Commitment Decisions with Partial Information Updating

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Abstract

In this paper, we extend the results of Ferguson [10] on an end-product manufacturer's choice of when to commit to an order quantity from its parts supplier. During the supplier's lead-time, information arrives about end product demand. This information reduces some of the forecast uncertainty. While the supplier must choose its production quantity of parts based on the original forecast, the manufacturer can wait to place its order from the supplier after observing the information update. We find that a manufacturer is sometimes better off with a contract requiring an early commitment to its order quantity, before the supplier commits resources. On the other hand, the supplier sometimes prefers a delayed commitment. The preferences depend upon the amount of demand uncertainty resolved by the information as well as which member of the supply chain sets the exchange price. We also show conditions where demand information updating is detrimental to both the manufacturer and the supplier.

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1 Introduction

Consider a situation where an End-Product Manufacturer (buyer) has a choice of two instances to place a binding order with its component supplier. The first instance occurs before the supplier commits resources to the production of the component. In this case, the supplier produces and delivers exactly the number of units ordered. The supplier experiences no demand risk, and the buyer is guaranteed to receive its exact order quantity. The second instance occurs after the time at which the supplier must commit to a component production quantity. By delaying its order until this time, the buyer learns additional information about demand that it can use to make a better order quantity decision. The buyer's ability to benefit from this information is limited, however, by the fact that its order quantity is now constrained by the (earlier) component production decision of the supplier. The supplier must choose its component production quantity without the benefit of the additional demand information and without a firm order commitment from the buyer. As a result, the supplier must now balance the risk of lost sales against the risk of producing excess components. We term the decision of "when to order" the commitment time frame. The commitment time frame determines how demand risk is shared between the supplier and the buyer – the shorter the commitment time frame, the more demand risk is shifted from the buyer to the supplier.

This paper studies how the choice of the component price and the commitment time frame influences the distribution of supply chain profits. We consider a series system consisting of a single supplier providing a component to a buyer. The buyer assembles a finished good using the component and sells it during a single selling season. The demand for the product is stochastic and occurs according to a distribution that is known by both firms. Information about demand arrives after the supplier's production decision but *before* the buyer assembles the final product. This updating ranges from no additional information to full knowledge about the upcoming demand. Without loss of generality, all unfulfilled demand is lost, and all leftover product must be scrapped at zero value.

This scenario broadly describes any product that has a multiple level supplier network, is

sold to the end-use customer in a make-to-stock or assemble-to-order environment, and whose producer makes production and ordering decisions in anticipation of the demand that occurs during a single selling season. Examples include fashion items and seasonal products. The intuition gained from our model, however, can be applied to a wider range of multi-echelon supply chains facing stochastic demand.

To model the effect of the buyer's commitment time frame, we investigate performance under two settings. In the first setting, the buyer commits to its order-quantity before the supplier commits any resources towards the production of the components. Here, the buyer assumes all of the demand risk. This is the equivalent to setting the "frozen zone" to its longest lead-time component or capacity decision. We phrase this choice of contract terms as an *early* commitment on the part of the buyer. In the second setting, the supplier is required to produce the component before the buyer commits to an order-quantity, forcing the supplier to decide upon purchase and production quantities when only a demand distribution is known. This *delayed* commitment by the buyer forces the supplier to share some of the demand risk.

We characterize the supply chain members' optimal ordering, production, and pricing decisions under both early and delayed commitment for any given level of information updating. For the early commitment case where the buyer makes all of the production decisions, we show that the buyer's expected profit is concave in the order quantity from the supplier. We also show that the optimal order quantity, and resulting expected profit, is non-decreasing in the amount of information updating. These results provide guidance for firms operating in dynamic and fast changing environments.

Given a choice, a supplier would typically choose a high price and an early commitment. The buyer, conversely, would prefer a low price and a latter commitment. Dell Computer, for instance, commits to an order-quantity with its suppliers only after receiving an actual customer order. The determination of these parameters normally depends on which supply chain party holds the most power in the relationship. For example, when Intel negotiates with a small personal computer manufacturer, the contract terms likely favor the supplier (Intel). On the other hand, when General Motors negotiates contract terms with one of its

smaller parts-suppliers, the contract terms favor the buyer (GM). Then again, when two small firms negotiate, the price is typically either determined exogenously, by the market, or the contract terms are weighted equally between the two firms.

The examples above illustrate different power structures of a supply chain. To model these different conditions, we investigate four price-setting scenarios. In the first scenario, the exchange price is determined exogenously, modeling a situation where neither the buyer nor the parts supplier can influence it. In the second setting, the supplier has the majority of the power in the supply chain and thus sets the price. We model this scenario by allowing the supplier to take a Stackelberg leader position in the contract negotiations. In the third scenario, the buyer assumes the Stackelberg leader role and forces a price upon the supplier. In the forth scenario, both members have equal power, which is modeled by finding an exchange price that results in equal profits.

We find that it is *not* always in the best interest of the buyer to delay its order-quantity commitment. Nor is it always in the best interest of the supplier to demand an early commitment. For example, when the buyer chooses the exchange price (scenario 3), we find that the buyer always prefers an early commitment. In the second scenario, the supplier always prefers a delayed commitment. These results are surprising - they contradict our intuition about who prefers what. In the other two scenarios, the commitment preference of both members depends upon the amount of demand uncertainty resolved by the information updating.

A second interesting finding involves the value of the demand information updating. We show that in a decentralized supply chain under delayed commitment and a low supplier margin, demand information updating actually *decreases* the expected profits for both the supplier and the buyer. This result stems from the fact that under delayed commitment, the supplier builds in anticipation of the buyer's order, before the additional demand information arrives. This finding contrast with most previous work on the value of information which assumes centralized control of the supply chain and finds that information is always valuable. The next few paragraphs provide a brief review of the literature and an outline for the rest of the paper.

Inventory models involving Bayesian forecast updates were first studied by Scarf [18]. Fisher and Raman [11] consider a two-period problem from the fashion goods industry where a manufacturer supplying a retailer determines a production quantity for each period given that the first period has a lower production cost but the second period has improved forecast information. Iyer and Bergen [15] investigate how the supply chain members' profits change when the retailer is allowed to delay its order from the manufacturer until better information is obtained about demand. This industry practice, termed "Quick Response", relies on forecast error reduction to reduce the variability of demand before the retailer commits to its order-quantity. Iyer and Bergen [15] show that Quick Response is not always Pareto improving. In particular, for small supplier margins the supplier is worse off but the buyer is better off with demand information updating. By allowing the supplier to act strategically, we show that both the supplier and buyer may be worse off with demand information updating. Work involving forecast updates over multiple periods include Tsay and Lovejoy [22] and Kaminsky and Swaminathan [16].

Several papers investigate multiple ordering opportunities where a delayed commitment can either be purchased up front as an option, or purchased latter at a higher per unit cost, possibly through a spot market. Brown and Lee [1] analyze a class of single-period, "pay-to-delay" capacity reservation contracts for the semiconductor manufacturing industry. In these contracts, the buyer is guaranteed a portion of the supplier's capacity as well as an additional amount that can be purchased at an extra price, after additional information about demand is obtained. Papers modeling a higher latter cost or spot markets include Donohue [7], Huang, Sethi and Yan [14], and Seifert, Thonemann and Hausman [19]. Serel, Dada and Moskowitz [20] and Golovachkina and Bradley [12] compare both capacity reservations and spot markets. Our model differs from the ones above in that we restrict the buyer to a single order opportunity; either early or delayed. While mathematically our restriction on the timing of the buyer's commitment may seem like two points on a continuum, in practice, these are often the only two possible choices. Many components, such as ASICS and printed circuit boards in the electronics industry, are unique to a specific product. Such components are not available on a spot market, nor is it possible to dramatically compress

their manufacturing lead-times. Thus, irrevocable quantity decisions must be made at the cumulative lead-time of the final product.

In a second contrast to our model, all of the previously mentioned papers assume the supplier either always provides the buyer with its order-quantity request, or builds nothing without a firm order commitment. We term this supplier behavior as "non-strategic". In contrast, we model a "strategic" supplier that builds without a firm commitment because he knows the underlying demand distribution and anticipates the buyer's order. Sharing the demand uncertainty comes at a price to the buyer however, as the supplier builds a quantity that maximizes his own expected profit. Papers modeling strategic supplier behavior include Lariviere and Porteus [17], Cachon and Lariviere [3], Gurnani and Tang [13], Deng and Yano [5][6], Erkoc and Wu [9], Taylor [21], Cachon [3], and Ferguson [10]. Of the above, Ferguson [10], Lariviere and Porteus [17], Taylor [21], and Cachon [3] are the closes in spirit to our paper. The later three only consider a single power structure for choosing the exchange price, concentrate on our special case of full information updating, and model a supplier-retailer supply chain where a second production opportunity is not possible after demand information is updated. Ferguson [10] provides a review of this literature and presents a model that includes the same power and supply chain structures as ours. His paper however, assumes that either all or none of the demand uncertainty is resolved with some predetermined probability. Our model, in contrast, allows the demand information to vary continuously from non-informative to full information. This approach offers deeper insights into how the value of the future information affects the supply chain members' choice of the commitment time.

In the next section we describe the model and in Section 3 we analyze the two commitment time frame choices under an exogenous exchange price. In Section 4 we look at the impact of information updating along with each supply chain member's commitment time preference for a given exchange price. In Section 5 we extend our analysis to three common supply chain power structures and in Section 6, we point out some managerial insights and directions for future research. All proofs are given in an appendix.

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2 Model Description

A single manufacturer (the buyer) assembles a finished good requiring a component, or a set of components, provided by a single supplier. Both the supplier and the end-product manufacturer (buyer) have positive manufacturing lead-times that are long compared to the single selling season of the end-product. The lead-time for the supplier is denoted by T_S and that of the buyer by T_B, so production of the components must begin at least $T_S + T_B$ periods prior to the final product demand realization. Thus, we have a two-stage newsvendor model. All information is shared between the two parties including prices, cost, and the demand distribution for the end-product.

The decision variables include the quantity of components produced by the supplier, P_S, the quantity of components ordered by the buyer, Q, and the quantity of final product assembled by the buyer, P_B. Because each final product contains exactly one component (or one component set), the final production quantity is constrained by the first two variables i.e. $P_B \leq min(P_S, Q)$. Time is counted backwards to the final product demand realization: The component production quantity decision, P_S, is made at $t = T_S + T_B$, the final product production quantity decision, P_B, is made at $t = T_B$, and final product demand occurs at $t = 0$. The mechanics of our model are not dependent on the actual lengths of the firms' lead-times and their lengths do not have to be equal. Thus, we simplify notation by designating two periods in which decisions must be made. Period 1 corresponds to the time period from $t = T_S + T_B$ until $t = T_B$ and period 2 represents the time period from $t = T_B$ to $t = 0$. Through our model we investigate the optimal choice for the timing of the buyer's order quantity decision, Q.

End-product demand is represented by the random variable Z, made up of two components i.e. $Z = X + Y$. The random variables X and Y are independent and have continuous distributions with pdfs $f_X(x)$ and $f_Y(y)$ and variances σ_X^2 and σ_Y^2. Let $F_X(x)$ $(F_Y(y))$ be the cdfs, $F_X^{-1}(x)$ $(F_Y^{-1}(y))$ the inverses, and define $\bar{F}_X(x) = 1 - F_X(x)$ $(\bar{F}_Y(y) = 1 - F_Y(y))$. The first component, X, represents the uncertainty that gets resolved before the buyer makes its production quantity decision where $E[X] = \mu$ and $\Pr[L_X \leq X \leq H_X] = 1$ for

$0 \leq L_X \leq H_X$. The second component, Y, represents residual demand uncertainty about the product where $E[Y] = 0$ and $\Pr[L_Y \leq Y \leq H_Y] = 1$ for $L_Y \leq 0 \leq H_Y$. We assume that demand is bounded from below by zero, i.e., $L_X + L_Y = 0$ (The assumptions about bounds are not strictly necessary; they are made for convenience only. We drop them in the numerical examples.) Before any uncertainty is resolved, Z has $E[Z] = \mu$ and $\Pr[L_X + L_Y \leq Z \leq H_X + H_Y] = 1$. Assume the distribution for Z has the strictly Increasing Generalized Failure Rate (IGFR) property. Many commonly used distributions have the IGFR property, including the Normal, the exponential, the gamma and the Weibull. After information arrives and the buyer observes $X = x$, conditioned end-product demand satisfies $E[Z|X = x] = x$ and $\Pr[Z \leq q|X = x] = \Pr[Y \leq q - x]$.

It is useful in our analysis to measure the amount of demand uncertainty that is reduced by the new information. We assign the parameter ρ to represent this amount where

$$\rho = \frac{Var(X)}{Var(X) + Var(Y)} \ . \tag{1}$$

Thus, a value of $\rho = 0$ represents the case where the new information provides no additional insight into the quantity of end-product demand. A value of $\rho = 1$ occurs when the incoming information provides the buyer with the exact quantity of end-product demand. In all cases, $0 \leq \rho \leq 1$.

The supplier faces a unit variable production cost of c_S. and sells the component to the buyer at an exchange price of w per unit. The specification of the exchange price occurs prior to any production by either party and depends upon the power structure of the supply chain. The buyer's unit variable production cost is c_B and each unit of product sold by the buyer to its customers generates revenue r, which is dictated by the buyer's market and is exogenous to our model. All leftover final product and components are scrapped at zero value. We consider only the most relevant and interesting case, where $r \geq w + c_B$ and $w \geq c_S$.

We solve this multiparty decision problem by backward induction, assuming that the decision maker at each step acts optimally, given the current information at hand and anticipating (likewise) optimal behavior by the decision maker in each subsequent step. Since

both firms are assumed risk neutral, each aims to maximize its expected profit. In both scenarios (early and delayed commitment), the exchange price is determined prior to the start of the game based on the power structure of the supply chain. For the two unbalanced power cases (supplier chooses price and buyer chooses price), the firm wielding the supply chain power selects a price that maximizes its expected profit assuming an optimal response to that price by the other firm. For the balanced power case (equal profit), the price is selected based upon the intersection of the two firms' optimal profit functions.

The major difference between the early and delayed commitment scenarios stems from the allocation of the demand risk. In particular, the early commitment scenario assigns all of the demand risk to the buyer, while the delayed commitment scenario shares the risk between the two firms. The actual percentage of the demand risk allocated to the supplier in the latter scenario is dependent upon the amount of demand forecast variance reduction that occurs due to the arrival of additional information.

3 Analysis

In this section, we present the firms' optimal ordering and production decisions for both an early and a delayed commitment when the exchange price is exogenous. There are many instances when an exchange price can be considered exogenous. This scenario is common when a supplier provides for multiple buyers and does not want to price discriminate between them or when the price is set by external market pressures posed by competing suppliers. In Section 5, we expand our study to include the cases when the exchange price is set under three different supply chain power structures.

3.1 Early Commitment: Q decided at $t = T_S + T_B$

First, we examine the case where the buyer takes on all of the demand risk. The supplier's order-quantity as well as its profit is deterministic, dependent only upon the buyer's order-quantity. The early commitment case involves the following timing of events:

1. The exchange price w is set based upon the power structure of the supply chain.

2. At the beginning of period 1, the buyer commits to order Q components from the supplier. The supplier produces $P_S = Q$ units and makes a profit equal to $(w - c_S)Q$.

3. After the buyer has committed to its order quantity but prior to the beginning of period 2, X is observed.

4. At the beginning of period 2, the buyer selects its final product *desired* optimal production quantity P_B^* based on its new revised forecast of demand. Final product production is constrained by the number of components available, Q. Thus, the buyer's *actual* production quantity is $P_B^A = \min(Q, P_B^*)$, where the superscript A represents the quantity actually produced. Any component surplus is scrapped at zero value.

5. At the end of period 2, market demand, Z, is revealed, and all revenues and cost are incurred.

Figure 1 illustrates the decisions made for the early commitment case over time.

*** Insert Figure 1 here ***

With the exchange price set, the only decision variables are the buyer's order-quantity *before* the demand information is received and the buyer's production quantity *after* receiving the demand information. The supplier plays a passive role in this scenario, as he will produce any quantity requested by the buyer as long as the exchange price covers its manufacturing cost i.e. $w > c_S$. As a result, the decision variable P_S does not appear in this scenario. We begin with the buyer's period 2 production decision and use backward induction to find its optimal period 1 order quantity. In the appendix, we provide an additional formulation of the problem that is easier to solve numerically.

The buyer's period 2 objective is

$$Max_{P_B}: \quad -c_B P_B + r E_Z[\min(P_B, Z)], \quad s.t. \ P_B \leq Q \ .$$

At the beginning of period 2, some demand information, $X = x$ is observed. Substituting $X = x$ into the objective and performing a change of variables by setting $k = P_B - x$ gives

$$Max_k : \quad -c_B(k+x) + rE_Y[\min(x+k, x+Y)] \quad s.t. \ k \le Q - x \ .$$

Rearranging terms gives

$$Max_k : \quad (r - c_B)x - c_Bk + rE_Y[\min(k, Y)] \quad s.t. \ k \le Q - x \ . \tag{2}$$

Note that once $X = x$ is observed, the mean of the demand distribution is updated to be x, thus k can be interpreted as the safety stock associated with a production choice P_B. Now define

$$H(k) = -c_Bk + rE_Y[\min(k, Y)]$$

as the variable expected profit given a safety stock of k. The objective of (2) is $(r - c_B)x + H(k)$ and $(r - c_B)x$ is a constant, so we can equivalently maximize $H(k)$, which is concave in k. Let k^* be the largest k that maximizes $H(k)$ globally, ignoring the constraint $k \le Q - x$. It is easy to show that

$$k^* = F_Y^{-1}\left(\frac{r - c_B}{r}\right),$$

where k^* is the unconstrained optimal safety stock, is independent of x, and is computed by solving $H'(k) = 0$. The buyer's constrained optimal level of final product safety stock is $\min\{k^*, Q - x\}$.

Next we relate the buyer's period 2 expected profit to the component order quantity, Q. We do so by substituting in the optimal value of k into (2). Define

$$\vec{\pi}(Q, x) = (r - c_B)x + H(\min\{k^*, Q - x\}) \ .$$

In the first period, both X and Y are unknown when the buyer makes its component order quantity decision. The period 1 objective is

$$Max_Q : \quad \pi_B(Q) = E_X[-wQ + \vec{\pi}(Q, X)] \ . \tag{3}$$

Proposition 1 *Under early commitment, (3) is concave in Q.*

(All proofs are in the Appendix.) Thus, the optimal Q is found by solving $\partial \pi(Q)/\partial Q = 0$. We refer to this optimal order quantity as Q^* and the buyer's optimal expected profit as π_B^*. Under early commitment, the supplier assumes no demand risk and makes a profit of

$$\pi_S^*(w) = (w - c_S)Q^* \quad . \tag{4}$$

The buyer's expected profit, π_B^*, is given by

$$\pi_B^*(w) = -wQ^* - c_B(\mu + k^*) + c_B L_X(Q^* - k^*) + r\mu - rL_Z(\mu + k^* - L_X(Q^* - k^*)), \tag{5}$$

where $L_X()$ and $L_Z()$ are the loss functions for their respective distributions. The resulting total system profit for the entire supply chain is $\pi_T^*(w) = \pi_B^*(w) + \pi_S^*(w)$. Figure 2 plots the buyer's and supplier's expected profit over all feasible exchange prices under early commitment given the following parameter values: $r = 100$, $c_B = c_S = 20$, $Z \tilde{} N(100, 20)$, and $\rho = 1$. The labels $w^S w^B$, and w^E point out the exchange prices selected by the three power structures described in Section 5.

*** Insert Figure 2 here ***

3.2 Delayed Commitment: Q decided at $t = T_B$

In this next scenario, the buyer and the supplier share the demand risk. The buyer commits to its order-quantity only after the supplier has produced the components and the buyer has learned more information about demand. The buyer's commitment time frame is now equal to its manufacturing lead-time only. The timing of events is the same as described in the early commitment section with two exceptions: 1) it is now the supplier that decides how many components to produce, and 2) the buyer does not place a formal order with the supplier until after receiving the information about demand. For clarification purposes, we distinguish our parameters and decision variables from the early commitment case through the placement of a tilde above the corresponding variable symbol. For instance, the variable k becomes $\tilde{k}$. Figure 3 illustrates the decisions made for the delayed commitment case over time.

*** Insert Figure 3 here ***

With a delayed commitment by the buyer, the supplier no longer plays a passive role in the decision making as he must now decide how many components to produce, prior to receiving a firm order-quantity commitment from the buyer. The decision variables are the supplier's production quantity, P_S, made *before* the information about demand is received, and the buyer's production quantity, $\tilde{P}_B$, made *after* receiving the additional information about end-product demand (since the buyer makes its component order quantity and final production quantity decisions at the same time, $\tilde{Q} = \tilde{P}_B$ so the decision variable $\tilde{Q}$ does not appear in this scenario). We begin, as before, with the buyer's period 2 production decision and work backwards to solve the supplier's period 1 production decision.

As in the previous case, partial demand information $X = x$ has been observed at the beginning of period 2. The buyer selects its final product production quantity, P_B, by maximizing its expected profit

$$Max \ \tilde{\pi}_B(\tilde{P}_B, x) = -(w + c_B)\tilde{P}_B + rE[\min(\tilde{P}_B, x + Y)] \ ,$$

We again make the change of variables $\tilde{k} = \tilde{P}_B - x$. The resulting objective function is concave in $\tilde{k}$, so first order conditions yield the optimal unconstrained production decision $\tilde{P}_B^* = x + \tilde{k}^*$ where

$$\tilde{k}^* = F_Y^{-1}\left(\frac{r - w - c_B}{r}\right) \ .$$

The buyer's actual production quantity is the minimum of its desired production quantity and the number of components available from the supplier, $\tilde{P}_B^A = \min(\tilde{P}_B^*, P_S)$.

The supplier's objective function at the beginning of period 1 is

$$Max_{\tilde{P}_S} \ \ \tilde{\pi}_S^*(\tilde{P}_S) = -c_S\tilde{P}_S + wE[\min(X + \tilde{k}^*, \tilde{P}_S)] \ . \tag{6}$$

Assuming that X is a continuous variable, (6) can be expressed as

$$Max \ \tilde{\pi}_S^*(\tilde{P}_S) = -c_S\tilde{P}_S + w\int_{L_X}^{\tilde{P}_S - \tilde{k}^*}(x + \tilde{k}^*)f_X(x)dx + w\int_{\tilde{P}_S - \tilde{k}^*}^{H_X}\tilde{P}_S f_X(x)dx \ .$$

Solving for first order conditions gives an optimal supplier's production quantity of

$$\tilde{P}_S^* = F_X^{-1}\left(\frac{w - c_S}{w}\right) + \tilde{k}^* \ . \tag{7}$$

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The supplier's expected profit for the delayed case is

$$\tilde{\pi}_S^*(w) = -c_S \tilde{P}_S^* + w E[\min(X + \tilde{k}^*, \tilde{P}_S^*)] , \tag{8}$$

and the buyer's expected profit is

$$\tilde{\pi}_B^*(w) = -(w + c_B) \left[(\mu + \tilde{k}^*) - L_X(\tilde{P}_S^* - \tilde{k}^*) \right] + r\mu - rL_Z(\mu + \tilde{k}^* - L_X(\tilde{P}_S^* - \tilde{k}^*)) . \tag{9}$$

The resulting total system profit for the entire supply chain is $\pi_T^{D,*}(w) = \tilde{\pi}_B^*(w) + \pi_S^*(w)$. Figure 4 plots the buyer's and supplier's expected profit under delayed commitment using the same parameter values as in the early commitment example. Once again, the labels $\tilde{w}^B$, $\tilde{w}^E$, and $\tilde{w}^S$ point out the exchange prices selected by the three power structures discussed in Section 5.

*** Insert Figure 4 here ***

4 Impact of Information Updating

In this section we investigate how the amount of information updating (i.e., the size of ρ) affects the buyer's first-period order quantity and the two players' profits under any given wholesale price. The generality of our model prevents us from obtaining closed form solutions under the early commitment case for the optimal order and production quantities when $0 < \rho < 1$. Closed form solutions can be obtained at the boundary cases corresponding to either no additional demand information or full information. In the no additional information case, the buyer faces the same demand distribution for both its ordering and production decisions. This situation is common for firms lacking sophisticated information systems or market analysis capabilities. Mathematically, this case is represented by $\rho = 0$ and we denote it through the use of the subscript NA (No Additional) on the decision variables. In the full information case, the buyer knows exactly what demand will be before making its final product production decision. This case represents the environment of assemble-to-order firms such as Dell Computer. Mathematically, this case is represented by $\rho = 1$ and we denote it through the use of the subscript FI (Full Information) on the decision variables.

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Table 1 summarizes the production quantity decisions made by the supplier and the buyer for No, Partial, and Full information updating.

	Early	Commitment	Delayed	Commitment
	Supplier $(Q^* = P_S^*)$	Buyer P_B^*	Supplier $\tilde{P}_S^*$	Buyer $\tilde{P}_B^*$
$\rho = 0$ (NA)	$F_Z^{-1}\left(\frac{r-w-c_B}{r}\right)$	$F_Z^{-1}\left(\frac{r-w-c_B}{r}\right)$	$F_Z^{-1}\left(\frac{r-w-c_B}{r}\right)$	$F_Z^{-1}\left(\frac{r-w-c_B}{r}\right)$
$1 > \rho > 0$	$(Q^* \geq Q_{NA}^*)$	$min[Q^*,$ $x + F_Y^{-1}\left(\frac{r-w-c_B}{r}\right)]$	$F_X^{-1}\left(\frac{w-c_S}{w}\right) +$ $F_Y^{-1}\left(\frac{r-w-c_B}{r}\right)$	$min[\tilde{P}_S^*,$ $x + F_Y^{-1}\left(\frac{r-w-c_B}{r}\right)]$
$\rho = 1$ (FI)	$F_X^{-1}\left(\frac{r-w-c_B}{r-c_B}\right)$	$min[Q^*, z]$	$F_X^{-1}\left(\frac{w-c_S}{w}\right)$	$min[\tilde{P}_S^*, z]$

Table 1: Production Quantities for Increasing Demand Information (ρ)

By comparing the solutions from Table 1, we can study the impact of ρ on the optimal decisions and expected profits. We do so for the special case of normally distributed demand. All results in this section assume that the initial variance of the demand distribution is equal under both boundary cases (that is, $Var(Y)$ for case $NA = Var(X)$ for case FI). The first result involves the optimal production quantity of components by the supplier. Under early commitment and any given exchange price, we show analytically that $Q^* \geq Q_{NA}^*$ and numerically that $Q_{FI}^* \geq Q^* \geq Q_{NA}^*$, where Q^* is the optimal order quantity of the general case $(0 \leq \rho \leq 1)$. Thus, as the buyer anticipates more information about demand becoming available in period two, she orders larger quantities of components from the supplier.

Proposition 2 *Under Normally distributed demand, early commitment and for any given exchange price w, we have $Q^* \geq Q_{NA}^*$.*

Table 2 gives the optimal order quantities for three different exchange prices, five increasing levels of variance reduction, and nine test cases. The test cases were chosen to demonstrate three states each of the buyer's cost and demand variance. In each case, the optimal Q increases with the amount of variance reduction. An intuitive argument for this result is that as ρ increases, the buyer is able to make better second period decisions which lowers its overage cost. The choice of Q^* depends on the balance of the firm's overage and underage cost. As the firm's overage cost decreases while its underage cost remains the same, then Q^* increases with ρ. Our numerical test also seem to indicate that $Q_{FI}^* \geq Q^* \geq Q_{NA}^*$.

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The next two propositions describe how the expected profits of the buyer and the supplier are affected by increases in the amount of variance reduction.

Proposition 3 *Under early commitment and for any given exchange price w, π_0^* and π_S^* are non-decreasing in ρ.*

Under early commitment, the buyer serves as the only quantity setting decision maker. As ρ increases, the buyer receives additional information about the final product demand. Suppose that the use of the additional information results in lower expected profits. Since the buyer is the only decision maker, she can always choose not to use the additional information and receive the same expected profit she would have received had the information not arrived. Thus, the buyer's profit is increasing with an increase in additional information, i.e. as ρ increases. Proposition 3 shows that the supplier's expected profit is non-decreasing in ρ as well. Since the supplier's profit (4) is monotonically increasing in Q^*, then if Q^* is non-decreasing in ρ, so is the supplier's profit. Thus, additional demand information seems to benefit both the supplier and the buyer under early commitment.

	Early			**Delayed**		
	$(Q^* = P_S^*)$	π_S^*	π_B^*	$\tilde{P}_S^*$	$\tilde{\pi}_S^*$	$\tilde{\pi}_B^*$
$\rho = 0\ (NA)$	104.0	208.1	5018	104.0	208.1	5018
$\rho = .50$	105.5	211.0	5230	84.0	154.8	4651
$\rho = 1\ (FI)$	112.0	223.9	5266	73.3	128.0	4202

Table 3: Effect of ρ on Prod Qnty and Profits under Early and Delayed Commitment
$(Z \~ N(100, 20),\ r = 100,\ c_B = 20,\ c_S = 20,\ w = 22)$

Under delayed commitment, additional demand information is sometimes detrimental to *both* the buyer and the supplier. In particular, below a certain threshold exchange price, the supplier's production quantity *decreases* with ρ, $(\tilde{P}_{B,FI}^* \leq \tilde{P}_{B,NA}^*)$. This is demonstrated through a numeric example in Table 3. For an exchange price of $w = 22$ and the same parameter values used to generate Figures 2 and 4, we give the supplier's production quantity, the supplier's expected profit, and the buyer's expected profit under early and delayed

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commitment for three values of ρ : No Additional Information ($\rho = 0$), Partial Information ($\rho = .5$), and Full Information ($\rho = 1$). As expected, all three variables increase with ρ under early commitment. What is less intuitive is the fact that they all decrease with ρ under delayed commitment. To understand why, observe the supplier's optimal production decisions under delayed commitment given in Table 1. With no information updating, the supplier builds the buyer's critical fractile solution since he faces no demand uncertainty risk in this case. With full information updating, the supplier builds its own critical fractile solution because he absorbs all of the demand uncertainty risk. Thus, when the supplier's margin is small then his fractile is also small so he produces less than the expected demand. Since the sales quantity for the entire supply chain is constrained by the production quantity of the supplier, both members are penalized by the supplier's profit maximizing decision. These observations are formalized in the following proposition.

Proposition 4 *Under delayed commitment, there exist exchange prices sufficiently close to c_S where $\tilde{\pi}^*_{S,FI} \leq \tilde{\pi}^*_{S,NA}$ and $\tilde{\pi}^*_{B,FI} \leq \tilde{\pi}^*_{B,NA}$ as well as exchange prices sufficiently close to $r - c_B$ where $\tilde{\pi}^*_{S,FI} \geq \tilde{\pi}^*_{S,NA}$ and $\tilde{\pi}^*_{B,FI} \geq \tilde{\pi}^*_{B,NA}$.*

Unlike the early commitment case, the supplier's profit under delayed commitment when $0 < \rho < 1$ is not bounded by the full and no additional information cases. In fact, our numerical results indicate that there often exists a range of exchange prices where the supplier's expected profit under partial information is lower than both the no additional and the full information cases, $\tilde{\pi}^*_{S,FI} > \pi^*_S$ and $\tilde{\pi}^*_{S,NA} > \pi^*_S$. Figure 5 plots the supplier's expected profit under delayed commitment for the full, partial, and no additional information cases. Notice that for exchange prices between 45 and 60, the supplier's expected profit with partial information is the lowest of the three. Table 4 summarizes the results from this section.

*** Insert Figure 5 here ***

	Early Commitment	**Delayed Commitment**
Buyer's Order Quantity Q^*	↑	NA
Buyer's Profit π^*_B	↑	↑↓ (depends on w)
Supplier's Profit π^*_S	↑	↑↓ (depends on w)

Table 4: Direction of Change Resulting from Increasing Demand Information (ρ)

4.1 Affect of Exogenous Exchange Price on Commitment Time Choice

In this section, we explore how the exogenous exchange price affects the choice of commitment time. Because we do not have a closed form expression for the Q^*, comparisons must be made numerically for cases where $0 < \rho < 1$. To do so, we took 100 random draws from the parameter sets: $\sigma_Z/\mu \in [.01, .4]$, $c_S/r \in [.1, .6]$, and $c_B/r \in [.1, r - c_S - .1]$. We then calculated the optimal expected profits under early and delayed commitment over the entire range of feasible exchange prices and $\rho = [0, .25, .5, .75, 1]$. The numerical observations listed below held true for all 100 cases tested.

For illustrative purposes, we combine the plots of the early commitment profits with those of the delayed commitment. Figure 6 shows the supplier's and buyer's expected profits as a function of the exchange price for both the early and delayed commitment cases when $\rho = 1$. Figure 7 plots these same functions when $\rho = 0.5$. Table 5 presents a subset of our numerical experiments for demonstration purposes.

<p style="text-align:center">*** Insert Figures 6 and 7 here ***</p>

When no information updating occurs, there is no difference in the expected profits of either firm between the early and delayed commitment cases. With some positive amount of information updating however, there exists an exchange price below which the buyer has higher expected profits with an early commitment and above which its profits are higher with a delayed commitment. This transition occurs where the buyer's profit curves cross as in Figures 6 and 7. We formalize this result in the following numerical conjecture.

Numerical Observation 1 *For Normally distributed demand and $\rho > 0$, there exists a maximum feasible exchange price $w^{t,B}$, below which $\pi_B^* \geq \tilde{\pi}_B^*$ and above which $\pi_B^* \leq \tilde{\pi}_B^*$.*

This result is surprising because general intuition leads one to expect the buyer to always prefer delaying its order quantity commitment until more information about demand is available. Under early commitment, the supplier accepts any order quantity from the buyer, regardless of its margin, as a guaranteed small profit is better than no profit. By delaying its order quantity however, the buyer subjects the supplier to sharing part of the

<p style="text-align:center">17</p>

demand risk. Facing an uncertain order quantity, the supplier reacts by producing a quantity based on its own critical fractile solution that approaches zero as its margin approaches zero. The buyer is then constrained from producing its desired end-product production quantity by the amount of components that the supplier produced. The $w^{t,B}$ row in Table 5 lists the exchange prices where the buyer's expected profit curves cross for four sample cases representing high and low levels of both the buyer's margin and the coefficient of variation of the demand distribution. The next conjecture states that the supplier's profit also has this characteristic although the transition occurs at a different exchange price.

Numerical Observation 2 *For Normally distributed demand and $\rho > 0$, there exists an exchange price $w^{t,S}$ such that $\pi_S^* \geq \tilde{\pi}_S^*$ for all feasible $w \leq w^{t,S}$ and $\pi_S^* \leq \tilde{\pi}_S^*$ for all feasible $w > w^{t,S}$.*

This is also a counter intuitive result as one expects the supplier to always prefer a deterministic order quantity over a probabilistic one. It turns out that under higher exchange prices, the reduced uncertainty about demand counters the effect of the lower buyer margins. Thus, the buyer purchases more units than she would if she were forced into an early commitment. As shown in section 3, under early commitment the buyer bases her order quantity on a critical fractile type solution that decreases with her margins. Under delayed commitment, the buyer is able to observe additional information about demand before selecting her order quantity. This is most obvious at the boundary case $\rho = 1$ where full information about demand arrives. Here, the buyer orders a quantity from the supplier equal to the (now) known demand amount and her order quantity is independent of the size of its margin because she no longer faces any of the demand risk. The $w^{t,S}$ row in Table 5 lists the exchange prices where the supplier's expected profit curves cross for our sample cases. If the expected profit curves never cross, then early commitment dominates over all feasible exchange prices. These cases, distinguished by the upper limit of the feasible exchange prices in our table, occur only in low supplier margin scenarios.

			μ	100	100	1000	1000
			c_S	20	60	20	60
$w^{t,B}$	Variance	25		31	75	30	74
	Reduction	50		30	72	29	72
	(%)	75		30	69	30	69
		100		31	65	31	65
$w^{t,S}$	Variance	25		75	80*	32	80*
	Reduction	50		68	80*	34	80*
	(%)	75		62	79	37	79
		100		57	74	57	74

Table 5: Exch Price where Early and Delayed Profits Cross: Buyer ($w^{t,B}$) and Supplier ($w^{t,S}$)

(Normal Dist, Total Variance = 400, Buyer's Price = 100, Buyer's Cost = 20)

* Profits under early commitment are always larger

5 Impact of Supply Chain Pricing Power

In this section, we explore the impact of different power structures in the supply chain by comparing the expected profits for both supply chain members under our three exchange price setting scenarios. We first characterize the optimal prices for each power structure under early and delayed commitment. We then provide a decision framework that allows us to conclude which commitment time choice results under each structure.

5.1 Power Structures Under Early Commitment

When the supplier wields the majority of the power in the supply chain and thus sets the exchange price, her objective function is

$$Max_w : \pi_S^*(w) = (w - c_S)Q^* \quad s.t. \quad \pi_B^*(w) \geq 0. \tag{10}$$

In this scenario, the supplier is setting a price for a buyer that orders based on a newsvendor solution. Let w^S represent the price that maximizes the supplier's expected profits under

early commitment. Because we have closed form expressions for Q^* when $\rho = 0$ and $\rho = 1$, we can solve for w^S in terms of Q^* :

$$w^S(Q^*)_{\rho=0} = r - c_B - r F_Z(Q^*) \quad \text{and} \quad w^S(Q^*)_{\rho=1} = r - c_B - (r - c_B) F_X(Q^*).$$

The price chosen by the supplier for our numerical example is denoted by w^S in Figure 2. Lariviere and Porteus [17] show that for the case of $\rho = 1$, the supplier's profit is unimodal in w if the demand distribution is IGFR. It is easy to show that the same is true for the $\rho = 0$ case. For intermeditate values, $0 < \rho < 1$, such properties are harder to show because of the lack of a closed form expression for Q^*. In Table 6, we offer a small numerical test to show how w^S changes with ρ. While the changes are small, the test indicates that w^S is non-increasing in ρ.

		μ	50	50	100	100	1000	1000
		c_S	20	60	20	60	20	60
	0		61.9	73.2	72.2	77.5	79.6	79.9
Variance	20		61.9	73.2	72.2	77.5	79.6	79.9
Reduction	50		61.5	73.2	72.1	77.5	79.6	79.9
(%)	80		61.4	73.2	72.0	77.5	79.6	79.9
	100		61.4	73.2	72.0	77.5	79.6	79.9

Table 6: Optimal Exchange Price When Supplier Sets The Price

(Normal Dist, Total Variance $= 400$, Buyer's Price $= 100$, Buyer's Cost $= 20$)

What happens when the buyer wields the pricing power in the supply chain and sets the selling price of the supplier? In the early commitment case, the buyer is assured of receiving its order-quantity as long as the supplier makes a positive profit. Since the buyer assumes all of the demand uncertainty risk regardless of the value of ρ, he sets a price incrementally larger than the supplier's marginal cost. The price chosen by the buyer for our numerical example is denoted by w^B in Figure 2.

What happens if the supplier and the buyer have equal price negotiating power? To model this scenario, we look at a simple heuristic that divides the profits among the supplier

and the buyer equally. This "equal profits" exchange price is found by setting $\pi_B^*(w) = \pi_S^*(w)$ and solving for the exchange price. For the normal distribution, there exist a unique exchange price where this occurs. Dividing profits equally is not an optimal or equilibrium mechanism, and it is rarely seen in the literature. It is not uncommon in practice, however, simply because it reflects a crude notion of fairness. The price chosen under equal power for our numerical example is denoted by w^E in Figure 2.

5.2   Power Structures Under Delayed Commitment

Under delayed commitment, when the supplier sets the price it must be high enough that the buyer makes a positive profit or she will not participate in the exchange. The supplier's objective is

$$Max_w : \tilde{\pi}_S^*(w) \quad s.t. \quad \tilde{\pi}_B^*(w) \geq 0 \ , \tag{11}$$

where $\pi_S^*(w)$ comes from equation (8) and $\tilde{\pi}_B^*(w)$ comes from equation (9). Let $\tilde{w}^S$ represent the price that maximizes the supplier's expected profits under delayed commitment. The supplier's optimal price in terms of $\tilde{P}_B^*$ is

$$\tilde{w}^S(\tilde{P}_B^*) = r - c_B - rF_Y(\tilde{P}_B^*).$$

This expression holds for all values of ρ. The supplier's profit is unimodal in w if the demand distribution is IGFR (Lariviere and Porteus [17]).

What about when the buyer sets the price? In the early commitment case, when the buyer sets the exchange price she sets it equal to the supplier's cost in the limit, capturing most of the supply chain profits. The supplier accepted such a low price because he did not share any of the demand uncertainty risk. Under delayed commitment, this is no longer the case. The price chosen by the buyer must now be high enough that the supplier makes a positive expected profit or he will not participate in the exchange. The buyer's objective is

$$Max_w : \tilde{\pi}_B^*(w) \quad s.t. \quad \tilde{\pi}_S^*(w) \geq 0 \ ,$$

where $\tilde{\pi}_B^*(w)$ comes from (9) and $\pi_S^*(w)$ from (8). Let $\tilde{w}^B$ represent the price that maximizes the buyer's expected profits under delayed commitment. The buyer's optimal price in terms

of $\tilde{P}_S^*$ is

$$\tilde{w}^B(\tilde{P}_S^*) = \frac{c_S}{1 - F_X(\tilde{P}_S^* - \tilde{k}^*)}. \tag{12}$$

This expression also holds for all values of ρ. Cachon [3] shows hat for the case of $\rho = 1$, the buyer's profit is unimodal in w if the demand distribution is IGFR His proof can be easily extended to our partial information updating scenario since the only difference is the inclusion of $\tilde{k}^*$ in (12), which is not a function of w.

As in the early commitment case, an "equal profits" exchange price under delayed commitment is found by equating the supplier's expected profit (8) with the expected profit of the buyer (9) and solving for the exchange price.

In the following subsections, we show how each supply chain member's preference for an early or delayed commitment changes for each of the three power structures investigated; supplier chooses price, buyer chooses price, and equal profits. We also show which commitment choice results under each structure. To do so, however, we must specify the decision framework and timing. First, the supply chain member(s) with price setting power propose two separate prices for each commitment time alternative (early or delayed). Next, we assume that the *buyer* always has the option of choosing an early commitment. While the buyer probably could not impose a choice of delayed commitment (without the supplier's willingness to accept such an arrangement), she *could* impose a choice of early commitment by simply choosing to place her order early even if the supplier were *willing to accept* (and even preferred) a delayed commitment. Realistically, it is hard to imagine situations where a supplier can justify *not* letting a buyer commit early to an order quantity. Lastly, the buyer chooses the price, and corresponding commitment time, that maximizes her expected profit.

5.3 Commitment Choice When The Supplier Chooses The Price

To analyze supply chains where the supplier is more powerful than the buyer, we consider the model in which the supplier chooses the wholesale price. The supplier first calculates $w^S(Q^*)$ and $\tilde{w}^S(\tilde{P}_B^*)$, then offers the choice between the the two prices to the buyer. The

buyer then chooses the price, and corresponding commitment time, that maximizes her expected profits.

Given an exchange price chosen by the supplier, the supplier prefers a delayed commitment while the buyer prefers an early commitment. Under early commitment, the buyer orders based on her critical fractile solution, forcing the supplier to select an exchange price that shares some of the supply chain profits with the buyer. Under delayed commitment, the buyer orders exactly the amount of components that she knows end-product demand will be, even when faced with very small margins (a guaranteed small profit is better than no profit). This allows the supplier, in the limit, to set the exchange price equal to the buyer's revenue minus her end-product production cost and capture the entire supply chain's profit.

When the supplier has pricing power in the supply chain, there exist conflicting preferences regarding the buyer's commitment timeframe. Since we assume the buyer has the final choice, a choice of early commitment results. To illustrate, consider the numerical example used to create Figure 7, a partial information updating scenario. The supplier first solves for his optimal price under early and delayed commitment, obtaining $w^S = 75$ and $\tilde{w}^S = 71$ respectively. The supplier then offers this set of prices to the buyer, who solves for her expected profit under each price/commitment time choice and chooses the larger. In this case, the buyer chooses a delayed commitment with $\tilde{w}^S = 71$, resulting in an expected profit of $\tilde{\pi}_B^* = 659$ compared to $\pi_B^* = 293$ if she had chosen the early commitment price.

5.4 Commitment Choice When The Buyer Chooses The Price

Given an exchange price chosen by the buyer and some positive amount of information updating, the buyer prefers an early commitment while the supplier prefers a delayed commitment. When the buyer chooses the exchange price under early commitment, she chooses a price equal to the supplier's cost plus a small margin ($w^B(Q^*) = c_S + \delta$) and extracts most of the supply chain's profit. Under delayed commitment however, the supplier produces based upon his critical fractile solution, equal to zero when the buyer chooses a price equal to the supplier's cost. The buyer is thus forced to give the supplier a margin large enough to induce him to produce the quantity of components that the buyer desires. This results

in the supplier making a positive profit and the buyer making a smaller profit under delayed commitment. As in the case of supplier pricing, when the buyer has pricing power the buyer prefers an early commitment while the supplier prefers a delayed commitment. Since we assume the buyer has the final choice, a choice of early commitment results.

5.5   Commitment Choice Under Equal Power

To analyze supply chains where both members have equal power, we consider the model in which the wholesale price is set through our equal profits heuristic. Let w^E represent the price that solves the equal profits objective under early commitment and $\tilde{w}^E$, the price that solves the equal profits objective under delayed commitment. We use a similar numerical experiment as the one described in the beginning of Section 4.1 to test our observation which compares the firms' expected profits when information updating occurs. The observation held true for all 100 cases tested. Table 6 provides a subset of our experiments for demonstration purposes.

Numerical Observation 3 *For Normally distributed demand and an exchange price that solves the equal profits condition,* $\left[\pi_S^*(w^E) = \pi_B^*(w^E)\right] - \left[\tilde{\pi}_S^*(\tilde{w}^E) = \tilde{\pi}_B^*(\tilde{w}^E)\right]$ *is decreasing in* ρ.

When the buyer and the supplier set an "equal profits" price, both members prefer an early commitment for small reductions in demand uncertainty. This is shown in Figure 7 for an example with $\rho = 0.5$ where the intersection of the early commitment profits results in a higher profit than the intersection of the delayed commitment profits. This preferences reverses however when $\rho = 1$. Table 6 lists the difference between the supply chain's total expected profit at the early equal profit price and at the delayed equal profit price for our sample cases.

μ		100	100	1000	1000
c_S		20	60	20	60
Variance	25	107	178	*	182
Reduction	50	81	166	*	161
(%)	75	-20	151	*	127
	100	-207	24	*	0

Table 6: Total Early Minus Total Delayed Profits at Equal Profit Price

(Normal Dist, Total Variance = 400, Buyer's Price = 100, Buyer's Cost = 20)

* No feasible equal profits price

The preceding results for the various pricing scenarios emphasize the important roles that the level of demand information updating and the supply chain power structure play in determining behavior and performance in the supply chain. In both of the extreme power cases (supplier chooses price and buyer chooses price), the two supply chain members prefer opposite commitment timeframes, and those preferences are independent of both who has pricing power and the amount of updated demand information that arrives at the end of period 1. In those cases, since the buyer can probably enforce its preference for early commitment, that is what is likely to result. However, when power is more equally distributed between the two supply chain members, the players prefer the same commitment timeframe, and this preference changes from early commitment to delayed commitment as the information contained in the demand update increases. Table 7 summarizes the results from this section below.

	Buyer's Preference	Supplier's Preference	Outcome
Supplier Pricing Power	Early	Delayed	Early
Buyer Pricing Power	Early	Delayed	Early
Equal Power*	Early for small ρ	Early for small ρ	Early for small ρ
	Delayed for large ρ	Delayed for large ρ	Delayed for large ρ

Table 7: Impact of Pricing Power and Information on Commitment Timeframe Preferences

*Based on Numeric Observations

6 Conclusions and Future Research

We have examined the implications surrounding the choice of the quantity commitment time by an end-product manufacturer (buyer) and its parts supplier in a stochastic demand environment under four different supply chain power structures. Our model captures the key trade-offs associated with the timing selection of the buyer's order quantity commitment in a supply chain. An early commitment by the buyer ensures that she will receive her order quantity and eliminates all of the demand risk of the supplier. However, an early commitment limits the buyer's ability to respond latter if new information is obtained about demand. Delayed commitment increases the buyer's flexibility but introduces the risk that the supplier (now facing its own demand risk) will not provide the buyer with her full order quantity request. We characterize each supply chain member's objective under the two commitment timing choices and provide properties for optimal decision making under demand information updating.

The choice of the commitment time leads to some interesting and surprising results. We find that the buyer is *not* always better off delaying its quantity commitment, even when there is no explicit cost associated with doing so. We also find conditions under which the supplier prefers a *delayed* commitment. In particular, we find that the choice of commitment time depends upon the power structure of the supply chain, as well as the amount of uncertainty over end-product demand that is reduced by additional information received before the buyer makes her production decisions. Regarding the value of information, we find that in a decentralized supply chain operating under delayed commitment and a low supplier margin, additional demand information leads to *lower* expected profits for *both* the supplier and the buyer. More specifically, our analysis yields the following managerial insights

1. Given an exogenous exchange price and no information updating, both members of the supply chain are indifferent to the choice of commitment time.

2. Given an exogenous exchange price and some positive amount of information updating, both the buyer and the supplier prefer an early commitment for low exchange prices and a delayed commitment for high exchange prices.

3. For decentralized supply chains operating under early commitment and Normally distributed demand, the order quantity from the buyer and both members' expected profits are non-decreasing in the amount of demand information updating.

4. For decentralized supply chains operating under delayed commitment, both members' expected profits decrease with the amount of demand information updating when the supplier's margin is small.

5. Given an exchange price chosen by the supplier and all levels of information updating, the buyer prefers an early commitment while the supplier prefers a delayed commitment.

6. Given an exchange price chosen by the buyer and all levels of information updating, the buyer prefers an early commitment while the supplier prefers a delayed commitment.

7. When the buyer and the supplier set an "equal profits" price, we observe numerically that both members prefer an early commitment for small reductions in demand uncertainty and a delayed commitment for large reductions.

This is a first step in understanding the impact of the quantity commitment time choices in a manufacturer-parts supplier contract. Future research issues include the impact of multiple suppliers, multiple buyers, multiple levels in the supply chain, and the inclusion of a rolling horizon time frame.

7 Appendix

Computational Formulation of Early Commitment Problem

In this section, we reformulate the buyer's problem in a manner that makes it easier to obtain numerical solutions. Knowing the buyer's optimal production quantity in period 2 will be $P_B^* = X + k^*$, the buyer's objective function at the beginning of period 1 is

$$Max_Q \ \pi_B(Q) = -wQ - c_B E[\min(X + k^*, Q)] + r E[\min(X + k^*, Q, Z)] \ . \qquad (13)$$

In order to express the buyer's objective function in terms of the distribution of the random variables X and Y, we first determine the limits of integration. For $\min(X + k^*, Q)$, we have two possibilities: $X < Q - k^*$ and $X \geq Q - k^*$. For $\min(X + k^*, Q, Z)$, we have six possibilities:

1. $Z \leq X + k^* \leq Q$: $Y \leq k^* \leq Q - X \rightarrow Y \leq k^*$ and $X \leq Q - k^*$

2. $Z \leq Q \leq X + k^*$: $Y \leq Q - X \leq k^* \rightarrow Y \leq Q - X$ and $Q - k^* \leq X$

3. $X + k^* \leq Z \leq Q$: $k^* \leq Y \leq Q - X \rightarrow k^* \leq Y$ and $Y \leq Q - X$

4. $X + k^* \leq Q \leq Z$: $-k^* \geq X - Q \geq -Y \rightarrow Q - X \leq Y$ and $X \leq Q - k^*$

5. $Q \leq Z \leq X + k^*$: $Q - X \leq Y \leq k^* \rightarrow Q - Y \leq X$ and $Y \leq k^*$

6. $Q \leq X + k^* \leq Z$: $Q - k^* \leq X \leq X + Y - k^* \rightarrow Q - k^* \leq X$ and $k^* \leq Y$

The objective function (13) can now be expressed as

$$Max_Q \ \pi_B(Q) = -wQ - c_B \int_{L_X}^{Q-k^*} (X + k^*) f_X(X) dX - c_B \int_{Q-k^*}^{H_X} Q f_X(X) dX +$$

$$r \int_{L_X}^{Q-k^*} \int_{L_Y}^{k^*} (X + Y) f_Y(Y) dY f_X(X) dX + r \int_{Q-k^*}^{H_X} \int_{L_Y}^{Q-X} (X + Y) f_Y(Y) dY f_X(X) dX +$$

$$r \int_{L_X}^{H_X} \int_{k^*}^{Q-X} (X + k^*) f_Y(Y) dY f_X(X) dX + r \int_{L_X}^{Q-k^*} \int_{Q-X}^{H_Y} (X + k^*) f_Y(Y) dY f_X(X) dX +$$

$$r \int_{L_Y}^{k^*} \int_{Q-Y}^{H_X} Q f_X(X) dX f_Y(Y) dY + r \int_{k^*}^{H_Y} \int_{Q-k^*}^{H_X} Q f_X(X) dX f_Y(Y) dY \quad \blacksquare.$$

Centralized Supply Chain Results

Here we explore the centralized supply chain, i.e., the supply chain where all decisions are made by a central planner with the objective of maximizing total supply chain expected profits. The centralized supply chain solutions are useful when making performance comparisons between our various scenarios and the results are used in several of the proofs. For clarification purposes, we distinguish our decision variables from the early and delayed commitment cases through the placement of a carat above the corresponding variable symbol. For instance, the parameter k becomes $\check{k}$.

As with the previous non-centralized cases, we begin with the central planner's period 2 decision of how much final product to produce, P_B. This decision is made after observing the additional demand information i.e. $X = x$. The centralized supply chain's second period

objective is

$$Max_{\check{P}_B} \quad \check{\pi}(\check{P}_B, x) = -c_B \check{P}_B + r E[\min(\check{P}_B, x + Y)] \quad .$$

Performing a similar analysis as in the earlier cases, we find that the centralized supply chain's optimal final product production quantity is $P_B^*(x) = x + \check{k}$ where

$$\check{k}^* = F_Y^{-1}\left(\frac{r - c_B}{r}\right) \quad .$$

The centralized supply chain's first period objective is

$$Max_{\check{P}_1} \quad \check{\pi}(\check{P}_S) = E[-c_S \check{P}_S - c_B \min(X + \check{k}^*, \check{P}_S) + r \min(X + \check{k}^*, \check{P}_S, Z)] \ . \tag{14}$$

As in the non-centralized case under early commitment, the objective function is concave in $\check{P}_1$. The optimal component production quantity, $\check{P}_1^*$, is found by solving $\partial \check{\pi}(\check{P}_S)/\partial \check{P}_S = 0$. The centralized supply chain's total expected profit is then

$$\check{\pi}^* = E[-c_S \check{P}_S^* - c_B \min(X + \check{k}^*, \check{P}_S^*) + r \min(X + \check{k}^*, \check{P}_S^*, Z)] \quad \blacksquare.$$

Proof of Proposition 1: $H(k)$ is concave in k. $H(Q - x)$ is concave in Q and $\partial H(Q - x)/\partial Q > 0$ as long as $Q - x < k^*$. Thus, $H(\min\{k^*, Q - x\})$ is concave for $Q - x < k^*$, after which it becomes a constant function and $H(\min\{k^*, Q - x\})$ is concave in Q. $\vec{\pi}(Q)$ is the sum of a linear function of x and $H(\min\{k^*, Q - x\})$, and thus is concave in Q. $\pi(Q)$ is the expectation over a linear function of Q and $\vec{\pi}(Q)$, so $\pi(Q)$ must also be concave in Q.$\blacksquare$

Proof of Proposition 2: To prove this proposition, we first construct a way to treat ρ as a parameter. Let U and V be independent random variables, each with mean 0 and variance 1. (For the moment these need not be normal.) Also, let σ^2 be the constant total variance of $Z = X + Y$. Define

$$\begin{aligned}
\alpha &= \sqrt{\rho} \\
\gamma &= \sqrt{1 - \rho} = \sqrt{1 - \alpha^2} \\
\beta &= \alpha/\gamma \quad .
\end{aligned}$$

Now construct

$$\begin{aligned}
X &= \mu + \sigma\alpha U \\
Y &= \sigma\gamma V \quad .
\end{aligned}$$

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Then, Z has the correct variance, and ρ is the correct ratio.

From (3) we have

$$\pi_B(Q) = E_X[-wQ + (r - c_0)X + H(\min\{k^*, Q - X\})] ,$$

where

$$H(k) = -c_0 k + r E_Y[\min\{k, Y\}]$$

and k^* minimizes H, i.e.,

$$k^* = F_Y^{-1}\left(\frac{r - c_0}{r}\right) .$$

Define

$$h(v) = -c_0 v + r E_V[\min\{v, V\}]$$

$$v^* = k^*/\sigma\gamma = F_V^{-1}\left(\frac{r - c_0}{r}\right) ,$$

so v^* minimizes h. Also, let

$$q = \frac{Q - \mu}{\sigma} .$$

Then,

$$
\begin{aligned}
\pi_B(Q) &= E_U[-w(\mu + \sigma q) + (r - c_0)(\mu + \sigma\alpha U) \\
&\quad + H(\min\{\sigma\gamma v^*, \sigma q - \sigma\alpha U\})] \\
&= (r - c_0 - w)\mu + E_U[-w\sigma q \\
&\quad + H(\sigma\gamma\min\{v^*, q/\gamma - \beta U\})] \\
&= (r - c_0 - w)\mu + \sigma\gamma E_U[-wq/\gamma \\
&\quad + h(\min\{v^*, q/\gamma - \beta U\})] \quad .
\end{aligned}
$$

Or, setting $\widetilde{q} = q/\gamma$ and $g(v) = h(\min\{v^*, v\})$,

$$\pi_B(Q) = (r - c_0 - w)\mu + \sigma\gamma E_U[-w\widetilde{q} + g(\widetilde{q} - \beta U)] \quad .$$

For any ρ, the optimal $\widetilde{q}^*$ minimizes this expectation, and from it we can recover the optimal $Q^* = \mu + \sigma\gamma\widetilde{q}^*$. The question is, how does $\widetilde{q}^*$ change with ρ?

Note that $\widetilde{q}^*$ solves

$$E_U[g'(\widetilde{q} - \beta U)] = w .$$

If g' were convex, then by Jensens's inequality this expectation would be increasing in β and hence in ρ. Thus, $\widetilde{q}^*$ would be increasing. But g' is not convex. It may be convex over parts of its domain, but not all. Anyway, γ is decreasing in ρ.

Let's consider small ρ. Consider the case of normal U and V. For small ρ and hence small β, $\Pr\{\widetilde{q}^*(0) - \beta U > v^*\}$ is negligible. So, the quantity to be minimized is approximately

$$
\begin{aligned}
& E_U[-w\widetilde{q} + h(\widetilde{q} - \beta U)] \\
={} & -w\widetilde{q} + E_U[-c_0(\widetilde{q} - \beta U) + rE_V[\min\{\widetilde{q} - \beta U, V\}]] \\
={} & -(w + c_0)\widetilde{q} + rE_{U,V}[\min\{\widetilde{q}, \beta U + V\}] .
\end{aligned}
$$

The random variable $\beta U + V$ is normal with mean 0 and variance

$$
\begin{aligned}
1 + \beta^2 &= 1 + \alpha^2/(1 - \alpha^2) \\
&= 1/\gamma^2 .
\end{aligned}
$$

So,

$$
\begin{aligned}
\widetilde{q}^*(\rho) &\approx (1/\gamma)\Phi^{-1}\left(\frac{r - w - c_0}{r}\right) \\
&= (1/\gamma)\widetilde{q}^*(0).
\end{aligned}
$$

Also,

$$
\begin{aligned}
Q^*(\rho) &= \mu + \sigma\gamma\widetilde{q}^*(\rho) \\
&\approx \mu + \sigma\gamma(1/\gamma)\widetilde{q}^*(0) \\
&= \mu + \sigma\widetilde{q}^*(0) = Q^*(0).
\end{aligned}
$$

Now, $h' \le g'$(because g truncates h at its maximum point). This approximation underestimates g' and hence $E_U[g'(\widetilde{q} - \beta U)]$. Hence, the actual $\widetilde{q}^*(\rho)$ is larger than $(1/\gamma)\widetilde{q}^*(0)$. Multiplying by γ yields something larger than $\widetilde{q}^*(0)$. Thus, the exact $Q^*(\rho)$ is larger than $Q^*(0)$.

Proof of Proposition 3: Buyer's expected profit: Suppose the buyer currently anticipates receiving an information update of $\rho = \tau$. The buyer can then expect profits of $\pi_B^*(\rho = \tau)$ Now

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assume that the buyer may obtain additional information, $\tau' > \tau$, before making its decisions. If $\pi_B^*(\rho = \tau) > \pi_B^*(\rho = \tau')$ then the buyer will know this and choose not to use the additional information in its decision making, thus making an expected profit of $\pi_B^*(\rho = \tau)$.

Supplier's expected profit: From (), the buyer's expected profit is increasing linearly in Q. From Proposition 2, $Q^*(\rho) > Q^*(0)$, so $\pi_S^*(\rho = \tau) > \pi_S^*(\rho = 0)$.

Proof of Proposition 4: The difference between the suppliers's expected profit under full information and delayed commitment minus its expected profit under no additional information and delayed commitment is

$$(\tilde{\pi}_{S,FI}^* - \tilde{\pi}_{S,NA}^*) = -c_S \tilde{P}_{S,FI}^* - (w - c_S)\tilde{P}_{S,NA}^* + wE[\min(\tilde{P}_{S,FI}^*, Z)] \ ,$$

a continuous function of w. From the boundary cases, $\tilde{P}_{S,NA}^* = \tilde{P}_{B,NA}^* = \mu + F_Y^{-1}\left(\frac{r - c_B - w}{r}\right)$ and $\tilde{P}_{1,FI}^* = F_X^{-1}\left(\frac{w - c_S}{w}\right) = F_Z^{-1}\left(\frac{w - c_S}{w}\right)$. For the NA case, $L_X = \mu = H_X$ and by assumption $\mu + L_Y = L_X + L_Y = 0$. Thus, as $w \to r - c_B$, $\tilde{P}_{S,NA}^* \to \mu + L_Y = 0$. The buyer makes zero margin in the limit but still faces demand uncertainty, thus he does not order. The supplier realizes this and does not produce. Also $\tilde{P}_{B,FI}^* \to F_X^{-1}\left(\frac{r - c_B - c_S}{r - c_B}\right) = F_Z^{-1}\left(\frac{r - c_B - c_S}{r - c_B}\right)$. The supplier's production quantity is positive here because the buyer faces no demand uncertainty and produces the known demand quantity, even as its margin approaches zero. Setting $w = r - c_B$ yields

$$(\tilde{\pi}_{B,FI}^* - \tilde{\pi}_{B,NA}^*) = (r - c_B)E[\min(\tilde{P}_{B,FI}^*, Z)] - c_S \tilde{P}_{B,FI}^*$$

$$= (r - c_B)\int_0^{\tilde{P}_{B,FI}^*} z f_Z(z)dz + (r - c_B)\tilde{P}_{B,FI}^*(1 - F(\tilde{P}_{B,FI}^*)) - c_S \tilde{P}_{B,FI}^*$$

$$= (r - c_B)\int_0^{\tilde{P}_{B,FI}^*} z f_Z(z)dz + (r - c_B)\tilde{P}_{B,FI}^*\left(\frac{c_S}{(r - c_B)}\right) - c_S \tilde{P}_{B,FI}^*$$

$$= (r - c_B)\int_0^{\tilde{P}_{B,FI}^*} z f_Z(z)dz > 0 \ .$$

Thus, for small $\delta > 0$, $\tilde{\pi}_{B,FI}^* > \tilde{\pi}_{B,NA}^*$ for $w > r - c_B - \delta$. If $w = c_S$, $\tilde{P}_{S,NA}^* = \mu + F_Y^{-1}\left(\frac{r - c_B - c_S}{r}\right) > \mu + L_Y = 0$, since in the NA case $\mu = L_X$ and by assumption $L_X + L_Y = 0$. Also, $\tilde{P}_{B,FI}^* = F_X^{-1}(0) = L_X = L_X + L_Y = 0$, since in the FI case $L_Y = H_Y = 0$. Since $\tilde{P}_{S,NA}^*$ and $\tilde{P}_{1,FI}^*$ are continuous in w, we have $\tilde{P}_{S,NA}^* > \tilde{P}_{B,FI}^*$ for exchange prices $c_S \leq w \leq c_S + \delta$ for some $\delta > 0$. Now

$$(\tilde{\pi}_{B,FI}^* - \tilde{\pi}_{B,NA}^*) = -w\tilde{P}_{B,NA}^* - c_S(\tilde{P}_{B1,FI}^* - \tilde{P}_{B,NA}^*) + wE[\min(\tilde{P}_{B,FI}^*, Z)]$$

$$\leq -w\tilde{P}^*_{B,NA} - c_S(\tilde{P}^*_{B,FI} - \tilde{P}^*_{B,NA}) + w\tilde{P}^*_{B,FI}$$

$$= (w - c_S)(\tilde{P}^*_{B,FI} - \tilde{P}^*_{B,NA}) \leq 0 \quad \text{for } c_S \leq w \leq c_S + \delta.$$

The proof for the buyer's expected profit follows similar logic. ∎

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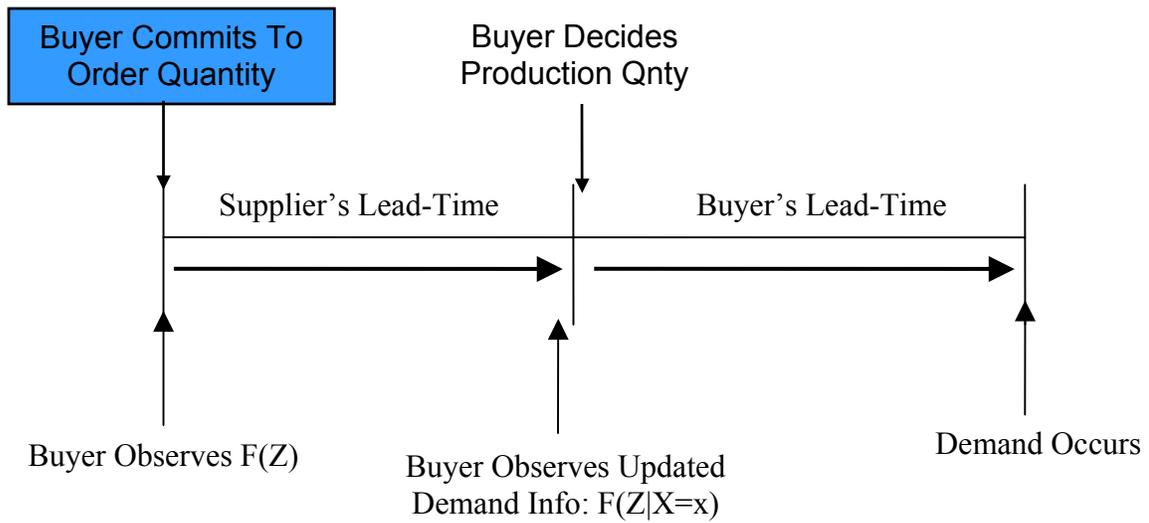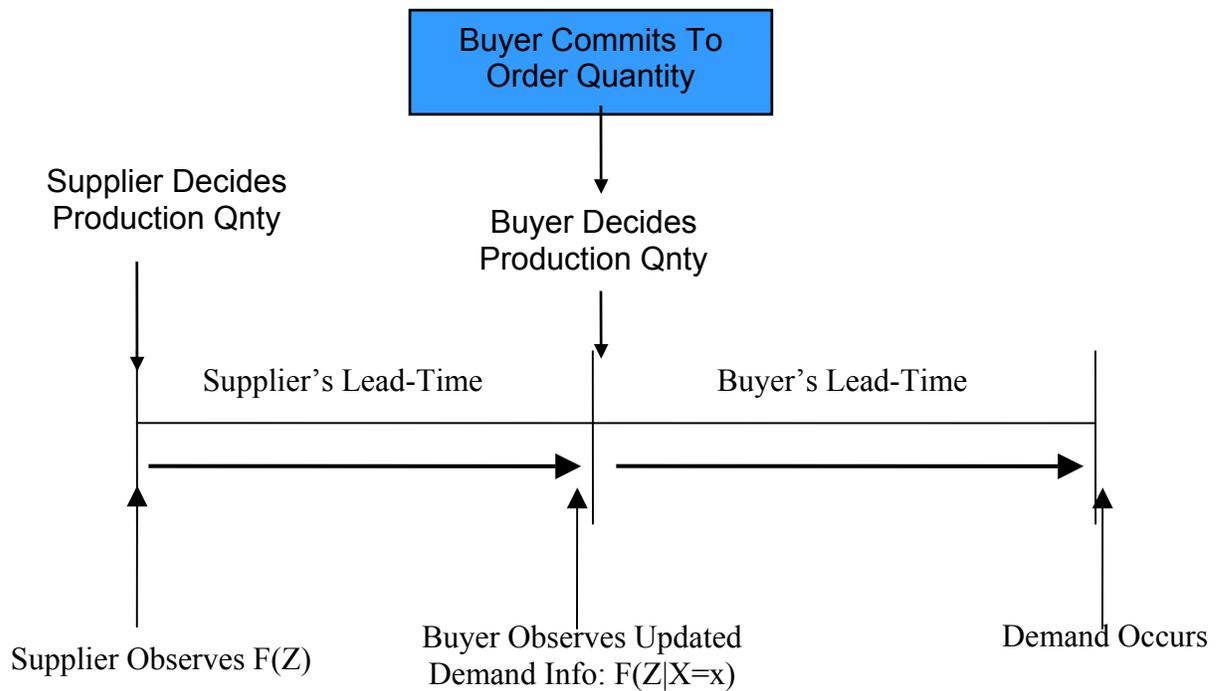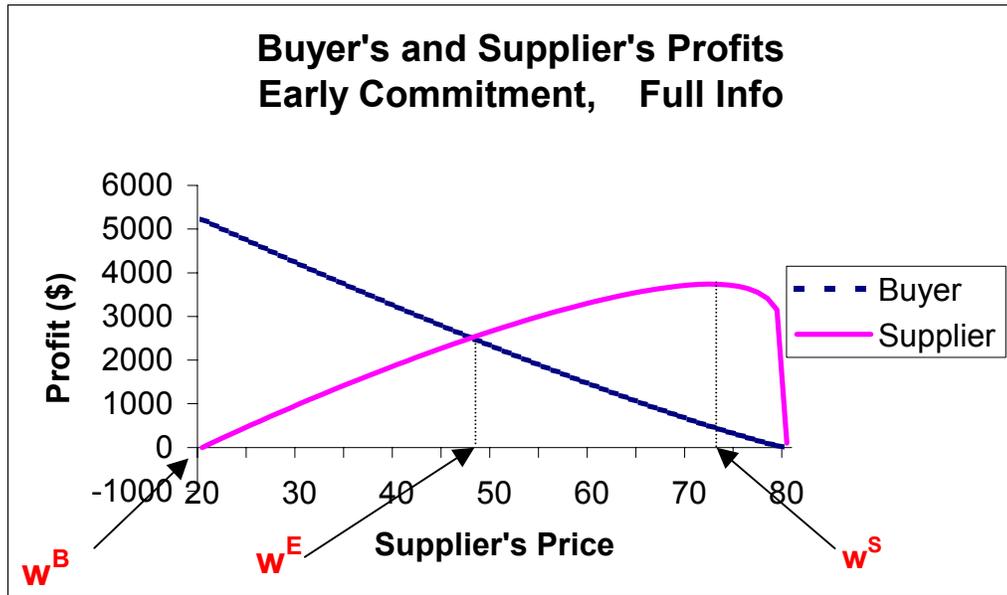Figure 1: Early Commitment

Figure 3: Delayed Commitment

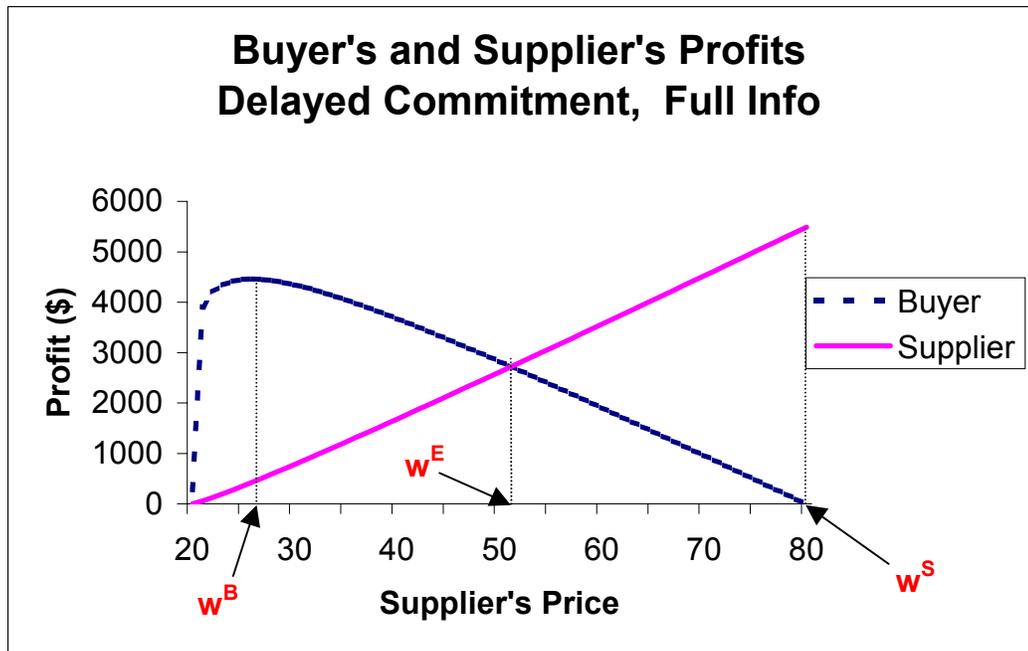**Figure 2:** $r = 100$, $c_1 = c_0 = 20$, $z \sim N(100,20)$

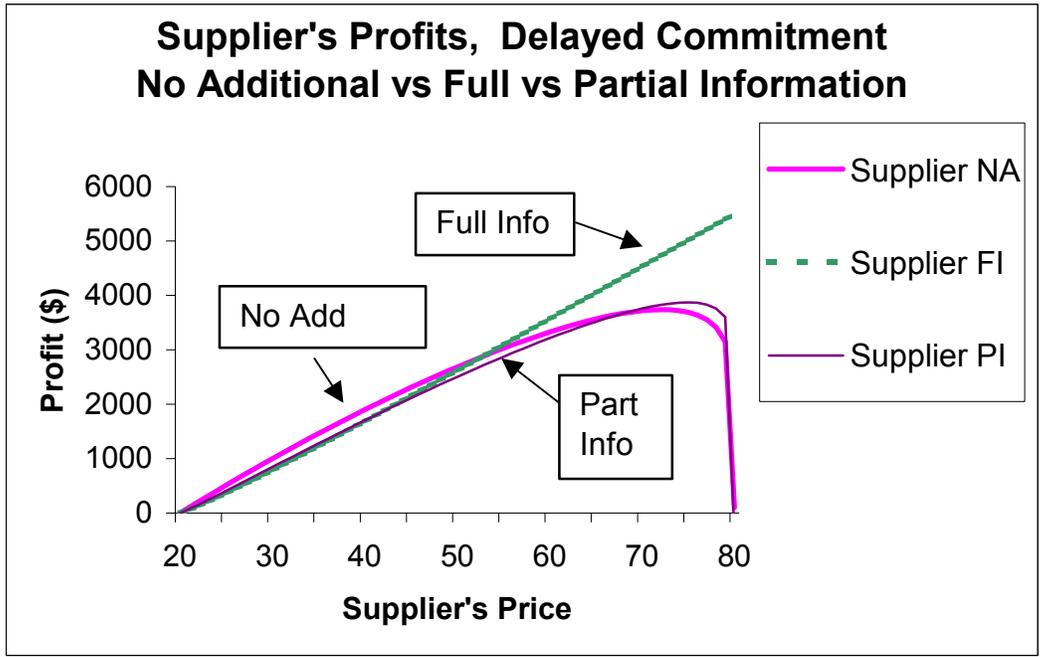**Figure 4:** $r = 100$, $c_1 = c_0 = 20$, $z \sim N(100,20)$

Figure 5: $r = 100$, $c_1 = c_0 = 20$, $z \sim N(100,20)$, $x \sim N(100, 14.14)$, $y \sim N(0, 14.14)$
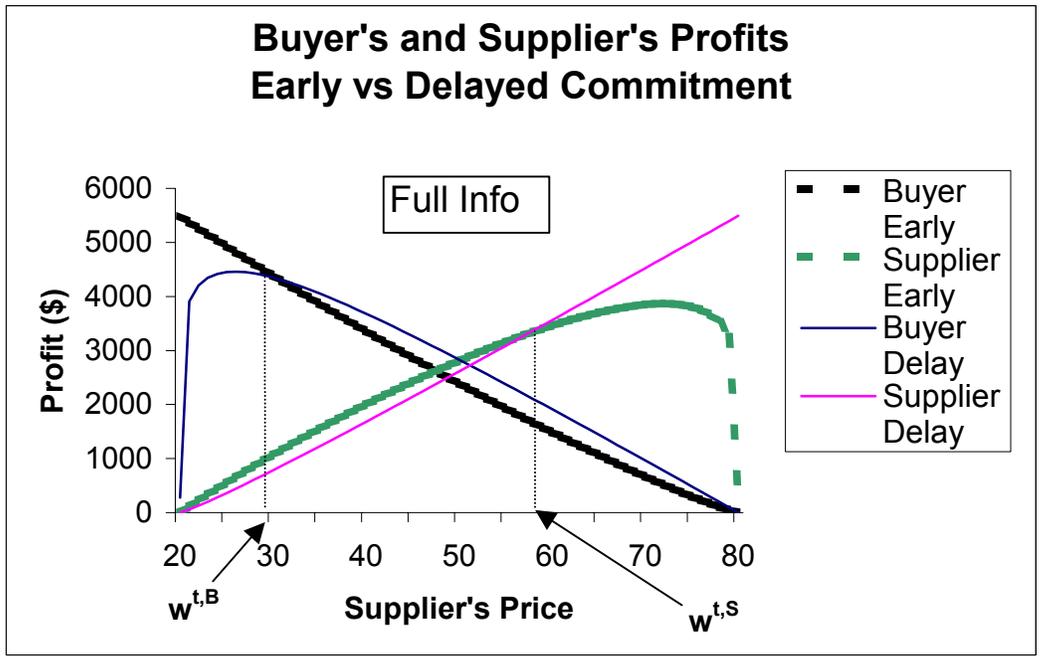
Figure 6: $r = 100$, $c_1 = c_0 = 20$, $z \sim N(100,20)$

Buyer's and Supplier's Profits
Early vs Delayed Commitment

Part Info

Legend:
- Buyer Early
- Supplier Early
- Buyer Delay
- Supplier Delay

Y-axis: Profit ($) — 0, 1000, 2000, 3000, 4000, 5000, 6000

X-axis: Supplier's Price — 20, 30, 40, 50, 60, 70, 80
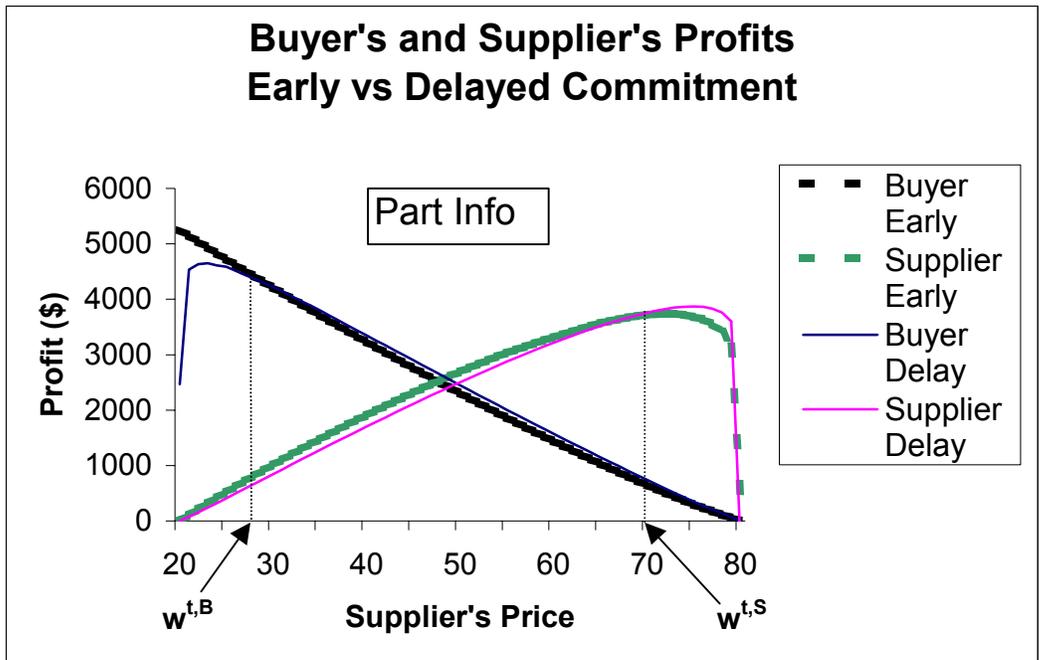
$w^{t,B}$ $w^{t,S}$

Figure 7: $r = 100$, $c_1 = c_0 = 20$, $z \sim N(100,20)$, $x \sim N(100, 14.14)$, $y \sim N(0, 14.14)$

			Total Variance	400	400	400	100	100	100	25	25	25
			Buyer's manf cost	20	40	60	20	40	60	20	40	60
		0	Q_{NA}	100	90	74	100	95	87	100	97	94
	Variance	25	Q^*	100	90	75	100	95	87	100	97	94
w = 30	Reduction	50	Q^*	101	91	76	101	96	88	100	98	94
	(%)	75	Q^*	103	94	78	101	97	89	100	98	94
		100	Q_{FI}	106	100	87	103	100	93	102	100	97
		0	Q_{NA}	90	74	-	95	87	-	97	94	-
	Variance	25	Q^*	90	74	-	95	87	-	97	94	-
w = 50	Reduction	50	Q^*	90	75	-	95	87	-	97	94	-
	(%)	75	Q^*	90	76	-	95	88	-	97	94	-
		100	Q_{FI}	94	81	-	97	90	-	98	95	-
		0	Q_{NA}	74	-	-	87	-	-	94	-	-
	Variance	25	Q^*	74	-	-	87	-	-	94	-	-
w = 70	Reduction	50	Q^*	74	-	-	87	-	-	94	-	-
	(%)	75	Q^*	75	-	-	87	-	-	94	-	-
		100	Q_{FI}	77	-	-	89	-	-	94	-	-

Table 2: Optimal First Period Order Qnty (Normal Dist, Mean = 100, Buyer's Price =100, Supplier's Cost =10)