Learning Submodular Mixtures; and Active/Semi-Supervised Learning

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Submodular functions are finding ever more application in machine learning.

They naturally represent and successfully solve the problem of document summarization.

They can be used to parameterize a class of joint active/semi-supervised learning algorithms.

A need for both fast submodular function maximization and minimization on large ground set sizes.
Acknowledgments

Joint work with my students:

Hui Lin

Andrew Guillory
Outline

1 Document Summarization
   - Background on Document Summarization
   - A Class of Submodular Functions for Document Summarization
   - Experimental Results
   - Learning Submodular Mixtures

2 Active/SSL
   - Basic Idea
   - Previous work: learning on graphs
   - More general setting using submodular functions
   - Experiments

3 Summary
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3. Summary
Document Summarization

- Proliferation of data and documents.
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- Document Summarization: Given a large collection of text documents, produce a short human-readable summary that accurately represents the documents.
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- Extractive Document Summarization: The summary is comprised of parts of the original.
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Extractive Document Summarization: The summary is comprised of parts of the original.

E.g., if $V$ is the set of all sentences in the documents, then $S \subseteq V$ is a candidate summary.
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- Proliferation of data and documents.
- Document Summarization: Given a large collection of text documents, produce a short human-readable summary that accurately represents the documents.
- Extractive Document Summarization: The summary is comprised of parts of the original.
- E.g., if \( V \) is the set of all sentences in the documents, then \( S \subset V \) is a candidate summary.
- If \( V \) is the set of all web pages, then \( S \subset V \) is a candidate summary.
∃ many well-established methods for document summarization
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Research in the natural language processing community have repeatedly but unknowingly used submodularity:

Carbongell & Goldstein, 1998; Filatova & Hatzivassiloglou, 2004; McDonald, 2007; Takamura & Okumura, 2009; Riedhammer et al., 2010; Shen & Li, 2010; Berg-Kirkpatrick et al., 2011

These researchers did not intentionally use submodularity, and they did not directly use submodular optimization. Occasionally, the greedy algorithm was used for optimization (e.g., Carbonell & Goldstein, 1998).

The majority of researchers defined heuristics involving non-submodular objectives, and either greedy or ILP optimization strategies.
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The objectives, for a given set of documents, are parameterized by actual summaries created by humans so they are not available for optimization in practice.
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- Again, these researchers did not intentionally use submodularity, and the objectives are often not monotone.
- The objectives, for a given set of documents, are parameterized by actual summaries created by humans so they are not available for optimization in practice.
- They are only available for evaluation, and they correlate quite well with manual human-generated summaries.
NIST’s ROUGE-N evaluation function

NIST’s ROUGE-N score is the standard evaluation measure, and it is polymatroidal:

\[
f_{\text{ROUGE-N}}(S) \triangleq \frac{\sum_{i=1}^{K} \sum_{e \in R_i} \min(c_e(S), r_{e,i})}{\sum_{i=1}^{K} \sum_{e \in R_i} r_{e,i}},
\]

where

- \(S\) is the candidate summary (a set of sentences extracted from the ground set \(V\))

\(c_e(S)\) is the number of times an \(n\)-gram \(e\) occurs in summary \(S\), clearly a modular function for each \(e\). \(R_i\) is the set of \(n\)-grams contained in the reference summary \(i\) (given \(K\) reference summaries). \(r_{e,i}\) is the number of times \(n\)-gram \(e\) occurs in reference summary \(i\).
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where

- $S$ is the candidate summary (a set of sentences extracted from the ground set $V$)
- $c_e : 2^V \rightarrow \mathbb{Z}_+$ is the number of times an $n$-gram $e$ occurs in summary $S$, clearly a modular function for each $e$. 
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- and \( r_{e,i} \) is the number of times \( n \)-gram \( e \) occurs in reference summary \( i \).
- Note again, ROUGE-N is unavailable to optimize directly.
The figure below represents the sentences of a document.

[Sentences of a document are represented visually with horizontal lines.]

The summary on the left is a subset of the summary on the right. Add new (blue) sentence to each of the two summaries. The marginal (incremental) benefit of adding the new (blue) sentence to the smaller (left) summary is no more than the marginal benefit of adding the new sentence to the larger (right) summary. This illustrates the concept of diminishing returns and submodularity.
Extractive Document Summarization

- We extract sentences (green) as a summary of the full document.
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**diminishing returns ↔ submodularity**
Problem setup

- The ground set $V$ corresponds to all the sentences in a document.
- Extractive document summarization: select a small subset $S \subseteq V$ that accurately represents the entirety (ground set $V$).
- The summary is usually required to be length-limited.
  - $c_i$: cost (e.g., the number of words in sentence $i$),
  - $b$: the budget (e.g., the largest length allowed),
  - knapsack constraint: $\sum_{i \in S} c_i \leq b$.
- A set function $f : 2^V \to \mathbb{R}$ measures the quality of the summary $S$,
- Thus, the summarization problem is formalized as:

Problem (Document Summarization Optimization Problem)

$$ S^* \in \arg\max_{S \subseteq V} f(S) \text{ subject to: } \sum_{i \in S} c_i \leq b. $$

(1)
A Practical Algorithm for Large-Scale Summarization

When $f$ is both **monotone** and **submodular**:

- A greedy algorithm with partial enumeration (Sviridenko, 2004), theoretical guarantee of near-optimal solution, but not practical for large data sets, $O(n^3)$. 

- Scalability: the argmax above can typically be solved by $O(\log n)$ calls of $f$, thanks to submodularity.

\[ k \leftarrow \arg\max_{i \in U} f(G \cup \{i\}) - f(G) \left(\frac{c_i}{r} \right). \]
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- A greedy algorithm (Lin and Bilmes, 2010): with worse theoretical guarantee but still constant factor $1 - 1/\sqrt{e} \approx 0.39$, and practical/scalable (e.g., Minoux trick still works)!  
  - We choose next element with largest ratio of gain over scaled cost:  

$$ k \leftarrow \arg\max_{i \in U} \frac{f(G \cup \{i\}) - f(G)}{c_i r}.$$  

(2)
When $f$ is both \textbf{monotone} and \textbf{submodular}:

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\]  

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- Scalability: the argmax above can typically be solved by $O(\log n)$ calls of $f$, thanks to submodularity
- Ex: integer linear programming (ILP) takes 17 hours vs. greedy which takes < 1 second!!
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3. Summary
The General Form of Our Submodular Functions

- Two properties of a good summary: relevance and non-redundancy.
- A redundancy penalty often violates monotonicity.
- Our approach: we positively reward diversity instead of negatively penalizing redundancy:

**Definition (The general form of our submodular functions)**

\[ f(S) = \mathcal{L}(S) + \lambda \mathcal{R}(S) \]

- \( \mathcal{L}(S) \) measures the coverage (or fidelity) of summary set \( S \) to the document.
- \( \mathcal{R}(S) \) rewards diversity in \( S \).
- \( \lambda \geq 0 \) is a trade-off coefficient.
Coverage function

Coverage Function

\[ \mathcal{L}(S) = \sum_{i \in V} \min \{ C_i(S), \alpha C_i(V) \} \]

- \( C_i : 2^V \rightarrow \mathbb{R} \) is monotone submodular, and measures how well \( i \) is covered by \( S \): \( \Rightarrow \mathcal{L}(S) \) is monotone submodular.
- \( 0 \leq \alpha \leq 1 \) is a threshold coefficient — sufficient coverage fraction.
- If \( \min\{C_i(S), \alpha C_i(V)\} = \alpha C_i(V) \), then sentence \( i \) is well covered by summary \( S \) (saturated).
- After saturation, further increases in \( C_i(S) \) won’t increase the objective function values (return diminishes).
- Therefore, new sentence added to \( S \) should focus on sentences that are not yet saturated, in order to increasing the objective function value.
Coverage function

\[ \mathcal{L}(S) = \sum_{i \in V} \min \{ C_i(S), \alpha C_i(V) \} \]

- \( C_i \) measures how well \( i \) is covered by \( S \).
- One simple possible \( C_i \) (that we use) is:
  \[ C_i(S) = \sum_{j \in S} w_{i,j}, \]
  where \( w_{i,j} \geq 0 \) measures the similarity between \( i \) and \( j \).
- With this \( C_i \), \( \mathcal{L}(S) \) is monotone submodular, as required.
Diversity reward function

\[ R(S) = \sum_{i=1}^{K} \sqrt{\sum_{j \in P_i \cap S} r_j}. \]

- \( P = \{ P_i : i = 1, \cdots, K \} \) is a partition of the ground set \( V \).
- \( r_j \geq 0: \text{singleton reward} \) of \( j \), which represents the importance of \( j \) to the summary.
- Square root over the sum of rewards of sentences belong to the same partition (diminishing returns).
- \( R(S) \) is monotone submodular as well.
Diversity Reward Function

Alternatively, we can utilize multiple partitions/clusterings $\mathcal{P}_1, \mathcal{P}_2, \ldots$, yielding diversity reward function for each one, and mix them together.

Multi-resolution Diversity Reward

$$
\mathcal{R}(S) = \lambda_1 \sum_{i=1}^{K_1} \sqrt{\sum_{j \in P_i^{(1)} \cap S} r_j} + \lambda_2 \sum_{i=1}^{K_2} \sqrt{\sum_{j \in P_i^{(2)} \cap S} r_j} + \cdots
$$
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- **Experimental Results**
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There are standard evaluation data sets for document summarization.

DUC Evaluation
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Researchers have continued to use these data sets since they are a widely-used standard and ∃ automatic evaluation strategy (ROUGE).
DUC Evaluation

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- Generic summarization (not in response to a query): DUC 2004
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Researchers have continued to use these data sets since they are a widely-used standard and ∃ automatic evaluation strategy (ROUGE).

Generic summarization (not in response to a query): DUC 2004

Query-based summarization (user issues a query and summary must be relevant to query): DUC 2005-2007 (more like web search/IR).
Generic Summarization

- DUC-04: generic summarization

**Table**: ROUGE-1 recall (R) and F-measure (F) results (%) on DUC-04. DUC-03 was used as development set.

<table>
<thead>
<tr>
<th>DUC-04</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1(S)$</td>
<td>39.03</td>
<td>38.65</td>
</tr>
<tr>
<td>$R_1(S)$</td>
<td>38.23</td>
<td>37.81</td>
</tr>
<tr>
<td>$L_1(S) + \lambda R_1(S)$</td>
<td><strong>39.35</strong></td>
<td><strong>38.90</strong></td>
</tr>
<tr>
<td>Takamura and Okumura (2009)</td>
<td>38.50</td>
<td>-</td>
</tr>
<tr>
<td>Wang et al. (2009)</td>
<td>39.07</td>
<td>-</td>
</tr>
<tr>
<td>Lin and Bilmes (2010)</td>
<td>-</td>
<td>38.39</td>
</tr>
<tr>
<td>Best system in DUC-04 (peer 65)</td>
<td>38.28</td>
<td>37.94</td>
</tr>
</tbody>
</table>

- Note: this was (in 2011) the best ROUGE-1 result ever reported on DUC-04.
Query-focused Summarization

- DUC-05,06,07: query-focused summarization
- For each document cluster, a title and a narrative (query) describing a user’s information need are provided.
- Nelder-Mead (derivative-free) for parameter training.
DUC-05 results

Table: ROUGE-2 recall (R) and F-measure (F) results (%)

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}_1(S) + \lambda R_Q(S)$</td>
<td>7.82</td>
<td>7.72</td>
</tr>
<tr>
<td>$\mathcal{L}<em>1(S) + \sum</em>{\kappa=1}^{3} \lambda_{\kappa} R_{Q,\kappa}(S)$</td>
<td><strong>8.19</strong></td>
<td><strong>8.13</strong></td>
</tr>
<tr>
<td>Daumé III and Marcu (2006)</td>
<td>6.98</td>
<td>-</td>
</tr>
<tr>
<td>Wei et al. (2010)</td>
<td>8.02</td>
<td>-</td>
</tr>
<tr>
<td>Best system in DUC-05 (peer 15)</td>
<td>7.44</td>
<td>7.43</td>
</tr>
</tbody>
</table>

- DUC-06 was used as training set for the objective function with single diversity reward.
- DUC-06 and 07 were used as training sets for the objective function with multi-resolution diversity reward.
- Note: this was (in 2011) the best ROUGE-2 result ever reported on DUC-05.
DUC-06 results

Table: ROUGE-2 recall (R) and F-measure (F) results (%)

<table>
<thead>
<tr>
<th>Formula</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}_1(S) + \lambda \mathcal{R}_Q(S)$</td>
<td>9.75</td>
<td>9.77</td>
</tr>
<tr>
<td>$\mathcal{L}<em>1(S) + \sum</em>{\kappa=1}^{3} \lambda_{\kappa} \mathcal{R}_{Q,\kappa}(S)$</td>
<td>9.81</td>
<td>9.82</td>
</tr>
<tr>
<td>Celikyilmaz and Hakkani-tür (2010)</td>
<td>9.10</td>
<td>-</td>
</tr>
<tr>
<td>Shen and Li (2010)</td>
<td>9.30</td>
<td>-</td>
</tr>
<tr>
<td>Best system in DUC-06 (peer 24)</td>
<td>9.51</td>
<td>9.51</td>
</tr>
</tbody>
</table>

- DUC-05 was used as training set for the objective function with single diversity reward.
- DUC-05 and 07 were used as training sets for the objective function with multi-resolution diversity reward.
- Note: this was (in 2011) the best ROUGE-2 result ever reported on DUC-06.
DUC-07 results

Table: ROUGE-2 recall (R) and F-measure (F) results (%)

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<thead>
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<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}_1(S) + \lambda R_Q(S)$</td>
<td>12.18</td>
<td>12.13</td>
</tr>
<tr>
<td>$\mathcal{L}<em>1(S) + \sum</em>{k=1}^3 \lambda_k R_{Q,k}(S)$</td>
<td>12.38</td>
<td>12.33</td>
</tr>
<tr>
<td>Toutanova et al. (2007)</td>
<td>11.89</td>
<td>11.89</td>
</tr>
<tr>
<td>Haghighi and Vanderwende (2009)</td>
<td>11.80</td>
<td>-</td>
</tr>
<tr>
<td>Celikyilmaz and Hakkani-tür (2010)</td>
<td>11.40</td>
<td>-</td>
</tr>
<tr>
<td>Best system in DUC-07 (peer 15), using web search</td>
<td>12.45</td>
<td>12.29</td>
</tr>
</tbody>
</table>

- DUC-05 was used as training set for the objective function with single diversity reward.
- DUC-05 and 06 were used as training sets for the objective function with multi-resolution diversity reward.
- Note: this was (in 2011) the best ROUGE-2 F-measure result ever reported on DUC-07, and best ROUGE-2 R without web search expansion.
Outline

1 Document Summarization
   - Background on Document Summarization
   - A Class of Submodular Functions for Document Summarization
   - Experimental Results
   - Learning Submodular Mixtures

2 Active/SSL
   - Basic Idea
   - Previous work: learning on graphs
   - More general setting using submodular functions
   - Experiments

3 Summary
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- Function class: conic combination of a finite number of submodular functions with known forms.
- ⇒ submodular mixtures, where each submodular function is a component of the mixture.
- A fairly rich class of monotone submodular functions can be represented as a submodular mixture of components with simple forms.
Structured Prediction in Machine Learning

- Given: a finite set of training pairs $D = \{(x^{(i)}, y^{(i)})\}_i$, where $x^{(i)} \in \mathcal{X}$, $y^{(i)} \in \mathcal{Y}$. 
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- Let $\ell: \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}_+$ be a loss function. I.e., $\ell_y(\hat{y})$ is cost of deciding $\hat{y}$ when truth is $y$.
- Empirical risk minimization: adjust $w$ so that $\sum_i \ell_y(h_w(x^{(i)}))$ is small subject to other conditions (e.g., regularization).
Structured Prediction: Approach

- Minimization via convex programming

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{T} \sum_t \xi_t + \frac{\lambda}{2} \|w\|^2 \\
\text{subject to} & \quad w^\top f_t(y^{(t)}) - w^\top f_t(y) \geq \ell_t(y) - \xi_t, \forall t, \forall y \in Y_t \\
& \quad \xi_t \geq 0, \forall t.
\end{align*}
\] (4)

where \( f_t(y) = f(x_t, y), \ell_t(y) = \ell_{y_t}(y), \)
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where \( f_t(y) = f(x_t, y), \ell_t(y) = \ell_{y_t}(y), \)

- Exponential number of constraints, due to the \( \forall y \in \mathcal{Y}_t \)
Structured Prediction: Approach with inference

- Constraints specified in inference form:

\[
\begin{align*}
\text{minimize}_{w, \xi_t} & \quad \frac{1}{T} \sum_t \xi_t + \frac{\lambda}{2} \|w\|^2 \\
\text{subject to} & \quad w^\top f_t(y^{(t)}) \geq \max_{y \in Y_t} \left( w^\top f_t(y) + \ell_t(y) \right) - \xi_t, \forall t \\
\xi_t & \geq 0, \forall t.
\end{align*}
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& \quad \xi_t \geq 0, \forall t.
\end{align*}
\] (7)

- Exponential set of constraints reduced to an embedded optimization problem, “inference.”
Structured Prediction: Unconstrained form

- Unconstrained form with hinge-loss

\[
\min_w \frac{1}{T} \sum_t \left[ \max_{y \in Y_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) \right] + \frac{\lambda}{2} \|w\|^2 \quad (10)
\]

This generalizes the hinge-loss function with loss as follows:

\[
\ell_{\text{hinge}}(h(x)) = \max_{y \in Y} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) \quad (11)
\]

\(\ell_{\text{hinge}}(h(x))\) convex in \(w\).
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- $\ell_y^{\text{hinge}}(h(\mathbf{x}))$ convex in $\mathbf{w}$. 
Structured Prediction: Subgradient

- Subgradient, evaluated at $w$, of the following

$$\max_{y \in Y_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) + \frac{\lambda}{2} \|w\|^2$$  \hspace{1cm} (12)

$$\text{can be found by computing}$$

$$y^* \in \arg\max_{y \in Y_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)})$$  \hspace{1cm} (13)

$$\text{and then finding subgradient of}$$

$$w^\top f_t(y^*) + \ell_t(y^*) - w^\top f_t(y^{(t)}) + \frac{\lambda}{2} \|w\|^2$$  \hspace{1cm} (14)

$$\text{which has the form}$$

$$f_t(y^*) - f_t(y^{(t)}) + \lambda w.$$  \hspace{1cm} (15)
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which has the form

$$f_t(y^*) - f_t(y^{(t)}) + \lambda w. \tag{15}$$
Structured Prediction: Subgradient Learning

**Algorithm 1:** Subgradient descent learning

**Input:** \( S = \{(x^{(t)}, y^{(t)})\}_{t=1}^T \) and a learning rate sequence \( \{\eta_t\}_{t=1}^T \).

\( w_0 = 0; \)

for \( t = 1, \cdots, T \) do

- **Loss augmented inference:** \( y^*_t \in \arg\max_{y \in Y_t} w^\top_{t-1} f_t(y) + \ell_t(y); \)

- Compute the subgradient: \( g_t = \lambda w_{t-1} + f_t(y^*_t) - f_t(y^{(t)}); \)

- Update the weights: \( w_t = w_{t-1} - \eta_t g_t; \)

**Return:** the averaged parameters \( \frac{1}{T} \sum_t w_t. \)
Structured Prediction: Approximate Inference

- Assumption has been $y_t^* \in \arg\max_{y \in Y_t} w_{t-1}^T f_t(y) + \ell_t(y)$ is exact.
Structured Prediction: Approximate Inference

- Assumption has been $y_t^* \in \arg\max_{y \in \mathcal{Y}_t} w^T_{t-1} f_t(y) + \ell_t(y)$ is exact.
- Past work has often required assumptions on $f$ (e.g., decomposability) and/or $\ell$ (e.g., Hamming loss) that makes the inference tractable.
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- If inference is approximate, the learning (and risk bounds) are not always guaranteed to exist (Kulesza & Pereira 2007) (loopy-belief propagation based inference dissolves the guarantee for perceptron learning).
- Fortunately, we show in our case (submodular maximization), that such bounds do exist.
Submodular Mixture Score Functions

Inference (decoding) problem in NLP:

- **Input:** \( x \in \mathcal{X} \)
- **Output:** \( y \in \mathcal{Y} \)
- **Score function:** \( s : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R} \)
- **Inference:** \( \text{argmax}_{y \in \mathcal{Y}} s(x, y) \)
- **Submodular mixture score:**

\[
s(x, y) = \sum_i w_i f_i(x, y) \quad (16)
\]

where \( w \geq 0 \) and \( \forall i, f_i : \mathcal{X} \times \mathcal{Y}_x \rightarrow \mathbb{R} \) is submodular on \( \mathcal{Y}_x \).

- \( f_i \): component of the mixture, which is given.
- \( w_i \): component weight, which is unknown
- **Goal:** supervised learning of component weights \( w \).
Submodular Components

Many possible components

- Weighted sums of weighted matroid rank functions (Shioura, 2012)

Given matroids $\mathcal{M}_i = (V, \mathcal{I}_i)$, and modular functions $m_i : 2^V \rightarrow \mathbb{R}_+$, class is:

$$f(S) = \sum_{i=1}^{M} w_i \max \{ m_i(I) : I \subseteq S, I \in \mathcal{I}_i \}$$  \hspace{1cm} (17)

can easily cover cover-like functions,
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This can easily cover cover-like functions, a broader class than \( M^\text{b} \)-concave functions.

- Truncation functions of the form \( f(S) = \min(m(S), \alpha) \) for modular \( m \).
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- Truncation functions of the form $f(S) = \min(m(S), \alpha)$ for modular $m$.

- Or any polymatroid functions, such as the ones we previously used in the fixed mixtures for summarization.
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- Key is that they need to be specified beforehand.
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- Truncation functions of the form $f(S) = \min(m(S), \alpha)$ for modular $m$.
- Or any polymatroid functions, such as the ones we previously used in the fixed mixtures for summarization.
- Key is that they need to be specified beforehand.
- Total number of available components $M$ must be fixed and finite.
Unconstrained form with hinge-loss

\[
\min_{w \geq 0} \frac{1}{T} \sum_t \left[ \max_{y \in \mathcal{Y}_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) \right] + \frac{\lambda}{2} \|w\|^2
\] (18)
Learning Submodular Mixtures: Unconstrained Form

- Unconstrained form with hinge-loss

\[
\min_{w \geq 0} \frac{1}{T} \sum_{t} \left[ \max_{y \in \mathcal{Y}_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) \right] + \frac{\lambda}{2} \|w\|^2 \quad (18)
\]

- To compute a subgradient, we must solve the following embedded optimization problem

\[
\max_{y \in \mathcal{Y}_t} \left( w^\top f_t(y) + \ell_t(y) \right) \quad (19)
\]
Learning Submodular Mixtures: Unconstrained Form

- Unconstrained form with hinge-loss

\[
\min_{\mathbf{w} \geq 0} \frac{1}{T} \sum_t \left[ \max_{y \in \mathcal{Y}_t} \left( \mathbf{w}^\top \mathbf{f}_t(y) + \ell_t(y) \right) - \mathbf{w}^\top \mathbf{f}_t(y^{(t)}) \right] + \frac{\lambda}{2} \|\mathbf{w}\|^2 \tag{18}
\]

- To compute a subgradient, we must solve the following embedded optimization problem

\[
\max_{y \in \mathcal{Y}_t} \left( \mathbf{w}^\top \mathbf{f}_t(y) + \ell_t(y) \right) \tag{19}
\]

- Convex in \(\mathbf{w}\), and \(\mathbf{w}^\top \mathbf{f}_t(y)\) presumably polymatroidal, but what about \(\ell_t(y)\)?
Learning Submodular Mixtures: Unconstrained Form

- Unconstrained form with hinge-loss

\[
\min_{\mathbf{w} \geq 0} \frac{1}{T} \sum_t \left[ \max_{\mathbf{y} \in \mathcal{Y}_t} \left( \mathbf{w}^\top \mathbf{f}_t(y) + \ell_t(y) \right) - \mathbf{w}^\top \mathbf{f}_t(y^{(t)}) \right] + \frac{\lambda}{2} \|\mathbf{w}\|^2 \quad (18)
\]

- To compute a subgradient, we must solve the following embedded optimization problem

\[
\max_{\mathbf{y} \in \mathcal{Y}_t} \left( \mathbf{w}^\top \mathbf{f}_t(y) + \ell_t(y) \right) \quad (19)
\]

- Convex in \( \mathbf{w} \), and \( \mathbf{w}^\top \mathbf{f}_t(y) \) presumably polymatroidal, but what about \( \ell_t(y) \)?

- Often one uses Hamming loss, but here lets assume that \( \ell_t(y) \) is at least polymatroidal (more soon on this).
Algorithm 2: Projected subgradient descent for learning submodular mixtures.

**Input**: $S = \{(x^{(t)}, y^{(t)})\}_{t=1}^T$ and a learning rate sequence $\{\eta_t\}_{t=1}^T$.

1. $w_0 = 0$;
2. for $t = 1, \cdots, T$ do
3.   Approximate inference: $\hat{y} \in \arg\max_{y \in \mathcal{Y}_t} w_{t-1}^\top f_t(y) + \ell_t(y)$;
4.   Compute the subgradient: $g_t = \lambda w_{t-1} + f_t(\hat{y}) - f_t(y^{(t)})$;
5.   Update the weights with projection: $w_t = \max(0, w_{t-1} - \eta_t g_t)$;

**Return**: the averaged parameters $\frac{1}{T} \sum_t w_t$.

- Note line 3, the approximate maximization (e.g., submodular maximization).
- Note line 5, the projection step, to ensure $w \geq 0$ which preserves submodularity.
Learning submodular mixtures with subgradient decent

**Theorem (Lin & Bilmes 2012)**

Assume $f_i, i = 1, \cdots, M$ are all upper-bounded by 1, $r_t(w) \leq B$, and $\|g_t\| \leq G$. Let $\hat{w}$ be the solution returned by Algorithm 2 using $\rho$-approximate inference with learning rate $\eta_t = \frac{2}{\lambda_t}$ and

$$\lambda = \frac{G}{M} \sqrt{\frac{2(1+\log T)}{T}}.$$  

Then for any $\delta > 0$ with probability at least $1 - \delta$,

$$\mathbb{E}_{(x,y) \sim D}[\mathcal{I}(h(x; \hat{w}))] \leq \frac{1}{\rho} \left( \frac{1}{T} \sum_{t=1}^{T} r_t(w^*) \right) + S(T),$$

where

$$S(T) = \frac{MG}{\rho} \sqrt{\frac{2(1+\log T)}{T}} + B \sqrt{\frac{2}{T} \log \frac{1}{\delta}} + \frac{1 - \rho}{\rho} M$$

and

$$r_t(w) = \max_{y \in \mathcal{Y}_t} \left( w^\top f_t(y) + \ell_t(y) \right) - w^\top f_t(y^{(t)}) \tag{20}$$
Risk bound, learning submodular mixtures with subgradient decent

**Theorem (Lin & Bilmes 2012 (summarized))**

\[
\mathbb{E}_{(x,y) \sim D}[\ell(h(x; \hat{w}), y)] \leq \frac{1}{\rho} \left( \frac{1}{n} \sum_{t=1}^{n} r_t(w^*) \right) + S(n),
\]

where

\[
S(n) = \frac{MG}{\rho} \sqrt{\frac{2(1 + \log n)}{n}} + B \sqrt{\frac{2}{n} \log \frac{1}{\delta}} + \frac{1 - \rho}{\rho} M
\]

- \(\mathbb{E}_{(x,y) \sim D}[\ell(h(x; \hat{w}), y)]\): risk of the approximately leaned model
- \(\frac{1}{n} \sum_{t=1}^{n} r_t(w^*)\): empirical risk of the model with **exact** learning
- \(\rho\): approximation ratio.
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- \( \frac{1}{n} \sum_{t=1}^{n} r_t(w^*) \): empirical risk of the model with exact learning
- \( \rho \): approximation ratio. \( \rho \approx 1 \) for greedy algorithm in budgeted submodular maximization
Submodular Components for Document Summarization

Components:

- **Coverage (fidelity) components**:

\[
f_\beta(S) = \frac{1}{|V|} \sum_{i \in V} \min \left\{ \frac{C_i(S)}{C_i(V)}, \beta \right\},
\]
Submodular Components for Document Summarization

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- **Diversity components, for partition \( \mathcal{P} = (P_1, \ldots, P_K) \):**
  \[
  f_{\mathcal{P},\alpha}(S) = \frac{\sum_{k=1}^{K} \left( \sum_{i \in S \cap P_k} r_i \right)^\alpha}{\sum_{k=1}^{K} \left( \sum_{i \in P_k} r_i \right)^\alpha},
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Submodular Components for Document Summarization

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- **Clustered facility location like components, for partition** \( \mathcal{P} \)
  \[ f_{\mathcal{P}}(S) = \frac{1}{K} \sum_{k=1}^{K} \max_{i \in S \cap P_k} r_i, \quad (21) \]
NIST’s ROUGE-N evaluation function

NIST’s ROUGE-N score is the standard evaluation measure, and it is polymatroidal:

\[ f_{\text{ROUGE-N}}(S) \triangleq \frac{\sum_{i=1}^{K} \sum_{e \in R_i} \min(c_e(S), r_{e,i})}{\sum_{i=1}^{K} \sum_{e \in R_i} r_{e,i}}, \]

where

- \( S \) is the candidate summary (a set of sentences extracted from the ground set \( V \))
- \( c_e : 2^V \rightarrow \mathbb{Z}_+ \) is the number of times an \( n \)-gram \( e \) occurs in summary \( S \), clearly a modular function for each \( e \).
- \( R_i \) is the set of \( n \)-grams contained in the reference summary \( i \) (given \( K \) reference summaries).
- and \( r_{e,i} \) is the number of times \( n \)-gram \( e \) occurs in reference summary \( i \).
- Note again, ROUGE-N is unavailable to optimize directly.
Loss Function $\ell$

- ROUGE-N can’t be used since it measures “accuracy” rather than loss.
Loss Function $\ell$

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- $1 - \text{ROUGE-N}$ is supermodular, sum would require a submodular-supermodular procedure (SSP) Narasimhan&Bilmes 2005 (demonstrates a need for good quality SSP).
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Surrogate loss: we use a form of “complement recall”, of the form:

\[ \ell_{ROUGE}(S) \triangleq \frac{\sum_{e \in \bar{R}} \min(c_e(S), r_e)}{\sum_{e \in \bar{R}} r_e}, \]  

(22)

where

\[ \bar{R} = N \setminus \bigcup_i R_i, \]  

(23)

and where \( N \) is the set of all the n-grams occur in the documents, and \( r_e \) is the number of times n-gram \( e \) occurs in the documents.
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and where $N$ is the set of all the n-grams occur in the documents, and $r_e$ is the number of times n-gram $e$ occurs in the documents.

$\ell_{\text{ROUGE}}$ is clearly polymatroidal.
Query-focused Summarization Results

DUC-05: DUC-06 and DUC-07 were used in submodular mixture learning

<table>
<thead>
<tr>
<th>ROUGE-2 Recall (%)</th>
<th>ROUGE-2 F-Measure (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td></td>
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<tr>
<td>7.8</td>
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<td>7.8</td>
<td></td>
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<tr>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td>8.8</td>
<td></td>
</tr>
</tbody>
</table>

Lin & Bilmes,
ACL 2011

DUC-05 best system Fidelity+Diversity
Query-focused Summarization Results

DUC-05: DUC-06 and DUC-07 were used in submodular mixture learning

- Lin & Bilmes, ACL 2011
- Lin & Bilmes, 2012 submitted

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</tr>
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<tbody>
<tr>
<td>DUC-05 best system</td>
<td>6.3</td>
<td>6.8</td>
</tr>
<tr>
<td>Fidelity+Diversity</td>
<td>7.3</td>
<td>7.8</td>
</tr>
<tr>
<td>Submodular mixture</td>
<td>8.3</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Bar chart showing performance metrics: ROUGE-2 Recall and F-Measure for different systems.
Query-focused Summarization Results

DUC-06: DUC-05 and DUC-07 were used in submodular mixture learning

### DUC-06 best system Fidelity+Diversity Submodular mixture

<table>
<thead>
<tr>
<th>ROUGE-2 Recall (%)</th>
<th>ROUGE-2 F-Measure (%)</th>
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</thead>
<tbody>
<tr>
<td>9.3</td>
<td>9.4</td>
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<tr>
<td>9.5</td>
<td>9.6</td>
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<tr>
<td>9.7</td>
<td>9.8</td>
</tr>
<tr>
<td>9.9</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Lin & Bilmes, ACL 2011

Lin & Bilmes, 2012 submitted
DUC-07: DUC-05 and DUC-06 were used in submodular mixture learning

Lin & Bilmes, ACL 2011
Lin & Bilmes, 2012 submitted
In general, the best results ever reported on DUC-04, 05, 06 and 07 (first reported here).

Lin & Bilmes, NAACL 2010

ROUGE-1 Recall (%)
ROUGE-1 F-Measure (%)
### Generic Summarization Results

In general, the best results ever reported on DUC-04, 05, 06 and 07 (first reported here).

- **Lin & Bilmes, NAACL 2010**
- **Lin & Bilmes, ACL 2011**
- **ROUGE-1 Recall (%)**
- **ROUGE-1 F-Measure (%)**

<table>
<thead>
<tr>
<th>Method</th>
<th>ROUGE-1 Recall (%)</th>
<th>ROUGE-1 F-Measure (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUC-04 best system</td>
<td>36.5</td>
<td></td>
</tr>
<tr>
<td>Non-monotone Submodular obj.</td>
<td>37.0</td>
<td></td>
</tr>
<tr>
<td>Submodular obj</td>
<td>37.5</td>
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<tr>
<td>Fidelity+Diversity</td>
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<tr>
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<td>40.0</td>
<td></td>
</tr>
<tr>
<td>ROUGE-1 F-Measure (%)</td>
<td>40.5</td>
<td></td>
</tr>
<tr>
<td>Fidelity+Diversity</td>
<td>41.0</td>
<td></td>
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Generic Summarization Results

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   - Background on Document Summarization
   - A Class of Submodular Functions for Document Summarization
   - Experimental Results
   - Learning Submodular Mixtures

2. Active/SSL
   - Basic Idea
   - Previous work: learning on graphs
   - More general setting using submodular functions
   - Experiments

3. Summary
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3 Summary
Active Learning and Transductive Semi-Supervised Learning

- Batch/Offline active learning: Given a set $V$ of unlabeled data items, the learner must choose a subset $L \subseteq V$ of the items that are to be labeled (and learnt from).
Active Learning and Transductive Semi-Supervised Learning

- **Batch/Offline active learning**: Given a set $V$ of unlabeled data items, the learner must choose a subset $L \subseteq V$ of the items that are to be labeled (and learnt from).

- **Transductive Semi-Supervised Learning**: Given a subset $L$ of data items that are already labeled, deduce the labels of all remaining items $V \setminus L$ without using any additional labels.
Active Learning and Transductive Semi-Supervised Learning

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- **Ideally do both, with a bound on the completion error.**
Batch/Offline active learning: Given a set $V$ of unlabeled data items, the learner must choose a subset $L \subseteq V$ of the items that are to be labeled (and learnt from).

Transductive Semi-Supervised Learning: Given a subset $L$ of data items that are already labeled, deduce the labels of all remaining items $V \setminus L$ without using any additional labels.

Ideally do both, with a bound on the completion error.

Very little work so far on methods that can do both.
For example, as represented by graph
Learner chooses a labeled set $L \subseteq V$
Nature reveals labels $y_L \in \{0, 1\}^L$
Learner predicts labels $\hat{y} \in \{0, 1\}$
Learner suffers loss $\|\hat{y} - y\|_1$

Predicted

Actual

$\|\hat{y} - y\|_1 = 2$ (24)
Basic Questions

- What should we assume about $y$?
- How should we predict $\hat{y}$ using $y_L$?
- How should we select $L$?
- How can we bound the error?
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3 Summary
Learning on graphs

- What should we assume about $y$?
- Standard assumption: small cut value
  - I.e., $\Phi(y) = \sum_{i<j} (y_i - y_j)^2 w_{ij}$ is small, where $w_{ij}$ measures similarity between item $i$ and $j$.
- This can be seen as exploiting “smoothness” assumption.

$\Phi(y) = 2$
Prediction on graphs

- How should we predict $\hat{y}$ using $y_L$?
- A standard approach: min-cut (Blum & Chawla 2001)
- Choose $\hat{y}$ to minimize $\Phi(\hat{y})$ s.t. $\hat{y}_L = y_L$
- Reduces to a standard min-cut computation
- This can be seen as a “smoothness” assumption about nature.
Active learning on graphs

- How should we select $L$?
- In previous work, we proposed the following objective

$$
\Psi(L) = \min_{T \subseteq V \setminus L, T \neq \emptyset} \frac{\Gamma(T)}{|T|} \quad (25)
$$

where $\Gamma(T)$ is the cut value between $T$ and $V \setminus T$. 

Small $\Psi(L)$ means an adversary can cut away many points from $L$ without cutting many edges.
Active learning on graphs

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$$\Psi(L) = \min_{T \subseteq V \setminus L : T \neq \emptyset} \frac{\Gamma(T)}{|T|}$$  \hspace{1cm} (25)

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- Small $\psi(L)$ means an adversary can cut away many points from $L$ without cutting many edges.

![Diagram](image-url)
Active learning on graphs

• How can we bound the error?

**Theorem**

Guillory & Bilmes 2009 Assume $\hat{y}$ minimizes $\Phi(\hat{y})$ subject to $\hat{y}_L = y_L$. Then

$$\|\hat{y} - y\|_1 \leq 2 \frac{\Phi(y)}{\Psi(L)}$$

(26)

• Intuition: Error $\leq \frac{\text{Complexity of true labels}}{\text{Quality of labeled set}}$

• Note: Deterministic bound, holds for adversarial labels.
Drawbacks of previous work

- Restricted to only graph based, min-cut learning.
- Not clear how to efficiently maximize $\Psi(L)$
  - Can compute in polynomial time (Guillory & Bilmes, 2009)
  - Only heuristic methods known for maximizing in general case.
- Not guaranteed that this bound is the right bound
More recent contributions

- Guillory & Bilmes, UAI 2011.
- A new more general bound on error, parameterized by an arbitrarily chosen submodular function.
- An active, semi-supervised learning method for approximately minimizing this bound.
- Proof that minimizing this bound exactly is NP-hard
- Theoretical evidence that this is the “right” bound.
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3 Summary
Submodular Functions For Learning

- $\Gamma(T)$ (cut value) is symmetric and submodular
- This makes $\Gamma(T)$ “nice” for learning
  - Easy to analyze
  - Can minimize exactly in polynomial time
- For other learning settings, other symmetric submodular functions make sense
  - Hypergraph cut is symmetric submodular
  - Symmetric mutual information is symmetric and submodular
  - An arbitrary submodular function $F$ (e.g., matroid rank) can be symmeterized

$$\Gamma(S) = F(S) + F(V \setminus S) - F(V) \quad (27)$$
Generalized Error Bound

**Theorem**

For any symmetric submodular $\Gamma(S)$, assume $\hat{y}$ minimizes $\Phi(\hat{y})$ subject to $\hat{y}_L = y_L$. Then

$$\|\hat{y} - y\|_1 \leq 2 \frac{\Phi(y)}{\Psi(L)}$$

(28)

- Here, $\Phi$ and $\Psi$ are defined in terms of the symmetric submodular function $\Gamma$, not graph cut.

$$\Phi(y) = \Gamma(V_{y=1}) \text{ and } \Psi(S) = \min_{T \subseteq V \setminus S : T \neq \emptyset} \frac{\Gamma(T)}{|T|}$$

(29)

- Each choice of $\Gamma$ gives different error bound (objective is “parameterized” by a submodular function).
- Minimizing $\Phi(\hat{y})$ s.t. $\hat{y}_L = y_L$ can be done in polynomial time (submodular function minimization).
Can we efficiently maximize $\Psi$?

- Two related problems
  1. Maximize $\Psi(L)$ subject to $|L| \leq k$
  2. Minimize $|L|$ subject to $\Psi(L) \geq \lambda$

- If $\Psi(L)$ were submodular, we could use well-known results of greedy procedure
  - $(1 - 1/e)$-approximation to 1. (Nemhauser et al. 1978)
  - $1 + \ln F(V)$ approximation for 2. (Wolsey 1981) for integer valued $F$

- Unfortunately, $\Psi(L)$ is not submodular.
Approximation result

- Define a surrogate objective $F_\lambda(S)$ s.t.
  \[
  F_\lambda(S) = \min_{T \subseteq V \setminus S: T \neq \emptyset} \Gamma(T) - \lambda |T|
  \]

- $F_\lambda(S)$ is monotone non-decreasing submodular
- Evaluating $F_\lambda(S)$ at $S$ requires SFM.
- $F_\lambda(S) \geq 0$ iff $\Psi(S) \geq \lambda$
- Can then use standard methods for $F_\lambda(S)$.

**Theorem**

*For any integral symmetric submodular function $\Gamma(S)$, integer $\lambda$, greedily maximizing $F_\lambda(L)$ gives $L$ with $\Phi(L) \geq \lambda$, and $|L| \leq (1 + \ln \lambda) \min_{L: \Phi(L) \geq \lambda} |L|$*
Can we do better?

- Is it possible to maximize $\Psi(L)$ exactly?
- Probably not, we show the problem is NP-complete
  - Holds also if $\Gamma(S)$ is even the cut function
  - Reduction from vertex cover on fixed degree graphs.
- Is there a strictly better bound?
- Not of the same form
  - No function larger than $\Psi(L)$ for which the bound of this form holds.
  - Suggests this is the “right” bound.
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3. Summary
Experiments: Movie Recommendation

- Which movies would a user rate to get accurate recommendations from collaborative filtering?
- We pose this problem as active learning over a hypergraph encoding user preferences using $\Gamma(S)$ set to hypergraph cut.
- Two hypergraph edges for each user.
  - Hypergraph edge connecting all movies a user likes
  - Hypergraph edge connecting all movies a user dislikes
- Partitions with low hypergraph cut value are consistent (on average) with user preferences.
Results on Movielens data

Movies Maximizing $\Psi(S)$
- Star Wars Ep. I
- Forrest Gump
- Wild Wild West (1999)
- The Blair Witch Project
- Titanic
- Mission: Impossible 2
- Babe
- The Rocky Horror Picture Show
- L.A. Confidential
- Mission to Mars
- Austin Powers
- Son in Law

Movies Rated Most Times
- American Beauty
- Star Wars Ep. IV
- Jurassic Park
- Fargo
- Titanic
- Mission: Impossible 2
- Babe
- The Rocky Horror Picture Show
- L.A. Confidential
- Mission to Mars
- Austin Powers
- Son in Law
- Star Wars Ep. V
- Star Wars Ep. VI
- Saving Private Ryan
- Terminator 2: Judgment Day
- The Matrix
- Back to the Future
- The Silence of the Lambs
- Men in Black
- Raiders of the Lost Ark
- The Sixth Sense
- Braveheart
- Shakespeare in Love
1 Document Summarization
  - Background on Document Summarization
  - A Class of Submodular Functions for Document Summarization
  - Experimental Results
  - Learning Submodular Mixtures

2 Active/SSL
  - Basic Idea
  - Previous work: learning on graphs
  - More general setting using submodular functions
  - Experiments

3 Summary
Submodular functions are finding ever more application in machine learning.

- They naturally represent and successfully solve the problem of document summarization.
- They can be used to parameterize a class of joint active/semi-supervised learning algorithms.
- A need for both fast submodular function maximization and minimization on large ground set sizes.