IDS comprises Positive and Negative Error Detectors, Flip-Flops 1 and 2, Positive and Negative Reset Pulse Generators, and the Storage Integrator. This Section is complete except for providing stabilized power and relay control for the analog amplifier in the Storage Integrator. The partial tests described below were carried out using a Berkeley 1048 operational amplifier and were intended primarily to test the general operation of the pulse circuitry.

In the IDS tests, the dead zone was adjusted to ± 200 millivolts. The size of this dead zone can easily be reduced to a very small value by adjustment of circuit values in the Error Detectors, but the wider zone was used here to give greater freedom from noise while checking the Reset Pulse Generators. In
operation the dead zone will be adjusted to ± 1 bit.

The amplitude of the reset pulses was adjusted to 15 volts and their duration set at 40 microseconds by the Timing Generator. The Storage Integrator utilized a 1-microfarad feedback capacitor with reset-pulse input resistors (R63, R67, R68, and R70, R74, R75 of Figure 2) adjusted to provide a gain of 41.7. Thus the bit size, referred to the Storage Integrator output, was 25 millivolts.

The x-input RC circuit utilized a 0.01-megohm resistor and 0.5-microfarad capacitor. This provided an x gain of 1/2 with a time constant of 5 milliseconds (or a break frequency of about 32 cps). The bit size referred to the x input was then 50 millivolts.

With the circuitry adjustments given above, the Incremental Detector Section was able to follow a 5-cps sine wave of approximately 15 volts peak. The only significant errors were those obtained as the input signal passed through zero. These errors were due to the large dead zone introduced at the Error Detectors.

At the present stage of development the Incremental Detector operates with a single analog amplifier. However, no critical examination of the threshold-level stability has been made. If further tests reveal drift problems in the Error Detectors, it may be necessary to provide a gain of 10 between the Storage Integrator and the Detectors. This will entail very little circuit change, since four TR-10 amplifiers are now mounted on the Electronic Generalized Integrator chassis.

2-3. Timing Generator

The Timing Generator includes the 100-Kc Oscillator and the Frequency Divider. This unit, shown schematically in Figure 3, furnishes interrogation pulses to the Error Detectors and establishes the reset-pulse duration by resetting Flip-Flops 1 and 2 at precise intervals. In the present configuration, the Frequency Divider counts by 8 to produce a clock rate of 12.5 kc. Since the Flip-Flop reset pulse lags the interrogation pulse by one-half cycle (see waveforms shown on Figure 1), the reset-pulse width is thereby fixed at 40 microseconds. Note that the "open" period of the y-Gates is also fixed by the Timing Generator at 40 microseconds.

The Timing Generator has been completed and tested. It exhibits no frequency
drift after normal Oven warm-up.

2-4. Temperature-Controlled Oven

The Oven used in the Electronic Generalized Integrator is constructed of aluminum lined with styrofoam to form a box approximately 5" x 4" x 6". This Oven houses four mounting boards which contain the following critical circuitry: 100-Kc Oscillator, Error Detectors, Reset Pulse Generators, and y-Gates.

A thermistor (Fenwal type GB32J2) is used as the temperature sensing element. Referring to Figure 4, the thermistor (R86) operates into a differential d-c amplifier (TR27, TR28, TR29, TR30, TR31) with the set point established by R88. Transistor TR32 controls the heater-resistor current in accordance with the d-c amplifier output signal. This Oven Heating Control system has been tested and found to control the oven temperature within 0.5°C of the set point after reaching the steady-state condition. Further smoothing of small temperature fluctuations is provided by aluminum-block heat sinks used with the critical transistors. No hysteresis in the steady-state value was detected over several days' operation.

Controlled heating for the oven is provided through eight 120-ohm, 2-watt resistors connected in parallel and distributed throughout the Oven. Experiment shows that an average dissipation of 13.2 watts is sufficient to maintain a temperature of 46°C with a mean room temperature of 23°C. In response to a step change in set point from 26°C to 46°C, the temperature was observed to rise to within 1°C of the steady-state value in about 80 minutes. Although this warm-up time could be decreased considerably by increasing the maximum power available, little would be gained since it is necessary to allow the transistor heat sinks to equilibrate with the ambient temperature.

2-5. Work Planned for Coming Quarter

The remaining work on the Electronic Generalized Integrator will proceed in the following steps:

(a) Completion of wiring for the TR-10 analog amplifiers.
(b) Detailed performance evaluation of the Incremental Detector Section and optimization of the design.
(c) Construction of the y-Gates.
(d) Performance evaluation of the y-Gates and optimization of the design.
(e) Tests of the Electronic Generalized Integrator system and optimization of the design.
It is anticipated that steps (a) through (c) will be completed during the coming quarter.
Figure 1: Block Diagram of Electronic Generalized Integrator
Figure 2: Schematic Diagram of Error Detectors, Reset Pulse Generators, Reset Pulse Indicators, and Associated Logic Circuitry
Figure 3: Schematic Diagram of Timing Generator
Figure 4: Schematic Diagram of Oven Heater Control System and Positive and Negative 8-Volt Power Supply Regulators
Chapter III
GENERATION OF NONSTATIONARY NOISE FOR ANALOG-COMPUTER MONTE CARLO STUDIES

Task II of GIT/EES Project A-588 concerns the investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian white noise. Efforts during the past quarter have centered about the preparation of a project technical note on fundamentals of nonstationary processes, further investigation of the separated-network techniques discussed in the previous Program Report, and a review of the open literature pertinent to Task II.

3-1. Covariance, Normalized Covariance, and Correlation Functions

The detailed attention given to the separated-network technique for nonstationary shaping has made it desirable to identify clearly and consistently a few important process functionals. For an arbitrary stochastic process, \( X(t) \), the following ensemble-statistic definitions will be used:

1. **Covariance Function**
   \[
   C_X(T,t) = E[(X(t) - \mu_X(t))(X(t+T) - \mu_X(t+T))]
   \]

2. **Normalized Covariance Function**
   \[
   \rho_X(T,t) = \frac{C_X(T,t)}{\sigma_X(t) \sigma_X(t+T)} \quad \text{(*)}
   \]

where

3. \( \mu_X(t) = E[X(t)] = \text{Mean of } X(t) \),

and

4. \( \sigma_X^2(t) = E[(X(t) - \mu_X(t))^2] = \text{Variance of } X(t) = C_X(0,t) \).

For a zero-mean process, the covariance function obviously reduces to the form

5. \[ C_X = E\{X(t)X(t+T)\} \] .

For a "wide-sense stationary" process—i.e., one in which, by definition, the covariance function is invariant with the instant of observation \( t \) and depends only on the time shift \( T \)—the normalized covariance function becomes

* Many authors refer to \( \rho_X \) as the "correlation coefficient." The term "normalized covariance" seems preferable in that it avoids potential conflict with "correlation function," which we use in accordance with the definition of Equation (7) below.

-10-
We will at this time adopt formally only one time-statistic definition pertaining to an arbitrary stochastic process \( X(t) \)—viz.,

\[
(6) \quad \rho_X(T) = \frac{C_X(T)}{\sigma_X^2} = \frac{C_X(T)}{C_X(0)}. 
\]

It should be noted that \( R_X(T) \) expresses the average lagged product taken over all time, for a particular "sample function" drawn from the population of possible \( X(t) \)'s. By comparison, \( C_X(T,t) \) may be considered the average lagged product taken over all sample functions, for a particular instant of time \( t \).

The functional described in Equation (1) is almost universally called the "covariance function," although a few authors use the terms "autocovariance" and "autovariance" to emphasize the distinction from the cross-variance function. However, the functionals of Equations (6) and (7) appear under a wide variety of names in the literature. In addition, the term "correlation function" is used by many writers to denote \( \text{E}\{X(t)X(t+T)\} \). By these writers, the "correlation function" (sometimes called the "autocorrelation function") is identical to the covariance function of Equation (1) for all zero-mean processes.

The nomenclature adopted here and to be employed hereinafter on this research project is that of Middleton, except that Middleton uses the term "correlation coefficient" to denote what we have called the "normalized covariance function" (Equation (6)).

3-2. Comments on the Separated-Network Technique

In the previous Program Report the difficulty of synthesizing general time-variable networks was discussed. It was suggested that the more tractable "separated network" might be sufficiently versatile to provide nonstationary shaping for the generation of a wide class of processes. Some limitations of the separated-network technique will be brought out in this Section.

The general separated network comprises a linear invariant filter followed

by a zero-memory time-variable circuit, as shown below.

The input process, $X$, is considered to be zero-mean Gaussian white noise, such as that produced by the standard noise generator, with a flat power spectral density extending well beyond the cutoff frequency of the Network $N$. This input is stationary and $N$ is time invariant; hence, $Y$ is zero-mean, Gaussian, and stationary, with a well-defined power spectrum given by

$$f_Y(\omega) = |N(j\omega)|^2 f_X(\omega).$$

Network $K$, which might be a standard function multiplier, produces from $Y$ the output process $Z$:

$$Z(t) = K(t) Y(t).$$

The synthesis problem is then as follows: Given the spectral height of the input $X$, and given the desired property of $Z$ to be produced, determine $N(j\omega)$ and $K(t)$.

Let the (constant) spectral height of $X$ be $f_X(\omega) = f_X^0$. The spectral density for $Y$ is therefore written

$$f_Y(\omega) = f_X^0 |N(j\omega)|^2.$$

By the Wiener-Khintchine theorem, the covariance of $Y$ may be expressed as

$$C_Y(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_Y(\omega) \cos \omega T \, d\omega = \frac{1}{2\pi} f_X^0 \int_{-\infty}^{\infty} |N(j\omega)|^2 \cos \omega T \, d\omega.$$
With no loss of generality, we can choose \( N(j\omega) \) so that \( C_Y(0) = 1 \).\(^*\) The covariance function of \( Z \) is given by

\[
C_Z(T,t) = K(t) K(t+T) C_Y(T),
\]

and \( K \) may be determined by setting \( T = 0 \) to obtain

\[
K^2(t) = C_Z(0,t) = \sigma_Z^2(t).
\]

From (11) we may write

\[
C_Y(T) = \frac{C_Z(T,t)}{\sigma_Z(t) \sigma_Z(t+T)},
\]

and from (2) we obtain

\[
C_Y(T) = \rho_Z(T,t).
\]

But \( C_Y \) is invariant with \( t \); hence, \( \rho_Z \) must be invariant with \( t \). Thus, use of the separated-network technique limits us to the generation of what may be called \textit{wide-sense stationary} processes. Specifically, by a "wide-sense stationary process" we mean a process in which the normalized covariance function is invariant with \( t \), being a function only of the time shift, \( T \). The covariance function of \( Y \) may then be written as

\[
C_Y(T) = \rho_Z(T).
\]

Now, since \( f_Y(\omega) \) is the Fourier transform of \( C_Y(T) \), we may express the requirements of \( N \) as

\[
|N(j\omega)|^2 = \frac{1}{T} \int_{-\infty}^{\infty} C_Y(T) \cos \omega T \, dT = \frac{1}{T} \int_{-\infty}^{\infty} \rho_Z(T) \cos \omega T \, dT.
\]

It is a relatively simple operation, though somewhat tedious, to demonstrate that the bivariate distribution of \( Z \) is jointly Gaussian with standard deviations \( \sigma_Z(t) = |K(t)| \sigma_Y(t) \) and \( \sigma_Z(t+T) = |K(t+T)| \sigma_Y(t+T) \). The resultant expression for the bivariate distribution of \( Z \) reveals also that the normalized

\* We assume at the outset that the integrals of Equation (10) exist. A sufficient condition is that the impulse response of \( N \) be a member of \( L_2 \) space.

-13-
covariance is equal to that of $Y$.

In summary, the separated-network technique may be used in conjunction with a standard noise generator to generate wide-sense stationoid processes having a specified normalized covariance function and a specified instantaneous variance. The design equations for the separated networks are

$$K_2(t) = \sigma_2^2(t)$$  \hspace{1cm} (12)

$$\left| N(j\omega) \right|^2 = \frac{1}{f_X} \int_{-\infty}^{\infty} \rho_2(T) \cos \omega T \,dT,$$  \hspace{1cm} (16)

where $f_X$ is the constant power-spectral height of the noise generator output.
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

I - An Experimental Electronic Generalized Integrator
II - Nonstationary Noise for Monte Carlo Studies

by

R. S. Johnson
and
F. R. Williamson

QUARTERLY PROGRAM REPORT
on
Contract No. NAS8-2473

12 March 1962 - 12 June 1962

For

GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia
DEVELOPMENT OF
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For

GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted: Robert S. Johnson
Project Director A-588

Approved: F. Dixon, Head
Special Problems Branch
Physical Sciences Division
1-1. Introduction

Georgia Tech Research Project No. A-588 was established on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at the George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project has been assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates under the Special Problems Branch of the Engineering Experiment Station's Physical Sciences Division.

1-2. Project Objectives

In accordance with decisions reached during the project organizational meeting with the Contracting Officer's Representative (Dr. W. P. Krause of MSFC), primary emphasis has been placed on areas A and B mentioned above. It was agreed that the following specific assignments would be undertaken initially:

Task I. Investigation of electronic techniques for integrating analog voltage signals with respect to arbitrary variables. The task includes construction of a working model of a generalized integrator suitable for use with existing analog equipment.

Task II. Investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian, band-limited, white noise.
1-3. Scope of the Report

During the past report period, essentially all Project effort has been directed toward Task I. The immediate Project objective is the completion of a working model of the Electronic Generalized Integrator (EGI) and the execution of performance and evaluation tests. This Program Report documents the work of the past quarter and presents the results of a series of preliminary EGI tests.
2-1. Outline of Work Completed to Date

As reported in the previous Program Report, at the beginning of the third report period the following work remained on the EGI:

(a) Completion of wiring for the TR-10 analog amplifiers.
(b) Detailed performance evaluation of the Incremental Detector Section.
(c) Construction of the y-Gates.
(d) Performance evaluation of the y-Gates and optimization of the design.
(e) Tests of the EGI system with minor design changes as indicated.

Items (a) through (c) have now been completed and the tests of (d) and (e) have been initiated. No further construction work is required and no major design changes are anticipated.

2-2. The Incremental Detector Section

The Incremental Detector Section, described in detail in the previous Program Report, has been completed and tested. These tests indicate satisfactory performance, although minor parameter changes are required to provide more convenient calibration.

At present, this Section is adjusted to operate as a free-running converter with a bit rate of 12.5 kc and a bit size of 25 millivolts. Two capacitive and two resistive input impedances are provided, the latter for use when x (rather than only x) is available in the external computer circuit. The dual inputs allow a scale-factor choice of x1 or 0.5 as appropriate to the range of variables involved.
Calibration of the Incremental Detector Section is presently accomplished by adjusting the reset pulse height to obtain the required number of bits after application of a known step voltage. In the test results cited below, the bit size is adjusted to 25 millivolts by requiring the generation of 600 bits under stimulus of a 30-volt step applied to the x0.5 input. To assure balance between the positive and negative bit sizes, the adjustments were made while repeatedly stepping back and forth between +15 volts and -15 volts. This bit size calibration is relatively stable, with variations less than one part per 600 over extended periods.

The dead zone is now adjusted to ±100 millivolts as measured at the Storage Integrator Output. The system has been operated with the dead zone adjusted to ±50 millivolts, but the larger figure provides greater freedom from noise with very little degradation of performance.

2-3. System Tests

A series of tests have been completed which provide information on drift, frequency response, and linearity of the overall EGI system. In each of these preliminary tests, the EGI was calibrated prior to the first measurement, but was not again adjusted for the remainder of the test.

(a) Drift and Offset Tests.—Two tests were performed with \( x = 0 \) to estimate the output error due to drift in the Storage Integrator and \( y \)-Gates. Each test comprised seven measurement runs of 200 seconds duration taken 10 minutes apart over a one-hour interval. The test results are shown below in Tables I and II.

<table>
<thead>
<tr>
<th>Table I: ( x = 0, \ y = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Run No.</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>
Table II: \( x = 0, \ y = 1 \)

<table>
<thead>
<tr>
<th>Run No.</th>
<th>EGI Output after 200 Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- 10 mv.</td>
</tr>
<tr>
<td>2</td>
<td>+ 25 mv.</td>
</tr>
<tr>
<td>3</td>
<td>+ 15 mv.</td>
</tr>
<tr>
<td>4</td>
<td>+ 5 mv.</td>
</tr>
<tr>
<td>5</td>
<td>+ 30 mv.</td>
</tr>
<tr>
<td>6</td>
<td>+ 55 mv.</td>
</tr>
<tr>
<td>7</td>
<td>+ 50 mv.</td>
</tr>
</tbody>
</table>

Taken as a whole, the tests indicate a drift rate of approximately 200 microvolts per second. Thus, with the present zero-adjustment circuitry, the useful computation time is limited to several minutes.

(b) Frequency Response Test.--The frequency response of the EGI system is limited by the rate at which the Incremental Detector Section can follow changes in the \( x \) input. With a bit rate of 12.5 kc and a bit size of 25 millivolts, the maximum slope without accumulation of error is 313 volts per second, corresponding to a frequency of

\[
f = \frac{313}{2\pi A},
\]

where \( A \) is the amplitude of the \( x \)-input sine wave. This theoretical frequency limit was checked empirically by observing the error voltage at the Storage Integrator output with an \( x \)-input sine wave of various amplitudes and frequencies. Partial results of this test are shown in Table III.

Table III: \( x = A \sin 2\pi ft \)

<table>
<thead>
<tr>
<th>Frequency (f)</th>
<th>Amplitude at Which Error Begins to Accumulate (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 cps</td>
<td>2.5 volts</td>
</tr>
<tr>
<td>9 cps</td>
<td>5.0 volts</td>
</tr>
<tr>
<td>5 cps</td>
<td>9.0 volts</td>
</tr>
<tr>
<td>1 cps</td>
<td>49.5 volts</td>
</tr>
</tbody>
</table>
The amplitude/frequency product for the present version of the instrument is seen to be approximately 50. This factor increases proportionally with bit size and bit rate.

(c) Linearity Test.—System linearity was tested by driving the x-input with a ramp, \( x = at \), while holding \( y \) fixed at various constant levels. The EGI output should then be a ramp with slope determined by the product of \( a \) and \( y \). Linearity with respect to the x-channel is determined by noting the departure from a straight line during any one run with \( y \) fixed. Linearity of the y-channel is estimated by observing the EGI output as \( x \) crosses a convenient value for several different \( y \)'s. The nonlinearity of the x-channel was too small to be sensible; data for the computation of \( y \) linearity is presented in Table IV.

<table>
<thead>
<tr>
<th>( y ) (volts)</th>
<th>EGI Output (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 5.0</td>
<td>- 42.7</td>
</tr>
<tr>
<td>- 4.0</td>
<td>- 34.0</td>
</tr>
<tr>
<td>- 3.0</td>
<td>- 25.5</td>
</tr>
<tr>
<td>- 2.0</td>
<td>- 17.0</td>
</tr>
<tr>
<td>- 1.0</td>
<td>- 8.4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ 1.0</td>
<td>+ 8.5</td>
</tr>
<tr>
<td>+ 2.0</td>
<td>+ 16.9</td>
</tr>
<tr>
<td>+ 3.0</td>
<td>+ 25.4</td>
</tr>
<tr>
<td>+ 4.0</td>
<td>+ 33.6</td>
</tr>
<tr>
<td>+ 5.0</td>
<td>+ 42.1</td>
</tr>
</tbody>
</table>

The maximum deviation from the least-squares straight line is 0.25 volts, representing approximately 0.3 per cent of full scale.

A series of measurements over the interval from - 0.5 to + 0.5 volts has shown a deviation from the best straight line of less than 0.1 per cent. Further measurements are planned for the full interval of \( y \) from - 10 to + 10 volts.
2-4. Work Planned for Final Quarter

During the coming quarter, efforts will be made to finalize the EGI circuitry and complete additional tests on system linearity and general performance. These tests will include operation with the ACL computer to gather data under operational conditions.

Several minor parameter changes are planned. These include adjustment of the system scale factor by selection of a suitable \( y \)-Integrator feedback capacitor, optimizing the \( y \)-Gate base current drive, and installing matched gating transistors. In addition, a more convenient calibration procedure will be developed.
DEVELOPMENT OF NEW METHODS AND
APPLICATIONS OF ANALOG COMPUTATION

I - An Experimental Electronic Generalized Integrator
(II - Nonstationary Noise for Monte Carlo Studies)

By F. R. Williamson, Jr.
and F. Dixon

Prepared for
George C. Marshall Space Flight Center
Huntsville, Alabama

Contract NAS8-2473

12 November

1962

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION
I - An Experimental Electronic Generalized Integrator
(II - Nonstationary Noise for Monte Carlo Studies)

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F. R. Williamson, Jr.
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F. Dixon

4th QUARTERLY PROGRAM REPORT
(12 June - 12 September 1962)
and
ANNUAL TECHNICAL SUMMARY
on
Contract No. NAS8-2473

For
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted:

F. Dixon
Project Director A-588, and
Head, Special Problems Branch
SUMMARY

Under Task I, an experimental electronic device (designated EGI) has been designed and constructed to perform integration of one analog voltage with respect to a second analog voltage or the time derivative. A description of the system and its operation is presented in this report, together with results of preliminary performance tests. The dependent-variable (Y) input for the EGI unit is seen to have a usable range of at least two decades. Based on a series of 60-second runs, the nominal accuracy rating would be 0.1 percent of full scale.

Tests to date have indicated that the Y Sampling Gate transistors experience parameter variations due to junction temperature changes related to the magnitude of the voltage being sampled. Moreover, the maximum usable voltage at the input of the Y Sampling Gates is lower than desired, as compared to the saturation voltage of the transistor switches. Adjustment of the offset voltage controls in the present sampling circuitry has been found to be quite critical. An easily adjusted gain control is needed on the EGI unit along with a convenient scale-factor definition for the output signal. Further operating tests are recommended to obtain a more complete performance evaluation of the present experimental system.

Work performed under Task II, dealing with nonstationary processes for Monte Carlo studies, is documented in a separate Technical Note.
CONTENTS

Title Page ................................................................. 1
SUMMARY ................................................................. ii

I. GENERAL INFORMATION
   1-1. Introduction .................................................. 1
   1-2. Program Outline ............................................. 1
   1-3. Activities During Quarterly Report Period ............. 2

II. ELECTRONIC GENERALIZED INTEGRATOR
   2-1. Generalized Integration ................................... 3
   2-2. The EGI System ............................................ 4
   2-3. Circuit Description ....................................... 6
   2-4. Operation of the EGI ..................................... 11
   2-5. Performance Data ......................................... 21
   2-6. Recommendations .......................................... 24

Figures (1 - 23) ..................................................... 25
Chapter I
GENERAL INFORMATION

1-1. Introduction

Georgia Tech Research Project No. A-588 was established under Contract NAS8-2473 on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project was assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates as a unit of the Special Problems Branch of the Engineering Experiment Station's Physical Sciences Division.

The basic contract was initiated for a term of one year, with an additional 45 days being allowed to cover preparation and submission of a Final Technical Summary Report. However, Modification No. 1, executed on 2 October 1962, authorized a no-cost extension in time of performance until 12 November 1962. A forthcoming Modification No. 2 is expected to provide for an additional year's work in the same or related areas of interest.

1-2. Program Outline

By agreement with the Contracting Officer's Representative, Dr. W. P. Krause of MSFC, efforts during the past year have been devoted to the following specific tasks under areas A and B:

Task I - Investigation of electronic techniques for integrating analog voltage signals with respect to arbitrary variables. This assignment includes construction of a working model of a generalized integrator suitable for use with existing analog computer equipment.

Task II - Investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies. Of particular interest is the problem of nonstationary shaping of Gaussian, band-limited, white noise.

-1-
Efforts on Task I, provided largely by Mr. Frank R. Williamson, Jr. (Assistant Research Engineer), have resulted in the development of an experimental electronic generalized integrator, designated EGI, which is fully described in the present report. Recent laboratory tests of this experimental unit indicate that a system of good overall performance and nominal 0.1% accuracy can be achieved through the incorporation of certain circuit modifications and improved components. However, it is recommended that additional tests be conducted first.

Under Task II of the program, technical reference material has been assembled covering the fundamentals of nonstationary stochastic process theory from the standpoint of the analog computer engineer. This effort, performed by Messrs. Robert S. Johnson (Research Engineer) and Ralph D. Loftin (Research Assistant), is documented in a special Technical Note (GIT/EES Report A588/T1), separate from the present Annual Technical Summary.

1-3. Activities During Quarterly Report Period

Work during the quarterly report period ending 12 September 1962 has included finalization of the EGI circuitry and continuation of various system performance tests, as described in Chapter II hereunder. In addition, a draft manuscript of the Technical Note on nonstationary stochastic process theory was completed.

On 3 August, Mr. R. S. Johnson visited Marshall Space Flight Center to review the program objectives and status with the Project Technical Officer, in anticipation of forthcoming personnel changes. On 10 August, Mr. R. D. Loftin terminated his employment at Georgia Tech in order to work with the Minneapolis-Honeywell Co. in Clearwater, Fla. On 16 August, Mr. R. S. Johnson terminated his employment at Georgia Tech in order to work with the General Electric Co. Computer Center in Huntsville, Ala. On 31 August, Mr. Johnson revisited Georgia Tech in company with Mr. Fred T. Shaver of MSFC to discuss the possibility of conducting preliminary manned space flight simulation studies utilizing the cockpit section from an F-86D Aircraft Trainer and Flight Simulator which had recently been released to Georgia Tech as surplus property from Dobbins Air Force Base. It is expected that this plan will be pursued under the contract extension.
Chapter II
ELECTRONIC GENERALIZED INTEGRATOR

2-1. Generalized Integration

The operation of the Electronic Generalized Integrator may be understood by examining the definition of a definite integral and the approximation used in instrumenting this function for the unit described in this report. Equation (2.1) expresses the definite integral of a function $Y(X)$ over the interval between $a$ and $b$. This is valid only if $Y(X)$ is continuous over the interval.

\[ \int_{a}^{b} Y(X) \, dX = \lim_{n \to \infty} \sum_{i=1}^{n} Y(X_i') \Delta X_i \]

where \[ \Delta X_i = X_i - X_{i-1} \]

and \[ X_{i-1} \leq X_i' \leq X_i \]

The limit is evaluated as $n$ is increased and as the maximum of the numbers $\Delta X_1, \ldots, \Delta X_n$ is made to approach zero. By assigning $\Delta X_i$ to be a constant (call it $\Delta X$) limited to a very small value, we may replace Equation (2.1) with the following approximation:

\[ \int_{a}^{b} Y(X) \, dX = \Delta X \sum_{i=1}^{n} Y(X_i') \]

Since both $X$ and $Y(X)$ will be functions of time in the intended application, the expression $Y(X_i')$ may be interpreted electronically as a sample of the analog of the function $Y(X)$ at the time when $X$ equals $X_i'$. Thus, the approximation made above is equivalent to a summation of successive samples of $Y(X)$ multiplied by the constant $\Delta X$. Moreover, from the fact that the variables $X$ and $Y(X)$ are functions of time, we may identify this constant $\Delta X$ with a constant time interval $\Delta t$, which will be the duration of each sample of the $Y(X)$ analog signal.

In order that the output of the Electronic Generalized Integrator may be an accurate approximation to the definite integral, the bandwidth of the $X$ analog input must be restricted so that the time required for a change in value of
amount $\Delta X$ is greater than the sampling time $\Delta t$. A similar restriction on the bandwidth of the $Y(X)$ input is required—namely, that this quantity can be considered constant during the period of one sample. The output of the EGI may then be written as

$$E(X,Y) = k \int_a^b Y(X) \, dX = k \Delta t \sum_{i=1}^n Y(X_i)$$

where $k$ represents a voltage scale factor. For nonmonotonic functions in $X$, the sign of the incremental change in the $X$ input must be incorporated inside the summation sign. This is effected in practice by sampling the negative of the $Y$ input signal.

No provision for "initial conditions" on the EGI output has been included in the present unit; consequently the following additional restriction is imposed on the output functions:

$$E(X_0,Y_0) = 0$$

where $X_0$ and $Y_0$ are the respective values of $X$ and $Y$ at the start of computation.

2-2. The EGI System

A block diagram of the experimental Electronic Generalized Integrator which has been built under this contract is presented in Figure 1. For purposes of discussion the system may be considered to comprise two major functional parts—one involving operations upon (or associated with) the $X$ analog input signal, the other involving operations on the $Y$ analog input signal. The first of these sections will be referred to as the Delta-X Detector, since its primary function is to examine the $X$ input signal and detect changes in magnitude of predetermined amount $\Delta X$. The second section includes gate circuits for sampling the $Y$ input signal (either $+Y$ or $-Y$, depending on the sign of $\Delta X$), plus an integrating circuit which sums these samples to furnish the desired EGI output.

The Delta-X Detector is a closed-loop configuration incorporating an operational amplifier with capacitive feedback for an analog storage element, this unit being identified as the Storage Integrator in Figure 1. The $X$ analog signal is applied to the grid through a capacitive input element. Thus, the analog signal appears at the output of the operational amplifier with a sign
inversion and with an amplitude determined by the magnitude of the feedback and input impedances used. A series resistor is included in the input impedance to the Storage Integrator to limit the bandwidth of the X analog input. This prevents conditions of extreme overload to the amplifier used in this circuit.

The output of the Storage Integrator is examined by two Error Detectors to determine when the magnitude of this voltage exceeds the value of a preset threshold level. The polarity of the output of the Storage Integrator is contained in the output signal of the particular Error Detector whose threshold level has been exceeded. The Error Detector output signal initiates a pulse of constant volt-second area from the corresponding Reset Pulse Generator. This reset pulse is summed into the grid of the Storage Integrator, being always of the proper polarity to produce a decrease in magnitude of the amplifier output voltage. The Delta-X Detector is thus a form of negative-feedback circuit with the summing point at the grid of the Storage Integrator and the error voltage appearing at the output of the amplifier. The duration of the reset pulses corresponds to the $\Delta t$ in Equation (2.3). The volt-second area of these pulses, taken in conjunction with the Storage Integrator gain factor, determines the value of $\Delta X$ in Equation (2.2). In essence, the Delta-X Detector constitutes a signal converter, yielding as output an incremental digital representation of the X analog input. This output information serves to operate the Y Sampling Gates but can also be read directly from the Positive and Negative Reset Pulse Indicator lamps.

A crystal oscillator and frequency divider are included in the Delta-X Detector section for the purpose of establishing the precise timing signals required in the operation of the unit. The length of the reset pulse is derived from this source, as well as the width of the gate used in sampling the Y input signal.

A part of the "analog program" not shown in the block diagram of Figure 1 inverts the Y input to the EGI and presents the normal and inverted signals respectively to the positive and negative Y Sampling Gates. When a change in the X signal of amount $\Delta X$ occurs, the appropriate Y Sampling Gate is opened for a period equal to $\Delta t$. The choice between a $+$Y or $-$Y sample is based on the direction of the change in the X input only and is independent of the polarity of the Y input. Successive input samples are summed in the Accumulating
Integrator and remain stored on the capacitor employed as feedback element of
the operational amplifier. The output of the Accumulating Integrator is in-
herently stair-step in form. The frequency of the step changes is dependent
on the rate of change of the X input, while the magnitude of the change is
dependent upon the value of the Y input at the instant that the Sampling Gate
is closed. With ΔX and Δt made very small, the output signal from the Accumu-
lating Integrator approaches a continuous function to approximate the definite
integral as expressed in Equation (2.1).

2-3. Circuit Description

The description of EGI circuitry which follows will be made with reference
to the schematic drawings of Figures 2 through 6 and to the layout drawings of
Figures 9 through 17. Figures 7 and 8 present photographic views of the equip-
ment proper, which should aid in identifying the location of the various con-
trols and circuits discussed in this section of the report.

Timing Generator

The Timing Generator, illustrated in Figure 2, includes the 100-Kc Crystal
Oscillator and the Frequency Dividers. It generates two output signals approxi-
mately 40 microseconds apart at a 12.5-kc rate which serve to synchronize the
operation of the various sections of the EGI; in particular, they control the
length of the reset pulse and the length of the Y analog sample. The stability
of these output signals is maintained by locating the crystal oscillator in a
constant-temperature oven.

One output signal from the Timing Generator is designated "A" in the schematic
diagram of Figure 2 and in the block diagram of Figure 1. The leading edge of
this signal resets two flip-flops that control the Reset Pulse Generators and
the Y Sampling Gates. The leading edge of the signal referred to as "A" in the
schematic and block diagrams is used to generate an interrogation pulse for the
Error Detectors. The leading edge of the "A" signal follows the leading edge
of the "A" signal by approximately 40 microseconds.

Error Detectors

The Error Detectors (Positive and Negative) examine the analog error
voltage delivered by the Storage Integrator to determine when its magnitude
exceeds a preset value. The polarity of the threshold level that has been exceeded is indicated by an output pulse from the appropriate Detector unit.

The Positive and Negative Error Detectors are, respectively, the stages including TR18 and TR14 shown in Figure 3. They are complementary circuits; that is, the NPN transistor in one is replaced by a PNP transistor in the other, with all voltages reversed in sign. Each Detector operates as a NOR logic device, the condition of the output being determined by the condition of two input signals. One of these is a square-wave pulse of approximately 5 microseconds duration that is supplied as an interrogation pulse to both detectors; the other input signal is the analog error voltage from the Storage Integrator. Thus, a pulse will appear at the output of one of the Detectors if and only if the analog error voltage exceeds the detector threshold level during the interrogation period. Any such output pulse is then amplified and the leading edge is used to set one of two flip-flop units, corresponding to the particular Error Detector which originated the pulse.

The precise value of the Error Detector threshold level is somewhat dependent upon the characteristics of the individual transistor used in the circuit. In order that the threshold level may be set to a desired value, a separate adjustable bias voltage is added to the analog error voltage at the input to each detector unit. In practice, the two threshold levels must be adjusted so that there is a small dead-zone in which no reset pulses will be generated. This is necessary to prevent triggering from noise. The magnitude of the dead-zone may be as small as two to four bits at the Storage Integrator output. In order to minimize shifts in the setting of the threshold level, the Error Detectors and subsequent amplifying stages have been enclosed in the constant-temperature oven.

Flip-Flops

Included in the Delta-X Detector section of the EGI are the circuits identified as Flip-Flop 1 and Flip-Flop 2 on the block diagram of Figure 1, which serve to control the outputs of the Reset Pulse Generators and the Y Sampling Gates. In the schematic diagram of Figure 3, Flip-Flop 1 is the circuit containing TR16 and TR17, while Flip-Flop 2 includes TR21 and TR22.
Both of the bistable circuits are reset by the leading edge of the common signal developed at the "A" output of the Timing Generator. This reset condition ensures that both of the Y Sampling Gates are in an open condition. The appearance of a pulse at the "Set" input to the Flip-Flop will reverse the state of the circuit and the conditions of the Reset Pulse Generator and Y Sampling Gate connected to that output. The output state of the Flip-Flops will be in the reset condition until the first interrogating pulse after the value of the error voltage has exceeded the threshold level of one of the Error Detectors. This output state then generates a reset pulse that reduces the magnitude of the error voltage. Simultaneously the proper Y Sampling Gate is opened. Approximately 40 microseconds after the change of state of the bistable circuit, it is returned to its original state by the reset pulse from the Timing Generator.

Reset Pulse Generators

The two Reset Pulse Generators contained in the Delta-X Detector are complementary transistor switches that control the reset current to the grid of the Storage Integrator. The binary type output of the preceding Flip-Flops ensures that each of these switches is either in the saturated (closed) condition or in the unsaturated (open) condition. The saturated condition of either switch grounds the input resistor from that switch to the grid of the Storage Integrator and inhibits the reset current from flowing to the amplifier. The unsaturated state of either switch allows the current to flow from the corresponding 15-volt supply to the grid of the operational amplifier. The duration of the reset pulses generated by the switches is controlled by the timing contained in the output signals of the Flip-Flops. The magnitude of the reset current from each generator is adjusted by means of the series potentiometer provided at its input to the Storage Integrator amplifier.

A transistor switch in the saturated condition exhibits a small but definite internal resistance, and hence has a small offset voltage associated with it. This is compensated for by the DC offset adjustment included in the emitter circuits of both of the Reset Pulse Generators. A biasing current passing through the emitter resistor is adjusted until the voltage seen at
the collector of the switch is minimum during the saturated condition. It is not necessary that this value be exactly zero since it may be cancelled by the offset of the other Reset Pulse Generator. However, it should be made as small as practical in order to reduce interaction of the offset bias control with the gain control from the output of the generator to the Storage Integrator. A higher resolution offset control is included among the front panel controls so that compensation may be made for any changes in the off condition output from these gates as well as changes in the zero of the operational amplifier used for the Storage Integrator.

Included in the front panel controls are separate indicator lights that show the presence of a reset pulse at the output of each pulse generator. These are useful indications to determine proper operation of the EGI during computing periods and while making adjustments on the gain of the Reset Pulse Generators and the gain of the Y Sampling Gates. A Zero/Operate switch is included in the circuit between the Flip-Flops and the Reset Pulse Generators to provide a zero-output condition of the gates during zero-offset adjustments. The Zero condition of this switch does not affect the condition of the Reset Pulse Indicator lamps.

**Oven Heater Control**

The Oven Heater Control is a DC amplifier which regulates the temperature of the oven located at the top rear of the EGI main chassis. The oven enclosure consists of an aluminum box approximately $5" \times 4" \times 6$", lined with styro-foam slabs for insulation. A schematic diagram of the heater control circuit appears in Figure (4).

The internal air temperature of the oven is sensed by a bead thermistor, which forms one leg of a balanced DC bridge. The output of the bridge is used as the differential input signal of the Oven Heater Control amplifier. The first two stages of this amplifier are located within the oven in order to increase the stability of the control circuit. The heater elements themselves consist of eight 120-ohm 2-watt resistors, connected in parallel and mounted so as to give an even distribution of heat throughout the enclosure.

Experimental data shows that an average input power of 13.2 watts is sufficient to maintain an air temperature of $46^\circ C$ in the oven with a mean room temperature of $23^\circ C$. The control system is capable of holding the oven
temperature within 0.5°C of the set point after reaching the steady-state condition. Further smoothing of small temperature fluctuations is achieved by means of aluminum-block heat sinks used with the critical transistors.

The oven is designed to maintain a temperature level between 45°C and 50°C for operation of the unit in a normal room-temperature environment. An adjustment is provided to change the balance point of the bridge containing the thermistor, thus allowing the regulated temperature to be altered.

8-Volt Regulators

Two series-type voltage-regulator circuits, as shown in Figure 4, are used to provide plus and minus eight volts for various biasing purposes in the EGI. These circuits obtain their power from the corresponding 15-volt supplies. Each unit contains its own voltage reference and feedback amplifier. A potentiometer control is included so that the output levels can be accurately adjusted to the desired magnitude of 8.00 volts.

Y Sampling Gates

The Y Sampling Gates are transformer-driven switching circuits utilizing paired transistors, as illustrated in the schematic diagram of Figure 5. The transformer connections to the transistors are so phased that an input signal to the primary of the transformers turns on both transistors of the switch and causes them to saturate. In the absence of an input signal, no base current is supplied to the transistors and they may be considered in the cutoff state. Since the two transistors are connected with their emitters together, they form a bidirectional switch that will accept signals of both polarities. If the positive and the negative gate circuits are matched, errors due to finite resistance of each gate circuit in its open condition should cancel out at the grid of the Accumulating Integrator. This follows from the fact that one gate circuit is connected to a voltage that is equal in magnitude but opposite in sign to that connected to the first gate.

Gain adjustment is provided for by the series potentiometer located in the Y analog input to each gate. This feature is used to balance the positive input against the negative input and to adjust the overall scale factor of the EGI. A DC offset control has also been provided which allows compensation for the offset voltage of the switches. Two additional offset controls with higher
resolution have been included on the front panel; their connections are shown in the schematic diagram of Figure 6.

The primary winding of each gate-driver transformer is in the collector circuit of a grounded-emitter amplifier. The current in this amplifier is controlled by the state of the flip-flop to which it is connected. A front-panel switch is provided to allow the inputs to both of the gates to be removed during periods of adjustment to the Accumulating Integrator. A germanium diode has been included across the primary of each gate-driver transformer to slow the current build-up in the transformer and prevent a saturation of the core before the "on" command of the input pulse has been completed. This diode is connected so that part of the current from the collector of the driving transistor is shunted around the transformer when the voltage across the winding exceeds the conducting voltage of the diode.

**Analog Program**

The term Analog Program is used to denote collectively the X and Y analog input attenuators, associated calibration circuitry, and four operational amplifiers with computing networks and zeroing controls, as shown in Figure 6. The amplifiers (designated A, B, C, and D) are plug-in units of the type used in the Pace TR-10 analog computer, and have been mounted as a group on top of the EGI main chassis. The recommended zero circuitry and zero/operate controls are located on the front panel immediately adjacent. These zero controls have not proven completely effective, so additional provisions for adjustment are included in the controls of the EGI.

Two relays located beneath the chassis are used for slaved operation of the EGI with a general-purpose analog computer. These relays control the reset circuits of the Storage Integrator and the Accumulating Integrator and provide the conditions of Reset, Compute, and Hold for the EGI.

2-4. Operation of the EGI

This section of the report covers operation of the EGI in conjunction with a general-purpose analog computer. Discussed in detail are the various operating controls, calibration procedures, and specifications for input and output connections to the unit.
Operating Controls

The location and function of individual operating controls are described in the paragraphs which follow. Reference is again made to the schematic diagrams and layout drawings for identification of these controls in the system circuitry. Figures 7 and 8 show the physical placement of each item.

**Operational Amplifier Zero/Operate Switches** - Located near the upper left corner of the front panel, these two toggle switches connect the operational amplifiers to high-gain inverting networks when in the Zero position and to computing networks when in the Operate position. They should be in the Operate position for normal operation of the EGI.

**Operational Amplifier Zero Controls** - These are four potentiometer controls located immediately above the Amplifier Zero/Operate switches, that provide the zero biasing current to the amplifiers. Adjustments should be made on these controls while the Zero/Operate switches are in the Zero position. Each should be set to produce minimum output voltage on the amplifier it controls. The amplifier outputs are accessible from the left side of the chassis. Zeroing of the amplifiers is not too critical, since provisions exist elsewhere in the EGI to compensate for small offset voltages in their outputs.

**Reset Pulse On/Off Switch** - This toggle switch is located on the lower left corner of the front panel and serves to inhibit the reset pulses when in the Off position.

**Positive and Negative Reset Pulse Indicator Lamps** - These are separate indicator lamps that are connected to the flip-flops controlling the output of the Positive and Negative Reset Pulse Generators. These lamps receive an impulse for each reset pulse that is generated.

**Gate Driver Test Points** - The gate driver test points are located on the front panel directly over the Positive and Negative Reset Pulse Indicator Lamps. They allow the commands to the Reset Pulse Generators to be monitored from this point. These test points have an output impedance of approximately 100 kilohms.

**Delta-X Detector Zero/Operate Switch** - This switch is located at the bottom center of the front panel. In the Zero position, it energizes the "A" relay and grounds the input to the Delta-X Detector. This places the Delta-X Detector in an operating condition for zeroing adjustments.
Delta-X Detector Zero Adjust - This is a ten-turn potentiometer (with locking wheel) located on the front panel adjacent to the Delta-X Detector Zero/Operate Switch. It provides an adjustable bias current to the grid of the Storage Integrator.

Y Gate Zero/Operate Switch - This switch is located on the lower right side of the front panel. When in the Zero position, it causes the inputs to the gate driver amplifiers to be grounded. It is used when correcting for the offset of the Y Sampling Gates, as noted below.

Y Gate Zero Adjust - This is a ten-turn potentiometer (with locking lever) located next to the Y Gate Zero Operate Switch. It provides an adjustable bias current to the grid of the Accumulating Integrator which allows compensation for the offset of the Y Sampling Gates and for any remaining zero offset in the output of the operational amplifier used for the Accumulating Integrator.

Calibrate/Operate Switch - This switch is located near the center of the front panel and provides an auxiliary control for the "A" and "B" relays used in the reset circuits of the Storage Integrator and the Accumulating Integrator. The Calibrate position of this switch energizes both the "A" and the "B" relays and overrides any commands at the external inputs.

Calibrate Step Switch - This is a push-button switch located next to the Calibrate/Operate Switch and is used in conjunction with the Calibrate mode of the Y Input Attenuator.

Positive and Negative Gate Offset Controls - These two potentiometer units are located on the front panel above the Y Gate zero controls. They are used to compensate for any DC offset in the Y Sampling Gates and in the output voltage of the two amplifiers driving the gates.

X Input Attenuator - This rotary switch is located near the center of the front panel and controls both the mode and the scale factor of the "X" input. The X-Input mode selects one of two capacitors as the input impedance for the Storage Integrator. The X-Input mode selects one of two resistors as the input impedance for the Storage Integrator, thus allowing the EGI to receive directly the derivative of a signal to be integrated, if this is already available. There is no restriction on the magnitude of the X input; however, its rate of change must be limited to prevent overload of the Storage
Integrator. For this same reason, the direct X input must also be restricted in magnitude.

Y Input Attenuator - This control is located to the right of the X Input Attenuator and allows the operator to have a choice of three attenuation values on the Y input signal. The product of the selected Y multiplication factor and the value of the Y input signal should not exceed 10 volts. This is necessary in order to avoid overloading the operational amplifiers which drive the Y Sampling Gates.

The Y Input Attenuator is incorporated in a rotary switch which has four additional positions, collectively designated the Calibrate Mode. When the switch is in either the Positive or Negative Gate Offset Zero position, the Y input to the operational amplifiers driving the Y Sampling Gates is grounded and a DC current (either positive or negative) will be supplied to the grid of the Storage Integrator upon pressing the Calibrate Step Switch. This causes the appropriate Y Sampling Gate to operate at a steady rate. If the Calibrate/Operate Switch is in the Calibrate position, the corresponding Gate Offset Control may be adjusted by observing the output of the Accumulating Integrator. The Negative Gate Offset Control is affected by the Positive Gate Offset Control, so the positive gate should be adjusted first.

Rotation of the Y Input Attenuator switch to the Calibration Check position causes a voltage to be applied to the Y Sampling Gates and allows an exponential step to be generated at the X input when the Calibrate Step Switch is pushed. This procedure may be used as a check on the operation of the EGI. Further, rotation of the Y Input Attenuator switch to the Drift Adjust position applies a voltage to the input of the Y Sampling Gates but leaves the input to the Delta-X Detector grounded. The output of the EGI then serves to indicate any drift generated in the Delta-X Detector as well as in the Accumulating Integrator.

Delta-X Detector Output Test Point - This test jack is located at the top of the front panel and is connected to the output of the Storage Integrator. It is useful in observing the error voltage of the Delta-X Detector.

Integrator Output Test Point - This test jack is located at the top of the front panel and is connected to the output of the Accumulating Integrator. It is equivalent to the EGI output terminal located at the rear of the chassis.
Positive and Negative Level Adjustments - These two potentiometers are located on top of the chassis adjacent to the front panel. They control the threshold levels of the Error Detectors, thereby allowing the dead zone of the Delta-X Detector to be adjusted. It is recommended that this dead zone be set on the range of ±50 to ±100 millivolts, referred to the Storage Integrator output.

Input Adjust Controls - Four potentiometers are located on the rear of the chassis for the purpose of accurately adjusting the X and Y input attenuation factors (Y multipliers of 1, .1, and .01; X multipliers of 1 and 2).

±8 Volt Regulator Adjustments - These two miniature-potentiometer controls (designated R118 and R108) are located on "Card A," containing the regulator circuits, which is illustrated in the layout drawing of Figure 9. Card A stands nearest the front panel as seen in Figure 8.

Oven Temperature Adjustment - The operating temperature of the oven is set by potentiometer R88, located in the oven on Card F, which includes the low-level stages of the Oven Heater Control amplifier. The layout drawing of this card is shown in Figure 14. The control itself is accessible through the top of the oven, as indicated in Figure 8.

Positive and Negative Reset Pulse Amplitude Adjustments - These two potentiometers are identified as R75 and R68 in Figure 14, and may also be reached through the top of the oven.

Positive and Negative Reset Pulse Offset Adjustments - These potentiometers, R73 and R66 in Figure 14, are mounted adjacent to the Reset Pulse Amplitude controls and are likewise accessible through the top of the oven.

Positive and Negative Y Gate Amplitude Adjustments - These potentiometers are located in the oven on Card H and are identified as R154 and R155 in the layout drawing of Figure 16. They serve to adjust the gain of the sampled inputs of the Accumulating Integrator.

Positive and Negative Y Gate Offset Adjustments - These potentiometers, designated R126 and R136 in Figure 16, are located adjacent to the Y Gate Amplitude controls. Both pairs of potentiometers are accessible through the top of the oven, as may be seen from Figure 8.
Calibration Procedures

The following recommended procedures for calibration are given to assist the operator in making certain adjustments on the EGI. These procedures are designed to minimize the interaction of the controls.

Delta-X Detector Zero Adjust - After a suitable warm-up period of about an hour, the Delta-X Detector should be adjusted to remove any drift in this section. The Delta-X Detector Zero/Operate switch should be placed in the Zero position and the output of the Storage Integrator monitored while the Delta-X Detector Zero Adjust control is adjusted until there is no change in the output of this section. The monitoring may be conveniently performed at the Delta-X Detector Output test jack. A general-purpose oscilloscope with a sensitivity of at least 100 millivolts per inch is suggested for this measurement. The Zero/Operate switch should be returned to its normal operating position after the adjustment has been completed.

Delta-X Detector Calibration - The bit size (reset pulse size) of the Delta-X Detector may be adjusted accurately to any chosen value within the range of the controls. However, it is more important that the positive and negative reset pulses be made equal in magnitude than that they be set to some particular level.

To make accurate adjustments on the reset pulse magnitude, a means must be provided to monitor the pulses accurately. The number of pulses generated for a known change in voltage at the X input of the EGI may be measured by a digital counter such as the Berkeley Universal Counter and Timer Model 5510. For monotonic changes in X, the reset pulses will be of the same polarity. These pulses may then be totaled by connecting the pulse counter to the proper Gate Driver test point (front-panel jack). The indicator lamp directly below the test point will indicate the polarity of the reset pulses being generated. The reset pulse size is given by dividing the known voltage change applied to the X input by the number of reset pulses resulting from that change.

The voltage change to the X input of the EGI may be generated by switching the input between two known levels of voltage. Depending upon the magnitude of the input step, it may be necessary to limit the rise time of the input by passing it through a passive low-pass filter in order to keep from
overloading the Delta-X Detector and hence causing errors in conversion. (The low-pass filter already incorporated in the X input circuitry will protect against overload for moderate steps.) The proper time constant for the filter is determined by observing the error voltage at the output of the Delta-X Detector (front-panel test jack), where the condition of an overload will cause the voltage to exceed the threshold level of the error detectors.

Positive and negative reset pulse amplitudes are established separately by stepping the input voltage in the proper direction as described above and adjusting the amplitude control until the desired number of reset pulses is counted at the associated test point. Between each adjustment the output of the Storage Integrator should be checked for drift which may be introduced by changes in the gate current. The reset pulse amplitudes should be adjusted to give approximately 50 millivolts per bit at the X input using a multiplier of 1. (Changing the input multiplier to 2 makes the reset pulse size 25 millivolts when referred to the EGI input.)

Y Sampling Gate Zero Adjustment - Before operation, any DC offset voltage in the sampling gates and in the operational amplifier used for the Accumulating Integrator should be balanced out. This may be done after a suitable warm-up period by placing the Calibrate/Operate switch in the Calibrate position, the Gate Zero/Operate switch in the Zero position, and observing the output of the EGI at the Integrator Output test point (front-panel jack). This output should be monitored with a general-purpose oscilloscope having a sensitivity of at least 50 millivolts per inch. The Y Gate Zero Adjust control should then be positioned until the output ceases to change. If this condition cannot be achieved with the front-panel controls, one or both of the controls contained in the oven should be adjusted.

The Y Input Attenuator should next be placed in the Positive Gate Offset Zero position and the Calibrate Step switch depressed while observing the output as described above. The Positive Gate Offset Control may then be adjusted until there is no change in the output when the Calibrate Step switch is closed. Upon completion of this adjustment, the Y Input Attenuator switch should be turned to the Negative Gate Offset Zero position and the same procedure performed on the Negative Gate Offset control. It is important that the positive gate be adjusted first since it affects the setting of the Negative
Gate Offset control. These zeroing procedures should be rechecked for maximum
accuracy and the switches then returned to their normal operating position.

**Y Sampling Gate Amplitude Adjustments** - The method that seems most
successful for setting the gains of the Y Sampling Gates utilizes a free-
running oscillator circuit of controlled amplitude and frequency. The well-
known sawtooth generator circuit shown in Figure 18 is suitable for the
purpose. When this waveform is applied to the X input of the EGI alone
with a constant voltage to the Y input, the same waveform should appear at
the output of the EGI. If the gains of the positive and negative sampling
gates are in error by the same amount, the amplitude of the output waveform
will be off. If the gain of one is higher than the other, the waveform will
be distorted in an upward or downward direction that can give the appearance
of a DC current being summed in with the signal at the grid of the Accumulating
Integrator. The gate amplitude controls should be adjusted so that the output
will reproduce the input waveform for several cycles and contain no offset
distortion. Reproducing the curves contained in Figure 20 is considered to
be a good test of the overall performance of the EGI.

The scale factor of the EGI has not been established at any particular
value. It was considered that the scale factor of this experimental system
could easily be adjusted at the output connections to an analog computer or
some recording device with which it might be used. Therefore, consideration
of the accuracy of scale factor adjustments is omitted in the performance
evaluation of the present unit.

**Input and Output Specifications**

The following discussion covers EGI input and output requirements, with
emphasis on operation of the unit in conjunction with a general-purpose
analog computer.

**Power Requirements** - The supply voltages required by the EGI are listed
in Table 1. The rear-panel connections for the inputs of these supplies are
given in Table 2. The plus and minus 15-volt supplies should be as stable
as practicable since they are used to furnish reference voltages for the
Reset Pulse Generators. The regulation and stability of the plus 30-volt
supply, which provided amplifier bias voltages, is not as critical. The
24-volt supplies used for heater power and relay power, as well as the
250-volt supply for lamp power, do not have to be regulated but they should be well filtered to eliminate the noise that might be introduced from these sources.

Table 1. Supply Voltage Requirements

<table>
<thead>
<tr>
<th>SUPPLY</th>
<th>CURRENT REQUIREMENTS</th>
<th>SUGGESTED REGULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plus 250 Volts DC</td>
<td>2 Milliamperes</td>
<td>None</td>
</tr>
<tr>
<td>Plus 30 Volts DC</td>
<td>15 Milliamperes</td>
<td>0.1 Percent or better</td>
</tr>
<tr>
<td>Plus 15 Volts DC</td>
<td>500 Milliamperes</td>
<td>0.05 Percent or better</td>
</tr>
<tr>
<td>Minus 15 Volts DC</td>
<td>500 Milliamperes</td>
<td>0.05 Percent or better</td>
</tr>
<tr>
<td>Minus 24 Volts DC (Heater)</td>
<td>2 Amperes</td>
<td>None</td>
</tr>
<tr>
<td>*24 Volts DC (Relay)</td>
<td>250 Milliamperes</td>
<td>None</td>
</tr>
<tr>
<td>6.3 Volts AC</td>
<td>250 Milliamperes</td>
<td>None</td>
</tr>
</tbody>
</table>

*Note: This supply may be connected either positive or negative.*

Table 2. Rear-Panel Terminal-Block Connections

<table>
<thead>
<tr>
<th>Terminal No.</th>
<th>Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Left Block)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Plus 250 Volts DC</td>
</tr>
<tr>
<td>2</td>
<td>Plus 30 Volts DC</td>
</tr>
<tr>
<td>3</td>
<td>Plus 15 Volts DC</td>
</tr>
<tr>
<td>4</td>
<td>Plus 8 Volts DC</td>
</tr>
<tr>
<td>5</td>
<td>Ground</td>
</tr>
<tr>
<td>6</td>
<td>Minus 8 Volts DC</td>
</tr>
<tr>
<td>7</td>
<td>Minus 15 Volts DC</td>
</tr>
<tr>
<td>8</td>
<td>Minus 24 Volts Heater Power</td>
</tr>
<tr>
<td>9</td>
<td>6.3 Volts AC</td>
</tr>
<tr>
<td>10</td>
<td>6.3 Volts AC</td>
</tr>
<tr>
<td>(Right Block)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Ground for Relay Control</td>
</tr>
<tr>
<td>12</td>
<td>Generalized Integrator Output</td>
</tr>
<tr>
<td>13</td>
<td>X Analog Input</td>
</tr>
<tr>
<td>14</td>
<td>Y Analog Input</td>
</tr>
<tr>
<td>15</td>
<td>(Blank)</td>
</tr>
<tr>
<td>16</td>
<td>Relay Power 24 Volts DC (Plus or Minus)</td>
</tr>
<tr>
<td>17</td>
<td>B-Relay Control Input</td>
</tr>
<tr>
<td>18</td>
<td>A-Relay Control Input</td>
</tr>
</tbody>
</table>
Grounding System - Separate ground wires are run from each of the EGI circuits to a common ground bus located below the chassis near the rear-panel terminal block. Separate external power-ground and signal-ground wires should also be used and connected to the common ground point (terminal 5). The present EGI unit has the ground bus tied to chassis ground for purposes of reducing noise.

Slave Relays - Two relays located under the chassis permit slaving the operation of the EGI to an analog computer. These relays designated "A" and "B", establish Reset, Compute, and Hold conditions through the following logic. With both the A and B relays de-energized, the EGI is in the Reset mode. It is switched to the Compute mode when both relays are energized; this is accomplished by grounding the A and the B relay control input only (terminal 17 to terminal 11). The Hold mode of operation is produced by grounding the B relay control input only (terminal 17 to terminal 18).

Signal Voltage Ranges - The magnitude of the X and the Y input voltages should not exceed 100 volts (plus or minus). Care should be taken to set the Y Input Attenuator so that the product of the Y multiplier used and the maximum value of the Y input voltage does not exceed 10 volts. This is the limit of the output voltage of the Pace TRIO operational amplifiers, which drive the Y Sampling Gates. The X and the Y inputs should be supplied from low-impedance sources to prevent errors due to loading; a driving-point impedance of 1 ohm or less is recommended.

The limit of the output voltage of the EGI is plus or minus 10 volts. If the output signal exceeds this value, errors may be introduced due to saturation effects in the operational amplifier used for the Accumulating Integrator. The linearity of the EGI output within the prescribed operating range is evident from the curves presented in Figure 20. An estimate of the usable range of the Y input to the sampling gates may also be obtained from this figure. It appears that a level of about four volts at the input to the Y Sampling Gates is all that can be accumulated without introducing errors due to nonlinearities in the presently used gate transistors.
2-5. Performance Data

Preliminary tests have been run with the EGI to demonstrate some performance capabilities of the present experimental model and to indicate where design improvements might be made. The results of these tests are summarized below.

X and X Inputs

The data recordings of Figure 19 illustrate the two modes available for the independent-variable input of the EGI. The input signals used in all of the tests were generated with the sawtooth oscillator circuit depicted in Figure 18.

From Equations (2-3) and (2-4) it is evident that if one supplies a constant Y input, the EGI output signal should accurately reproduce the X input signal, except for a possible change in sign and amplitude. Curve A-2 in Figure 19 illustrates the fidelity with which Curve A-1 is reproduced in this basic mode of operation. Curve A-3 shows (on a greatly expanded scale) the error contained in the output curve due to incomplete zeroing of the DC offset voltage of the Y Sampling Gates. Comparison may be made between this curve and Curve E in Figure 20, where a more critical adjustment of the offset controls was made.

The lower pair of curves in Figure 19 illustrate the use of an X signal to furnish the independent-variable input for the EGI. The derivative signal presented as Curve B-1 of the figure was obtained directly from the input to the integrator within the sawtooth function generator. This signal was fed to the EGI through the X input, which causes an integration in time to be effected prior to the main process of incremental digital conversion. This time integration is achieved by employing a resistive (rather than capacitive) input element to the grid of the Storage Integrator. The EGI output waveform obtained with the above X signal appears as Curve B-2. This output would be identical to Curve A-3 except for the fact that the gain of the resistive input differed slightly from that of the capacitive input. (This discrepancy will be corrected for future tests by changing the value of the resistor used.) No adjustments were made to the controls of the EGI during the runs shown in Figure 19.
Y Input Range

Figure 20 presents results of a brief series of runs which provide some indication of the useful working range of the Y analog signal at the input of the EGI. The same sawtooth waveform was used for the X analog signal in every run of this series, thereby minimizing the possible effect of errors introduced by the Delta-X Detector. Each run was made with a different constant-amplitude Y input, the output being attenuated accordingly so as to produce the same recorded level. The attenuators used had a tolerance of better than 0.1 percent. Curve E of Figure 20 indicates that this was essentially no zero error in the output signals recorded during this test. Extra care was taken in adjusting the EGI controls before the series of runs was started, but no changes were made at any time in its course.

As noted from Curves F, G, and H of Figure 20, the sign of the output waveform reflects the polarity of the Y input signal, in accordance with Equation (2-3). It was found that the Y input level could be varied over a range of two decades for both polarities of inputs without appreciable effect on the normalized output signals. The zero output case, Curve E, shows an error of less than one millivolt during the run. In Curves D and F there appears to be a negative drift amounting to one or two millivolts at the end of the 60 second run. This maximum error is not large when referred to the "full-scale" output swing of -4 to +4 volts represented by Curves B and H.

Function Generation

Figures 21 through 23 present data illustrating use of the EGI to generate various functions of time. For the first set of curves in Figure 21 a negative ramp function was used for the Y analog signal and the sawtooth waveform again served as the independent variable X. The resulting output function, Curve A-3, is readily verified by a piecewise numerical solution. For the second set of curves in Figure 21 the sawtooth waveform was replaced by a cosine. An analytical expression for the output function can be readily written in this case:

\[
(Fig. 21-B) \quad Y = -at, \quad X = b \cos \omega t, \quad \int_{-22-}^{Y} dX = \frac{ab}{\omega} (\sin \omega t - \omega t \cos \omega t).
\]
A related group of functions is presented in Figure 22. The output curves here are described by the following mathematical forms:

(Fig. 22-B) \[ Y = a, \quad X = b \cos \omega t, \quad \int Y \, dX = ab (\cos \omega t - 1); \]

(Fig. 22-C) \[ Y = a \cos \omega t, \quad X = b \cos \omega t, \quad \int Y \, dX = \frac{ab}{2} (\cos 2\omega t - 1); \]

(Fig. 22-D) \[ Y = a \sin \omega t, \quad X = b \cos \omega t, \quad \int Y \, dX = \frac{ab}{2} (\omega t - \sin 2\omega t). \]

The EGI control settings used in generating the functions shown in Figures 21 and 22 were identical to those used in generating the curves of Figure 20.

Figure 23 shows the result of applying the same sawtooth function to both the X and the Y inputs simultaneously. Curve A of this figure is a plot of the input voltage against the output voltage of the EGI unit; this trace represents two complete cycles of the input function. Curve B and Curve C show these same input and output signals plotted separately against time. Due to the nature of the function generated, any errors that are stored in the Accumulating Integrator during one direction of the sweep of the independent variable may not be removed by sweeping this variable in the opposite direction. Such errors may be introduced by changes in the offset voltage of the Y Sampling Gates. Also errors can be introduced by any phase difference in the signal arriving at the X input and the signal arriving at the input to the Y Sampling Gates. This latter error may result from the low-pass filter built around the operational amplifiers driving the Y Sampling Gates; however, the limiting factor found with the present EGI unit is that the transistors used in the sampling gates show drift problems due to temperature changes. The temperature of the junction of the gate transistors will change slightly with the current flowing through the gate. This change in temperature will affect the operating characteristics of the transistors (e.g., the saturation voltage and collector leakage current) which in turn may introduce errors into the function being generated. For the test illustrated in Figure 23 the Sampling Gate offset controls were changed slightly from the settings previously used, in order to compensate.
for the temperature-induced errors. However, some residual effect is still evident in the fact that the retrace of the output signal does not lie on itself (see Curve A).

2-6. Recommendations

It is felt that further improvement of the EGI should be made in the area of the Y Sampling Gates. One obvious step would be to replace the present sampling-gate transistors with a matched pair in each gate. This matching would eliminate some of the problem caused by temperature change of the transistor junctions with changing magnitude of the Y input signal. To further reduce the temperature effect, better heat transfer should be achieved between the junctions and the constant-temperature environment of the oven. It might be possible, for example, to obtain transistors with significantly lower thermal resistance between the junction and case. A better heat-sink design should also be attempted.

Further reduction of power in the sampling gates by circuit changes should be considered. An approach to this might be to use a second set of positive and negative sampling gates which would operate on an alternating basis with the present set, thereby reducing the total number of samples per gate by a factor of two for any given function. The power dissipated in each transistor is approximately equal to the product of the square of the average gate current and the saturation resistance of the gate transistor. The alternate pair of units would thus allow the power dissipated in the gates to be reduced by a factor of four, which should significantly reduce the junction temperature fluctuations due to the Y signal magnitude.

It is recommended that a more detailed set of operating tests and performance evaluation of the EGI be carried out prior to the proposed modifications.
Figure 1: Block Diagram of Electronic Generalized Integrator
Figure 2. Schematic Diagram of Timing Generator
Figure 3. Schematic Diagram of Error Detectors, Reset Pulse Generators, Reset Pulse Indicators, and Associated Logic Circuitry
Figure 1: Schematic Diagram of Oven Heater Control System and Positive and Negative 8-Volt Power Supply Regulators
Figure 5. Schematic Diagram of Y Sampling Gates and Gate Drivers
Figure 6. Schematic Diagram of Input Switching Circuitry and Analog Program
Figure 7. Front and Rear Views of Electronic Generalized Integrator
Figure 8. Top Views of BGI with Oven Cover In Place and Removed
Figure 10. Circuit Layout Drawing, Card B
Figure 11. Circuit Layout Drawing, Card C
Figure 12. Circuit Layout Drawing, Card D
Figure 13. Circuit Layout Drawing, Card E
Figure 14. Circuit Layout Drawing, Card F
Figure 15. Circuit Layout Drawing, Card G
Figure 16. Circuit Layout Drawing, Card H
Figure 17. Circuit Layout Drawing, Card I
Figure 18. Schematic Diagram of Sawtooth Function Generator
Figure 19. Comparison of EGI Alternative Modes of Operation
Figure 20. Test Results Illustrating Linearity of Y Sampling Gates
Figure 21. Function Generation with the EGI, Examples 1 and 2
Figure 22. Function Generation with the EGI, Example 3
Figure 23. Function Generation with the EGI, Example 4
5th QUARTERLY PROGRAM REPORT

PROJECT NO. A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

By F. Dixon

Prepared for
George C. Marshall Space Flight Center
Huntsville, Alabama

Contract NAS8-2473

12 September - 12 December

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

5th QUARTERLY PROGRAM REPORT

on

Contract No. NAS8-2473

12 September - 12 December 1962

For

GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted:

F. Dixon
Project Director A-588, and
Head, Special Problems Branch
1. PROGRAM OUTLINE

Georgia Tech Research Project No. A-588 was established under Contract NAS8-2473 on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at the George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project was assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates as a unit of the Special Problems Branch of the Engineering Experiment Station's Physical Sciences Division.

By agreement with the Contracting Officer's Representative, Dr. W. P. Krause of MSFC, efforts during the first year were devoted to the following specific tasks under areas A and B:

I - Design and construction of an experimental electronic generalized integrator (EGI) suitable for use with existing analog equipment.
II - Investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies.

Results of these efforts are documented in an Annual Technical Summary (GIT/EES Report A588/134) and a special Technical Note (GIT/EES Report A588/T1).

Modification No. 1 to the basic contract, executed on 2 October 1962, authorized a no-cost extension in time until 12 November 1962. Modification No. 2 provided funds for an additional twelve months' period of performance, with the Scope of Work revised as follows:

A - Generalized Analog Integrator. Continue experiments with the generalized integrator developed under this contract to determine its capabilities as a practical analog computing device.
B - Manned Space Flight Simulation. Investigate methods and techniques for simulating manned space flight and develop minor electronic equipment as required to implement suggested approaches.
2. ACTIVITIES DURING CURRENT REPORT PERIOD

2-1. EGI Documentation

Efforts during September were limited to the preparation of electrical diagrams and a detailed description of the generalized integrator for inclusion in a "final" technical summary report. Completion of this report was delayed because of uncertainties as to the duration, funding, and future status of the project.

2-2. Modification of F-86D Flight Simulator Components

On 22 August 1962 Georgia Tech received at its Chamblee warehouse the F-86D Aircraft Trainer and Flight Simulator (USAF Stock Nr. 6930-61-1, Serial Nr. 52-32) which had been released as surplus property from Dobbins AFB, Georgia. This equipment was obtained as a donation from the U.S. Government with the understanding that it could be cannibalized by sections for educational and research purposes, in accordance with a letter request dated 1 March 1961 to Ogden Air Material Area, Hill AFB, Utah.

Following the project visit on 31 August 1962 by Mr. F. T. Shaver, (MSFC) and Mr. R. S. Johnson (G.E., Co., Huntsville), consideration was given to possible utilization of the F-86D cockpit section for preliminary manned space flight simulation studies at MSFC. The pertinent technical requirements were further discussed during a visit to Georgia Tech by Messrs. F. L. Vinz and J. A. Waller (MSFC) with Mr. R. S. Johnson on 14 September. It was subsequently agreed that the project would make appropriate modifications to the cockpit and would design, assemble, and wire all auxiliary apparatus needed to permit operation of the desired set of panel instruments and navigational controls in conjunction with a general-purpose dc analog computer at MSFC.

The major F-86D Flight Simulator components were transported to project headquarters at Georgia Tech Research Area 4 in early October. Technical planning visits in connection with the modification program were made by Mr. R. S. Johnson alone on 8 October and by Mr. Johnson with Messrs. F. L. Vinz and J. P. Pavlick (MSFC) on 19 October. Assistance in component wiring
and assembly was subsequently provided by Mr. Roy Hewett (G. E. Co., Huntsville). An additional visit was made by Mr. F. L. Vinz of MSFC on 1 November to help check out portions of the equipment and establish final installation requirements.

2-3. Transmission of Flight Simulator to MSFC

By letter dated 2 November 1962 the NASA Contracting Officer in Huntsville was officially advised that the following F-86D Flight Simulator components would be made available by Georgia Tech to the MSFC Computation Division on a one-year renewable loan basis:

(a) Instrumented cockpit section (5 ft. x 8 ft. x 7 ft. high) complete with motor-driven canopy and hood

(b) One 4-door cabinet (3 ft. x 5 ft. x 7-1/2 ft. high) containing two racks of electronic apparatus, blower unit, and four 50-ft. cable assemblies to connect with items (a) and (c)

(c) Relay panel with six repeater instruments similar to those in cockpit to be operated from item (b).

Items (a) and (b) were shipped on 13 November via North American Van Lines and unloaded at Redstone Arsenal the following day. Item (c), together with wiring diagrams and miscellaneous spare parts, was delivered on 15 November by Mr. F. R. Williamson of Georgia Tech, who also assisted with installation and checkout of the equipment at MSFC. A second visit was made by Mr. Williamson and Mr. J. W. Robertson on 6 December to deliver additional spare parts and help with equipment calibration procedures and adjustments.
3. PLANNED ACTIVITIES FOR NEXT REPORT PERIOD

(1) Georgia Tech project personnel will continue to assist the Flight Simulation Branch at MSFC, as needed, in providing optimal instrumentation of the F-86D Flight Simulation cockpit for immediate studies of orbital docking or other manned space flight maneuvers.

(2) In the event that no new equipment development assignments arise in connection with the manned space flight program, attention will be given to further testing and possible modification of the Electronic Generalized Integrator.
6th QUARTERLY PROGRAM REPORT

PROJECT NO. A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

Prepared for
George C. Marshall Space Flight Center
Huntsville, Alabama

Contract NAS8-2473

12 December 1962 - 12 March 1963

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

6th QUARTERLY PROGRAM REPORT
on
Contract No. NAS8-21473

12 December 1962 - 12 March 1963

For
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted:

F. Dixon
Project Director A-588, and
Head, Special Problems Branch
1. PROGRAM OUTLINE

Georgia Tech Research Project No. A-588 was established under Contract NAS8-2473 on 12 September 1961 to assist the Flight Simulation Branch of the Computation Division at the George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within the following areas of interest:

A - Development of a Generalized Integrator
B - Analog-Computer Statistical Analysis
C - Analog-Computer Partial Differential Equation Solution

This project was assigned to the Georgia Tech Analog Computer Laboratory (ACL), which operates as a unit of the Special Problems Branch of the Engineering Experiment Station's Physical Sciences Division.

By agreement with the Contracting Officer's Representative, Dr. W. P. Krause of MSFC, efforts during the first year were devoted to the following specific tasks under areas A and B:

I - Design and construction of an experimental electronic generalized integrator (EGI) suitable for use with existing analog equipment.

II - Investigation of analog techniques for the generation of nonstationary noise voltages to be used in analog Monte Carlo studies.

Results of these efforts are documented in an Annual Technical Summary (GIT/EES Report A588/P4) and a special Technical Note (GIT/EES Report A588/T1).

Modification No. 1 to the basic contract, executed on 2 October 1962, authorized a no-cost extension in time until 12 November 1962. Modification No. 2 provided funds for an additional twelve months' period of performance, with the Scope of Work revised as follows:

A - Generalized Analog Integrator. Continue experiments with the generalized integrator developed under this contract to determine its capabilities as a practical analog computing device.

B - Manned Space Flight Simulation. Investigate methods and techniques for simulating manned space flight and develop minor electronic equipment as required to implement suggested approaches.
2. ACTIVITIES DURING CURRENT REPORT PERIOD

2-1. Generalized Analog Integrator

Quarterly work on the experimental electronic generalized integrator (EGI) has included efforts to make a semi-permanent installation of the unit with the analog computer facility so that a more extensive program of operational tests might be performed as machine time becomes available. Particular attention had to be given to the grounding system in order to insure the most accurate results possible.

The necessary cabling to convert the EGI to the analog computer has been completed. However, one of the PACE TR-10 operational amplifiers used in the EGI developed defects and had to be repaired with temporary substitute parts which do not provide completely satisfactory operation.

Efforts to restore the system to its previous level of performance and to extend the evaluation tests will be continued. Further consideration will also be given to the possibility of improving the transistor circuitry used in the Y Sampling Gates of the EGI, as discussed in the Annual Technical Summary Report.

2-2. Manned Space Flight Simulation

On 12 January, Mr. R. S. Johnson (G.E. Co., Huntsville) visited Georgia Tech to discuss requirements for an area display system to be used in simulating terminal control of a lunar landing vehicle. On 15 January Messrs. F. Dixon and F. R. Williamson visited MSFC to attend a project planning session on this topic, and at the same time to provide further advisory assistance with the operation of instruments in the F-86D Flight Simulator cockpit.

It appears that an optical projection scheme to be implemented in the Flight Simulation Branch at MSFC will meet the immediate requirements for simulating lunar-surface approach. However, the Georgia Tech project should consider the feasibility of designing an electronic system which will generate artificial targets and grid lines, with suitable control of spacing and orien-
tation, for display on a television tube. The idea originally proposed at the January meeting still seems to have merit—namely, to record on magnetic tape, in standard book-scan sequence, the surface target information for some maximum area of interest in a lunar descent. This tape, formed into a loop, would then serve as a stored map or a master raster, to be scanned repetitively at the appropriate picture rate (typically once every three seconds). By suitable synchronizing circuitry, any component area or subraster could be read off the tape for presentation on the display tube. Gating of this read-out would be controlled by an analog computer programmed so as to take into account the instantaneous altitude and attitude angles of the lunar viewing device. Further attention must be given to the various technical problems which are recognized in this concept.

It is understood that negotiations are currently underway to transfer to NASA the F-86D Flight Simulator components that were furnished by Georgia Tech in November 1962 on a one-year renewable loan basis. This proposed transfer of title appears to be in the best interests of all concerned.
7th QUARTERLY PROGRAM REPORT

PROJECT NO. A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

Prepared for
George C. Marshall Space Flight Center
Huntsville, Alabama

Contract NAS8-2473

12 March - 12 June

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
DEVELOPMENT OF
NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

7th QUARTERLY PROGRAM REPORT
on
Contract No. NAS8-2473

12 March - 12 June 1963

For
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

Submitted:

F. Dixon
Project Director A-588, and
Head, Special Problems Branch
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2. ACTIVITIES DURING CURRENT REPORT PERIOD

2-1. Generalized Analog Integrator

Relatively little effort was applied to the EGI test program during this quarter. However, in anticipation of modifying the Y Sampling Gate circuitry, procurement was made of some Fairchild type FSP38 transistors, which are expected to provide a substantial improvement in performance over the presently used type 2N495. Each FSP38 contains two electrically isolated but thermally connected transistors (similar in type to the 2N708), and hence would replace a pair of 2N495's which, in the present EGI, are clamped 1/2-inch apart between aluminum blocks for heat-sinking purposes. The FSP38's were designed for high-speed saturated switching operation, and have the desirable features of low leakage current and extended current gain through the microampere region. Tests with these new transistors in the EGI will be attempted during the next quarter.

On 23 May, F. Dixon and F. R. Williamson visited MSFC for a project planning session, and at that time obtained a spare PACE TR-10 operational amplifier on a contract-loan basis to facilitate work with the EGI. It should also be mentioned that plans are underway to move the main bank of analog computer equipment from Research Area 4 to a more central location in the Electrical Engineering Building on the Georgia Tech campus. Future tests of the EGI will therefore be conducted with the smaller but newer Donner Model 3100 system, which is being assigned for full-time use on instrumentation development and research in the Special Problems Branch.

2-2. Special Study of Passive Element Computers

During the visit to MSFC on 23 May, discussions were held concerning the proposal by Mr. Frank L. Vinz (M-COMP-SA) for two additional tasks to be assigned to Georgia Tech under the current contract (see Memo to M-COMP-S dated 13 May 1962). The first of these, designated "Comparison of Passive Element Computers with Other Computers," involves the study and analysis of a fairly extensive body of literature, coupled with the procurement of up-to-date cost and applications data from users and
manufactures of passive element computers. The task objective is to assemble in report form such information as will assist the Computation Division at MSFC in determining whether or not a passive element computer should be purchased.

Mr. Richard E. Bryan (Research Assistant) is currently performing the survey of available literature and will prepare a summary thereof during the next quarter. The primary references are five text-books on analog computation, in which some 150 pertinent journal articles have been cited. A breakdown by subject indicates past applications of passive element computers in the analysis and simulation of power distribution, mechanical structures, heat flow, fluid flow, quantum mechanics, physiological systems, particle diffusion, nuclear reactor kinetics, chemical processes, and waveguides and cavity resonators. From the standpoint of mathematical forms, the problems covered include linear and nonlinear ordinary differential equations, various types of partial differential equations, so-called "secular equations," and systems of linear algebraic equations. A recent ASTIA report bibliography on "PERT" (AD-297 800) has provided some leads apropos of the possible application of passive element computers to the solution of network problems from the field of scientific management control.

Upon completion of the basic literature survey, a visit will be made to the Lockheed/Georgia Division plant in Marietta, Ga., to confer with personnel there who have had first-hand experience in the operation and maintenance of a large-scale network analyzer.

2-3. Manned Space Flight Simulation

The second of the tasks submitted for assignment to Georgia Tech during the project meeting in May is designated "Electronic Control of Closed-Circuit Television for Visual Simulation." A preliminary examination has been made of the MSFC Internal Note IN-M-COMP-63-4 which was furnished as background material. It is believed that with the time and money remaining on this contract, a laboratory unit could be built to demonstrate the feasibility of controlling TV picture size and deflection by electronic rather than servo means. However, another visit to MSFC should be scheduled to discuss details of the proposed approach before this task is undertaken.
8TH QUARTERLY PROGRAM REPORT

PROJECT NO. A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

A - Generalized Analog Integrator
B - Manned Space Flight Simulation

By F. Dixon

Contract NAS8-2473

12 June to 12 September 1963

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama
GEORGIA INSTITUTE OF TECHNOLOGY
Engineering Experiment Station
Atlanta, Georgia

8TH QUARTERLY PROGRAM REPORT
PROJECT A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION
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Modification No. 1 to the basic contract, executed on 2 October 1962, authorized a no-cost extension in time until 12 November 1962. Modification No. 2 provided funds for an additional twelve months' period of performance, with the Scope of Work revised as follows:

A - Generalized Analog Integrator. Continue experiments with the generalized integrator developed under this contract to determine its capabilities as a practical analog computing device.

B - Manned Space Flight Simulation. Investigate methods and techniques for simulating manned space flight and develop minor electronic equipment as required to implement suggested approaches.
2. ACTIVITIES DURING CURRENT REPORT PERIOD

2-1. Special Study of Passive Element Computers

Most of the effort expended on this project during the current quarterly period has been in connection with the "Comparison of Passive Element Computers with Other Computers"--the first of the special task assignments outlined in Mr. Frank L. Vinz's memo of 13 May 1963 to M-COMP-S. The literature survey, which was initiated immediately following the project conference at MSFC on 23 May, has now been completed. The results of this survey are incorporated in Project A-588 Technical Note No. 2, "Outline of Network Analyzer Applications" by Richard E. Bryan, 15 August 1963, seven copies of which have been forwarded to the NASA Contracting Office in Huntsville for distribution.

On 1 August, project staff members visited Mr. Sam Robinson of the Analog Computation Group at Lockheed/Georgia (Marietta, Ga.) to obtain more direct information concerning the operation and maintenance of a passive-element computer. The Lockheed facility consists of one medium-size DAEAC (Direct Analogy Electric Analog Computer) system, produced by Computer Engineering Associates in Pasadena, California. The original design of the DAEAC equipment is credited to Dr. G. D. McCann of the California Institute of Technology.

The Lockheed unit, which represents an investment of approximately $500,000, is comfortably housed in a 30- by 40-foot room, adjacent to the main analog computer laboratory. The equipment is divided into several small "bays", each provided with banks of resistors, capacitors, inductors, transformers, and input equipment, interconnected through patch panels and controlled from a single console. Resistors are variable from 1 ohm to 10 kilohms in 1-ohm steps; capacitors are variable from 1 millimicrofarad to 1.11 microfarad in 1-millimicrofarad steps; inductors are variable from 1 millihenry to 1 henry in 1-millihenry steps; transformer turns ratios may be established with 1% accuracy. Also, a small supply of fixed 10-henry inductors and 10-microfarad capacitors are kept on hand. The system utilizes current generators and operational amplifiers to simulate forcing functions, and is capable of both repetitive and one-shot operation.

Although the DAEAC would be classed as a general-purpose machine, it has been used at Lockheed almost exclusively for structural analysis. Static analysis is carried out by exciting an L-C network with constant-amplitude ac current and
reading the amplitudes (i.e., loads and moments) at various points of interest. Dynamic analysis begins with the construction of a network analog of the structure to be analyzed using R, L and C components and transformers. A cell-by-cell procedure is followed in developing the analog, and a direct programming procedure is used which eliminates the need for writing equations. Using various input or forcing functions, the structure is then analyzed for vibration and flutter characteristics.

Programming a typical structure (such as an aircraft wing) requires on the order of two months. Once the programming has been completed, computer analysis of the structure usually proceeds without interruption for some three to six months. This scheduling policy eliminates the need for storing computer programs over long periods of time, which could become quite expensive owing to the high cost of patchboards and the number required per problem. Wind-tunnel analysis of a structural model is normally carried out in parallel with the computer analysis, with close coordination being maintained between the two groups.

To operate and maintain the Lockheed DAEAC requires on the average five full-time and one half-time personnel—namely, two analog-computer engineers, three structures engineers, and one analog-computer maintenance technician for half-time servicing.

A DAEAC system may be expanded more or less indefinitely by adding bays as required. For a system approximately twice as large as the present Lockheed unit, it would be desirable to have two control consoles, with each bay independently controlled from both. This would allow the analysis of either a single large structure by itself or else two separate average-size structures concurrently.

2-2. Generalized Analog Integrator

As mentioned in the preceding quarterly report, it is hoped that improvements in the performance of the Y Sampling Gate of the EGI may be realized by using Fairchild type FSP38 transistors in place of the original type 2N495 transistors. Since the 2N495 is a p-n-p unit while the FSP38 (combining two 2N708's) is of the n-p-n variety, some redesign of the driving circuitry was required for this change. The necessary modifications have now been made and the FSP38's installed. Other minor problems continue to plague the effort, however, and no satisfactory test runs are as yet recorded.
On 15 August, F. Dixon, J. W. Robertson and F. R. Williamson of Georgia Tech visited MSFC to discuss plans for the work on "Electronic Control of Closed-Circuit Television for Visual Simulation", the second of the task assignments outlined in Mr. Frank L. Vinz's memo of 13 May to M-COMP-S. In view of the uncertainties concerning future requirements for visual displays for simulation of manned space flight, and taking into account the concurrent development of other possible approaches at MSFC, it was decided that Georgia Tech should proceed with a limited experimental effort utilizing an available surplus television receiver (Philharmonic Model 54TW21). The objective would be to uncover and solve the various general problems that might be involved in controlling TV raster size and position from analog voltages.

As of the close of the current quarterly period, no laboratory work on this problem has yet been started. Considering the limited time and funds remaining under the contract, it may prove desirable to delay this effort until after the contract has been renewed (November 12) and the project has completed its forthcoming move from Research Area 4 to the new Nuclear Research Center.
QUARTERLY PROGRAM REPORT 64-1

PROJECT A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

F. Dixon and J. L. Hammond, Jr.

Contract NAS8-2473

12 December 1963 to 12 March 1964

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama
DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

By

F. Dixon and J. L. Hammond, Jr.

Contract NAS8-2473

12 December 1963 to 12 March 1964

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER
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Huntsville, Alabama
INTRODUCTION

Program Outline

Georgia Tech Research Project No. A-588 was established under Contract NAS8-2473 on 12 September 1961 in the Special Problems Branch of the Engineering Experiment Station's Physical Sciences Division. The overall project aim has been to assist the Flight Simulation Branch of the Computation Division at George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within several areas of mutual interest. By agreement with the Contracting Officer's Representative, Dr. W. P. Krause of MSFC, efforts during the first year were devoted to 1) design and construction of an experimental electronic generalized integrator for potential application as an analog computing component, and 2) investigation of analog techniques for the generation of nonstationary noise voltages to be used in Monte Carlo studies. Results of these efforts are documented in the initial Annual Technical Summary (Report A588/P1) and in Technical Note No. 1 (Report A588/P1), both dated 12 November 1962.

Following a no-cost extension of the program until 12 November 1962, the contract was renewed for an additional twelve months to cover a) continued experimentation with the generalized integrator, and b) assistance to MSFC on problems connected with manned space flight simulation. After a further no-cost extension until 12 December 1963, the contract was renewed for another twelve months with the Scope of Work revised to read as follows.

A. Simulation of Manned Space Flight. The Contractor will continue the work already started under Contract NAS8-2473.

B. The Contractor will in consideration of work done with the generalized integrator concentrate on an investigation and on improvement of electronic switches for analog computation, which are the key elements of the generalized integrator as well as of the analog incremental converter and of dual time scale analog computation.

C. Analytical investigations of methods for generating nonstationary noise of specified spectral and amplitude distribution for Monte Carlo simulation studies.

Project Organization

The first two of the above assigned tasks are being performed in the Special Problems Branch proper, which now has its Headquarters and Instrumentation Laboratory located in the new Nuclear Research Center (at 6th
Street and Atlantic Drive, N. W.) on the Georgia Tech campus. The third task is being performed largely by personnel associated with the School of Electrical Engineering, and utilizes facilities of the main Analog Computer Laboratory which is operated by the Special Problems Branch in the E.E. Building.

For purposes of administrative convenience, the project staff has been divided into two functional groups, presently organized as follows:

**Group 1 - Tasks A and B (Instrumentation Lab)**
- Mr. Frederick Dixon (Senior Research Physicist) - Project Director
- Mr. Frank R. Williamson (Assistant Research Engineer)
- Mr. John W. Robertson (Research Assistant)

**Group 2 - Task C (School of E.E. and Analog Computer Lab)**
- Dr. Joseph L. Hammond, Jr. (Associate Professor) - Group Coordinator
- Dr. Roger P. Webb (Assistant Professor)
- Dr. Thomas M. White (Associate Professor)
- Mr. Clifford 0. Guffee (Graduate Teaching Assistant)
- Mr. Richard E. Bryan (Graduate Research Assistant)

Progress reports for the two groups will normally be presented in separate sections of each Quarterly Program Report.
Simulation of Manned Space Flight

At the start of the current report period, the only active assignment under Task A of the contract program was on "Electronic Control of Closed-Circuit Television for Visual Simulation." Existing plans were to modify an available surplus television receiver (Philharmonic Model 54TW21) and devise additional circuitry, as needed, in order to demonstrate the feasibility of controlling TV raster size and position from suitable analog voltages. The disruption of laboratory operations due to relocation of the Special Problems Branch from Research Area 4 to the Nuclear Research Center prevented any major effort being applied to this assignment for most of the quarter. A telephone call from Dr. Krause in early March indicated that the successful development at MSFC of other devices for visual simulation made it unnecessary to proceed further with the TV control problem, and this effort has therefore been abandoned.

Electronic Switches for Analog Computation

The immediate objective under Task B of the contract program has been to survey the state of the art in electronic switching techniques. Listed below are the various devices identified in this category for which manufacturers' data sheets and/or technical articles have thus far been compiled.

(1) Diode gates
(2) Transistor gates
(3) Hot Carrier Diode gates
(4) Unifet gates
(5) Dual Transistor gates
(6) Hall Effect gates
(7) Photoconductor gates
(8) Phototransistor gates
(9) Photodiode gates

Companies represented by product information on one or more of these types of devices are as follows:

Amperex
Edgerton, Gemerschauen, and Greer
Fairchild Semiconductor
General Electric Co.
General Instrument
General Transistors
Hewlett-Packard Associates
Hughes Semiconductor Division
Pacific Semiconductor Inc.
Raytheon Co.
Siliconix Inc.
Sperry Semiconductor
Sprague Electric Co. (2)
Sylvania Electronic Tubes (7)
Texas Instruments (1,2,5,8)

Listed next are sources of useful technical articles or descriptive literature on some of the devices in question:

Bell Telephone Mfg. Co., Antwerp, Belgium (1,3)
Columbia University Electronics Research Lab (1,3)
Convair (2,5,6)
Defense Research Board Telecommunications Establishment, Ottawa, Canada (1,3)
English Electric Co., Rugby, England (1,3)
General Electric Co. (1,3)
Hermes Electronic Co. (1,3)
Hughes Semiconductor Division (2,5)
Leeds and Northrup Co. (2,5)
Siliconix, Inc. (4)
Texas Instruments, Inc. (1,2,3,5)
Westinghouse Electric Corp. (2,5)

The following textbook and journal references, dealing primarily with diode and transistor gates, have been found useful:

Chance, B. et al (ed.), Waveforms, MIT Radiation Laboratory Series, Vol. 19, McGraw-Hill Book Co., 1949. (Ch. 10 and Sec. 11.3)


Millman, J. and Taub, H., Pulse and Digital Circuits, McGraw-Hill Book Co., 1956. (Ch. 14)

It is expected that the basic literature survey will be extended to include a more thorough library search during the next quarter. It is also planned that laboratory experimental studies will be initiated along lines to be discussed with the Project Officer during his proposed visit to Georgia Tech in April.
GROUP 2 PROGRESS REPORT

Objectives

The first objective adopted under Task C is to formulate a method for generating a nonstationary random process with prescribed first and second statistical moments. The procedure is to be implemented with standard analog computer components. Two parallel areas are presently being developed, namely:

**Area 1** - The development of a synthesis technique for constructing an analog computer circuit which will operate on Gaussian white noise to produce an output process with prescribed first and second moments, and **Area 2** - A consideration of the problems associated with processing random signals to determine their moments and other useful statistical properties.

Guffee and Webb are presently working in Area 1, while Bryan, White and Hammond are working in Area 2.

Current Activities

During the current report period a considerable amount of attention was devoted to planning the course of the work for the first two quarters, and to reviewing the available literature pertinent to Task C of the contract.

Plans for the first two quarters, mentioned briefly above, were discussed in detail with the Project Officer, Dr. W. P. Krause, during a visit to Marshall Space Flight Center on February 26. A list of references surveyed and found to contain pertinent material is appended to this section. Attention should be called to the references by Webb (1), Kaplan (3), (4) and Coddington et al. (6) which provide a theoretical background for the work being done in Area 1. The papers by Tukey (32), Feriet (34) and McCausland (35) will be studied further, and may be useful for certain aspects of the work in Area 2.

The following specific progress was made in the two areas. In Area 1, the synthesis method described by Webb (1) in his thesis (Appendices I, II, and III) was studied in detail and plans for implementing this method with analog computer components were made. Analog computer circuits which will generate several specific covariance functions for nonstationary processes were designed.

In Area 2, detailed plans have been made for processing a nonstationary random process produced by the analog computer so as to determine the first
two statistical moments of the actual process. Work is well along toward having the necessary equipment in operation.

Plans for Next Quarter

Early in the next quarterly report period, the computer circuits for generating specific covariance functions, referred to above, will be patched into the computer. The processing equipment now being assembled and tested will then be used to determine the first two moments of the actual generated process. This work will be followed by the simulation of processes with more complicated covariance functions.

Concurrent with the above work, some attention will be given to the problem of processing a recorded sample from a nonstationary noise process in such a way as to obtain data from which a computer circuit to simulate the noise can be built.

The project staff for the second report period will remain the same as for the first period, although the total number of hours devoted to the project is expected to increase by a small amount. It is anticipated that a considerably larger amount of time can be devoted to the project during the summer quarter.

Present plans call for issuing a technical note, covering the first phases of the contract, early in the third report period. This technical note will contain a tutorial section considering, among other things, the relation between power spectral density and correlation functions.

References for Task C


QUARTERLY PROGRAM REPORT Q-2
PROJECT A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION
F. Dixon and J. L. Hammond, Jr.

Contract NAS8-2473

12 March to 12 June 1964

Prepared for
GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

By

F. Dixon and J. L. Hammond, Jr.

Contract NAS8-2473

12 March to 12 June 1964

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER
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Huntsville, Alabama
INTRODUCTION

Program Outline

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- Dr. Thomas M. White (Associate Professor)
- Mr. Clifford O. Guffee (Graduate Teaching Assistant)
- Mr. Richard E. Bryan (Graduate Research Assistant)

Progress reports for the two groups will normally be presented in separate sections of each Quarterly Program Report.

**Conferences**

Dr. W. K. Polstorff visited Georgia Tech on 17 April for a program planning conference with each of the Project A-588 groups.

F. R. Williamson, Jr. (Group 1) and J. L. Hammond, T. M. White, and R. E. Bryan (Group 2) visited Marshall Space Flight Center on 5 June to attend technical sessions of the Southeastern Simulation Council and hold supplementary project discussions with Dr. Polstorff and other personnel at the MSFC Flight Simulation Branch. During the morning meeting of the Simulation Council, Mr. Bryan presented a paper entitled "Generation of Non-Stationary Stochastic Processes," covering several aspects of the work performed under Task C of the contract.
GROUP 1 PROGRESS REPORT

Simulation of Manned Space Flight.

There are no active assignments under Task A of the contract program at the present time.

Electronic Switches for Analog Computation

Literature Survey

A review has been made of all issues of the Defense Documentation Center (DDC) Technical Abstract Bulletins (TAB's) since 1 January 1963 for research reports of possible interest on electronic switching techniques and applications. Seven items uncovered from this source are being procured through the Georgia Tech Library. An additional 13 items (listed in the "limited distribution" section of the TAB's) have been identified for preliminary evaluation by the Project Officer at MSFC.

A review has been made of The Engineering Index for the years 1961 and 1962. The following pertinent articles have thus far been uncovered from this and other references to the open literature:


The following circuits have been tentatively selected for further analysis and possible breadboard evaluation:

(a) a modified version of the Guennou chopper (see diagram below);
(b) a nonsaturating transistor switch (see Reference 3 above);
(c) a microvolt transistor bridge (see Reference 6 above);
(d) a field-effect transistor switch;
(e) a six-diode bridge using improved diodes (see Reference 9 above);
(f) a photoconductor switch.

Preliminary laboratory experiments have been performed with the modified, direct-coupled, Guennou chopper circuit depicted below. This version of the circuit appears suitable for non-repetitive switching applications, as in the area of analog computer programming of interest under the contract. Tests have been made to determine variations in offset voltage, feedthrough, leakage, and gain with various settings of load resistance \( R_l \), balance pot \( R_2 \), and shunt resistance \( R_1, R_3 \) for the case of either randomly selected transistors or transistors matched as regards drive current for minimum offset voltage. Thus far, only type 2N697 transistors have been tried, and the results seemed little better than those obtainable with the simplest single transistor switch. However, improvements in the present method of matching transistors and the use of other transistor types for this circuit may yield more promising results.

During the coming quarter, greater attention will be given to the six-diode bridge circuit (item e above), as suggested by Dr. Polstorff in his letter of 15 May and further discussed in the conference on 5 June.
Current Activities

In the preceding quarterly report, two major areas of investigation were identified under Task C of the contract, namely: Area 1 - the development of a synthesis technique for constructing an analog computer circuit which will operate on Gaussian white noise to produce an output process with prescribed first and second moments, and Area 2 - a consideration of the problems associated with processing random signals to determine their moments and other useful statistical properties. During the present report period, the work identified as Area 1 and certain aspects of Area 2 were completed. A Technical Note describing this work is being prepared for publication in the next report period.

The Technical Note is to include sections on (a) the synthesis procedure for constructing analog computer circuits to generate nonstationary random processes from stationary noise inputs, (b) the instrumentation required to measure the parameters of nonstationary random processes produced by the computer, and (c) examples of the use of the synthesis procedure and the instrumentation. This material may be briefly outlined as follows.

The synthesis procedure makes possible the design of a computer circuit which accepts as its input a Gaussian stationary process with constant power spectral density and produces as its output a nonstationary process with prescribed first and second moments. The first moment or mean of the random process can be generated separately and incorporated with the final output process in a straightforward way. Therefore, the major part of the synthesis procedure applies to generating a random process with zero mean and prescribed second moment or covariance.

This latter procedure involves the following four steps. (1) Approximate the prescribed covariance function with a series expansion. (2) Write a general differential equation for the linear time-varying operation to be performed by the computer on the stationary noise input as

\[ L_t \{x(t)\} = N_t \{y(t)\} \]
where $x(t)$ is the desired output random process, $y(t)$ is the stationary white noise input, and $L_t$ and $N_t$ are linear differential operators. (3) Use a technique, which will be described in detail in the Technical Note, to determine $L_t$ and $N_t$ from the expansion of the given covariance function. (4) Synthesize a computer circuit from the differential equation of step 2. The number of multipliers in the resulting computer circuit will be proportional to the number of terms necessary for a good series approximation to the covariance function in step 1.

A procedure for sampling nonstationary noise processes generated on the computer, recording the samples on paper tape, and processing the recorded results with a general purpose digital computer has been developed and will be described in the Technical Note.

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**Plans for Next Quarter**

Group 2 efforts will be expanded during the summer quarter with the following personnel assignments:

- Dr. D. L. Finn, Professor of E.E. (one-half time)
- Dr. J. L. Hammond, Jr., Associate Professor of E.E. (one-half time)
- Dr. R. P. Webb, Assistant Professor of E.E. (one-third time)
- Dr. T. M. White, Associate Professor of E.E. (full time)
- Mr. R. E. Bryan, Graduate Research Assistant (full time)
- Mr. J. S. Gray, Graduate Assistant (one-half time)
- Mr. J. W. Petway, Graduate Assistant (one-half time)
- Mr. J. H. Schlag, Graduate Assistant (one-half time)

As indicated earlier, work in Area 1 is essentially completed. Efforts during the next report period will be devoted to Area 2 and a new area, which will be identified as Area 3 - Hybrid Computation, to be added by the Sponsor to Task C.

Planned activities for the next report period include the following:

(a) Completion of a Technical Note

(b) Measurement of Nonstationary Random Processes - This will include work directed toward processing measured data on covariance functions to obtain series expansions appropriate for use with the synthesis procedure developed under Area 1, and work relating to statistical accuracy of measurement and sensitivity of results to small changes in measured data.
(c) Properties and Representations of Nonstationary Random Processes -
A study of the best representations for nonstationary random processes, properties of nonstationary processes which violate common assumptions made for stationary processes, effects of nonstationary processes on physical devices, and related topics.

(d) Problems in Hybrid Computation - An attempt will be made to formulate several approaches to problems in hybrid computation. The first problem to be considered is that of determining a criterion for the necessary speed of a digital computer to be used with an analog computer in solving a given class of problems. Consideration is being given to two approaches, one involving a statistical performance criterion, and the other being an attempt to obtain useful deterministic bounds on the complexity of possible output analog signals so that sampling rates for a given accuracy can be specified.
QUARTERLY PROGRAM REPORT 64-3
PROJECT A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION
F. Dixon and J. L. Hammond, Jr.

Contract NASA-2473

12 June to 12 September 1964

Prepared for
GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
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Engineering Experiment Station
Atlanta, Georgia

QUARTERLY PROGRAM REPORT 64-3
PROJECT A-588

DEVELOPMENT OF NEW METHODS AND APPLICATIONS OF ANALOG COMPUTATION

By
F. Dixon and J. L. Hammond, Jr.

Contract NAS6-2473

12 June to 12 September 1964

Prepared for
GEORGE C. MARSHALL SPACE FLIGHT CENTER
National Aeronautics and Space Administration
Huntsville, Alabama
INTRODUCTION

Program Outline

Georgia Tech Research Project No. A-588 was established under Contract NA88-2473 on 12 September 1961 in the Special Problems Branch of the Engineering Experiment Station’s Physical Sciences Division. The overall project aim has been to assist the Flight Simulation Branch of the Computation Division at George C. Marshall Space Flight Center (MSFC) in the investigation and development of new methods and applications of analog computation within several areas of mutual interest. By agreement with the Contracting Officer’s Representative, Dr. W. K. Polstorff of MSFC, efforts during the first year were devoted to 1) design and construction of an experimental electronic generalized integrator for potential application as an analog computing component, and 2) investigation of analog techniques for the generation of nonstationary noise voltages to be used in Monte Carlo studies. Results of these efforts are documented in the initial Annual Technical Summary (Report A588/PL) and in Technical Note No. 1 (Report A588/T1), both dated 12 November 1962.

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C. Analytical investigations of methods for generating nonstationary noise of specified spectral and amplitude distribution for Monte Carlo simulation studies.

The following assignment was recently added by Contract Modification No. 5:

D. Investigations of Combined Digital Analog Computation will also be performed; a theoretical study of the speed requirement for the digital computer in a hybrid simulation of a control system which shows the relations between the power spectrum of the solution, the sampling rate, the RMS error, and the maximum absolute error is to be made.

1
Project Organization

For purposes of administrative convenience, the project staff has been divided into two functional groups. Group 1 comprises personnel of the Special Problems Branch Instrumentation Laboratory, presently located in the Nuclear Research Center and operating under the immediate supervision of the Branch Head and Project Director, Mr. Frederick Dixon. This group has responsibility for Tasks A and B outlined above.

Group 2, which is concerned with Tasks C and D above, consists primarily of personnel associated with the School of Electrical Engineering and operates under the direction of Dr. J. L. Hammond, Jr. This group also utilizes facilities of the main Analog Computer Laboratory, which is maintained by the Special Problems Branch in the E.E. Building.

Progress reports for the two groups are presented under separate sections below.
GROUP 1 PROGRESS REPORT

General

There are at present no active assignments under Task A of the contract program (Simulation of Manned Space Flight). The Project A-588 Group 1 effort has therefore been devoted to Task B—investigation and improvement of electronic switches for analog computation. The bulk of this effort has been provided by Mr. J. W. Robertson (Research Assistant) under the guidance of Mr. F. R. Williamson, Jr. (Assistant Research Engineer).

Work during the current report period has included (a) continuance of the literature search, (b) experimental evaluation of nonsaturating transistor switches, (c) evaluation of the "complementary microvolt bridge," (d) investigation of improvements on the Guennou chopper, and (e) preparation for tests of the "six-diode bridge."

Literature Search

A selected bibliography of pertinent journal articles, technical brochure material, and research documents on electronic switching is included as an Appendix to the present quarterly report. The entries have been subdivided under the following topic headings: (1) diode gates, (2) transistor gates, (3) bidirectional transistor gates, (4) field effect transistor gates, (5) Hall generator gates, (6) photo gates, (7) varicap gates, (8) integrated circuit gates, (9) surveys, (10) commercially available gates, (11) gate applications.

A continuing search of the DDC Technical Abstract Bulletins has failed to uncover any new items of interest. However, four of the AD documents previously requested through the Project Officer at MSFC were received from Redstone Scientific Information Center on an extended load basis (see Appendix entries 2.2, 9.1, 9.3, and 10.1). Copies of two additional AD documents were obtained from DDC by direct order through the Georgia Tech Library (entries 2.6 and 2.14).

A review of The Engineering Index for 1963 has yielded several new journal references of interest—including Appendix items 1.2, 2.7, 2.8, 9.2, and 9.4. Other entries contained in the current bibliography are from the preceding quarterly report or from more recent periodicals and cross references to the earlier literature.
Note: All transistors - 2N697.
All diodes - IN916. (IN458As substituted with no noticeable change in performance.)

**TWO-TRANSISTOR NON-SATURATING SWITCH**

**FOUR-TRANSISTOR NON-SATURATING SWITCH**
Evaluation of Nonsaturating Transistor Switches

The investigation of nonsaturating transistor switches described in this section has been largely derived from and guided by Reference 2.5 of the Appendix. The two-transistor and four-transistor circuit configurations which were tested are shown on the opposite page, along with the specially designed driver used for this study.

Basic Offset and Leakage Tests

The driver input was chosen so as to cause the switch to open and close at a rate of several kilocycles per second with approximately equal open and closed periods. Gate output voltages \( e_o \) were recorded at different levels of gate input voltage \( e_i \). Typical readings for the four-transistor switch are illustrated below.

\[
\begin{align*}
\text{\( e_i = 0 \)} & : & \begin{array}{c}
500 \mu V \\
1 \text{mv}
\end{array} & e_o \\
0 & & 0
\end{align*}
\]

\[
\begin{align*}
\text{\( e_i = 5.5 \text{v} \)} & : & \begin{array}{c}
700 \mu V \\
5.0 \text{v}
\end{array} & e_o \\
0 & & 0
\end{align*}
\]

The following performance data may be determined from these measurements.

- **Offset Voltage:** \( V_o = 500 \) microvolts
- **Drive Leakage:** \( L_d = 1 \) millivolt
- **Off-Voltage Gain:** \( A_{V(\text{off})} = \frac{700 \mu V - 500 \mu V}{5.5 \text{v}} \)
- **On-Voltage Gain:** \( A_{V(\text{on})} = \frac{5.0 \text{v} - 500 \mu V}{5.5 \text{v}} \)

The two-transistor switch proved inferior to the four-transistor switch in all respects, so only the latter unit was considered for further evaluation.

Computer Switching Test

In order to obtain a more definitive test of the four-transistor switch for possible use in a computer system, the program shown below was set up on the Donner 3100 Analog Computer. (This program represents a modified version of the test arrangement given in Reference 2.5) When the switch is open the program functions as an integrator, with the gate leakage and offset being compensated for by an input to the integrating amplifier. When the switch is closed, the program functions as a low-pass summer; thus the voltage on the capacitor is rapidly brought to the value of the initial condition input.
The limiter in this program is provided to prevent the input to the switch from becoming too high.

Performance of the four-transistor switch in this computer arrangement was checked by integrating a zero input with several values of initial conditions, and several values of input voltage with zero initial conditions. Observing the departure from the straight line that would result if the switch had no leakage gives a measure of performance of the gate. A typical value of drift with zero initial conditions and zero input is 0.1 volts after 65 seconds. Repeatability seems to be very good with this gate.

Evaluation of the Complementary Microvolt Bridge

The low-level switching circuit shown below was derived from the work reported in Reference 2.12. Basic tests of this unit were run in a fashion similar to those described for the nonsaturating transistor switches. Comparable results were obtained, except that the leakage was so low as to be undetectable by the simple computer method outlined above. A different program was therefore devised, as shown on the next page.
With the I.C. set to zero and with the leakage compensation adjusted to give the same output voltage at the end of 65 seconds for an input of either plus or minus 0.1 volt, the output is recorded on an x-y plotter. The switch is then removed from the program and the output again recorded. Subtracting the values of the two recordings at any given time gives the output voltage due to the switch leakage. The tracings appear to be linear after a short period (5 seconds or so), indicating that the leakage from the gate is of constant magnitude.

The constant leakage should theoretically be perfectly correctable. In order to calculate switch offset, the output voltage produced by the gate may be divided by the time at which this is measured and by the gain of the integrator. In the case under consideration, ten runs were averaged at a point, first with the switch in the program and then with it removed, the offset being calculated from these averaged readings. Taking into account the nominally 0.1-percent accuracy of the computer, the average offset was determined to be 119 microvolts maximum.

**Improvements on the Guennou Chopper Switch**

Further efforts have been made to evaluate the modified, direct-coupled, Guennou chopper circuit described in the preceding quarterly report. In an attempt to find more suitable transistors for this switch, a Tektronix Model 575 Transistor Curve Tracer was used to test the following transistor types: 2N220, 4044, 408, 706, 1132, 1179, 1306, 1605A, 2102, and 2613. The transistors were placed on the Curve Tracer in the inverse mode, and inverse current gain was determined by extending the linear region above the knee of the
\( V_{ce} = -I_c \) characteristic to the \( V_{ce} \) axis \( (\beta_i = I_c/I_b, V_{ce} = 0) \). Unfortunately, no transistors with a high enough \( \beta_i \) were found.

In order to achieve a possible satisfactory solution, it is recommended that silicon transistors especially intended for inverse operation be obtained.

**Preparation for Tests of Six-Diode Bridge**

The G.E. 1N443 diodes recommended by Dr. Polstorff for use in the diode gate of Reference 1.7 have been ordered but not yet received. Fairchild 1N3595 diodes selected as another good prospect for the gate are in hand, and a six-diode bridge is under test using these in place of the 1N443's.
Introduction

During the current report period the following persons contributed to the program:

Dr. D. L. Finn, Professor of E.E. (one-half time)
Dr. J. L. Hammond, Jr., Associate Professor of E.E. (one-half time)
Dr. R. P. Webb, Assistant Professor of E.E. (one-third time)
Dr. T. M. White, Associate Professor of E.E. (full time)
Mr. R. E. Bryan, Graduate Research Assistant (full time)
Mr. J. S. Gray, Graduate Assistant (one-half time)
Mr. J. W. Petway, Graduate Assistant (one-half time)
Mr. J. H. Schlag, Graduate Assistant (one-half time)

As stated in Quarterly Program Report 64-2, work during the current report period can be classified in four major areas, namely:

(a) Completion of a Technical Note covering work on generation of random processes,
(b) Work in the area of measurements on nonstationary random processes,
(c) A study of the properties and representations of nonstationary random processes,
(d) Work on problems pertaining to hybrid computation.

Progress in each of these areas will be discussed below.

Completion of Technical Note

During this report period Technical Note 3 entitled "Generation and Measurement of Nonstationary Random Processes" was published and forwarded to the sponsor. This Technical Note covers the details of a synthesis procedure for constructing prescribed first and second moments from a stationary white noise input. The Note also discusses implementation of an analog computer to use such a noise generator, and methods for computing the error caused by using a limited number of samples to approximate a statistical average. Several complete examples illustrating the use of the synthesis procedure, implementation on an analog computer, and computation of statistical errors are included.

Measurements on Nonstationary Random Processes

Work in this area has been carried on by Dr. Webb. The objective during the current report period has been to develop a method for processing data obtained from measurements on a nonstationary random process in such a way as to obtain a series expansion appropriate for use with the synthesis procedure described in Technical Note 3. This work is not yet completed.
Properties and Representations of Nonstationary Random Processes

The primary responsibility for this area of work has been with Dr. Finn. One objective of studies made during the current report period has been to establish precise definitions for the type of random processes which are necessary in simulation studies of missile and rocket flights. This objective has been accomplished. A class of nonstationary random processes which includes those of interest in simulation studies has been carefully defined, and certain mathematical properties of members of this class have been investigated. Applications of this work to the problems of measurement and computer synthesis of random processes of interest in trajectory simulations are currently being investigated.

Hybrid Computation

In response to the request from the sponsor to add error analysis for hybrid computation to the scope of the contract, a considerable amount of effort has been expended in this area. The objectives during the current report period have been: (1) to perform a careful survey of the literature in this area, (2) to investigate some elementary problems both analytically and through analog simulation, and (3) to formulate one or more methods of attack on the error problem. Drs. Hammond and White assisted by Gray, Peilway and Schlag have been active in this area.

The literature survey has been completed. At the present time a Technical Note reviewing the literature on error analysis for hybrid computation and related areas is being prepared. This Note will be published during the next report period.

Several elementary problems involving one and two computer loops containing an analog integrator and sample-and-hold and delay elements have been investigated both theoretically and through analog simulation. For these simple cases several of the error analysis techniques described in the literature have been compared. In simple linear cases the root shifting technique of accounting for errors due to sample-and-hold with delay seems effective. The work of applying specific techniques has not yet progressed to a consideration of problems sufficiently complex to be considered practical.

Work on the problem of formulating methods of attack on the error analysis problem is currently active. Two approaches are under consideration: one
based on the work of Miller and Murray\textsuperscript{1}, involves writing an expression for
the combined analog and digital errors and expanding this in a power series;
the other approach aims at separating the analog and digital elements of a
given program and characterizing these elements independently. When the
approaches have been more completely formulated, the sponsor will be provided
with a detailed discussion.

\textbf{Plans for Next Quarter}

The project staff for next quarter will be reduced to the following:
Dr. J. L. Hammond, Jr., Associate Professor of E.E. (one-half time)
Dr. R. P. Webb, Assistant Professor of E.E. (one-tenth time)
Dr. T. M. White, Associate Professor of E.E. (one-tenth time)
Mr. J. W. Petway, Graduate Assistant (one-half time)
Mr. J. H. Schalg, Graduate Assistant (one-quarter time)

Objectives for the next quarter are the following: (a) Completion of a
Technical Note surveying the literature on error analysis for hybrid computa-
tion and related areas; (b) Further theoretical and experimental work on meas-
urements of nonstationary random processes; and (c) Formulation of two methods
of attack on the problem of error analysis for analog computation and work toward
implementing these approaches. Area (c) will receive the largest amount of at-
tention.

A visit with the sponsor in Huntsville is planned for some time in October.
The purpose of this visit will be to discuss Technical Note 3, to discuss the
preliminary work in the area of hybrid error analysis, and to plan future work.

\textsuperscript{1}Miller, K. S., and Murray, "Dynamic System Studies: Error Analysis for
APPENDIX: SELECTED REFERENCES ON ELECTRONIC SWITCHING

1. Diode Gates


2. Transistor Gates


3. Bidirectional Transistor Gates

4. Field Effect Transistor Gates
5. Hall Generator Gates


6. Photo Gates


7. Varicap Gates


8. Integrated Circuit Gates


9. Surveys


10. Commercially Available Gates

11. Gate Applications