Project Title: Analysis and Evaluation of Range Instrumentation Techniques

Project No.: A-1678

Project Director: Dr. E. K. Reedy

Sponsor: Procurement Directorate - White Sands Missile Range, New Mexico, 88002

Effective 9-11-74  Estimated to run until 3-10-75 (Work Period)

Type Agreement: Contr. DAAD07-75-C-0025  Amount: $52,096


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DEFENSE PRIORITY RATING: DO-S1 under DMS Reg. 1

Assigned to SENSOR SYSTEMS Division

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Sue Corbin
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Project Title: Analysis and Evaluation of Range Instrumentation Techniques

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Effective Termination Date: 7/11/75 (Contract Expiration)

Clearance of Accounting Charges: 7/31/75

Grant/Contract Closeout Actions Remaining: Final Invoice & Closing Documents
                                         Final Report of Inventions
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Monthly Progress Report No. 1
Commanding Officer
U.S. Army White Sands Missile Range
Attention: Mr. Robert E. Green, STEWS-ID-S
Building 1506
White Sands Missile Range, New Mexico 88002

Reference: Contract No. DAAD07-75-C-0025, "Analysis and Evaluation of Range Instrumentation."

Subject: Technical Report (Monthly Progress Report) No. 2 covering the period 11 October through 10 November 1974

Gentlemen:

This is the second progress report under the referenced contract and covers the period 11 October through 10 November 1974. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study will be to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

(1) 10 feet in position, any axis
(2) 5 feet per second in velocity
(3) 5 feet per second per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible.

On 11 September 1974, E. K. Reedy and G. W. Ewell visited the WSMR facilities in New Mexico to discuss with Mr. R. E. Green, Dr. A. L. Gilbert, and others the detailed definition of range instrumentation requirements, tracking accuracy bounds, system constraints, etc. The discussion resulted in the following additional information. Some refinement of our original concept of the requirements has resulted from later discussions.

(1) Target will probably be one SRAM or similar type missile, flying at 200-1000 feet, and at speeds ranging from subsonic to supersonic.
(2) Target may carry a radar beacon transponder.
(3) Data format will be compatible with standard telemetry equipment.
(4) Data processors available include IBM 360/65 and Univac 1108 computers.
Instrumentation payload on board target should be 1000 lbs. or less, and should fit within a space equivalent to a cylinder 3 feet in diameter and 15 feet long. Instrumentation pod should be capable of being hung beneath an aircraft on a standard bomb rack or instrumentation pod mount. Interchangability between different aircraft is desirable.

Clear weather and low humidity can be expected.

The first system to be examined should comprise at least four ground based transponders, a fifth, radar transponder on the target, an airborne radar which tracks the target transponder, an airborne interrogator which tracks the ground-based transponders, and an inertial measurement unit in the airborne platform.

A more complete description of the general operational requirements for a low-altitude tracking system is given in Attachment I. This attachment also contains a brief description of the proposed system. These requirements were reviewed with Mr. Robert Green, WSMR Technical Monitor for this contract, during his 19 November 1974 visit to GIT.

The study effort during the first two months has been devoted to a review of reports concerned with radio reference systems, precision navigation systems, airborne radar systems, transponders, Kalman filters, and signal processing.

An extensive bibliography and a number of reports were received from WSMR. Our request for permission to access DDC controlled reports has not yet been granted, however. The request for approval to access DDC has been resubmitted. Direct access through the technical monitor appears to be the most feasible route to obtain necessary information.

Attachment II to this status report includes both a detailed task outline and task and time schedule based on completing the effort within a 7-month performance. A no-cost time extension of 3-months has been requested and is currently being reviewed by WSMR.

The radio reference systems reviewed thus far have included:

1. AROD: coded CW modulation type.
2. Unified S-band ranging system for Saturn V vehicles; coded CW.
3. SHIRAN; multiple tone FM.
4. PLRS, NAVELEX, coded subcarrier, burst transmission.
5. RMS-2/DCS; hybrid system, pulsed carrier RRS with IMU.
6. CIRIS; hybrid system--multiple tone FM RRS, with IMU. Kalman filter for estimation using combined signals.

The major components of the error budget, for the range signals of the RRS, include:

1. Survey of ground-based transponder locations:
   (2 parts per $10^6$ with distance measured from reference),
2. Range:
   a. Vector magnitude of scale factor propagation error, due to imperfect modeling of atmosphere.
b. Geometric "blow-up" factor when converting range measurement to position (depends on geometry and number of ground-based sensors)

c. Random variation of index of refraction along propagation path.

d. Uncompensated equipment delays, phase errors, equipment noise, digitization and quantization errors (in multitone FM system) or threshold level uncertainty (in pulsed carrier systems).

e. Multipath effects.
   1. In multitone FM systems, multipath error is inversely proportional to the deviation ratio.
   2. In pulsed carrier systems, assuming sufficient signal power and precise gating, multipath error is negligible.

f. Data processing.

(3) Range Rate: (same general errors as for range measurements)

a. Scale factor error
b. Geometric "blow up" factor
c. Random effects of atmosphere
d. Equipment errors
e. Multipath effects—dynamic effects of multipath on frequency shift of carrier.
f. Data processing.

The realization of certain system objectives could reduce the magnitude of the errors that have been listed. These objectives are as follows:

(1) A sufficient number of ground-based transponders must cover the desired coverage area, with spacing and dispersal such that
   (a) the range measurement scale factor error is kept small by keeping the maximum range distances small; and/or
   (b) the elevation angles from transponders to airborne platform are always in the vicinity of 35°, in order to minimize the geometric blow-up factors.

(2) Equipment errors must be reduced through better design; e.g., more precise oscillators.

(3) Multipath errors can be reduced by screening antennas from reflecting points, or by proper modulation parameters.

(4) Kalman filters must be made optimum.

Only the effect of randomness of the index of refraction along the path of propagation is uncontrollable. Operation during calm meteorological conditions can minimize this source of error.

Errors in the IMU component of a hybrid system are largely determined by equipment errors associated with the barometric altimeter, gyros, and accelerometers.

The time constants of time varying equipment errors are usually hundreds of seconds. The hybrid system may be crudely viewed either as a stable baro/inertial system updated periodically by radio reference estimates of position and velocity, or as a rapidly sampled radio reference system with
smoothing of the output by the baro/inertial system acting as a low-pass filter. Of course, combining both system outputs by Kalman filter estimation/digital processing techniques constitutes a considerably more sophisticated hybrid system.

Study and analysis of available airborne radar equipment and operational requirements for the airborne radar portion of the proposed low altitude tracking system have recently been initiated. Preliminary signal-to-noise ratio calculations and tracking error estimates have been made.

During the next reporting period, the analysis of tracking radar requirements will be continued along with investigation of position location systems for locating the airborne platform. Efforts at developing an overall system error budget by combining individual component errors will be commenced.

Respectfully submitted.

E.K. Reedy,
Project Director

EKR:jm

cc: A-1678 File
OPERATIONAL REQUIREMENTS FOR LOW ALTITUDE TRACKING SYSTEM

I. General Requirement:

An instrumentation tracking system is required to provide accurate trajectory data on test vehicles flying at low-altitudes anywhere on White Sands Missile Range.

II. Target: (Test Vehicle)

A. Type: Missiles, RPV's, A/C, etc., but probably typified by SRAM (cruise missile)

B. Number: Single Target

C. Velocity: Both subsonic & Supersonic targets must be considered.

D. Altitude: 200 feet to 1000 feet AGL typical

E. Expected RCS: 5 - 15 dBsm typical

F. Target Instrumentation: Radar Transponder

III. Coverage/Operational Scenario:

A. Area: Test vehicle located anywhere on (over) WSMR; future expansion to include Green River Corridor.

B. Terrain: Desert & Mountainous

C. Weather: Clear with low humidity; little or no rain

IV. System Error Requirements:

A. Position Measurement Accuracy: 10 feet, any axis

B. Velocity Measurement Accuracy: 5 feet/second

C. Acceleration Measurement Accuracy: 5 feet/second^2

V. Data Format and Processing:

A. Format: Standard telemetry equipment compatible

B. Computation Equipment: IBM 360/65 (on-line with telemetry system) and UNIVAC 1108.

C. Philosophy: Utilize ground-based processors to maximum extent possible
VI. Airborne Instrumentation:

A. **Test Vehicle Target**: Limited to transponder
   (relatively small package)

B. **Other A/B Instrumentation**: Packaged in a standard bomb-rack
   pod, weighing no more than approximately 1000 pounds, and having
   dimensions of 15 feet long by 3-foot diameter cylinder. Pod should
   be completely interchangeable between aircraft.

C. **Availability**: Instrumentation currently within the military/commercial
   inventory should be used to the maximum extent possible.

D. **Maintenance & Calibration**: Prime Considerations

VII. Ground-Based Instrumentation:

A. **Mobility/Transportability**: Equipment should be as small and
   transportable as possible consistent with other system constraints.

B. **Unattended Operation**: Ground-Based instrumentation may be
   required to operate at remote locations and unattended.

C. **Survey Error**: On the WSMR, ground-based instrumentation can be
   located to an accuracy of 2 parts per million.

D. **Availability**: Instrumentation currently within the military/commercial
   inventory should be used to the maximum extent possible.

E. **Maintenance & Calibration**: Prime considerations

VIII. Proposed System Configuration:

A. **Primary Components**:

   1. Position location system consisting of at least 4 ground-based
      transponders, an airborne interrogator, data processor, and an
      inertial navigation system.

   2. Airborne instrumentation platform (probably an aircraft)

   3. Airborne Radar capable of acquiring and tracking the test
      vehicle target.

   4. Radar transponder on-board the test vehicle.

B. **Operation**:

   By providing measurement of range and range rate (nominally) between
   the A/B interrogator and 4 ground-based transponders, the position
   measurement system establishes an estimate of the position of the
airborne platform. A second, independent estimate of position is obtained from the inertial navigation unit carried on-board the airborne platform. Combining the two independent position estimates improves the overall position estimate. Position data on the test vehicle relative to the airborne platform is obtained from the A/B radar measurements of range and angle. Radar attitude stabilization is obtained from the inertial navigation unit also. The transponder on board the test vehicle target improves the received S/N and allows the target return to be separated from the ground return.

C. Specific System Parameters:

1. A/B platform altitude is approximately 40,000 feet AGL.
2. A/B platform velocity is approximately the same as test vehicle.
3. Slant range from A/B platform to target more than 40K feet but less than 100K feet. (40K' X 90K')
4. Expect A/B radar to operate above X-band.
5. Elevation Angle for position location system should be optimized to approximately 35°.
PROPOSED LOW-ALTITUDE TRACKING SYSTEM

Airborne Platform
(with interrogator, A/B radar, IMU, & altimeter)

Test Vehicle
(with radar transponder)

Position Location Transponders
Tasks

I. Review and analyze WSMR requirements for a low-altitude tracking system

II. Develop set of performance goals and operational scenario for the low-altitude tracking system.

III. Determine performance limitations of proposed system (airborne platform position location system, airborne phased-array radar, and test vehicle transponder) and ability to meet performance goals.

A. Airborne platform position location system studies
   1. Review current operational position location equipment and compile performance specifications (includes limited analysis of INS and Altimeter System in addition to R & RRS)
   2. Relate attainable performance characteristics to platform position location accuracy requirements.

B. Airborne phased array radar & associated transponder studies
   1. Review current operational A/B radar (both phased array & mechanically scanned) and transponder equipment and compile performance specifications.
   2. Relate attainable A/B radar performance to test vehicle location requirements (requires system error analysis as discussed in C below).
   3. Determine if A/B radar requirements are within the current state-of-the-art.

C. System Accuracy Studies
   1. System combined accuracy analysis (determine requirements on A/B radar).
   2. Assume perfect knowledge of A/B platform position and attitude, and determine requirements on A/B radar.
   3. Develop required specification for A/B radar.

D. Evaluate the capability of proposed system to meet low-altitude tracking requirements with current state-of-the-art equipment.
IV. Alternative Concepts

A. Develop alternative low-altitude tracking concepts and evaluate to maximum extent possible.

B. Compare projected performance to system requirements and proposed system.

V. Make recommendations on system configuration for WSMR low-altitude tracking system and estimate expected performance.

VI. Prepare Final Engineering Report.
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<th>TASKS</th>
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<td>I. Review &amp; Analyze Requirements</td>
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- Task Initiation: ○
- Task Duration: ○—○
- Task Completion: ○
- Report Due: △

TASK AND TIME SCHEDULE FOR PROJECT A-1678,
"Analysis and Evaluation of Range Instrumentation Techniques"
Gentlemen:

This is the third progress report under the referenced contract and covers the period 11 November through 10 December 1974. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study will be to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible. Further system constraints were outlined in the second progress report dated 21 November.

The review of reports and technical literature was continued during this reporting period, with emphasis on a search for an airborne radar system that has the capability of detecting a transponder and measuring its angular position with the required accuracy. The characteristics of existing airborne radars reflect the basis of their system design requirements, and few if any of them have been designed to the stringent requirements set forth for this project. The angle measurement accuracy required is about 0.1 milliradian. No airborne radars with the ability to measure angles to one tenth milliradian are presently known to us, and it seems unlikely that minor modifications to near-suitable radar systems will be satisfactory.
For an example of an angle measuring radar, consider the AN/FPS-16, which is usually considered to measure angular data to about 0.1 or 0.02 milliradian. It has a 12 foot antenna and a 1.1 degree, (19.2 milliradian) beamwidth. It appears that measurement has been achieved to 0.001 or 0.005 of a beamwidth, and this is for skin tracking. Now an antenna one fifth this size could be made to fit into an airborne pod. If the signal to noise ratio were raised through the use of a transponder on the target of sufficient power, then the 5.5 degree beam of the smaller antenna could, in principal, be made to track and measure angles to 0.1 milliradian like the FPS-16.

Suppose, now, an airborne radar is discovered in inventory or "on the shelf" that has the proper configuration, but is capable of measuring only to one milliradian, or more likely, several milliradians. (It would be a very special development that resulted in a radar tracking a smaller fraction of a beamwidth than the FPS-16). The considerations of the preceding paragraph suggest that there is no really fundamental limitation to tracking to 0.1 milliradian with a 2.4 foot antenna, provided the tracked signal is strong enough. However, no minor modification of the hypothetical airborne radar will bring this off. This radar was designed for one milliradian accuracy. Its radome, gimbal systems, servos, and data circuits are so designed, and one would have to essentially start over to upgrade the accuracy by a factor of 10. Radome distortions that were acceptable will no longer be so. Fifteen or sixteen bit encoders, rather than 12 bit ones would be required on the gimbals, and the data handling would have to be so redesigned. The mechanical structure would likely have to be stiffened, and the bearings reconsidered. These items are not minor modifications.

The notion of a phased array, if one were located, would require similar redesign. We cannot be specific about the items that would require redesign because no standard way of realizing angle measurement with a phased array has evolved, but there is no reason to suppose the redesign and development would be any less a major effort.

In short, it may be within the state-of-the-art to develop an airborne radar that will measure angles to 0.1 milliradian, but we are pessimistic about finding one already developed for another purpose that can be adapted to the requirements of this project without essentially going through the development process all over again.

The effects of the angles of elevation from the ground based transponders which are used to determine the position of the tracking aircraft are serious. The error for ranging signals, as compiled in the CALSPAN report, is 6.9 ft. using a CUBIC transponder and 5.7 feet using RMS-2/DCS pulse leading edge transponders. The geometrical "blow-up" factor, with four ground-based transponders, and angles of 25 or 55 degrees would multiply these errors by about 1.7, so that the CUBIC system would have 11.5 ft. RMS error and the RMS-2/DCS would have 9.5 ft. RMS error. Even if the largest sources of multipath error for the CUBIC and leading edge position ambiguity for the RMS-2/DCS systems--are reduced to 2 ft. RMS each, the total errors would be 9.5 ft. RMS and 6.5 ft. RMS respectively.
The request for a 3-month, no-cost, extension has been approved by WSMR. An amended time schedule, which will replace the schedule in the second monthly progress report, is included as Attachment II.

Respectfully submitted,

E. K. Reedy,
Project Director

EKR: jm

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○ Task Initiation ○——○ Task Duration ○ Task Completion △ Report Due

TASK AND TIME SCHEDULE FOR PROJECT A-1678,
"Analysis and Evaluation of Range Instrumentation Techniques"
Commanding Officer
U. S. Army White Sands Missile Range
Attention: Mr. Robert E. Green, STEWS-ID-S
Building 1506
White Sands Missile Range, New Mexico 88002

Reference: Contract No. DAAD07-75-C-0025, "Analysis and Evaluation of Range Instrumentation"


Gentlemen:

This is the fourth progress report under the referenced contract and covers the period 11 December 1974 through 10 January 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study will be to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible. Further system constraints were outlined in the second progress report dated 21 November 1974.

The review of reports and technical literature was continued during this reporting period, and the analysis of the pointing requirements of an airborne radar which would track the low-flying target is now being pursued. Some suggestion of the system parameters that may emerge from the analysis can be found in an article by F. E. Hoge, "Integrated Laser/Radar Satellite Ranging and Tracking System", Applied Optics, October, 1974, pages 2352-2358. The paper describes the combination of a laser
with an AN/FPQ-6 radar, and a digital computing and data recording facility, all ground-based at NASA, Wallops Island.

The radar is utilized to track geodesic satellites GEOS-I and GEOS-II, with slant range and angle errors of ± 2 meters and ± 15 seconds of arc. The tracking pedestal is capable of 0.05 milliradian (10.3 seconds of arc) precision. The aiming encoders are nineteen bits, corresponding to 1.24 seconds of arc, the finest division of the control signal. The antenna is mounted on a hydrostatic azimuth plane bearing. Hydraulic drive motors move the antenna about its azimuth and elevation axes.

The AN/FPQ-6 operates at "C-band" (5.2 gigahertz) with an 8.8 meter parabolic reflector. It is capable of tracking a one square meter target to beyond 1000 kilometers. Scaling the AN/FPQ-6 to 1.5 meter reflector would require operating at Ka-band, or about 30 gigahertz. The pointing error of ± 15 seconds of arc would correspond to 0.073 milliradians, for a position error at a 20-mile tracking slant range of ± 7.7 feet. The airborne radar would thus have to have a pointing accuracy as good as, or better than, the Wallops Island, fixed installation AN/FPQ-6 satellite tracker. The beamwidth of a 30 GHz radar, with an antenna aperture of 1.5 meters (59 inches), would be about 0.5 degrees, as shown in Figure 1. Higher frequency operation (to reduce the antenna size) would encounter problems of propagated energy absorption by water vapor and other atmospheric gases.

A geometric analysis has been made of the pointing accuracy that is required in tracking a low-flying target, from an aircraft that flies at various altitudes. The assumptions (see Figure 2) for the analysis were:

1. Location error of tracking aircraft ± 5 ft., each axis
2. Total error budget for location of target 10 ft., rms
3. Aximuth and elevation tracking errors are equal $\sigma_{\theta_1} = \sigma_{\theta_2}$
4. Target altitude is negligible.
5. Maximum elevation tracking angle is 45° below horizontal $\phi_1 = 135^\circ$
Figure 1. Beamwidth Versus Aperture at Millimeter Frequencies for two Sidelobe Levels.
Figure 2. Target Location Relative to Moving Platform
(6) Ranging accuracy, tracker-to-target \( \sigma_R = \pm 2 \) ft.

(7) Constant refractive index along line of sight

The required angular tracking accuracy is then:

<table>
<thead>
<tr>
<th>Platform Altitude (feet)</th>
<th>Pointing Accuracy ( \phi_1 ) (millirad)</th>
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<td>10,000</td>
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A second analysis included atmospheric effects, but did not include the position errors associated with the tracking airplane.

It is well known that electromagnetic waves travel along curved rather than straight line paths while passing through the troposphere. This refraction of the waves produces range errors and elevation angle errors in radar position locating systems. An estimate of the magnitude of this effect for the WSMR geometry was obtained using curves presented in Barton [1] that were derived using the NBS Exponential Reference Atmosphere model.

It was assumed that the aircraft would be flying at an altitude of from 15,000 to 60,000 feet above the local terrain and that the angle from the aircraft to the target could be anywhere from \(-110^\circ\) to \(-140^\circ\). (This situation would describe both the aircraft/ground-base transponder situation and the aircraft/missile situation.) Some of the propagation errors associated with this range of parameters are listed below. From the table it can be seen that the effects of atmospheric refraction on propagation errors must be taken into account in determining the requirements for pointing accuracy. For a track airplane at 40,000 feet, the angle error

---

due to atmosphere is about 0.4 millirad. The tracking angle would have to be corrected by use of an atmospheric model of refractive index; residual uncertainty would be about 0.15 milliradian, which is twice the allowable error as determined in the geometric analysis.

Preparations are being made to visit a number of companies and military installations to obtain information about equipment and systems used to locate and track aircraft, missiles, and ground vehicles. The information sought will include:

1. Error sources
2. Error magnitudes
3. Methods of reducing errors
   a. Directional antenna patterns
   b. Modifications to electronics
   c. Alternative systems.
4. Cost estimates

Sites included in visit plans are:

<table>
<thead>
<tr>
<th>Aircraft Altitude (ft)</th>
<th>Angle to Target (degrees)</th>
<th>Slant Range to Aircraft (ft)</th>
<th>Slant Range Error (ft)</th>
<th>Standard Deviation of Range Error (Millirad)</th>
<th>Elevation Angle Error (Millirad)</th>
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</table>
Analyses of alternative approaches to the WSMR tracking problem have been initiated.

Respectfully submitted,

Spurgeon L. Robinette
Project Director

Approved:

Edward K. Reedy
Manager, Special Programs Office
Sensor Systems Division
25 February 1975

Commanding Officer
U. S. Army White Sands Missile Range
Attention: Mr. Robert E. Green, STEWS-ID-S
Building 1506
White Sands Missile Range, New Mexico 88002

Reference: Contract No. DAAD07-C-0025, "Analysis and Evaluation of Range Instrumentation"


Gentlemen:

This is the fifth progress report under the above referenced contract and covers the period 11 January through 10 February 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study is to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible.

During the week of February 3 through February 7, Mr. S. L. Robinette and Dr. J. E. Rhodes visited some of the commercial manufacturers of tracking systems which are presently installed or being installed on military ranges. The manufacturers included:

1. Litton Guidance and Control Systems, Los Angeles, California
2. Hughes Ground Systems Group, Los Angeles, California
3. General Dynamics, Electronics Division, San Diego, California
4. Cubic Corporation, San Diego, California

Visits were also made to:

1. Yuma Proving Grounds, Yuma, Arizona
2. White Sands Missile Range, New Mexico
Dr. R. D. Hayes joined Mr. Robinette and Dr. Rhodes for the WSMR visit and discussions.

Systems reviewed included:

1. RMS/SCORE (General Dynamics)
2. RMS-2/DCS (General Dynamics, Hughes)
3. PLRS (General Dynamics, Hughes)
4. ARIS (Litton)
5. PATS (Yuma Proving Ground, Sylvania)
6. Air Combat Maneuvering Range (Cubic)

The critical points of a precision range measurement system for low-flying targets were discussed with engineers and scientists at the sites visited. These critical points have been identified as:

1. Airborne reference platform attitude error.
2. Pointing angle error, platform to target.
3. Multipath error.
4. Equipment signal delay error.
5. Cost.

Among the systems discussed, only the laser ranging Precision Automated Tracking System (PATS) appears to be capable of meeting the measurement accuracy goals of WSMR. At Yuma Proving Ground, PATS was observed tracking a helicopter at a range of 35,000 to 40,000 feet. The lock-on jitter was judged (from the TV presentation) to average about +5 feet in azimuth—even less in elevation. PATS appears to be capable of meeting the WSMR requirements, as a network of ground-based laser ranging units, without an airborne tracker. Sylvania, the manufacturer of the PATS unit, states that the "absolute accuracy" is ±0.1 milliradians azimuth and elevation, and ±2 feet in range for targets at 65,000 feet distance. It can be assumed that atmosphere effects are not included in these accuracy specifications; nonetheless, PATS appears to be capable of meeting the WSMR accuracy requirements—at least for ranges of 1000 to 50,000 feet.

System advantages of laser tracking over radar and radio ranging systems include:

1. Vanishingly small multipath error.
2. Elimination of transponder delay error. (The transponder is a mirror.)
3. Small pointing error
4. Real time data acquisition

The disadvantages of a laser ground-based network of PATS units are:

1. High cost: $550,000 to $575,000 each.
2. Two operators are presently required at each PATS site; although additional automation might be developed to eliminate this requirement.
3. A coordinating voice or data network would be needed to effect hand-off of the target, and to coordinate data acquisition.
4. A target will be "lost" behind terrain obstructions during some parts of its run.
Instead of a completely ground-based laser tracking system, it might be feasible to adapt PATS (or develop another laser system) for the airborne pod, substituting laser tracking for the radio beacon tracking suggested by WSMR. The single airborne laser system would cost less than an all-ground-based laser network. The airborne laser would also decrease the probability of loss of target behind terrain obstructions.

PATS at present uses an operator to acquire the target, by way of a closed circuit television system, and manual control of the pointing direction of the laser/vidicon mount. When the target has been approximately centered (manually) on the TV screen, the operator switches the system to automatic tracking of the laser beam reflected from the target. Mounting a laser tracking system in the airborne pod would require either automating the acquisition mode, or adding a video link to permit an operator (on the ground or in the aircraft) to acquire the target by remote control of the laser/vidicon mount.

The pointing error of the airborne PATS would be degraded by the attitude error of the airborne reference platform; however PATS appears to be capable of greater precision than a radar antenna that would fit within the pod.

A phased array antenna, measuring 10 feet by 2 feet, could conceptually be set into the bottom of an instrumentation pod. Let us assume that the array operates at 17 GHz and produces a monopulse, steerable beam. The "sum" beam would be fan shaped, and the angular "thickness" of the beam (assuming cosine function illumination) measured in the direction of the long axis of the pod, would be

\[ \theta_{3\text{dB}} = \frac{68.7\lambda}{\text{Length of Antenna}} \text{ degrees} \]
\[ = 0.4 \text{ degrees} \]

If we now assume a tracking capability of one-fiftieth the beamwidth, the tracking error in the direction of the flight path, with the beam at radar boresight (pointing directly downward during level flight) would be;

\[ \Delta\theta = \frac{0.4}{50} \text{ degrees} \]
\[ = 0.14 \text{ millirad} \]

It can also be shown that the tracking resolution in the direction normal to the flight path, with the beam at antenna boresight, is:

\[ \Delta\theta = 0.68 \text{ millirad} \]

When the beam center is deflected from boresight in order to point at a target, the beam is degraded by broadening:

\[ \theta_{3\text{dB}}(\phi) = \theta_{3\text{dB}} \text{ at } \phi = 0^\circ \]
\[ \frac{1}{\cos \phi} \]
The tracking resolution is also degraded:

\[ \Delta \phi_f = \frac{0.14}{\cos \phi_f} \text{ millirad} \]

where

\[ \phi_f = \text{deflection angle component projected on the flight path.} \]

and

\[ \Delta \theta_n = \frac{0.68}{\cos \phi_n} \text{ millirad} \]

\[ \phi_n = \text{deflection angle component normal to flight path} \]

A specification of 10 feet vector error for tracking the low-flying target defines an error angle:

\[ \Delta \psi = \tan^{-1}(10/R) \]

The specification error angle will depend on the altitude difference between aircraft and target, and on the pointing angle, \( \phi \). Table I compares the tracking angular resolution of the postulated 10 foot by 2 foot antenna with specification error angles, for various beam pointing angles. For the values computed, the chase aircraft is assumed to be flying a path that parallels the target path. Atmospheric effects and airborne platform attitude errors are not included in Table I values.

From the Table, it is seen that the resolution angle in the direction of flight would be less than the specification error angles for beam deflections less than 40 degrees when the aircraft is at 40,000 feet altitude. With the aircraft at 30,000 feet, the deflections can be ± 50 degrees. The system would fail to meet the 10 foot error specifications in the left or right directions, even when the aircraft is directly over the target.

To meet the specified error for the left/right directions, the resolution would have to be improved by a factor of five—by (1) reducing the radar wavelength, (2) increasing the width of the antenna, (3) decreasing the resolution angle to less than one-fiftieth of the 3 dB beamwidth, or (4) by a combination of these measures. For example, the radar frequency might be increased to 35 GHz, the width of the antenna to 4 feet, and the resolving fraction of the 3 dB beamwidth to one-sixtieth, to produce a five-fold improvement in radar tracking resolution. Even then, however, the added errors caused by variations and uncertainty of the atmospheric index of refraction, the uncertainty of the airborne platform attitude and position, and the errors in the measurement of the radar pointing angle would probably degrade the left/right determination of target position to be larger than the 10 foot specification, except perhaps within about ± 10° look-down angle.

To summarize our conclusions at this point:
### Table I

**Angular Errors Equivalent to the Specified Position Error (5 Foot Vector Error) Compared with 17 GHz Radar Resolution Angles**

<table>
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<tr>
<th>Look-Down Angle, $\phi$ (degrees)</th>
<th>Angular Error Due to 10 Foot Position Error</th>
<th>Resolution Angle</th>
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<td>40,000 Feet Altitude</td>
<td>30,000 Feet Altitude</td>
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<tr>
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<td>Slant Range (10^3 feet)</td>
<td>Error Angle, $\Delta\psi$ (millirad)</td>
</tr>
<tr>
<td>$\pm 50$</td>
<td>62.2</td>
<td>0.161</td>
</tr>
<tr>
<td>$\pm 45$</td>
<td>56.6</td>
<td>0.177</td>
</tr>
<tr>
<td>$\pm 40$</td>
<td>52.2</td>
<td>0.192</td>
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<tr>
<td>$\pm 30$</td>
<td>46.2</td>
<td>0.217</td>
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<tr>
<td>$\pm 20$</td>
<td>42.6</td>
<td>0.235</td>
</tr>
<tr>
<td>$\pm 10$</td>
<td>40.6</td>
<td>0.246</td>
</tr>
<tr>
<td>$0$</td>
<td>40.0</td>
<td>0.250</td>
</tr>
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</table>
1. No airborne radar has been found which will support instrumentation for tracking low-flying targets to 10 feet of position.

2. A conceptual pod-mounted airborne radar antenna, which might be capable of determining the position of a low-flying target within 10 feet in the direction of flight of the tracking aircraft, would be about 10 feet by 2 feet. It could be operated at 17 GHz, or a higher frequency. It would require a monopulse beam, and the system would be capable of resolving better than one fiftieth of the 3 dB beamwidth. The tracking angle, measured from the downward direction, would be restricted to about ± 45° forward or backward. To obtain the resolution needed to determine the target within 10 feet in a plane orthogonal to the flight path of the tracking aircraft, the antenna might be widened to 4 feet and the frequency raised to 35 GHz. The radar tracking resolution would have to be equal to or smaller than one-sixtieth of the 3 dB beamwidth, even in this case.

3. Laser tracking, because of the higher frequencies involved, appears to be logical choice over radar for tracking low-flying targets to the accuracy specified by WSMR. Coherence of the radiation may permit the design of systems superior even to cine-theodolites, which are today’s standards for range instrumentation. Laser systems would permit real-time data processing. This feature alone would probably make laser tracking more cost/effective than cine-theodolites. The contribution of atmospheric turbulence to system error must be analyzed. The advantage of a laser system in the infrared region is that its aperture, on the order of 10 cm, is tens of thousands of wavelengths in diameter. By comparison, the 2 foot dimension of the postulated 17 GHz radar antenna would be equivalent to 30 wavelengths.

In the next reporting period an attempt will be made to quantify the pointing errors of an airborne tracker.

The effects of atmospheric perturbations and discontinuities in the index of refraction along a slant range line-of-sight path are being explored. These effects will be examined for both microwave and optical frequencies.

The radio reference systems examined thus far will be compared on a basis of systematic error sources.

Respectfully submitted,

Spurgeon L. Robinette/
Project Director

SLR: jm
Approved: / /
Edward K. Reedy, Head
Systems Technology Branch
Sensor Systems Division
Commanding Officer
U. S. Army White Sands Missile Range
White Sands Missile Range, New Mexico 88002

Attention: Mr. Robert E. Green, STEWS-ID-S, Building 1506

Reference: Contract No. DAAD07-C-0025, "Analysis and Evaluation of Range Instrumentation"

Subject: Technical Report (Monthly Progress Report) No. 6 covering the period 11 February through 10 March 1975

Gentlemen:

This is the sixth progress report under the above referenced contract and covers the period 11 February through 10 March 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study is to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

(1) 10 feet in position, any axis
(2) 5 feet per second in velocity
(3) 5 feet per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible.

The analysis of systematic sources of ranging errors has been continued. In the fifth progress report the angular pointing resolution of an airborne radar tracking a low flying target was presented. The radar postulated for that analysis operated at 17 GHz and had a 2-foot by 10-foot phased array antenna. The range in elevation-angle-error, for beam deflections from directly downward to 40 degrees in the forward direction was 0.14 to 0.18 milliradians; and for beam deflections from directly downward to 40 degrees in a direction transverse to the aircraft heading, was 0.68 to 0.88 milliradians. For a five-foot position error of the target, at 40 degrees deflection, with the airborne radar at 40,000 feet, the total angular error cannot exceed 0.1 milliradians.
Other sources of pointing angle error for the airborne radar include (1) the error in the determination of attitude of the airborne reference, (2) deflections between the reference platform and the antenna, (3) the sighting angle error due to the variation of the index of refraction of the atmosphere along the path of the radar signal.

The range in elevation angle errors caused by uncertainty in the determination of the attitude of the reference platform has been estimated [1] to be 0.05 to 0.11 milliradians.

Barton [2] has analyzed the effect on the sighting angle of the varying atmospheric index of refraction between the earth and a target. The long term uncorrected error, approximately 0.1 to 0.4 milliradians, can be largely compensated. The residual error would be approximately 0.03 to 0.05 milliradians for forward deflections and 0.05 to 0.07 milliradians for deflections normal to the flight path.

The effect of the radar beamwidth is seen to be a critical error source. Increasing the frequency to 90 GHz would reduce the radar angular errors by a factor of 5.3. If the width of the antenna were doubled also (to 4 feet), the maximum radar beam error would be 0.08 milliradians.

In the next reporting period, the writing of the final report will commence. A working outline for the final report is:

1. Introduction
2. Existing Systems and Equipment
3. Workable System
4. Conclusions and Recommendations
5. References
6. Appendices

No existing equipment has been found that will meet all the measurement specifications under the full range of target speeds and range geometry. Even the 17 GHz, phased array radar postulated in the foregoing represents something close to the state-of-the-art, and not equipment already available. Some parameters of airborne radar equipment that would approach the specified accuracy have been discussed in the fifth and sixth letter reports.


A system for measuring altitude of a subsonic target with the required accuracy of 10 feet, and horizontal position to relaxed specifications, is being investigated.

Based on our analyses, further investigations of 90 GHz radar and laser trackers seem to be required to fully establish the potential of the CIRIS type airborne tracker to meet the WSMR instrumentation needs.

Respectfully submitted,

Spurgeon L. Robinette
Project Director

SLR:gh

Approved:

Edward K. Reedy, Head
Systems Technology Branch
Sensor Systems Division
Commanding Officer
U.S. Army White Sands Missile Range
Attention: Mr. Robert E. Green, STAWS-ID-S
Building 1506
White Sands Missile Range, New Mexico 88002

Reference: Contract No. DAAD07-75-C-0025, "Analysis and Evaluation of Range Instrumentation"


Gentlemen:

This is the seventh progress report under the referenced contract and covers the period 11 March 1975 through 10 April 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study will be to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

(1) 10 feet in position, any axis
(2) 5 feet per second in velocity
(3) 5 feet per second per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible. Further system constraints were outlined in the second progress report dated 21 November 1974.

As stated in the sixth progress report and in a telephone report to you, the writing of the final report was begun, and has continued during this reporting period. The authorization to obtain classified information was received only a few weeks before its terminating date, which was our original contract termination date (March). As discussed with you earlier, the difficulties in obtaining information reduce the
effectiveness of short-time studies such as this originally was scheduled.

This progress report is late because of our misunderstanding concerning the reporting requirements during the no-cost contract extension period.

Respectfully submitted,

Spurgeon L. Robinette
Project Director

SLR: jm

Approved:

Edward K. Reedy, Head
Systems Technology Branch
Sensor Systems Division
Gentlemen:

This is the eighth progress report under the above referenced contract and covers the period 11 April through 10 May 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study is to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible.

The writing of the final report was continued in this reporting period.

An analysis of a multilateration system, with range and range rate measured from ground sites to the low-flying missile, from the ground sites to a high reference "chase" airplane, and from the chase ship to the low-flying missile was initiated. Only the missile's horizontal position information obtained by ground to low-flying target would be retained.
Commanding Officer
U. S. Army White Sands Missile Range
White Sands Missile Range, New Mexico 88002

Attention: Mr. Robert E. Green, STEWS-ID-S, Building 1506

Reference: Contract No. DAAD07-75-C-0025, "Analysis and Evaluation of Range Instrumentation"

Subject: Technical Report (Monthly Progress Report) No. 9 covering the period 11 May through 10 June 1975

Gentlemen:

This is the ninth progress report under the above referenced contract and covers the period 11 May through 10 June 1975. The project status, objectives met, work performed, and information obtained during the reporting period are summarized below.

The scope of the study is to define a feasible, conceptual instrumentation system to track low-flying targets anywhere on the White Sands Missile Range. The accuracy objectives are:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second per second in acceleration

Constraints for the system include the use of available military or commercial equipment to the maximum extent possible.

The writing of the final report was continued and it is anticipated that it will be completed as scheduled.

One of the major realizations that has emerged from the study has been that, after correction is made for atmospheric index of refraction, the residual errors of pointing angle measurements tend to affect the estimation of position of a target more strongly than do the errors in range measurements.
This is because:

1. The path of the phase front of an electromagnetic signal transmitted between two points is such as to minimize the propagation time from transmitter to receiver.

2. The measured angle of arrival or of transmission of the signal is strongly influenced by localized, small structure variations of the index of refraction, particularly those variations that are near to the receiver or the transmitter.

The systems analyses for this study have been completed, and this is the last status report. We have enjoyed working with WSMR and hope our study will be of some benefit in the search for a satisfactory system for measuring performance of low-flying missiles.

Respectfully submitted,

Spurgeon L. Robinette
Project Director

Approved:

Edward K. Reedy, Head
Systems Technology Branch
Sensor Systems Division
FINAL TECHNICAL REPORT

INSTRUMENTATION TECHNIQUES FOR TRACKING LOW-FLYING VEHICLES

EES/GIT PROJECT A-1678

Prepared for
UNITED STATES ARMY
WHITE SANDS MISSILE RANGE
NEW MEXICO 88002

Under
CONTRACT DAAD07-75-C-0025

By
S. L. Robinette, J. E. Rhodes, Jr.,
R. D. Wetherington, E. K. Reedy,
and R. D. Hayes

15 July 1975

ENGINEERING EXPERIMENT STATION
Georgia Institute of Technology
Atlanta, Georgia 30332
### Performance measurement techniques for terrain avoidance vehicles


### An analysis and evaluation has been made of available range instrumentation which would permit White Sands Missile Range to measure performance of low-flying missiles and aircraft, with the following accuracy objectives:

1. 10 feet in position, any axis
2. 5 feet per second, in velocity
3. 5 feet per second in acceleration

(continued)
A configuration was analyzed which used range measurements from ground sites to determine the position of an overflying aircraft, and tracking (measurements of range and pointing angles from the aircraft to the test vehicle) to determine the position of the low-flying vehicle. An inertial measurement unit, an altimeter, and a digital processor in the aircraft would establish attitude of the airborne reference system. No available airborne tracking equipment was found which would meet the White Sands Missile Range requirements.

Both millimeter and laser airborne radars were evaluated as candidates for device development programs, to perform the function of airborne tracking.

The possibility was examined of using an available Ku band airborne radar to determine altitude with 10 foot accuracy, the assumption being that higher horizontal position errors (=50 feet) could be tolerated.

A ground based laser radar network, and a multilateration technique were analyzed. The latter would require range measurements from ground sites to the low-flying target, from the ground sites to an overflying aircraft, and from the aircraft to the low-flying target.
Final Technical Report
EES/GIT Project A-1678

by

S. L. Robinette, J. E. Rhodes, Jr., R. D. Wetherington
E. K. Reedy, and R. D. Hayes

Prepared for

United States Army
White Sands Missile Range
New Mexico 88002

under
Contract DAAD07-75-C-0025

15 July 1975
INSTRUMENTATION TECHNIQUES FOR TRACKING LOW-FLYING VEHICLES

by

S. L. Robinette, J. E. Rhodes, Jr., R. D. Wetherington
E. K. Reedy and R. D. Hayes

ABSTRACT

An analysis and evaluation has been made of available range instrumentation which would permit White Sands Missile Range to measure performance of low-flying missiles and aircraft, with the following accuracy objectives:

1. 10 feet in position, any axis
2. 5 feet per second in velocity
3. 5 feet per second^2 in acceleration

A configuration was analyzed which used range measurements from ground sites to determine the position of an overflying aircraft, and tracking (measurements of range and pointing angles from the aircraft to the test vehicle) to determine the position of the low-flying vehicle. An inertial measurement unit, an altimeter, and a digital processor in the aircraft would establish attitude of the airborne reference system. No available airborne tracking equipment was found which would meet the White Sands Missile Range requirements.

Both millimeter and laser airborne radars were evaluated as candidates for device development programs, to perform the function of airborne tracking.

The possibility was examined of using an available Ku band airborne radar, to determine altitude with 10 foot accuracy, the assumption being that higher horizontal position errors (± 50 feet) could be tolerated.
A ground based laser radar network, and a multilateration technique were analyzed. The latter would require range measurements from ground sites to the low-flying target, from the ground sites to an overflying aircraft, and from the aircraft to the low-flying target.

The results of the analyses indicated that a feasibility study of the multilateration technique should be performed. If more detailed analyses support our findings, a prototype, abbreviated system should be developed, implemented, and tested; and if tests prove the system feasible it should be installed by WSMR.

In the event that the multilateration technique proves to be infeasible (because of excessive terrain masking or multipath effects, for example) it has been recommended that a prototype airborne millimeter or laser tracking radar be developed, be incorporated in a commercially available radio reference system, and then be tested with a minimum of four ground sites.
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1. INTRODUCTION

There is at present no capability for determining the position, velocity, and acceleration of low flying targets with the accuracy which is increasingly being requested by users of White Sands Missile Range (WSMR). A recent survey of requests for tests (see Appendix A) shows that out of twenty-nine tests there were twenty that were to be flown between zero and two thousand feet above ground level. Six of the twenty-nine users requested position measurement to within five to ten feet, but eleven requested accuracies of one to five feet. The requests for accuracy of velocity measurements for twelve of the programs were one to five feet per second, and for six of the programs velocity accuracy was specified as one-half inch to one foot per second. Similar stringent requests were made for accuracies in acceleration measurements. They ranged from one inch to ten feet per second per second.

The need for measurements on low flying targets is not new. Studies at White Sands Missile Range in 1967 and 1970 showed that the (unattainable) measurement accuracy requirements have been at a consistent level for nearly ten years. The following were suggested (see Appendix B) as realistic design goals for testing low-flying vehicles:

- Coverage - 200 feet above ground level
- Position measurement accuracy - 10 feet, any axis
- Velocity measurement accuracy - 5 feet per second, any axis
- Acceleration measurement accuracy - 5 feet per second per second, any axis
- Data output - digital format compatible with WSMR telemetry and computer equipment.

It should be noted that these requirements do not satisfy the most stringent requests cited above. They do, however, represent values that are deemed realizable, or almost realizable, with today's technology. Appendix C details the set of constraints and requirements of a WSMR measurement system for testing low-flying missiles and aircraft.

A specific measurement system configuration was suggested by WSMR (Appendix B). As shown in Figure 1, it would include an airborne radar from which to measure the position, velocity, and acceleration of a low-
Radio ranging signals, outputs of inertial measurement unit, altimeter, signal processor: produce estimates of altitude, position, velocity, acceleration of airborne reference.

Radio ranging signals: measure range and range rate from ground sites to airborne reference.

Figure 1. Overflying Aircraft Tracking a Low-Flying Missile.
flying vehicle. The position, velocity, and acceleration of the airborne reference platform would be determined by range and range rate measurements from accurately surveyed ground based stations. The range and range rate signals would be combined with platform attitude signals derived from an inertial navigation unit (INU), and with an altitude signal from a barometric altimeter, to establish the position, velocity, acceleration, and attitude of the airborne reference platform with respect to fixed, ground reference coordinates.

The airborne platform would fly at an altitude sufficiently high to be located by a small number of ground based units, and to permit a subsonic aircraft to track a supersonic vehicle over the length of the range—about one hundred and twenty miles. It was suggested that a small, phased-array radar antenna in the aircraft could track a transponder mounted on the low-flying test vehicle. For system flexibility, the airborne equipment would be mounted in a pod that could be attached to standard Air Force pod hangers. The pod could be a cylinder as large as three feet in diameter and fifteen feet long (Appendix C). The weight budget could be 1000 pounds. The equipment in the pod would include (1) range and range rate measuring devices, (2) an inertial measurement unit, (3) an altimeter, (4) a small digital computer, and (5) a tracking radar. The first four components are found in an existing system, CIRIS (for Completely Integrated Reference Instrumentation System) [1-4].

The purpose of the Georgia Tech study has been to determine:

(1) Will the system of Figure 1, using existing commercial or military equipment mounted as described above, enable White Sands Missile Range to measure the performance of low-flying missiles or aircraft, which have speeds ranging from subsonic to supersonic?

(2) If the system of Figure 1 will not meet the specified accuracy requirements in all axes and over a subsonic to supersonic range of speeds, will it meet the required measurement accuracy in the vertical axis, at subsonic speeds?

(3) Is it feasible to modify the system of Figure 1 to meet the required measurement accuracies?
The following sections of this report detail the findings of the Georgia Tech study, which can be summarized:

(1) Existing airborne equipment was sought to implement the system of Figure 1 so as to meet the accuracy requirements in all axes. Chapter 2 details the search. The CIRIS type of system (see Appendix D) which includes the ground based transponders of Figure 1, and all of the airborne components except the tracking radar, is available and error analysis indicates the system is capable of locating the tracking aircraft with an error of about 5.8 feet, any axis. No airborne radar, for the final link between the low-flying target and the tracking aircraft, was found that would meet the space constraints and the accuracy requirements as defined by WSMR.

(2) The most critical component of the system of Figure 1 is the airborne radar or other sensor which would track the low-flying test vehicle. When no existing airborne radar was found that would meet all requirements, studies were made to determine the feasibility of developing an airborne radar. Alternative approaches, which depart in varying degrees from the Figure 1 concept, were also examined. System parameters for conventional and laser airborne radars are developed in Chapter 3. Three alternatives to the Figure 1 system are described in Chapter 4. The alternatives are:

- A ground based network of laser radars.
- A Ku band radar in the tracking aircraft which would determine altitude of the test vehicle within 10 feet RMS, but horizontal position error would exceed 40 feet RMS in each horizontal axis.
- Multilateration, with radio ranging to the low-altitude target and to the high-altitude aircraft from ground based stations. Range would also be measured to the low-flying target from the high-altitude aircraft.

A fourth multilateration system that would have utilized three CIRIS type airborne reference platforms was discarded because of anticipated operational and scheduling difficulties.

Attention is focused on position error in the following studies, rather than velocity and acceleration errors. This was justified (for the Chapter 3 treatment) by observing that in an analysis of CIRIS [1], velocity errors were two orders of magnitude smaller than position errors. In the Chapter 4 analyses, time did not permit an extension to velocity and acceleration errors.
2. THE SEARCH FOR EXISTING MEASUREMENT SYSTEMS AND EQUIPMENT

In the search for existing systems and equipment which would permit White Sands Missile Range to measure the performance of low-flying vehicles, emphasis was placed on the approach embodied in Figure 1. The functions of the components of the system [1-6] can be outlined:

(1) A radio ranging system (RRS) measures range and range rate from surveyed ground sites to the tracking aircraft.

(2) An airborne inertial measurement unit (IMU) measures acceleration and velocity of its three inertial axes.

(3) A computer in the aircraft combines the RRS and IMU measurements and computes an estimate of position, velocity, and acceleration of the airborne reference coordinates, with respect to the coordinates of the ground sites.

(4) An airborne subsystem measures range and angular directions from the airborne reference coordinates to the low-flying vehicle under test.

(5) A transponder on the low-flying vehicle enhances its "visibility" by increasing the signal-to-noise ratio of the return signal.

The search consisted of a review of reports, papers, and text books on CIRIS type systems [1-6], radio ranging systems (RRS) [6-14], inertial measurement units (IMU) [15-17], barometric altimeters [18], Kalman filter [19-23], airborne radar [24-27], phased array antennas [26, 28-31], atmospheric refraction [1,2,24,32-34], and radar transponders [24-27,35]. Trips were made to the manufacturers of radio position location systems and to Yuma Proving Grounds and White Sands Missile Range to discuss the capabilities and limitations of existing equipment.

Analyses indicated that the constraint on the dimensions of the antenna, which would have to fit within a pod 3 feet in diameter and 15 feet long, and the accuracy requirements ruled out existing airborne radars. It was, however, concluded that CIRIS, which includes the RRS, IMU, barometric
altimeter, and airborne computer with Kalman filter, could determine the location of the tracking aircraft with sufficient accuracy if a large enough number of ground based transponders (18) is used.

**CIRIS**

A version of CIRIS has been assembled and test flown at Holloman Air Force Base. It is reported [36] to be consistently meeting its design goals, which were [1]:

Position: 12.5 feet RMS vector error  
Velocity: 0.05 feet/sec RMS vector error  
Attitude: 26 sec RMS vector error*

The geographic coverage, using four ground based transponders, was simulated in a very thorough computer analysis [1] as an isosceles trapezoid measuring 150 and 100 nautical miles on the two parallel sides, and 90 nautical miles on each of the other two. A race track course for an aircraft at 30,000 feet, flying between the two sites separated by 150 nautical miles, was simulated. Using the most optimistic results under the assumption that it represents the best use of measured data by the CIRIS system, it is found that the position error would be about 14 feet, RMS vector, when the aircraft is near one of the transponders. This result can serve as a starting point for estimating errors in a CIRIS type system for the WSMR mission.

Decreasing the distance between the ground based transponder sites would reduce the effect of range scale factor error (which is caused by atmospheric refraction), and the effect of geometric dilution of precision (GDOP). An adjustment can be made to the CIRIS simulation data to account for such a reduction of distance between ground based transponders.

The CIRIS simulation assumed an error of 10 ppm in the survey of its four ground sites, but WSMR survey accuracies approach 2 ppm [37]. The results of the simulation can also be adjusted to reflect the higher survey accuracies.

There is in the literature a considerable range in estimates of the errors caused by multipath and equipment delays. The CIRIS simulation used 3 feet RMS; another report [9] used 6.9 feet RMS, which is probably valid for low (below 10 degrees) elevation angles.

---

* sec = arc second
Reducing the CIRIS vector error of 14 feet to account for a ten-to-one reduction in maximum range and a five-to-one increase in survey accuracy, but increasing the error to allow for the more pessimistic estimate of 6.9 feet equipment and multipath errors, the new vector error for radio ranging with transponders about 15 miles apart would be 10 feet RMS, or 5.8 feet RMS per axis.

If the WSMR system is based on a network of squares with a ground site transponder at each corner (see Figure 2) and if the side of each square (80,000 feet) is twice the height of the aircraft above ground level (40,000 feet), the GDOP factors in vertical and horizontal axes as the aircraft crosses over the center of the square would be the minimum value possible, 0.867 [6]. Low GDOP factors could be maintained as the aircraft approaches the midpoint of a side that is common to two squares if six rather than four transponders are queried. At the crossover point, the GDOP factors would be 0.61 (vertical) and 0.775 (horizontal). For a complete coverage of the 30 by 120 mile range with the 80,000 foot squares, 27 ground based transponders would be required; but only 18 would be required for a corridor as shown in Figure 2. It appears that the CIRIS system now operating at Holloman AFB should be capable of determining the position of an airborne reference flying at 40,000 feet, with a vector error less than 10 feet RMS, or 5.8 feet per axis, with the transponder configuration of Figure 2.

The reference platform position error of 10 feet RMS vector, ascribed here to CIRIS, will be used in the following chapter to determine the allowable error budget for the link from the tracking aircraft to the low-flying target, as conceptualized in Figure 1. There are a number of position locating systems that are similar to CIRIS. These are described in Appendix D. Only CIRIS was (1) designed specifically to effect the location of an airborne instrumentation reference platform, with accuracies approaching the needs of WSMR, and (2) completed and successfully tested.

* GDOP = geometrical dilution of precision
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Figure 2. WSMR Ground Sites for Transponders in a CIRIS-Type System.
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3. DESIGN GOALS FOR AIRBORNE RADAR

When no existing radar system or equipment was found which would meet the system constraints and the WSMR accuracy requirements (detailed in Appendix A), a series of studies was initiated to determine the parameters needed for such a radar system. The feasibility of airborne conventional radar and laser radar systems are examined in this chapter. The analysis does not attempt to be rigorous but only seeks to determine feasibility, using deliberately conservative estimates of error.

Assuming that the WSMR position error requirement, stated as "10 feet in any axis" is RMS error, the total allowable vector error would be 17.3 feet RMS. It was estimated in Chapter 2 that using the CIRIS radio ranging and IMU system and the transponder configuration of Figure 2 the error in the position of an airborne reference would be 10 feet RMS vector. The vector error budget for the design goals of a radar tracker would then be

\[ \sigma = \sqrt{(17.3)^2 - (10)^2} \]

= 14.1 feet RMS, vector error budget for aircraft-to-target tracking system.

where

17.3 feet = vector error budget, target position
10 feet = vector error budget, tracking aircraft position

The tracking error budget for the airborne tracking system in each of three axes is

\[ \sigma_T = \frac{14.1}{\sqrt{3}} \]

= 8.14 feet RMS, each axis.

The sources of error in tracking from the airborne reference include:

(1) Uncertainty in the reference system attitude.
(2) Residual range and pointing angle errors arising from atmospheric refraction (after correction is made for meteorological conditions).

(3) Fluctuation range and pointing angle errors caused by local variations in atmospheric refraction.

(4) Radar resolution and equipment errors.

From the CIRIS simulation [1], the attitude uncertainty may be estimated (after making adjustments for smaller distances) as 22.8 sec

or 0.11 milliradian RMS vector. This could be budgeted as 0.078 milliradians in each of two pointing axes:

\[
\sigma_{A1} = \sigma_{B1} = \sqrt{\frac{1}{2} (0.11)^2} \text{ milliradian} = 0.078 \text{ milliradian}
\]

The convention adopted here is illustrated in Figure 3. The angle, A, measures the orientation of a vertical plane rotated about the vertical reference axis, Z, of the airborne platform, and A is measured from a reference axis which is independent of the aircraft heading (for convenience it could be true north). The angle B is measured from the true vertical downward direction, which is also independent of the aircraft attitude. The radar antenna needed for the conceptual system of Figure 1 could be assumed to use electrical corrective signals from the CIRIS system which decouple the aircraft's motion. Range is measured radially as in Figure 3 along the line-of-sight.

There is an error in the angle B caused by atmospheric refractivity, \(N_s\) (\(N_s\) is about 315). If uncorrected, the bias error is [24]:

\[
\sigma_{B2}^{(\text{uncorrected})} = N_s \cot (90 - B) \ \mu \text{rad} = N_s \tan B \ \mu \text{rad}
\]

If the angle B is restricted to be less than 55 degrees from antenna bore-sight, the maximum uncorrected error for B is 1.43 \(N_s\) microradians. It is
Figure 3. Configuration for Analysis of Airborne Tracker Errors.
assumed that correction can be made using the NBS atmospheric model to reduce the error to 10% of the uncorrected value [24] so that,

\[ \sigma_{B2} \leq (1.43)(315)(0.1) \mu \text{rad} \]
\[ \sigma_{B2} \leq 0.045 \text{ milliradian} \]

There is no corresponding bias error in the angle, A. There are, however, additional errors in both angles, A and B, caused by local fluctuations of refractivity, which can be estimated to be [8]:

\[ \sigma_{A3} \approx 0.037 \text{ milliradian} \]
\[ \sigma_{B3} \approx 0.037 \text{ milliradian} \]

A fourth error is attributable to the resolution of the radar beam and to equipment errors. It is some fraction of the 3-dB width of the beam. To estimate this error, consider [24] the AN/FPS-16, a ground based instrumentation radar which operates at 5.4 to 5.9 GHz. It has a 12-foot diameter reflector, a monopulse feed, and a \( \theta \) beam width of 1.1 degrees, or 19.2 milliradians. Its accuracy is characterized as nominally 0.1 mrad RMS bias and 0.1 mrad RMS noise, for a total of 0.14 mrad RMS, which is equivalent to 0.007 times the 3-dB beam width. It would be difficult to achieve the resolution and accuracy of the FPS-16 in an airborne radar even if the frequency is increased and the antenna scaled to fit in a 3-foot pod. The radome, the shorter wavelength, the necessarily lighter gimbal mounting, and the stringent requirements on servo drives would all add to both the bias and the noise errors in an airborne, small radar antenna.

The value of the transponder which would be mounted in the low-flying target can be seen from the following expression [24] for the error of a monopulse tracking radar:

\[ \sigma_{b} = \frac{\theta}{k_m \sqrt{2nS/N}} \]
where

\[ \sigma_\theta = \text{error in estimate of pointing angle} \]

\[ \theta = \text{3-dB beam width of the beam} \]

\[ k_m = 1.63 \]

\[ = \text{normalized monopulse slope} \]

\[ n = \text{number of pulses integrated to estimate the pointing angle} \]

\[ S/N = \text{signal-to-noise ratio of received signal} \]

The upper limit on the pulse rate is range dependent, to avoid ambiguity, and would in our case be less than 4000 pps. At 600 mph the target would travel about 0.88 feet during four pulse intervals. For \( n = 4 \), and \( S/N = 20 \) dB,

\[
\sigma_\theta = \frac{\theta}{1.63 \sqrt{8}(100)}
\]

\[ = 0.022 \theta \]

A reasonable design goal would appear to be that the errors in an airborne, mechanically pointed, millimeter radar would be about three times the FPS-16 errors, or 0.02 times the 3-dB radar beam width. This agrees with other estimates of realizable radar system accuracies of approximately one-fiftieth of the 3-dB beam width [38].

The 3-dB beam width of a monopulse radar, assuming \((\cosine)^2\) illumination, is [24]:

\[
\theta = \frac{1.44\lambda}{w} \text{ radians}
\]

where \( \lambda = \text{radar wavelength, meters} \)

\[ w = \text{aperture width, meters} \]

The tracking error, based on an estimate of one-fiftieth of the beam width, is:
\[ \sigma_{A4} = \sigma_{B4} \]

\[ = \frac{28.8 \lambda}{w} \text{ milliradian} \]

If the antenna were a phased array, \( \sigma_{B4} \) would be increased by a foreshortening of the effective width of the antenna as the angle, \( B \), increases:

\[ \sigma_{B4} \text{ (phased array)} = \frac{28.8 \lambda}{w \cos B} \]

Since it is not known whether a phased array antenna is feasible at 70 GHz and 95 GHz, only mechanically pointed antennas will be considered. If a phased array antenna is employed, the accuracy would be degraded by beam-spreading as the beam is deflected. If the antenna is a steered reflector, there is no beam spreading effect.

The total error for angle \( A \) is:

\[ \sigma_{AT} = \left[ (\sigma_{A1})^2 + (\sigma_{A3})^2 + (\sigma_{A4})^2 \right]^{1/2}, \]

and the total error for angle \( B \) is:

\[ \sigma_{BT} = \left[ (\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2 \right]^{1/2} \]

The range measurement error for an airborne radar would include all of the error sources of the range measurement in CIRIS, except that the errors in detecting the leading edge of the radar transponder pulse would be substituted for the multipath errors of the CW/DME system used in CIRIS.

A conservative estimate [9] of the error in a pulse leading edge ranging system is 5.7 feet, for a range of 120,000 feet. For a range of 100,000 feet, two distance-related error sources would be reduced, and the error in ranging to the lesser distance is estimated to be 5.6 feet. (A 1964 analysis of bias and noise static errors [25] estimated 4.5 feet RMS error can be achieved with an AN/FPS-16 radar.)
For small angles, the position errors at ranges less than $10^5$ feet can be approximated by:

\[ \Delta_A = R\sigma_{AT} \text{ feet} \]
\[ \Delta_B = R\sigma_{BT} \text{ feet} \]
\[ \Delta_R < 5.6 \text{ feet} \]

where,

- $R$ = range to target, feet
  = $H/\cos B$

- $\sigma_{AT}$, $\sigma_{BT}$ = angular errors, radians

$\Delta_R$ is based on a conservative estimate of range error of a pulse leading edge system [9]

$H$ = height of airborne platform above target

$B$ = LOS angle, measured from downward, vertical

It was estimated at the beginning of this analysis that the error budget for each axis of the target position is 8.14 feet, so the design criterion for the horizontal error due to angle $A$ is

\[ \Delta_A < 8.14 \text{ feet} \]

The horizontal error due to angle $F$ for a one-meter antenna can be expressed as:

\[ (\Delta_A)^2 = (R)^2(\sigma_{AT})^2 \]
\[ = R^2\left[\left(\sigma_{A1}\right)^2 + \left(\sigma_{A3}\right)^2 + \left(\sigma_{A4}\right)^2\right] \]
\[ = \frac{(40,000)^2}{(\cos B)^2} \left[\left(0.078\right)^2 + \left(0.037\right)^2 + \left(29\lambda\right)^2\right] \times 10^{-6} \]
\[ = \frac{1600}{(\cos B)^2} \left[0.0061 + 0.0014 + (29\lambda)^2\right] \]

The minimum error, at a given wavelength, occurs with $B = 0^\circ$, $\cos B = 1$. The criterion for minimum horizontal error is:

17
\[ (\Delta_A)^2_{\text{min}} \leq (8.14)^2 = 66.3 \]

therefore,

\[ (29\lambda)^2 \leq \frac{66.3}{1600} - 0.0075 \]

and

\[ (29\lambda)^2 \leq (0.184)^2 \]

\[ \lambda \leq \frac{0.184}{29} \]

\[ \leq 0.64 \times 10^{-2} \]

The minimum frequency to meet this error criterion is:

\[ f \geq \frac{3 \times 10^8}{0.64 \times 10^{-2}} \]

\[ \geq 4.7 \times 10^{10} \]

\[ \geq 47 \text{ GHz} \]

Both 70 GHz and 95 GHz radars and a laser radar will be examined.

The vertical measurement of position is the most critical measurement for low-flying, terrain avoidance vehicles; therefore, the vertical error, \( \Delta_Z \), is the most critical error. It can be seen that the vertical components of \( \Delta_R \) and \( \Delta_B \), in the vertical plane that contains the line-of-sight, are:

\[ \Delta_R (\text{Z component}) = \Delta_R \cos B \]

\[ \Delta_B (\text{Z component}) = \Delta_B \sin B \]
Because it is orthogonal to Z, $\Delta_A$ has no effect on $\Delta_Z$. The criterion for vertical error can be written,

$$\Delta_Z = \sqrt{(\Delta_R \cos B)^2 + (\Delta_B \sin B)^2} \leq 8.14 \text{ feet}$$

or,

$$(\Delta_Z)^2 = (5.6 \cos B)^2 + (R \sin B)^2 \left[ (\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2 \right] \leq 66.3$$

Noting that $R = \frac{H}{\cos B}$, where $H$ = height of tracking aircraft above target, substitution may be made that:

$$R \sin B = H \tan B$$

and

$$(\Delta_Z)^2 = (5.6 \cos B)^2 + (H \tan B)^2 \left[ (\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2 \right] \leq 66.3$$

This criterion will be tested for $H = 40,000$ feet, first letting the operating frequency be 95 GHz, then 70 GHz.

The horizontal error in the vertical plane which contains the line-of-sight is orthogonal to $\Delta_A$, and is:

$$\Delta_{h,r} = \sqrt{(\Delta_R \sin B)^2 + (\Delta_B \cos B)^2}$$

$$= \sqrt{(\Delta_R \sin B)^2 + \left[ (R \cos B)(\sigma_{BT}) \right]^2}$$

and, letting $H = R \cos B = 40,000$ feet

$$(\Delta_{h,r})^2 = (5.6 \sin B)^2 + (40,000)^2 \left[ (0.078)^2 + (0.045)^2 + (0.037)^2 \right. + \left. (\sigma_{B4})^2 \right] x 10^{-6}$$

$$= 31.4 \sin^2 B + 1600 \left[ 0.0095 + (\sigma_{B4})^2 \right]$$
Since $\left(\Delta_{h,r}\right)^2 \leq 66.3$, the criterion for this horizontal error is:

$$31.4 \sin^2 B + 1600 (\sigma_{B4})^2 \leq 66.3$$

$$31.4 \sin^2 B + 1600 (\sigma_{B4})^2 \leq 51.1$$

$$\sin^2 B + 51(\sigma_{B4})^2 \leq 1.63$$

Noting that $\sin^2 B$ increases as $B$ increases from zero, it is clearly sufficient to use this criterion along with the vertical error criterion to establish a maximum value for the deflection angle, $B$.

95 GHz Radar Design Parameters

For 55° deflection and $f = 95$ GHz, the values of the error components of the angle $B$ are:

$$\sigma_{B1} = 0.078 \text{ milliradian}$$

$$\sigma_{B2} \leq 0.045 \text{ milliradian}$$

$$\sigma_{B3} = 0.037 \text{ milliradian}$$

and

$$\sigma_{B4} = \frac{(0.0288)(3)(10^8)}{(1)(95)(10^9)} = 0.091 \text{ milliradian}$$

Testing the error criterion, with $B = 55^\circ$, $f = 95$ GHz, and $H = 40,000$ feet,

$$\left(\Delta_z\right)^2 = [(5.6)(0.574)]^2 + [(40,000)(1.428)]^2 \left[(0.078)^2 + (0.045)^2 + (0.037)^2 + (0.091)^2\right] \times 10^{-6}$$

$$= 10.3 + (1600)(2.04) \left[0.0061 + 0.0020 + 0.0014 + 0.0083\right]$$

$$= 10.3 + 58.1$$

$$= 68.4 \leq 66.3$$
However, with a mechanically steered antenna holding the target at boresight, the vertical error criterion can be shown to be met if the deflection angle, \( \theta \), is 54 degrees or less. When \( \theta = 54^\circ \),

\[
(\Delta z)^2 = [(5.6)(0.588)]^2 + (1600)(1.376)^2 \cdot 0.0178
\]

\[= 10.8 + 53.9 \]

\[= 64.7 < 66.3 \]

Testing the criterion for horizontal error, when \( \theta = 55^\circ \) and \( f = 95 \) GHz:

\[
\sin^2 \theta + 51(\sigma_{B4})^2 = (0.819)^2 + 51(0.091)^2
\]

\[= 0.671 + (0.51(0.83)) \]

\[= 1.09 < 1.69 \]

It is seen that at an operating frequency of 95 GHz, if a 1 meter antenna is used and the altitude of the CIRIS platform is 40,000 above the low-flying target, and if the line-of-sight deflection angle remains less than 54 degrees, the vertical target position error will not exceed 10 feet, RMS. Range from the airborne platform to the low-flying target would be 68,000 feet maximum.

70 GHz Radar Design Parameters

If the operating frequency of the airborne radar is 70 GHz, \( \sigma_{B4} \) will be increased:

\[
\sigma_{B4} = (0.091) \left( \frac{90}{70} \right)
\]

\[= 0.118 \text{ millirad.} \]

Testing the vertical error criterion for 70 GHz operating frequency, and \( \theta = 50^\circ \) degrees,
\[
(\Delta Z)^2 = \left[(5.6)(0.643)\right]^2 + (1600)(1.19)^2 \left[0.0061 + 0.0020 + 0.0014 + (0.118)^2\right] = 13.0 + (2272)(0.0234) = 13.0 + 53.2 = 66.2 < 66.3
\]

The horizontal criterion for \( f = 70 \text{ GHz} \) and \( B = 50 \text{ degrees} \) can also be tested:

\[
\sin^2 B + 51(\sigma_{B4})^2 = 0.587 + 0.710 = 1.30 < 1.64
\]

**Laser Radar**

A conceptual airborne laser radar boresighted with a conventional radar is suggested by a recent article which describes such a combination in a ground based installation [39] and by a description of a laser spacecraft communication system [40]. The conventional radar would be used for target acquisition, and final target lock-on and dish steering would be performed by the laser. The design parameters for the laser radar can be estimated.

The altitude error estimated for the CIRIS type reference platform was:

\[
\sigma_{B1} = 0.078 \text{ milliradian}
\]

The errors due to atmospheric refraction can be extrapolated by noting how the atmospheric model for radio frequencies differs from the model for optical frequencies [34]. For radio frequencies, the atmospheric index of refraction is:

\[
n_R = 1 + \left[ \frac{77.6P + 3.73 \times 10^5 e}{T^2} \right] \times 10^{-6}
\]
where

\( n_R \) = index of refraction at radio frequencies
P = atmospheric pressure, millibars
T = temperature, degrees Kelvin
e = partial pressure of water vapor, millibars

For optical and infrared frequencies the index of refraction is:

\[
n_0 = 1 + \left[ \frac{77.6P}{T} + \frac{0.584P}{T\lambda^2} \right] \times 10^{-6}
\]

where
\( \lambda \) = wavelength, microns
\( n_0 \) = index of refraction at optical frequencies

Noting that the expressions differ in the last term only, let

\[
\frac{0.584P}{T\lambda^2} = \frac{3.73 \times 10^5 e}{T^2}
\]

and solve for \( \lambda \) to obtain an expression that represents equivalence of water vapor and wavelength effects on refractivity:

\[
\lambda = 1.25 \times 10^{-3} \sqrt[3]{\frac{PT}{e}}
\]

Letting \( P, T, \) and \( e \) have the following ranges:

\[
950 < P < 1050 \text{ mb}
\]
\[
1 < e < 30 \text{ mb}
\]
\[
274 < T < 318 \text{ deg K}
\]

the corresponding range in \( \lambda \) would then be:

\[
0.12 < \lambda < 0.72 \text{ microns.}
\]

This represents the range of optical wavelengths which contribute to atmospheric refractivity the same error as contributed by water vapor.
from 1 to 30 mb partial pressure. Wavelengths longer than 0.72 microns will have less effect on refractivity than 0.1 percent water vapor. The radio frequency model which was used to estimate position errors of the CIRIS type airborne reference assumed standard atmospheric conditions of temperature (288°K), pressure (1013 mb), and about 1.0 percent water vapor, or a partial pressure of 10.2 millibars (relative humidity 60 percent). The refractive index for radio frequencies under these conditions is:

$$n_R = 1 + 77.6 \left[ \frac{1013}{288} + \frac{(4807)(10.2)}{(288)^2} \right] \times 10^{-6}$$

$$= 1 + 77.6 \left( 3.518 + 0.5911 \right) \times 10^{-6}$$

If the water vapor content were reduced by a factor of ten, to one millibar, the refractive index would be

$$n_{R(dry)} = 1 + 77.6 \left( 3.518 \times 0.0591 \right) \times 10^{-6}$$

and the error term for conventional radar would in this lower R.H. case be reduced by 12.9%. If the laser radar operates at 1.06 micron, the error for refraction of the atmosphere can be extrapolated from the estimates of refractivity error for radar:

$$\sigma_{B2} \leq (0.045)(0.875)$$

$$= 0.039$$

$$\sigma_{B3} \leq (0.037)(0.875)$$

$$= 0.032$$

The estimate of the beam width error in the case of radar included pointing resolution error, equipment error, multipath effects, etc. It would be possible, though not desirable, to make the laser beam extremely sharp. Instead, it should be made broad enough to maintain illumination of a retroreflector mounted on the target. The beam width requirement
would be defined by the optics of the system, the detector, and the system
dynamics. The basic RMS pointing error, in a diffraction limited system,
has been expressed as [40]:

$$\sigma_\theta = \frac{1.22\lambda}{D \cdot (S/N)} \text{ radian}$$

where
- $\lambda =$ wavelength, meters, = $1.06 \times 10^{-6}$
- $D =$ aperture of optics, meters
- $S/N =$ signal-to-noise ratio, assumed to be greater than 10.

Thus,

$$\sigma_\theta < \frac{1.3 \times 10^{-4}}{D} \text{ millirad}$$

and if $D > 10 \text{ cm}$,

$$\sigma_\theta < 0.001 \text{ milliradian}$$

As we have noted, however, the error that is equivalent to the radar
beam width error would include equipment errors. The "absolute accuracy"
specification for a commercially available ground based laser radar, PATS
(Precision Automated Tracking System), is:

$$\sigma_{B4} = 0.1 \text{ milliradian}.$$  

The retroreflector used by PATS is 3 inches in diameter, which subtends an
arc at a range of 50,000 feet of only 0.005 milliradian. Improvement of
$\sigma_{B4}$ to 0.02 milliradian instead of 0.1 milliradian might be feasible, but
system dynamics (control) errors would probably be limiting.

The range error for laser radar should be considerably less than the
5.6 feet assumed for radar, because there is no delay uncertainty associated
with the transponder, and because the range/timing pulse can be of the order
of one to ten nanoseconds. The primary sources of range error are the resid-
ual error from the corrected atmospheric index of refraction, plus counter 
logic (start/stop) uncertainty. These sources, in PATS, produce an "absolute" 
range error specification of 2 feet at 65,000 feet.

The vertical error criterion of 8.14 feet can now be tested, letting 
B = 60°, and using the following error values:

\[ \sigma_{B_1} = 0.078 \text{ milliradian} \]
\[ \sigma_{B_2} = 0.039 \text{ milliradian} \]
\[ \sigma_{B_3} = 0.032 \text{ milliradian} \]
\[ \sigma_{B_4} = 0.1 \text{ milliradian} \]

\[ \sigma_z = \sqrt{(\Delta_R \cos B)^2 + (\Delta_B \sin B)^2} \leq 8.14 \]
\[ = \sqrt{(\Delta_R \cos B)^2 + (H \tan B \sigma_{BT})^2} \]

\[ (\Delta_z)^2 = [(2)(0.5)]^2 + (1600)(3) [(0.078)^2 + (0.039)^2 + (0.032)^2 + (0.1)^2] \]
\[ = 1 + 89.4 \leq 66.3 \]

The test fails for B = 60°, but for B = 55 degrees:

\[ (\Delta_z)^2 = [(2)(0.574)]^2 + (3263)(0.0186) \]
\[ = 1.3 + 60.7 \]
\[ = 62.0 < 66.3 \]

With a laser radar, deflection from vertical downward direction would 
have to be less than 55 degrees to keep the vertical error in the target 
position less than 10 feet RMS.
System Constraints

One consequence of the restrictions on the angle B is that the ratio of the speed of the target vehicle to the speed of the tracking aircraft is also restricted. Assume, for example, the tracking aircraft is 40,000 feet above ground level, the tracking radar is 70 GHz, and the test flight begins with the low-flying vehicle 48,000 feet (horizontal distance) behind the tracking aircraft (B = 50°). The target would pass under the tracking aircraft at midcourse and at the end of a 600,000-foot run would be 48,000 feet in the lead, only if the target is 16 percent faster than the tracker. If the target is too fast it will pull too far ahead, the angle B would exceed 50°, and the error in the estimate of target altitude would become excessive.

Summary

It has been established that if the system errors are as estimated, a 70 GHz radar, a 95 GHz radar, or a laser radar would permit measurements of position of a low-flying target from a CIRIS type airborne platform, with errors in each of three orthogonal axes (two horizontal and one vertical) less than 10 feet, RMS. Table I summarizes the system constraints on line-of-sight angles.

Some of the sources of error, such as the servos which point the radar reflector and the delay uncertainty of the transponder mounted in the low-flying target, have been assumed to be included in the estimates of $\sigma_B$, and $\Delta_R$. If the error estimates are too small, the angular deflection, B, would have to be reduced, the frequency of the operation raised, or the operating altitude lowered. These parameters, together with their effects on the necessary ground based transponder configuration, constitute trade-off components.

A system constraint that has been established is that the speed of the low-flying target cannot greatly exceed the speed of the tracking aircraft because the angle of the line-of-sight from the platform to the target must be held within about 50 degrees from the downward vertical direction. With this constraint a 600 mph aircraft could track a target only if the target does not exceed 715 mph.
TABLE I

DESIGN CONSTRAINTS FOR AIRBORNE RADARS THAT PERMIT MEASURING POSITION OF LOW-FLYING TARGE WITH ACCURACY OF 17.3 FEET, RMS, VECTOR

<table>
<thead>
<tr>
<th>Frequency/Wavelength</th>
<th>Antenna</th>
<th>Maximum LOS Angle Degrees</th>
<th>Maximum Range</th>
<th>Maximum Ground Projection of Range</th>
<th>Ratio of Target Speed to CIRIS Platform Speed*</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 GHz</td>
<td>Reflector</td>
<td>54</td>
<td>1.70H</td>
<td>1.38H</td>
<td>1.22:1</td>
</tr>
<tr>
<td>70 GHz</td>
<td>Reflector</td>
<td>50</td>
<td>1.55H</td>
<td>1.19H</td>
<td>1.19:1</td>
</tr>
<tr>
<td>1.06 micron</td>
<td>Laser</td>
<td>55</td>
<td>1.74H</td>
<td>1.43H</td>
<td>1.24:1</td>
</tr>
</tbody>
</table>

*Assumes H = 40,000 feet, total run = 600,000 feet.
The most influential vertical error component is a scale factor multiplier, $H \tan B$, where $H$ is the height of the tracking aircraft above the target, and $B$ is the angle of the line-of-sight measured from downward vertical. This is a range related error, inasmuch as $H \tan B$ is the projection onto the ground of the range from the tracking aircraft to the low-flying target. If the maximum deflection of $B$ is $55^\circ$, and the tracking aircraft is at 40,000 feet AGL, maximum range to target is about 57,000 feet.

Appendix E describes state-of-the-art 9.5 GHz and 70 GHz prototype radars that have been fabricated and evaluated by Georgia Tech. The state-of-the-art in laser ranging and tracking is described in the literature [39,40].

R and D Program

To develop an airborne radar tracking system, for closing the link from aircraft to target in the system of Figure 1, will require a major R & D effort. A significant part of that effort would be devoted to the development of a tracking antenna and radome, for mounting in an airborne pod. The pointing mechanism would have to be capable of pointing the antenna with an accuracy of about $10 \text{ sec}$ in each of two orthogonal axes, and reading out the angles with comparable accuracy. This is approximately the performance level of the AN/FPS-16 radar.

The advantages of a phased array antenna for tracking low-flying vehicles are enumerated in Appendix B, and those advantages would probably outweigh the drawback of beam spreading and loss of resolution as the beam is deflected from normal to the array plane. However, there are no phased array millimeter wavelength antennas known to the authors. The development of such an antenna would be a higher risk and would cost more than the development of a steered dish antenna. Indeed, it would first have to be determined that phased array antenna elements are available or feasible at 70 GHz or 95 GHz.

The development of an airborne laser tracking radar to be boresighted with a Ku band acquisition radar would be an R and D effort comparable to developing an airborne millimeter radar.
4. SYSTEMS BASED ON AVAILABLE EQUIPMENT

Three systems have been conceived that could partially meet the WSMR requirements using available equipment. One of these is a totally ground based system using the Sylvania laser tracking radar (PATS), which is commercially available. The second conceptual system would be like that of Figure 1, with a Ku band radar in the tracking aircraft, or some means of maintaining the tracker as near as possible directly above the low-flying vehicle. The third system would be a multilateration ranging system; target position would be computed from radio range signals between the target and at least three ground stations, between the target and the airborne platform, and between the airborne platform and the ground stations.

The ground-based laser radar network could meet the WSMR requirements for accuracy, at a cost that would be high but perhaps not beyond reason. The station keeping Ku band radar would permit ranging measurements of the altitude of the low-flying vehicle within the WSMR accuracy requirements, but horizontal location errors would be on the order of 40 feet RMS. An analysis of errors in the "pyramidal" multilateration system indicates acceptable accuracy can be obtained, with "good" atmospheric conditions.

Ground Based Laser Tracking System

The Sylvania laser radar (PATS) specifies a range accuracy of 2 feet and an angular resolution of 0.1 milliradian about two axes out to 65,000 feet of range. The tracking rate is about 30 degrees per second.

A PATS laser radar has been observed tracking a helicopter at about 36,000 feet range, alternately against sky and desert background. The laser radar was sometimes depressed below horizontal. Acquisition was accomplished with the aid of a video camera which was boresighted with the laser. The operator first nominally centered the target on a TV screen with joystick control, then he activated laser lock-on.

The jitter of the target as seen on the TV display appeared to be about 5 feet. This corresponds to a pointing angle error of 1/7 milliradian.
A video recording made during earlier tests showed PATS tracking a mortar shell (fitted with a retroreflector) at the same distance of 36,000 feet.

A system consisting of nine or ten such laser radars positioned every 10 miles along the test flight path, and set back about 4 miles, would meet the WSMR requirements for location accuracy.

The cost of one tracker is about $600,000, but the unit cost of several such systems could be considerably less, enough so to consider such a system.

Each laser radar station would locate the target vehicle with respect to its own position. Range, azimuth, and elevation are measured. The reduction of these data to location of the target on a master coordinate system is a modest computation which must be made at each station in order to supply the next station orders for acquisition.

A minimum layout of 9 or 10 trackers, while not covering the whole of WSMR, would provide a test corridor which could include a number of alternative paths.

As supplied to Yuma, the PATS angular tracking rate (500mr/sec) is adequate for targets to above Mach two if the crossing range is kept greater than about 5000 feet.

At the laser wavelength of 1.06 microns there is no multipath error, because surface roughness scatters the signal instead of reflecting it, and the retroreflector is a very efficient transponder. Range scale error should be only about a foot at 10^5 feet, after bias error due to atmospheric refraction has been corrected by using a model for the atmosphere. Angular error associated with refraction and scintillation will limit the system when the air is unstable over the line-of-sight, and it may be that the times of worst atmospheric conditions must be avoided. (This would be true for conventional radar systems, too.) About ninety percent of the bias error can be removed by refractivity correction, and straightforward smoothing of the target trajectory will reduce the effects of residual fluctuations due to local atmospheric turbulence.

It may be desirable to record both the angular encoder outputs and the error signal, rather than have the trajectory smoothed by the response
of the servo systems and the smoothing filters that are used in the present installation. Subsequent smoothing of the recorded data could be more effective than real time smoothing. The tape recording format of PATS is compatible with computer processing.

If modest improvements can be made to reduce the pointing angle errors below the 0.1 milliradian that the manufacturer claims, the PATS laser units could replace theodolites as basic range instruments. Data turn-around time could be greatly reduced.

Overhead Tracking, Ku Band

The measurement of altitude of low-flying, terrain avoidance vehicles is usually more critical than the determination of horizontal position coordinates. If the accuracy requirement for horizontal position determination is relaxed, the frequency of the radar for the system described in Chapter 3 could be as low as 17 GHz. The antenna could also be smaller than one meter, say 0.5 meter. It could also be rigidly mounted and pointed nominally downward, with a display for the pilot which would enable him to maintain the aircraft very nearly over the low-flying vehicle. When sighting directly down, the vertical error in the ranging signal could be estimated as 5.6 feet, using the same assumptions as in Chapter 3. The error normal to the line-of-sight would be:

\[ \Delta_B = H_0 \sigma_{BT} \]

where

\[ \sigma_{BT} = \sqrt{(\sigma_{B1})^2 + (\sigma_{B2})^2 + (\sigma_{B3})^2 + (\sigma_{B4})^2} \] radians

and

\[ \sigma_{B1} = 0.078 \text{ milliradian} \]
\[ \sigma_{B2} = 0.045 \text{ milliradian} \]
\[ \sigma_{B3} = 0.037 \text{ milliradian} \]
\[ \sigma_{B4} = \frac{(1.44)(3)(10^8)}{(50)(0.5)(17.5)(10^9)} = 1 \text{ milliradian} \]
Therefore, if $H = 40,000$ feet

$$
\Delta B = 40 \sqrt{(0.078)^2 + (0.045)^2 + (0.037)^2 + (1.0)^2}
$$

$$
= 40 \sqrt{(0.0061 + 0.0020 + 0.0014 + 1.0)}
$$

$$
\approx 40 \text{ feet}
$$

As the beam deflects from directly downward, for small angle $\theta$ the vertical error criterion is:

$$
(A_z)^2 = (5.6 \cos \theta)^2 + (40 \sin \theta)^2
$$

$$
= \cos^2 \theta \left[(5.6)^2 + (40 \tan \theta)^2\right]
$$

$$
\leq 66.3
$$

But for $\theta \leq 10^\circ$, $\cos^2 \theta \approx 1.0$, and

$$
\tan \theta \leq 0.148
$$

$$
\theta \leq 8.4^\circ
$$

By using a sufficient number of ground sites, placed to minimize vertical error in determining the position of the tracking aircraft, the error in estimating the altitude of the low-flying vehicle can be held to less than 10 feet RMS. Eighteen ground sites placed at the corners of squares as shown in Figure 2 would seem to be sufficient for a flight-path corridor.

The following section examines the possibility of utilizing only ranging measurements, from ground sites to the target and to the over-flying aircraft, and also from the aircraft to the target.
Multilateration with Two Vehicles

If one attempts to determine the position of a low-flying vehicle from ground-based radio ranging signals, five categories of error sources affect the accuracy. One error category is associated with the modulation type. If FSK FM is used, multipath causes phase error in the demodulated signal, and hence causes error in range estimates. If pulse modulation is used, the pulse becomes distorted in transmission and detection error results. The second category of error is associated with equipment noise and delays. A third source of error is related to the inability to completely compensate for the atmospheric index of refraction. This error is directly proportional to range; hence, it is a scale factor error. The fourth error category is due to fluctuations of the index of refraction along the path of propagation. It is related to measurement time interval. The effect is negligible for sampling rates faster than one per minute, and for distances less than 100 miles. The fifth error category is survey error, which produces uncertainty in the location of ground stations.

The technique of Figure 1 would use multilateration to estimate the position of an over-flying reference aircraft, and would estimate the position of the target relative to the reference aircraft by radar measurements of range and pointing angles. Analysis of this system, previously discussed, has shown that present state-of-the-art airborne radar equipment cannot meet the error specifications desired.

Another approach that is attractive would use multilateration from ground based stations to locate both the over-flying reference aircraft and the low-flying target, with only the horizontal coordinate estimates of the target being retained. The target altitude could then be calculated from range measurements made from the chase ship to the target. Such a system would avoid the large uncertainties associated with angular measurements made with available airborne radar. The following analysis considers such a system and uses a physical approach to develop estimates of the errors.

It will be shown that a radio ranging, multilateration approach can yield measurements of position of the low-flying target with 10-foot RMS accuracy in any axis. The aircraft must stay within about 20,000 feet, horizontally, of an overhead position above the target to maintain the desired altitude measurement accuracy.
The over-flying aircraft will be located by lateration from only three stations in the example presented. Redundant measurements from additional ground stations could improve the position estimates.

The test vehicle will be located by lateration from ground stations but only its horizontal position will be further utilized, the altitude being too poor a result to retain.

Thus, six range signals locate the aircraft in space and the test vehicle horizontally. A seventh range signal from the aircraft to the test vehicle permits the altitude of the test craft to be determined.

The propagation of each range measurement uncertainty into this altitude determination will be developed. An estimate of precision of the horizontal location will also be made.

In actual use such a system should perform better than the analysis indicates because redundant data will sometimes be taken from more than three ground stations at a time. Also, after the altitude of the test craft is determined as described, the horizontal location would be recalculated, again using the original information but this time with a redundant range to a station. Such iteration would improve the results.

Propagation of Uncertainties

When the effect of an uncertainty in range from a ground station to an airborne vehicle affects a derived quantity, one needs to know the sensitivity of the latter to the former. In our case the derived quantity will be a position coordinate of the vehicle such as altitude. If a mathematical expression is available relating altitude to range from the three stations then the sensitivity is represented by \( \frac{\partial Z}{\partial r_k} \), where \( Z \) is the altitude and \( r_k \) is the range from the station in question.

In order to avoid developing a complete expression \( Z = f(r_1, r_2, r_3) \), and to keep a strong physical meaning attached to each step, a computation of \( \frac{\partial Z}{\partial r} \) will be made directly. Figure 4 shows any three ground stations A, B and C. We seek \( \frac{\partial Z}{\partial r} = \left( \frac{\partial Z}{\partial r_k} \right) A r_B r_C \). Physically, if \( r_B \) and \( r_C \) are maintained fixed, then the position of the intersection of \( r_A \), \( r_B \), and \( r_C \) at D must lie on a circle in a vertical plane that is centered on the line joining B and C.
Figure 4. Configuration for Analysis of Multilateration System Errors.
The radius is the line of fixed length, c; c is determined by the sides of the triangle DBC, all three sides of which are of fixed length, and by the requirement that the line pass through D and be perpendicular to BC. Considering B and C to be on level ground for this idealization, BC is a horizontal line and the plane in which c is restrained to turn is a vertical plane.

In order to write analytic expressions a coordinate system has to be selected. Choose Z as vertical, the origin at station A, the direction Y parallel to BC. The plane containing line c is a plane described by Y = const. D is constrained to a circle on this plane as shown in Figure 4. The equation of this circle is,

\[ c^2 = (e - X)^2 + Z^2 \]  

(1)

where e is the X-coordinate of stations B and C.

When \( r_A = r \) is determined, D is fixed, locating our vehicle. Determination of r fixes D on a sphere of radius r centered at 0, (point A) given by

\[ r^2 = x^2 + y^2 + z^2 \]  

(2)

A condition satisfying (1) and (2) simultaneously is a relation associated with the position of D, the intersection of the sphere with the circle.

There are many manipulations of (1) and (2) that will give the following results. In our analysis Y, c, e are considered fixed quantities, while r is allowed to vary. It is clear that any variation is r is accomplished by variation in both X and Z, so uncertainty in Z arising out of uncertainty in Z arising out of uncertainty in r is correlated with uncertainty in X.

The mutual dependence of Z and X can be derived by differentiating (1),

\[ 0 = -2(e - X) + 2Z \frac{\partial Z}{\partial X}, \]

\[ \frac{\partial Z}{\partial X} = \frac{e - X}{Z} \]  

(3)
The dependence of $X$ on $r$ can be obtained by differentiating the expression obtained by substituting the value of $(X^2 + Z^2)$ from (2) into (1),

$$c^2 = e^2 - 2eX + r^2 - Y^2,$$

$$0 = -2e \frac{\partial X}{\partial r} + 2r,$$  \hspace{1cm} (4)

$$\frac{\partial X}{\partial r} = \frac{r}{e}.$$  \hspace{1cm} (4)

Combining equations (4) and (3) gives

$$\frac{\partial Z}{\partial r} = \frac{e - X}{e} \frac{r}{Z},$$

$$\frac{\partial Z}{\partial r} = \frac{1}{\sin \alpha} \frac{e - X}{e},$$  \hspace{1cm} (5)

where $\alpha$ is the elevation angle of $D$ from the ground station, as shown in Figure 4. Since all effects due to $r$ lie in the plane $Y =$ const, then

$$\frac{\partial Y}{\partial r} = 0$$  \hspace{1cm} (6)

All horizontal uncertainty associated with range measurement from a particular station lies in a direction perpendicular to the line joining the other two stations.

In computing the uncertainty of position of the airborne craft at $D$, there will be uncertainties in $r$, which must be summed up, such as the scale factor error due to uncertainty of the index of refraction over the transmission path, the errors due to measurement instrument noise and multipath, and the error due to instrument bias.
Another source of error is in the survey of the ground station itself. The effect of the survey error on the location of the origin in Figure 4, should the displacement be along an arc centered under D and perpendicular to $\rho$, will have no effect on the computation of D. The measured r will have the same value and the same uncertainties as if it were measured from the assumed origin, and the resulting computation of the location of D will not be affected.

Should the dislocation of the origin be along $\rho$ then the r being measured is not the r from the assumed origin. Figure 5 is a sketch of the vertical plane containing both $\rho$ and r. The displacement of the station along $\rho$ increases the measured r by

$$\delta r = \delta \rho \cos \alpha$$  \hspace{1cm} (7)

The effect on the computations will be the same as mismeasuring r by the amount given by (7). Ordinarily, survey errors are uncorrelated with range measurements, so an uncorrelated uncertainty of the amount indicated in (7) must be added to other errors associated with r.

If there is vertical as well as horizontal survey uncertainty, its component parallel to the range vector will contribute to position computation just like the parallel component of horizontal survey error, (7). No vertical survey error will be included in this analysis.

The partial derivatives, and all the contributions to $\delta r$ just discussed are not limited to the coordinate system of Figure 4; it is only necessary that their proper geometrical meanings be recognized. To get the full uncertainty in the altitude, $Z$, and the uncertainties in horizontal position, it will be necessary to account for the other two stations, whose range measurements are partially correlated, as will be seen. To do this, we select a single coordinate system, $(x, y, z)$ for all three stations. For each station we compute the uncertainty in $x_A$, $y_A$, and $z_A$, the coordinates of the aircraft in this system, due to each of the stations.
Figure 5. Effect of Displacement (Survey Error) of Ground Site.
Thus

\[ \delta x_A = \frac{\partial x_A}{\partial r_1} \delta r_1 + \frac{\partial x_A}{\partial r_2} \delta r_2 + \frac{\partial x_A}{\partial r_3} \delta r_3 \]  

(8)

and likewise for \( \delta y_A \) and \( \delta z_A \). The subscript \( A \) refers to the overflying aircraft. Thus, \( r_{A2} \) refers to the range to the aircraft from station 2.

Here \( \frac{\partial x_A}{\partial r_1} \) is the component of \( \frac{\partial X}{\partial r} \), of (4), parallel to the x axis, etc.

Use of expressions like (8) to compute the covariance elements of the aircraft position will be described below. As developed, \( \frac{\partial x_A}{\partial r_1} \), etc., are differential expressions for displacements arising from any source.

Our present interest is to estimate the quality of a location of the test vehicle below the aircraft, rather than the location of the aircraft itself. The position of the test vehicle will first be determined as was that of the airplane, although the vertical uncertainty will be very large due to the low values of \( a \). This altitude uncertainty will be so great that the vertical determination will be abandoned, and used no further in the computations; and only the horizontal position estimates will be retained.

The uncertainties in the ranges from the three ground stations to the test vehicle will probably be larger than the errors in ranging to the aircraft because of the greater uncertainty due to multipath.

Altitude of the test vehicle will be determined from an additional radio ranging from the overflying aircraft. Its altitude will be given by

\[ z_T = z_A - r_a \cos \beta \]  

(9)

where \( \beta \) is the angle between the vector from the aircraft to the test craft and the vertical. The range from the aircraft to the test vehicle is \( r_a \) and the aircraft altitude is \( z_A \). We need (9) in the form where it depends upon the seven range measurements used to compute it. Let the horizontal displacement of the test vehicle relative to the aircraft be

\[ h = \sqrt{x_T^2 + y_T^2} = \sqrt{(x_T - x_A)^2 + (y_T - y_A)^2} \]  

(10)
then
\[ \beta = \sin^{-1} \frac{h}{r_a} \]  

(11)

and (7) becomes
\[ z_T = z_a - r_a \cos \left( \sin^{-1} \frac{\sqrt{\xi^2 + \eta^2}}{r_a} \right) \]

(12)

The partial derivatives of interest come out of (12):

\[ \frac{\partial z_T}{\partial z_a} = 1 \]  

(13a)

\[ \frac{\partial z_T}{\partial r_a} = -\cos \beta - r_a \left( \sin \sin^{-1} \frac{\sqrt{\xi^2 + \eta^2}}{r_a} \right) \left( \frac{\sqrt{\xi^2 + \eta^2}}{r_a \sqrt{1 - \frac{\xi^2 + \eta^2}{r_a^2}}} \right) \]

(13b)

\[ = -\cos \beta - \frac{\sin^2 \beta}{\cos \beta} = -\frac{1}{\cos \beta} (\cos^2 \beta + \sin^2 \beta) = -\frac{1}{\cos \beta} \]

\[ \frac{\partial z_T}{\partial \xi} = r_a \sin \beta \frac{\xi}{\sqrt{1 - \frac{\xi^2 + \eta^2}{r_a^2}}} \frac{\sqrt{\xi^2 + \eta^2}}{r_a \sqrt{\xi^2 + \eta^2}} = \frac{\xi}{r_a \cos \beta} \]  

(14)

Similarly,

\[ \frac{\partial z_T}{\partial \eta} = \frac{\eta}{r_a \cos \beta} \]  

(15)

An uncertainty in \( z_T \) can be expressed,

\[ \delta z_T = \left( \frac{\partial z_T}{\partial z_a} \right) \delta z_a + \left( \frac{\partial z_T}{\partial r_a} \right) \delta r_a + \left( \frac{\partial z_T}{\partial \xi} \right) \delta \xi + \left( \frac{\partial z_T}{\partial \eta} \right) \delta \eta \]  

(16)
\[ \delta z_T = (\delta z_a) - \frac{1}{\cos \beta} \delta r_a + \frac{c}{r_a \cos \beta} \delta z_c + \frac{\eta}{r_a \cos \beta} \delta \eta \]  

(16)

where \( \delta z_a \), \( \delta z_c \), and \( \delta \eta \) need to be expanded in terms of the six ranges measured from the three stations.

**The Measurement Uncertainties**

The range uncertainty from a ground station to the overflying aircraft can be due to several sources. For any one station

\[ \delta r_A = \delta r_{A,N} + \delta r_{A,B} + \delta r_{A,R} + \delta r_{A,S} \]

\[ = \delta r_{A,N} + \delta r_{A,B} + \delta r_{A,R} + \delta \rho \cos \alpha \]  

(17)

where subscript B refers to bias in the instrument zeros due to adjustment tolerances and drift, N to noise and multipath, S to station survey uncertainty, and R to scale factor error associated with uncertainty in the index of refraction over the path.

There will be a differential expression like (17) for each of the three ranges to the aircraft, and likewise for each of the three ranges to the test craft. The range, \( r_a \), from the overflying aircraft to the test vehicle has a similar expression except there is no survey error term.

Twenty-seven sources of uncertainty have been identified, four for each of six range measurements from the ground stations, and three for the range measurement between the two vehicles. It would be straightforward but unwieldy to expand the differential expressions for \( \delta z_T \), \( \delta x_A \), etc. in terms of these 27 uncertainties. Rather, we will collapse the 27 into the covariance elements of the seven range measurements and develop the uncertainty in vehicle location from the expressions already set down.

**Correlations**

The four sources of uncertainty identified in (17) are independent in any one range measurement, and therefore no correlation between them is to be expected. Also, no correlation is to be expected between sources of
different type in different range measurements. However, certain correlations do exist between errors arising from the same type source, in the different range measurements, and these correlations are too strong to be ignored.

Noise in the querying device and in the transponder lead to timing errors and therefore to range errors. Different range measurements will normally be made at different times, so there will be no correlation of the noise of one range measurement with that of another.

Multipath consists of addition of direct path ranging signals with reflected signals. The reflected signals can pull the time measurement either way, depending on their phase. This is true when complex coding such as PSK/FM is employed, and also when very short pulse signals are used. Multipath error depends in a complex way, but strongly, on the detailed geometry of the location of the interrogator and the transponder, and on the surrounding topographical feature. Correlation of multipath errors between range measurements over different paths is not expected.

Noise and multipath can be lumped together in the computations which follow because they constitute all of the sources considered that correlate with no other sources.

The three remaining sources, while not correlating with each other, are correlated between some paths. The most obvious is survey error of the stations. We envision range measurements to the overflying aircraft and to the test vehicle from each of the ground stations. The test vehicle will be at nearly the same azimuth from any one station, so there is strong correlation between $\delta r_{A,S}$ and $\delta r_{T,S}$ from any one station. Whatever the error is, the two will be proportional to one another and have the same sign, therefore a correlation coefficient of one can be used. (It is possible to account for the slightly different expected azimuth angles by using the average cosine of the azimuth difference, instead of using one; but this average cosine will typically exceed 0.95 in the system under consideration.)

Bias error in any range measurement is associated with alignment tolerances and subsequent circuit drifts which vary time delays in the transponder, the interrogating transmitter, and the receiver. Each range to be measured will share a transponder or interrogator with another range, and the bias in the
common device will correlate between these ranges. The expected magnitude of bias in a transponder will not equal that in an interrogator, but this can be dealt with by dividing bias error into two separate source types, one for transponder, one for interrogator. Wherever a common instrument is involved in two range measurements the correlation will be one. Alternately proper fractional correlation coefficients can be assigned between ranges with a common instrument without dividing bias error into two types. The latter is done in the examples which follow, where for illustrative purposes it is assumed that the bias tolerance in both the transponders and the interrogators is the same. In our example, the appropriate correlation coefficient is one half for each of two ranges which have one instrument common to their measurements.

Range scale factor uncertainty is associated with the inability to model index of refraction of the atmosphere precisely. For paths involving a large range of altitude some of the parameters of the atmospheric model are adjusted to match measured meteorological data. The errors in measurement and the failure of the model to match the atmosphere, averaged horizontally at each level, cause the resulting scale factor errors in range measurements to correlate. On the other hand, local variations in the atmospheric index of refraction will not correlate. Some fractional correlation between all $\delta_{A,R}$'s and $\delta_{T,R}$'s is therefore expected. Information is not presently in hand to assert what this correlation should be. Some not unreasonable values have been selected for the illustrative examples as follows:

- For two paths to the high flying aircraft, through many level strata of the atmosphere, range scale correlation = 0.4.
- For two paths from two ground stations to the low flying test vehicle (nearly horizontal paths over widely different surfaces), range scale correlation = 0.1.
- For a nearly horizontal path from one ground station to the test vehicle and an elevated path to the overflying aircraft from a different ground station, range scale factor correlation = 0.1.
For a horizontal path to the test vehicle and an elevated path to the overflying aircraft from the same ground station, range scale factor correlation = 0.2.

When a basis for better correlation coefficients for range scale factor errors is available, then they can be incorporated into the analysis just as these values will be in the examples which follow.

Configuration for the Analysis

Envisioned is two nominally parallel lines of ground stations, one on each side of a test corridor. To idealize this for analysis the ground stations have all been placed at the same altitude at the apexes of equilateral triangles as in Figure 6. The triangles were chosen to have sides 120,000 feet (20 n.m.) long. An analysis for three ground stations ranging at any one time corresponds to an analysis of the interior of any one of the triangles. Figure 7 is one equilateral triangle in which 5 points, D, E, F, G, H form a suitable collection for evaluating the concept. These 5 points correspond to 4 points every 20 n.m. along a path one third of the way from one row of ground stations to the other row, and likewise along a path halfway between the rows.

The many familiar relations associated with these points make the geometrical analysis easy to develop, while adequately illustrating the concepts. Table II lists the ranges to a low-flying target at each point and ranges to an aircraft overflying the point 30,000 feet above. Also, factors that appear in some of the partial derivatives developed above are tabulated.

The Covariance of the Measured Ranges

In order to compute expected uncertainty in the location of aircraft and test vehicle it will be necessary to have expected squares of the uncertainties in the seven measured ranges and the expected products of uncertainty for the 21 pairs of different measured ranges. These covariance elements have been assembled for the specific case of point E, Figure 7, and are laid out in Table III.

Table III consists of 27 rows and 27 columns, associated with the 27 sources of uncertainty. The four (or three, in the case of $r_a$) sources
Page intentionally left blank
Figure 6. WSMR Ground Sites for Two-Vehicle Multilateration System.
Figure 7. Vehicle Locations Used in Multilateration Error Analyses.
### TABLE II
PARAMETERS FOR MULTILATERATION ERROR ANALYSIS

<table>
<thead>
<tr>
<th>PATH</th>
<th>HORIZONTAL RANGE (ft)</th>
<th>SLANT RANGE (ft)</th>
<th>$\alpha$ (deg)</th>
<th>$\frac{e - X}{e}$ (Target)</th>
<th>$\frac{r}{e}$ (Aircraft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-D</td>
<td>69,282</td>
<td>75,498</td>
<td>23.4132</td>
<td>0.3333</td>
<td>0.6667</td>
</tr>
<tr>
<td>B-D</td>
<td>69,282</td>
<td>75,498</td>
<td>23.4132</td>
<td>0.3333</td>
<td>0.6667</td>
</tr>
<tr>
<td>C-D</td>
<td>69,282</td>
<td>75,498</td>
<td>23.4132</td>
<td>0.3333</td>
<td>0.6667</td>
</tr>
<tr>
<td>A-E</td>
<td>105,830</td>
<td>110,000</td>
<td>15.8266</td>
<td>0.0</td>
<td>1.0184</td>
</tr>
<tr>
<td>B-E</td>
<td>40,000</td>
<td>50,000</td>
<td>36.8699</td>
<td>0.6667</td>
<td>0.3849</td>
</tr>
<tr>
<td>C-E</td>
<td>80,000</td>
<td>85,440</td>
<td>20.5560</td>
<td>0.3333</td>
<td>0.7698</td>
</tr>
<tr>
<td>A-F</td>
<td>91,652</td>
<td>96,437</td>
<td>18.1246</td>
<td>0.1667</td>
<td>0.8819</td>
</tr>
<tr>
<td>B-F</td>
<td>91,652</td>
<td>96,437</td>
<td>18.1246</td>
<td>0.1667</td>
<td>0.8819</td>
</tr>
<tr>
<td>C-F</td>
<td>34,641</td>
<td>45,826</td>
<td>40.8934</td>
<td>0.6667</td>
<td>0.3333</td>
</tr>
<tr>
<td>A-G</td>
<td>79,373</td>
<td>84,853</td>
<td>20.7047</td>
<td>0.2500</td>
<td>0.7638</td>
</tr>
<tr>
<td>B-G</td>
<td>79,373</td>
<td>84,853</td>
<td>20.7047</td>
<td>0.2500</td>
<td>0.7638</td>
</tr>
<tr>
<td>C-G</td>
<td>51,962</td>
<td>60,000</td>
<td>30.0000</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
<tr>
<td>A-H</td>
<td>103,923</td>
<td>108,166</td>
<td>16.1021</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>B-H</td>
<td>60,000</td>
<td>67,082</td>
<td>26.5651</td>
<td>0.5000</td>
<td>0.5774</td>
</tr>
<tr>
<td>C-H</td>
<td>60,000</td>
<td>67,082</td>
<td>26.5651</td>
<td>0.5000</td>
<td>0.5774</td>
</tr>
</tbody>
</table>
### TABLE III
MULTILATERATION RANGE ERROR MATRIX, POINT E

<table>
<thead>
<tr>
<th></th>
<th>r_{A1}</th>
<th>r_{A2}</th>
<th>r_{A3}</th>
<th>r_{T1}</th>
<th>r_{T2}</th>
<th>r_{T3}</th>
<th>r_{N}</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.21</td>
<td>0.22</td>
<td>0.3759</td>
<td>0.2328</td>
<td>0.0440</td>
<td></td>
<td>0.0880</td>
</tr>
<tr>
<td>S</td>
<td>1.8512</td>
<td>0.0440</td>
<td>0.1367</td>
<td>0.0600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r_{A1}</td>
<td>1/2</td>
<td>9</td>
<td>4.5</td>
<td>0.25</td>
<td>0.1709</td>
<td>0.0579</td>
<td>0.0400</td>
</tr>
<tr>
<td>r_{A2}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>0.7300</td>
<td>0.0994</td>
<td>0.0342</td>
<td>0.1367</td>
</tr>
<tr>
<td>r_{A3}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>1.1200</td>
<td>0.0423</td>
<td>0.0847</td>
<td>0.0317</td>
</tr>
<tr>
<td>r_{T1}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>1.1200</td>
<td>0.0423</td>
<td>0.0847</td>
<td>0.0317</td>
</tr>
<tr>
<td>r_{T2}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>1.1200</td>
<td>0.0423</td>
<td>0.0847</td>
<td>0.0317</td>
</tr>
<tr>
<td>r_{T3}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>1.1200</td>
<td>0.0423</td>
<td>0.0847</td>
<td>0.0317</td>
</tr>
<tr>
<td>r_{N}</td>
<td>1/2</td>
<td>1/2</td>
<td>9</td>
<td>1.1200</td>
<td>0.0423</td>
<td>0.0847</td>
<td>0.0317</td>
</tr>
</tbody>
</table>

Note: Along the diagonal are the variances associated with 27 sources of uncertainty to seven range measurements. The source labels are N, noise and multipath; B, bias; R, range scale; S, survey. A refers to the overflying aircraft, T to the test vehicle and r_{A} to the range between those vehicles. Below the diagonal are the non-zero correlation coefficients between the several sources, and above the diagonal are the expected values of the uncertainty products.
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associated with a particular range go with a group of adjacent columns and adjacent rows as the labels indicate.

Along the diagonal of the table are tabulated the squared uncertainties associated with each source. The values are representative of the experience indicated in the many interviews conducted and reports examined during this contract. If specific equipment is contemplated, the uncertainties associated with that equipment should be substituted and the computations carried forward as indicated. If the numerical choices here are deemed appropriate, then the numerical results can be accepted, but a primary purpose of this example is to clearly set down how the calculations should be carried out.

For noise and multipath the squared uncertainty has been chosen to be 9 ft\(^2\) for all ranges at high elevation, i.e., the four ranges involving the overflying aircraft. For the three nearly horizontal ranges, 20 ft\(^2\) has been selected. There is considerable controversy about the magnitude of multipath uncertainty. Clearly, it varies with the terrain, and in any case, the antenna patterns of the several antennas employed are important in limiting its value.

For survey error the figure of 2 feet, vector uncertainty, has been verbally suggested to us for locations on WSMR. If this is taken as 2 feet horizontal, then the component in any one direction will be \(\sqrt{2}\) feet. Only one component of the survey error contributes to effective range error. According to (7) this gives \(2 \cos^2 \alpha \text{ ft}^2\) for the square of uncertainty of ranges to the overflying aircraft and 2 \(\text{ft}^2\) for ranges to the test aircraft at nominally zero elevation.

The squared error due to bias has been taken as 9 \(\text{ft}^2\) and this includes bias in both the interrogating device and the transponder.

All range scale factor uncertainties are taken as the square of \(10^{-5}\) times the range involved, a figure that seems acceptable to most of the interviewees and report authors.

The range measurements, themselves, are going to be made as unbiased as possible, so the expected value of any range error is zero; but the expected values of the squares of errors (arising out of the 27 sources) are not zero. In an ensemble of range measurements, the individual errors are expected to average zero, but the squares, always positive, will average some positive
quantity, as is reflected in the diagonal values chosen above. Below the diagonal in Table III have been placed the correlation coefficients discussed above. The blank elements correspond to zero correlation. Above the diagonal are the corresponding expectation value terms, themselves \((\text{in ft}^2)\). They are the products of the roots of the corresponding diagonal terms times the associated correlation coefficient. When there is zero correlation the average value of one error term in an ensemble is zero for every specific value of the other, and hence, the average value in the ensemble, that is, the expected value of the product, is zero.

When the correlation is high, as is the case with survey error, every error in the range to one vehicle is proportional to the error in the range to the other vehicle when the ranges are measured from the same ground station. The expected value of the product is the root of the product of corresponding the diagonal terms.

When correlation is partial the correlation coefficient is multiplied by the root of the product of the two diagonal terms. Physically one can envision that the error stems from a sum of "sub-sources," some uncorrelated and some correlated, as was discussed in the case of bias, where part of the error arose in the common instrument, which was fully correlated, while the rest was uncorrelated. Only the correlated part of this sum contributes to the off-diagonal expectation values, the rest averaging zero in an ensemble. In the case of survey error, if the average cosine between the azimuth to the overflying aircraft and the test vehicle is taken as the correlation coefficient, a number close to unity in the configuration being considered, then the component of the survey error of one azimuth, which is parallel to the other azimuth is fully correlated, while the perpendicular component is uncorrelated.

We could work directly with these 27 diagonal members and 36 non-zero off-diagonal members, but it is more convenient to collapse Table III into Table IV. Table IV contains the expectation values of the squares of errors for measurements of each of the 7 ranges along the diagonal and the expected values of the products of the errors above the axis. These turn out to be
### TABLE IV
MULTILATERATION RANGE ERROR MATRIX, COLLAPSED, POINT E

<table>
<thead>
<tr>
<th></th>
<th>( r_{A1} )</th>
<th>( r_{A2} )</th>
<th>( r_{A3} )</th>
<th>( r_{T1} )</th>
<th>( r_{T2} )</th>
<th>( r_{T3} )</th>
<th>( r_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{A1} )</td>
<td>21.0612</td>
<td>4.7200</td>
<td>4.8759</td>
<td>6.6570</td>
<td>0.0440</td>
<td>0.0880</td>
<td>0.1320</td>
</tr>
<tr>
<td>( r_{A2} )</td>
<td>19.5300</td>
<td>4.6709</td>
<td>0.0529</td>
<td>6.1400</td>
<td>0.0400</td>
<td>0.0600</td>
<td></td>
</tr>
<tr>
<td>( r_{A3} )</td>
<td>20.4834</td>
<td>0.0904</td>
<td>0.0342</td>
<td>6.5094</td>
<td>0.1025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{T1} )</td>
<td>32.1200</td>
<td>4.5423</td>
<td>4.5847</td>
<td>4.5317</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{T2} )</td>
<td>31.1600</td>
<td>4.5320</td>
<td>4.5120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{T3} )</td>
<td>31.6400</td>
<td>4.5240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_a )</td>
<td>18.0900</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Along the diagonal are the variances for the seven range measurements. Off diagonal are the expected values of the products of the uncertainties. The matrix is symmetrical, the lower triangle mirroring the upper.
the sums of the terms in the corresponding intermediate sized rectangles in Table III. The reasons for this are as follows:

A differential relation of the form (17) expresses any linear deviation of one of the seven measured ranges. The product of two such relations (including the product of one range differential by itself) expresses how the several error sources combine to form the product. To get the expectation value of an overall product, each product of differentials should be replaced by its expectation value. Thus, in the product,

\[ (\delta r_{A_2})^2 = (\delta r_{A_2,N}) + 2(\delta r_{A_2,N})(\delta r_{A_2,B}) + \cdots, \]

only the squared terms on the right have non-zero expectation values according to Table III and the expectation value is,

\[ E[\delta r_{A_2}] = \sigma_{r_{A_2}}^2 + \sigma_{r_{A_2,B}}^2 + \sigma_{r_{A_2,R}}^2 + \sigma_{r_{A_2,S}}^2 \tag{18} \]

the sum of the four diagonal terms associated with \( r_{A_1} \).

In the product differential,

\[ (\delta r_{A_1})(\delta r_{T_1}) = (\delta r_{A_1,N})(\delta r_{T_1,N}) + (\delta r_{A_1,N})(\delta r_{T_1,B}) + \cdots + (\delta r_{A_1,R})(\delta r_{T_1,B}) + \cdots + (\delta r_{A_1,S})(\delta r_{T_1,S}) \]

only three of the products on the right have non-zero expectation values, and,

\[ E(\delta r_{A_1, \delta r_{T_1}}) = \sigma_{r_{A_1,T_1}}^2 + \sigma_{r_{A_1,B}} r_{T_1,B} + \sigma_{r_{A_1,R}} r_{T_1,R} + \sigma_{r_{A_1,S}} r_{T_1,S} \tag{19} \]

Thus, Table IV is composed of the sums of the contents of the numbers inside the intermediate sized rectangles of Table III that are situated above and along the diagonal. This would be the proper rule for collapsing Table III.
even if there were off diagonal terms within one of these intermediate sized rectangles, provided the off diagonal terms on both sides of the diagonal were included in the sum.

The correlation coefficients associated with Table IV could have been calculated. They are not of great interest since they lack the simple association with the physical system that the coefficients in the lower half of Table III have.

Aircraft Location Uncertainties

Above point E on Figure 7 the differentials of the position of an aircraft are, from (4), (5) and Table II,

\[
\delta x_A = \frac{r_{A1}}{e} \cos 30^\circ \delta r_{A1} - \frac{r_{A2}}{e} \cos 30^\circ \delta r_{A2} \tag{20}
\]

\[
\delta y_A = \frac{r_{A1}}{e} \sin 30^\circ \delta r_{A1} + \frac{r_{A2}}{e} \sin 30^\circ \delta r_{A2} - \frac{r_{A3}}{e} \delta r_{A3} \tag{21}
\]

\[
\delta z_A = \frac{e - x_1}{e \sin \alpha_1} \delta r_{A1} + \frac{e - x_2}{e \sin \alpha_2} \delta r_{A2} + \frac{e - x_3}{e \sin \alpha_3} \delta r_{A3} \tag{22}
\]

Thirty degrees is the angle between the x direction and the perpendicular to the lines joining stations B and C and that joining A and C. Signs in (20) and (21) are chosen appropriate to Figure 7. The subscripts on the \( \delta r \)'s are 1, 2, 3 standing for the ranges from stations at A, B, C, respectively, in Figure 4.

Numerical values for \( r/e \) and \( (e-X)/e \) are to be taken from Table II. For all cases in the table, e is the altitude of the equilateral triangle in Figure 4. X is the component of the horizontal range in a direction perpendicular to the line joining the other two ground stations, or is the slant range and \( \alpha \) is the elevation angle of the aircraft, which has been assigned the altitude of 30,000 feet.

The "covariance matrix" elements of the aircraft location consist of the expectation values of \((\delta x_A)^2\), \((\delta x_A \delta y_A)\) etc. These are computed by forming appropriate products of (20), (21) and (22) with each other, then replacing products of \( \delta r \)'s on the right by their expectation values from Table IV, thus,
\[
(\delta x_A)(\delta y_A) = \{(1.0585)(0.8660)\delta r_1 - (0.4811)(0.8660)\delta r_2\}
\]
\[
x\{(1.0585)(0.5)\delta r_1 + (0.4811)(0.5)\delta r_2 - (0.8221)\delta r_3\}
\]
\[
= 0.4851(\delta r_1)^2 + (0.2205-0.2205)(\delta r_1 \delta r_2)
\]
\[
- (0.7536)(\delta r_1 \delta r_3) -(0.1002)(\delta r_2)^2 + (0.3425)(\delta r_2 \delta r_3)
\]

To obtain \(\sigma_{x_A y_A}\), the products on the right of (23) are replaced by their expectation values from Table IV. The result is,

\[
\sigma_{x_A y_A} = (0.4851)(21.0612) - (0.7536)(4.8759)
\]
\[
- (0.1002)(19.5300) + (0.3425)(4.6709)
\]
\[
= 6.1867 \text{ ft}^2
\]

The other elements associated with aircraft location at point E are similarly calculated to yield

\[
P = \begin{pmatrix}
\sigma^2_{x_A} & \sigma_{x_A y_A} & \sigma_{x_A z_A} \\
\sigma^2_{y_A} & \sigma_{y_A z_A} \\
\sigma^2_{z_A}
\end{pmatrix} = \begin{pmatrix}
17.4830 = (4.1813)^2 & 6.1867 & -1.8367 \\
15.9857 = (3.9982)^2 & -8.7386 \\
52.4231 = (7.2402)^2
\end{pmatrix}
\]

The elements of (25) are, indeed, the expectation values of the several products for any one determination of the aircraft's position. The diagonal members are the mean square values of uncertainty in the three coordinate directions. The six elements can be visualized as an ellipsoid inscribed in a rectangular box extending \(\pm \sigma_x\), \(\pm \sigma_y\), \(\pm \sigma_z\) in the three coordinate directions. The orientation of the ellipsoid is given by the three off-diagonal terms.
The equation for the ellipse is

$$X^T P^{-1} X = 1$$  \hfill (26)

where $X$ is any radius vector to the surface of the ellipse and $P^{-1}$ is the matrix inverse to $P$ so that $P^{-1} P = I$, the unit matrix. In terms of the six elements of $P$ in (25), the ellipse, (26) is

$$\begin{align*}
&\left(\sigma_{yz}^2 - \sigma_{yz}^2\right)x^2 + \left(\sigma_{xz}^2 - \sigma_{xz}^2\right)y^2 + \left(\sigma_{xy}^2 - \sigma_{xy}^2\right)z^2 \\
&+ 2\left(\sigma_{xy}^2 - \sigma_{xy}^2\right)xy + 2\left(\sigma_{xy}^2 - \sigma_{xy}^2\right)xz \\
&+ 2\left(\sigma_{xy}^2 - \sigma_{xy}^2\right)yz
\end{align*}$$  \hfill (27)

Its structure is evident if the coordinate axes are rotated to $(x',y',z')$ where $P$ and $P^{-1}$ are diagonal matrices. The new coordinates align with the principal axes of the ellipsoid. In those coordinates, and in terms of the three new non-zero elements of $P$, (26) and (27) become,

$$\begin{align*}
\frac{x'^2}{\sigma_{x'}^2} + \frac{y'^2}{\sigma_{y'}^2} + \frac{z'^2}{\sigma_{z'}^2} = 1
\end{align*}$$  \hfill (28)

The separation of any two parallel planes tangent to the ellipsoid is twice the RMS uncertainty in the direction perpendicular to the planes. The shape of the ellipsoid thereby indicates the relative likelihood of position error in a given direction.

**Test Vehicle Location**

Our primary interest is in the location of the test vehicle rather than the overflying aircraft. It is located nominally under the aircraft, and for the discussions to follow the elements in Table IV associated with the $r_T$'s and with $r_a$ will be taken to be unchanged for non-zero values of $\zeta$ and $\eta,$
that is, when the test vehicle is located not directly under the aircraft, but displaced horizontally by, say, no more than about 20,000 feet.

Expressions for \( \delta x_T \) and \( \delta y_T \) are exactly like (20) and (21) with \( T \) replacing \( A \) in the subscripts. The expressions for \( \delta z_T \) is (16) when it is written with \( \delta x_A \) substituted from (22) and \( \delta x = \delta x_T - \delta x_A \) substituted from (20) and its counterpart for \( \delta x_T \). Likewise, \( \delta \eta \) should be replaced by (21) and its counterpart for \( \delta y_T \). With these three expressions the six covariance elements of the target location can be computed analogous to the procedure for the aircraft location. The procedure is straightforward but tedious because of the relatively large number of terms.

For point E of Figure 7, after entering the appropriate values,

\[
\delta x_T = 0.8819 \delta r_{T1} - 0.333 \delta r_{T2} \tag{29}
\]

\[
\delta y_T = 0.5092 \delta r_{T1} + 0.1925 \delta r_{T2} - 0.7698 \delta r_{T3} \tag{30}
\]

\[
\delta z_T = 1.1111 \delta r_{A2} + 0.9493 \delta r_{A3} - \frac{1}{\cos \beta} \delta r_a \tag{31}
\]

\[
+ \frac{\xi}{r_a \cos \beta} \left\{ -0.9167 \delta r_{A1} + 0.4166 \delta r_{A2} + 0.8819 \delta r_{T1} - 0.3333 \delta r_{T2} \right\}
\]

\[
+ \frac{\eta}{r_a \cos \beta} \left\{ -0.5293 \delta r_{A1} - 0.2406 \delta r_{A2} + 0.8221 \delta r_{A3} \right\}
\]

\[
+ 0.5092 \delta r_{T1} + 0.1925 \delta r_{T2} - 0.7698 \delta r_{T3} \right\}
\]

There is no particular interest in the shape of the error ellipsoid. Its general size is evident from its diagonal terms. As with the aircraft, \( \sigma_{T_x}^2 \) is the square of (29) with differential products on the right replaced by expectation values from Table IV. Likewise, \( \sigma_{T_y}^2 \) from (30). These lead to

\[
\sigma_{T_x}^2 = 25.7725 \text{ ft}^2; \quad \sigma_{T_x} = 5.0767 \text{ ft.}
\]

\[
\sigma_{T_y}^2 = 24.1856 \text{ ft}^2; \quad \sigma_{T_y} = 4.9179 \text{ ft.}
\]
The RMS value of vertical error is much more important to our considerations and \( \sigma_{Tz}^2 \) has a form that depends on \( \xi, \eta \) and the value of \( \beta \) that goes with the horizontal displacement between test vehicle and aircraft. \( \beta \) is given by (11) or by,

\[
\tan \beta = \frac{\sqrt{\xi^2 + \eta^2}}{\text{altitude of aircraft}}
\]  

(32)

When (31) is squared and the products of \( \delta r \)'s on the right replaced by appropriate values from Table IV, then,

\[
\sigma_{Tz}^2 = 52.4231 - \frac{0.3279}{\cos \beta} + \frac{18.0900}{\cos^2 \beta}
\]

\[+ \xi \left(-0.6408 - \frac{4.7933}{\cos \beta}\right)\]

\[+ \eta \left(10.6813 + \frac{0.6130}{\cos \beta}\right)\]

\[+ 30.8526 \xi' \xi'\]

\[+ 27.9263 \eta' \eta'\]

\[+ 23.0751 \xi' \eta'\]

where

\[
\xi' = \frac{\xi}{r_a \cos \beta} = \frac{\xi}{\text{Altitude of aircraft}}
\]

\[
\eta' = \frac{\eta}{r_a \cos \beta} = \frac{\eta}{\text{Altitude of aircraft}}
\]

When the test vehicle is directly under the aircraft, \( \xi = \eta = 0 \), and \( \beta = 0 \), and

\( (\sigma_{Tz}^2)_{\xi=\eta=0} = 70.1852 \text{ ft}^2 \); \( (\sigma_{Tz}^2)_{\xi=\eta=0} = 8.3777 \text{ ft} \)
When the test vehicle is not directly under the aircraft, \( \sigma_{Tz} \) increases. It is desirable to know how far from the overhead position the aircraft can be without having \( \sigma_{Tz} \) exceed some specified value. One can insert such a value into (33) and compute \( \xi' \) as a function of \( \eta' \) (or vice-versa) to find the horizontal extent of the region where the vertical measurement will be in some sense satisfactory. This will reveal how closely the aircraft must locate over the test vehicle.

To the extent that \( \beta \) is constant in (33) with \( (\sigma_{Tz}^2) \) fixed, the solution represents an ellipse in the \( (\xi',\eta') \) plane. The term \( 18.0900/\cos\beta \) varies rapidly enough with \( \xi' \) and \( \eta' \) that it is not profitable to discuss this ellipse. Rather, (32) and (33) can be solved directly for a series of values of one variable. Figure 8 consists of plots for \( \sigma_{Tz} = 10 \) ft. Inside the contours, \( \sigma_{Tz} \) is less than 10. The points on the contour correspond to the dimensionless value of \( \xi' \) and \( \eta' \) multiplied by the altitude of the aircraft, 30,000 feet in this example. The closest approach of the contour of \( \sigma_{Tz} \) to the origin is about 0.65, which amounts to 19,500 feet for the aircraft at 30,000 feet, Point E.

Point D, Figure 7, has its symmetry reflected in the resulting covariance elements. All off diagonal terms are zero, and,

\[
\begin{align*}
\sigma_{Ax} & = 3.5062 \text{ ft.} \\
\sigma_{Ay} & = 3.5062 \text{ ft.} \\
\sigma_{Az} & = 7.9200 \text{ ft.} \\
\sigma_{Tx} & = 4.2375 \text{ ft.} \\
\sigma_{Ty} & = 4.2375 \text{ ft.}
\end{align*}
\]

Right under the aircraft,

\[
(\sigma_{Tz})_{\xi=\eta=0} = 8.9644 \text{ ft.}
\]
The contour of horizontal positions for which \( \sigma_{z_T} = 10 \) feet is a circle of radius 0.7110, in the \((\xi', \eta')\) plane, which amounts to 21,330 feet or 3.56 nm. Data for the other three points of Figure 7 are contained in Table V and Figure 8.

**Discussion of Lateration Results**

**Correlations.** One striking item that came out in the foregoing analysis is the very strong correlations between some of the error sources. The use of an instrument common to more than one range measurement is the source of correlation between certain bias terms and likewise, between station survey errors. The magnitude of these two sources of correlation is clear; the choices of correlation coefficients in the examples are appropriate, and the adjustments to match a particular system in which bias in the transponders may have a different expected value from those in the querying devices are straightforward.

Correlation associated with range scale uncertainty is less clear. The choices in the examples are not unreasonable ones, but no experience is in hand to guide the choice of the correlation coefficients. Certainly the correlation exists. There is a temptation to take comfort in the expected correlations of range scale, through a layered, partially known atmosphere, up to an overflying aircraft and downward to a test vehicle. Indeed, this correlation tends to cancel the uncertainty in the altitude of the test vehicle, but examinations of Table III will reveal that range scale error is one of the smaller sources of uncertainty. The bias correlation between the three ranges up to the overflying aircraft is also strong, and pertains to a source of greater uncertainty. This correlation accentuates the uncertainty in the altitude of the overflying aircraft, and through this, the altitude of the test vehicle. On balance the correlations result in greater uncertainty in the altitude of the test vehicle.

The correlations result in smaller variances in horizontal directions. The correlated lengthening (or shortening) of the three ranges to either vehicle due to bias in the transponder carried by the vehicle and the correlation of scale uncertainty is responsible for this.
TABLE V
COVARIANCE MATRICES OF AIRCRAFT LOCATION AND TARGET LOCATION
AT POINTS E THROUGH H IN FIGURE 7
(Target Directly Under Aircraft)

<table>
<thead>
<tr>
<th>Point F</th>
<th>Point G</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A = \begin{pmatrix} 20.4948 &amp; 0 &amp; 0 \ 10.9626 &amp; 6.2512 &amp; 44.9828 \ 31.7935 &amp; 0 &amp; 0 \end{pmatrix} ]</td>
<td>[ A = \begin{pmatrix} 15.6817 &amp; 0 &amp; 0 \ 10.6071 &amp; 3.1174 &amp; 58.4212 \ 23.6881 &amp; 0 &amp; 0 \end{pmatrix} ]</td>
</tr>
<tr>
<td>[ T = \begin{pmatrix} 14.9603 &amp; -1.3653 \ 62.7128 \end{pmatrix} ]</td>
<td>[ T = \begin{pmatrix} 21.8434 &amp; 1.3788 \ 76.0793 \end{pmatrix} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point H</th>
<th>Point E (Lasers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ A = \begin{pmatrix} 18.5140 &amp; 5.1424 &amp; -5.8005 \ 12.5758 &amp; -3.3481 &amp; 61.8212 \ 27.9476 &amp; 8.3921 &amp; -5.1130 \end{pmatrix} ]</td>
<td>[ A = \begin{pmatrix} 10.6116 &amp; 4.1229 &amp; -3.9975 \ 10.3005 &amp; -5.3753 &amp; 21.3209 \ 9.5882 &amp; 4.2082 &amp; 0.0632 \end{pmatrix} ]</td>
</tr>
<tr>
<td>[ T = \begin{pmatrix} 18.2595 &amp; -2.9522 \ 79.5512 \end{pmatrix} ]</td>
<td>[ T = \begin{pmatrix} 9.3818 &amp; -0.0370 \ 29.1282 \end{pmatrix} ]</td>
</tr>
</tbody>
</table>
Figure 8. Bounds on Overflying Aircraft Position for 10-foot RMS Vertical Error--Radio Ranging.
The variances of the three aircraft location coordinates about point E are repeated in Table VI along with the values that result if all correlations are ignored. Likewise, similar variances are given for the test vehicle directly under the aircraft.

Comparison of the numbers in Table VI suggests that better knowledge about the range scale correlations is not going to dramatically change the numerical results of these examples.

**TABLE VI**

MULTILATERATION MATRIX DIAGONAL (POSITION) ELEMENTS, WITH AND WITHOUT CORRELATIONS

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Variances, $\sigma^2$, and Their Roots, $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correlations Included</td>
</tr>
<tr>
<td>$X_A$</td>
<td>17.4830</td>
</tr>
<tr>
<td>$Y_A$</td>
<td>15.9857</td>
</tr>
<tr>
<td>$Z_A$</td>
<td>52.4231</td>
</tr>
<tr>
<td>$X_T$</td>
<td>25.7725</td>
</tr>
<tr>
<td>$Y_T$</td>
<td>24.1856</td>
</tr>
<tr>
<td>$Z_T$</td>
<td>70.1852</td>
</tr>
</tbody>
</table>

Magnitudes of Sources of Range Uncertainty. In the examples the values chosen for range uncertainties are representative of state-of-the-art, or at least what the state-of-the-art is thought to be by the persons interviewed and the authors whose papers were read for this study. The firmness of the several values differs, however, and deserves some discussion. The detailed methods of computation here set down can, of course, be applied to any revisions of the values of range error sources.

There is extensive experience with the effects of circuit noise and the stability of trigger circuits. The effects of these on the behavior of a
timing instrument can be and have been measured under controlled conditions, so the effects of noise and instrument bias are probably accurate.

The range scale error would be three parts in $10^4$ if no allowance were made for the presence of the atmosphere at sea level. A reasonably good model of the atmosphere that does not change with time should reduce the uncertainty to three in $10^5$; and if meteorological data is skillfully applied, one part in $10^5$ is to be expected. Certainly the range scale error need not be as large as three in $10^5$, and it is unlikely that it can be kept to three in $10^6$.

Site survey error is not usually as small as two feet over a test range extending 50 or 60 miles each way from its middle. WSMR is very special in this regard, and the choice may well be realistic.

The examples in this analysis dealt only with horizontal survey errors. Vertical errors can be simply included and how to do so was laid out. In any event, it is the expected component of survey error parallel to the range vector that will enter the computations.

The most controversial of the sources is multipath, particularly at lower elevation angles, as envisioned for determining the horizontal position of the test vehicle. The consequences of the low angle multipath variance (20 ft$^2$ including instrument noise, but not instrument bias) being too small should be understood. Certainly it will increase the horizontal uncertainty of the test vehicle, but this system of location was analyzed with the notion that the horizontal location specifications for the test vehicle could often be relaxed if only its altitude uncertainty could be maintained.

If the overflying aircraft is directly over the test vehicle, then the horizontal uncertainty of the test vehicle does not enter into the evaluation of its altitude uncertainty. This can be seen from Equation (16). There the last two terms are,

$$\frac{\xi \delta \xi}{r_a \cos \phi} + \frac{n \delta n}{r_a \cos \phi} = \frac{\delta (\xi^2 + n^2)}{2 \text{ (Aircraft Altitude)}} = \frac{\delta h^2}{2 \text{ (Aircraft Altitude)}}$$

$$= \frac{h \delta h}{\text{Aircraft Altitude}} = \tan \phi \delta h$$

69
When $\beta$ is small, the effect of uncertainty in the horizontal position, $h$, of the test vehicle relative to the aircraft does not affect the altitude determination. From Table V the altitude variance $z_T$ at Point E for this condition is 70 ft$^2$, while the horizontal variance of $h$ is of the order of $G^2 + a^2 + g^2 \approx 67$ ft$^2$.

Now to keep $c_{Tz}$ within 10 feet, the contribution of the term in (34) must not exceed 30 ft$^2$. That is,

$$\tan^2 \beta (67 \text{ ft}^2) < 30 \text{ ft}^2$$

$$\beta < 33.8 \text{ degrees}.$$ 

This is inexact, neglecting some correlations, but it gives a fair estimate of the size of the contour in Figure 8. The closest approach of the contour to the origin corresponds to $\beta = 34.2$ degrees, the furthest to 48.2 degrees.

The consequence of less favorable multipath conditions on the ranges to the low-flying vehicle is that the overflying aircraft must maintain its position over the test vehicle more accurately to keep its altitude determination within limits. This follows even if the increased horizontal position uncertainty can be tolerated.

Signal coding in the ranging equipment can possibly reduce the effect of multipath on range measurements. The interaction is complex, and not widely understood. With any equipment proper attention to the antenna will reduce the non-direct energy that is the multipath reflection. The Cubic [10] antenna has been configured to concentrate the gain above the horizon for the ground stations. The siting of all antennas should be given adequate consideration; and at some sites the antennas may have to be very carefully designed to minimize multipath errors.

Likewise, the siting of ranging antennas on the vehicle is important. On the test vehicles, particularly, it may be impractical to measure radiation patterns and do all that is literally possible, so the best skill and intuition should be employed.
Usefulness of the System. Examination of the computed data at the several points examined indicate that the uncertainty in locating the test vehicle does not vary radically from point to point inside the equilateral triangle. The precision with which an overflying aircraft has to stay with the test vehicle also is reasonably constant. On-line estimation of the test vehicle's position relative to the aircraft will usually have to be used to maintain station, either through presentation to the pilot, or to an autopilot.

The requirement that the aircraft stay over the test vehicle restricts the precise measurements to a small portion of WSMR for any one test flight, or to test vehicles that the aircraft can follow, which may mean restriction to subsonic test flights.

The original goal of 10 feet RMS uncertainty in each axis can be met by this system even when the aircraft is not directly over the test vehicle. This analysis was carried forward in the belief that the altitude of the test vehicle might often be the really critical measurement. It has been shown that multilateration would permit altitude measurements with 10 foot accuracy, with the overfly conditions in Figure 8.

There are two refinements that could improve the performance of the system. At most positions over WSMR more than three ground stations can range on the test crafts. Redundant data, if properly employed, will improve the results; but this would be a minor improvement, especially on the altitude of the test vehicle. When the additional data is most likely to be available is near the cross-over between one triangle and the next, that is, near points E or H of Figure 7. At E or H the computed altitude of the overflying aircraft depends entirely on ranges from stations B and C. Range from A, or from the fourth station, the third corner of the other triangle containing B and C, can add little to the precision of the altitude of the aircraft.

Improvement in the two horizontal estimates will enhance the test vehicle altitude estimate when it is not directly under the aircraft. However, at points interior to the triangle, like D, G, or F, Figure 7, range to a fourth station is greater than 20 n.m.; and the signal-
to-noise ratio will at such a range (depending on designs of the ranging devices) begin to degrade the information. Redundancy would probably be of little value.

More important than redundant range data is smoothing of the computed trajectory. The foregoing analysis addresses only a single position determination by seven range measurements. In practice, the seven measurements will not be simultaneous, and computation of the position at any one time must take this into account. If a high data rate can be maintained, then it will be practical to smooth the trajectory and still keep the details of the actual motion of the vehicle. Smoothing will average down the effects of noise, multipath, and small scale fluctuations of the atmospheric refractivity. It cannot change effects of bias, station survey, or large scale, slowly changing uncertainties in the atmospheric refractivity. This averaging can be significant since noise and multipath constitute a large fraction of the range measurement uncertainty.

Reduction of multipath error would enhance the system accuracy. If the ranging system is a pulse leading edge system, with pulses on the order of 10 nanoseconds, and with peak power increased to maintain the energy in each pulse high, the ranging error due to signal reflections could be reduced.

**Multilateration with Laser Ranging Devices**

There is presently a proliferation of laser ranging devices being developed for military and non-military needs.

A review of the errors in the above analysis of a radio ranging, multilateration system indicates several ways that a laser based multilateration system could improve accuracy in determining the position of a low-flying test vehicle.

Multipath error could be greatly reduced by virtue of the enhanced ratio of desired (retroreflected) to undesired (stray reflected and scattered) signal. Furthermore, the laser pulse length can easily be held to less than 10 nanoseconds.

Equipment delay uncertainties would be reduced because the "transponders" would be retroreflectors.
The scale factor error would be reduced at least by 10 percent, and a two-wavelength method of measuring atmospheric refraction effects might permit further reduction [42].

The errors of a laser-based system can be estimated by assuming the following error source effects:

- Noise and multipath: \(4 \text{ ft}^2\), all ranges.
- Bias: \(4 \text{ ft}^2\).
- Range scale factor: \(0.81 \times 10^{-10} \text{ ft}^2\).
- Survey error: \(2 \cos^2 \alpha \text{ ft}^2\) to aircraft.
  \(2 \text{ ft}^2\) to target.

The correlation values may be assumed to be the same as in the radio ranging analysis, except for the correlation of bias uncertainty. Four separate lasers and range measuring devices could be mounted in the overflying aircraft, and one at each ground site; or alternatively and more practically, two lasers and range measuring devices could be located at each ground site, and one in the aircraft. The absence of a common bias delay in different range measurements reduces the bias correlation to practically zero.

Using the above values, with the target and aircraft both located directly over Point E the position errors are:

\[
\begin{align*}
\sigma_{Ax} &= 3.2575 \text{ ft.} \\
\sigma_{Tx} &= 3.0965 \text{ ft.} \\
\sigma_{Ay} &= 3.2094 \text{ ft.} \\
\sigma_{Ty} &= 3.0630 \text{ ft.} \\
\sigma_{Az} &= 4.6175 \text{ ft.} \\
\sigma_{Tz} &= 5.3971 \text{ ft.}
\end{align*}
\]

Figure 9 is a plot of the "overfly" contour -- the ground projection of the limit within which the aircraft must hold its overflying pattern to keep the target altitude error within 10 feet RMS. The closest point of the contour to Point E corresponds to a value of the look-down angle (measured from the vertical downward direction) of 50.4 degrees, which means that the overflying aircraft can stray at least as far as 36,300 feet from directly over the low-flying vehicle, without causing the error in estimation of the altitude of the low-flying vehicle to exceed 10 feet RMS.
Overflying Aircraft at 30,000 Feet Above Low-Flying Missile, Which Is Over Point E.

Figure 9. Bounds on Overflying Aircraft Position for 10-foot RMS Vertical Error--Laser Ranging.
5. CONCLUSIONS

The primary conclusion that has been reached in this research is that the system shown in conceptualization in Figure 1 is feasible for precision measurements of position, velocity, and acceleration of low-flying missiles and aircraft at White Sands Missile Range. The goal of the contract, however, was to find available equipment for a system like that in Figure 1 which would enable WSMR to make precision measurements. Part of this system is available; part would have to be developed. That part which is available has been termed in this report a CIRIS-type system. A CIRIS-type system includes ground-based reference transponders which are positioned with very high accuracy through ground survey (accurate to two parts per million). On board the high altitude aircraft is a radio range measurement set, a high-accuracy inertial measurement unit, a barometric altimeter, and a computer. The radio reference system makes measurements of range and range rate from the ground based transponders. The inertial measurement unit estimates position and attitude of the airborne platform from its gyroscopes and accelerometers. An estimate of altitude of the airborne platform is derived from a barometric altimeter. These estimates of position, velocity, and acceleration are combined or computed by an "optimum" algorithm in the digital computer.

A CIRIS-type system exists at Holloman Air Force Base, and is reported to have met its original specifications [36]. The conclusion reached in this report is that this system, with a sufficient number of ground based transponders spaced in square grids 80,000 feet on a side, would be capable of determining the position of the airborne reference platform within about 5.8 feet RMS, any axis. The attitude of the reference platform can be determined within about 22 arc seconds.

No airborne radar or other system was found, however, which would permit range and pointing angle measurements from the airborne reference platform to the low-flying missile with sufficient accuracy to meet the overall specifications of 10 feet RMS position, any axis, for the low-flying missile. The conclusion is that such a system is feasible; however, it would have to be developed. The R&D program necessary to develop the
airborne radar (which would have to operate at 70 GHz or 95 GHz) would be a major undertaking. The radar transmitter, the antenna, the radome, and the pointing mechanism and circuitry, as well as the transponder which would be mounted in the target missile, would have to be developed, and prototypes would have to be constructed. Georgia Tech has had experience in the development of both 70 and 95 GHz radar systems, but these were not airborne systems. They were designed for ground vehicles such as armored personnel carriers and surface effect vehicles.

One alternative to the use of conventional radar in the link between airborne platform and target, as shown in Figure 1, would be a laser radar. This alternative has been analyzed in this report. The laser could be mounted in boresight with a 17 GHz radar. The latter would serve to acquire the target, and the laser radar component would lock on and track a retro-reflector mounted on the low-flying missile.

Another alternative concept would be to track the low-flying target entirely from ground based positions using an available ground based laser radar system, PATS, which is manufactured by Sylvania. PATS was observed in action at Yuma Proving Grounds. In the test at Yuma, a helicopter was tracked at a distance of about 36,000 feet with an estimated error of about 5 feet RMS. PATS consists of a YAG laser which is boresighted with a telescopic video camera. The laser and camera tube are mounted elevation over azimuth. The video display on a closed circuit television screen is used to acquire the target, through joystick control of the laser radar mount. When the target is approximately centered on the video display screen, the operator transfers control to the automatic tracking mode of the laser radar. Accuracy specifications for PATS is 0.1 milliradians in each of the two axes, elevation and azimuth, and two feet in range up to 65,000 feet. Maximum range for this system is said to be 100,000 feet. Some nine or ten PATS laser radars could track low-flying targets over most of White Sands Missile Range. The elevation angle of a laser radar can be depressed below horizontal because the reflectivity of the earth is much, much less than the reflectivity of the retroreflector mounted on the target. Multipath is thus no problem with laser radar, except perhaps over water. The effect of atmospheric index of refraction on angle measurement error has not been defined, however.
The laser radars could also be considered as replacements for the WSMR theodolite cameras, which are presently the basic instrumentation for the range. The laser radar units are capable of making measurements at low altitudes whereas the theodolites are not. The laser radars also permit real time or almost real time data turn-around, whereas the theodolites require days or even weeks for data turn-around.

A third alternative system, which would employ existing Ku-band radar, has also been described in this report. The airborne reference platform would have to maintain station within 8 degrees of vertical over the low-flying target. The down-looking radar would serve to measure range to the target. So long as the angle of the line-of-sight from reference platform to target is within 8 degrees of vertical, the overall error in vertical position of the target would be within 10 feet RMS. The reference platform would be positioned by a CIRIS-type system described in Chapter 2. The horizontal error would be on the order of 40 feet, because of the low frequency of the Ku-band radar. The usefulness of this concept is that existing equipment, with some modifications, could be used. The assumption is that the most important position coordinate of the target is altitude.

The fourth concept which has been examined is a total range measurement, or multilateration concept. In this scheme range (and range rate) would be measured from ground based sites to the target, as well as to the airborne reference platform. In the measurements from ground sites, only the information concerning the horizontal position coordinates of the target would be retained; the altitude data would be discarded because it would be known to be inaccurate. The range and range rate measurements from the airborne platform (40,000 feet AGL) to the target would be the source of information for the estimate of target altitude.

The analysis of the multilateration approach indicated that the low-flying target position could be determined with an error less than 10 feet RMS, any axis, using radio ranging measurements. Extension of the analysis to laser ranging devices indicated that errors of 5 feet RMS, any axis, may be feasible.
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6. RECOMMENDATIONS

The need for adequate instrumentation to measure the performance of low-flying missiles and aircraft is unquestionable. It was found that the Figure 1 system cannot be fully implemented with available equipment; the airborne radar conceptualized in Figure 1 would have to be a 70 GHz or a 95 GHz radar or a laser radar. Analyses were made of the system requirements of airborne millimeter and laser radars, which would have to be developed.

In addition to three airborne radar, four other systems were analyzed. All seven potential systems are listed in the decision matrix, Table VII. Of the seven, only PATS is immediately available, and its cost is high. Furthermore, the pointing angle error for PATS low-angle tracking has not been established. However, a PATS system could replace ciné theodolites, giving WSMR immediate data turnaround. PATS would probably extend WSMR measurement capability to altitudes lower than the theodolites can handle.

The development cost of the multilateration system which has been analyzed in this report would be relatively small. Indeed, it is believed that a number of moderate improvements could be made in the RMS/micro B equipment that would reduce errors to levels that would permit better than 10 foot RMS position accuracy in a multilateration system. A multilateration system using laser ranging might permit position determination within 5 feet.

A complete system design study of a multilateration system is recommended. Both radio and laser ranging would be examined. Various interrogator/transponder configurations, and the consequent telemetry and data reduction needs would be evaluated.

If there are positive results from the multilateration study, prototype ranging equipment would be developed, and an experimental, abbreviated, multilateration system would be implemented and tested.

In the event that the system design study of the multilateration system indicates the approach is not feasible, it would be recommended that attention be shifted to 95 GHz and laser radars to be used in conjunction with an airborne platform and ground-based position determining system. First a review would
# Table VII

## Decision Table for Recommendations

<table>
<thead>
<tr>
<th>DECISION FACTORS</th>
<th>MULTILATERATION</th>
<th>CIRIS/TRACKER</th>
<th>PATS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radio Ranging</td>
<td>Laser Ranging</td>
<td>Laser 95 GHz</td>
</tr>
<tr>
<td>Order of</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td>Recommendation</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

### Reasons for Rankings

- **Lowest R&D and system cost**
- **Highest potential accuracy**
- **Variation of index of refraction along LOS causes more position uncertainty through errors in angle measurement than through errors in range measurement.**
- **No water vapor refractive effect**
- **Increasing beamwidth error**
- **Limited to altitude measurements.**
- **Highest system cost.**
- **Pointing angle error has not been determined.**

### Counteracting Factors

- **Terrain sensitive: requires LOS from 3 ground sites to test vehicle (and to overflying aircraft) over most of the flight.**
- **Limited R&D cost of airborne trackers**
- **Could replace theodolites.**
- **Fast data turnaround.**
be made of airborne laser trackers, to determine the R and D gap between existing equipment capabilities and the WSMR requirements. An assessment would then be made of relative cost-benefit of a prototype laser, versus a 95 GHz, radar.

A prototype airborne tracking radar would then be developed, and incorporated in a purchased CIRIS-type system (ARIS, for example). An abbreviated system of ground sites would be implemented, and the system would be tested.

Regardless of which system reaches prototype realization—a multilateration or a CIRIS/airborne tracker system—the next stage of the recommended program would be a range-wide system design. It would include system control, telemetry network, data processing, and other system features. The resulting design would be a basis for requests for bids from manufacturers for a complete system which would enable WSMR to measure performance of low-flying missiles and aircraft.
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7. REFERENCES


11. "PLRS/Position Location Reporting System," General Dynamics Electronics Division and Hughes Aircraft Co.


83


18. MIL P-38005, Altimeters, pitot tubes, and static ports.


36. Telecon with Mr. Richard A. Pearson, Holloman Air Force Base, New Mexico.

37. Mr. R. E. Green, during project conference at Georgia Tech.


APPENDICES
APPENDIX A

LOW ALTITUDE TRACKING PROBLEM DEFINITION
(WSMR Internal Memorandum)

Background

US Army White Sands Missile Range has a long standing need to provide trajectory measurements on targets that sustain flight at low altitude. The requirements were summarized and documented in 1967, (reference 1). Low level intrusion is a very attractive offensive and counter offensive tactic. The Department of Defense has invested much time and money in recent years to develop guidance and control technology for low level intrusion weapon delivery systems. These developments have included: high quality inertial guidance systems, terrain matching systems, and terrain avoidance control systems. The success of these developments has intensified the need to develop instrumentation that can be used to evaluate weapon systems employing the new technology. Experience with complex weapon systems in the Vietnam conflict has shown that testing in a more realistic environment is required to assure operational effectiveness. Weapons that worked well under benign test conditions failed completely in combat. This indicates that low level intrusion weapons need to be tested in an environment approximating that which would be encountered in actual deployment. The development of such a capability at USAWSMR would not duplicate a function provided by any other DOD test facility. The development of this unique capability for USAWSMR should enhance position of the Range by providing a new capability that is needed by all services and not available at alternate locations.

Measurement Environment

In considering the measurement environment it is assumed that USAWSMR will cooperate to the fullest extent possible with providing test data under realistic conditions consistent with safety requirements. This implies that the measurement environment should duplicate the distances and types of terrain that might be encountered in actual combat situations. The actual
combat environment can be visualized by considering targets which US Forces might be required to engage using low level intrusion techniques. In the present world situation, desert terrain, mountainous terrain, wooded hills, and jungle areas are all likely areas where weapons of this type might be deployed. Some objectives might require flights over hundreds of miles across varied terrain. The response of a weapon delivery system to such an environment cannot be adequately tested by short flights over level terrain. Hence the required measurement environment for low altitude tracking is a large area with varied terrain. Vehicles that sustain flight at low altitude must be relative large in order to carry sufficient fuel to complete the flight. This means that the vehicle is large enough to carry a transponder.

Measurement Accuracy

A survey of the projects currently assigned to USAWSMR, that require low altitude tracking, was conducted to determine the measurement accuracy need for present weapons system technology. It is recognized that the measurement accuracy requirements stated in the UDS are not always a completely accurate statement of needs. It does represent the only official record of what is needed and is the basis for commitment of USAWSMR resources for testing. The survey indicated that 29 test programs (see Table A-1) currently being conducted at USAWSMR require support for sustained flights below 10,000 feet. The following summarizes the existing requirements for low altitude flight measurements.

UDS Low Altitude Requirements Summary

<table>
<thead>
<tr>
<th>Minimum Altitude Flown</th>
<th>0-200 ft</th>
<th>200-500 ft</th>
<th>500-2000 ft</th>
<th>2000-10,000 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Projects</td>
<td>8</td>
<td>10</td>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum Range Flown</th>
<th>10-20 mi</th>
<th>20-50 mi</th>
<th>50-100 mi</th>
<th>&gt;100 mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Projects</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position Measurement Accuracy Required</th>
<th>1-5 ft</th>
<th>5-10 ft</th>
<th>10-50 ft</th>
<th>&gt;50 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Projects</td>
<td>11</td>
<td>6</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>
Velocity Measurement Accuracy Required

<table>
<thead>
<tr>
<th>Accuracy Required</th>
<th>No. of Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05-1 ft/sec</td>
<td>6</td>
</tr>
<tr>
<td>1-5 ft/sec</td>
<td>12</td>
</tr>
<tr>
<td>5-10 ft/sec</td>
<td>2</td>
</tr>
<tr>
<td>&gt;10 ft/sec</td>
<td>7</td>
</tr>
</tbody>
</table>

Acceleration Measurement Accuracy Required

<table>
<thead>
<tr>
<th>Accuracy Required</th>
<th>No. of Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1-1 ft/sec^2</td>
<td>6</td>
</tr>
<tr>
<td>1-10 ft/sec^2</td>
<td>6</td>
</tr>
<tr>
<td>10-50 ft/sec^2</td>
<td>3</td>
</tr>
<tr>
<td>&gt;50 ft/sec^2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A-1

Current Projects Having Low Altitude Flight Measurement Requirements

<table>
<thead>
<tr>
<th>UDS#</th>
<th>NAME</th>
<th>UDS#</th>
<th>NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>MQM-61</td>
<td>486</td>
<td>ASM TEST</td>
</tr>
<tr>
<td>89</td>
<td>SAM-D</td>
<td>520</td>
<td>PAVE DEUCE</td>
</tr>
<tr>
<td>105</td>
<td>FAAAR</td>
<td>521</td>
<td>AQM-34U TERCOM DEMO</td>
</tr>
<tr>
<td>124</td>
<td>HAWK</td>
<td>379</td>
<td>A/C IN NAV SYS</td>
</tr>
<tr>
<td>147</td>
<td>NV 123</td>
<td>420</td>
<td>LORAINS</td>
</tr>
<tr>
<td>151</td>
<td>MQM 34D</td>
<td>449</td>
<td>AF-EMP</td>
</tr>
<tr>
<td>152</td>
<td>TALOS LAST</td>
<td>492</td>
<td>621B FIELD TESTS</td>
</tr>
<tr>
<td>157</td>
<td>HITVAL</td>
<td>495</td>
<td>DEFENSE SUPPRESSION</td>
</tr>
<tr>
<td>158</td>
<td>MODEL 1089</td>
<td>518</td>
<td>INHI FLT TEST</td>
</tr>
<tr>
<td>160</td>
<td>YAQM-37A</td>
<td>522</td>
<td>BI NAV TEST</td>
</tr>
<tr>
<td>301</td>
<td>HAWK/HP</td>
<td>713</td>
<td>CEFIRM LEADER</td>
</tr>
<tr>
<td>364</td>
<td>BQM-34A</td>
<td>808</td>
<td>NAV AIR WPNS TEST</td>
</tr>
<tr>
<td>374</td>
<td>HOUND DOG</td>
<td>833</td>
<td>F 14 FLT TEST</td>
</tr>
<tr>
<td>452</td>
<td>MAVERICK</td>
<td>953</td>
<td>EMP SIM</td>
</tr>
<tr>
<td>464</td>
<td>SRAM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The projects used in compiling this summary are identified in Table A-1. Comparison of the above summary with similar summaries compiled in 1967 (reference 1) and 1970 (reference 2) indicate that these requirements have
remained at a consistent level for the past six years. If USAWSMR is to respond to the need for more realistic testing, it is imperative that a capability for meeting these requirements be provided. It is suggested that the following be established as design goals for a trajectory measurement system to meet this need:

Coverage: Provide data on targets flying 200 feet AGL over the USAWSMR area with future expansion capability to include the USAWSMR-Green River corridor.

Position Measurement Accuracy: 10 feet

Velocity Measurement Accuracy: 5 feet/second

Acceleration Measurement Accuracy: 5 feet/second²

Data Output: Digital data available for use in flight control and flight safety applications.

Operational Environment

The system purchased to meet the need for low altitude tracking should provide for simple reliable operation at reasonable cost. It should be assembled from proven component subsystems not requiring research or further development apart from system integration. Ease of maintenance and calibration are important attributes for the system and should be prominent factors in system design. The personnel requirements for system operation should be minimized consistent with maintaining reasonable system cost.

Summary

This report has provided a discussion of the background and present requirements for an instrumentation system that furnished low altitude flight measurements. The information included indicates that the need for such a system has existed for at least the past six years. The problem is expected to continue for the foreseeable future. The report also shows that a low altitude tracking capability would provide USAWSMR with a unique testing asset.

(Signed)

ROBERT E. GREEN
Mathematician
A-4
REFERENCES
(Appendix A)

APPENDIX B

AIRBORNE TRACKING SYSTEM
FOR
LOW ALTITUDE TRACKING
(WSMR Internal Memorandum)

I. Introduction

USAWSMR has an established need for a measurement system to provide trajectory data on vehicles that sustain flight at low altitude. The requirement is for a system that can provide data on vehicles flying 200 feet AGL anywhere on the Range. The required trajectory data accuracies are:

- Position: 10 feet
- Velocity: 5 feet/second
- Acceleration: 5 feet/second^2

The vehicles tested under these conditions will carry beacons or transponders as tracking aids. The purpose of this report is to describe a tracking system that could be procured to meet these requirements. The proposed system will be described along with its operation, expected accuracy, and estimated cost. Possible alternate uses for the system or its components will also be discussed.

II. Airborne Tracking System Description

The proposed configuration for an airborne tracking system consists of three major components:

- An airborne platform location system.
- A small phased array radar.
- A test vehicle transponder.

The first two items are to be mounted in a pod that can be attached to an aircraft using standard Air Force pod hangers if possible. The test vehicle transponder will be mounted in the object being tested while flying at low altitude. The airborne platform location system will be used to locate the
position of the instrumentation pod being carried on a high flying aircraft while the phased array radar will track the transponder on the test object from the pod. The suggested equipment configuration consists of the following.

A. Airborne Platform Location System

The airborne platform location system proposed configuration includes a range and range rate measuring system and a high quality inertial navigation system. This configuration provides two independent estimates of aircraft position with different types of error statistics. This should provide better information than can be achieved by combining two systems with similar error statistics. The range and range rate system will be required to produce accurate measurements that can be processed for real time display. The required performance is two feet accuracy in range and .1 foot/second in range rate sampled five times per second. The recommended configuration for the range and range rate measurement system is an airborne interrogator that provides simultaneous measurements to at least four ground transponders. This type of system is recommended for the following reasons:

1. Studies performed by the Air Force indicate that system accuracy cannot be met unless simultaneous measurements are performed (1).
2. Airborne tracking systems of this type have been fabricated for similar applications (2).
3. This configuration allows the measurement of Doppler velocity without the need to transmit a reference frequency between ground stations. This eliminates a major source of error and expense for Doppler measurement systems.
4. Present technology allows such a system to be packaged for airborne application.
5. Measurements to four ground stations provide some redundancy for greater reliability and error estimation.
6. The range rate measurements can be used to significantly reduce the noise content of the range data thereby enhancing the accuracy of platform location. The velocity information is required to provide update information for the inertial navigation system. This results in much improved accuracy from an inertial navigation system.(1).

The inertial navigation system should be one that is presently in the operational inventory for DOD aircraft. This obviates the need for development in an area where the Range has very limited expertise. The inertial navigation system will play a dual role in the airborne tracking system. The data from it will be used to estimate the position of the airborne platform and the orientation of the phased array radar. The inertial navigation system will also be interfaced with the phased array radar for an altimeter input.

B. A Small Phased Array Radar

The small phased array radar will be used to look down from the airborne platform to track a transponder equipped target flying near the ground. The phased array radar approach is chosen since its electronic agility eliminates the need for three-axis stabilization required for a mechanical tracking device. The multiple target tracking capability of the phased array eliminates the need for a separate altimeter in the system and also makes in-flight calibration of the radar system practical. The radar can be used as an altimeter by directing a beam down from the airborne platform. The inertial navigation system can be used to determine the downward direction. In-flight calibration of the radar can be accomplished by locating additional radar transponders at the location of the range and range rate measurements system transponders. Interrogation of these transponders should provide accurate inflight calibration of the phased array radar system. It is recommended that the radar operate at K-band using an array with an aperture of approximately 36 inches in diameter. It is suggested that the radar be equipped with a four target capability for the required operational flexibility.
Each track channel should provide for either beacon or skin tracking. The present Air Force inventory of airborne phased array radars should be investigated to determine if available equipment could be adapted to meet this need.

C. A Test Vehicle Transponder

The test vehicle transponder is used to separate the tracked target from the radar echo returned from the ground and to increase the tracking range capability of the system. A conventional K-band radar beacon should provide the desired target enhancement for tracking in the low altitude environment. For this application, the antenna should be mounted on top of the test vehicle to provide the required coverage and limit the amount of power illuminating the ground. Transponders for this application should be readily available from industry.

III. System Operational Concept

A test conducted using an airborne tracking system will require an aircraft to carry the instrumentation pod and a test vehicle transponder mounted in the test object. The flight of the instrumentation aircraft and the test object must be coordinated so that the separation between the two does not exceed the tracking range of the phased array radar. The ground transponders are placed along the flight path of the instrumentation aircraft in a pattern that minimizes errors due to system geometry. It is suggested that the instrumentation aircraft be operated at an altitude of approximately 40,000 feet. The Air Force maintains a fairly large inventory of aircraft that can be operated at this altitude. The suggestion for mounting the system in a pod that can be carried by whatever aircraft is available is an attempt to avoid being restricted to a single plane that may not be available when needed. In an actual test operation the instrumentation aircraft will be flown over a prescribed course to coincide with the launching of the test object. The platform location system will be used to
determine the position of the aircraft. The phased array radar will then acquire and track the test vehicle. The relative location of the two aircraft at the beginning of the test will be a function of the relative speeds that are flown. If the instrumentation aircraft can fly at approximately the same speed as the test vehicle, then the test might begin with the test vehicle slightly ahead of the instrumentation aircraft. If the test vehicle flies much faster than the instrumentation aircraft then the instrumentation aircraft would be positioned ahead of the test vehicle in order to maximize the amount of time that the test vehicle will remain within range of the radar. Initial acquisition techniques will require further investigation. Possible alternatives include the use of the aircraft bomb sight by the pilot, calibrated to direct a search by the phased array radar. Initial acquisition might also be provided from information generated by ground based instrumentation. It is suggested that the data processing performed in the airborne tracking system be limited to that required for effective operation and control. The remainder of the data processing can be performed by the USAWSMR UNIVAC 1108 computing system. The requirement to mount the equipment in a pod necessitates minimizing size and weight of the airborne equipment. It is suggested that the data generated by the airborne tracking system be transmitted to the ground for processing using standard telemetry equipment. The telemetry system is designed for reception of such data and provides for direct entry into existing computing facilities. The use of the radar as an altimeter requires that the altitude of the point measured on the ground must be known. The presently available maps of the USAWSMR area should provide sufficient accuracy for this purpose. It is suggested that a task be initiated to develop a method of reducing the map information to digital data for use with an airborne tracking system.

IV. Range and Range Rate Transponder Deployment

The optimum elevation angle for a range and range rate tracking system is approximately 35 degrees (3). Using this criterion, twelve transponders deployed on the Range would provide very good geometry over the entire area for an aircraft flying at 40,000 feet altitude. The system can be used with
the transponders spaced further apart resulting in somewhat lower accuracy. The inertial navigation system can be used to provide data when the system is in unfavorable geometric locations. It is suggested that the initial system procurement acquire four transponders with the remaining eight being acquired after the system has been acceptance tested.

The following suggested transponder locations have been chosen from maps of the USAWSMR area and represent a typical deployment. Inspection of these sites may indicate that some are not operationally suitable. The proposed site locations are:

A. Four station optimal geometry for checkout.
   1. UPDOC
   2. TWO BUTTES
   3. Chuck
   4. 3.5 miles west of SW 30

B. Four station Range wide coverage.
   1. UPDOC
   2. COWAN
   3. D-5
   4. SOTIM 3

C. Twelve station range wide coverage.
   1. UPDOC
   2. TWO BUTTES
   3. CHUCK
   4. 3.5 miles west of SW 30
   5. COWAN
   6. D-5 (on Gunsight Peak or Salinas Peak)
   7. Along RR9 17 miles east of SALINAS
   8. SOTIM 3
   9. MARTIN Ranch
10. OSCURA RC
11. TIFF
12. Intersection RR9 and Highway 380

It is expected that a narrow corridor from USAWSMR to Green River can be instrumented with these twelve transponders. Further analysis is required to determine the best transponder deployment for this application.

V. System Error Budget

The following system error budget is estimated based on available information. It should be recognized that this information is preliminary and will be refined as the system is more completely defined.

A. Range and Range Rate System
   1. Range measurement accuracy 2 feet
   2. Range rate measurement accuracy 1 ft/sec
   3. System position measurement accuracy 1.5 feet

B. Inertial Navigation System
   1. Position measurement accuracy .5 foot
   2. Attitude measurement accuracy 10 sec

C. Altimeter System
   1. Radar altimeter measurement accuracy 5 feet
   2. Map location and height accuracy 3 feet

D. Airborne Platform Location System
   Estimated accuracy of the position and attitude obtained by combining A, B, and
   Position accuracy 2 feet
   Attitude accuracy 10 sec

E. Phased Array Radar System
   1. Range measurement accuracy 5 feet
2. Angle measurement accuracy 20 sec
3. Position measurement accuracy relative to the airborne platform for slant range to a target of 100,000 feet.

F. Geodetic Measurement Systems
Position measurement accuracy 2 PPM

G. Airborne Tracking System
Position measurement accuracy for slant range to target of 100,000 feet.

It is expected that position data of the quality indicated can be differentiated to provide velocity and acceleration data of the quality specified for the system.

VI. System Cost Estimate

The cost estimate furnished here is preliminary and should be replaced by a more careful and detailed engineering cost estimate.

A. Range and Range Rate Tracking System
One Airborne interrogator and four transponders $1,000,000.00

B. Inertial Navigation System
200,000.00

C. Phased Array Radar
1,500,000.00

D. Systems Integration and Packaging
500,000.00

TOTAL $3,200,000.00

VII. Alternate Uses

The acquisition of an airborne tracking system would enhance the USAWSMR capability to support testing in other areas besides low altitude tracking. The ability to use the system for other applications increases the system utilization and permits the amortization of system cost over a greater percentage of the testing workload. The Airborne Tracking System
or components thereof could be applied to the following USAWSMR testing problems:

- Aircraft flight testing
- Range instrumentation calibration
- Air to air missile testing
- Low altitude drone control
- Near launch missile tracking

The first three items of this list are applications for using the system as configured for low altitude tracking. For aircraft flight testing, the system could be attached to the vehicle being tested. The normal operation of the airborne platform location system would perform the required function. As indicated earlier, the transponders could be redeployed to cover greater distances. The lower accuracy achieved would be sufficient to meet many user requirements. The Airborne Tracking System could function as a standard for Range instrumentation calibration. The accuracy specified for the airborne platform location system is sufficient to identify bias errors in present Range instrumentation. The use of such a system for calibration should improve the performance of present instrumentation by providing a tool that can be used to reduce bias errors significantly. An airborne tracking system would provide a cost effective method of calibration since some calibrations could be performed when the system was being used as test instrumentation. It could be used for calibration of most types of USAWSMR instrumentation and the cost of special calibration flights would not be excessive. The Airborne Tracking System should provide a significant improvement in Range capability to support air-to-air missile launches and intercepts. This is particularly true for high altitude tests of these small sized missiles. If the system can be successfully pod mounted, it might be possible to carry this instrumentation on the missile launching aircraft. Operation of the phased array radar at the short ranges involved should provide good quality data for scoring air-to-air intercepts. An airborne tracking system is also potentially useful as a device for controlling drones flying at low altitude. The system could be interfaced with the Vega drone control system to provide the required control functions. This application requires that the original system be modified by adding the Vega control system. The last suggested alternate use of the
The proposed system is for near launch missile tracking. This application would require only the phased array radar portion of the system. The small size and electronic agility of the radar make it possible to locate the equipment near a missile launcher. The radar could provide data very early in a missile flight. The data provided could be used for flight safety monitoring, direction of other instruments, and metric measurement data. The application of an Airborne Tracking System for these uses in addition to low altitude tracking indicate that it is a cost effective solution to the problem.

VIII. Requirements Summary for Alternate Uses

A brief summary of requirements for the four alternate uses identified is included to show that the Airborne Tracking System can be useful in meeting these needs.

A. Aircraft Flight Testing Measurement Accuracy Requirements:

<table>
<thead>
<tr>
<th>Function</th>
<th>Accuracy Required</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Highest</td>
</tr>
<tr>
<td>Position</td>
<td>2.0 ft</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.10 ft/sec</td>
</tr>
<tr>
<td>Acceleration</td>
<td>10.0 ft/sec²</td>
</tr>
<tr>
<td>Attitude</td>
<td>0.05 deg</td>
</tr>
</tbody>
</table>

Aircraft flight testing represents a significant portion of the USAWSMR workload. It is estimated that 20 per cent of the workload is aircraft flight testing.

B. Range Instrumentation Calibration

This function deals not with amount of workload but with quality of results furnished to Range customers. The required calibration accuracy levels for each major measurement instrumentation system is included in this table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Instrument Type</th>
<th>Accuracy Required</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contraves</td>
<td>Askania</td>
</tr>
<tr>
<td>Position</td>
<td>1.0 ft</td>
<td>1.0 ft</td>
</tr>
<tr>
<td>Velocity</td>
<td>1.2 ft/sec</td>
<td>1.0 ft/sec²</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1.6 ft/sec²</td>
<td>1.2 ft/sec²</td>
</tr>
<tr>
<td>Attitude</td>
<td>1.0 deg</td>
<td>1.0 deg</td>
</tr>
</tbody>
</table>

B-10
C. Air-to-Air Missile Testing Measurement Accuracy Requirements:

<table>
<thead>
<tr>
<th>Function</th>
<th>Highest</th>
<th>Lowest</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>1.0 ft</td>
<td>10.0 ft</td>
<td>5.0 ft</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.20 ft/sec</td>
<td>5.0 ft/sec</td>
<td>1.0 ft/sec</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1.0 ft/sec²</td>
<td>16.0 ft/sec²</td>
<td>3.20 ft/sec²</td>
</tr>
</tbody>
</table>

This category represents a fairly small number of Range customers, but is usually afforded a high priority due to its importance to the defense effort.

D. Low Altitude Drone Control Requirements

The Range has not yet been requested to provide data for the low altitude drone control function. It is estimated that to control a drone flying 200 feet AGL, that data accurate to 50 feet would be required.

E. Near Launch Missile Tracking Measurement Accuracy Requirements:

<table>
<thead>
<tr>
<th>Function</th>
<th>Highest</th>
<th>Lowest</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>0.15 ft</td>
<td>100 ft</td>
<td>5.0 ft</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.10 ft/sec²</td>
<td>100 ft/sec²</td>
<td>5.0 ft/sec²</td>
</tr>
<tr>
<td>Acceleration</td>
<td>0.01 ft/sec²</td>
<td>32.0 ft/sec²</td>
<td>7.5 ft/sec²</td>
</tr>
</tbody>
</table>

This category also represents a significant portion of the USAWSMR workload. It is estimated that 30 per cent of the Range customers require near launch tracking data. Data is presently being provided using Fixed Cameras. This method of data collection is slow and expensive. If the phased array radar could be used for half of these projects, the saving would be significant in both time and money.

IX. Summary

This paper has presented a description of an Airborne Tracking System as a proposed solution to USAWSMR low altitude tracking problem. The system proposed will meet the requirements stated in the problem statement. The Airborne Tracking System can be used to instrument flights over hilly and mountainous terrain as well as over flat terrain. It appears that the system can be expected to provide the required measurement accuracy. Besides
providing the required capability for low altitude tracking, the Airborne Tracking System equipment could be applied to other Range measurement problems. Hence an Airborne Tracking System would provide a workable cost effective method of meeting the USAWSMR low altitude tracking requirements.
APPENDIX C

REQUIREMENTS FOR A SYSTEM TO MEASURE PERFORMANCE OF LOW-FLYING MISSILES AND AIRCRAFT

The following outline specifies the WSMR requirements for a system to measure low-flying test vehicles:

I. General Requirement:

An instrumentation tracking system is required to provide accurate trajectory data on test vehicles flying at low-altitudes anywhere on White Sands Missile Range.

II. Target: (Test Vehicle)

A. Type: Missiles, RPV's, A/C, etc., but probably typified by SRAM (cruise missile).
B. Number: Single target.
C. Velocity: Both subsonic and supersonic targets must be considered.
D. Altitude: 200 feet to 1000 feet AGL typical.
E. Expected RCS: 5 - 15 dBsm typical.
F. Target Instrumentation: Radar transponder.

III. Coverage/Operational Scenario:

A. Area: Test vehicle located anywhere on (over) WSRM; future expansion to include Green River Corridor.
B. Terrain: Desert & mountainous.
C. Weather: Clear with low humidity; little or no rain.

IV. System Error Requirements:

A. Position Measurement Accuracy: 10 feet, any axis.
B. Velocity Measurement Accuracy: 5 feet/second.
C. Acceleration Measurement Accuracy: 5 feet/second².
V. Data Format and Processing:
   A. Format: Standard telemetry equipment compatible.
   B. Computation Equipment: IBM 360/65 (on-line with telemetry system) and UNIVAC 1108.
   C. Philosophy: Utilize ground-based processors to maximum extent possible.

VI. Airborne Instrumentation:
   A. Test Vehicle Target: Limited to transponder (relatively small package).
   B. Other A/B Instrumentation: Packaged in a standard bomb-rack pod, weighing no more than approximately 1000 pounds, and having dimensions of 15 feet long by 3-foot diameter cylinder. Pod should be completely interchangeable between aircraft.
   C. Availability: Instrumentation currently within the military/commercial inventory should be used to the maximum extent possible.
   D. Maintenance and Calibration: Prime considerations.

VII. Ground-Based Instrumentation:
   A. Mobility/Transportability: Equipment should be as small and transportable as possible consistent with other system constraints.
   B. Unattended Operation: Ground-based instrumentation may be required to operate at remote locations and unattended.
   C. Survey Error: On the WSMR, ground-based instrumentation can be located to an accuracy of 2 parts per million.
   D. Availability: Instrumentation currently within the military/commercial inventory should be used to the maximum extent possible.
   E. Maintenance and Calibration: Prime considerations.

VIII. Proposed System Configuration:
   A. Primary Components:
      1. Position location system consisting of at least 4 ground-based transponders, an airborne interrogator, data processor, and an inertial navigation system.
      2. Airborne instrumentation platform (probably an aircraft).
3. Airborne radar capable of acquiring and tracking the test vehicle target.
4. Radar transponder on-board the test vehicle.

B. Operation: By providing measurement of range and range rate (nominally) between the A/B interrogator and 4 ground-based transponders, the position measurement system establishes an estimate of the position of the airborne platform. A second, independent estimate of position is obtained from the inertial navigation unit carried on-board the airborne platform. Combining the two independent position estimates improves the overall position estimate. Position data on the test vehicle relative to the airborne platform is obtained from the A/B radar measurements of range and angle. Radar attitude stabilization is obtained from the inertial navigation unit also. The transponder on board the test vehicle target improves the received S/N and allows the target return to be separated from the ground return.

C. Specific System Parameters:
1. A/B platform altitude is approximately 40,000 feet AGL.
2. A/B platform velocity is approximately the same as test vehicle.
3. Slant range from A/B platform to target more than 40K feet but less than 100K feet.
4. Expect A/B radar to operate above X-band.
5. Elevation angle for position location system should be optimized to approximately 35°.
APPENDIX D

POSITION LOCATING SYSTEMS

The following outline is a condensation of the characteristics of a number of position locating systems. It includes systems with ranging only, as well as systems combining range and inertial measurement. One system is a laser radar which measures azimuth, elevation, and range to the target from a ground based site.

I. AROD (RRS)

A. Data Source/References (all by Motorola):

1. AROD Test and Feasibility Demonstration Program Definition.
2. AROD Vehicle Tracking Receiver Design.
3. AROD System Concept.
4. AROD System Test Model.
5. AROD Test Model Hardware.
6. AROD Flight Demonstration Proposal.
7. AROD Flight Demonstration Test Report [12].

B. Operational Description:

1. Three or more ground-based, completely automatic transponders.
2. Space vehicle based interrogator.
3. Space vehicle based computer.
4. Range modulation: + 90° phase shift, PN code.
6. Acquisition: 2 sec.
7. PN code: Low clock rate for acquisition; high clock rate for tracking.
   Length: $6.084 \times 10^6$ count equivalent.
   Down link: 2.214 GHz.
   Up link: 1.800 GHz.
   Command: 137.5 MHz.
   Transponders: 60.
   S-Band: 20W.
VHF: 6W.
Threshold: -126 dBm.
Dynamic range: 27 dB.
N. F.: 8.3 dB
Power Required:
Interr: 143W.
Trans.: 220W.
Tracking BW: Range, 4-5 Hz; carrier, 200 Hz.
Signal: As strong as -70 dBm degrades the performance.

C. Employment (Scenario):

Range and range-rate measurements from space vehicle to ground are transmitted on a turn-around S-band link. Range is determined from two-way time delay; range rate, from Doppler shift of S-band carrier frequency. PN code length assures no ambiguity within 3.042 x 10^6 m. Transponders are phase locked loop tracking type--not easily adapted to multiple interrogators.

Interrogation of three transponders is simultaneous, while fourth is being acquired. Pick-up and drop are automatic, controlled by range.

D. Principal Sources and Magnitudes of Error:

1. Range to position geometrical blow-up error (GDOP) 10 times the range error.
2. Survey error 1 x 10^-5.
3. Altitude measurement error (negligible if calibrated during line crossing).
4. Equipment error 0.7 feet RMS.
5. Atmospheric propagation velocity error, 6 x 10^-6.

E. Accuracy Specifications (bench test):

Range
Resolution 0.25m
Accuracy ± 0.5m (0.75m, with 26 dB co-channel interference)
R max, 2 x 10^6m (unambiguous range)
Range Rate
Resolution 0.02 m/s
Accuracy + 0.015 m/s
\( R_{\text{max}} + 1.2 \times 10^4 \) m/s
\( \dot{R}_{\text{max}} 450 \) m/s² (for 20 sec)

F. Cost Estimate:
10's of thousands of dollars for transponders (1967).

G. Availability: No working system exists.

II. CIRIS, Litton/Cubic CR-100 (RRS, IMU, Kalman)

A. Data Source/References:
1. CIRIS Design Evaluation Report [1].
2. Precision Ranging System, CR-100 brochure.
4. Telephone conversations with:
   (a) Richard Pearson, Holloman AFB.
   (b) Bard Crawford, TASC.
   (c) Visit to Litton and Cubic.
5. Post-Flight Filtering and Smoothing of CIRIS Inertial and Precision Ranging Data [4].

B. Operational Description:
2. Airborne interrogator.
3. IMU: Litton AN/ASN-86, with Navigation Computer Unit, which uses barometric altimeter input to vertical channel.

C. Employment:
Sequential interrogation from airborne reference platform of the ground site transponders, at rate of 5 sec per transponder, 15 sec for three units. Range and range rate are obtained at same time in each interrogation. Dropout of one transponder and pick-up of another, to get optimum location accuracy, is possible. The
RRS may be viewed as reinitializing the IMU, or the IMU can be viewed as a smoothing filter to give continuity of data between RRS interrogations. A 10-state Kalman filter permits the hybrid system to be more accurate than either component (IMU or RRS) alone, provided the filter is properly designed. This implies good prior knowledge of the characteristics of the sources of error.

D. Principal Sources of Error:
   1. IMU sensors (gyros and accelerometers).
   2. Attitude readout.
   3. Range measurement (scale factor and atmospheric disturbances).
   4. Range rate measurement.
   5. Barometric measurement.
   7. Computer mechanizations.

E. Accuracy Specifications:
   - Position: 12.5 feet RMS (150 mile maximum spacing between transponders)
   - Velocity: 0.05 ft/sec RMS
   - Attitude: 15 sec/axis RMS

F. Cost Estimate:
   - $100,000 for each transponder (space shuttle version)
   - $1,500,000 for airborne unit

G. Generic system is operational at Holloman AFB.

III. AC Carousel/Cubic CR-100 (RRS, IMU, Kalman)
A. Data Sources/References: Same as II.
B. Operational Description:
   1. RRS: Cubic CR-100, range only, ground based.
   2. Airborne interrogator.
   3. IMU: AC Carousel IV, with 32-speed resolver for azimuth readout.
   4. Northrup NDC-1051A computer.
C. Employment:

Same as II, except that Kalman filter has 22 states (including 3 for survey errors when in "survey mode"). Every fifth measurement is the output of an altimeter, and four transponders are interrogated cyclically rather than three. Every 10 seconds the system is updated by a single scalar measurement, so 50 seconds is the period of an interrogation cycle.

D. Principal Sources of Error:

Same as II, but with bias tip rate in place of azimuth gyro scale factor error and certain other sensor errors. Absence of range rate information as an independent measurement affects error distributions and magnitudes.

E. Accuracy Specification: Same as II.

F. Cost Estimate: Same as CIRIS/Litton/Cubic.

G. Generic system is operational at Holloman AFB.

IV. SHIRAN (RRS)

A. Data Source:

1. Motorola report [2].
2. SHIRAN Geodetic Survey System, Electronic [13].

B. Operational Description:

1. 3 GHz.
2. 4 of 6 transponders at a time.
3. 500 miles capability.
4. Preflight calibration (pole beacon).
5. Interrogation: 12 millisecond, each station, every 0.1 sec.
6. 4 sinewave frequencies, lowest gives least significant digit = 500 miles.
7. \( \phi \) modulation \( \pm 12 \) rad.
8. RF BW = 35 MHz.
9. Continuous range tracker, 10 samples/sec input, 22 bits @ 5/sec (110 bits/sec) output.
11. Transmitter power: 20W, airborne and ground units.
12. Antenna gain: A/C, 8 dB; ground, 18 dB.
13. Transponder: 250 lb. 50 foot pole mounted, must be monitored.

C. **Employment Scenario:**

   Aerial surveying. Calibration by pole beacon; line crossings for range and altitude calibration during flight. Adaptable to multiple users and to slaving.

D. **Principal Sources of Error:**

   1. Atmospheric effect of index of refraction along propagation path.
   3. Calibration errors.

E. **Accuracy (measured):**

   1. Position resolution: 9 inches.
   2. Position accuracy: 3 m (includes propagation and survey error).

F. **Cost Estimate:**

   $25,000 for each of six transponders.
   $200,000 for airborne interrogator.

G. **Availability:** Exists, has military designation, AN/ASQ-32.

V. PLRS Hughes/Gen. Dynamics, (RRS)

A. **Data Source/References:** Notes [11].

B. **Operational Description:**

   1. One master unit, one sub-master unit.
   2. Many man-packed, surface vehicular, and airborne units.
   3. Range measurements.
   4. Trilateration computes three dimensional position.
   5. Unit display of position, navigation, related information.
6. Time slot reporting.
7. 100 message types.
8. 1.875 second reporting cycle (frame).
9. 9 millisecond range/message time slot, 900 per frame.
10. Aircraft reporting cycle: 2 seconds at 15 per second maximum rate for a mix of users.

C. Employment:
   Tactical data support system for command and control of deployed amphibious assault forces. Capacity: 370 users.

D. Principal Error Sources:
   Probably equipment, since accuracy specifications are poor.

E. Accuracy Specifications:

<table>
<thead>
<tr>
<th>Zone A</th>
<th>Az</th>
<th>El</th>
</tr>
</thead>
<tbody>
<tr>
<td>slow, fixed wing a/c</td>
<td>50 m</td>
<td>50 m</td>
</tr>
<tr>
<td>high speed</td>
<td>200 m</td>
<td>200 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zone B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>slow</td>
<td>200 m</td>
<td>200 m</td>
</tr>
<tr>
<td>high speed</td>
<td>400 m</td>
<td>400 m</td>
</tr>
</tbody>
</table>

F. Cost Estimate: Several million dollars for a full system.

G. Availability: Operating system at Navelex, Fleet Marines.

VI. RMS-2/DCS (Range Measuring System/Data Collection System), General Dynamics

A. Data Source/References: Notes, brochure [7].

B. Operational Description:
   1. Fixed and mobile interrogation (A units).
   2. Relay (D units).
   3. One centralized, computer interfaced (C unit).
   4. Range [C/A or D:9km; A/B:64km(LOS)]. By command from C unit, A unit interrogates B unit by sending a ranging pulse, measuring time to response, sending 15-bit number to C unit.
5. Time Slot: 0.744 percent duty cycle/B unit.
6. WWV synchronization.

C. Employment:

Cylinder 20 miles diameter, 20,000 feet altitude. Men, vehicles, aircraft. Position and communication. C unit uses semi-trailer (10 tons) scaffold tower, parabola and omni antennas. Full computer/terminal equipment. Power required: 18 kW. A-station has erected tower and unmanned electronics.

D. Principal Sources of Error:

1. Survey errors for C and A units.
2. Propagation errors.
3. Equipment errors, A/B units, including A unit clocks.

E. Accuracy Specifications:

1. Position, ±3 meters, with respect to known reference, in x, y, z coordinates.
2. Precision of ranging: ±2 meters.
3. Clock must thus have pulse jitter less than ±7 nanoseconds.
4. ±20 meters reported as experienced Yuma Proving Ground.

F. Cost Estimate:

C unit: $350,000
A unit: $50,000
Micro B unit: $35,000 each

G. Availability: Operational at Yuma Proving Ground.

VII. RMS/SCORE, General Dynamics (RRS, IMU, Kalman)

A. Data Source/Reference:

1. Trip reports.
2. Brochures [8].
B. Operational Description:

Same as item VI, RMS-2/DCS, with additions of SCORE (Simulated Combat Operations Range Equipment), large scale computer capability and large screen 3-D real time display. SCORE has an aircraft sub-system which includes:

- IMU (strapdown)
- Signal conditioner
- Micro B transducer
- Antenna and radome
- Air data unit

C. Employment:

Extends RMS-2/DCS from primarily locating ground based equipment and low-flying support aircraft to include high-flying aircraft.

D. Principal Sources of Error: IMU, and same as in item VI.

E. Accuracy Specifications:

- Position: 25 feet any axis
- Velocity: 15 feet/sec.

IMU:

- Accelerometer bias (3σ): $2 \times 10^{-3} g$.
- Accelerometer misalignment (3σ): 205 sec.

Flight test errors (in good, transitional, and bad geometry regions):

- X and Y: ± 4 meters
- Z: ± 6, 8, 10 meters
- Roll and pitch: ± 1 degree
- Yaw: 1.5, 2.0, 2.5 degrees

F. Cost Estimates:

- SCORE pod: $100,000
- Micro B unit: $35,000 each
- C unit: 350,000 each
- A unit: 50,000 each

G. Availability: Can be ordered.
VIII. ARIS (Airborne Range Instrumentation System) Litton (RRS, IMU, computer)

A. Data Source/Reference:

1. Trip report.
2. Brochure [5].

B. Operational Description:

SUU - 16, gun type pod, 22 inches diameter, 15 feet long, 800 pounds.
IMU: AN-92 INU.
Computer: ASN-92 ANCU
Pitot tube probe.
Air pressure transducer.
Interrogator.
Recorder.
Power supply and control.
1.6 GHz interrogator.
Cubic CR-100 ground sited transponders.

C. Employment:

High precision bomb scoring.
Quick data turn-around.
One-day preparation.
Unmanned ground transponders.
Base maintenance.

D. Principal Sources of Error: Same as CIRIS.

E. Accuracy Specifications:

Position: 5 feet.
Velocity: 0.5 feet/sec.

F. Cost Estimates:

Pod: $350,000.
Transponders: $12,000 each.
Ground data terminal: $50,000.
Support equipment: $30,000.
G. Availability: Operating at Eglin AFB. Can be purchased.

IX. PATS, Sylvania (laser radar; azimuth, elevation, range)

A. Data Source/References:
   1. Trip report.
   2. Brochure.

B. Operational Description:

   YAG, 1.06 micron wavelength.
   Tracking laser.
   Elevation over azimuth mount, ground based.
   Retroreflector fastened to target.
   Joystick acquisition.
   Video camera co-mounted with laser for aid in acquisition.
   Minicomputer.
   Video recorder.
   X-Y plotter.
   Range counter.
   Logic control unit.
   Instrument van.

C. Employment:

   Tracks mortar shell, helicopter, aircraft.
   Maximum range, 100,000 feet.
   Data rate: 10, 20, 50, 100/sec.
   Coverage: Azimuth, ±170 degrees; elevation, -5 to +85 degrees.
   Slewing characteristics: 0.5 rad/sec, 0.08 rad/sec², azimuth and elevation.
   Display: Range, 1-foot increments; elevation and azimuth, 1 degree increments.
   Field of view: Video, 5 to 20 degrees (zoom); laser, acquisition, 3 millirad.
   Set-up: 1 hour.
D. Principal Sources of Error:

Atmospheric refraction.
Optics mechanical error.
Servo and readout resolution.

E. Accuracy Specified (up to 65,000 feet):

Range, + 2 feet.
Azimuth and elevation, 0.1 milliradian

F. Cost Estimate: $600,000, complete with instrument van.

G. Availability: Operational at Yuma Proving Ground.

X. A-7E Navigation and Weapon Delivery System (RRS, IMU)


B. Operational Description:

IMU: AN/ASN-90(V).
Doppler Radar Set (DRS): AN/APN-190(V).
Forward Looking Radar (FLR): AN/AFQ-126(V).
Air Data Computer (ADC): CP-953/AJQ.
Heads Up Display: AN/AVQ-7(V).
Projected Map Display: AN/ASN-99.
Tactical Computer Set (TC-2): AN/ASN-91(V).


D. Principal Sources of Error: Not discussed.

E. Accuracy Specifications:

1. Probably not stringent because missiles require only rough aiming if they are homing devices.
2. Gun aiming probably uses feedback, miss distance error signal.

F. Cost Estimate: Not given.

G. Availability: All equipment in military arsenal.
XI. ACRS (Air Combat Maneuvering Range) Cubic (RRS, IMU, ground based data reduction and graphics display)

A. Data Sources/References: Trip report.

B. Operational Description:

- Strapdown IMU.
- Six ground based transponders.
- Telemetry, Yuma Marine Air Station to Miramar Naval Air Station, San Diego.
- Data reduction, recording at Miramar.
- Graphics display on large screen CRT, with variable aspect, terrain, dynamics of encounter, scoring, time, printout availability.
- Airborne equipment in sidewinder pod.

C. Employment:

- Real (mock) dogfight recording, instant debriefing, detailed analysis of combat (32 reasons for a miss are available). Graphics from cockpit of "friend" or "foe", or any point external to action.

D. Principal Sources of Error: Same as for item VIII, ARIS.

E. Accuracy Specifications: Not given, probably same as item VIII, ARIS.

F. Cost Estimate:

- Ground based transponder: $65,000 to $80,000 each.
- Pod: $350,000.
- Ground equipment: $1,500,000.

G. Availability: Operational at Air Marine Station, Yuma, and Miramar Naval Air Station, San Diego.
Table E-I lists the known (as of June 1974) U. S. radars in the frequency region 70 to 140 GHz (F. B. Dyer and E. K. Reedy, "Millimeter Wave Radars," 1974 IEEE S-MTT International Microwave Symposium Proceedings Georgia Institute of Technology, Atlanta, Georgia, June, 1974). Georgia Tech developed and fabricated prototypes of five of the radars listed.

The analysis in Chapter 3 indicated that a 70 GHz or 95 GHz radar mounted in an airborne pod with the components of a CIRIS-type reference platform locating system would permit the measurement of performance of low-flying missiles within the accuracy required by WSMR.

Prototype 95 GHz Radar

Georgia Tech has developed a number of millimeter radars. These will be described here to indicate the state-of-the-art. The first to be described is an instrumented, calibrated short pulse measurement radar operating at approximately 95 GHz. Major parameters of this radar are summarized in Table E-II. It is housed in a small, protective container which consists of two separate compartments; one containing the magnetron and modulator, shown in Figure E-1. The receiver is behind the antenna shown in Figure E-1. The packaging approach combines the desirable level of isolation of the functions needed to minimize interaction and interference problems with good portability and accessibility. Sufficient space was provided in the package to allow the radar to be used in a number of different experiments. The overall system configuration is shown in block diagram form in Figure E-2.

This radar could possibly be adapted for mounting in an airborne pod. The research and development effort would include modifications of the packaging to meet environment requirements. The R&D would also include the design and fabrication of a larger, steerable antenna, the means for acquiring and automatically tracking the transponder on the low-flying vehicle, the means for readout of angular directions and range to the target, and a compatible transponder. This seems to be a feasible, though difficult, electromechanical R&D task.
<table>
<thead>
<tr>
<th>Identification</th>
<th>Application</th>
<th>Frequency</th>
<th>Sponsor &amp; Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>AN/APQ-62</td>
<td>Side Looking Mapping Radar</td>
<td>70 GHz</td>
<td>WPAFB &amp; TRC Raytheon</td>
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<tr>
<td>JR-9</td>
<td>Search Mapping Radar</td>
<td>70 GHz</td>
<td></td>
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<tr>
<td>AN/BPS-8</td>
<td>Search Radar</td>
<td>70 GHz</td>
<td>USAECOM &amp; Georgia Institute of Technology</td>
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<tr>
<td>AN/MPS-29</td>
<td>Search and Surveillance Radar</td>
<td>70 GHz</td>
<td>USAECOM &amp; Nordon Division, United Aircraft</td>
</tr>
<tr>
<td>Experimental</td>
<td>Aircraft Obstacle Avoidance</td>
<td>70 GHz</td>
<td>NADC, Warminster, Pa.</td>
</tr>
<tr>
<td></td>
<td>Aircraft Instrument Landing</td>
<td>70 GHz</td>
<td>Aerospace Corp., El Segundo, Cal.</td>
</tr>
<tr>
<td></td>
<td>Obstacle Avoidance</td>
<td>95 GHz</td>
<td>Navy &amp; Applied Physics Lab, Silver Springs, Md.</td>
</tr>
<tr>
<td>Experimental</td>
<td>Sea Clutter Measurement</td>
<td>95 GHz</td>
<td>WPAFB &amp; Goodyear Aerospace Corp.</td>
</tr>
<tr>
<td>Experimental</td>
<td>Space Object Identification</td>
<td>95 GHz</td>
<td>Ballistic Research Labs, Aberdeen Proving Grounds, Md.</td>
</tr>
<tr>
<td>Experimental</td>
<td>Arctic Terrain Avoidance</td>
<td>95 GHz</td>
<td>Harry Diamond Laboratories and Georgia Institute of Technology</td>
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<td>Experimental</td>
<td>Airborne Applications; Instrument</td>
<td>95 GHz</td>
<td>Georgia Institute of Technology</td>
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<td>Landing, Short Range Weapon</td>
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<td>Experimental</td>
<td>Low Altitude Aircraft Tracking;</td>
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<tr>
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<td>Target Acquisition, Basic Milli-</td>
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<tr>
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<td>meter Wave Radar Studies</td>
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<tr>
<td>Experimental</td>
<td>Noise Modulated Radar for Clutter</td>
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<td>Suppression for FM/CW for High</td>
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<tr>
<td></td>
<td>Range Resolution</td>
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<td>Experimental</td>
<td>Bistatic CW Radar for Cross Section</td>
<td>140 GHz</td>
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<td>Rapid Scan</td>
<td>Measurements</td>
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<td>Norden Division, United Aircraft</td>
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<td>Command Fusing, Fire-Control, and</td>
<td></td>
<td>Applied Physics Lab and Georgia Institute of Technology</td>
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<td>Experimental</td>
<td>Navigation</td>
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<td>Experimental</td>
<td>Instrumentation for Basic Milli-</td>
<td>95 GHz,</td>
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<td></td>
<td>meter Radar Studies; Backscatter</td>
<td>and 70</td>
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<td>Studies, etc.</td>
<td>GHz</td>
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<tr>
<td>Experimental</td>
<td>Monopulse Tracking Investigations</td>
<td>70 GHz</td>
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<td>Experimental</td>
<td>Arctic Terrain Avoidance</td>
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<td>Parameter</td>
<td>Description</td>
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<tr>
<td>-------------------------------</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>95 GHz (Nom)</td>
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</tr>
<tr>
<td>Peak Power</td>
<td>6 kW</td>
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</tr>
<tr>
<td>Pulse Width</td>
<td>50 ns or 10 ns</td>
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</tr>
<tr>
<td>PRF</td>
<td>0-4000 pps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna Type</td>
<td>Paraboloid (Cassegrain)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>.70°</td>
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<td></td>
</tr>
<tr>
<td>Elevation Beamwidth</td>
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</tr>
<tr>
<td>Gain</td>
<td>47.1 dB</td>
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<tr>
<td>Polarization</td>
<td>H or V</td>
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<td></td>
</tr>
<tr>
<td>IF Center Frequency</td>
<td>60 MHz or 160 MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF Bandwidth</td>
<td>20 MHz or 100 MHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF Response</td>
<td>Logarithmic (linear available)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noise Figure</td>
<td>15 dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>70 dB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Display Type</td>
<td>A-scope, B-scope, PPI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dimensions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cabinets</td>
<td>36 x 36 x 30 inches</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna Dish</td>
<td>12 inches diameter</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure E-1. Georgia Tech Experimental 95 GHz Radar, GT-M.
Figure E-2. Simplified block diagram of Georgia Tech 95 GHz radar, GT-M.
Several risk elements would have to be resolved before undertaking the development of the radar. They include the assessment of errors of the radome and the positioning and resolution errors if the antenna is to be a steered dish. Other risk elements would involve the method used in acquiring the target, and the automatic control loop used for tracking the target.

The beam of the antenna shown in Figure E-1 (0.7 degrees) is too broad because the dish is smaller than the one meter dimension found in this study to be required. One-fiftieth of 0.7 degrees is 0.24 milliradian, but our study has estimated the allowable resolution error to be 0.096 milliradian. The antenna diameter would thus have to be on the order of 2.5 times the diameter shown in Table E-II, or 30 inches.

The second Georgia Tech prototype 95 GHz radar system is described in Table E-III. The program under which this radar was developed required a fan-beam scanning antenna which is shown in Figure E-3. The thickness of the fan beam, 2 milliradians, is about 2.4 times smaller than the WSMR requirement we have estimated.

Prototype 70 GHz Radar

The AN/MPS-29 combat surveillance radar is a rapid-scan radar system designed, developed, tested, and evaluated by Georgia Tech for the U. S. Army Electronics Command during 1957-1960. The primary intent of this research effort was to develop and evaluate the performance of an experimental 70-GHz ground surveillance radar to provide high resolution display of ground targets at short ranges. A unique rapid-scan antenna was developed for this application. The scanning antenna for the AN/MPS-29 would be too wide (5 feet) for the WSMR requirement. It consisted of a geodesic Luneberg lens for azimuthal collimation and a modified parabolic cylinder for vertical collimation and beam shaping. The characteristics of the AN/MPS-29 are shown in Table E-IV and Table E-V.

A smaller version of the AN/MPS-29 was constructed, to mount in an armored personnel carrier. The smaller antenna is shown in Figure E-IV and described in Table E-VI.
<table>
<thead>
<tr>
<th><strong>Electrical</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency range</td>
<td>93.0 - 97.0 GHz</td>
</tr>
<tr>
<td>Broad-plane beamwidth (E-plane)</td>
<td>1.5°</td>
</tr>
<tr>
<td>Narrow-plane beamwidth (H-plane)</td>
<td>0.11° (2 mrad) or less</td>
</tr>
<tr>
<td>Scanning in narrow-beam plane (H-plane)</td>
<td>± 1.0°</td>
</tr>
<tr>
<td>Sidelobe level</td>
<td>-20 dB wrt main beam</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear (in non-scan plane)</td>
</tr>
<tr>
<td>Power</td>
<td>6.0 kW peak</td>
</tr>
<tr>
<td>Gain</td>
<td>48.0 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Environmental</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Velocity</td>
<td>20 mph</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-20 to +80°F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Scanner</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Scan speed</td>
<td>0 to 50 scans/sec continuously variable</td>
</tr>
<tr>
<td>Scanner position readout accuracy</td>
<td>0.1 mrad</td>
</tr>
<tr>
<td>Prime power</td>
<td>115 volt, 5 amp single phase AC, 60 Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Mechanical</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant frequency</td>
<td>15 Hz minimum; vertical or horizontal mounting</td>
</tr>
<tr>
<td>Weight w/o transmitter or adapter</td>
<td>575 lbs.</td>
</tr>
<tr>
<td>G-loading</td>
<td>3 G max</td>
</tr>
</tbody>
</table>
Figure E-3. Fan Beam, 95 GHz Radar Antenna, Designed and Fabricated by Georgia Tech.
### TABLE E-IV

SYSTEM PARAMETERS FOR THE AN/MPS-29

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>70 GHz</td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>0.2° (3.5 mils)</td>
</tr>
<tr>
<td>Elevation Beamwidth</td>
<td>0.3° (shaped to -4° of elevation)</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>0.05 μsec (7.5 meters)</td>
</tr>
<tr>
<td>Polarization</td>
<td>Vertical</td>
</tr>
<tr>
<td>Scan Rate</td>
<td>20 Scans per second</td>
</tr>
<tr>
<td>Scan Sector (Azimuth)</td>
<td>30° (150 Beamwidths)</td>
</tr>
<tr>
<td>PRF</td>
<td>10,000 pps</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>54.7 dB</td>
</tr>
<tr>
<td>Transmitter Power</td>
<td>15 kW</td>
</tr>
<tr>
<td>Receiver Noise Figure</td>
<td>18 dB</td>
</tr>
<tr>
<td>Doppler</td>
<td>Noncoherent</td>
</tr>
</tbody>
</table>

### TABLE E-V

MAXIMUM RANGE FOR DETECTION OF TARGETS WITH AN/MPS-29

**B-Scope-Display**

<table>
<thead>
<tr>
<th>Target</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking man</td>
<td>5 km</td>
</tr>
<tr>
<td>Light vehicles</td>
<td>10 - 15 km</td>
</tr>
<tr>
<td>2-1/2 ton truck</td>
<td>18 km</td>
</tr>
<tr>
<td>Helicopter (H-19)</td>
<td>15 km</td>
</tr>
</tbody>
</table>

**Aural Display**

<table>
<thead>
<tr>
<th>Target</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking man</td>
<td>8 km</td>
</tr>
<tr>
<td>8 walking men</td>
<td>10 km</td>
</tr>
</tbody>
</table>
Figure E-4. Rapid Scan, Combat Surveillance Radar Antenna, AN/MPS-29, Designed and Developed by Georgia Tech.
TABLE E-VI

ANTENNA PARAMETERS FOR THE ARMORED PERSONNEL CARRIER'S
FOLDED GEODESIC LUNEBERG LENS ANTENNA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Frequency</td>
<td>70 GHz</td>
</tr>
<tr>
<td>Azimuth Beamwidth</td>
<td>0.55°</td>
</tr>
<tr>
<td>Elevation Beamwidth</td>
<td>Shaped (Positionable in elevation from -10° to 20°)</td>
</tr>
<tr>
<td>Polarization</td>
<td>Vertical</td>
</tr>
<tr>
<td>Scan Rate</td>
<td>1 Scan/Minute (Min)</td>
</tr>
<tr>
<td></td>
<td>70 Scans/Sec (Max)</td>
</tr>
<tr>
<td>Scan Sector (Azimuth)</td>
<td>45° (± 22 1/2° about boresight). Boresight may be varied over 360° of azimuth.</td>
</tr>
<tr>
<td>Antenna Gain (including losses)</td>
<td>43.2 dB</td>
</tr>
<tr>
<td>Dimensions</td>
<td>24-inch diameter by 3.5-inch height</td>
</tr>
</tbody>
</table>