A FREQUENCY SELECTIVE POWER METER

FOR MILLIMETER WAVES

by

M. W. LONG

Department of the Navy
Office of Naval Research
Contract No. Nonr-991(10)
NR 372 - 781

Project A-701

(A Continuation of Project A-390)

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia
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CONTENTS

Semi-Annual Status Report.
April 1--October 22, 1963.

Final Report.
July 29, 1964.
22 October 1963

Dr. Arnold Shostak
Head, Electronics Branch
Office of Naval Research, Code 427
Main Navy Building
Washington 25, D. C.

Subject: Semi-Annual Status Report, Contract Nonr-991(10), NR 372-781

Dear Dr. Shostak:

This is the first report on the subject contract, and it summarizes the research since 1 April 1963. At that time Contract Nonr-991(10) superseded Contract Nonr-991(07). The last technical report prepared under Contract Nonr-991(07) was Technical Report No. 4.

Technical Report No. 4 describes a new technique for microwave radiometry which can be used for measuring the transmission properties of media over a wide range of frequencies, and for determining the absolute sensitivity of detectors over a wide range of frequencies. The report also discusses the possibility for determining the power level of coherent sources as a function of frequency. Effort on the current contract has been directed toward research needed to further develop the concept of a tunable, highly sensitive power meter based on the interference principle. Most of the effort to date has been directed toward modifying the existing interference device (operates within the 1.3-m wavelength region) for power measurements. Limited measurements have been

made to demonstrate the practicability of the technique. Some consideration has also been given to determining the suitability of the technique for other wave-

lengths.

**Introduction**

Technical Report No. 4 includes a detailed description of operating principles for the interference device. The tuned audio amplifier (or other frequency analyzer) in conjunction with the varying path length serves as a band-pass filter. Thus, the audio amplifier output caused by a wide band microwave noise source is proportional to \( B^2 \), where \( B \) is the effective microwave bandwidth which is controlled by audio bandwidth. The magnitude of the peaks in the amplifier output caused by a coherent source is also proportional to source power; therefore, comparison of peak output with that caused by a calibrated noise source provides a method for determining the power level of a coherent source. In principle, power measuring devices can be calibrated as a function of frequency with a modulated noise source and a tunable microwave band-pass filter. Such measurements are not currently possible in the low millimeter region because band-pass filters with tunable, but known, pass-band characteristics are not available.

The power meter discussed herein is basically different from conventional power meters employing a barretter or thermistor in which the r-f power measurement is based upon the substitution of a d-c or audio power for r-f power. In that type of power meter there is an inherent calibration error caused by the uncertainty in the so-called "calibration factor" for the detector. Calibration factor is defined as the ratio of substituted d-c power in the detector element to the microwave power incident upon the detector; thus, the calibration factor
is often an unknown "factor" which in general depends on frequency, even within a given waveguide band. For the technique presently under investigation, microwave power from the noise source is compared with the unknown coherent microwave power. The noise source temperature can be ultimately determined with a thermometer and thus noise source power is attainable.

**Power Measurements at 70 Ge**

The material below describes progress in using the trombone device shown in Figure 3 of Technical Report No. 4 for measuring power levels of low-level coherent signals in the 1-4 cm wavelength region. Waves from a coherent source, when passed through the trombone device, are modulated sinusoidally. This means that a discontinuity in waveform is produced when trombone motion is reversed at the end of each stroke. The discontinuity is less troublesome for the noise source because percentage modulation is smaller than for the coherent source, and it approaches a very small value at the end of each stroke. Thus for measuring the coherent source, a dead-time must be introduced so that the 30-cps Doppler wave can decay before the trombone cycle is reversed, otherwise a prohibitive interference is produced. To accomplish this, a novel mechanical device was developed by Mr. J. C. Butterworth which causes the trombone to be stopped at the end of the cycle so that the amplified interference wave can decay before the cycle is reversed. The similarity of the spectrum of a pulsed Doppler wave generated by trombone movement and the spectrum of other pulsed waves should be noted. Since it is known that spectral width is of the order of reciprocal of pulse length, it is seen that sweep length ultimately controls resolution. For example, the trombone sweep length as presently operated is approximately 1/2 second and the Doppler frequency is 31 cps. Thus, the
limiting fractional resolution with this sweep is approximately 1/15.

In an effort to acquire a feel for the requirements in making low level power measurements, the absolute power level was determined for the 70 Ge output of a crystal harmonic generator driven by a CK 283 klystron operating at 35 Ge. The harmonic generator was of a conventional cross-guide type consisting of RG 96/U and RG 98/U waveguide and a modified type 1N53 crystal diode. The crystal diode was driven so that crystal current was between 0.5 ma and 1.0 ma.

The audio amplifier bandwidth was increased from 4 cps to 6 cps to permit the discontinuity in the Doppler component of the coherent wave to decay prior to reinitiation of trombone motion. The 6-cps bandwidth in conjunction with a center frequency of 31 cps results in an equivalent microwave bandwidth or resolution of 13.5 Ge (19%). The standard of reference for these measurements was a Roger White noise source and the detector was an evacuated barretter; these components are described in Technical Report No. 4. For the system bandwidth of 13.5 Ge, equivalent noise source power* is 2.7 x 10⁻⁹ watt. Output level with noise source and with coherent generator were compared by displaying audio amplifier output on a Brush paper tape recorder. The peak output level caused by the coherent source was set equal to the peak output level caused by the noise source by attenuating the coherent signal 14.5 db with a FXR model M164A microwave attenuator. Thus, the interference technique was used to measure

*Equivalent noise source power is equal to kT B, where k is Boltzmann's constant, T is noise temperature, and B is bandwidth.
the output level of a very weak (75 millimicrowatts) coherent source.

It is shown in Technical Report No. 4 that the percentage microwave bandwidth (resolution) is equal to percentage audio bandwidth. This is the feature of the interference technique which results in the device giving the effect of a tunable microwave filter. In that report results were included to demonstrate that, within experimental accuracy, measured microwave resolution was equal to percentage amplifier bandwidth. The correspondence between audio bandwidth and microwave bandwidth has been further investigated for the case in which the source is coherent to determine the plausibility of developing a tunable power meter. In order to accomplish this test, the harmonic generator was operated at a fixed frequency and the response curve of peak detector output was measured as a function of trombone velocity. These results also indicate that the percentage microwave resolution is approximately equal to the percentage audio bandwidth.

**Other Modes of Operation**

For a given mechanical configuration, the minimum audio amplifier bandwidth is limited to a value which permits the Doppler wave to decay to a negligible value during the reversal time. (If desired, noise bandwidth can be reduced without changing the resolution by adding integration after audio detection.) Because of the limitation imposed upon audio bandwidth discussed above, it might be surmised that system microwave resolution (equivalent to ratio of audio amplifier bandwidth to center audio frequency) could only be improved by increasing trombone speed and amplifier center frequency. Recall that detection of the microwave noise source is possible only with a relatively sensitive detector. The response time of our detector, an evacuated barretter, limits sensitivity
if the modulation rate is increased much above the 30-cps rate. Even if a sensitive, fast detector were available, it appears impractical from the mechanical standpoint to increase Doppler frequency by the order of magnitude or more that might be desired. Some thought has been given to using a relatively wide band audio amplifier, which would require less trombone dead time, and increasing signal frequency by multiplication. With this concept a spectrum analyzer might provide tunability and good resolution. This same technique would, in principle, also permit the development of a tunable, high resolution radiometer. In calibrating the meter with a noise source, the spectrum analyzer would have to be operated with a relatively wide bandwidth. The analyzer bandwidth could then be switched to a narrow bandwidth mode for high resolution operation. Nonlinearities associated with frequency multiplication are not understood by the author, and it is planned that this matter will be considered further.

Some consideration has been given to determining the practicality of using the interference technique for absolute power measurements at lower microwave frequencies. Well developed power measuring techniques exist within much of the microwave region, but even so there is need for highly sensitive power meters and for tunable power meters. To use this technique one must have adequate sensitivity to detect a microwave noise source, and it is shown below that this would be more difficult at lower frequencies than it is at 70 Mc.

For a detector operating in the square-law region, minimum detectable power, \( \Delta P_{\text{min}} \), varies as

\[
\Delta P_{\text{min}} = K \sqrt{B_a},
\]
where \( K \) is the minimum detectable change in power referred to a 1-ops audio bandwidth, and \( B_a \) is the audio bandwidth. Thus from Equation 9 of Technical Report No. 4, minimum detectable noise temperature can be determined from the following relationship:

\[
K \sqrt{B_a} = \frac{k}{4} kTB .
\]

In the above equation, \( T \) is noise source temperature expressed in degrees Kelvin, \( k \) is Boltzmann's constant, and \( B \) is microwave bandwidth. If \( F \) is defined as microwave frequency and \( R \) is defined as fractional microwave resolution, the above equation may be expressed as

\[
K \sqrt{B_a} = \frac{k}{4} kTRF .
\]

Noise sources available throughout the microwave region and for frequencies at least up to the 79 km region have temperatures of approximately 15,000 \(^\circ\)K, and detector sensitivities within this range are also nearly constant and are of the order of \(10^{-10}\) watt per cycle per second of audio bandwidth. It may be seen from the above equation that reasonable resolutions can be obtained at the lower frequencies only by using very narrow audio bandwidths. Thus it would seem impractical to use noise source calibration at the lower frequencies. However, since detector efficiencies are relatively well known within much of the microwave region, it would sometimes be possible to use d-c calibration, thus eliminating the necessity of detecting a noise source.
Future Program and Fiscal Data

It is planned that future research will be devoted to:

1. continuation of effort to obtain an improved method for measuring power by the interference technique in the 70-Gc region, and

2. continuation of analytical studies to determine optimum operating parameters for the interference technique.

The termination date for Contract Nonr-991(10) is 31 March 1964. It is anticipated that all studies will be completed within the contract period and budget.

Respectfully submitted:

M. W. Long, Chief
Electronics Division

cc: Mr. R. J. Whitcomb, OHR ResRep/Atlanta

bc: J. C. Butterworth
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M. W. Long

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Department of the Navy
Office of Naval Research
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29 July 1964

Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia
A FREQUENCY SELECTIVE POWER METER
FOR MILLIMETER WAVES

By

M. W. LONG

29 July 1964
### GLOSSARY

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<td>B</td>
<td>Microwave bandwidth</td>
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<td>B&lt;sub&gt;a&lt;/sub&gt;</td>
<td>Audio amplifier bandwidth</td>
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<tr>
<td>f&lt;sub&gt;a&lt;/sub&gt;</td>
<td>Audio amplifier center frequency</td>
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<td>F</td>
<td>Center of microwave passband of width B</td>
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<td>Minimum detectable power for a 1-cps audio bandwidth</td>
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<td>L</td>
<td>Maximum change in path length</td>
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<td>P&lt;sub&gt;min&lt;/sub&gt;</td>
<td>Minimum detectable power</td>
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<td>P&lt;sub&gt;i&lt;/sub&gt;</td>
<td>Power level of coherent microwave input signal</td>
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<td>R</td>
<td>Responsivity of detector</td>
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<td>R&lt;sup&gt;∞&lt;/sup&gt;</td>
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<td>s</td>
<td>Speed of path length change</td>
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<td>T</td>
<td>Equivalent black body temperature of noise source</td>
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<td>V&lt;sub&gt;d&lt;/sub&gt;</td>
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<td>δ</td>
<td>Path length difference</td>
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<td>λ&lt;sub&gt;g&lt;/sub&gt;</td>
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I. ABSTRACT

This is the final report on Office of Naval Research Contract Task Nonr-991(10) and is the only technical report that was published for the task which has been active for the period 1 April 1963 through 31 July 1964. The report contains a summary of all work accomplished under the task, and this work was an outgrowth of research reported in Technical Report No. 4 on Contract Task Nonr-991(07) and dated 3 May 1963. That report, which was prepared by M. W. Long and J. C. Butterworth, was entitled "New Technique for Microwave Radiometry," and was subsequently published in the September 1963 issue of the IEEE Transactions on Microwave Theory and Techniques.

An interference modulation technique for microwave radiometry is described. The technique is considered for use in the development of a frequency selective power meter for operation over a wide range of frequencies. A device using this technique does not require the use of a coherent source, and the technique is applicable to wavelengths well into the low millimeter region.

A tunable radiometer using interference modulation has been operated as a frequency selective power meter in the 4-mm region. As a source of microwaves for system calibration, a gaseous noise source having a known temperature was used. Measurements were made to determine the power output level from a crystal harmonic generator, and the 4-mm harmonic was also used with the system when operated with a frequency resolution between half power points of 8% to investigate system selectivity and spurious responses.
II. INTRODUCTION

A measurements device based on an interference technique has been operated as a continuously tunable, frequency selective power meter capable of detecting the output of weak coherent sources. A previous report\(^1\) described measurements using the interference technique to determine detector sensitivity, without a coherent microwave source, as a function of frequency.

The heart of the device is, in essence, a continuously tunable microwave band-pass filter. Filtering is accomplished through the use of microwave interference and filtering in the audio range. The device has been operated throughout the 50-90 gc region where 4-mm components could be used, but the technique should be useful throughout the low millimeter-wave region. Operation is based on the fact that there is a one-to-one correspondence between mechanical speed and the frequency of Doppler components. Power from a microwave source is divided between two paths by a waveguide T and is recombined with a T before reaching the detector. Doppler components are produced by varying one of the paths with a trombone-shaped section which is moved back and forth by the drive rod at a nearly constant speed. Thus, an interference modulation frequency is produced which is uniquely related to the wavelength within the waveguide. The audio frequency modulation spectrum at the detector for a wide band microwave source will be continuous; thus, a tuned audio amplifier (or other frequency analyzer) in conjunction with the varying path length will serve as a band-pass filter.

Since the magnitude of the peaks in the amplifier output caused by a coherent source is proportional to source power, comparison of peak output with that caused by a calibrated noise source provides a convenient method for determining the power level of a weak coherent source. In principle, power measuring devices can be calibrated as a function of frequency with a modulated noise source and a tunable microwave band-pass filter. Such measurements are not currently possible in the low millimeter region because band-pass filters with variable but known center frequency and effective bandwidth are not available.

The power meter discussed herein is basically different from conventional power meters employing a barretter or thermistor in which the r-f power measurement is based upon the substitution of a d-c or audio power for r-f power. In that type of power meter there is an inherent calibration error caused by the uncertainty in the so-called "calibration factor" for the detector. Calibration factor is defined as the ratio of substituted d-c power in the detector element to the microwave power incident upon the detector; thus, the calibration factor is often an unknown "factor" which in general depends on frequency, even within a given waveguide band. For the technique presently under investigation, microwave power from the noise source is compared with the unknown coherent microwave power. The noise source temperature can be ultimately determined with a thermometer and thus noise source power is determinable.
III. PRINCIPLES OF OPERATION

System selectivity is obtained because there is a one-to-one correspondence between mechanical speed and the frequency of Doppler components. Assume that the difference in path length between Paths 1 and 2 of Figure 1 is varied at a constant speed \( s \); then an interference modulation frequency \( f_a \) would be produced which is related to the wavelength within the waveguide \( \lambda_g \) by the equation

\[
f_a = \frac{s}{\lambda_g} \tag{1}
\]

Thus, the tuned audio amplifier in conjunction with the varying path length will serve as a microwave band-pass filter because the amplifier output will depend on those frequency components corresponding to \( \lambda_g \) of Equation 1.

Assume that power from the microwave input signal of Figure 1 is divided equally between Paths 1 and 2, and is recombined before reaching the detector. Let the difference in lengths of Paths 1 and 2 be designated \( \delta \). Then for any single microwave frequency, power reaching the detector, \( P_d \), expressed in terms of input power, \( P_i \), is

\[
P_d = \frac{P_i}{2} \left[ 1 + \cos \frac{2\pi \delta}{\lambda_g} \right], \tag{2}
\]

where \( \lambda_g \) is the guide wavelength corresponding to the microwave frequency \( F \).

For an air-filled waveguide, guide wavelength and frequency are related by the well-known equation

\[
\lambda_g = \frac{c/F}{\sqrt{1 - \left(\frac{c/F}{\lambda_c}\right)^2}}, \tag{3}
\]
Figure 1. Equivalent circuit of interference modulation device.
where \( c \) is free space propagation velocity and \( \lambda_c \) is guide cutoff wavelength. Notice that the detector power is equal to the total source power when the path difference is zero, and that it changes periodically with path length.

Assume that the difference in lengths of Paths 1 and 2 is varied in a uniform and continuous manner. From Equation 2, the power incident upon the detector with a coherent microwave input signal will consist of a constant term plus a sinusoidal term. When operated at low power levels (square-law region), detector output voltage \( V_d \) is related to incident microwave power \( P_d \) by

\[
V_d = RP_d.
\]  

(4)

For a given detector the responsivity \( R \) is approximated by a constant. Thus, the voltage out of an amplifier tuned to the Doppler frequency (Equation 1) will be a sinusoidal wave having a maximum at the time the path length difference is zero. Let \( t = 0 \) correspond to the time at which the path length difference is zero. Then, from Equations 1, 2, and 4, a coherent input signal will cause a carrier voltage at the detector output that may be expressed as

\[
v = \frac{P_d R}{2} \cos \omega_a t = E_o \cos \omega_a t,
\]  

(5)

where

\[
\omega_a = \frac{2\pi s}{\lambda_g}.
\]

In order to visualize amplifier waveform, now assume that the difference in lengths of Paths 1 and 2 is varied by sequentially lengthening one of the paths, by stopping the motion during the reversal period, and by shortening
one of the paths. The starting and stopping action limits system resolution by square-wave modulating a Doppler "carrier" of frequency $s/\lambda_g$. Thus, the voltage waveform out of a tuned amplifier following the detector will contain pulses occurring periodically.

A. Detection of a Coherent Source

1. Interpulse Period and Amplifier Bandwidth

From Equation 5, uniform motion of the trombone will modulate a coherent microwave so that the output voltage of a detector is a constant audio signal of frequency $f_a$ which equals $s/\lambda_g$. Therefore, if trombone motion persists for a time period $t_o$, the detector output voltage will be a pulse of length $t_o$ and frequency $s/\lambda_g$. Assume that the interpulse period is equal to the pulse length as illustrated in Figure 2; this allows the same time for the envelope of the audio amplifier output to build up as to decay. Under the conditions described above

$t_o$ is the time for which uniform trombone motion of speed, $s$, exists, and $T$ is equal to $2t_o$ and is the period of the square-wave modulated audio frequency signal to be amplified.

There are relatively stringent requirements on the amplifier passband characteristics to assure that transient responses for the coherent source and for the noise source will not cause serious calibration errors. As an example regarding the effects of bandwidth on pulse interference, assume that the amplifier passband corresponds to that of a simple R-L-C tuned circuit. As a compromise between quality of pulse reproduction and signal-to-noise ratio, let the value of $B_a$ be between $1/t_o$ and $2/t_o$. The time constant for the exponential curves forming the edges of the output-pulse envelope is
Figure 2. Pulse response of audio amplifier.
related\(^2\) to 3-db bandwidth by

\[ \tau = \frac{1}{\frac{1}{\pi B_a}}. \]  

(6)

Thus, for a bandwidth between \(1/\tau_0\) and \(2/\tau_0\), there will be between three and six time constants within the dead time, which would usually result in negligible pulse interference. A word of caution: Care must be exercised to assure no pulse interference because effects of pulse-to-pulse interference might not be negligible for bandwidths close to \(1/\tau_0\).

Irregularities in phase and amplitude characteristics that are associated with some multituned amplifiers might cause deleterious calibration errors. The objective is to provide an amplifier for which the value of the peak output caused by a coherent input source of a given power will be close to that caused by a noise source of equal power (contained within a determinable bandwidth). One requirement for the amplifier is that the peak output voltage for a pulsed audio carrier (see Figure 2) must closely approximate the peak for the carrier when unmodulated. This necessitates that the amplifier build up to a peak without overshoot. Another requirement is that the time delay for each frequency within the passband be the same (linear phase versus frequency characteristics).

The phase and amplitude characteristics are well known for audio

amplifiers using a single R-L-C circuit$^3$ or a twin-T network$^4$ in a degenerative feedback loop. For these amplifiers the phase characteristics are linear over the center of the passband, and the transient responses to coherent input signals and noise source inputs are expected to result in only negligible calibration errors. Requirements on amplifier characteristics are also discussed in Section A3 and B1 of this chapter and Section C of Chapter IV.

2. Frequency Spectrum at Output of Microwave Detector

In accordance with Equation 5, let $E_0 \cos \omega t$ represent detector output that would exist for uniform and continuous changes in path length and a coherent microwave input signal. The frequency spectrum of a square-wave modulated detector output can be obtained from well-known Fourier series for video pulses. Regard the audio frequency output pulse as a product of square video pulses (Figure 3A) and the cosine wave which would be generated for continuous motion (Figure 3B). Multiplying instantaneous amplitudes of the waveforms of Figures 3A and 3B yields a cosine wave of peak value $E_0$ during the pulse interval and zero in the interval between pulses.

The relationships between the parameters of the video pulse of Figure 3A and the frequency spectrum of Figure 3C are well known$^5$ and are illustrated in Figure 3D. The pulse shape determines the shape of the envelope curve. For a rectangular pulse, the envelope always has the form of $\sin \pi t / t$ for which

$^3$See page 352 of reference 2.


$^5$See, for example, Chapter VI of reference 2.
Figure 3. Fourier components of carrier-frequency pulse.
the spacing of the zeros is $1/t_0$. The repetition frequency determines the spacing (which is always $1/T$) representing the Fourier components but has no effect on the envelope curve drawn through the ends of the lines.

In practice, the modulation function will not be a perfectly periodic rectangular train of waves (the wave train will have jitter), because of mechanical differences between the outgoing and incoming strokes.

With good mechanical practices, it is expected that the spectrum for the sweeps to the right would be virtually periodic; similarly, the spectrum for sweeps to the left would be periodic and like the spectrum for sweeps to the right. However, some jitter still might be introduced by slight differences in registration for the beginning of each sweep and for slight mechanical imperfections along the sweep. Substantial interference between sweeps will be caused by small differences (fractions of a guide wavelength) in the mechanical lengths from the electrical center (zero path length difference) to the terminal points because this will vary the amount of constructive or destructive interference in the Fourier components in the two sets of spectra. For the work reported herein, no attempt has been made to provide registration of the terminal points with respect to the zero path length center; instead, it has been assumed that the systems under discussion are those for which the amplifier output signal decays to a sufficiently small value that there is in essence no sweep-to-sweep interference. With this point of view, it is not necessary to become involved with the many details of the individual spectral

---

components under the envelope. Therefore, the analysis is based on system response to independent pulses, the spectrum envelopes for which are identical to that for the perfectly idealized periodic case.

For the interference power meter, microwave frequency selection is accomplished by filtering in the audio range. Therefore, the sidebands generated by discontinuous motion cause an uncertainty in frequency, that is, a loss in system frequency resolution. Since detector output voltage is proportional to microwave power, the $\sin X/X$ curve of Figure 3D should be interpreted as a power spectrum. The frequency interval $1/t_o$ is used throughout this report as being representative of pulse spectrum width; note from Figure 3D that this is roughly the frequency separation between points having half the power level of the major lobe crest (carrier wave).

It has been found (see Section A3 of this chapter) that the amplifier passband will adequately shape the spectrum for the purposes of this investigation. However, in certain cases it might be desirable to reduce the lobes of the upper and lower sidebands to improve the frequency rejection properties of an interference power meter. The edges of the amplitude function corresponding to Figure 3A could be tapered by controlling amplifier gain with a potentiometer that controls gain as a function of trombone position (path length). Effects of pulse shaping and ultimate improvements to be expected in the frequency spectrum of the detector output are discussed in Appendix B.

3. System Resolution and Length of Trombone Travel

System resolution is controlled by amplifier bandwidth and width of the pulse spectrum of the microwave detector output. From Figure 3D it may be seen that the spectrum width for a pulsed carrier is of the order of the
reciprocal of the length of the pulse; therefore, in no case can resolution at 
the audio amplifier be less than 1/\( t_0 \). Because of this, minimum system resolu-
tion may be expressed in terms of overall path length change, \( L \). Since \( L \) is 
equal to \( s/t_0 \), minimum attainable fractional resolution, \( \mathcal{R}_{\text{min}} \), may be expressed as

\[
\mathcal{R}_{\text{min}} = \frac{1/\( t_0 \)}{f_a} = \frac{s/L}{s/\lambda g} = \frac{\lambda g}{L} .
\]

Thus, fractional system resolution will always be limited so as to exceed the 
reciprocal of path length change expressed in terms of guide wavelength. On 
the other hand, if amplifier bandwidth \( B_a \) is much larger than the spectrum of 
the pulse generated by trombone motion (approximately \( 1/\( t_0 \) \)), then system 
resolution \( \mathcal{R} \) can be expressed in terms of amplifier center frequency as

\( B_a/f_a \).

From the above, it is clear that resolution will be nearly equal to the 
wider bandwidth (amplifier bandwidth or pulse spectrum) if one of the band-
widths greatly exceeds the other. In order to make efficient use of time, and 
thus in principle approach optimum signal-to-noise ratio for a given observa-
tion time, amplifier bandwidth\(^7\) should be between \( 1/\( t_0 \) \) and \( 2/\( t_0 \) \). Thus, it is 
desirable to operate with parameters such that the two pertinent bandwidths are 
of comparable magnitude. The fact that effective bandwidth is expected to ex-
ceed the largest of the two pertinent bandwidths (\( B_a \) and \( 1/\( t_0 \) \)), and to

\(^7\)For a discussion of dependence of signal-to-noise ratio on amplifier band-
width, see J. L. Lawson and G. E. Uhlenbeck, "Threshold Signals," McGraw-
Hill Book Company, Inc., New York, p. 201; 1950. Also, to avoid problems 
with pulse-to-pulse interference, the bandwidth should not be less than \( 1/\( t_0 \) \).
approximately equal to the largest bandwidth providing it is much greater than
the smaller one, suggests that system resolution might correspond roughly to
the square root of the sums of the squares of \( B_a \) and \( 1/t_o \).

The data shown in Figure 4 illustrate the manner in which the resultant
bandwidth is affected by the combination of amplifier bandwidth and spectrum
bandwidth. The measurements were made by reading the peak output signal on a
General Radio Type 1232A tuned amplifier and null detector; this meter reading
is proportional to the audio amplifier output voltage. The pulsed audio
signals were obtained by pulsing a Hewlett-Packard Type 200J oscillator for
the various output frequencies shown. It may be seen from the measured c-w
response that the amplifier, which is reported to have a nominal bandwidth of
5%, when tuned to 22 cycles has a bandwidth between 0.707 points of approxi-
mately 0.8 cps. In reading the meter it was planned that the overshoot that
occurs for each pulse would be ignored, and the meter would be read for each
pulse after the meter had settled down. For the pulse lengths of 1 second
and 3/4 second, the combination of amplifier build-up time and meter damping
were such that only a peak (probably meter overshoot) was observed; therefore,
Figure 4 is of value only in a qualitative sense.

From Figure 4 it can be seen that for \( B_a t_o \), sufficiently large, the am-
plifier meter can be read to give a signal whose amplitude is approximately
that for c-w operation. Therefore, we see that for an amplifier bandwidth
slightly in excess of the reciprocal of pulse length, the system bandwidth is
sufficiently wide to give us almost complete signal build-up for each pulse.
In addition, it should be noted that there is only a small difference between
the effective bandwidths for c-w and for the 1-1/2 second pulse.

For the purposes of this report, system resolution will be defined as
Figure 4. Peak output response of audio amplifier.
that bandwidth between frequencies for which amplifier peak voltage has dropped to one-half the maximum value. This fractional system resolution will be represented by the symbol $R_{1/2}$; since peak output voltage is proportional to microwave power, $R_{1/2}$ corresponds to a half power microwave resolution. This resolution roughly corresponds to the minimum frequency separation for which it is possible to discern through the use of this instrument the existence of two signals of equal magnitude. It should be noted from the figure that $R_{1/2}$ is approximately 50 per cent larger than the 0.707 bandwidth. Therefore, for values of $B$ between 1 and 2, a reasonable estimate is that half power resolution $R_{1/2}$ will fall between the limits

$$2 \frac{\lambda g}{L} \leq R_{1/2} \leq 3 \frac{\lambda g}{L}.$$  

(8)

B. Detection of a Noise Source

1. Amplifier Output for Noise Source

Let $t$ equal to zero correspond to the time at which the path length difference is zero. Then, from Equation 5, a coherent microwave input signal will cause a carrier of angular frequency $\omega$ at the detector output that may be expressed as

$$v = \frac{P R}{2} \cos\omega t,$$

(9)

where

$$\omega = 2\pi f = \frac{2\pi s}{\lambda g} = \frac{2\pi s}{(\lambda g/\lambda_c)c} F.$$

Similarly, for several sources of equal magnitude
where $P_1$ is now the sum of the powers of the individual coherent sources. Thus, it is seen that at $t = 0$, the carrier voltage caused by several sources of total power $P_1$ is equal to the voltage which would be caused by a single source of power $P_1$. The output voltage caused by a noise source can be thought of as being generated by an infinity of infinitesimal sources within a microwave band. Since passbands of interest are narrow, it is reasonable to assume that the ratio $\lambda_g/\lambda$, detector responsivity $R$, and noise temperature are constant. The assumption that $\lambda_g/\lambda$ is constant implies (see expression for $\omega$ in Equation 9) that the ratio of microwave frequency $F$ to audio frequency $f$ is also constant.

Assume that there is an equal time delay for each frequency within the amplifier passband. Then, Equation 10 can be used to determine the peak voltage $V_o$ which exists at $t = 0$ (neglecting constant amplifier delay time) for a noise source. Assume that the complete microwave spectrum is covered by $N$ narrow-band noise sources which do not overlap in frequency. Designate the incremental noise bandwidth of each fictitious source by the symbol $\delta F$; this width may be expressed in terms of the corresponding incremental audio width, $\delta f$, as $(F/f)\delta f$. Let $G$ represent audio amplifier gain, then

$$V_o = \frac{kTR}{2} \sum_{n=1}^{N} \left[ \sum_{n=1}^{N} \cos \omega_n t \right] ,$$

which, in the limit as $\delta F$ approaches zero, becomes
\[ V_o = \frac{kTR}{2} \left( \frac{F}{f} \right) \int_0^\infty G(f)df \]  

(11)

It is well known\(^8\) that \( \int_0^\infty G(f)df \) is equal to the product \( G_{\text{max}} B_N \), where \( B_N \) is the noise bandwidth of an amplifier and \( G_{\text{max}} \) is the maximum of the gain versus frequency characteristic.

The bandwidth \( B_N \) of an amplifier can be measured but it is usually more convenient to measure the frequency interval between points at which the amplifier output voltage is 0.707 times its maximum value; this half power amplifier bandwidth is denoted by \( B_a \). The difference between \( B_a \) and \( B_N \) depends on the shape of the response curve, but the values\(^9\) for the two types of bandwidth are usually nearly alike.

By comparing Equations 11 and 9 it may be seen that the peak output voltage caused by a noise source is proportional to noise power and the peak output is equal to that caused by a coherent source of power \( kT(F/f)B_N \). For most passband characteristics, the half power amplifier bandwidth \( B_a \) is a good approximation for \( B_N \).

The requirements for a real amplifier to give a transient response that will permit a power meter to be calibrated on the basis of peak reading has been outlined in Section A1. An idealized amplifier with a rectangular passband of width \( B_a \) is now assumed in order to simplify further analysis. Also assume that there is equal time delay for each frequency within the passband.

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\(^8\) See page 101 of reference 7.

\(^9\) See page 177 of reference 7.
Then, the voltage out of a unity gain amplifier near the sweep center can be expressed as

\[ v = \frac{kT(F/f)B_a}{2} R \left( \sin \frac{B_a}{2} t \right) \cos \omega_a t \]  

(12)

where \( \omega_a = \frac{2\pi s}{\lambda_g} \). Here \( \lambda_g \) represents the guide wavelength corresponding to the band center.

Near each envelope peak where values of \( t \) are sufficiently small (by definition) that

\[ \sin \left( \frac{B_a}{2} t \right) \] can be approximated by \( \left( \frac{B_a}{2} t \right) \),

amplifier output voltage may be expressed as

\[ v = \frac{kT(B_a/f)R}{2} \cos \omega_a t \]  

(13)

Notice that Equation 13 indicates that the magnitude of the peaks in amplifier waveform will be directly proportional to the power \( kT(B_a/f) \). This output voltage agrees with that indicated by Equation 11 if \( B_N \) is set equal to \( B_a \).

---

2. System Resolution and Useful Source Power Available

The magnitude of the peaks in the amplifier waveform is directly proportional to the power contained in a microwave band of width \( F(B_a/f) \). This is because the detector output spectrum will be uniformly distributed in frequency independent of the overall path length change. Thus, the bandwidth that should be used for system calibration is \( F(B_a/f) \) where \( B_a \) represents amplifier noise bandwidth.

The overall path length change does, however, control system resolution by pulse modulating (see Equation 7) all of the Doppler signals of the microwave noise source spectrum. The resulting resolution is a complicated function of amplifier bandwidth and overall change in path length. Considered below is the case for which amplifier bandwidth is equal to \( 2/t_o \) or greater. In this case system resolution will be controlled almost entirely by amplifier bandwidth.

Let \( f_1 \) and \( f_2 \) be the lower and upper frequency limits, respectively, of an idealized rectangular audio passband \( B_a \). Then the lower and upper microwave frequency limits to which the amplifier responds, \( F_1 \) and \( F_2 \), may be expressed as

\[
F_2 \left[ \frac{\lambda_2}{\lambda_{g2}} - \frac{\lambda_1}{\lambda_{g1}} \right] \frac{s}{c} = f_2 - f_1
\]

where \( \lambda_1 \) and \( \lambda_2 \) are free space wavelengths for the frequencies \( F_1 \) and \( F_2 \),

\( \lambda_{g1} \) and \( \lambda_{g2} \) are guide wavelengths for the frequencies \( F_1 \) and \( F_2 \),

\( s \) is the rate of change in path length, and

\( c \) is the free space propagation velocity.

Let \( F \) represent the center of the microwave spectrum, for which the Doppler
frequency equals amplifier center frequency \( f_a \), and let \( B \) and \( B_a \) represent
the microwave and audio bandwidths, respectively. Since the passbands of
interest are narrow, \( \lambda_1/\lambda_2 \) is assumed equal to \( \lambda_2/\lambda_2 \). With this assumption, it is seen that the relation

\[
\frac{B}{F} = \frac{B_a}{f_a}
\]  

(14)

is a good approximation for narrow passbands. In other words, the percentage microwave bandwidth resolved by the system when using a noise source is equal to the percentage audio bandwidth.

Detector output voltage with a coherent source can be expressed (Equation 5) as \( P_1(R/2)\cos \omega_t \). From Equation 13 it may be seen that detector power from an incoherent source near the interval of maximum output is \( kTFB_a(R/2f)\cos \omega_t \). Noise power from a matched microwave load can be determined from the approximation 11

\[
P = kTB,
\]  

(15)

where \( k \) is Boltzmann's constant \((1.38 \times 10^{-23} \text{ Joules per degree Kelvin})\), \( T \) is the temperature of the load in degrees Kelvin, and \( P \) is the power in watts available in the bandwidth \( B \) in cycles per second. Thus, equivalent source power for the case of an incoherent source is equal to the total noise power,

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11 This approximation is usually valid for the millimeter region; it causes only a small error in calculations for the submillimeter region providing low temperatures are not involved, and it is not usually valid for infrared. For example, see G. R. Nicoll, "The Measurement of Thermal and Similar Radiations at Millimeter Wavelengths," Proceedings of the Institute of Electrical Engineers, vol. 104, pp. 519-527; September, 1957.
kT\frac{B_a}{f_a}, contained within the resolution of the system. Assume that an audio amplifier having a noise bandwidth of 5% is used. Let the change in path length and audio amplifier center frequency be such that the system responds to a band centered at 70 gc; then from Equations 14 and 15 we find that the system responds to a microwave band \( B \) as follows:

\[
B = \frac{B_a}{f_a} F = 3.5 \, \text{gc}
\]

Under these conditions and with the noise source employed (\( 14,500^\circ \text{K} \)), the equivalent power \( kT_B \) is \( 7 \times 10^{-10} \) watt. In comparison with power levels for coherent sources, the equivalent power is small but useful if a sensitive detector is employed.
IV. EXPERIMENTAL CONSIDERATIONS

A. Experimental System

Figure 1 illustrates the microwave circuit used for measuring the power of low level coherent sources in the 4-mm wavelength region. A trombone device provides the variable path length required; it and other major system elements may be seen in Figure 5. The power is divided by waveguide T's, and the detector shown is a PRD-634 barretter mounted in an evacuated PRD-632 holder. The calibrated source is a commercially available 4-mm noise source.

The mechanism that drives the waveguide trombone back and forth may also be seen in Figure 5. With this drive mechanism, tuning to a desired microwave frequency can be accomplished without changing the period. This is done by adjusting the length of the travel and thus changing the speed of path length change (see Appendix A). Waves from a coherent source, when passed through the trombone device, are modulated sinusoidally except when trombone motion is reversed at the end of each stroke. To avoid prohibitive interference caused by the discontinuity in waveform at the end of each stroke, an adjustable dead time is introduced so that the audio frequency Doppler wave can decay before the trombone cycle is reversed.

The power dividers are E-plane T's in which the output branches near the

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12 For a discussion on improvements in sensitivity that results from operating the barretter in an evacuated atmosphere, see reference 1.

13 Noise source Model GNW-V18, for which the noise temperature was given as 14,500°K ± 1800°K in private communications with R. White, Roger White Electron Devices, Inc.
Figure 5. Trombone version of interference device.
junction are half the height of the input branch. The output branches are gradually tapered up to the size of RG 98/U guide. The movable section of the trombone is constructed of round tubing with an inside dimension of 0.189 inch. Tubing was used because of the availability of slide fit telescoping brass tubing and the relative ease of fabricating a bend. Operation of the system could be limited by the presence of higher order modes that might be generated within the bend. For this reason, a rectangular trombone was constructed. The movable section was fabricated of RG 98/U waveguide, and it slides over rectangular waveguide fabricated of 0.003-inch brass stock. No discernible differences were observed between the system performance obtained with the circular and rectangular trombone sections.

1. Wide Bandwidth Measurements

In the initial effort to acquire a feel for the requirements in making low level power measurements, the absolute power level was determined for the 70-gc output of a crystal harmonic generator driven by a QK 288 klystron operating at 35 gc. The harmonic generator was of a conventional cross-guide type consisting of RG 96/U and RG 98/U waveguide and a modified type LN53 crystal diode. The crystal diode was driven so that crystal current was between 0.5 ma and 1.0 ma. The microwave detector used was a PRD-634 barretter mounted in an evacuated PRD-632 holder. The trombone was driven back and forth at a nearly constant speed except for the reversal time. The period of the mechanical motion was 2 seconds, i.e., pulses occurred at the rate of 1 per second. With this drive mechanism, tuning to a desired microwave frequency of observation was accomplished by adjusting the length of the travel of the trombone without changing the period.
The trombone was operated so that the Doppler frequency for 70 gc was 31 cps, and the trombone transit time and dead times were approximately 1/2 second; therefore, the fractional spectrum width (see Section A2, Chapter III) was 2/31. The audio amplifier bandwidth used was 6 cps to permit the discontinuity in the Doppler component of the coherent wave to decay prior to re-initiation of trombone motion. The 6-cps bandwidth in conjunction with a center frequency of 31 cps results in a fractional amplifier bandwidth of 6/31 or 19%. Since the pulse spectrum caused by trombone motion is considerably narrower than this, the approximate microwave resolution would be 19% or 13.5 gc.

The standard of reference for these measurements was a Roger White noise source. For the system noise bandwidth of 13.5 gc, equivalent noise source power is $2.7 \times 10^{-9}$ watt. Output level with noise source and with coherent generator were compared by displaying audio amplifier output on a Brush paper tape recorder. The peak output level caused by the coherent source was set equal to the peak output level caused by the noise source by attenuating the coherent signal 14.5 db with a FXR model M164A microwave attenuator. Thus, the interference technique was used to measure the output level of a very weak (76 millimicrowatts) coherent source which was isolated from the power meter by 14 db of attenuation.

2. Narrow Bandwidth Measurements

An experiment on system resolution was designed around the amplifier and output meter in the General Radio Type 1232A tuned amplifier and null detector because this amplifier is tunable and has a relatively narrow bandwidth. The manufacturer's specifications indicate that the nominal bandwidth
is 5% throughout the tuning range. As a compromise between signal-to-noise ratio and system resolution, a trombone displacement of 3 inches (6-inch path length change) was selected for operation at 70 gc where the trombone guide wavelength is 0.2 inch. Equation 7 indicates that the spectrum width will be 3.3%.

The drive mechanism was operated with a period of 6 seconds and the trombone transit time was made equal to the trombone dead time. For this mode of operation, a 1-1/2 second pulsed, 23-cps carrier occurs at the amplifier input once every 3 seconds. Figure 4 shows the results of measuring the frequency response of the amplifier only with a 1-1/2 second input pulse.

To determine system resolution, the 70-gc output from the harmonic generator described in the previous section was measured with a Type 1N53 diode detector tuned to 70 gc. The diode detector was used because at the particular time that these measurements were made a good 4-mm barreteter was not available within the laboratory. Microwave attenuators were used at the input T and at the output T to minimize possible effects of reflections external to the interference modulator. Figure 6 shows the peak amplifier output meter reading as a function of amplifier center frequency between 20 cps, the lowest frequency to which the amplifier tunes, and 40 cps. As discussed in the next section, spurious responses are generated by internal microwave reflections in audio frequency bands for which the band edges differ by a factor of 2 or more. In terms of microwave frequency, the audio frequencies of 20 cps and 40 cps correspond, respectively, to 64 gc and a frequency slightly in excess of 100 gc.

Figure 6 indicates that the 1/2 voltage points were separated by approximately 1.8 cps. Thus, in terms of the microwaves, the 1/2-power points
Figure 6. Amplifier output for 70-gc input signal to system.
(amplifier voltage equal to \(1/2\) maximum value because for a square-law detector output voltage is directly proportional to microwave power) are separated by approximately 8% of the center frequency. The 1.8 cps half-voltage width is consistent with Equation 8 and indicates that the half-power system resolution \(\sqrt{\frac{R}{2}}\) is 2.4 \(\lambda_g/L\).

There is not a contradiction in the fact that the diode detector bandwidth, which is too narrow for general usefulness with a power meter, is considerably less than 8%. The detector was tuned to 70 gc, the only microwave frequency present within the system. The finite resolution is caused by the ever present sidebands generated in the audio range and represents the frequencies to which a complete system that employs a broadband detector, such as a barretter, will respond.

3. Spurious Responses

If the amplifier bandwidth were narrow compared with the pulse spectrum width \(\left(B_a \ll \frac{1}{T_o}\right)\), the amplifier output as a function of center frequency would reproduce the envelope of the pulse spectrum. Thus, there would be numerous peaks observable for a single microwave oscillator and signals from several microwave sources would therefore be confused and indistinguishable. This effect is somewhat similar to the appearance on a scanning radar display of false targets caused by antenna side lobes. For the system described herein, the amplifier bandwidth is wider than the lobes of the pulse spectrum envelope; therefore, spectrum envelope lobes (false targets) are not observable when the amplifier center frequency is moved across the pulse spectrum. Figure 7 shows the response of a General Radio Type 1232A tuned amplifier and null detector to pulsed, audio waves. The audio signals were
Figure 7  Wide-band response of amplifier and spurious signals produced by interference device.
obtained from a Hewlett-Packard Type 200J audio oscillator which was tuned to 22.5 cps and modulated with rectangular pulses occurring once every 6 seconds. Note that the response approaches asymptotically a small value (approximately 2% for Figure 7) for the higher frequencies; this was observable for frequencies up to approximately 100 cps. No differences were observed in the tail above 25 cps for the modulations used: c-w, 1-1/2 second pulses, and 3/4 second pulses. In this experiment the pulse envelopes were sufficiently "washed out" that false signals did not appear.

Spurious responses can also be generated by reflections from microwave components. The reflections at the power dividers as seen from Paths 1 and 2 of Figure 1 will cause audio signals to be generated that are harmonics of the carrier frequency. Assume that Path 1 of Figure 1 is the variable path length. The wave that is reflected so that it traverses Path 1 three times causes a second order modulation at three times the carrier frequency. A wave within Path 1 that is reflected to the left off the power divider at the right, and propagates through the power divider at the left, will produce a modulation at twice the carrier frequency when combined (by traversing Path 2) with the waves in Path 2. Since these harmonics are far removed from the carrier, they are expected to be troublesome only in special cases.

Spurious responses caused by reflections within the interference modulator were measured. The operating conditions used were the same as those described in Section A2 of this chapter. Measured output as a function of amplifier center frequency corresponded to that obtained with the pulsed audio oscillator, except for the additional prominences represented by the dashed lines in Figure 7. Note that the microwave test showed additional output clustered in groups centered near audio frequencies equal to twice and three
times the frequency of the dominant peak caused by the 70-gc signal. The broadness of the two clusters is not surprising because amplifier bandwidth is larger for the higher frequencies (percentage bandwidth for the General Radio amplifier is independent of frequency). The cause of the individual peaks within the cluster has not been explained.

A check was made to determine if the spurious signals were caused by higher order harmonics from the harmonic generator. To accomplish this a double tapered waveguide, necked down so that the calculated cutoff frequency is 75 gc, was used as a high-pass filter. No signals were received when the filter was inserted which indicates all output was caused by frequencies below 75 gc.

B. Sensitivity and Observation Time

Let minimum detectable change in power $\Delta P_{\text{min}}$ for a detector be defined as that change in signal power for which the rms value of signal voltage out of the tuned amplifier is equal to the rms value of output noise. For a detector operating in the square-law region, minimum detectable power $\Delta P_{\text{min}}$ is equal to $K \sqrt{B_a}$, where $K$ is the minimum detectable change in power referred to a 1-cps audio bandwidth and $B_a$ is the audio bandwidth. For most bolometers and modulation waveforms, $K$ is of the order of $10^{-9}$ or $10^{-10}$ watt. For the modulation waveform used herein (Equation 2), the change in microwave power incident upon the detector is equal to source power. Therefore, minimum detectable power $P_{\text{min}}$ can be expressed as

$$P_{\text{min}} = K \sqrt{B_a}.$$ (16)
Since the power output is assumed to be indicated by a peak reading meter which follows the audio amplifier, the minimum observation time is equal to the pulse-to-pulse period, $2t_0$, because the maximum signal (assuming signal much greater than noise) occurs only once during each stroke. In practice, observation time can exceed one period because several peaks can be averaged and taken as one observation -- this would improve measurement accuracy by the square root of the number of peaks used for one observation.

C. Calibration Errors

1. General

The interference modulation technique provides a very sensitive method for measuring power; the ultimate limit of minimum detectable power for the system is set by noise and is about $10^{-10}$ watt for an evacuated barretter\(^{14}\) used with a 1-cps audio bandwidth. This power level is several orders of magnitude smaller than can be measured when barretters or thermistors are used as one arm of a bridge in a conventional power meter.

Since the magnitude of the peaks in the amplifier output caused by a noise source will be proportional to the peaks caused by a comparable power level from a coherent source, comparison of peak output with that caused by a calibrated noise source provides a method for determining the power level of a weak unknown coherent (or incoherent) source. A basic limitation of this method of calibration is that one must have adequate sensitivity to detect a microwave noise source for which the available noise power is generally small. In certain circumstances it may be desirable to improve sensitivity by

\(^{14}\) See page 394 of reference 1.
reducing noise bandwidth without changing system resolution by adding integra-
tion after audio detection. If it is assumed that detector efficiencies are
adequately well known, it would be possible to use d-c or a-c calibration
(method used in conventional power meters), thus eliminating the necessity
for detecting a noise source.

Irregularities in the phase and amplitude characteristics that are as-
sociated with some multituded amplifiers will cause serious calibration er-
rors. For single-tuned amplifiers, the phase characteristics are linear over
the center of the passband and the transient responses to input signals are
expected to result in only negligible errors. The bandwidth to be used to
calculate equivalent microwave power for a noise source calibration is the
amplifier noise bandwidth (see Equation 11). For single-tuned amplifiers
this bandwidth is expected to closely approximate the conventional 3-db
amplifier bandwidth, i.e., the frequency separation between 0.707 voltage
points. In order to improve calibration in certain cases, it might be de-
sirable to calculate amplifier responses by a Fourier analysis using known
values of amplifier phase and amplitude characteristics.

Assume that the noise source temperature, noise bandwidth, and the at-
tenuation of calibrated attenuators are accurately known; further, assume
that the reading errors are negligible. The remaining major sources of error
include (1) uncertainty in calibration with a noise source because of output
meter movement caused by thermal noise of the amplifier-detector combination,
(2) uncertainty in reading the signal output caused by the amplifier-detector
combination, and (3) waveguide reflection losses. Ohmic waveguide losses are
relatively insensitive to frequency and therefore the error caused by a dif-
ference in loss seen by the band of useful frequencies available from the
noise source and the frequency from the coherent source is negligible. On the other hand care must be taken to account for reflection losses because the average reflection loss for a band of frequencies may differ from that for a single frequency.

2. Errors Caused by Amplifier Noise

Noise voltage out of the narrow-band amplifier is indistinguishable\textsuperscript{15} from the signal voltage except for the fact of the time of occurrence. Since the bandwidth of the amplifier controls the ring time, the relative phase of the noise voltage and signal voltage will be constant during a given signal peak. However, when all peaks are considered, the relative phase of noise and signal voltage will be random so that noise will either add to or subtract from the signal voltage. Because of this, the uncertainty in peak meter reading caused by noise depends to some extent on the relative magnitude of signal power and noise power.

Consider the case in which the signal voltage out of the audio amplifier is large compared to the rms (standard deviation) of amplifier noise voltage. In this case the rms value of signal plus noise will be twice that of the rms value of noise alone. Thus, the rms value of signal plus noise varies over a range differing by a factor of 2, i.e., the rms value with signal large compared to noise is twice that of noise alone.

For the usual power meter case, the calibration signal is larger than amplifier noise and the uncertainty (caused by amplifier noise) in signal is comparable to minimum detectable power of the system. This becomes clear when

\textsuperscript{15}For example, see Figure 4 of reference 1.
it is realized that minimum detectable power by definition is that power for
which the signal voltage is comparable to noise. Thus, minimum detectable
power, \( P_{\text{min}} \), is a reasonably accurate estimate of rms error due to uncertainty
caused by meter flicker.

Suppose the system were operated with no appreciable noise integration
except for that provided by the audio amplifier bandwidth and with the follow-
ing parameters:

\[
\begin{align*}
f_a &= 30 \text{ cps} \\
B_a &= 1.5 \text{ cps} \\
F &= 70 \text{ gc} \\
K &= 10^{-10} \text{ w/cps} \\
T &= 14,500^\circ\text{K}
\end{align*}
\]

Then from Equations 16 and 15 minimum detectable power would be \( 1.2 \times 10^{-10} \)
watt and \( kTB \) would be \( 7 \times 10^{-10} \) watt. The rms value of meter fluctuations
would be comparable to \( P_{\text{min}} \); therefore, the error in measuring \( kTB \) would be
0.8 db. Barretter detectors are broad-band devices and are, therefore,
relatively insensitive to microwave frequency. Suppose that the audio band-
width were increased during the time that the meter was being calibrated with
a noise source. If in so doing \( B_a \) were increased to 6 cps, \( kTB \) would be in-
creased by a factor of 4 and \( P_{\text{min}} \) would be increased by a factor of 2. For
these conditions the rms of meter fluctuations would be reduced so that the
meter calibration error would be approximately 0.4 db. If integration were
used in the meter circuit comparable to the original 1.5-cps audio bandwidth,
the increase in amplifier bandwidth by a factor of 4 would increase \( kTB \) by a
factor of 4 as before, but \( P_{\text{min}} \) would be unchanged. Therefore, in this case
rms meter fluctuation during calibration would be reduced by a factor of 4 and
the meter calibration error would be only 0.2 db.

Meter flicker will also exist when the unknown coherent power is read.
Since this flicker error is independent of the one discussed above, the ex-
pected error due to noise is the square root of the sum of the squares of the
two errors. Because of the low microwave noise power generally available, one
would expect that the error in reading the output caused by a coherent source
would normally be negligible compared with that for measuring the noise source.

As previously discussed, a basic limitation of this method of calibration
is that one must have adequate sensitivity to detect the microwave noise
source. It has been previously reported\textsuperscript{16} that the minimum detectable power
has been improved by 14 db by removing the air surrounding this detecting
element, and the sensitivities used above assumed that the barretter was
evacuated. Without this added sensitivity, the signal that would be produced
by the noise source would only be perceptible above amplifier noise for the
most sensitive set of conditions discussed above. Therefore, further trade-
off of system resolution and/or noise bandwidth would be required to get
adequate calibration sensitivity with a conventionally operated barretter.

The noise source calibration discussed above is basically different from
that used in conventional power meters in which the microwave power measure-
ment is based upon the substitution of a d-c or audio power for microwave
power. If this type of calibration were used, there would be an inherent
calibration error caused by the uncertainty in the so-called "calibration
factor" for the detector. Calibration factor is defined as the ratio of sub-
stituted power in the detector element to the microwave power incident upon
the detector; thus, calibration factor is often an unknown "factor" which in
general depends on frequency.

\textsuperscript{16} See reference 1.
D. Discussion

Figure 8 illustrates the microwave circuit of the first modulation device which was constructed and operated. The moving short circuit was driven back and forth in a waveguide so that the speed was essentially constant except during the reversal time. System performance was poor outside of the design region of the hybrid ring because the percentage modulation caused by the moving short was low; a somewhat wider bandwidth was later obtained by replacing the hybrid ring with a magic T. In order to obtain wideband performance, the trombone modulator shown in Figure 5 was developed.

Data from various manufacturers indicate that topwall and sidewall 3-db hybrid couplers have suitable broadband performance for use as an interference modulator (in conjunction with a moving short) over a complete waveguide band. It is expected that the electrical performance of these junctions would be sufficient to minimize the spurious responses observed during this investigation. Furthermore, use of such a junction would permit the development of a considerably more compact system. These junctions have simple configurations which can be readily electroformed and should, therefore, be easily fabricated for any of the waveguide bands within the millimeter wavelength region.

For some applications it would be desirable to operate over an extremely wide band of frequencies should a sensitive, wideband detector be available. A highly sensitive wideband detector that would be useful for laboratory measurements is currently in production by Mullard Ltd. It would seem that the

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Figure 8. Block diagram of interference modulation device as first constructed.
best type of power divider for a wideband system extending over much of the millimeter region would consist of junctions fabricated by using a thin conducting septum centered within a waveguide.

\[18\]

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\(18\) See Figure 8.4-1(a), George C. Southworth, "Principles and Applications of Waveguide Transmission," D. Van Nostrand Company, Inc., Princeton, New Jersey, p. 234; 1950.
V. CONCLUSIONS

A technique using interference modulation has been shown to be practicable for developing a highly sensitive, frequency selective power meter. The technique can also be used for measuring the sensitivity of detectors, allowing sensitivity measurements to be made without the use of a coherent source.

Theoretical and experimental investigations have shown that the interference modulation technique provides a very sensitive method for measuring power; the limit on minimum detectable power is set by detector noise and is about $10^{-10}$ watt if an evacuated barretter is used with a 1-cps audio bandwidth. This power level is several orders of magnitude smaller than can be measured when barretters or thermistors are used as one arm of a bridge in a conventional power meter.

Interference modulation was produced by varying the relative difference in two waveguide paths by sequentially lengthening one of the paths, by stopping its motion during the reversal period, and then by shortening the path. Frequency selectivity is obtained because there is a one-to-one correspondence between guide wavelength and frequency of the Doppler components generated by the changing path length. Thus, the effect of microwave filtering can be obtained through the use of interference modulation and filtering audio frequency components at the detector output. The nonuniform motion caused by the starting and stopping action associated with mechanically reversing the waveguide direction of travel limits system resolution by generating sidebands of the Doppler carrier. The ultimate limit of frequency resolution is controlled by overall path length change and practical limits are set by this path length change in combination with amplifier bandwidth. Microwave bandwidths of 5 to 10 per cent have been readily attainable.
Calibration is obtained by comparing power meter output caused by a microwave noise source with that caused by the coherent source of unknown power. The power of the noise source can be calculated by standard techniques because the system bandwidth can be readily determined.

The power meter is basically different from conventional power meters employing a barretter or thermistor in which the r-f power calibration is based on a comparison of the output caused by d-c or audio power. The meter can also be used in that mode of operation but there would then exist the usual inherent calibration error caused by uncertainty in detector efficiency. A basic limitation of using noise source calibration is that one must have adequate sensitivity to detect a noise source; however, sensitivities obtainable with response times suitable for laboratory devices are better than those obtainable by other techniques. The interference-type power meter has frequency selectivity that permits the determination of power output as a function of harmonic number from low level harmonic generators.
VI. ACKNOWLEDGMENTS

The author expresses his appreciation to E. R. Flynt, R. M. Goodman, Jr., and W. K. Rivers, Jr., for numerous profitable discussions regarding this work. A special note of appreciation is expressed for the diligent efforts of J. C. Butterworth, for without his dedicated pursuits the research reported herein could not have been accomplished.

Respectfully submitted:

M. W. Long, Chief
Electronics Division
VII. APPENDICES

A. Modulation Device

1. Drive Mechanism

The mechanism that drives the waveguide trombone back and forth may be seen in Figure 5. With this drive mechanism, tuning to a desired microwave frequency can be accomplished without changing the period by adjusting the length of the travel and thus altering the speed of path length change. The length of travel is continuously adjustable over the range of 1/2 to 6 inches; mechanical periods of 6, 4, and 2 seconds are available by appropriate combinations of pulleys. Since the path length changes by twice the trombone displacement, the speeds of path length change available with 6 inches of travel are 4, 6, and 12 inches per second. Figure 9 is included to facilitate the calculation of audio frequency produced by trombone motion as a function of microwave frequency.

A multi-groove cam cylinder with a flywheel attached is used as the basis of the traversing mechanism. The cam cylinder is driven at constant speed by a synchronous motor through a combination worm gear reductor and belt drive. By changing drive pulleys, the cam period can be varied so that the cam follower will be moved at a constant linear speed of 6, 3, and 2 inches per second. The cam follower, or shoe, engaging the cylindrical cam is constrained by a first traversing guide to reciprocate along a straight line parallel to the cam cylinder axis and is mounted in a slide which is attached to and parallel to a pivoting arm which reciprocates through a constant arc in a horizontal plane. The trombone device drive slide is constrained by a second traversing guide, located on the base plate under the pivot arm, to move along
Figure 9. Guide wavelength versus frequency.
a straight line parallel to the first traversing guide and is driven by an attached cam follower which rides in a groove in the bottom of the pivot arm. By varying the position of the second traversing guide in relation to the pivot arm, the stroke of the trombone device drive can be varied between 1/2 inch and 6 inches. The linear velocity of the trombone device drive slide is thereby varied proportionately.

The trombone is moved through its selected stroke, at the proportional speed of that stroke, by means of a thrust wire which passes through an extension of the trombone device drive slide. Two thrust collars are mounted on the thrust wire, one on each side of the slide extension. As the trombone drive slide moves forward through its stroke, the "push" thrust collar comes in contact with the slide extension and the trombone is pushed to its minimum path length position. The direction of movement of the trombone drive slide reverses and the slide extension moves back along the thrust wire until it contacts the "pull" thrust collar; the slide continues along its back stroke and pulls the trombone to its maximum path length position. The spacing between the two thrust collars is adjusted to obtain the desired dwell time in the trombone stroke cycle.

2. Power Dividers

The power dividers are E-plane T's in which the output branches near the junction are half the height of the input branch. The output branches were gradually tapered up to the size of RG 98/U guide. Figure 10 shows a scaled version of a T made of RG 91/U guide. In this scaled version all linear dimensions differ from the actual T by a factor of 4.2. Therefore, performance as a function of frequency for the RG 98/U dividers can
Figure 10. Scaled version of waveguide T.
be surmised from experimental data by multiplying the frequency axis by the factor 4.2. Thus, the data in Figure 11 for the frequency range of 11.5-19.0 gc should be indicative of RG 98/U divider performance in the 48-80 gc range. Measurements were made in the 11.5-19.0 gc region instead of the higher frequency region because of the availability of test equipment.

Figure 11 includes the maximum and minimum VSWR's obtained looking into Port A with Ports B and C terminated with sliding loads. Because of the very wide frequency range, two test setups were required. One consisted of X-band components and the other of K_u-band components. For the measurements in the X-band region, the T was coupled to the test setup by means of a waveguide taper from RG 91/U guide to RG 52/U guide. The sliding loads were designed for K_u-band, but reflections were adequately low even within the X-band region. For each load the maximum VSWR exists at the low frequency end of the band of interest. For one load the maximum VSWR was 1.07, and for the other the maximum VSWR was 1.03.

Measurements were also made on the difference in power division between Ports B and C, with Port A energized and with Ports B and C terminated. The measurements indicated an essentially random variation of power difference versus frequency. Over the wide frequency range of interest the power difference never exceeded 1/2 db, and the difference was less than 1/4 db for more than half the frequency range.

3. Transmission Characteristics

Because of the symmetry of the modulator arrangement with T's back to back, it is clear that for a zero phase difference there is complete transmission between the input port and the output port of the combined arrangement of the T's and the trombone. On the other hand, because of the
Figure 11. VSWR versus frequency of T.
symmetry it is clear that when the phase difference in the two paths is 180°, the transmission is zero (100% reflection). Since, in principle, there are no dissipative elements, the power that is not transmitted is reflected. Note that second order effects on modulation are caused by waves reflecting at each end of the variable path. The result of multiple reflections is a modulation at multiples of the fundamental frequency; effects of multiple reflections are discussed in the section entitled Spurious Responses.

B. Control of Pulse Spectrum

For the mode of operation described in this report, amplifier bandwidth used is sufficiently large to optimize signal-to-noise ratio. In this case amplifier bandwidth is comparable to spectrum width, 1/t_o, or slightly larger, and the individual minor lobe peaks of the envelope of Figure 3D are not discernible.

As illustrated by Equation 7, the smallest fractional system resolution attainable is equal to the reciprocal of over-all path length change expressed in terms of guide wavelength. This limiting resolution would be approached if amplifier bandwidth were narrow compared with 1/t_o. As a trade-off for this narrow system resolution, the amplifier output as a function of center frequency would reproduce the envelope of the pulse spectrum (Figure 3D) if the output of a single coherent source were observed. Thus, there would be numerous peaks observable for a single microwave source and signals from several sources would be confused and indistinguishable. Should an amplifier bandwidth small in comparison with 1/t_o be used, it might be desirable to reduce the lobes by tapering the edges of the amplitude function corresponding to Figure 3A -- this could be accomplished by controlling amplifier gain with
a potentiometer that controls gain as a function of trombone position. The ultimate improvements to be expected from pulse shaping is the subject of this section.

The frequency spectrum of a pulsed carrier is determined from the Fourier transform of the amplitude function (video pulse). In a like manner the Fourier transform provides a convenient method for finding the field patterns of line-source antenna apertures. Specifically, the field pattern can be formulated as the Fourier transform of the aperture distribution. The problem of interest here is to ascertain practical, attainable levels for frequency spectra; as noted above, this is closely related to predicted attainable side-lobe levels for microwave antennas which are illuminated by constant phase but with tapered amplitude distributions. An aperture amplitude distribution which results in a radiated power level X db below the main lobe (constant phase across aperture) corresponds to a frequency spectrum for which the detector output voltage would be X db below the center of the frequency spectrum (constant frequency, i.e., constant trombone speed). However, for the power meter the voltage spectrum is proportional to r-f power because the detector is a square-law device. Therefore, if the video pulse were of the same shape as the amplitude distribution for an antenna, the signal rejection would be only X/2 db.

Taylor has considered the problem of constructing a line source with


an optimum compromise between beamwidth and sidelobe level (analogous to mini-
mizing the main lobe width of the frequency spectrum for a given envelope level
of nearby sidebands). Taylor's objective was to design a space factor with
the narrowest beamwidth and lowest sidelobes consistent with the available
aperture. In his analysis, an ideal space factor was formulated which has
uniform sidelobes. Further, Taylor showed that a practical line source can
have a space factor whose characteristics approach the characteristics of
this ideal space factor arbitrarily close and that the ideal cannot be imitated
exactly without resorting to super-gain (analogous to requiring a pulse
spectrum containing infinite energy). The theoretical work by Taylor would be
highly useful for the engineer interested in details of trade-off between main-
lobe width and side-lobe level. Figure 7 of Taylor's paper is helpful for
obtaining a feel regarding the interdependence between side-lobe level and
main-lobe width. It may be seen that for the extremely stringent amplitude
distribution which in theory provides a 50-db antenna sidelobe level, the half
power width of the main lobe would exceed the width of a 13-db level (uniform
amplitude distribution) by a factor of approximately 2.

Antenna side lobes are introduced into this discussion because microwave
engineers have devoted a great deal of effort toward reducing the side lobe
levels of antennas, and therefore it provides a convenient analogy for con-
jecturing about attainable frequency rejection capabilities should amplitude
tapering be used for the power meter. For example, it is quite difficult to
develop antennas for which the first side lobes are down 40 db below the
peak of the main beam, but side-lobe levels of 30 db are readily attainable.
In terms of the power meter (under the assumption that amplifier bandwidth
is much narrower than the pulse spectrum), this means that power spectra
should be attainable for which signals at spurious frequencies are down 15
db. The control of the amplitude distributions of a pulse would appear to
be easier than the control of amplitude distribution for an antenna; thus,
it seems plausible that the close-in sidebands for the power meter could be
suppressed to 20 db below the peak of the frequency envelope. Therefore,
for the case of an amplifier with a bandwidth narrow compared with 1/t_0,
careful control of amplitude tapering provides the opportunity for
suppressing the close-in sidebands by approximately 14 db below that which
would exist if no amplitude tapering were used. This amplitude tapering
would broaden the system resolution by a factor of somewhat less than 2.
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