Analysis of Longshore Sediment Transport on Beaches

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SUMMARY

The present study investigates longshore sediment transport for a variety of bathymetric and wave conditions using the National Oceanic Partnership Program (NOPP) NearCoM Model. The model is used to determine the effects of wave shape and bathymetry changes on the resulting longshore sediment transport. The wave drivers, REF/DIF 1 and REF/DIF S, are used to assess the effects of monochromatic and spectral waves on longshore sediment transport, respectively. SHORECIRC is used as the circulation module and four different sediment transport models are used. Longshore transport comparisons are made with and without skewed orbital velocities in the shear stress and current velocities. It is found that the addition of skewed orbital velocities in shear stress and transport formulations increases longshore sediment transport by increasing time-varying effective shear stress. The addition of skewed orbital velocities greatly increases the transport due to advection by waves.

The localized longshore sediment transport is calculated using a generic physics based method and formulas by Bagnold, Bailard, and Bowen, Watanabe, and Ribberink. The transport results for each scenario are compared to the total transport CERC, Kamphuis, and GENESIS formulas. The bathymetries tested include an equilibrium beach profile, cusped beach profiles, and barred beach profiles with different bar locations. The longshore transport on an equilibrium beach profile is modeled for a 0.2 mm and 0.4 mm grain size and transport is compared to the CERC formula. The longshore sediment transport for d=0.2 mm is larger than d=0.4 mm when wave power is small, but as wave power increases the transport for the larger grain size dominates. The transport is also affected by the addition of cusps and bars on an equilibrium beach profile. The barred beach is modified to compare transport between waves breaking at the bar, before the bar, and after the bar. The features affect the transport when the wave powers are small, but as wave heights increase the cusp and bar features induce little change on the longshore sediment transport.
CHAPTER I

INTRODUCTION

Sediment transport is an area of interest to developers, engineers, researchers, and landowners because of the impact of the transport on coastlines. Evidence of longshore sediment transport is usually quite pronounced at inlet structures as often seen by deposition on one side of the channel and erosion on the downdrift side. Longshore transport can impede shipping channels by filling them with material, which will eventually require dredging. Longshore transport will typically change directions seasonally, or on even shorter time scales such as storm events. The net drift is the sum of the directional components of the transport, while the gross transport is the sum of the transport magnitudes. If gross transport is high, dredging may be required for an inlet or channel (Dean and Dalrymple, 2002). Knowledge of the gross transport is extremely important before construction of harbors and channels as it will directly affect dredging or sand transfer costs.

Many man-made and natural features impede longshore sediment transport or dampen wave power. Examples include breakwaters, groins, headlands, and seawalls. The design of hard engineering structures, dredging plans, and sand transfer facilities rely on accurate longshore sediment transport predictions.

Longshore sediment transport can also have environmental implications. Nutrients can bind to sediment and be transported in ocean, estuarine, and river environments. Sediment transport models can also be used to show the progression of sediment-bonded contaminants. Hydrophobic contaminants such as polycyclic aromatic hydrocarbons (PAHs) and polychlorinated biphenyls (PCBs) will bind to sediment and follow the same transport path (Schneider et al., 2002). These contaminants are of particular concern because they have been shown to cause cancer, and affect the immune system, nervous system, and endocrine system in animals (Environmental Protection Agency, 2002). Sediment transport research
CROSS-SHORE TRANSPORT

LONGSHORE TRANSPORT

SHORELINE

**Figure 1:** Longshore and cross-shore transport diagram.

is conducted experimentally in rivers, oceans, and laboratories, and theoretically using analytical and numerical models. Throughout the years, many methods have been developed for field measurements and modeling which will be discussed in the following sections.

### 1.1 Longshore Sediment Transport Measurement History

Many methods are used to measure total longshore sediment transport and the hydrodynamic conditions that drive sediment transport. This section highlights methods for measuring the longshore transport including physical, acoustic, and optical methods. The most commonly used longshore sediment transport field measurement techniques include sand tracers, impoundments, and streamer sediment traps. Less commonly used longshore sediment measurement techniques include measuring morphology change, suspended sediment pumping, instantaneous suspended sediment sampling, and trench infilling (Wang and Kraus, 1999).

Bedload transport occurs in the sheet flow or rolled along the bottom, and suspended load is suspended sediment in the water column. Longshore and cross-shore indicate components of sediment transport as shown in Figure 1. Velocity and concentration indicate sediment velocity and sediment concentration in the water column. Transport indicates the ability of the method to directly measure total transport.

Fluorescent tagging of sand grains is a method to measure longshore transport by coating
native sand with dye and measuring the time progression of the center of gravity of the tracer. The sand movement is measured by collecting volume samples from a specified grid. The volume samples are then tested for tracer concentration using techniques such as ultraviolet light (UV) analysis. Tracer concentration contours can then be developed from the UV results (Komar and Inman, 1970). Relatively recent tracer experiments were performed by Galvin (1987), Madsen (1987), Madsen (1989), and Drapeau et al. (1991).

Another method used to find sediment transport rate utilizes a short-term impoundment as performed by Wang and Kraus (1999). Impoundment studies quantify the coastline change by blocking sediment movement with a shore structure perpendicular to the shoreline. The structure can be a long-term impoundment created by a permanent coastal structure or a short-term impoundment created by a temporary structure. The long-term impoundment leads to sediment transport rates averaged over months, while the short-term impoundment leads to rates averaged over hours. An advantage of the short-term impoundment is the ability to conduct a mass balance with updrift accumulation and downdrift erosion at the impoundment. The short-term impoundment was found to work best under low wave energy conditions (Wang and Kraus, 1999). Earlier short-term impoundment tests were performed by Johnson (1957), Bruno and Gable (1977), Bruno et al. (1981a), Bruno et al. (1981b), Dean et al. (1983), Dean et al. (1987), and Dean (1989).

Streamer sediment traps measure the relative spatial differences in transport rates, but they are inaccurate for measuring total transport quantities. A vertical setup of samplers can be used to collect sediment, which is then weighed. More advanced methods for measuring concentration of sediment include acoustic and optical techniques. Table 1 summarizes the measurable characteristics for each method.

Successfully modeling longshore sediment transport requires validation of the model with measured field data. Major field campaigns and associated data collection techniques are listed in Table 2 with the list of testing parameters. The table indicates if measurements were taken of the water currents, bathymetric profiles, wave pressures, sediment transport, edge waves, shear waves, and/or bar migration.

Schoonees and Theron (1993) reviewed the field data available for longshore sediment
**Table 1:** Sediment transport measurement techniques and capabilities for large-scale traps, tracers, optics, and acoustics adapted from White (1998).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Large-Scale Traps</th>
<th>Tracers</th>
<th>Optics</th>
<th>Acoustics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bedload</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Suspension</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Longshore</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Cross-shore</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Velocity</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Concentration</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Transport</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

transport measurements. The data search provides 273 points for bulk (or total) transport rates, and a point rating was given to each data set. Data sets which did not include simultaneous measurements of transport and wave characteristics were eliminated from the review. The points were measured with a variety of methods including beach fill, deposition in trap, sand bypassing, accretion at groin, accretion at breakwater, pump sampler, and tracers.

Wang *et al.* (1998) measured total longshore sediment transport rate by using streamer traps at 29 locations and then comparing the field data to empirical formulas. They found that the empirical predictions by the CERC formula were too high for the low wave energy observed in the field measurements.

**Table 2:** Longshore sediment transport field data collection. Information collected includes currents (1), profiles (2), wave pressure (3), sediment transport (4), edge waves (5), shear waves (6), and bar migration (7).

<table>
<thead>
<tr>
<th>Source</th>
<th>Location</th>
<th>Date</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearshore Sediment Transport Study</td>
<td>Torrey Pines,CA</td>
<td>1979</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td>Nearshore Sediment Transport Study</td>
<td>Santa Barbara,CA</td>
<td>1981</td>
<td>1,2,3,4</td>
</tr>
<tr>
<td>DUCK</td>
<td>Duck, NC</td>
<td>1984</td>
<td>4,6,7</td>
</tr>
<tr>
<td>SUPERDUCK</td>
<td>Duck,NC</td>
<td>1987</td>
<td>4,6,7</td>
</tr>
<tr>
<td>DELILAH</td>
<td>Duck,NC</td>
<td>1991</td>
<td>4,6,7</td>
</tr>
<tr>
<td>DUCK94</td>
<td>Duck,NC</td>
<td>1994</td>
<td>4,6,7</td>
</tr>
<tr>
<td>Sandy Duck</td>
<td>Duck, NC</td>
<td>1996</td>
<td>4,6,7</td>
</tr>
</tbody>
</table>
1.2 Longshore Sediment Transport Modeling

The simplest longshore sediment transport modeling techniques are based on empirical equations developed from scores of data. Alternatively, the longshore sediment transport can be computed with equations governing the physical processes of both the hydrodynamics and sediment transport. These sediment transport formulations were initially developed for rivers and extended to the ocean. With time, the formulations have evolved to include the effects of waves and currents in the ocean as opposed to strictly currents in a river environment. Early models such as Meyer-Peter and Mueller (1948), Einstein (1950) and Engelund and Hansen (1967) are based on the relationship between shear stress and sediment transport rate. These formulas are still used for calculating sediment transport in rivers, but more sophisticated formulations have been created for the ocean environment due to more complicated hydrodynamics (Ribberink, 1998).

1.2.1 Total Longshore Sediment Transport Calculations

Total longshore sediment transport is the volumetric sediment transport rate. Longshore transport equations which calculate localized sediment transport at a specific point must be integrated across the domain in order to produce total longshore sediment transport. A commonly used total longshore transport equation is the Komar and Inman (1970), or CERC (Coastal Engineering Research Center), formula (CERC, 1984). The CERC formula relates immersed-weight sediment transport rate to the longshore component of wave energy flux by,

\[ I_i = K P_i \]

(1)

and

\[ P_i = (ECn \cos \alpha \sin \alpha)_b \]

(2)

where \( I_i \) is the immersed-weight sediment transport rate, \( E \) is the wave energy density given by \( E = \frac{\rho g H_b^2}{8} \), \( C \) is shallow water wave celerity given by \( \sqrt{gh_b} \), \( n \) is the ratio of wave group speed to individual wave celerity, \( \alpha \) is the wave angle, \( b \) denotes the breakpoint, and \( K \)
is an empirical sediment transport coefficient. The immersed-weight sediment transport is
related to the volumetric sediment transport rate \( Q_i \) as follows,

\[
Q_i = \frac{I_i}{\rho(s - 1)g(1 - p)}.
\]  

(3)

The constant \( K \), in Equation (1), has been the subject of much debate. The value
of \( K \) for different situations has been explored by Komar and Inman (1970), Kamphuis
\textit{et al.} (1986), and Schoonees and Theron (1994). The value of \( K \) with respect to grain size
was explored by del Valle \textit{et al.} (1993). The value of \( K \) for longshore transport measured
by a short-term impoundment was measured by Wang and Kraus (1999). Komar and
Inman (1970) developed the CERC equation after performing field measurements of wave
and current parameters in the surf zone and measuring the resulting longshore sediment
transport using a tracer.

It is important to note that the CERC formula has no dependence on grain size. This
relationship was developed from Komar and Inman’s field data for the grain size range of
175 to 600 microns. Komar and Inman propose that the lack of dependence on sediment
size suggests that bed load longshore sediment is dominant over suspended load sediment
transport. The Komar and Inman studies at two different beaches, El Moreno Beach and
Silver Strand Beach, indicated a \( K \) value of 0.77. Haas and Hanes (2004) also found that
the CERC formula constant has weak dependence on grain size. They found that the
various effects and influences of sediment size tend to cancel out resulting in little overall
dependence on sediment size. The range of sediment size testing for the current research
does not extend beyond the CERC formula range.

Komar (1988) later compiled \( K \) values from field data for longshore sediment transport
rates. The field data median diameter ranges from 0.18 to 1.0 mm as shown in Table 3.
As seen in Table 3, the \( K \) value is variable for different bathymetric and hydrodynamic
conditions. It is expected that the \( K \) value will decrease as the sediment size increases
sizes from 0.33 to 2.0 mm. They found that \( K \) does decrease with increasing \( D_{50} \). del Valle
\textit{et al.} (1993) compiled their data with other sources and found a relationship for \( K \) is as

6
Table 3: Field data summary for K value in CERC formula. 'Source' is the reference for the field data, 'location' indicates where the measurement was conducted, $d_{50}$ is the median grain size, 'Data Points' represents the number of points sampled, and 'K' is the average empirical value for the data with the range in parentheses.

<table>
<thead>
<tr>
<th>Source</th>
<th>Location</th>
<th>$d_{50}$ (mm)</th>
<th>Data Points</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watts (1953)</td>
<td>Lake Worth, FL</td>
<td>0.40</td>
<td>4</td>
<td>0.89 (.73-1.03)</td>
</tr>
<tr>
<td>Caldwell (1956)</td>
<td>Anaheim, CA</td>
<td>0.40</td>
<td>6</td>
<td>0.63 (.16-1.65)</td>
</tr>
<tr>
<td>Moore, Cole (1960)</td>
<td>Cp. Thompson, AK</td>
<td>1.0</td>
<td>1</td>
<td>0.18</td>
</tr>
<tr>
<td>Komar, Inman (1970)</td>
<td>El Moreno, Mexico</td>
<td>0.60</td>
<td>8</td>
<td>0.82 (.48-1.15)</td>
</tr>
<tr>
<td>Knoth, Nummedal (1977)</td>
<td>Bull Is, SC</td>
<td>0.18</td>
<td>5</td>
<td>0.62 (.23-1.0)</td>
</tr>
<tr>
<td>Duane, James (1980)</td>
<td>Pt. Mugu, CA</td>
<td>0.15</td>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td>Inman et al. (1980)</td>
<td>Torrey Pines, CA</td>
<td>0.20</td>
<td>2</td>
<td>0.69 (.26-1.34)</td>
</tr>
<tr>
<td>Duane, James (1980)</td>
<td>Pt. Mugu, CA</td>
<td>0.15</td>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td>Bruno et al. (1981)</td>
<td>Channel Islands, CA</td>
<td>0.20</td>
<td>7</td>
<td>0.87 (.42-1.5)</td>
</tr>
<tr>
<td>Dean et al. (1982)</td>
<td>Santa Barbara, CA</td>
<td>0.22</td>
<td>7</td>
<td>1.15 (.32-1.63)</td>
</tr>
<tr>
<td>Dean et al. (1987)</td>
<td>Rudee Inlet, VA</td>
<td>0.3</td>
<td>3</td>
<td>1.00 (.84-1.09)</td>
</tr>
</tbody>
</table>

follows,

$$ K = 1.4 \times \exp(-2.5D_{50}). $$ (4)

The CERC formula is a widely used longshore sediment transport formulation. Other commonly used formulas include energetics and physics based sediment equations that take into account grain size diameter by directly including the diameter or fall velocity. The potential errors in practical application of the CERC formula were evaluated by Bodge and Kraus (1991). When using significant wave height, K is equal to 0.32 according to the U.S. Army Corps of Engineers (2001). In the present study, model results are compared to the CERC formula for validation. Model results are also compared to the Kamphuis (1990) empirical formula and the long term shoreline change model GENESIS (2001) (U.S. Army Corps of Engineers, 2001).

The CERC, Kamphuis, and GENESIS formulas each have an empirical component meaning that they were calibrated through observations and/or experimentation. The CERC formulation is shown in Equations 1, 2, and 3. Kamphuis (1990) established a predictive relationship for longshore sediment transport after performing dimensional analysis.
and calibrating an equation against laboratory data. The Kamphuis formulation includes many descriptive parameters that are not included in the CERC formula such as beach slope, sediment size, wave period, and wavelength. The formulation is as follows,

\[ Q = \frac{\rho H_b^3}{T} - 0.0013 m^{0.75} \left( \frac{H_b}{L_\infty} \right)^{-1.25} \left( \frac{H_b}{D_{50}} \right)^{0.25} \sin^{0.6}(2\theta_b) \]  

(5)

where \( H_b \) is the significant breaking wave height, \( D_{50} \) is the mean sediment size, \( T \) is the wave period, \( m \) is the beach slope, and \( L_\infty \) is the deep water wavelength.

The GENESIS model (GENErated model for Simulating Shoreline change) was developed by contributors Hanson (1987), Hanson and Kraus (1989), and Gravens et al. (1991). The model calculates shoreline evolution resulting from spatial and temporal gradients in longshore sediment transport as influenced by breaking waves. The empirical predictive formula for longshore sediment transport in GENESIS is given by the following,

\[ Q_l = H_b^2 C_{gb} \left( a_1 \sin 2\alpha_b - a_2 \cos \alpha_b \frac{dH_b}{dy} \right). \]  

(6)

The nondimensional parameters \( a_1 \) and \( a_2 \) are given by,

\[ a_1 = \frac{K_1}{16 \left( \frac{\alpha_b}{p} - 1 \right) (1 - p)} \]  

(7)

and

\[ a_2 = \frac{K_2}{8 \left( \frac{\alpha_b}{p} - 1 \right) (1 - p)m} \]  

(8)

where \( H_b \) is the RMS breaking wave height, \( K_1 \) and \( K_2 \) are empirical coefficients, \( m \) is the average bottom slope from the shoreline to the depth of active longshore sediment transport, \( \frac{dH_b}{dy} \) is the longshore gradient of RMS wave height at breaking, and \( p \) is the sediment porosity.

When \( K_1 \) is equal to 0.7 and the longshore gradient is zero, the GENESIS equation is the same as the CERC equation.

### 1.2.2 Numerical Methods for Calculating Longshore Sediment Transport

Deigaard et al. (1986) model suspended sediment transport in the breaking zone by using a one-dimensional turbulence model without convective terms. Morfett (1991) presents an approach to numerically model longshore transport of sand in the surf zone. Christensen
et al. (2000) model sediment transport under breaking waves by using a Navier-Stokes solver, a free surface model, a turbulence model, and a sediment transport model. Christensen et al. (2000) used the model to simulate sediment transport resulting from both spilling and plunging breakers.

Most sediment transport equations require the definition of a threshold for the initiation of sediment movement. Shields (1936) was one of the first to relate sediment grain size to the shear stress exerted by the fluid flow. Komar and Miller (1973) took Shields’ formulation a step further to calculate the threshold of sediment movement under oscillatory water waves. They find that for grain sizes under 0.05 cm the threshold for motion is reached while the boundary layer is still laminar. When grain sizes exceed 0.05 cm, the sediment motion is not initiated until the boundary layer becomes turbulent. Seymour (1985) further studied the effects of a threshold for incipient motion on sediment transport by waves. Seymour used a simple longshore sediment transport formula to determine the magnitude of threshold effects for spectral and monochromatic waves. He found that the threshold effects significantly changed the transport estimates under certain surf zone conditions, and large net transport differences result from monochromatic versus spectral waves.

Briand and Kamphuis (1993) created a quasi-3D model which describes long-crested waves in the surf zone. The model calculates current velocities for sediment transport rate calculations. The model was tested for regular and irregular wave fields for a variety of wave conditions and sand sizes. The model solves for reference concentration and total swash zone sediment transport. The model requires different calibration for regular and random waves. The calibration coefficient, $K_r$, is 5.0 for regular waves and 10.0 for random waves, and $K_Q$ is 0.55 for regular waves and 0.90 for random waves. The model is based on the assumption that the time-averaged quasi-3D velocities can be multiplied by the time-averaged sediment concentration profile (Briand and Kamphuis, 1993).

Watanabe et al. (1986) created a three-dimensional numerical model for computing circulation, waves, and transport and compared the results with experimental data. Watanabe’s model calculates the wave field by using combined refraction, diffraction, and breaking nearshore wave computations. Watanabe’s model uses three submodels for waves, nearshore
currents, and sediment transport calculations which is similar in structure to the model structure in the current research. Watanabe models only a small domain (less than 20 square meters) and the model is inefficient compared to current computational standards.

Recent numerical models are composed of a wave, current, and sediment module working together as shown in Figure 2. The wave model provides the wave heights, directions, cross-shore and longshore water volume fluxes, and radiation stresses. The current, or circulation, module uses the wave module output to calculate wave-induced currents at each grid point in the domain. The sediment transport module then uses the hydrodynamic information to calculate the sediment transport rate (or vectors), and it outputs a new bathymetry to the wave module. This module setup is utilized in the present study.

1.3 Longshore Sediment Transport and Wave Properties

Different wave shapes can have a significant effect on sediment transport. Simple models use a linear monochromatic wave which does not accurately represent a natural wave which is frequently nonlinear in and near the surf zone. Bodge and Kraus (1991) found that the longshore wave power, $P_l$, is significantly higher for linear theory than for nonlinear waves. They used stream function wave theory to show that wave power $P_l$ is less for a nonlinear
wave than a wave calculated by linear wave theory. This effect is more pronounced as the breaking depth to deep wave wavelength ratio \( \frac{d_A}{L_0} \) becomes smaller (i.e. for long period or small waves) (Bodge and Kraus, 1991).

Ozhan (1983) studied the effect of breaker type on longshore sediment transport. The longshore sediment transport was measured using regular waves with plunging, collapsing, and surging breaker types. The transport was evaluated on a plane beach with a 1:10 slope using the CERC formula. The breaking-generated turbulence and vortices of a plunging breaker put sediment into suspension. A collapsing breaker carries sediment in the longshore direction by a saw-tooth motion in the swash zone. A spilling breaker suspends sediment through the boundary layer turbulence (Ozhan, 1983). Ozhan’s experiment indicates that among two breakers with the same amount of longshore wave energy flux, the collapsing breaker has a greater rate of sediment transport than the plunging one. The surging breaker longshore sediment transport rate is between the rates of the collapsing and spilling breakers. Pedersen et al. (1995) study the sediment transport effects induced by plunging breakers. The plunging breaker is represented by superimposing a jet on a nonbreaking wave in a numerical model.

Kamphuis (1991) performed wave basin experiments and measured alongshore sediment transport rates under regular (or monochromatic) and irregular (or spectral) waves. Gentile (2000) describes a procedure to evaluate the longshore sediment transport due to random waves. Karambas and Koutitas (2002) developed a model for surf and swash zone morphology evolution induced by nonlinear waves using a numerical solution to Boussinesq-type equations. The Boussinesq model provides bottom velocity, undertow, wave energy dissipation and other hydrodynamic information to the sediment transport/morphology model.

Because the strength of the longshore current can greatly affect the longshore sediment transport, the influence of rollers on longshore currents is an important process to be investigated. Osiecki and Dally (1996) developed a computational model to determine the depth-averaged currents with the effects of the surface roller generated by wave breaking. The presence of the roller induced turbulence increases the peak longshore current current and shifts the maximum closer to the shoreline. After wave breaking, there is an increase
in longshore current when the roller induced turbulence is included. Therefore, the roller
turbulence effect will play a role in modeling longshore sediment transport and has been
included in the present research.

1.4 Outline of Present Work

The purpose of the present research is to show sediment transport predictions made with the
Nearshore Community Model, or NearCoM, using numerous sediment transport formulae
and a variety of idealized wave and bathymetric conditions. All results are compared to
several empirical longshore sediment transport formulas.

Chapter 2 presents an overview of the numerical modeling. Details are presented for
the SHORECIRC circulation model including discussion of the governing equations. Wave
driver model information is given describing REF/DIF1 and REF/DIF S. The sediment
transport formulas which are used in the sediment transport module are presented and de-
scribed. The formulas discussed include the CERC formula and other well-known empirical
and process based formulas.

Chapter 3 describes methods for calculating skewness which include empirical, theo-
retical, and numerical methods as well as effects of wave shape on hydrodynamics and
longshore sediment transport. The skewness components in SHORECIRC shear stress and
the sediment transport velocities are reviewed. The effects of skewness on sediment trans-
port are discussed for each sediment transport formula. Finally, the causes for changes due
to skewness on advection components, bottom velocities, and mean current are discussed.

Chapter 4 describes sediment transport with monochromatic versus spectral wave drivers.
The differences in hydrodynamics between the monochromatic and spectral wave driver
cases are explored. Hydrodynamic properties analyzed include wave height, radiation
stresses, wave setup and setup, longshore currents, and shear stresses. The changes in
longshore sediment transport due to random versus monochromatic waves are described.

Chapter 5 describes the effects of grain size and alongshore bathymetric variations on
longshore sediment transport. Longshore sediment transport is modeled for equilibrium
beach profiles with 0.2 mm and 0.4 mm as the median grain size. A cusped beach is used
as the model bathymetry with varying cusp heights and cusp wavelengths. A barred beach is also used and transport is modeled with waves breaking at the bar, before the bar, and after the bar. A nondimensional parameter analysis is presented and trends are discussed.

Finally, Chapter 6 summarizes results of the present study. Conclusions are presented on the effect of including additional skewness terms in the shear stress and current velocity transport calculations. The effect of a monochromatic wave driver versus spectral wave driver on hydrodynamics and longshore transport are discussed. The ability of the model to describe transport on beaches with alongshore variations is described. Finally, the comparisons to empirical longshore sediment transport predictions are discussed.
CHAPTER II

MODEL DESCRIPTION

The numerical model in the present research is used to determine nearshore waves, currents, and sediment transport. The Nearshore Community Model (NearCoM) developed by NOPP, provides the framework (Master) for the connection of wave modules, circulation modules, and sediment transport modules. The wave modules in the NearCoM model utilized in this study include REF/DIF 1 and REF/DIF S. The circulation module is consistently SHORECIRC (SC). The sediment transport modules utilize a generic transport formula developed by Haas and Hanes (2004) (HH), a formula similar to Bagnold (1966), Bowen (1980) and Bailard (1981) formulae (BBB), a Watanabe (1992) type formula (W), and a Ribberink (1998) (R) type formula. Table 4 summarizes the different models used in conjunction with the NearCoM Model for this study.

2.1 Numerical Modeling Background

The Master program within NearCoM controls the interaction between the circulation, wave, and sediment transport modules. The NearCoM Program is designed to make the modules interchangeable. For instance, a user can choose a circulation module such as the Princeton Ocean Model (POM) (Mellor, 2004) or SHORECIRC (SC) (Svendsen et al., 2002), a wave module (e.g. REF/DIF 1 or REF/DIF S), and a sediment transport module. Therefore, the model can be optimized for a particular application.

Table 4: Module components utilized from the NearCoM Program.

<table>
<thead>
<tr>
<th>Wave Module</th>
<th>Circulation Module</th>
<th>Sediment Module</th>
</tr>
</thead>
<tbody>
<tr>
<td>REF/DIF 1</td>
<td>SC</td>
<td>HH</td>
</tr>
<tr>
<td>REF/DIF S</td>
<td></td>
<td>BBB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R</td>
</tr>
<tr>
<td></td>
<td></td>
<td>W</td>
</tr>
</tbody>
</table>
Figure 3: Definition sketch for wave components and vertical variation.

Many physical parameters must be defined to correctly implement the model. Figures 3 and 4 are definition sketches for hydrodynamic parameters and axis orientation. SWL is the still water level, $\zeta$ is the instantaneous free water surface relative to the SWL, MWL is the mean water level or $\bar{\zeta}$, $h$ is the depth from the MWL, $h_o$ is the depth from the SWL, $H$ is the wave height, $Q_{wa}$ is the volume flux due to short wave motion from the elevation of the wave trough to $\zeta$, $z$ increases positively upward from the SWL, $x$ increases from the offshore boundary (defined for each model domain) shoreward, $y$ is in the alongshore direction, $u$, $v$ and $w$ are the velocities in the $x$, $y$ and $z$ directions respectively, and the current velocity profile components $V_\alpha$, $V_{ma}$, and $V_{da}$ are shown. $V_\alpha$ is the current velocity, $V_{ma}$ is the depth uniform current velocity, and $V_{da}$ is the depth-varying current velocity.

Figure 4: Plan view definition sketch for directions and incoming wave angle.

The numerical models solve equations cannot be solved analytically. The wave model
must calculate the radiation stresses, wave heights, and wave directions. The radiation 
stress is the momentum flux due to the presence of waves which drive the currents and mean 
water level variations, or setup and setdown. The concept of radiation stress producing the 
longshore current was introduced by Longuet-Higgins (1970). The integral form of the 
radiation stresses \( S_{\alpha\beta} \) used by Phillips (1977) is given by,

\[
S_{\alpha\beta} \equiv \int_{-h_0}^{\zeta} \left( p\delta_{\alpha\beta} + \rho u_{\alpha w} u_{\beta w} \right) dz - \delta_{\alpha\beta} \frac{1}{2} \rho gh^2 - \rho \frac{Q_{\alpha w} Q_{\beta w}}{h} \tag{9}
\]

where \( \alpha \) and \( \beta \) are directional indices, \( u_{\alpha w} \) is the horizontal shortwave-induced velocity, and 
p is pressure. After applying linear wave theory, the radiation stress equations become,

\[
S_{xx} = E \left[ n(\cos^2 \theta + 1) - \frac{1}{2} \right] \\
S_{yy} = E \left[ n(\sin^2 \theta + 1) - \frac{1}{2} \right] \\
S_{xy} = S_{yx} = \frac{E}{2} n \sin 2\theta. \tag{10}
\]

The subscript 'xx' represents the x-momentum flux in the x-direction; the subscript 'yy' 
represents the y-momentum flux in the y-direction; the subscript 'xy' represents the x-
momentum flux in the y-direction or vice versa. \( E \) is the wave energy given by \( \frac{1}{8} \rho g H^2 \) and 
n is given by,

\[
n = \frac{1}{2} \left( 1 + \frac{2kh}{\sinh 2kh} \right) \tag{11}
\]

With the progression of computing power, numerical models have become less time 
consuming and more readily applied. Numerical circulation, wave, and sediment transport 
models have all been improved from their initial states. The early circulation models only 
involved two dimensions, that is the longshore and cross-shore directions, without including 
depth-varying currents. The depth-uniform circulation models are based on solving the 
continuity and momentum equations and are respectively given by,

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial Q_{\alpha}}{\partial x_\alpha} = 0 \tag{12}
\]

and

\[
\rho \frac{\partial Q_{\beta}}{\partial t} + \frac{\partial}{\partial x_\alpha} \left( \rho \frac{Q_{\alpha} Q_{\beta}}{h} + S_{\alpha\beta} - \int_{-h_0}^{\zeta} \tau_{\alpha\beta} dz \right) = -\rho g h \frac{\partial \zeta}{\partial x_\beta} + \tau_{\beta}^{S} - \tau_{\beta}^{B} \tag{13}
\]
where \( t \) is time, \( g \) is gravity, \( \rho \) is water density, \( Q_\alpha \) and \( Q_\beta \) are volume fluxes in the \( \alpha \) and \( \beta \) directions, \( S_{\alpha\beta} \) is the radiation stress, \( \tau_{\alpha\beta} \) is the Reynolds stress, \( \tau_{\beta}^S \) is the surface shear stress, and \( \tau_{\beta}^B \) is the bottom shear stress.

If the model assumes steady-state and longshore uniformity, the continuity, \( x \)-momentum, and \( y \)-momentum equations respectively simplify to:

\[
\frac{\partial Q_x}{\partial x} = 0, \tag{14}
\]

\[
g(h_0 + \zeta) \frac{d\zeta}{dx} = -\frac{1}{\rho} \frac{dS_{xx}}{dx}, \tag{15}
\]

and

\[
\frac{dS_{xy}}{dx} - h \frac{d\tau_{xy}}{dx} + \tau_{y}^B = 0. \tag{16}
\]

For the cross-shore momentum equation (\( x \)-momentum) in Equation (15), changes in the cross-shore components of radiation stress are balanced by the change in mean water level (Longuet-Higgins and Stewart, 1963; Bowen et al., 1968). The longshore momentum (\( y \)-momentum) in Equation (16) shows the balance between changes in the longshore radiation stress component and the bottom shear stress with a redistribution of currents by the Reynolds stresses (Svendsen and Petrevu, 1995).

The first vertical variation added to the 2D local models was undertow in the cross-shore direction (Svendsen et al., 1987). Svendsen and Lorenz (1989) resolved the vertical variation of the longshore current. Quasi-3D models developed from this point, and the term refers to the combination of the depth integrated solutions with a locally varying vertical structure. The first of the quasi-3D models was developed by De Vriend and Stive (1987). The importance of the vertical resolution in the quasi-3D models is in the lateral mixing, or the effect of the horizontal interaction with the vertical structure (Svendsen and Petrevu, 1995). The lateral mixing mechanism is very important for correctly modeling nearshore circulation. By including vertical variation in the currents, the current can be resolved near the ocean bed which is particularly important in sediment transport calculations.
2.2 Circulation Module Background

SHORECIRC is a nearshore circulation model, and it is the only circulation model used with the NearCoM program for the current research. SHORECIRC is more advanced than traditional nearshore models because it calculates the horizontal momentum exchange due to depth variation whereas in many other models this parameter is typically empirical and artificial. Utilizing uniform horizontal currents is not a realistic assumption in the nearshore region. The quasi-3D model combines the effects of the vertical structure of the currents with the 2D horizontal model for nearshore circulation. The 3D solution for the depth varying currents is obtained by a semi-analytical solution, while the 2D depth-integrated horizontal equations are solved numerically. The SHORECIRC model has been verified with many data sets from laboratory experiments (Haas et al., 1998; Haas and Svendsen, 2000, 2002; Svendsen et al., 2003; Qin and Svendsen, 2003) and from the DELILAH field experiment (Svendsen et al., 1997; Van Dongeren et al., 2003).

The governing equations for the SHORECIRC circulation model are shown in tensor notation where $\alpha$ and $\beta$ represent horizontal Cartesian coordinate directions. The instantaneous total fluid velocity $u_\alpha(x, y, z, t)$ is split into three components and is given by,

$$u_\alpha = u'_\alpha + u_\omega \alpha + V_\alpha$$  \hspace{1cm} (17)

where $u'_\alpha$ is the turbulent velocity component, $u_\omega \alpha$ is the wave component defined so that $\pi_w = 0$ below the trough level, and $V_\alpha$ is the current velocity (typically depth-varying). $Q_\alpha$ is the total volume flux, and is given by

$$Q_\alpha = \overline{\int_{-h_0}^{\zeta} u_\alpha dz}$$  \hspace{1cm} (18)

where the overbar represents a wave-average. The volume flux due to short wave motion, $Q_{\omega \alpha}$, is given by,

$$Q_{\omega \alpha} = \overline{\int_{-h_0}^{\zeta} u_{\omega \alpha} dz} = \int_{\zeta}^{\zeta} u_{\omega \alpha} dz$$  \hspace{1cm} (19)

where $\zeta_t$ is the elevation of the wave trough. The total volume flux can then be written as,

$$Q_\alpha = \int_{-h_0}^{\zeta} V_\alpha dz + Q_{\omega \alpha}$$  \hspace{1cm} (20)
The current velocity is separated into a depth-uniform part, \( V_{ma} \), and a depth-varying component, \( V_{da}(z) \) as follows,
\[
V_a = V_{ma} + V_{da}(z)
\]
where
\[
V_{ma} = \frac{Q_a - Q_{wa}}{h_0 + \zeta},
\]
and therefore
\[
\int_{h_0}^{\zeta} V_{da} dz = 0.
\]

The governing equations for SHORECIRC are derived from the Navier-Stokes equations. Using the above definitions, these equations are depth-integrated and wave averaged and the appropriate surface and bottom boundary conditions are applied. After much manipulation, the governing equations are then given by the continuity equation,
\[
\frac{\partial \zeta}{\partial t} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0,
\]
the cross-shore momentum equation,
\[
\begin{align*}
\frac{\partial Q_x}{\partial t} &+ \frac{\partial}{\partial x} \left( \frac{Q_x^2}{h} + M_{xx} \right) + \frac{\partial}{\partial y} \left( \frac{Q_x Q_y}{h} + M_{xy} \right) - \frac{\partial}{\partial x} h \left[ (2D_{xx} + B_{xx}) \frac{\partial Q_x}{\partial x} \right] \\
&+ 2D_{xy} \frac{\partial}{\partial y} \left( \frac{Q_x}{h} + B_{xx} \frac{\partial Q_y}{\partial y} \right) - \frac{\partial}{\partial y} h \left[ (D_{xy} + B_{xy}) \frac{\partial Q_x}{\partial x} + D_{xx} \frac{\partial Q_y}{\partial x} \right] \\
&+ (D_{xy} + B_{xy}) \frac{\partial}{\partial x} \left( \frac{Q_y}{h} + A_{xx} \frac{Q_x}{h} + A_{xy} \frac{Q_y}{h} \right) + \frac{\partial}{\partial y} \left[ A_{xx} \frac{Q_x}{h} + A_{xy} \frac{Q_y}{h} \right] \\
&= -gh \frac{\partial \zeta}{\partial x} - \frac{1}{\rho} \left( \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right) + \frac{1}{\rho} \left( \frac{\partial}{\partial x} \int_{h_0}^{\zeta} \tau_{xx} dz + \frac{\partial}{\partial y} \int_{h_0}^{\zeta} \tau_{xy} dz \right) + \frac{\tau_S - \tau_B}{\rho},
\end{align*}
\]
and the longshore momentum equation,
\[
\begin{align*}
\frac{\partial Q_y}{\partial t} &+ \frac{\partial}{\partial x} \left( \frac{Q_x Q_y}{h} + M_{xy} \right) + \frac{\partial}{\partial y} \left( \frac{Q_y^2}{h} + M_{yy} \right) - \frac{\partial}{\partial x} h \left[ (2D_{xy} + B_{xy}) \frac{\partial Q_y}{\partial x} \right] \\
&+ D_{xx} \frac{\partial}{\partial x} \left( \frac{Q_y}{h} + (D_{xy} + B_{xy}) \frac{\partial Q_x}{\partial y} \right) - \frac{\partial}{\partial y} h \left[ (D_{xy} + B_{xy}) \frac{\partial Q_y}{\partial x} + 2D_{xy} \frac{\partial Q_x}{\partial x} \right] \\
&+ (2D_{yy} + B_{yy}) \frac{\partial}{\partial y} \left( \frac{Q_y}{h} + A_{xx} \frac{Q_x}{h} + A_{xy} \frac{Q_y}{h} \right) + \frac{\partial}{\partial y} \left[ A_{xx} \frac{Q_x}{h} + A_{xy} \frac{Q_y}{h} \right] \\
&= -gh \frac{\partial \zeta}{\partial y} - \frac{1}{\rho} \left( \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \right) + \frac{1}{\rho} \left( \frac{\partial}{\partial x} \int_{h_0}^{\zeta} \tau_{xy} dz + \frac{\partial}{\partial y} \int_{h_0}^{\zeta} \tau_{yy} dz \right) + \frac{\tau_S - \tau_B}{\rho},
\end{align*}
\]
where \( M_{\alpha\beta}, D_{\alpha\gamma}, B_{\alpha\beta}, \) and \( A_{\alpha\beta\gamma} \) are 3D dispersive mixing coefficients, \( \tau_{\alpha\beta} \) is the Reynolds stress, \( \tau_S \) is the surface shear stress, and \( \tau_B \) is the bottom shear stress. Details for the
derivation and definitions of the three-dimensional terms are found in Haas and Svendsen (2000). SHORECIRC utilizes the forcing and mass flux calculations provided by the wave modules which are described in Section 2.3.

SHORECIRC short wave averaged bottom shear stress is given by the general shear stress formula for combined waves and current flow,

\[
\tau_{\alpha}^H(t) = \frac{1}{2} \rho f_{cw}(u_{0,\alpha}(t) + V_{b,\alpha})(u_{0,\alpha}(t) + V_{b,\alpha}).
\] (27)

After assuming linear wave theory the shear stress is given by,

\[
\tau_{\alpha}^H = \frac{1}{2} \rho f_{cw} u_0 (\beta_1 V_{b,\alpha} + \beta_2 u_{0,\alpha})
\] (28)

where \( u_{0,\alpha}(t) \) is the bottom velocity in the wave motion, \( V_b \) is the bottom velocity in the current motion, and \( f_{cw} \) is the bottom friction factor which can be spatially varying but is assumed constant with time. \( \beta_1 \) and \( \beta_2 \) are the weight factors given by,

\[
\beta_1 = \left[ \left( \frac{V_b}{u_0} \right)^2 + 2 \frac{V_b}{u_0} \cos \theta \cos \mu \cos \theta \right]^{1/2}
\] (29)

and

\[
\beta_2 = \cos \theta \left[ \left( \frac{V_b}{u_0} \right)^2 + 2 \frac{V_b}{u_0} \cos \theta \cos \mu \cos \theta \right]^{1/2}.
\] (30)

The friction factor is considered constant and is calculated from the formula by Nielsen (1992) which assumes that the bottom roughness is due to bedload transport and is given by,

\[
f_w = \exp \left[ 5.213 \left( \frac{r}{a_o} \right)^{0.191} - 5.977 \right]
\] (31)

where \( f_w \) is the friction factor, \( r \) is the bottom roughness height, and \( a_o \) is the amplitude of the bottom orbital excursion. The bottom roughness is given by,

\[
r = 170d_{50} \sqrt{\theta_{2.5} - 0.05},
\] (32)

and \( \theta_{2.5} \) is determined by,

\[
\theta_{2.5} = \frac{0.5f_w'(a_o \omega)^2}{(s - 1)g_d_{50}}
\] (33)

where \( \omega \) is the wave frequency, and \( f_w' \) is found from Equation (31) with \( r = 2.5d_{50} \).
2.2.1 SHORECIRC Depth Variations

Many circulation models only calculate two-dimensional flow (in the longshore and cross-shore directions) and empirically or artificially account for lateral mixing. SHORECIRC is a quasi-3D model because it semi-analytically solves for the vertical variation of the current profiles and the depth-integrated 2D horizontal equations. Using the quasi-3D SHORECIRC approach, the 3D dispersive mixing coefficients can be resolved in terms of the depth uniform flow properties. The dispersive mixing accounts for a large portion of the lateral mixing, and consequently is very important in the nearshore region (Svendsen et al., 2002). A representation of the velocity close to the bed is also important for sediment transport. The solution for the depth-varying profiles begins with the local non-integrated horizontal momentum equations. The velocity is split between (mean) current and short-wave velocities and wave-averaged assuming hydrostatic pressure variation. The currents are split again into depth uniform and depth varying parts with Equation (21) as seen in Figure 3.

An eddy viscosity model is used to represent the turbulent stresses. The boundary conditions of matching shear stresses at the bottom and specifying no net integrated flow, as seen in Equation (23), are used to close the problem. After much manipulation and assuming quasi-steady flow, the solution results in a polynomial expression for the depth-varying currents in terms of the depth-integrated currents. The detailed general solution for the depth-varying current is found in Putrevu and Svendsen (1999), and the slightly simplified version applied in SC in Haas and Svendsen (2000).

2.2.2 Numerical Solution

SHORECIRC uses a finite difference scheme for discretizing the governing equations. A central finite difference scheme is utilized to represent spatial derivatives. The SHORECIRC spatial finite difference scheme is fourth order in the grid size except for the diffusive terms which are second order (Svendsen et al., 2002) to improve model stability. The time derivatives are solved using a Predictor-Corrector scheme with the Adams-Bashforth predictor and the Adams-Moulton corrector (Svendsen et al., 2002).
Boundary conditions are required at the offshore, shoreline, and lateral boundaries. In the current research, the lateral boundaries are considered periodic. A periodic boundary condition means that the instantaneous flow at each point of one of the cross-shore boundaries is mirrored at the equivalent point at the other cross-shore boundary (Svendsen et al., 2002). The offshore boundary condition is always an absorbing/generating boundary condition (Van Dongeren and Svendsen, 1997). The shoreline boundary condition is no flux/direct wall for the equilibrium beach profile cases. For the cusped beach bathymetries, the shoreline boundary condition is no flux following the still water line. For this case, SHORECIRC identifies the location of the shoreline by sweeping through each row in the cross-shore direction to find the first point where the depth is below the minimum depth (Svendsen et al., 2002).

2.3 Wave Driver Module Background

Waves can be modeled by a variety of different numerical programs. Monochromatic wave drivers utilize a single frequency, or wave period, to represent the incoming wave condition. Spectral wave drivers are created to more accurately simulate a random sea by using a spectrum of wave frequencies and wave directions for the incoming wave condition. In this study, two different wave drivers are coupled to the NearCoM program and the different effects on longshore sediment transport are explored.

2.3.1 REF/DIF 1 Monochromatic Wave Driver

REF/DIF 1 by Kirby and Dalrymple (1994) is a short wave driver which determines the short wave motions and calculates the short-wave averaged forcing that drives the currents and infragravity waves for the circulation module (SHORECIRC). The REF/DIF 1 wave driver is based on the parabolic approximation of the extended mild-slope equation, and it accounts for refraction, diffraction, shoaling, and breaking phenomena. This monochromatic wave driver assumes a Svendsen type roller (Svendsen, 1984) and a wave breaking decay model by Dally et al. (1985).
2.3.2 REF/DIF S Spectral Wave Driver

REF/DIF S is a weakly nonlinear model which combines refraction and diffraction to simulate the behavior of random waves over irregular bottom bathymetry. The spectral model incorporates the effects of shoaling, refraction, energy dissipation, and diffraction (Kirby and Ozkan, 1994). REF/DIF S generates a two-dimensional spectrum that combines a frequency spectrum and a spreading function. REF/DIF S uses a diffraction model by Mei and Tuck (1980) and a multiple scale approach by Yue and Mei (1980) in nonlinear parabolic form. The parabolic wave model is solved using a combination of forward and central finite difference schemes (Kirby and Ozkan, 1994). REF/DIF S allows for a choice of wave height decay models by Thornton and Guza (1983) and Battjes and Janssen (1978). The decay model by Thornton and Guza (1983) utilizes a bore dissipation model. The decay model by Battjes and Janssen (1978) predicts the breaking induced energy dissipation by multiplying the energy dissipation of a single breaking wave by the probability of occurrence. The occurrence is predicted from a Rayleigh distribution as described further in Chapter 4. The spectral wave driver also offers a selection between a local roller model by Lippmann et al. (1996) and Svendsen (1984) and an evolving roller by Stive and DeVriend (1994).

2.4 Sediment Transport Module Formulas

The sediment transport module for NearCoM has been developed during the current research. The sediment transport is calculated with four different sediment transport formulas. Sediment transport formulas are frequently based on an energetics approach such as Bagnold (1963) or a probabilistic approach such as Einstein (1972). The longshore sediment transport formulas utilized in this study include:

- HH: A physics-based generic sediment transport formula which was first developed by Haas and Hanes (2004).
- BBB: An energetics approach to sediment transport was developed to include bedload and suspended load by Bagnold (1966), Bowen (1980) and Bailard (1981).
• W: An energy dissipation method which includes critical shear stress by Watanabe (1992).

The longshore sediment transport predictions from each formula are compared for most hydrodynamic situations described in Section 2.5.

2.4.1 Generic Sediment Transport

A simple local sediment transport equation is used to evaluate the response of the longshore sediment transport under different hydrodynamic and physical situations. The formula developed by Haas and Hanes (2004) characterizes the sediment flux by the product of the suspension due to bed shear stress and the advection due to the total velocity. The HH transport is written as follows,

$$\tilde{q}_{HH} = \frac{C_1 f_w}{g} |\tilde{u}|^2 \tilde{u}(t)$$

with $\tilde{u}(t) = \tilde{u}_w(t) + \tilde{V}_b(t)$ and where $C_1$ is a constant, $f_w$ is the friction factor, $g$ is gravity, $\tilde{u}_w$ is the near bottom orbital velocity, $\tilde{V}_b$ is the near bottom current velocity and the overbar represents the time-averaging over a short wave period.

If the shear stress is written as,

$$|\tilde{\tau}| = \frac{1}{2} \rho |\tilde{u}|^2$$

and the effective shear stress is given by,

$$\tau_{eff} = |\tilde{\tau}| - \tau_{cr}$$

where $\tau_{cr}$ is the critical shear stress (Equation (38)), then the longshore sediment transport equation can then be rewritten as,

$$q_y = \frac{2C_1}{\rho g} [\tau_{eff} u_{wy} + \tau_{eff} V_y].$$

The $\tau_{eff} u_{wy}$ term represents the advection due to waves, and the $\tau_{eff} V_y$ term represents the advection due to the mean current. The critical shear stress is calculated from Nielsen (1992) by using Shields parameter,

$$\tau_{cr} = \rho (s - 1) g \theta_{cr}$$
where \( s \) is the specific gravity of the sediment, \( d \) is the grain size diameter, and \( \theta_{cr} \) is the threshold for incipient motion given by 0.05 for the test scenarios.

### 2.4.2 Bailard, Bowen, Bagnold Longshore Sediment Transport

Bagnold (1966), Bowen (1980), and Bailard (1981) contributed to an energetics approach for sediment transport due to waves. The immersed-weight longshore sediment transport rate is calculated by,

\[
i_{BBB} = \frac{e_s(1 - e_b)\rho \overline{f_w}}{W_o} \left| \frac{\overline{u}}{\overline{u}} \right|^3 u_y + \frac{e_b \rho \overline{f_w}}{\tan \phi} \left| \frac{\overline{u}}{\overline{u}} \right|^2 u_y,
\]

where \( W_o \) is the fall velocity, \( u_y \) is the longshore component of the instantaneous bottom velocity, \( e_s \) is the suspended load efficiency factor, \( e_b \) is the bed load efficiency factor, and \( \tan \phi \) is the internal friction angle. The volumetric sediment transport rate can then be found by dividing \( i_{BBB} \) by \( \rho(s - 1)g(1 - p) \). Equation (39) was formulated by Bailard after modifications to Bagnold’s energetics formulation, Bowen (1980) previously modified Bagnold’s energetics formula.

The initial energetics formula by Bagnold (1966) separates the sediment transport into two modes: (1) bedload which is supported by grain to grain interactions and (2) suspended load supported by the fluid by turbulent diffusion. Modifications by Bailard (1981) calculate total load (i.e. bedload and suspended load) for time-varying flow over a sloping bed.

### 2.4.3 Ribberink Longshore Sediment Transport

Ribberink (1998) developed a wave-averaged longshore transport formula based on a bedload sheet-flow model. The formula by Ribberink (1998) assumes the instantaneous solid flux is proportional to a formulation of the difference between time-dependent shear stress and the critical shear stress (Camaden and Larroute, 2003). The model is based on instantaneous shear stresses and is shown in the following equations:

\[
q_R = m \sqrt{(s - 1)gd_{50}^3} \left( | \theta' | - | - \theta_{cr} | \frac{\theta_y}{\theta'} \right)
\]

where

\[
| \theta' | = \frac{\tau}{\rho(s - 1)gd_{50}}
\]
\[
\theta' = \frac{\tau_y}{\rho(s - 1)g d_{50}} 
\]

where \(d_{50}\) is the median grain size, and \(m\) and \(n\) are constants respectively equal to 11 and 1.65. The Ribberink formula was calibrated for sediment transport on a plane beach regime where the suspended sediment was negligible (Camenen and Larroude, 2003). The effect of this calibration on longshore sediment transport will be described on the equilibrium beach profile, cusped beach, and barred beach.

### 2.4.4 Watanabe Longshore Sediment Transport

Watanabe (1992) based a longshore sediment transport equation on energy dissipation as shown by,

\[
q_W = A_c \left( \frac{\tau_{b, \text{max}}}{\rho g} - \tau_{c,r} \right) V_{my} 
\]

where \(A_c\) is a constant set at 2.0, \(\tau_{b, \text{max}}\) is the maximum instantaneous bottom shear stress from wave and current combination flow, and \(\tau_{c,r}\) is the critical bottom shear stress which is based on the Shields parameter in Equation (38).

### 2.5 Suite of Test Conditions

All tests are performed on equilibrium beach profiles (Dean and Dalrymple (2002)) except the barred and cusped beach cases in Chapter 5 which incorporate additional features onto the equilibrium beach profiles. Figure 5 shows the equilibrium beach profile for the sediment diameter test cases of 0.2 mm and 0.4 mm. The equilibrium beach profile is calculated by,

\[
\text{Depth} = Ar^n 
\]

where \(A=0.1\) for a sediment diameter of 0.2 mm and \(A=0.145\) for a sediment diameter of 0.4 mm and \(n\) is \(\frac{2}{3}\) (Dean and Dalrymple, 2002).

When an input parameter or case is changed, a suite of tests is run for the case. The suite includes the combination of wave heights of 0.25, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 meters and incoming wave angles of 3, 6, 9, 12, and 15 degrees. The wave period for the smaller
Figure 5: Example equilibrium beach profile for test cases for sediment diameter size of 0.2 mm (solid) and 0.4 mm (dashed).

wave heights of 0.10 and 0.25 m is 4 seconds. The wave period for wave heights from 0.5 m to 3 m is 10 seconds.

The variability of wave height and wave angle allows for a thorough comparison with established laboratory measurements, field data, and empirical formula predictions. Input parameter and case changes include wave driver type, grain size, bathymetric variability, velocity skewness, transport skewness, and longshore sediment transport formula. Wind and tidal effects are not considered.
CHAPTER III

EFFECT OF WAVE SHAPE ON LONGSHORE
SEDIMENT TRANSPORT

Waves are frequently described by linear wave theory, first developed by Airy (1845), to describe the displacement of the water surface from the mean water level. Linear wave theory assumes an incompressible fluid, irrotational fluid motion, impermeable flat bottom, and small amplitude waves (Dean and Dalrymple, 2002). Linear wave theory simplifies the kinematic and dynamic free surface boundary conditions by linearizing the equations. Applying linear wave theory results in a cosine shape repeated over the wavelength \((L)\). A measure of nonlinearity of a wave which will be explored in this chapter is called skewness, which is related to the cube of the velocity, and it is a function of the wave number \((k = 2\pi/L)\) and the amplitude of a wave \((a)\). For linear wave theory, the wave steepness \((ka)\) is assumed to be small and higher order terms are not included in wave properties. For more advanced theories, higher order terms of wave steepness are maintained thereby increasing the nonlinearity of a wave. The addition of skewness is important to sediment transport because sediment transport is proportional to the cubed orbital velocity \((u^3)\) assuming mean flow is zero. The mean value of the cubed orbital velocity is zero for linear waves, and it is nonzero for skewed waves which therefore produces a net transport.

An example demonstrating the differences between the surface of a linear and nonlinear water wave is shown in Figure 6. Bottom shear stress and bed frictional characteristics are extremely important parameters for calculating sediment transport. Bed shear stress is generated by combined wave and current motion, and it causes sediment motion. This study investigates the differences between linear and skewed velocities, and the effects on the bottom shear stress and the impact on the hydrodynamics and longshore sediment transport.
Figure 6: Comparison of a linear (black) and nonlinear (red) waves and definition sketch for wave properties. Definition labels refer to linear wave shape (adapted from Dean and Dalrymple (2002)).

3.1 Skewed Time Series

Bottom shear stress and bed frictional characteristics are extremely important parameters for calculating sediment transport. Bed shear stress is generated by combined wave and current motion, and it causes sediment motion. This study investigates the differences between linear and skewed velocities and the effects on the bottom shear stress.

According to linear wave theory, velocity in the horizontal direction is given as,

\[ u(x, z, t) = \alpha(z) \cos(kx - \omega t) \]  \hspace{1cm} (45)

where \( k = \frac{2\pi}{L} \), \( \omega \) is angular frequency, \( t \) is time, and

\[ \alpha(z) = \frac{H \omega}{2 \sinh (kh)} \cosh (kz). \]  \hspace{1cm} (46)

Physically, the skewness is a measurement of the asymmetry of the flow and is indicative of the shape of the wave. When calculating skewness from measured time series, the time-averaged cubed velocity is divided by the root-mean-square cubed velocity. Skewness can be calculated for a time series \( Y \) by,

\[ \text{skewness} = \frac{\sum_{i=1}^{N} (Y_i - \overline{Y})^3}{(N - 1)s^3} \]  \hspace{1cm} (47)

where \( \overline{Y} \) is the mean, \( s \) is the standard deviation, and \( N \) is the number of data points. For nonlinear waves, the higher order terms cannot be neglected, and therefore the solution can
be obtained using Fourier series as follows,

\[ u(x, z, t) = \sum_{n=1}^{N} \alpha_n(z) \cos[n(kx - \omega t)] + \sum_{n=1}^{N} \beta_n(z) \sin[n(kx - \omega t)] \] (48)

where \( \alpha_n(z) \) and \( \beta_n(z) \) are series coefficients.

### 3.1.1 Methods for Computing Skewness

Many methods have been developed to calculate skewed velocities. Elgar and Guza (1986) used Boussinesq equations to calculate skewness of shoaling waves. Doering and Bowen (1995) used an empirical calculation from bispectral analysis to find velocity skewness of cross-shore flow. Streamfunction wave theory was developed by Dean (1965) to provide a representation of a nonlinear gravity water wave. It is less computationally expensive than prior higher order theories (Dean and Dalrymple, 1984). The stream function solution determines coefficients in a Stokes expansion to satisfy the dynamic free surface boundary condition using a numerical procedure (Dalrymple and Solana, 1986). Dean (1965) found that the stream function method is more accurate than linear solutions if the wave height is greater than 50 percent of the breaking height. Nonlinear errors become smaller than linear errors for larger wave heights. Dalrymple (1974) further expanded upon the stream function solution by Dean (1965) to to include a linear shear current.

The stream function wave solution was found by Dalrymple and Solana (1986) to have non-unique solutions under certain conditions which resulted in triple-crested waves. To eliminate this problem, three options were determined: (1) Examining water surface displacement until a single-crested solution is obtained, (2) constrain water surface so it decreases monotonically from crest to trough, or (3) increment the N order solution from a small number.

Rienecker and Fenton (1981) developed a Fourier approximation method for steady water waves to determine the progression of steady periodic waves on irrotational flow. The results for fluid velocities compare well with experimental data. The problem with the streamfunction and the method by Rienecker and Fenton (1981) is the applicability of the methods for wave breaking. The methods do not work well in small depths and for wave
breaking, therefore making them inadequate for the present analysis which is focused on the nearshore region with breaking waves in shallow water.

The aforementioned methods in this section for calculating skewness are not chosen for the present research due to their performance in the nearshore region under breaking wave conditions. The method chosen to calculate skewness is by Doering et al. (2000) which uses an empirical formula as opposed to theoretical methods based on the Boussinesq or full Navier-Stokes equation which are computationally intensive. The empirical equation by Doering et al. (2000) defines the skewness as a function of the Ursell number, significant wave height, mean water depth, wavelength, deep water wavelength, mean wave period, and surf similarity parameter in the following,

$$skewness = \tanh[1 + \frac{2.582}{\lambda/h} + \sqrt{|\xi - 0.207| + \sqrt{-0.22 + |\xi \tanh(U_r) + 2.527\lambda/h| + \frac{\lambda/h}{U_r^2}}} + |\frac{\lambda/h}{U_r} - 0.218|] (49)$$

where \( U_r \) is the Ursell number given by \( \frac{H_s \lambda^2}{h^2} \), \( H_s \) is significant wave height, \( h \) is mean water depth, \( \lambda \) is wavelength given by \( \lambda_0 \tanh(\frac{2\eta h}{\lambda_0}) \), \( \lambda_0 \) is deep water wave length given by \( \frac{g}{2\pi}T_m^2 \), \( T_m \) is mean wave period, and \( \xi \) is the surf similarity parameter given by \( \frac{\tan \beta}{\sqrt{H_s/\lambda_0}} \). The expression by Doering et al. (2000) for velocity skewness under shoaling and breaking waves was derived by fitting an analytic curve to available skewness data. The equation is validated by comparing it to field measurements for a broad range of hydrodynamic conditions including data from Terschelling, The Netherlands, Sandyduck97, and Duck94.

The velocity time series for the skewness model is calculated using the method by Elfrink et al. (2002) for calculating near bed orbital velocities under irregular waves. The method describes the time variation of near bed orbital velocities of individual waves in irregular wave trains (Elfrink et al., 2002). It is based on an evolutionary algorithm procedure which reduces the data set by calculating mean values of skewness for predefined classes of independent parameters. The method was developed with data mining techniques to derive expressions for orbital velocity parameters for individual waves in irregular wave trains.
3.1.2 Incorporation of Skewness in NearCoM Model

The original SHORECIRC code utilizes the generalized wave-averaged bottom shear stress for combined waves and currents which is written as,

\[
\bar{\tau}_b = \frac{1}{2} \rho \overline{(U + \bar{u}_w)} |U + \bar{u}_w|.
\]  

(50)

Linear wave theory is assumed for the linear bottom shear stress formulation for solving Equation (50) resulting in Equation (28) in Chapter 2. For the incorporation of skewness into the bottom shear stress formulation, Equation (50) is solved using a skewed wave velocity time series as opposed to a wave velocity calculated with Equation (50) using linear wave theory.

The present research utilizes the method by Elfrink et al. (2002) using the skewness determined by Doering et al. (2000) to calculate the orbital velocities. The code in SHORECIRC is modified to include a time-averaged bottom shear stress with skewed (or nonlinear) wave velocity terms. The skewed bottom orbital wave velocities computed by the method by Elfrink et al. (2002) are used in Equation (50) which is time-averaged numerically to produce a time-averaged shear stress. The resulting effects on hydrodynamic and longshore transport are explored in great detail in Sections 3.2 and 3.3.

3.2 Effect of Skewness on Hydrodynamics

The hydrodynamic results such as longshore currents and shear stress are graphically and qualitatively described for linear and skewed cases in this section. The inclusion of skewed wave velocities in the shear stress results in significant changes compared to the cases with linear wave theory wave velocities.

Cotton and Stansby (2000) studied the bed frictional characteristics in a turbulent flow driven by nonlinear waves. They found that the nonlinear wave friction factor for \( \frac{H}{\lambda} = 0.3 \) is 4 to 13 percent lower than the linear value. For \( \frac{H}{\lambda} = 0.6 \), the nonlinear friction factor value lies between 2 to 10 percent above the linear case. They also found that a lower wave power is associated with the nonlinear case for bed friction factor, but the nonlinear waves also produce a lower relative decrease in wave power \( \frac{\Delta E_I'}{E_I'} \).
In the present study, the skewed wave velocities and the skewness distributions are first compared to the linear cases. The effect the skewed wave velocities have on the mean and depth-varying currents is then presented.

3.2.1 Examples of Skewed Velocity and Skewness Distribution

The hydrodynamic properties of test cases using linear wave velocities (linear) and nonlinear wave velocities (skewed) in the shear stress are shown in Figure 7 for a wave height of 1.5 meters and wave angle of 6 degrees. The hydrodynamics for the linear and skewed cases are very similar. The waves, and hence the radiation stresses and wave volume flux, are identical therefore producing the same setup and cross-shore return flow. However, the longshore current shows differences between the linear and skewed cases. The longshore current in the linear case is stronger than the longshore current in the skewed case while maintaining the same shape.

![Graphs showing hydrodynamic conditions for linear and skewed cases](image)

**Figure 7:** Hydrodynamic conditions for linear (black) and skewed (red) cases with $H=1.5$ meters and angle=6 degrees. Overlap by red indicates identical hydrodynamics.
The difference in longshore current can be explained by examining the momentum balance. Figures 8(a) and 8(b) show a comparison between the case of linear and skewed momentum balances. In the cross-shore direction, the most dominant terms in the momentum balance are the $S_{xx}$ radiation stress gradient and the pressure gradient, $d\zeta/dx$. In the longshore direction, the most dominant terms in the momentum balance are the $S_{xy}$ radiation stress gradient and the bottom friction. The differences between the linear and skewed case look subtle on this scale, therefore the shear stress for the linear and skewed cases are compared in more detail in Figure 9. The shear stress in the cross-shore direction is changing substantially between the linear and skewed cases. However, the longshore shear stress changes minimally for the linear and skewed cases. It is interesting to note that the skewed shear stress for smaller wave heights such as 0.25 m is larger than the shear stress in the linear calculation. Skewed shear stresses remain larger than the shear stresses for all linear cases tested with a wave height of 0.25 m (tests included wave angles of 3, 6, 9, 12, and 15 degrees). Shear stresses for linear tests begin to exceed skewed shear stress for a wave height of 0.5 m and wave angle of 3 degrees.

The magnitude of the difference between the linear and skewed shear stresses for different wave heights and angles can be explained by understanding the time variations of the skewed velocities. Figure 10 shows the time-varying orbital velocity for a breaking wave and a nonbreaking wave for a linear case and a skewed case. It is evident from the figure that the linear case leads to opposite and equal magnitudes in the trough and the crest. However, the orbital velocity magnitude for the skewed case is not equal in the trough and the crest. The crest becomes more narrow and peaked while the trough becomes broader and flatter. Because orbital velocity is used in the shear stress formulation, the differences affect the shear stress and consequently the longshore sediment transport calculations.

Figure 11 shows a comparison of skewness magnitude for each wave height with a 6 degree wave angle. The total skewness values range from approximately -0.15 to 0.60. The effect of wave angle change on skewness magnitude is negligible, and therefore only wave height variations are explored. As wave height increases, peak skewness moves closer to the breaking point (i.e. $h/h_b = 1$) and the magnitude of skewness becomes larger. This
Figure 8: Momentum balances for $H=1.5$ m and angle=6 degrees showing x-momentum, y-momentum, and water depth for (a) linear and (b) skewed cases. The momentum plots include radiation stress gradients, 3D mixing terms, bottom shear stress, turbulent mixing, wave pressure, and convergence terms.

Figure 9: Shear stress for $H=1.5$ m and angle=6 degrees for the cross-shore direction (black) and longshore direction (red) for linear (solid) and skewed (dashed) cases.
Figure 10: Time-varying $u_w$ for linear (solid) and skewed (dashed) for nonbreaking (black) ($x=176$ m) and breaking waves (red) ($x=332$ m) with $H=1.5$ m and angle=6 degrees.

indicates that a wave becomes more nonlinear, or peakedness increases, with increasing wave height. The accuracy of the skewness is not independently verified in this study. However, because the method for computing the skewness comes from a data fit of an extensive data set with similar conditions to the present study, the skewness estimates are deemed fairly reliable.

3.2.2 Effect of Skewness on Mean and Depth-Varying Currents

The mean bottom current in the cross-shore and longshore directions, $u_b$ and $v_b$, are affected by the shear stress. Therefore, any shear stress differences caused by including the skewed velocities will affect the mean current. Figure 12 shows the cross-shore and longshore currents at the bed for the linear and skewed cases. When skewed velocities are added to shear stress, the cross-shore currents have larger magnitudes before breaking and similar magnitudes after breaking. For skewed velocities, the longshore currents are similar before breaking, and the longshore current becomes weaker after breaking.

To better understand what is causing the longshore bottom velocity to become more negative for the skewed case, the depth-varying currents are explored in more detail. The depth-varying currents for the linear case and skewed case are compared in Figure 13. The
**Figure 11:** Skewness comparison with angle=6 degrees and $H=0.25$ m (blue), 0.5 m (red), 1.0 m (green), 1.5 m (cyan), 2.0 m (magenta), 2.5 m (black), and 3 m (orange).

**Figure 12:** Cross-shore variation of mean bottom cross-shore (black) and longshore (red) currents for linear (solid) and skewed (dashed) cases for $H=1.5$ m and angle=3 degrees.
ability to conduct the depth-varying analysis highlights the advantages of the 3-dimensional attributes of SHORECIRC. In this figure, the depth-varying cross-shore and longshore currents are shown at two different cross-shore positions. The top panel represents the velocities offshore and the bottom panel represents the velocities in the surf zone. The depth mean undertow, \( u_m \), must be the same for the linear and skewed cases because the volume flux, \( Q_w \), must be the same as given by,

\[
u_m = \frac{-Q_w}{h}.
\]

However, the velocity is different at the bottom of the vertical profile. The change is related to the change in shear stress as seen in Figure 9. The solution for the velocity profile requires a balance in the curvature of the depth-varying velocity with the bottom shear stress to satisfy the bottom boundary condition (Haas and Svendsen, 2000),

\[
\nu \frac{\partial u}{\partial z} \bigg|_{z = -h_o} = \tau_b.
\]

Hence, the bottom velocity for the skewed case is larger at the bottom because it has less curvature than the linear case due to weaker shear stresses at the offshore positions. As the position moves inside the surf zone, the linear and skewed profiles are very similar. This similarity in cross-shore velocities in the surf zone is reflected by the similarity for the shear stresses in Figure 9.

For the longshore depth-varying currents, the linear and skewed profiles are similar until inside the surf zone. Inside the surf zone, the magnitudes of the current are different, but the curvature is almost identical. The similar curvature is because the radiation stress gradient is the same for each case leading to similar shear stress for each case (as verified by Figure 9). Because the shear stress is similar, the curvature of the lines must be similar as given by the bottom boundary condition in Equation (52).

The change of depth-mean longshore current in the surf zone is therefore contributing to the change in the mean bottom current. The orbital velocity part of the shear stress for the skewed case is enhanced by the skewness terms, and because the overall magnitude of the shear stress is the same for the linear and skewed cases, the longshore velocity must
Figure 13: Cross-shore (left) and longshore (right) depth-varying velocities for H=1.5 m and angle of 6 degrees for linear (black) and skewed (red) cases showing the total velocity.

decrease to maintain a constant shear stress magnitude. The decrease of the longshore velocity magnitude is indicated by the leftward shift of the velocity profile in Figure 13.

3.3 Effect of Skewness on Longshore Transport

In this section, the total longshore sediment transport is compared for the cases with linear wave velocities in the shear stress and transport (linear) and skewed wave velocities in the shear stress and transport (skewed). To better understand why the total sediment transport is changing, the components of transport are compared. Descriptions of the sediment transport equations used for the comparison are found in Chapter 2.

3.3.1 Overview of Change in Transport

It has been determined that the hydrodynamics are changed when skewed velocities are used in the shear stress calculations. Therefore, a change in the longshore sediment transport is expected because the transport is a function of the hydrodynamics. In addition,
incorporating the skewed velocity directly into the transport will make a difference as well. To first verify this assumption, the cross-shore variations of the longshore transport are investigated. The sediment transport is divided into advection by the waves and current in the HH longshore sediment transport equation, as seen in Chapter 2, and restated here,

\[ q_y = \frac{2C_1}{\rho g} \left[ \tau_{eff} \frac{\partial u_{wy}}{\partial y} + \tau_{eff} V_y \right]. \] (53)

The \( \tau_{eff} \frac{\partial u_{wy}}{\partial y} \) term represents the advection due to waves, and the \( \tau_{eff} V_y \) term represents the advection due to the mean current.

After the cross-shore variations of longshore transport are found, the total longshore transport is determined by integrating the cross-shore varying transport across the domain. Section 3.3.1.2 reviews the CERC formula and compares the total transport determined by the HH, BBB, R, and W sediment transport formulas for both the linear and skewed cases.

### 3.3.1.1 Cross-Shore Variations of Transport

The variation of longshore sediment transport due to advection by waves is shown in Figures 14 and 15. For the transport due to advection by waves, the linear case always produces a negative sediment transport. For small wave heights seen in Figure 14, the skewed cases result in a larger negative transport than the linear cases close to shore. For variations in wave angle seen in Figure 15, the magnitude of transport becomes larger as wave angle increases. For the linear case, the transport becomes more negative. For the skewed case the transport becomes more positive. In the offshore region, the skewed cases have a positive transport while the linear cases are always negative.

Figures 16 and 17 show the longshore transport due to advection by mean current for each wave height and wave angle combination. The sediment transport due to advection by waves is approximately an order of magnitude smaller than the advection by mean current for each case. For the longshore transport due to advection by the mean current, the small and medium wave heights result in smaller transport for the skewed case than the linear case before breaking, but the skewed case has transport larger than the linear case after breaking. The longshore transport due to mean current increases with an increase in wave angle, and the linear and skewed cases react similarly.
**Figure 14:** Cross-shore variation of longshore transport due to waves with angle=6 degrees and \( H=0.25 \) (blue), \( H=1.5 \) m (red), \( H=3 \) m (black) for the linear case (solid) and skewed case (dashed).

**Figure 15:** Cross-shore variation of longshore transport due to waves for \( H=1.5 \) m and angle=3 degrees (blue), angle=9 degrees (red), angle=15 degrees (black) for the linear case (solid) and skewed case (dashed).
Figure 16: Cross-shore variation of longshore transport due to mean current with angle=6 degrees, $H=0.25$ (blue), $H=1.5$ m (red), $H=3$ m (black) for the linear case (solid) and skewed case (dashed).

Figure 17: Cross-shore variation of longshore transport due to mean current for $H=1.5$ m and angle=3 degrees (blue), angle=9 degrees (red), angle=15 degrees (black) for linear case (solid) and skewed case (dashed).
Figure 18: Cross-shore variation of total longshore transport with angle=6 degrees, H=0.25 (blue), H=1.5 m (red), H=3 m (black) for the linear case (solid) and skewed case (dashed).

Figures 18 and 19 show the total longshore transport for varying wave height and wave angle combinations respectively. The total transport is the combination of the advection by mean current and waves. The same trends that were evident for the sediment transport due to the mean current occur for the total transport because the mean current advection contributes more to transport than the advection by waves.

3.3.1.2 Skewness Effects on Total Transport

The total transport for each case is found by integrating the longshore sediment transport across the nearshore region. Figures 20 through 27 show log-log plots of the total transport for each wave height and wave angle combination for the linear and skewed cases for each transport method. The log-log plots show the longshore component of wave power, $P_l$, on the x-axis and total transport in m$^3$/day on the y-axis. These plots represent total transport from the two explored cases: 1) linear wave velocities in the shear stress and transport (linear) and 2) skewed wave velocities in the shear stress and transport (skewed).

The log-log plots show the model predictions, the CERC formula, and experimental data as tabulated by Komar (1998). The experimental data are collected from experiments involving sand accumulation at jetties and breakwaters, sand tracers, and sediment traps.
Figure 19: Cross-shore variation of total longshore transport for $H=1.5$ m with angle=3 degrees (blue), angle=9 degrees (red), angle=15 degrees (black) for the linear case (solid) and skewed case (dashed).

Experimental data for sand accumulation is used from work by Watts (1953), Caldwell (1956), Dean and Seymour (1987), Moore and Cole (1960), Bruno et al. (1981b), Walton and Bruno (1989), and Kamphuis (1991). Experimental data for sand tracer studies is used from work by Komar and Inman (1970), Inman et al. (1980), and Kraus et al. (1982). Experimental data collected by sediment traps is used from work by Lee (1975) and Kana and Ward (1980).

A best fit line to the model output is found by fitting the equation,

$$I = Q_d / \rho (s - 1) g (1 - p) = KP_i^N$$  \hspace{1cm} (54)

to the modeled data in a least squares sense to find optimal values of $K$ and $N$. For the log-log plots the power $N$ corresponds to the slope of the best fit line for the model output. Alternatively, $N$ is constrained to be 1.0 similar to the CERC formula and a ratio $R$ is defined as,

$$R = \frac{K_f P_i}{I}$$  \hspace{1cm} (55)

where $I$ is the immersed weight transport and $P_i$ is the longshore wave power from the model. $K_f$ is found by minimizing the deviations of this ratio from a value of 1.0 in a least
squares sense. Differences in $K_f$ values are a direct indicator of changes in total longshore transport.

The value of the variance of $R$ is indicative of the deviation of the model output from the best fit line. A variance value of zero indicates that the standard deviation of the data from the best fit line is zero. An increase in variance value indicates that there is more spread in the data series.

The term 'Equivalent $K$' is used later to compare model output to the CERC $K$ value. The equivalent $K$ value is calculated according the following,

$$K = \frac{Q_l \rho (s - 1) g (1 - p)}{P_1}$$

and is calculated using the model output from specific wave conditions. $K$ and $K_f$ differ from the equivalent $K$ because they take into account transport from each wave height and angle combination, while the equivalent $K$ calculates the transport for a specific wave condition only.

The properties of the best fit lines from the model results are as shown in Table 5. The slope value, $N$, in Table 5 provides a measure of the proximity of the analytical formula prediction (HH, BBB, R, W) to the CERC formula power of 1.0. Table 5 and Figures 20 through 27 show that the total transport for each analytical sediment transport equation predicts higher total sediment transport for the skewed case. The addition of the skewed velocities in the shear stress and transport calculations increases the equivalent $K_f$ value for HH by 18.9 percent, BBB by 23.2 percent, R by 26.0 percent, and W by 67.4 percent. The magnitude of increase in longshore sediment transport by adding skewed terms to each of the formulas highlights the importance of accounting for higher order terms affecting the wave shape. The variance is closest to zero for the linear BBB case indicating that the scatter in the model output is the lowest. The highest variance in the data is for the skewed Watanabe case. There is an increase in variance for the skewed cases for each formula except the HH formula.
Figure 20: Total longshore transport for linear case for HH formula.

Figure 21: Total longshore transport for skewed case for HH formula.
Figure 22: Total longshore transport for linear case for BBB formula.

Figure 23: Total longshore for skewed case transport for BBB formula.
Figure 24: Total longshore transport for linear case for R formula.

Figure 25: Total longshore transport for skewed case for R formula.
Figure 26: Total longshore transport for linear case for W formula.

Figure 27: Total longshore transport for skewed case for W formula.
Table 5: Slopes for transport formulas HH, BBB, R, and W for linear and skewed cases where $N$ is the slope of the best fit line through the model results from Equation (54), and $K$ is the equivalent coefficient for the best fit line through the model results.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Case</th>
<th>$N$</th>
<th>$K_f$</th>
<th>$K$</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>Linear</td>
<td>0.953</td>
<td>0.787</td>
<td>0.610</td>
<td>0.041</td>
</tr>
<tr>
<td>HH</td>
<td>Skewed</td>
<td>0.965</td>
<td>0.971</td>
<td>0.800</td>
<td>0.032</td>
</tr>
<tr>
<td>BBB</td>
<td>Linear</td>
<td>1.023</td>
<td>0.484</td>
<td>0.547</td>
<td>0.020</td>
</tr>
<tr>
<td>BBB</td>
<td>Skewed</td>
<td>1.058</td>
<td>0.630</td>
<td>0.860</td>
<td>0.079</td>
</tr>
<tr>
<td>R</td>
<td>Linear</td>
<td>1.002</td>
<td>0.386</td>
<td>0.391</td>
<td>0.021</td>
</tr>
<tr>
<td>R</td>
<td>Skewed</td>
<td>1.016</td>
<td>0.522</td>
<td>0.568</td>
<td>0.033</td>
</tr>
<tr>
<td>W</td>
<td>Linear</td>
<td>0.959</td>
<td>3.180</td>
<td>2.543</td>
<td>0.027</td>
</tr>
<tr>
<td>W</td>
<td>Skewed</td>
<td>0.782</td>
<td>9.740</td>
<td>2.915</td>
<td>0.297</td>
</tr>
</tbody>
</table>

3.3.2 Causes for Transport Changes

It has been shown that by including skewness terms in the shear stress and transport terms, there is a significant effect on longshore sediment transport. It has also been determined that both transport due to advection by waves and mean current in the HH formulation are significantly affected. The analysis of the transport terms uses an additional combination of skewness terms. As previously discussed, the linear case describes linear shear stress and linear transport terms and the skewed case describes skewed velocities included in the shear stress and the transport terms. An additional case is referred to as the skewed transport only case and describes skewed velocity time series included in the transport calculations but not in the shear stress calculations in the circulation module. The addition of the new case facilitates discussion and helps decipher the changes in hydrodynamics and changes in the transport.

3.3.2.1 Advection by Current

The advection by the mean current is proportional to $\tau_{eff} V_y$. Figure 28(a) shows the cross-shore variation of effective shear stress, and Figure 28(b) shows the longshore bottom velocity for the linear and skewed cases. The figures are designed to clarify how the effective shear stress, $\tau_{eff}$, is increasing while the longshore bottom velocity, $V_b$, is decreasing for the skewed case. It can be concluded that using skewed velocities in the shear stress and
transport term decreases the longshore bottom velocity and increases the effective shear stress. The combination of the two terms results in a higher longshore sediment transport for the skewed case. Figure 29(a) shows the longshore transport due to advection by the mean current, and Figure 29(b) shows the product of $\tau_{eff}$ and $V_b$ which is proportional to advection by mean current term. The figures show that the linear case leads to the smallest transport after wave breaking ($h/h_b=1$). The skewed transport only case produces the highest transport across the entire domain. When the skewed orbital velocities are added to the the shear stress as well as the transport (i.e. the skewed case), the transport decreases.

By comparing Figure 28(b) and Figure 29(a), it can be seen that the difference in longshore transport due to advection by mean current between the skewed and skewed transport only cases is due to the change in the longshore bottom velocity since $\tau_{eff}$ is the same. The transport for the skewed case is smaller than the linear case before breaking because the effective shear stress and longshore bottom velocity are smaller. The transport for the skewed case is larger than the linear case after and at breaking because the effect of the increase in effective shear stress is smaller than the effect of decreasing the longshore bottom velocity.

3.3.2.2 Advection by Waves

The $\tau_{eff} u_{w}$ term in the HH transport formula is proportional to the transport due to wave advection. Figure 30 shows the HH longshore sediment transport due to advection by waves for the linear, skewed transport only, and skewed cases. The skewed transport only term results in the largest amount of longshore transport due to advection by the waves as seen in Figure 30. When the skewed time series is included in the shear stress and the transport (skewed case), the longshore transport due to the waves becomes smaller than the skewed transport only case. Although the transport is smaller than the skewed transport only case, it is still larger than the linear case which is negative everywhere.

The time-varying effective shear stress and orbital velocity are important in explaining the onshore/offshore differences in the wave transport. As seen in previous sections, the
Figure 28: (a) Cross-shore variation of effective shear stress and (b) longshore bottom velocity for linear case (solid black), skewed transport only (red), and skewed case (dashed black) for $H=1.5$ m and angle=6 degrees. In panel (b) the skewed transport only case falls on the linear case.

Figure 29: (a) Longshore transport due to advection by mean current and (b) product of $V_0$ and $\tau_{eff}$ for linear case (blue), skewed transport only case (black), and skewed case (red) for $H=1.5$ m and angle=6 degrees.
orbital velocity is a part of the effective shear stress and the effective shear stress is part of the sediment transport calculations as seen in Equation (53).

Figures 31(a) and 31(b) show the time-varying orbital velocity and effective shear stress for the offshore position. Figures 32(a) and 32(b) show the time-varying orbital velocity and effective shear stress for the position inside the surf zone. For both of the cross-shore locations shown in Figures 31(a) and 32(a) the effective shear stress for the linear case is larger in the trough of the wave. For the skewed case, the effective shear stress is larger at the crest of the wave for the offshore case and is larger at the trough for the surf zone case.

To better understand the cause for the differences in crest and trough magnitudes, the way the individual velocities add up vectorially needs to be understood. Graphical vector addition is used to demonstrate the differences between the effective shear stress in the crest and trough for the linear and skewed cases in Figure 33. For the linear case, the orbital velocity is equal in the crest (positive) and the trough (negative) as indicated by the red vectors. The longshore current is then added to the orbital velocity in the positive direction and is indicated by the blue $V_y$ vector. The cross-shore current (or undertow) is added in the negative direction and is indicated by the green $V_x$ vector. The resultant
Figure 31: Offshore (x=176 m) time-varying effective shear stress (green) and orbital velocity (blue) for (a) linear case and (b) skewed case for H=1.5 m and angle=6 degrees.

The vector addition illustration for the skewed case is shown on the right-hand side of Figure 33. For the skewed case, the orbital velocity at the crest is larger than the trough as indicated by the $u_w$ red vectors. The $V_x$ and $V_y$ vectors are the same as the linear case. The resultant total velocity vector, and hence the effective shear stress vector is therefore larger in the crest than in the trough. For the breaking waves, effective shear stress in the crest is not larger than the trough because the magnitude of the orbital velocity is more similar in the trough and crest.

The transport due to waves is calculated based on the product of the magnitude of the effective shear stress and the wave orbital velocity. The direction of the transport is governed by the direction of the orbital velocity, positive under the crest and negative under the trough. For the linear case the time varying magnitude of the transport is governed only by the magnitude of the effective shear stress since the orbital velocity is symmetric. Therefore, the transport is larger under the trough than under the crest since the effective shear stress is larger under the trough. The time averaged transport will then be in the
Figure 32: Inside surf zone ($x=332$ m) time-varying effective shear stress (green) and orbital velocity (blue) for $H=1.5$ m and angle=6 degrees for (a) linear case and (b) skewed case.

Figure 33: Vector addition analysis for effective shear stress for linear case (left) and skewed case (right).
negative direction since the transport is larger under the trough and is in the negative direction. For the skewed case, since the magnitude of the orbital velocity is not equal under the crest and trough, the magnitude of the transport is proportional to both the velocity and the effective stress. Hence the direction of the time averaged transport is generally positive for the regions where the effective shear stress is larger under the crest, and negative where the effective shear stress is larger under the trough.

3.4 Summary

This chapter investigates the effect of wave shape on hydrodynamics and longshore sediment transport using linear wave theory solutions and solutions with skewed orbital velocity time series. The skewed velocity time series lead to a smaller longshore velocity than the linear case, but the effective shear stress is higher than the linear case. An analysis of depth-varying currents shows the cause for differences between the cross-shore and longshore currents for the linear and skewed cases. The combination of a lower longshore velocity and higher effective shear stress ultimately leads to higher longshore sediment transport.

Overall, more longshore sediment transport is caused by advection due to mean current than by advection due to waves. The changes in the advection components are a result of a combination of the changes in the longshore bottom velocity and the orbital velocity. For a range of wave heights and wave angles, the skewed case results in more longshore transport than the linear case. Total transport is calculated for the HH, BBB, R, and W models and it is shown that larger total longshore transport is predicted for the skewed case for each analytical formula.

The reasons for the differences are explored through an analysis of the time-varying orbital velocity and effective shear stress. The analysis of the individual transport terms for advection by waves shows how the orbital velocity in the crest and the trough combine with the mean current to result in differences in the effective shear stress which contribute to the transport changes.
CHAPTER IV

SEDIMENT TRANSPORT WITH A SPECTRAL WAVE DRIVER

The NearCoM model allows the user to select between wave driver modules, and this chapter focuses on the resulting hydrodynamics and longshore sediment transport from monochromatic and spectral wave drivers. The monochromatic wave driver describes a single wave frequency and direction, and the spectral wave driver utilizes a spectrum of frequencies and directions to more accurately simulate a random sea. The monochromatic and spectral wave driver explored in the present research are REF/DIF 1 and REF/DIF S respectively.

4.1 Spectral Wave Driver Background

The spectral wave driver, REF/DIF S, is coupled to the NearCoM Program to compare the hydrodynamic and resulting longshore sediment transport to those of the monochromatic wave driver, REF/DIF 1. The short wave drivers determine the short wave motions and calculate the forcing that drives the currents. As described in Chapter 2, the REF/DIF 1 wave driver is based on the parabolic approximation of the mild-slope equation, and accounts for refraction, diffraction, shoaling, and breaking phenomena. The monochromatic wave driver, REF/DIF 1, assumes a roller by Svendsen (1984) and a decay model by Dally et al. (1985). The use of spectral wave drivers is preferred to monochromatic wave drivers because a monochromatic wave driver only incorporates short wave motion defined by one frequency and a single breaking wave height which is not realistic for a natural beach (Svendsen et al., 2002). The spectral wave models utilize a spectrum of frequencies which more accurately represents a random sea.

The spectral wave driver, REF/DIF S, uses a distribution of wave heights and frequencies to simulate a random sea, REF/DIF S uses a wave breaking model by Thornton and Guza (1983) to compute statistical wave breaking properties for each spatial step. Statistical
properties include percentages of breaking waves and dominant wave frequency (i.e. $1/T$). Thornton and Guza (1983) found that the Rayleigh distribution effectively describes the random nature of wave heights in a single-parameter transformation model based on energy flux balance.

Thornton and Guza (1983) analyzed prior methods for the transformation of wave height distribution, and they concluded that:

- The use of monochromatic, nonlinear theories to represent shoaling of random waves is unjustifiable.

- The model by Battjes and Janssen (1978) does not accurately represent the measured wave height probability density function (pdf).

The model developed by Thornton and Guza (1983) expands upon the model by Battjes and Janssen (1978) by more accurately describing the transformation of wave height probability density functions, root-mean-square (RMS) wave height, and by considering bottom friction in the dissipation function.

REF/DIF S is a weakly nonlinear model which combines refraction and diffraction to simulate the behavior of random waves over irregular bottom bathymetry. The spectral model incorporates the effects of shoaling, refraction, energy dissipation, and diffraction (Kirby and Ozkan, 1994). The REF/DIF S model is incorporated into the NearCoM model and provides the wave-averaged properties necessary to drive the circulation module (Kaihatu, 2001). The REF/DIF S model also allows for a choice of roller models.

Roller models explain the volume of water which rushes down the face of a breaking wave. A roller is used to represent the front face of a wave feature where the water is tumbling down and the area of roller is carried by the wave at the velocity of the wave as shown in Figure 34. The incorporation of the roller is important because it can influence longshore currents, therefore influencing the longshore sediment transport. Osiecki and Dally (1996) found that the presence of a roller increased longshore currents. The inclusion of rollers increases the radiation stress due the the additional force exerted on the water column surface. The roller properties are also used to modify the short-wave mass flux for
the model (Kaihatu, 2001).

The roller model by Svendsen (1984) defines this volume of water as a detached body of fluid which is on the face of the wave and separated from the actual wave form. The inclusion of the roller model is important for nearshore hydrodynamics. Roller dissipation is calculated from the balance of energy flux in the surf zone (Lippmann et al., 1996). The roller model by Stive and DeVriend (1994) is based on a dynamic description where the roller change is described by an ordinary differential equation which has a memory of conditions upstream of the calculation point (Kaihatu, 2001). The effects of the different roller models coupled with different decay models is discussed in the summary.

The local roller description by Lippmann et al. (1996) is developed for randomly breaking waves over arbitrary bathymetry. The model uses the roller theory by Svendsen (1984) to specify the dissipation function, and includes roller energy gradients in the energy flux balance. The root-mean-square wave heights are found using numerical iteration across the surf zone, and the model's free parameters include the mean angle to vertical of the wave/roller interface and the breaking wave saturation value. Both free parameters fittings are consistent with field data (Lippmann et al., 1996).

REF/DIF S can be set up with a combination of roller and decay choices. Three different roller and roller model combinations can be used with the spectral wave driver.

- Spectral Case 1: Thornton and Guza (1983) decay with no roller effects
• Spectral Case 3: Evolving roller by Stive and DeVriend (1994), Battjes and Janssen (1978) decay

Each spectral case is run for the suite of tests listed in Section 2.5. The following standard input parameters are used for the spectral wave driver set ups (with the exception of the sensitivity analysis in Section 4.7):

• Linear dispersion relationship,

• Open initial lateral boundary condition,

• Peak frequency=0.1 Hz (H=0.5,1.0,1.5,2.0,2.5,3.0 m) Peak frequency=0.25 Hz (H=0.1,0.25 m),

• Max spectral frequency=5 Hz,

• Peak enhancement factor (PF)= 10; Produces a narrow or broad frequency spectrum (i.e. 1=broad, 20=narrow),

• Number of frequency components= 50,

• Directional Spreading factor (SF)=10; Produces a narrow or broad directional spreading (i.e. 5=narrow, 20=broad (Kaihatu, 2001)),

• Number of direction components=50,

• Gamma=0.6; Thornton and Guza (1983) decay mechanism parameter,

• b=1; Thornton and Guza (1983) decay mechanism parameter, and

• $\sigma_{\text{decay}}=10$; Interface angle between the roller and the wave.

4.2 Matching Wave Conditions for Monochromatic and Spectral Wave Drivers

The monochromatic and spectral model runs are compared by matching RMS wave heights at the offshore boundary. Because the wave heights are matched at the offshore boundary, the breaking wave heights are not exactly equivalent. Considering that much sediment
transport is the result of breaking waves, it is prudent to compare this to the transport resulting from matching the wave conditions at breaking. The effect of the different matching criteria is explored in this section. The resulting longshore sediment transport is compared for the two wave height matching criteria. Matching the wave heights at the offshore boundary is ultimately chosen as the matching requirement.

Figure 35 shows the wave height distributions for offshore matching versus matching the wave heights at breaking for the monochromatic and spectral wave drivers. Matching the RMS wave heights at breaking causes a significant difference in offshore matching, or deep water wave conditions. Figure 36 shows the longshore sediment as computed with the HH model transport resulting from the different wave matching conditions. By matching the wave heights at breaking, the monochromatic and spectral wave drivers result in more similar total longshore transport. Although the wave match at breaking results in more similar transport magnitudes, matching wave heights at the offshore boundary is the equivalent of matching deep water wave conditions. The wave drivers are fundamentally different in the way wave heights are treated as they move to the nearshore region, and the wave drivers are better compared with the same conditions at the offshore boundary. Most of the analysis is performed based on the actual breaking wave height so the differences are accounted for in the HH, BBB, R, and W sediment transport changes between the monochromatic and spectral wave driver as discussed further in Section 4.5.

4.3 Hydrodynamic Differences Between Monochromatic and Spectral Wave Drivers

This section explores the differences in hydrodynamic conditions between the monochromatic and spectral wave drivers. The hydrodynamics which are compared include wave height, longshore currents, x-momentum wave flux in the onshore direction, and the x-momentum wave flux in the y-direction (measure of obliquity of wave). This section also studies the radiation stress calculation methods used by each wave driver.
Figure 35: Wave height distribution for matching monochromatic offshore (solid), matching monochromatic at breaking (dashed), and spectral (dash-dot) for H=1.5 m and angle=6 degrees.

Figure 36: Longshore transport with HH for H=1.5 m and angle=6 degrees for offshore matching wave height (solid), breaking wave height match (dashed), and spectral case (dash-dot).
4.3.1 Wave Driver Comparison for Wave Heights

For each incoming wave height, the shape of each profile and relative wave height differences between the different drivers for each case are similar. As seen in Figure 37, the monochromatic wave has the highest wave height until the breaking point, and then the spectral wave drivers have the highest wave heights closer to shore. Spectral cases 1 and 2 are identical for each wave height and wave angle because they use the same breaking model. Spectral Case 3 has the smallest wave height until the wave comes very close to the shoreline. This indicates that the decay model by Battjes and Janssen (1978) or the evolving roller by Stive and DeVriend (1994) creates these differences.

It can be seen from the wave heights plotted in Figure 37 that the monochromatic wave case peaks high above the spectral wave drivers, but then decreases more rapidly than the spectral models. The spectral wave drivers lead to a more gradual progression of the waves which is closer to reality on natural beaches. The monochromatic wave driver produces a fixed break point for the waves, while the spectral wave driver produces a spectrum of breaking wave heights which do not all break at the same point. The graduated breaking in the spectral wave driver produces a more smoothed effect for the wave height distribution.

4.3.2 Wave Driver Comparison for Radiation Stresses

Radiation stresses from both wave drivers are compared in this section. Radiation stresses are important because it is the momentum flux of the waves that controls wave setup and setdown and drives the currents. The wave momentum fluxes are proportional to wave energy, and the momentum flux must be balanced in the surf zone by forces acting in the opposite direction. $S_{xx}$ is the x-momentum flux in the onshore direction, $S_{xy}$ is the x-momentum flux in the y-direction, and $S_{yy}$ is the y-momentum in the longshore direction (Dean and Dalrymple, 2002).

In the monochromatic wave driver, REF/DIF 1, the radiation stresses outside the surf zone are calculated by,

$$S_m = \int_{-h_0}^{h} \rho u^2_\eta dz,$$  \hfill (57)
Figure 37: Wave height comparison for H=1.0 m and wave angle=6 degrees for Spectral Case 1 and 2 (dashed), Spectral Case 3 (dot-dash), and monochromatic (solid).

and

\[ S_p = -\int_{-h_0}^{\zeta} \rho u_w^2 dz + \frac{1}{2} \rho g \zeta^2 \]  \hspace{1cm} (58)

where \( u_w = |u_w| \). \( S_{xx} \), \( S_{xy} \), and \( S_{yy} \) are then given by,

\[ S_{xx} = S_m \cos^2 \alpha + S_p, \]  \hspace{1cm} (59)

\[ S_{xy} = S_m \cos \alpha \sin \alpha, \]  \hspace{1cm} (60)

and

\[ S_{yy} = S_m \sin^2 \alpha + S_p. \]  \hspace{1cm} (61)

\( S_m \) represents the wave force due to momentum in the direction of wave propagation, and \( S_p \) represents the wave force due to pressure.

In the spectral wave driver, REF/DIF S, Kaihatu (2001) shows that the radiation stresses are calculated from a summation and average of individual wave components by,

\[ S_{xx} = \frac{1}{2} \rho g \sum_{n=1}^{N} |A_n|^2 \left[ \frac{C_{p,n}}{C_n} \left( 1 + \cos^2 \theta_n \right) - \frac{1}{2} \right], \]  \hspace{1cm} (62)
\[ S_{yy} = \frac{1}{2} \rho g \sum_{n=1}^{N} \left| A_n \right|^2 \left[ \frac{C_{g,n}}{C_n} (1 + \sin^2 \theta_n) - \frac{1}{2} \right], \]  

and

\[ S_{xy} = \frac{1}{4} \rho g \sum_{n=1}^{N} \frac{C_{g,n}}{C_n} \left| A_n \right|^2 (\sin 2\theta_n). \]

where \( A_n \) is the complex wave amplitude. The gradient in radiation stresses drives the currents and since \( \frac{\partial S_{yy}}{\partial y} = 0 \) for the cases presented here, \( S_{yy} \) will not be examined further.

Figure 38 shows the \( S_{xy} \) radiation stresses for each REF/DIF S case. The radiation stress is the highest for Case 3 and smallest for Case 1 which indicates that the roller is increasing the x-momentum flux in the alongshore direction. Figures 39 and 40 show the effect of variation of wave angle on the \( S_{xx} \) and \( S_{xy} \) radiation stresses respectively. The radiation stresses for the monochromatic and spectral cases are shown on the same graph. The \( S_{xy} \) radiation stress increases with increasing wave angle for both wave drivers, and \( S_{xx} \) decreases slightly with increasing wave angle as expected. With increasing wave angle, radiation stress is stronger in the longshore direction, therefore increasing x- and y- momentum in the longshore direction and decreasing x-momentum in the cross-shore direction. The angle change most greatly affects the magnitude of the \( S_{xy} \) radiation stress, and there is a greater resulting radiation stress change for increasing wave angle for the monochromatic wave driver.

Figures 41 and 42 show the effect of variation of wave height on radiation stresses. Again, the monochromatic and spectral wave driver results are included on the same plot for the full range of wave heights. Although the radiation stress magnitudes are slightly different for the monochromatic and spectral wave drivers, the percentage of radiation stress change is similar for increases in wave height.

For \( S_{xx} \) and \( S_{xy} \) the higher wave height results in higher radiation stresses which is expected because a larger wave generates more momentum flux since radiation stress is proportional to \( H^2 \). The monochromatic test case results in the highest radiation stresses in each direction. The radiation stress for the monochromatic case is slightly larger because a single frequency wave is used and not a spectrum. Therefore the stress is more concentrated at each specific point. For the spectral cases, it can be concluded that the Spectral Case 3
Figure 38: $S_{xy}$ radiation stresses with $H=1.0$ m and wave angle=6 degrees for Spectral Case 1 (black), Spectral Case 2 (blue), and Spectral Case 3 (red).

leads to the largest x-momentum flux in the alongshore direction owing to the addition of the roller.

The radiation stresses display differences between the monochromatic and spectral wave drivers. This difference arises from the calculation of the radiation stresses within REF/DIF 1 versus REF/DIF S (as seen in Equations (57) through (64)). The differences in radiation stress magnitudes between the monochromatic and spectral wave driver cases require further investigation of the radiation stress calculation methods. The spectral wave driver code is manipulated to use the root-mean-square wave height in Equations (57) through (60) as opposed to the summation of each wave component property in Equations (62) through (64). All spectral cases are run with local roller dissipation by Svendsen (1984) and Lippmann et al. (1996), Thornton and Guza decay, and roller effects modifying radiation stresses.

Comparisons of the monochromatic, spectral, and modified spectral wave driver radiation stress variations are shown in Figures 43 and 44 for a range of wave heights. The resulting variations for the currents and longshore sediment transport are compared without changing any coefficients. The longshore sediment transport is calculated using the HH formula. Comparisons are shown for wave heights of 0.5 m, 1.5 m, and 3 m with a 6 degree
Figure 39: Angle variation effects on $S_{xx}$ for skewed case with monochromatic (solid) and spectral wave driver (dashed) for H=1 m with angle of 3 degrees (red), 9 degrees (blue), 15 degrees (green).

Figure 40: Angle variation effects on $S_{xy}$ for skewed case with monochromatic (solid) and spectral wave driver (dashed) for H=1 m with angle of 3 degrees (red), 9 degrees (blue), 15 degrees (green).
Figure 41: Wave height variation effects on $S_{xy}$ for skewed case with monochromatic (solid) and spectral wave driver (dashed) with angle=6 degrees with wave height 0.5 m (red), 1.0 m (green), 1.5 m (black), 2.0 m (magenta), 2.5 m (blue), 3.0 m (cyan). The x-axis is varying for each case because the domain size is changing.

Figure 42: Wave height variation effects on $S_{xy}$ for skewed case with monochromatic (solid) and spectral wave driver (dashed) with angle=6 degrees with wave height 0.5 m (red), 1.0 m (green), 1.5 m (black), 2.0 m (magenta), 2.5 m (blue), 3.0 m (cyan). The x-axis is varying for each case because the domain size is changing.
Figure 43: Radiation stresses for monochromatic (black), spectral (blue), and $H_{rms}$ spectral (red) for $S_{xx}$ (solid), $S_{xy}$ (dashed) for $H=0.5$ m and angle=6 degrees.

incoming wave angle. Figure 45(a) shows the longshore currents resulting from the radiation stress calculation change in the wave drivers. Finally, Figure 45(b) shows the resulting longshore sediment transport resulting from the radiation stress calculation change. The figures similar in transport regardless of the change in radiation stress calculation method. Using the $H_{rms}$ value to calculate radiation stresses for the spectral model has minimal effects on the longshore current, and consequently the longshore transport. However, the monochromatic wave driver produces slightly larger currents and transport due to the larger RMS wave height.

Feddersen (2004) compared the true radiation stress measured by a current meter or directional wave buoy to the radiation stress predicted from a narrow-banded approximation. He found that the narrow-banded approximation systematically predicted higher $S_{xx}$ and $S_{xy}$ radiation stress components. For the present research, the narrow-banded spectral wave driver closely approximates the monochromatic case, and is characterized by higher radiation stresses than the broad-banded spectral case as seen in Figure 43.
4.4 Longshore Sediment Transport Calibration for HH Longshore Sediment Transport for Spectral Waves

It is apparent from Figure 45(b) that the HH longshore sediment transport leads to smaller transport when used with a spectral wave driver versus a monochromatic wave driver. The HH formula is given by,

\[ q_{HH} = \frac{C_1 f_w}{g} |\bar{u}^2| \bar{u}(t). \]  

(65)

For the monochromatic wave driver, the \(C_1\) calibration constant is equal to 1. In order to bring the spectral wave induced longshore sediment transport to the same magnitude of empirical formulas and data measurements, the calibration constant, \(C_1\) must be equal to 1.3 for the HH model. The following figures of total transport, Figures 46 and 47, show transport with \(C_1=1\) and \(C_1=1.3\) respectively. Clearly changing \(C_2\) to be equal to 1.3 brings the model predictions closer in line to the CERC formula prediction. All further HH longshore sediment transport analysis will utilize a calibration constant of \(C_1=1.3\) for the spectral wave driver.

The HH transport model coefficients resemble those found by the model by Briand and
Figure 45: (a) Longshore current comparison and (b) longshore transport comparison for monochromatic (solid), spectral (dashed), and $H_{rms}$ spectral (dash-dot) $H=1.5$ m and angle=6 degrees.

Kamphuis (1993) which was tested for regular and irregular wave fields for a variety of wave conditions and sand sizes. The model finds reference concentration and total swash sediment transport. The model requires different calibration constants depending on whether the waves are regular or random. The calibration coefficient, $K_u$, is 5.0 for regular waves and 10.0 for random waves, and $K_Q$ is 0.55 for regular waves and 0.90 for random waves.

4.5 Longshore Sediment Transport Differences Between Monochromatic and Spectral Cases

The hydrodynamic differences between the wave drivers indicate that the longshore sediment transport will also be affected. The change in the $S_{xy}$ radiation stress and hence the longshore current will have an effect on the longshore sediment transport. As previously discussed in Chapter 3, the HH transport due to mean current is given by,

$$q_y \text{ current} = \frac{C_1 f_w}{\rho g} \tau_{eff} V_y,$$

and the HH transport due to waves is given by,

$$q_y \text{ waves} = \frac{C_1 f_w}{\rho g} \tau_{eff} u_{wy}.$$

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Figure 46: Total HH longshore transport for skewed spectral wave driver with $C_1=1$.

Figure 47: Total HH longshore transport for skewed spectral wave driver with $C_1=1.3$. 
Figures 48(a) and 48(b) show the longshore transport due to advection by waves and mean current for the spectral monochromatic and spectral skewed cases. The advection due to waves is almost an order of magnitude smaller than the advection by mean current, and the advection due to waves shows more shape changes between the different cases than the advection due to mean current. Although the transport due to advection by current is larger than advection by waves, the magnitude of change between the linear and skewed cases is highest for the transport due to waves. To illustrate this point, the total transport was found for the linear and skewed cases in Figures 48(a) and 48(b). The difference in transport from the linear to skewed case for transport due to waves is $+0.0285$ m$^3$/s, while the transport change from linear to skewed case for transport due to current is $+0.0045$ m$^3$/s. Therefore, while the total transport for advection due to waves is smaller than by currents, the magnitude of the change between the linear and skewed cases is significant. For advection due to waves for the skewed case, the monochromatic wave driver shows negative transport nearshore while the spectral driver is positive over the entire domain. However, for the linear cases the wave advection transport is always negative as previously seen in Chapter 3.

![Graphs of longshore transport](image.png)

**Figure 48:** Cross-shore variation of longshore transport (a) due to waves and (b) due to current for monochromatic (solid) and spectral (dashed) wave drivers for linear (black) and skewed (red) cases for $H=1.5$ m and angle=6 degrees.
Figure 49 shows the combined longshore transport due to waves and current using the HH model with both calibration constants \((C_1 = 1 \text{ or } 1.3)\). With the calibration, the transport magnitude for the monochromatic and spectral wave drivers are similar, however the peaks of the transport and the width of significant transport are different. The peak of the transport for the monochromatic wave driver is higher than the spectral wave driver, but the transport is over a smaller region of the surf zone. The difference is attributed to the wave height and wave angle differences between a monochromatic and a spectral wave driver. Consequently, the radiation stresses are different and the longshore transport mechanisms, which are driven by the longshore velocity, and the near bottom orbital velocity are different. The spectral wave driver is considered more appropriate because a random sea does not usually produce such a defined cross-shore peak of wave height and consequently a strong peak of longshore transport on an equilibrium beach profile.

![Graph](image)

**Figure 49:** Cross-shore variation of total longshore transport for skewed monochromatic (solid), skewed spectral \(C_1=1.0\) (red) and skewed spectral \(C_1=1.3\) (red-dash) wave driver for \(H=1.5\) m and angle=6 degrees.

The longshore sediment transport is calculated using several longshore sediment transport formulas and results from the monochromatic and spectral wave drivers are compared. Figure 50 shows the transport from all formulas for the spectral wave driver. These cases all have a wave height of 1.5 m and an incoming wave angle of 6 degrees for both the linear
Figure 50: Cross-shore variation of spectral longshore transport comparison for H=1.5 m and angle=6 degrees for HH with $C_1=1.3$ (black), BBB (red), R (blue), and W (green) for skewed cases.

and skewed cases.

For each sediment transport formula, the longshore transport resulting from the spectral wave driver is lower than the monochromatic wave driver. As shown in previous sections, the decrease is due to the changes in the hydrodynamics. The interest in this section is the trends seen for each formula in response to the change in wave driver. The HH, BBB, and R formulas each display a decrease in sediment transport and an increase in the area of the surf zone area on which the transport is occurring. These differences are expected due to the radiation stress changes and the resulting current and orbital velocity changes.

4.6 Comparison of Total Transport Between Model Results and Empirical Formulas

This section compares the total longshore sediment transport from various combinations of the model to several empirical formulas. The empirical formulas, previously mentioned in Chapter 1, include the CERC formula (CERC, 1984), the Kamphuis (1990) empirical formula, and the longshore transport equation from the long term shoreline change model GENESIS (U.S. Army Corps of Engineers, 2001). The CERC formula has been the only total transport empirical method of comparison until this point in the research. As indicated
previously, when the longshore gradient in the GENESIS formula is zero \( (dH_b/dy = 0) \), the GENESIS and CERC formulas predict the same amount of total longshore sediment transport (assuming the coefficients are identical). For each case shown in Chapter 4, the model output is generated for an equilibrium beach profile, so the longshore gradient is zero and the GENESIS and CERC formulas result in the same transport.

The CERC, Kamphuis, and GENESIS formulas each have an empirical component meaning that it is derived through observations and/or experimentation. As in Chapter 3, the total longshore transport is plotted versus longshore wave power on log-log plots in Figures 51 through 54. These figures show the total longshore sediment transport from calculations for the full range of tests for each formula (i.e. for wave height range from 0.25 m to 3.0 m and wave angle range from 3 to 15 degrees). Results are shown for the fully skewed monochromatic and spectral wave driver cases.

For the monochromatic and spectral wave drivers, the GENESIS model is the same as the CERC formula if the longshore gradient is zero. Therefore, only the CERC formula is shown on Figures 51 through 54. The Kamphuis model predicts lower transport than the GENESIS and CERC formulas. For the spectral wave driver, the CERC model is the most similar to the HH and W transport results. The BBB and R transport predictions are more similar to the Kamphuis model for low wave power, and to the CERC model for higher wave powers. Table 6 summarizes the most similar empirical model for each sediment transport formula output.

**Table 6:** Most similar empirical formula for each analytical sediment transport resulting from the monochromatic and spectral wave drivers. The table corresponds to Figures 51 through 54.

<table>
<thead>
<tr>
<th>Transport</th>
<th>Driver</th>
<th>Low Wave Power</th>
<th>High Wave Power</th>
<th>Figure</th>
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</thead>
<tbody>
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<td>HH</td>
<td>Spectral</td>
<td>CERC</td>
<td>CERC</td>
<td>51</td>
</tr>
<tr>
<td>BBB</td>
<td>Spectral</td>
<td>Kamphuis</td>
<td>CERC</td>
<td>52</td>
</tr>
<tr>
<td>R</td>
<td>Spectral</td>
<td>Kamphuis</td>
<td>CERC</td>
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</tr>
<tr>
<td>W</td>
<td>Spectral</td>
<td>CERC</td>
<td>CERC</td>
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Figure 51: Empirical formulas and data measurements for spectral wave driver for skewed case for HH model transport.

Figure 52: Empirical formulas and data measurements for spectral wave driver for skewed case for BBB model transport.
Figure 53: Empirical formulas and data measurements for spectral wave driver for skewed case for R model transport.

Figure 54: Empirical formulas and data measurements for spectral wave driver for skewed case for W model transport.
4.7 Sensitivity Analysis- Effects of Spreading and Peak Enhancement Factors

The effects of the directional spreading factor (SF) and the peak enhancement factor (PF) in the REF/DIF S Wave Driver are tested. All prior model runs were conducted with a spreading factor of 10 and a peak enhancement factor of 10. The effects of changing the spreading factor and the peak enhancement factor on the longshore sediment transport are evaluated here. The peak enhancement factor controls the bandwidth of the spectrum. As the spectrum becomes more narrow-banded, the hydrodynamics and transport should more closely resemble monochromatic wave driver results. A peak enhancement factor of 2 creates a broad frequency spectra while a peak enhancement factor of 20 creates a narrow frequency spectra (Kirby and Ozkan, 1994). The directional spreading parameter can be selected to produce a narrow or broad directional range. A broad directional spectrum is used to identify waves approaching the coast from many directions (40=broad), while a narrow directional spectrum tightly centers the waves around the primary wave direction (5=narrow) (Minerals Management Service, 1999).

Figure 55 shows variation of longshore transport for different values of the peak enhancement factor. As the peak enhancement factor increases, the longshore sediment transport increases. Figure 56 shows variation of longshore transport for different values the directional spreading factor. When changes are made to either the peak enhancement factor or the directional spreading factor, the other term remains fixed at 10. The HH transport formula is used to compute the transport and includes a calibration parameter of \( C_1 = 1.3 \) for the spectral wave driver and \( C_1 = 1 \) for the monochromatic wave driver.

As the spreading factor increases, the resulting longshore sediment transport decreases. This indicates that as the waves are more focused in one direction, the transport is higher than spreading the focus of the waves. Therefore, for the low (SF=5) spreading factor, the transport is the closest to the monochromatic waves result which utilizes only one wave direction.
**Figure 55:** Cross-shore variation of longshore transport for $H$=1.5 m and angle=6 degrees for transport due to monochromatic waves (purple), and for spectral waves with PF=20 (red), PF=10 (blue), PF=5 (green), and PF=2 (black).

**Figure 56:** Cross-shore variation of longshore transport for $H$=1.5 m and angle=6 degrees for transport due to monochromatic waves (purple) and spectral waves for increasing directional spreading factor starting at 5 (green) and incrementing by 5 to 40 (red).
Figure 57 shows the longshore transport created by monochromatic waves, a narrow-banded spectrum, and a broad-banded spectrum. As a spectrum becomes more narrow-banded, the longshore sediment transport rate becomes more similar to that of the the monochromatic wave driver case. Conversely, as a spectrum becomes more broad-banded, the longshore sediment transport rate diverges from the monochromatic case. This converging of the monochromatic and spectral wave drivers for smaller directional spreading is also confirmed by Vincent and Briggs (1989). They established how well monochromatic waves represented irregular waves for a case of strong bathymetric induced wave convergence. They found that the monochromatic waves provided a difference of less than 5-10 percent error in wave heights compared to the irregular wave cases if there is no directional spread and small waves.

![Graph showing longshore transport](image)

**Figure 57:** Cross-shore variation of longshore transport for H=1.5 m and angle=6 degrees for monochromatic transport (purple), a narrow-banded case with SF=5 and PF=20 (blue), and a broad-banded case with SF=40,PF=2 (green).

### 4.8 Summary

The spectral wave drivers produce a more realistic representation of hydrodynamics than the monochromatic wave driver. The spectral case involving the evolving roller description by Stive and DeVriend (1994) and decay by Battjes and Janssen (1978) results in the
lowest longshore radiation stress ($S_{yy}$) and highest cross-shore ($S_{xx}$) and $S_{xy}$ radiation stress. The HH model is recalibrated for the spectral wave driver which results in $C_1 = 1.3$ for the spectral wave driver cases. Other formulas do not explicitly take the wave driver differences into account. The BBB, W, and R formulas predict different transport for each wave driver because the hydrodynamics are changing. The most significant hydrodynamic change between the wave drivers is the wave height distribution and therefore the radiation stresses which consequently affect the currents and the near bottom orbital velocity. The current and near bottom velocity are directly incorporated into the sediment transport formulas.
CHAPTER V

EFFECTS OF GRAIN SIZE AND BATHYMETRIC VARIATIONS

This chapter explores the effects of grain size and bathymetric variability on the longshore sediment transport. The grain size variations include two grain sizes, 0.2 mm and 0.4 mm, which in turn change the steepness of the equilibrium beach profile. The bathymetric variations include cusped beaches with different cusp heights and longshore lengths and barred beaches with varying bar heights and relative cross-shore locations.

5.1 Grain Size Effects on Longshore Sediment Transport

The CERC formula, which the sediment transport results are compared to, is not dependent on grain size. A larger grain size will typically lead to a steeper beach slope. With the same domain size in the test runs, the offshore depth increases with the larger grain size of 0.4 mm due to the steeper beach shape. The hydrodynamic and longshore transport response to grain size changes has been modeled by Haas and Hanes (2004) using a monochromatic wave driver with orbital velocities calculated by linear wave theory. The present study expands upon their research to include skewed orbital velocity time series and by incorporating a spectral wave driver.

5.1.1 Sediment Size Model Run Overview

Variations in hydrodynamic and longshore transport resulting from the sediment size are examined for a large range of hydrodynamic conditions. Grain sizes of 0.2 mm and 0.4 mm are used with the HH model for the linear and skewed case with both the monochromatic and spectral wave drivers. The BBB, R, and W sediment transport formulas use both d=0.2 mm and 0.4 mm for the skewed case with the spectral wave driver and for d=0.2 mm for the linear and skewed cases with the monochromatic wave driver. A summary of model
runs is presented in Table 7. Each model run suite consists of wave heights from 0.1 m to 3.0 m and incoming wave angles from 3 degrees to 15 degrees.

**Table 7:** Summary of tests conducted for varying wave and sediment characteristics for the spectral wave driver. 'Test' indicates whether skewed orbital velocity time series are included, and HH, BBB, R and W are the sediment transport expressions. Columns marked with an 'X' indicate the test is conducted for that formula.

<table>
<thead>
<tr>
<th>Test</th>
<th>Grain Size (mm)</th>
<th>HH</th>
<th>BBB</th>
<th>R</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.2</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.4</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewed</td>
<td>0.2</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Skewed</td>
<td>0.4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

5.1.2 Effects of Grain Size on Transport with Monochromatic and Spectral Wave Drivers

Figure 58 compares the longshore sediment transport for the spectral wave driver for the linear and skewed cases. When the grain size diameter is increased, the peak transport is higher and the peak moves closer to the shoreline which is similar to the monochromatic case. However, for the 0.2 mm grain size, the transport acts over a larger portion of the surf zone. Interestingly, when the transport is integrated over the surf zone, the total longshore sediment transport for each grain size is similar with the larger grain size leading to slightly lower transport for this particular case. The longshore sediment transport resulting from the larger grain size is about 80 percent of the transport resulting from the 0.2 mm grain size.

Figures 59(a) and 59(b) show each sediment transport formula (HH, BBB, R, and W) for the skewed spectral wave driver case with H= 1.5 m and wave angle equal to 6 degrees. Figure 59(a) shows the sediment transport for d=0.2 mm, and Figure 59(b) shows the sediment transport for d=0.4 mm. For the median grain size of 0.2 mm, the W and calibrated HH formulas predict the highest transport, BBB predicts less than HH, and R predicts the lowest transport. When the grain size is increased to 0.4 mm, the R formula responds strongly and predicts higher transport than the BBB formula. This is because the R formula includes grain size to the third power (Equation (40)), while BBB only includes grain size.
as a function of fall velocity (Equation (39)).

5.1.3 Hydrodynamics

This section compares the hydrodynamic changes resulting from the two grain sizes for the spectral wave driver for the skewed orbital velocity cases. For the two grain size cases, the radiation stress gradient and bottom shear stress are significantly affected by the change in diameter. The diameter change directly affects the friction factor (Equation (31)), raising it from 0.024 to 0.03 with the increase in grain size diameter. The diameter change also changes the slope of the beach according to the equilibrium beach profile formulation.

The momentum balances for each grain size diameter for the skewed spectral case are shown in Figure 60. The same trends for the longshore shear stress and the radiation stress gradient are observed for the spectral wave driver. The radiation stress gradient has a higher peak for the d=0.4 mm case because the larger grain size causes a steeper beach slope which results in a larger gradient. Because the breaking wave heights and therefore the radiation stresses are equivalent and the waves break closer to the shoreline, the radiation stress gradient must be larger. The increase in radiation stress gradient is then balanced
Figure 59: Longshore transport for $H=1.5$ m and angle=6 degrees for skewed spectral case for calibrated HH (blue), BBB (red), R (green), and W (black) for (a) $d=0.2$ mm and (b) $d=0.4$ mm.

by an increase in the longshore shear stress.

5.1.4 Total Transport

The total longshore transport is found by integrating the longshore sediment transport across the nearshore region as previously described in Chapter 3. The total transport as a function of wave power is shown in Figures 61 through 65 for all the transport formulas for both grain sizes. Figures 61(a) and 61(b) show the linear HH total transport for the spectral wave driver cases for $d=0.2$ mm and $d=0.4$ mm respectively. Figures 62(a) and 62(b) show the skewed HH total transport for the spectral wave driver cases with both grain sizes. Figures 63(a) and 63(b) show the skewed case BBB total transport for the spectral wave driver cases for the two diameter sizes. For the skewed case with the spectral wave driver, the BBB formula predicts higher longshore sediment transport for the smaller grain size of 0.2 mm than the 0.4 mm grain size. The $K$ value for the larger grain size is only 60 percent of the $K$ value from the smaller grain size. The $K$ value is introduced in Section 3.3.1.2 and is calculated according the following,

$$K = \frac{Q_l \rho (s - 1) g (1 - p)}{P_l}$$

(68)
Figure 60: Bottom friction and radiation stress components of the longshore momentum balance for $d=0.2$ mm (solid) and $d=0.4$ mm (dashed) for $H=1.5$ m and angle=3 degrees for spectral wave driver with skewed case.
Figures 61(a) and 61(b) show the total longshore sediment transport for the spectral wave driver cases for both diameter sizes. The Ribberink formula predicts higher total sediment transport with the larger grain size diameter of 0.4 mm. Figures 65(a) and 65(b) show the skewed case W mean current induced total transport for the spectral wave driver cases for both diameter sizes. Longshore sediment transport caused by the mean current for the Watanabe formula increases with an increase in grain size by 6 percent.

Table 8 summarizes the slope and transport coefficients for the best fit lines from Figures 61 to 65 as defined in Section 3.3.1.2 for each modeled sediment transport formula for the monochromatic and spectral wave drivers. From Table 8, it is shown that the W and HH formulas predict the highest sediment transport. The R formula predicts the lowest relative sediment transport, and W predicts the highest. All of the slopes, N, are fairly close to 1.0 indicating that the predictive formulas behave similarly for changes in wave power.

The K value for the best fit slope is increasing for the linear HH case, the R case, and the W case. The increase of the K value results because the higher grain size changes the slope of the best fit line. The increase in the slope of the best fit line indicates that the higher wave powers are creating more transport for d=0.4 mm and the smaller wave powers.
**Figure 62:** Total longshore sediment transport for (a) $d=0.2$ mm skewed HH calibrated formula with spectral wave driver and (b) $d=0.4$ mm skewed HH calibrated formula with spectral wave driver.

**Figure 63:** Total longshore sediment transport for (a) $d=0.2$ mm skewed BBB formula with spectral wave driver and (b) $d=0.4$ mm skewed BBB formula with spectral wave driver.
Figure 64: Total longshore sediment transport for (a) d=0.2 mm skewed R formula with spectral wave driver and (b) d=0.4 mm skewed R formula with spectral wave driver.

Figure 65: Total longshore sediment transport for (a) d=0.2 mm skewed W formula with spectral wave driver and (b) d=0.4 mm skewed W formula with spectral wave driver.
Table 8: Summary of tests conducted for varying wave and sediment characteristics for the spectral wave driver. Transport indicates the analytical sediment transport equation, the case indicates linear or skewed orbital velocities, \( N \) is the slope of the best fit line to the model predictions, \( K \) is the CERC formula coefficient equivalent, and \( K_f \) is the equivalent \( K \) value for \( N=1.0 \).

<table>
<thead>
<tr>
<th>Transport</th>
<th>Case</th>
<th>d (mm)</th>
<th>( N )</th>
<th>( K_f )</th>
<th>( K )</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>Linear</td>
<td>0.2</td>
<td>0.882</td>
<td>0.620</td>
<td>0.321</td>
<td>0.226</td>
</tr>
<tr>
<td>HH</td>
<td>Linear</td>
<td>0.4</td>
<td>0.940</td>
<td>0.521</td>
<td>0.375</td>
<td>0.051</td>
</tr>
<tr>
<td>HH</td>
<td>Skewed</td>
<td>0.2</td>
<td>0.959</td>
<td>0.831</td>
<td>0.659</td>
<td>0.055</td>
</tr>
<tr>
<td>HH</td>
<td>Skewed</td>
<td>0.4</td>
<td>0.989</td>
<td>0.692</td>
<td>0.651</td>
<td>0.020</td>
</tr>
<tr>
<td>BBB</td>
<td>Skewed</td>
<td>0.2</td>
<td>1.050</td>
<td>0.268</td>
<td>0.355</td>
<td>0.063</td>
</tr>
<tr>
<td>BBB</td>
<td>Skewed</td>
<td>0.4</td>
<td>1.041</td>
<td>0.168</td>
<td>0.211</td>
<td>0.036</td>
</tr>
<tr>
<td>R</td>
<td>Skewed</td>
<td>0.2</td>
<td>1.009</td>
<td>0.223</td>
<td>0.235</td>
<td>0.035</td>
</tr>
<tr>
<td>R</td>
<td>Skewed</td>
<td>0.4</td>
<td>1.038</td>
<td>0.194</td>
<td>0.239</td>
<td>0.033</td>
</tr>
<tr>
<td>W</td>
<td>Skewed</td>
<td>0.2</td>
<td>0.901</td>
<td>1.834</td>
<td>1.056</td>
<td>0.146</td>
</tr>
<tr>
<td>W</td>
<td>Skewed</td>
<td>0.4</td>
<td>0.924</td>
<td>1.704</td>
<td>1.124</td>
<td>0.052</td>
</tr>
</tbody>
</table>

are creating less transport for \( d=0.4 \) mm. However, the additional transport for the higher wave powers is not enough to compensate for the reduced transport for the small wave powers leading to a net decrease in sediment transport for the larger grain sizes. For each transport formula, the higher grain size leads to less variance in the model output. The HH linear formula for \( d=0.2 \) mm displays the highest variance, while the HH skewed formula for \( d=0.4 \) mm displays the lowest variance.

To explain this change, the transport for a high power wave and a low power wave for both grain sizes are shown in Figures 66(a) and 66(b). For the high power case, the total transport for \( d=0.2 \) mm is 0.621 m\(^3\)/s and for \( d=0.4 \) mm the transport is 0.616 m\(^3\)/s. For the low wave power case, the total transport for \( d=0.2 \) mm is 0.0024 m\(^3\)/s and for \( d=0.4 \) mm the total transport is 0.0015 m\(^3\)/s. The Figures 66(a) and 66(b) combined with the total transport values validates the conclusion that the the smaller grain size leads to more transport each wave power case, but the overall change in transport due to grain size is not large.

To investigate the wave term more thoroughly, Figures 68(a), 68(b), 69(a), and 69(b) show the time-varying properties of the product of wave orbital velocity and effective shear stress, which is proportional to the transport due to advection by waves. From Figure 68(a)

91
it can be concluded that the transport due to advection by waves for $d=0.4$ mm is larger than $d=0.2$ mm in the surf zone region. However, in the offshore region shown in Figure 68(b), the wave advection transport for $d=0.2$ mm is larger than the $d=0.4$ mm transport. This is because for the large grain size the beach is steeper, therefore the depth is larger leading to less skewness and reduced wave transport. Accounting for the nearshore and offshore information, the overall wave advection changes shown in Figure 67(b) are caused by the differences in orbital velocity and effective shear stress for each grain size. In Figures 69(a) and 69(b), the product of orbital velocity and effective shear stress is larger for $d=0.4$ mm in the surf zone region, but the product for $d=0.4$ mm is larger in the offshore region.

There have been many attempts to describe the coefficient, $K$, in the CERC formula for varying grain sizes. A more recent attempt by del Valle et al. (1993) was formulated after evaluating sediment transport rates from aerial photographs. They found a decreasing trend in the empirical coefficient $K$ with an increase in sediment size. The following formulation for CERC coefficient $K$ resulted from their study,

$$K = 1.4 \exp(-2.5D_{50})$$

(69)

where $D_{50}$ is given as millimeters.
Figure 67: Longshore HH sediment transport for calibrated linear spectral (solid) and skewed spectral (dashed) cases for d=0.2 mm (black) and d=0.4 mm (red) for transport due to advection for H=1.5 m and angle= 6 degrees by (a) current and (b) waves.

Figure 68: Product of time-varying orbital velocity, $u_w$, and effective shear stress, $\tau_{eff}$, for d=0.2 mm (blue) and 0.4 mm (red) for the skewed spectral case for H=1.5 m and angle= 6 degrees for a position (a) inside the surf zone and (b) offshore.
**Figure 69:** Product of time-varying orbital velocity, $u_w$, and effective shear stress, $\tau_{eff}$, for $d=0.2$ mm (blue) and $0.4$ mm (red) for the linear spectral case for $H=1.5$ m and angle= 6 degrees for a position (a) inside the surf zone and (b) offshore.

For the cases in the present study, the formula by del Valle et al. (1993) predicts that $K = 0.85$ for $d=0.2$ mm and $K = 0.515$ for $d=0.4$ mm. This predicted decrease in sediment transport is only qualitatively supported by the HH and BBB sediment transport formulas. The R and W models predict higher transport with increasing grain size, therefore not following the $K$ value prediction by del Valle et al. (1993). The addition of the skewed velocities in the transport formulas and the shear stress formulations shows that there is little overall dependence on grain size even though the skewness terms do modify the hydrodynamics. The present results including the skewed orbital velocity do not change the conclusions found by Haas and Hanes (2004) who solely explored the effect of grain size on the linear case.

### 5.2 Cusped and Barred Beaches

Variation in grain size is an example of a sediment characteristic which can change the shape of a beach and has been shown to affect the total longshore sediment transport. Other nearshore features also have effects on hydrodynamics and longshore sediment transport. Bars, cusps, ripples, megaripples, ridges, sand waves, and swash marks on beaches are
nearshore features that may have an impact on longshore sediment transport and hydrodynamics (Werner and Fink, 1993). Cusped beaches are hypothesized to form from alongshore standing waves on beaches. The cusps result from a combination of accretion at the horns of the cusp and erosion at the cusp bays (Inman and Guza, 1982).

Definition sketches for the geometry of cusped and barred beaches used in the present study are shown in Figures 70 and 71. The cusped and barred beach bathymetries are held constant allowing no erosion or accretion of the shape, but calculating localized transport rates across the bathymetry. The cusped beach bathymetry is created using the following equation adapted from Rogers et al. (2002),

\[ h_o(x, y) = A x^n + h_{\text{min}} + \varepsilon \cos (k y \exp(-\frac{x}{\lambda})) \]  \hspace{1cm} (70)

where \( k = \frac{2\pi}{L_c} \), \( h_{\text{min}} \) is the depth at the shoreline, \( L_c \) is the cusp length, \( \varepsilon \) is the cusp amplitude, and \( \lambda \) is the cusp decay term. An example of the cusped beach bathymetry is shown in Figure 72. The barred beach bathymetry is formulated by,

\[ h_o = A x^n + h_b \exp \left[ 16 \log(0.05) \left( \frac{x - W_b}{W} \right)^4 \right] \]  \hspace{1cm} (71)

where \( h_b \) is the bar height, \( W \) is the width of the bar, and \( W_b \) is the distance of the bar from the shoreline. The bar height, \( h_b \), is typically 0.5 m, but is 0.3 m for the spectral wave driven cases with the bar close to shore. The bar width, \( W \), is 20 m, and \( W_b \) is varied to change the bar location. An example of the barred beach bathymetry is shown in Figure 73. \( W_s \) is the width of the surf zone, and the bar location is defined by \( W_b/W_s \). For \( W_b/W_s = 1.0 \), the waves are breaking at the bar because the width of the surf zone is equal to the distance of the bar from the shoreline. For \( W_b/W_s = 1.5 \), the waves are breaking onshore of the bar location. For \( W_b/W_s = 0.5 \) the waves are breaking offshore of the bar location.
Figure 70: Vertical section of cusped beach profile view definition sketch.

Figure 71: Barred beach side view definition sketch.
Figure 72: Cusped beach bathymetry for $\varepsilon=0.25$ m and $L=25$ m.

Figure 73: Barred beach bathymetry for $H=1.0$ m and angle=3 degrees with $W_b/W_s=1$, $H_{bar}=0.5$ m, and $W=20$ m.
5.2.1 Effects of a Cusped Beach on Longshore Sediment Transport

The cusp amplitude ($\varepsilon$) is varied from 0.10 m to 0.50 m with corresponding changes in cusp length ($L_c$) from 10 m to 50 m. Longshore sediment transport is calculated on the cusped beach for each scenario and is compared to the CERC formula. Because the cusped beach is not uniform in the longshore direction as the previous bathymetries have been, the integration across the nearshore region is slightly different. The transport is integrated for each cross-section to produce a longshore varying total transport for that cross-section. The breaking wave height is determined at each longshore position which is used to calculate transport for that cross-section also generating a longshore varying total transport for each cross-section. The transport at each cross-section is then averaged together to determine the overall total transport.

Figures 74, 75, and 76 show the total longshore sediment transport for each cusped beach bathymetry scenario with a range of hydrodynamics from wave height of 0.5 m to 3.0 m and wave angle of 3 to 15 degrees. The figures show the HH model output and the CERC and GENESIS empirical formulas with the best fit line for the HH model output. A summary of the best fit line slopes and $K$ values is shown in Table 9 as defined earlier in Section 3.3.1.2. In Figures 75 and 76, the cusped beach with $\varepsilon=0.10$ m produces slightly higher transport than the cusped beach with $\varepsilon=0.50$ m. The variance in the model output decreases with an increase in cusp height.

<table>
<thead>
<tr>
<th>$\varepsilon$ (m)</th>
<th>$L_c$ (m)</th>
<th>d (mm)</th>
<th>$N$</th>
<th>$K$</th>
<th>$K_f$</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>10</td>
<td>0.2</td>
<td>0.905</td>
<td>0.667</td>
<td>1.300</td>
<td>0.032</td>
</tr>
<tr>
<td>0.25</td>
<td>25</td>
<td>0.2</td>
<td>0.922</td>
<td>0.658</td>
<td>0.1135</td>
<td>0.026</td>
</tr>
<tr>
<td>0.50</td>
<td>50</td>
<td>0.2</td>
<td>0.933</td>
<td>0.646</td>
<td>1.031</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

Table 9: Summary of tests conducted for cusped beaches total longshore transport computed with HH for the spectral wave driver where $\varepsilon$ is the cusp height, $L_c$ is the cusp length, d is the grain size, $N$ and $K$ correspond to the best fit line for the model output, and $K_f$ is the $K$ value for $N$ fixed at 1.0.

The cusped beaches present an interesting case for an application of the GENESIS empirical formula. As seen in Equation (6), the GENESIS formula includes a term, $\frac{dH}{dy}$, which
Figure 74: Total HH longshore sediment transport for skewed case for $\varepsilon=0.25$ m, $L=25$ m, decay=25 with spectral wave driver.

Figure 75: Total HH longshore sediment transport for skewed case for $\varepsilon=0.10$ m, $L=10$ m, decay=25 with spectral wave driver.
Figure 76: Total HH longshore sediment transport for skewed case for $\varepsilon=0.50$ m, $L=50$ m, decay=10 with spectral wave driver.

takes into account the longshore gradient of the wave height on the cusped bathymetry. Figure 77 shows the gradient of the breaking wave height across the longshore for a fixed wave condition for each cusp height. For the equilibrium and barred beach profiles, the gradient term is zero because there is no alongshore variation. The GENESIS transport in Figures 74 through 76 more closely predicts the model output than the CERC prediction. The proximity of the GENESIS and model output in Figures 74 through 76 can be expected after seeing Figure 79(a). In Figure 79(a), the GENESIS formula and model output show more similar longshore variabilities than the Kamphuis or CERC formulas, and the formulas have similar resulting longshore transport.

Figure 78 shows a $K$ equivalent value for each wave height transport calculation plotted against the breaking wave height normalized by the minimum wave height condition. The $K$ equivalent value is a measure of the relative transport given by,

$$K = \frac{Q_i \rho (s - 1) g (1 - p)}{P_i},$$

(72)
**Figure 77:** $dH_b/dy$ Term for GENESIS longshore sediment transport formula for cusped beaches with $\varepsilon=0.10$ m (blue), $\varepsilon=0.25$ m (red), and $\varepsilon=0.50$ m (black) for $H=1.5$ m.

The figure reveals that the cusp heights do not have different effects on the sediment transport as the wave heights become larger. When the wave heights are small (i.e. when $H_b/H_{\text{min}}$ is small), $\varepsilon=0.25$ m leads to less transport than the $\varepsilon=0.10$ m case. When the wave heights become larger, the cusp height difference does not have an effect on transport.

Figures 79(a) and 79(b) show the longshore variability of the transport for the cusped beach for the HH model output and the CERC, Kamphuis, and GENESIS empirical formulas. The figures show that the GENESIS and model output are the only formulas that show similar structure of longshore variability due to the variability in the cusp bathymetry. The Kamphuis and CERC formula show some weak longshore variability due to variation in breaking wave height, but the magnitude and phase of the variability is much smaller than the model output and GENESIS formula.

Figures 80(a) and 80(b) show the longshore variation of the transport for the skewed monochromatic cases for the model output for several wave conditions. The figures show the resulting transport from different wave heights and wave angles. Increasing the wave angle increases the transport as expected while maintaining the same longshore variability. Increasing wave height increases transport and changes the alongshore variability. In Figure
Figure 78: $K$ equivalent value for $\varepsilon=0.10$ m (red) and $\varepsilon=0.25$ m (blue) and $H_{\text{min}}=0.5$ m.

Figure 79: Longshore variation of longshore transport with spectral wave driver for $\varepsilon=0.25$ m, $L=25$ m and decay=25 for HH model output (green), GENESIS (black), Kamphuis (red), and CERC (blue) for $H=1.0$ m and (a) angle=3 degrees and (b) angle=12 degrees,
**Figure 80:** Cusped beach longshore HH transport variability with spectral wave driver with $\varepsilon=0.25$ m for angle= 3 degrees (blue), angle=9 degrees (red), angle=15 degrees (green) with (a) $H=0.5$ m and (b) $H=3$ m.

**Figure 81:** Velocity vector plot for $\varepsilon=0.25$ m, $L_c=25$ m, $H=0.5$ m and angle=3 degrees.

**Figure 82:** Velocity vector plot for $\varepsilon=0.25$ m, $L_c=25$ m, $H=0.5$ m and angle=9 degrees.
Figure 83: Velocity vector plot for \( \varepsilon=0.10 \) m, \( L_c=10 \) m, \( H=0.5 \) m and angle=3 degrees.

Figure 84: Velocity vector plot for \( \varepsilon=0.25 \) m, \( L_c=25 \) m, \( H=1 \) m and angle=9 degrees.

80(a), the case with the smaller angle has negative transport at some longshore locations. The reason for the negative transport can be explained by looking at the mean current distribution in Figures 81, 82, 83, and 84 which show the velocity vectors for two of the cases with varying wave angle and \( \varepsilon=0.25 \) m in Figure 80(a).

For the small angle, eddies are forming, which leads to a negative transport at some longshore areas as seen in Figure 80(a). The negative transport for the small angle is also seen in the longshore variability in Figure 80(a). As the wave angle increases, as seen in Figure 82, the eddies are no longer occurring because the larger angle has increased the alongshore velocity enough to prevent flow reversal. Therefore, the transport is always positive as also seen in Figure 80(a) and the total transport remains positive. Figure 84 shows the velocity field for an increased wave height. When the wave height is increased, the cusp has less effect on the transport, because the transport extends over a larger portion of the domain.

Figure 85 shows the total HH transport with the alongshore direction normalized by the cusp length, \( L_c \). The cusp height is varied from \( \varepsilon=0.10 \) m to 0.25 m. When \( \varepsilon=0.10 \) m, the
transport is always positive. However, when $\varepsilon=0.25$ m the transport is negative at some longshore points. Again, looking at the velocity distributions in Figure 83 for the smaller cusps ($\varepsilon=0.10$ m), it is apparent that the flow reversal is much weaker than the case with the larger cusps ($\varepsilon=0.25$ m) in Figure 81. Therefore it can be concluded that the larger cusps lead to more flow reversals and hence less total longshore transport.

![Figure 85: Total HH transport from model output for spectral wave driver for cusped beaches with $H=0.5$ m for angle $= 3$ degrees with $\varepsilon=0.10$ m (black) and $\varepsilon=0.25$ m (red).](image)

### 5.2.2 Effects of a Barred Beach on Longshore Sediment Transport

The effects of sand bars on longshore transport is examined in this subsection. For one suite of tests, the location of the bar on the barred beach is specified to ensure that each wave height breaks on the bar ($W_b/W_s=1.0$). Additional barred beach bathymetries are then designed to have the wave breaking before ($W_b/W_s=0.5$) or after the bar ($W_b/W_s=1.5$) to determine the effects of breaking location on longshore sediment transport. An example of the variation in wave breaking locations is shown in Figure 86.

Figures 87, 88, and 89 show the total longshore transport for the barred beach cases with the monochromatic wave driver. Table 10 summarizes the parameters for the best fit of the model data for the barred beach cases. From Table 10, the relative amount of transport predicted by the model for each bathymetry can be compared. For the barred
Figure 86: Barred beach bathymetry (solid) with waves (dashed) for $W_b/W_s=1.5$ (red), $W_b/W_s=1.0$ (black), and $W_b/W_s=0.5$ (blue) for $H=1.0$ m.

Table 10: Summary of tests conducted for barred beaches with a monochromatic wave driver with $d=0.2$ mm where $W_b/W_s$ indicates the bar location, $N$ and $K$ correspond to the best fit line through the model output, $K_f$ is the $K$ value for a fixed slope of 1.0. The data for an equilibrium beach profile without a bar is indicated by $W_b/W_s=0$.

<table>
<thead>
<tr>
<th>$W_b/W_s$</th>
<th>$N$</th>
<th>$K$</th>
<th>$K_f$</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.957</td>
<td>0.682</td>
<td>0.919</td>
<td>0.0121</td>
</tr>
<tr>
<td>0.5</td>
<td>0.855</td>
<td>0.802</td>
<td>2.168</td>
<td>0.0597</td>
</tr>
<tr>
<td>1.5</td>
<td>0.916</td>
<td>0.739</td>
<td>1.315</td>
<td>0.0253</td>
</tr>
<tr>
<td>0</td>
<td>0.897</td>
<td>0.720</td>
<td>1.079</td>
<td>0.0144</td>
</tr>
</tbody>
</table>

beach, the monochromatic wave driver with the $W_b/W_s=0.5$ produces the highest amount of transport, while the $W_b/W_s=1.0$ case produces the lowest. For the spectral wave driver, the $W_b/W_s=1.0$ case produces the highest amount of transport, while the $W_b/W_s=0.5$ case produces the lowest transport. For the monochromatic case with $W_b/W_s=0.5$, the slope of the best fit line is the most mild. The mild slope indicates that the bar is lowering transport for high wave powers and raising transport for lower wave powers. The highest variance is found for the $W_b/W_s=0.5$ case, while the lowest variance is found for the $W_b/W_s=1.0$ case.

Figures 90, 91, and 92 show the value of $K$ equivalent plotted against $H_b/H_{bar}$ for the monochromatic wave driver. Each figure also has the equivalent $K$ value for an equilibrium
Figure 87: Total longshore transport for barred beach for $W_b/W_s=1.0$ for skewed case with monochromatic wave driver.

Figure 88: Total longshore transport for barred beach for $W_b/W_s=0.5$ for skewed case with monochromatic wave driver.
beach profile without a bar. For the identical incoming wave conditions, the equilibrium beach profile transport is used as a baseline to evaluate the effect of the barred beach and bar location on longshore transport.

As seen in Figures 90, 91, and 92, the barred beach transport becomes more similar to the equilibrium beach profile as the wave heights become larger. The difference indicates a threshold in transport which is affected by the bar, and when the threshold is exceeded with larger wave heights the bar no longer affects transport. Referring to Table 10 for the monochromatic case, the slope of the best fit line indicates that we should see the largest effects for smaller wave powers (i.e. smaller $H_b/H_{bar}$) for the bar location of $W_b/W_s=0.5$. As seen in Figure 90, the transport for smaller wave powers on the barred beach is higher than on the equilibrium beach profile, then as the wave become larger (i.e. larger $H_b/H_{bar}$), the transport is lower than the equilibrium beach profile. Hence the slope of the best fit line is reduced. As seen in Figure 91, the change is because the smaller wave powers are having less of an effect on the transport, and the higher wave powers are having the same effect as the case with $W_b/W_s=0.5$. For the $W_b/W_s=1.0$ bar location the smaller wave powers have an even further decrease in transport than the $W_b/W_s=1.5$ case, and the high wave powers

Figure 89: Total longshore transport for barred beach for $W_b/W_s=1.5$ for skewed case with monochromatic wave driver.
Figure 90: $K$ equivalent value for barred beach (red) for $W_b/W_s=0.5$ and equilibrium non-barred beach (blue) with monochromatic wave driver.

have the same decrease as before. Therefore, it can be concluded that the higher wave heights have similar effects for each bar location because (as in the cusped beach case), the bathymetry is having less effect on the transport for the higher wave powers. The reactivity of transport to the small wave powers distinguishes the bar locations. The bar located at $W_b/W_s=0.5$ results in the highest transport, then $W_b/W_s=1.5$, and finally the $W_b/W_s=1.0$ location produces the least transport out of the bar locations.
Figure 91: $K$ equivalent value for barred beach (red) for $W_b/W_s=1.5$ and equilibrium non-barred beach (blue) with monochromatic wave driver.

Figure 92: $K$ equivalent value for barred beach (red) for $W_b/W_s=1.0$ and equilibrium non-barred beach (blue) with monochromatic wave driver.
5.3 Summary

The effect of the bathymetric variations on longshore sediment transport is examined with both the skewed case and the monochromatic and spectral wave drivers. The HH and BBB formulas predict slightly smaller transport for the larger grain size (d=0.4 mm) for the skewed case. The R and W formulas predict slightly higher transport for the larger grain size. However, overall the sediment size dependence seems to be weak. The cusped beach produces reduced total transport for larger cusp heights for the small wave heights. As the wave heights become larger on a cusped beach, the changes in the cusp height have little effect on the transport.

The barred beach is tested for the monochromatic wave driver for bar locations $W_b/W_s=0.5$, $W_b/W_s=1.0$, and $W_b/W_s=1.5$. The $W_b/W_s=0.5$ produces the largest amount of longshore sediment transport, and therefore an increase over the transport on an equilibrium beach profile. The barred beaches show an increase in $K$ with an increase in $\frac{H_b}{H_{bar}}$. As wave heights become larger, the bar has less effects on the transport because the wave power is high enough to produce transport which is not affected by the bar. The bar location where transport most resembles that of an equilibrium beach profile is the $W_b/W_s=1.0$ case.
CHAPTER VI

CONCLUSIONS

The prediction of longshore sediment transport by numerical modeling is important for beach nourishment projects, coastal development plans, dredging operations, and additional engineering applications. The present research has reviewed previous methods for modeling longshore sediment transport and outlined the modeling methods available. The effect of skewness on longshore sediment transport are evaluated for a variety of wave conditions. Variations in hydrodynamics and the resulting effects on longshore sediment transport is evaluated for monochromatic and spectral wave drivers. Finally, the effect of grain size and bathymetric variations on longshore sediment transport are analyzed.

The NearCoM nearshore model allows a user to select between different modules to define the wave driver, circulation, and sediment transport formulation. The present study utilizes both monochromatic and spectral wave drivers, the SHORECIRC circulation module, and the (1) Haas and Hanes, (2) Bagnold, Bowen, and Bailard, (3) Ribberink, and (4) Watanabe transport formulas. The monochromatic wave driver, REF/DIF 1, utilizes a single frequency and wave angle, while the spectral wave driver, REF/DIF S, utilizes a spectrum of frequencies and wave angles. Longshore transport is calculated using all the different transport formulas and compared to the CERC, Kamphuis, and GENESIS empirical formulas and field measurements.

The following highlights were discovered during the present research which will be expanded upon:

- Including skewed velocities in shear stress and transport calculations increases total longshore sediment transport;

- The spectral wave driver results in slightly less transport than the monochromatic wave driver due to the differences in breaking RMS wave height;
• Longshore sediment transport is slightly affected by a change in grain size when the
  slope of the beach is a function of grain size;

• Transport is affected by features such as cusps and bars, but the effects that the
  features have on transport are decreased as wave height increases.

All the shear stress and sediment transport equations are formulated to include skewed
orbital velocities to evaluate the effects of skewed wave shape on longshore sediment trans-
port. An empirical method for determining skewed orbital velocities is incorporated into the
model, and the velocities are used in the formulation of the shear stress and transport. By
including the skewed velocities, the time-varying orbital velocity and effective shear stress
are larger in the crest than in the trough of the wave. The skewed velocity time series leads
to a smaller longshore velocity than the linear case, but the effective shear stress is higher
than the linear case. The combination of a lower longshore velocity and a higher effective
shear stress ultimately leads to more longshore sediment transport.

The transport due to advection by waves increases from the addition of skewed orbital
velocities, and the transport due to advection by current increases slightly due to an en-
hanced effective shear stress. The longshore transport is integrated across the surf zone to
determine the total transport for each wave condition. For all of the sediment transport
formulas, the skewed case predicts significantly higher transport than the linear case. There-
fore, the addition of the skewed orbital velocities to incorporate the effects of wave shape is
not a negligible addition. Including the skewed orbital velocity more closely represents an
actual wave and causes significant changes in longshore sediment transport.

The effect of a monochromatic and spectral wave driver on longshore sediment transport
is investigated using REF/DIF 1 and REF/DIF S. The largest differences between the two
models is the variation of the wave heights, and consequently the radiation stresses. The
Haas and Hanes transport model is calibrated using the CERC formula and field and lab
measurements to produce a transport coefficient of 1 for the monochromatic wave driver
and 1.3 for the spectral wave driver. The Bagnold, Bowen, and Bailard, Ribberink, and
Watanabe transport formulas predict lower longshore sediment transport for the spectral
wave driver due to the hydrodynamic changes between the wave drivers. The model results are compared to the CERC and Kamphuis empirical formulas. For the equilibrium beach profile tested for the monochromatic and spectral cases, the GENESIS and CERC models are the same because the longshore gradient is zero. The CERC and GENESIS formulas most closely match each analytical formula prediction for the higher wave powers, but the Kamphuis formula most closely matches the Bagnold, Bowen, and Bailard and Ribberink transport for lower wave powers. For the spectral wave driver, the narrow-banded frequency spectrum results in higher transport than the broad-banded spectrum and more closely resembles the monochromatic transport.

Grain size and bathymetric variations also have impacts on longshore sediment transport. The bathymetric variations are tested with the skewed case and the monochromatic and spectral wave drivers. An analysis of the effects of grain size variation on longshore sediment transport reveals different reactions for small and large wave powers. For small wave powers, the transport for $d=0.2$ mm dominates, however as wave power increases the $d=0.4$ mm case leads to more transport. Overall, the changes in longshore transport due to the increase in sediment size is relatively small.

Barred and cusped beaches are developed to test for bathymetric variations, with the cusped beach involving alongshore variations. The cusped and barred beaches both affect sediment transport with smaller wave powers, but as the wave power becomes larger the cusps and bars have little effect on the longshore sediment transport. For small wave angles on the cusped beach, the transport becomes negative at some longshore positions due to the formation of eddies, however the total transport remains positive. The cusped beach produces the highest total transport for the smallest cusp height of $\varepsilon=0.10$ m. The barred beach is tested for the monochromatic wave driver for bar locations of $W_b/W_s=0.5$, $W_b/W_s=1.0$, and $W_b/W_s=1.5$. The transport on the barred beach is compared to transport on an equilibrium beach profile. For larger wave power, the bar has minimal effects on the total longshore transport. For smaller wave power, the bar location affects the longshore sediment transport. The largest transport results from the bar location at $W_b/W_s=0.5$.

Aspects that could be expanded upon would be testing all scenarios possible with the
NearCoM modules. Additional circulation modules and wave driver modules could be incorporated to determine the effects on longshore sediment transport. The next natural step for the analysis is to evaluate the same effects on the cross-shore sediment transport. A more extensive dimensional analysis can be conducted on more longshore variation scenarios to determine the impact on sediment transport. These results have been compared to empirical formulas based on many longshore sediment transport measurements, but data from specifically designed field or laboratory experiments would help validate the model results.
Bibliography


