AIRSPACE ANALYSIS AND DESIGN
BY DATA AGGREGATION
AND LEAN MODEL SYNTHESIS

A Thesis
Presented to
The Academic Faculty

by

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In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the
Daniel Guggenheim School of Aerospace Engineering

Georgia Institute of Technology
August 2013

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AIRSPACE ANALYSIS AND DESIGN
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Date Approved: 26 June 2013
“The factory of the future will have only two employees, a man and a dog. The man will be there to feed the dog. The dog will be there to keep the man from touching the equipment.”

Warren G. Bennis,
Distinguished Professor of Business Administration,
Marshall School of Business,
University of Southern California
ACKNOWLEDGEMENTS

I would like to thank the people who allowed this work to come to fruition. First of all, to my committee members, four of whom have also served as advisors and have funded my research at one point or another. Thank you all for your mentoring and support during my four years at Tech. Interacting with you and adapting to your different research interests, work methods, personalities, and extraordinary intellectual abilities has developed and matured me. My time with you has been defining and is something I will always hold dear.

Dr Feigh, thank you for trusting to hire me early on in your career, thank you for allowing me to express myself, encouraging my academic initiatives within and outside of Aerospace Engineering, keeping track of all my research ideas while pushing me to go to the essential in my work. Dr Pritchett, thank you for trusting my abilities as you did, for funding me even as I was taking business school classes, and for your guidance and support over the last two years. Dr Feron, thank you for always finding the time to talk to me, be it about research or life; conceptual discussions with you have allowed me to make sense of much of my work. Dr Clarke, thank you for bringing your good nature to every one of our research meetings, and for providing great insights that allowed me to link my mathematical interests with ATC engineering realities. Dr Delahaye, every one of our encounters and conversations at various conferences were a treat; thank you for agreeing to be part of this committee, for reviewing and helping improve my research, and for subjecting yourself to the annoyance of a teleconference defense.

Next, I would like to thank my CEC labmates, especially Alex for being there for me during quals and Zarrin for being so supportive and dedicated to her friends. I
also want to mention William, Scottie-Beth and So Young whom it was great to get
to know in the lab, and especially outside of it. William, I would have never tried
underwater hockey had it not been for you! I also thank everyone else in the CEC,
not mentioning you by name doesn’t mean I didn’t appreciate your contribution to
making the lab a great place to spend my days (and sometimes nights and weekends).

I could not skip some of the folks in the ATL/FMC, especially Adan, Maxime,
Manu, and Aude. Working with you and getting to know you has been a privilege,
and a lot of the research in this thesis could not have happened without your direct
or indirect contributions. Also, thank you Tim for spending so much of your time
helping me prepare for quals, and even more time the summer after that chatting
with me!

To my family, especially my parents, my grandmother, my brother, and Stefan,
thank you for always being so encouraging, so supportive, and for helping me keep
my perspective on the “real world”. To my friends in Atlanta and elsewhere, thank
you for the good times we’ve had. It’s been sad to see the (mostly) French contingent
roll through as the years went by, especially Greg, Colin, Etienne, Arnaud, Thomas
C., and Aurelien. It was great having all you guys here. Ludo, thank you for being
the best roommate one could have, and good luck as you go on the last stretch in
your thesis. Last but not least, a special thanks to the lovely Birney for her kindness
and support in stressful times, for always encouraging me, and for helping me enjoy
life with our many great trips and outings.

***************
Air traffic demand is growing. New methods of airspace design are required that can enable new designs, do not depend on current operations, and can also support quantifiable performance goals. The main goal of this thesis is to develop methods to model inherent safety and control cost so that these can be included as principal objectives of airspace design, in support of prior work which examines capacity. The first contribution of the thesis is to demonstrate two applications of airspace analysis and design: assessing the inherent safety and control cost of the airspace. Two results are shown, a model which estimates control cost depending on autonomy allocation and traffic volume, and the characterization of inherent safety conditions which prevent unsafe trajectories. The effects of autonomy ratio and traffic volume on control cost emerge from a Monte Carlo simulation of air traffic in an airspace sector. A maximum likelihood estimation identifies the Poisson process to be the best stochastic model for control cost. Recommendations are made to support control-cost-centered airspace design. A novel method to reliably generate collision avoidance advisories, in piloted simulations, by the widely-used Traffic Alert and Collision Avoidance System (TCAS) is used to construct unsafe trajectory clusters. Results show that the inherent safety of routes can be characterized, determined, and predicted by relatively simple convex polyhedra (albeit multi-dimensional and involving spatial and kinematic information). Results also provide direct trade-off relations between spatial and kinematic constraints on route geometries that preserve safety. Accounting for these clusters thus supports safety-centered airspace design. The second contribution of the thesis is a general methodology that generalizes unifying principles from these two demonstrations. The proposed methodology has three
steps: aggregate data, synthesize lean model, and guide design. The use of lean models is a result of a natural flowdown from the airspace view to the requirements. The scope of the lean model is situated at a level of granularity that identifies the macroscopic effects of operational changes on the strategic level. The lean model technique maps low-level changes to high-level properties and provides predictive results. The use of lean models allows the mapping of design variables (route geometry, autonomy allocation) to design evaluation metrics (inherent safety, control cost).
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>SUMMARY</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xii</td>
</tr>
<tr>
<td>I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem statement</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Scope</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Objectives and contributions</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Position within a systems engineering perspective</td>
<td>6</td>
</tr>
<tr>
<td>1.6 Thesis outline</td>
<td>7</td>
</tr>
<tr>
<td>II REFRAMING THE AIRSPACE DESIGN PROBLEM</td>
<td>9</td>
</tr>
<tr>
<td>2.1 Traditional capacity-centric view</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Challenge of including taskload and safety in airspace design</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Proposed concept of operation</td>
<td>13</td>
</tr>
<tr>
<td>2.3.1 Present organization</td>
<td>13</td>
</tr>
<tr>
<td>2.3.2 Transition from centralization to decentralization</td>
<td>16</td>
</tr>
<tr>
<td>2.4 Envisioned airspace paradigm</td>
<td>20</td>
</tr>
<tr>
<td>2.4.1 Airspace attributes</td>
<td>21</td>
</tr>
<tr>
<td>2.4.2 Inherent safety</td>
<td>22</td>
</tr>
<tr>
<td>2.4.3 Control cost</td>
<td>25</td>
</tr>
<tr>
<td>2.4.4 Summary and justification of the reframed problem</td>
<td>27</td>
</tr>
<tr>
<td>III MODEL AND APPLICATION OF CONTROL COST</td>
<td>30</td>
</tr>
<tr>
<td>3.1 Background</td>
<td>32</td>
</tr>
</tbody>
</table>
4.4.2 Flow characterization ........................................ 103
4.5 Summary ....................................................... 104

V UNIFYING PRINCIPLES AND PROPOSED METHOD .... 107
5.1 The problem of current airspace design .................. 107
5.2 Making sense of big data sources ......................... 109
5.3 The gap between analysis and design .................... 112
5.4 Proposed methodology ....................................... 114

VI CONCLUSION .................................................... 118
6.1 Suggested extension to capacity ......................... 121
6.2 Suggested future directions .............................. 123

Appendices ......................................................... 125

APPENDIX A — THEORETICAL DERIVATIONS OF CONTROL
COST PROBABILITY .............................................. 125

REFERENCES ..................................................... 132
# LIST OF TABLES

1. Ornstein-Uhlenbeck aircraft model parameters ........................................ 41
2. Johnson $S_U$ parameters for FTE fit ......................................................... 43
3. Johnson $S_U$ quartiles ................................................................................. 44
4. FTE statistical moments .................................................................................. 46
5. List of distributions used in maximum likelihood estimation ....................... 65
6. Intruder commanded equations ...................................................................... 84
7. Intruder aircraft guidance dynamics ............................................................... 85
8. Intruder states in RRT search example ......................................................... 92
9. Experiment design and RRT method reliability ............................................ 99
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The airspace construct</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Systems engineering decomposition</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>NAS centers</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Airspace classes</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>Aircraft separation</td>
<td>23</td>
</tr>
<tr>
<td>6</td>
<td>Trajectory safety models</td>
<td>24</td>
</tr>
<tr>
<td>7</td>
<td>State-based and intent-based safety</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>Sources of control cost</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>Airspace simulator diagram</td>
<td>31</td>
</tr>
<tr>
<td>10</td>
<td>Autonomous navigation module diagram</td>
<td>38</td>
</tr>
<tr>
<td>11</td>
<td>Simulated FTE histograms and theoretical density functions</td>
<td>44</td>
</tr>
<tr>
<td>12</td>
<td>Autonomous flow scheduling module diagram</td>
<td>47</td>
</tr>
<tr>
<td>13</td>
<td>CD&amp;R module diagram</td>
<td>49</td>
</tr>
<tr>
<td>14</td>
<td>Controlled traffic module diagram</td>
<td>51</td>
</tr>
<tr>
<td>15</td>
<td>Route network limited to sector ZOB49</td>
<td>53</td>
</tr>
<tr>
<td>16</td>
<td>Cleveland center graph</td>
<td>54</td>
</tr>
<tr>
<td>17</td>
<td>Extracted subnetworks</td>
<td>54</td>
</tr>
<tr>
<td>18</td>
<td>Traffic distribution in sector</td>
<td>55</td>
</tr>
<tr>
<td>19</td>
<td>Simulated control cost</td>
<td>57</td>
</tr>
<tr>
<td>20</td>
<td>Simulated task rate</td>
<td>58</td>
</tr>
<tr>
<td>21</td>
<td>Sensitivity analysis</td>
<td>59</td>
</tr>
<tr>
<td>22</td>
<td>Simulated probability densities</td>
<td>66</td>
</tr>
<tr>
<td>23</td>
<td>Goodness of fit at 1X traffic</td>
<td>67</td>
</tr>
<tr>
<td>24</td>
<td>Goodness of fit at 2X traffic</td>
<td>68</td>
</tr>
<tr>
<td>25</td>
<td>Control cost: Poisson process parameter fit</td>
<td>69</td>
</tr>
<tr>
<td>26</td>
<td>Task rate: Exponential process parameter fit</td>
<td>70</td>
</tr>
<tr>
<td>27</td>
<td>Tree of possible initial TCAS RAs</td>
<td>79</td>
</tr>
<tr>
<td>28</td>
<td>Tree of all possible TCAS RAs .................................. 80</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Intruder trajectory relative to the ownership in a typical scenario ... 82</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Intruder aircraft commanded trajectory .......................... 83</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Generic control model ............................................ 85</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Trajectory to RA direct and inverse mappings ...................... 86</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Construction of the RRT in the trajectory waypoint space .......... 91</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Horizontal plane traffic situation ................................ 92</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Trajectory space partition by RA .................................. 93</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>RA locations in validation experiment ............................. 95</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Expected vs Observed RAs in all 4 studies ........................ 97</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Accuracy of RA generation ......................................... 98</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Trajectories resulting in Climb RA ................................ 100</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Phase space plots for Climb RA trajectories ....................... 101</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Cluster characterization for Climb RA ............................. 101</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Unsafe areas in the airspace ..................................... 102</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Dallas Bump geometry .............................................. 103</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>Trade-off between climb rate and flow separation ................. 105</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

1.1 Motivation

Air traffic demand is growing. It is widely believed that air traffic in the US and Europe will reach capacity limits in the near future [88]. With limited airspace capacity to handle traffic, the forecast points to a National Airspace System (NAS) with ever-increasing congestion.

Many technological improvements are being developed and implemented to address the capacity issues faced in the NAS. Technological innovations such as ADS-B and datalinks have the potential to expand the capabilities of, and reduce the constraints on stakeholders of the system. For example, ADS-B provides detailed information to pilots about their surrounding traffic, and thus could potentially relieve controllers of their roles in traffic surveillance, conflict detection and resolution, or more general route planning and traffic separation [44]. Much of the focus has been on these new technologies, which are poised to overcome present-day systemic limitations.

But solely focusing on novel technology development leads to a limited perspective. In the past, technology was indeed the limiting factor of achievable operational concepts. Current air traffic operations have slowly evolved with incremental and localized changes, without fundamentally overhauling their paradigm since the advent of radar. Present operations are thus predicated by historical technology constraints [42].

However, technological capability is no longer the primary concern. Adapting operations to technology should therefore not be a goal in itself. Many possible
computerized systems can be imagined and implemented, and a reasoned process to support informed design decisions is needed. Air traffic operations must change to ensure performance, rather than focus on integrating technology. For example, it has been observed that traffic demands are spatially and temporally heterogeneous, at times leaving substantial regions of the NAS with underutilized resources while capacities become saturated in other regions [174]. Yet current regulations and procedures do not provide enough flexibility to dynamically transfer resources. To achieve performance goals, systemic paradigm shifts must therefore occur in the way airspace is operated. Some of the presumed requirements are modified route infrastructures and procedures that give more autonomy to aircraft and flight crews, and rely on higher-precision trajectories and operations [45].

Thus, this thesis focuses on the key challenge of altering airspace design. Current airspace is structured by airways and jet routes [48]. However, as more aircraft travel along these paths, their limitations become apparent with bottlenecks and delays forming at intersections. In addition to increasing numbers of traditional aircraft, the near-term emergence of new types of aircraft such as very light jets and unmanned aerial systems is likely to contribute to significant increase in the density and complexity of air traffic as well as in the operational requirements [47]. These changes contribute to rendering present airspace design inadequate.

Methods for airspace design must account for such new concepts of operations. While most if not all of the existing systematic approaches to airspace design are grounded in current operations [48], the Federal Aviation Administration has been promoting innovation in the structure and operation of the NAS, focusing on improving the airspace capacity along with efficiency and the enhancement of mobility. Defined under the coordination of the Joint Planning and Development Office [77], and supported by significant efforts from the FAA [45] and other agencies, the NextGen concept of operations involves a paradigm shift from a fixed airspace structure to a
fluid airspace. Thus, there is a need for methods that can enable new designs, do not rely on current operations in their assumptions, and can also support quantifiable performance goals. To that end, predictive and parsimonious models of airspace under new paradigms are needed.

1.2 Problem statement

This thesis focuses on problem of airspace design. Paradigm changes must occur in the way airspace is operated. Consequently, airspace design methods and models should have three characteristics: integrate additional objectives, support a range of concepts of operations, and be predictive.

First, models that can integrate additional design objectives differ from past approaches. In the past, airspace planning has focused on capacity and delay improvements through flow management, with the inclusion of taskload and safety as constraints [85]. In such past approaches, complexity limitations and the ambiguity of possible metrics have been identified as factors limiting the scope of airspace design. Future airspace design methods must overcome these limitations and include taskload and safety alongside capacity as core objectives of airspace design.

Second, to support a range of potential future concepts of operation, design methods and models must not be pinned to current operations. However, no systematic design methods exist which are not conceptually reliant on current operations [27]. Indeed, methods of air traffic assessment are directly related to past and present operational expertise [63]. Where design methods speculate with future concepts of operation, an a priori design of airspace is assumed and constructs of safety or taskload are estimated.

Third, predictive models are needed to guide airspace design and enable new design methods. These methods of airspace design must involve a more advanced degree of formalization and logical deduction to predict system behavior and map
interscale effects on the system. For this, predictive and parsimonious models of airspace under new paradigms are a necessary step.

This thesis introduces the concept of lean models and demonstrates their use in airspace design. Lean models are defined by four characteristics: they are compact descriptors, have forecasting ability, can guide systematic airspace design, and require low computational cost for use online.

1.3 Scope

This section provides a general definition of the concepts of the research, and the scope of the thesis.

The term *airspace* refers to an abstract construct defined by route geometry, autonomy allocation, and traffic levels. The geographical extent of the airspace in this thesis corresponds to approximately a large airspace sector, roughly 150 nautical miles across. The focus is on the enroute portion of flight, and is concerned with strategic decisions. The design scale is at the route and flow level, not that of individual aircraft. For the purpose of the thesis, the flows in the airspace are modeled as the edges of a network. The flow intersections are nodes in the network. Figure 1 illustrates the spatial extent of the airspace.

Two design variables parametrize this vision of the airspace: the geometry of the flows, and the delegated autonomy and authority for separation. The flow geometry captures the spatial position of the flow centroids, the velocity constraints that aircraft must follow inside the flow, the bounds for tolerated deviation away from the centroid, and the traffic volume supported by each flow. The autonomy captures the capability, authority and responsibility for trajectory determination by the aircraft. The routing decisions are on a strategic level. More specifically, aircraft are able to select the edges they wish to follow through a limited network of flows.

Three metrics are relevant to the perspective taken in the thesis: control cost,
inherent safety, and capacity. Control cost captures the requirements to control the airspace system, to avoid degradation and to maintain stability. Inherent safety is a characteristic of a route structure which is insensitive to predictable errors and deviations in aircraft separation. Capacity is the amount of traffic the airspace can support. The design variables and metrics are further discussed in Section 2.4.

1.4 Objectives and contributions

The main goal of this thesis is to propose and apply a methodology and models for airspace design that are not reliant on the current operational paradigm, for example by including additional objectives in the airspace design. The work is based on a concept of operations that is realistic in the medium-term and accounts for practical concerns. The premise of the thesis is to consider capacity as one of several objectives of airspace design. Traditionally safety and taskload have been modeled as constraints. However, it is possible to design the airspace for inherent safety and control cost as objectives alongside capacity. By adopting this premise, we go from optimizing capacity of the airspace to including triple factors of inherent safety,
control cost, and capacity as objectives.

**Contribution 1:** Demonstration of a model which estimates control cost depending on autonomy allocation and traffic volume, and a method for generating a route structure with inherent safety to prevent unsafe trajectories.

**Contribution 2:** A methodology where data mining and aggregation are used offline to synthesize models or classes of properties of the system. These compact descriptors can then be used to design the airspace and rapidly iterate. Their main benefit is requiring scarce computational resources for use online. The offline generation can be heavy but only needs to be performed once.

The research presented in this thesis has led to several publications. At the time of writing, the author of this thesis is the main author on one journal paper which has been accepted for publication [123], one journal paper which is under review [121], and three conference papers which he has presented [120,122,124]. The author of the thesis is also a main co-author on a fourth conference paper [100], and has contributed to the research resulting in three additional conference papers [132–134], for which he is listed as co-author.

1.5 **Position within a systems engineering perspective**

This thesis is meant to support airspace design, and must therefore consider complex systems of systems (SoS). A SoS is defined as “a set or arrangement of systems that results when independent and useful systems are integrated into a larger system that delivers unique capabilities. Both individual systems and SoS conform to the accepted definition of a system in that each consists of parts, relationships, and a whole that is greater than the sum of the parts” [32].

To describe system development lifecycles, systems engineering practitioners have formalized different models [22,49]. A generic vertical decomposition going from the
most conceptual to the most applied level of abstraction starts with defining the requirements, designing the architecture, designing the detailed components, then integrating the system, and finishing by the verification and validation.

Figure 2 shows how this thesis fits within a systems engineering endeavor. The thesis scope presented in Section 1.3 situates the research in the left arm of the decomposition. The proposed concept of operations and triple objectives of design discussed in Chapters 1 and 2 reframe the airspace design problem, thus essentially establishing new requirements. The two contributions of the thesis follow at a more applied level. The demonstrations in Chapters 3 and 4 make up the detailed design, while the unifying methodology in Chapter 5 is a formalization of the architecture.

1.6 Thesis outline

The thesis begins by reframing the airspace design problem in Chapter 2. To support this novel formulation, the proposed concept of operations is defined. Traditionally
safety and taskload have been hard constraints, recognized as essential but not explicitly modeled as objective functions. The thesis argues in favor of a paradigm shift. Under a new perspective, the metrics of inherent safety and control cost, as well as capacity, can be proactively accounted for in airspace design.

The thesis introduces the first contribution through the demonstrations in Chapters 3 and 4, respectively. The thesis shows, by example, how control cost and inherent safety can become objectives in airspace design. The scope of the work is restricted to the context of the aforementioned concept of operations. Two predictive technical solutions, for control cost and inherent safety, are shown to support this objective.

The thesis continues with its second contribution in Chapter 5. This chapter formulates of unifying principles from the two applications and defines a methodology. By definition, a methodology is a guideline system for solving a problem, with specific components such as postulates, techniques and tools. This methodology supports airspace design by constructing compact models which can be used to identify problems and limitations or recommend improvements. The methodology synthesizes data, extracts simple models, and reduces the problem complexity to manipulate design parameters. It is this methodology which enables the examples successfully implemented for inherent safety and control cost. The thesis ends with concluding remarks in Chapter 6.
This chapter presents a reformulation of the airspace design problem. Traditionally, improving capacity has been focus of airspace design, with safety and taskload serving as constraints. The capacity-centric traditional view is discussed in Section 2.1. This thesis shows it is possible to design the airspace for additional objectives, although Section 2.2 explains that including constructs such as taskload and safety in the design process is not an easy problem. To that end, a concept of operations with two metrics, inherent safety and control cost, are introduced in Section 2.3.

2.1 Traditional capacity-centric view

Capacity improvements have historically been the motivation for technological and operational changes. By improving capacity, it is expected that present and future demand can be accommodated with a reduction in delays and savings in associated costs such as fuel expenses. The estimation of enroute sector capacity is especially important in high-density traffic areas, since congestion has been shown to increase the sensitivity to disturbance of traffic flow management performance [162]. Approaches that recognize the concern of capacity limits have focused on flow management algorithms which optimize ground delay, route selection, and airborne holding times for individual aircraft [85]. For these approaches to work, a clear understanding of capacity is however required.

The traditional view in the literature is that capacity is constrained by controller workload, also sometimes called dynamic density. This dependency is visible in estimations of capacity, which commonly rely on fast-time computer simulations using...
models of controller workload. At times, capacity estimations are also verified by subject matter experts such as active or retired controllers [97]. In these cases, capacity evaluation remains strongly correlated to current or past concepts of operation.

Moreover, even when attempting to create generic metrics and methodologies which can be applied to different concepts of operation, research at EUROCONTROL has focused on evaluating sustainable throughput for a given maximum acceptable delay [63]. While each sector currently has an aircraft capacity limit, called the Monitor Alert Parameter (MAP), detecting where efficiency may be degraded [48], new and more refined surrogate metrics such as information processing load are being developed to replace MAP and thus address capacity evaluation on a more systemic level. Indeed, by design, MAP is not meant to be a measure of airspace capacity, but rather a performance threshold which could indicate excessive traffic density.

In fact, research has shown that capacity and delay are not solely related to traffic demand. Weather causes approximately 70% of delays in the NAS and is therefore considered by some to be the most fundamental limiting factor of airspace capacity [84]. A more accurate measure of capacity of a sector would therefore need to be modeled as an uncertain predictive measure which depends on the complexity of the traffic flows and on weather conditions [163]. Stochastic models have been used to model weather uncertainty. These models can be overlayed onto airspace geometries to define analytical, probabilistic expressions of weather constraints that limit access to the sector and reduce the available navigation space [104].

While the modeling and definition of capacity has taken several paths, its use is consistent in the literature. A multitude of possible optimization solutions remain solely focused on designing airspace around capacity improvements. Capacity improvements have been sought through solutions such as flexible traffic management, coupled traffic and weather prediction, and improved situational awareness between agents in the NAS [84]. Simulations have suggested that planning actions in the
Airspace should aim to replicate well-structured queuing models [162]. Airspace planning optimizations have proposed fuel-optimal flight plans satisfying workload, safety, and equity constraints which guarantee that controller workload in each sector is held under a permissible limit, that any potential conflicts are routinely resolvable, and that the various airlines derive equitable levels of benefits [151].

Yet in all of these approaches, the perspective is always that of designing airspace around capacity improvements, with taskload seen as a fixed constraint. Comprehensive models of the NAS which define it as a multi-path, steady-state network of queues conclude that its maximum capacity is the sum of the maximum airport operational rates constrained by taskload limitations [33]. Likewise, proposed resectorization solutions resort to geometric partition optimization and model taskload as problem constraints [10].

2.2 Challenge of including taskload and safety in airspace design

To accomplish the goals of NextGen and SESAR, control cost and inherent safety need to be accounted for in the design stage. This approach implies a break in perspective from traditional practices. Conventionally, capacity has been the declared objective of airspace design, with safety and taskload considered as imperative constraints on operations, procedures, and technologies. A distinction will be made in Section 3.1.1 between taskload - understood to be related to human intervention - and control cost - a broader concept which includes human and automated interventions.

The NextGen concept involves an operational paradigm shift from a fixed airspace structure to a fluid airspace able to cope with increasing demand and to withstand severe perturbations with minimal degradation [45]. Capacity improvements are the economic motivator for the NextGen program, as well as for SESAR. However, capacity improvements can be viewed as trade-offs with the safety and control cost inherent to the design. When viewed collectively as design objectives, airspace designs may be
identified that improve all of inherent safety, control cost, and capacity, rather than viewing safety and control cost as constraints on capacity.

Airspace design must therefore change its dominant paradigm and fully include control cost and inherent safety as objective functions. A systematic design process considers trade-offs between performance levels, life cycle costs, operational benefits, and technical and functional integration risks, driven by performance objectives such as capacity, safety, affordability and environmental impact, has long been advocated and called for [55] but has yet to be implemented.

Indeed, the importance of the safety-driven and human-centered design has been duly noted in the literature. Proponents have argued that integrating new components into an airspace management system should account for these two principles, allowing for critical properties to be designed into the system from the start [95]. When these perspectives are ignored, after-the-fact safety assessment leads to iterative model adjustment instead of redesign. Likewise technology-centric airspace design that assumes the human cannot handle the control cost of the airspace, yet leaves the human supervising automation, may lead to accidents easily blamed on pilot or controller error when in fact the mistakes lie with the designer [117].

A proposed solution in the direction of designing airspace according to additional objectives has been the full resectorization of the airspace in order to optimize taskload allocation between sectors [172]. This approach, however, falls short of suggesting the more fundamental redesign. Compared to current practices, it is only a slight improvement in that it takes a more global view instead of the localized, center-based perspective.

This thesis advocates the reconsideration of control cost and inherent safety as principal objectives integrated in the early process of airspace design alongside capacity. The empirical applications in Chapters 3 and 4 show how these factors can be evaluated using lean models and result in conclusions about airspace design. The
specific motivation of this work is to move away from treating safety and control cost as constraints, but rather providing a way to control them in the airspace design phase. This motivation leads directly into the two applications shown in Chapters 3 and 4.

2.3 Proposed concept of operation

2.3.1 Present organization

Air Traffic Management (ATM) is the service ensuring the movement of aircraft in the airspace. ATM can be broken down based on the forecast and action time horizon into two components: Air Traffic Control (ATC) and Traffic Flow Management (TFM). ATC is the tactical safety separation service. ATC look-ahead time horizon when interacting with a given aircraft is usually under 15 minutes. TFM is the strategic traffic allocation service which manages flows and allocates routes to demand. TFM look-ahead time horizon ranges from an hour to as much as months.

To fulfill its function, ATM has three essential capabilities: Communication, Navigation and Surveillance (CNS). Communication occurs between aircraft, pilots, and air traffic controllers. Navigation is the capability of aircraft to follow a given trajectory, with prescribed precision. Surveillance is the capability of air traffic controllers to monitor the position of the aircraft relative to each other, and relative to terrain or restricted areas.

In the United States, the National Airspace System (NAS) is partitioned in twenty-four Air Route Traffic Control Centers (ARTCC). Twenty of these cover the continental U.S., two additional centers handle traffic over and around Hawaii and Alaska, and two more are responsible for the territories of Puerto Rico and Guam. Each of these centers is further subdivided into different sectors, as shown in Figure 3 [1]. The sectors are staffed by different sets of controllers and use distinct radio frequencies for communication with the aircraft. As aircraft traverse the NAS and are in
different stages of their flight from takeoff to landing, pilots must switch frequencies and communicate to tower, terminal radar control TRACON, several ARTCCs, again TRACON, and tower.

![Figure 3: NAS centers](image)

The boundaries are seldom redefined other than merging adjacent sectors during off-peak hours such as late-night shifts. Resectorization only occurs in the current system at the traffic managers’ initiatives, and is performed in a way aimed at balancing controller workload and increasing the level of perceived traffic structure. Sector shapes thus usually account for major traffic flows. The purpose of current airspace management is primarily to avoid controller overload. Each sector has an aircraft capacity limit, called the Monitor Alert Parameter (MAP), notifying when such overload might be possible [48].

The primary building blocks of the current airspace are sectors, routes, fixes, and different airspace classes [48]. Class A airspace is the high-altitude enroute airspace between 18,000 feet (flight level 180) and 60,000 feet (FL 600) and requires filing of an instrument flight rules (IFR) flight plan. Class B airspace is the airspace below
10,000 feet around the busiest airports, and is designed on a case by case basis. Class C airspace is beneath 4,000 feet around medium-sized airports with a control tower and radar. Class D airspace is beneath 2,500 feet around airports with a control tower. Class E airspace is all controlled airspace that is not one of the other classes between A-D. It is used by aircraft transiting between the terminal and high altitude enroute airspace, and makes up most of the current low altitude en-route airspace. Class G airspace is uncontrolled airspace. Figure 4 shows the classes of airspace.

![Airspace-at-a-Glance](image)

**Figure 4:** Airspace classes [3]

Two types of routes comprise the network on which aircraft travel: jet routes (J) in Class A airspace and victor (V) routes in Class E airspace. The route network connects ground-based navigational aids (NavAids) and at times forces aircraft to move in a zigzag fashion. When compared to wind-optimal or great circle routes, the total length of all victor routes currently in use is 9.3% greater, and that of the 303 jet routes is 4.7% greater [59].

Current ATM infrastructure is becoming obsolete, and two major programs are being developed to overhaul this aging system. The programs are Next Generation
Air Transportation System (NextGen) - introduced by the Joint Planning and Development Office (JPDO) in the USA [77] - and Single European Sky ATM Research (SESAR) - introduced by SESAR Joint Undertaking (SJU) [149] in the EU. With these programs, ATM moves toward leveraging new technologies such as digital communication, satellite navigation, and other improvements which reduce inefficiencies.

2.3.2 Transition from centralization to decentralization

Current instrument flight rules (IFR) operations are centralized. Traffic data is gathered at a central point via radar or radio, and conflict detection and resolution (CD&R) is performed at the centralized point by a human air traffic controller, who then transmits instructions by radio to the entire sector, verbally specifying which aircraft they are destined for. Aircraft are expected to comply unless safety concerns arise. Some suggested future concepts of operation have sought to maintain this organization whilst replacing the controller with ground-based automation which centrally coordinates navigation in the airspace and broadcasts clearances via datalink [38]. These centralized concepts rely mainly on accurate trajectory prediction and following.

Conversely, many speculative decentralized concepts of operation have revolved around the ideal of free flight [37], in which aircraft have the authority to select their own trajectories, subject to some constraints to ensure separation. Under this concept of operation, on-board CD&R was considered to be key. A copious amount of research inspired by free flight has therefore been carried out since the 1990s on the topic of CD&R [87]. In current operations, a decentralized approach is only found in VFR operations where pilots are responsible for avoiding other aircraft.

Current regulations notwithstanding, nearly twenty years after it was first mentioned [119] true free flight continues to seem a far-term concept at best. Of more
relevance to this thesis is the literature concerned with the near- to mid-term timeline, grounded to some extent in existing or foreseeable practices. Significant research surrounds future concepts of ATM with varying degrees of autonomy, where some aircraft can fly under a centralized mode, while others fly under a decentralized mode of control [89]. The allocation of control modes need not be fixed for an aircraft and may dynamically vary depending on internal or external circumstances, as can the specific function allocation between modes depending on the respective intents and time horizons (tactical or strategic) of each task. In general, it has been accepted that a centralized paradigm would provide more equitable and wide-ranging optimality of the airspace if complete and accurate information exists about current aircraft locations and predictions of future position. Conversely, a decentralized paradigm is thought to provide more individualized optimality, to distribute workload, and to be more resilient in the face of degradation [86].

Preliminary testing of autonomous navigation scenarios showed that airborne separation could potentially be safely performed with five times the typical traffic density compared to today’s NAS [28]. However, some form of supervisory control of separation by controllers remains the most viable concept for separation assurance and collision avoidance [36]. Along those lines, NASA has proposed three major concepts that progressively introduce autonomy into operation, transitioning from current practices without abruptly removing the human controller from the system.

Autonomous Flight Management (AFM) is a concept which supports demand-scalable capacity, user flexibility and autonomy [169]. It is envisioned AFM operations would operate in a shared airspace with the existing ground-controlled flight operations. The concept is supported by Airborne Separation Assistance System (ASAS) technologies which typically allow the aircraft to manage certain components of its 4-D trajectory within the set of operational constraints negotiated with the Air Navigation Service Provider. To certify participant aircraft to rigorous and
appropriate performance standards, the existing Required Navigation Performance (RNP) construct could be extended into a dynamic standard [168].

Function allocation concepts have been compared in terms of separation assurance in high density airspace [170], with homogeneous airborne and ground-based approaches. Where comparisons were possible, no substantial differences in performance or operator acceptance were observed. Mean schedule conformance and flight path deviation were considered adequate for both approaches. Conflict detection warning times and resolution times were mostly adequate, but certain conflict situations were detected too late to be resolved in a timely manner, which led in both experiments to situations compromising safety and where workload was rated as unacceptable.

Another NASA concept in its initial deployment stages is the Distributed Air/Ground Traffic Management concept (DAG-TM), also enabled by ASAS. ASAS systems enable pilots to resolve near-term conflicts even with lower separation minima (3 NM instead of 5 NM). With prototype systems, an improvement was found in threat proximity and risk mitigation when pilots followed tactical guidance cues provided by ASAS, although the compliance rate remained relatively low [8]. Performance and reported acceptance of the use of procedures relying on airborne separation assistance and trajectory management tools was also tested [29]. A lognormal distribution was found to be a good fit for the pilot response delay. Additionally, it was determined that the total pilot response times were well within ranges verified during prior stress tests and no safety impact in terms of loss of separation was observed. Recommendations for the interface design of an integrated weather and traffic information cockpit display that would improve compliance have been formulated [26].

The feasibility of the DAG-TM concept was tested by means of a human-in-the-loop study of autonomous aircraft operations in a highly constrained airspace [83]. Results showed that reducing the lateral separation standard had no adverse effect on the pilot ability to meet traffic flow management constraints, and had the positive
effect of decreasing the incidence of cascading conflicts. In near-term conflicts, the pilots’ ability to mitigate risk depended more on their compliance with the tactical resolution guidance than on the size of the conflict detection zone. The use of priority rules improved the pilots’ ability to conform to airspace constraints but was not critical for ensuring separation in over-constrained scenarios. Among the conclusions applicable to cockpit traffic advisory systems, it was found that broadcasting the commanded trajectory (i.e. the actual 4-D command law from the avionics) rather than the proposed but unachievable FMS flight plan, would reduce unnecessary maneuvering and the adverse interactions between aircraft resulting from false conflict alerts.

A third concept related to AFM and DAG-TM is Cooperative ATM (Co-ATM) which was formalized to describe the first stages of introducing autonomy into ATM operations. Co-ATM envisions coexisting present-day and future semi-autonomous operations [128], and distinguishes between sector controllers who control conventional aircraft along predictable flight paths and area controllers who coordinate strategic trajectory changes with the flight crews of equipped aircraft. The airspace is shared between the two, and coordination occurs via data link. Area controllers rely on automation support for conflict detection and resolution and for traffic flow management. The automation performs routine tasks such as hand-offs. Properly equipped aircraft may be cleared to operate at different levels of autonomy, including self-separation. Initial testing of this concept has shown potential for increasing en-route capacity [130], however the research has yet to reach a maturity level sufficient to generalize its predictions.

Finally, other research introduces a more theoretical distributed approach to manage traffic complexity while preserving aircraft trajectory flexibility. The underlying hypothesis of this work is that autonomously preserving trajectory flexibility is sufficient to naturally avoid excessive traffic complexity on an aggregated scale [66].
Furthermore, an assumption is that individual trajectory flexibility can be increased by collaboratively minimizing trajectory constraints, which in turn does not jeopardize the intended air traffic management objectives.

2.4 *Envisioned airspace paradigm*

This thesis allows for two navigation modes that may co-exist: autonomous flows in which aircraft self-separate, and conventional routes which rely on direct air traffic control guidance. For autonomous flows, a human or automated controller must intervene to return aircraft that have reached the tolerance bounds of their 4-D trajectories. Assuming formally correct automation is responsible for establishing 4-D trajectories, imprecision still arises from the unavoidable discrepancy between an algorithmic model and the physical world, a phenomenon known as total system error (TSE). For controlled routes, a controller must intervene to correct trajectories and prevent a loss of minimal separation.

While other sources of uncertainty may exist in the full-scale system - such as equipment reliability and human controller awareness - the aforementioned variabilities are inherent to even idealized operational settings and provide baseline estimates.

Over time, as many aircraft traverse the sector, the cumulative effect of several improbable events may ultimately require the controller or automation to intervene and realign an aircraft with its intended 4-D trajectory. The volatility of trajectory deviations is a measure of variability for the oversight activity the controller must provide.

Conversely, in conventional airspace, controllers must themselves perform conflict detection and resolution (CD&R) work and directly intervene to communicate trajectory changes. The variability of arrival schedules into the sector, and that of the occurrence of intersecting trajectories, translates into a measure of variability for the CD&R activity, that the controller must provide.
The following sub-sections describe the implication of this paradigm for the airspace, inherent safety, and control cost.

2.4.1 Airspace attributes

The airspace refers to an abstract construct defined by route geometry, autonomy allocation, and traffic levels. The model of airspace parametrized by the aforementioned items has been previously used in the literature [72]. The geographical extent of the airspace in this thesis corresponds to approximately an airspace sector, roughly 150 nautical miles across. The thesis assumes no resectorization, and does not investigate redesigning the boundaries specifically. The research also does not attempt a comprehensive mathematical formulation of the airspace as a dynamical system.

Autonomy defines an aircraft’s capability, authority and responsibility for trajectory determination. Routing decisions are of two types: strategic route definition and tactical trajectory determination. This thesis only focuses on the strategic level. Aircraft are authorized to determine their trajectories through a limited network of flows. The network is a model for routes in the airspace, segregated between autonomous and directly controlled routes. The degree of autonomy, defined by the size of the autonomous subnetwork within the whole sector network, is one of the factors defining an airspace structure. In this research, two navigation modes coexist: autonomous flows which function by self-separating aircraft navigation, and conventional routes which are reliant on direct air traffic control guidance.

With changing operational paradigms requiring 4-D navigation, such as flow corridors, the nature of air traffic patterns and the complexity of traffic will shift. Different sources of uncertainty and variability may emerge, such as the imprecision of autonomous trajectory determination [125], the occurrence of scheduling errors [9], or the evolving traffic distribution between points of entry and exit [58]. Relative to current operations, controllers will thus be required to interact with the airspace in
new ways.

The disparate effects of various airspace configurations are yet to be understood on an aggregated scale. Control cost and inherent safety descriptors of the airspace that are constructed in this thesis emerge from the granular level of aircraft-by-aircraft behavior. The aircraft granular behavior is translated into a more macroscopic system view through a network model of routes.

### 2.4.2 Inherent safety

Inherent safety refers in this thesis to a route structure which is insensitive to predictable errors and deviations, regarding aircraft separation and operation. A conflict is a loss of minimal separation between aircraft, as defined by FAA regulations. Typically, minimal separation is 3-5 nautical miles depending on the phase of flight.

The traditional view of airspace design sees safety as a constraint on allowable states. Safety has been and continues to be a foremost concern of air traffic, with a clear metric in its symptoms: Airspace is generally considered to be safe if no conflicts and collisions occur. Yet safety has an unclear definition in its roots. Is it a matter of system reliability [94]? An emergent property [150]? Because of the unclear characterization, safety-driven design in the airspace only exists for IFR terminal approach procedures, through the FAA TERPS [41] and ICAO PANS-OPS [68].

For that reason, an adequate metric must be defined. A first-order measure of safety is the physical distance between aircraft. But separation is a static measure that falls short of giving the full picture. Indeed, how separation evolves is affected by traffic density, trajectory geometry, and trajectory imprecision or risk of deviations from the prescribed intent and nominal conditions. Figure 5 shows two circumstance with the same separation. Intuitively, however, it is clear that the diverging aircraft present safer conditions than the converging aircraft. A more complete assessment of safety requires a complex and eluding construct meant to describe a dynamic
situation.

(a) Diverging aircraft  (b) Converging aircraft

**Figure 5:** Aircraft separation

In the literature many possible surrogates such as hockey pucks, ovaloids, trumpet shapes have been used in order to characterize safe trajectories [87]. Figure 6 shows three types of models: nominal, worst case, and probabilistic [51, 87]. In the nominal case, perfect information and trajectory accuracy are assumed. Aircraft avoid conflict as long as they do not directly cross each other’s paths closer than the minimum separation distance. In the worst case, it is assumed that an aircraft could deviate anywhere within their turning radius at any given time. Therefore, conflicts are avoided if other aircraft do not come within a range less than the minimum separation distance, over a wide range of bearings. In the probabilistic case, position and trajectory uncertainties are modeled by random variables, and conflict risk becomes a fuzzy measure determined by a probability density [51].

The traditional description of airspace safety, static or probabilistic, is intent-based. Separation in the airspace is assured if aircraft follow their prescribed routes (with some level of accuracy). This thesis seeks to find routes that are inherently safe in terms of being robust to potential deviations without trying to assess the probabilities of those deviations. Therefore, another quantifiable surrogate for safety is needed. In this thesis, the Traffic Alert and Collision Avoidance System (TCAS)
TCAS logic evaluates conflicts in state-based conditions instead of the more traditional intent-based view. A set of trajectories are safe - and so is ultimately the airspace - if following nominal trajectories does not result in TCAS advisories being triggered. Figure 7 shows navigation conditions which are safe in an intent-based perspective (if aircraft follow their routes and turn), but not in a state-based view (their present velocity vector leads to a collision).

The use of TCAS logic thus provides a path toward ensuring airspace resilience by testing the intent-based safety of nominal conditions with the state-based safety of dynamical operation. Routes can therefore be designed in a way such that no collisions or near misses could occur within a sensible interval (30 seconds to a minute). In previous studies where routes were redesigned for new types of operations, such as for continuous descent arrivals (CDA), designers used nominal conditions with large tolerance bounds to ensure separation [139].

Using TCAS as a proxy of safety allows the designer to anticipate and proactively avoid the propagation of cascading deviations: a TCAS advisory means that following an initial deviation, two aircraft are now in unsafe circumstances and must coordinate a maneuver. In cases of high-density traffic, these two maneuvers may trigger
This thesis considers that precise route planning and flow scheduling under nominal conditions would improve separation assurance and thus reduce the need for controller interventions. However, this thesis asserts that safety is not only about being able to follow routes with extreme precision, even if such routes are nominally safe according to instantaneous static separation assurances. The thesis assumes that these nominal standards are reliably defined at a broader spatial and temporal resolution, which is outside the scope of the thesis. The reduction of controller-requested conflict avoidance maneuvers is a consequence of nominal separation assurance. This thesis instead focuses on degraded conditions. Indeed, for many reasons ranging from human error to equipment failure, nominal conditions can degrade [51] and routings or clearances can be breached. It is in that case that the TCAS-based description of safety is relevant. In terms of the allowable separation provided by TCAS, it has been shown that spacing using this system is primarily limited by aircraft dynamic response rather than uncertainty of data [142]. Therefore, resorting to TCAS as a surrogate to distinguish dangerous from benign trajectories makes sense.

### 2.4.3 Control cost

The control cost is a construct which identifies the requirements necessary to control a system. In this research, the evaluation of control cost pertaining to an airspace
structure refers to the rate of required interventions by the controller or automation to avoid degradation and maintain stability. The control cost changes when allocating partial autonomy to some flows (allowing some aircraft to self-separate) while retaining control of others (requiring ATC control as in current practices). The control cost also changes due to increasing traffic volumes.

The traditional view of control cost has been limited to human controller taskload. Research has studied the effect of airspace configurations and traffic patterns on taskload. Sources of variation include traffic complexity, quality of equipment, individual differences, and cognitive strategies [105]. In the traditional view, taskload has been a constraint on allowable states of airspace design.

Conversely, this thesis introduces control cost as an objective to guide airspace design. Control cost has two sources depending on navigation modes: deviations in autonomous airspace, and conflict resolution in controlled airspace. For autonomous flows which function by self-separating aircraft navigation, the controller or automation must intervene to return aircraft that have reached the tolerance bounds of their 4-D trajectories. Assuming formally correct automation is responsible for establishing 4-D trajectories, the imprecision comes from the unavoidable discrepancy between an algorithmic model and the physical world, a phenomenon known as total system error (TSE). For controlled routes which are reliant on direct air traffic control guidance, a controller must intervene to correct trajectories and prevent a loss of minimal separation. Figure 8 shows the two sources of control cost.

In autonomous space, both the human controller and automation may intervene when aircraft deviate outside of their tolerance bounds. Centralized automation on the ground may take on the task of detecting such deviations and send a corrective message to the pilot via datalink. It is however not likely to be implemented in the near future given current regulatory environment and legacy equipage. First, the automation that separates aircraft in the autonomous flow is located on the aircraft,
not on the ground. Thus the data that the onboard autonomous system is using is actually different from the data the controller or ground-based automation sees. Second, even in such circumstances where centralized ground automation would directly communicate with aircraft, experiments have identified controllers’ reticence at completely giving up oversight of aircraft routing and conflict resolution [129]. Other studies have shown that controllers’ workload increases when autonomous aircraft deviate without signaling intent [37]. Thus, even if the automation would actually be tasked with communication, it is likely that controller’s attention will nonetheless be solicited. Therefore, control cost in both cases has an impact on human performance.

2.4.4 Summary and justification of the reframed problem

A wide range of alternative concepts of operation are possible and have been considered in the literature - ranging from ground-based automated centralization to full free flight [11, 13]. This thesis examines the specific paradigm defined by partial allocation of routes between autonomous 4-D flows and controlled space.
The merit of this concept is its grounding in a real-world setting. In any system, transitions are the hardest to manage. The concept of operations is a plausible evolution of air traffic operations, perhaps even a necessary step toward operations which deliver on the promises of NextGen or SESAR. To some extent, modern operations such as continuous descent approaches (CDA) and RNP already delegate some level of autonomy to the aircraft [7]. Yet while technological advances and modern capabilities are considerable, operational changes lag behind.

The work is placed within a defined concept of operations, focused around enroute traffic. The concept of operations involves segregated autonomous and controlled regions of the airspace, distinguished by separate subnetworks. From a control standpoint, the concept can be referred to as mixed centralized-decentralized control. From a human factors perspective, the concept involves a mixed active-passive role of the controller.

The objective of this work is to design airspace for inherent safety and control cost. This chapter reviewed how safety and control cost have been used descriptively. The approach taken in the thesis uses them prescriptively, and expands the approach of airspace design centered on capacity improvements. Since NextGen and SESAR state economic performance goals, capacity is one objective which continues to be relevant, but inherent safety and control cost must also be accounted for in design process for several reasons.

First, additional metrics such as the aforementioned ones would not easily specified if determined after-the-fact. Instead, the concept and methods introduced in the thesis allow an explicit trade-off. Furthermore, focusing airspace design on capacity improvements restricts stakeholders’ perspectives. A limited perspective in turn has repercussions on the types of technologies which are considered of interest, and leads to a poorly-defined technology-centric design. Other limitations can also result from a narrow focus on capacity improvements, such as limited types of operational changes.
are envisioned and considered. As a result, limited paths become apparent in the system evolution process, rendering the task of reaching performance goals stated by the NextGen concept of operations ever more arduous.
CHAPTER III

MODEL AND APPLICATION OF CONTROL COST

This chapter demonstrates airspace design centered on control cost, based on research published by the author [121,122]. This chapter introduces tools to clarify the control cost of allocating partial autonomy to some flows while retaining control of others, and also the expected control cost due to increasing traffic volumes. Control cost is measured by the resulting requirements needed to maintain the part-autonomous and part-controlled structure, i.e. the rate of required interventions, by the controller or automation, that ensure safety of the airspace.

A lean model of control cost is constructed in the form of a one-parameter stochastic process that allows the mapping of design variables (specifically, autonomy allocation) to design evaluation metrics (specifically, control cost). Lean models are defined by four characteristics: they are compact descriptors, have forecasting ability, can guide systematic airspace design, and require low computational cost for use online.

The research envisions two navigation modes coexisting: autonomous flows which function by self-separating aircraft navigation, and conventional routes which are reliant on direct air traffic control guidance. In autonomous flows, a stochastic model is used to estimate the probability of individual aircraft deviation away from the prescribed 4-D trajectories and associated bounds. Over time, as many aircraft traverse the sector, it is the cumulative effect of several improbable events that ultimately may require a controller to intervene and realign an aircraft with its intended 4-D trajectory. The volatility of trajectory deviations is a measure of variability for the oversight activity the controller must provide.
Conversely, in conventional airspace, controllers must themselves perform conflict detection and resolution (CD&R) work and directly intervene to communicate trajectory changes. The variability of arrival schedules into the sector and of the occurrence of intersecting trajectories translates into a measure of variability for the CD&R activity the controller must provide.

This chapter presents the result of a Monte Carlo simulation method, which incorporates stochastic models of both autonomous and controlled routes. A diagram of the airspace simulation architecture constructed for this research is shown in Figure 9.

![Airspace simulator diagram](image)

**Figure 9:** Airspace simulator diagram

An overview of the state of the art for topics relevant to the work is given in Section 3.1. In Section 3.2.1, we present the navigation model used to characterize the aircraft behavior along the autonomous flows, as well as reasonable numerical values of the model parameters taken from the literature. Section 3.2.2 provides an overview of the other simulator models used in this work, such as the autonomous flow scheduling model, the conflict detection and resolution CD&R simulation, and the traffic model used on controlled routes. This network structure is used to partition the airspace between autonomous and directed traffic. In Section 3.3, we provide the results of the Monte Carlo simulation. Insights from the model are discussed in Section 3.4. Section 3.5 concludes the chapter by suggesting optimal policies for autonomy allocation in the airspace and discussing some of the insights obtained from this simulation, including
a proposed Poisson process as the lean model for control cost. The Poisson process is found to be the maximum likelihood model. The formulas and theoretical derivations used in this chapter are detailed in Appendix A.

3.1 Background

This section reviews the literature and state of the art relevant for the control cost application. After a discussion of the traditional view of control cost, the concept of flow corridors is presented, a discussion on aircraft navigation models and conflict detection and resolution algorithms is carried out. The section ends with a review of air traffic flow modeling.

3.1.1 Traditional view of control cost

In most research meant to quantify the requirements of a human controller interacting with the airspace system, taskload and workload have often been used interchangeably. In the commonly accepted definition, workload is a mental construct which cannot be directly observed [78]. The construct of controller workload has been extensively studied [35], and measurements related to workload are grouped into three types: objective performance measures for primary or secondary tasks, subjective measures, and physiological measures. Very diverse approaches have been tried [40, 136]: subjective self-rated questionnaires adapted from NASA TLX [56], psycho-physiological measures, contextual, environmental and behavioral studies, action and event counts, or complexity modeling [105]. Other models of taskload requirements include aggregated temporal cost by tasks involved in controlling traffic [146]. Yet the literature to date has not provided a model that is compact, accurate, and general.

This thesis is restricted to a simple and easily observable metric well suited to the scope of this research: control cost. Control cost can be manifested as controller taskload or automation interventions. In both cases, control cost is defined as the rate
of required interventions that ensure safety of the airspace. We recognize that control
cost thus defined is only a subset of aggregated taskload relating to all controller
activities, and also a subset of the construct of workload [5]. Early in airspace design,
however, only control cost can be estimated.

It is known that taskload is related to interactions with the airspace and control
strategies [143]. Yet the research concerning the stochastic modeling of controller
interaction contains significant gaps: older research has shown that subjective conflict
occurrence as perceived by controllers approximately adheres to a Poisson distribution
[34]; the influence of air traffic controllers on the aircraft separation had been modeled
as a Gaussian noise which has led to the expression of constraints expected to improve
throughput [73].

Similarly, upper bounds on feasible resolution rates at flow intersections appear
in [159]. However, each of these approaches only focuses on single, specific details
of a controller’s interaction with air traffic. Many approaches have attempted to
characterize traffic effect on workload on a more aggregate level. Some examples
include deriving capacity from notions of airspace structure [72] or through metrics
of complexity such as dynamic density [91].

Research has also identified a need to understand the effect of changing airspace
configurations and traffic patterns on the taskload of air traffic controllers. Four
main factors have been shown to influence taskload: the primary factor is air traf-
cic complexity, an ill-defined construct. Mediating factors are quality of equipment,
individual differences, and cognitive strategies [105]. Complexity refers to a descrip-
tion of air traffic patterns and sector characteristics and has produced much research
seeking to best quantify it. A well-known metric to that end is dynamic density [153].
Quality of equipment considers the accuracy of radar systems or the usability of user
displays and interfaces. Individual differences account for anxiety levels, personality,
age, and experience. Cognitive strategies are the approaches used by controller to
mentally process the requirements of handling traffic.

Cognitive strategies have been extensively studied [65], and it has long been known that controllers rely on underlying airspace structure to reduce the cognitive complexity of their tasks. Airspace structures are believed to influence controllers’ mental models in a way that simplifies planning, implementing, monitoring, and evaluating tactical situations, as well as maintaining situational awareness [141]. Yet little research attempts to apply taskload models in a proactive manner to design airspace. The traditional perspective is that of a causal dependency going from airspace design to its consequence on control cost. The possible feedback which would promote airspace design with taskload in mind is not addressed.

Taskload is related to airspace configurations through estimations of sector capacity that usually call on controller taskload models in fast-time computer simulations. A review has identified two major models in use, the Reorganized ATC mathematical simulator (RAMS) and the Total Airspace Airport Modeller (TAAM). RAMS is used in the UK, Spain, Portugal, Italy, and Sweden. TAAM is used in Germany, Switzerland, France, Canada, Japan and the US. These models are discrete event simulation models which consider a number of defined events such as entry into and exit from the sector, conflict detection, and conflict resolution. A different approach is the Sector Design and Analysis Tool (SDAT) developed by the FAA. SDAT is an analytical model of controller workload built from historical radar tracking and radio communication data, without consideration of conflicts [97]. Yet in both analytic, stochastic approaches, as well as more qualitative, operational perspectives, taskload has been modeled as a constraint in airspace design solutions instead of being modeled as an objective to guide airspace developments.
3.1.2 Flow corridors

The flow corridor concept introduced in the NextGen concept of operations [77] describes aircraft following high-precision 4-D-metered trajectories and managing their in-trail separation. Similarly, the SESAR goal [149] of trajectory-based operations (TBO) relies on 4-D trajectories with traffic flows that are deconflicted before flight in order to reduce tactical interventions during flight. The TBO concept is operationalized as RNP trajectories enabled by equipment certified to operate within probabilistic tolerance bounds [69]. In the future, the standards are expected to converge on more uniformly stringent requirements, which will impact the oversight requirements [43].

Under future operations, flow corridors will rely on self-separation, a capability achieved by fully leveraging Automatic Dependent Surveillance-Broadcast (ADS-B) [44]. ADS-B equipment allows for the construction of high-density, high-altitude enroute corridors. Relative to conventional controlled airspace, flow corridors allow multiple parallel lanes of traffic, require self-separation by means of advanced avionics supporting 4-D trajectories [107], and use dynamic activation rules to add or remove corridor structures with changing demand [173]. Preliminary testing [28] of autonomous navigation showed that airborne separation could potentially be safely performed with up to five times current traffic volumes. The validity of these concepts and results is examined in this research.

3.1.3 Aircraft navigation models

Much of the trajectory variability research invokes position uncertainty. The classic characterization of position uncertainty models the error as normally distributed with a time-wise linear standard deviation of the along-track error and a constant standard deviation of the cross-track error [115]. This model has been expanded to represent aircraft oscillations around their prescribed trajectories using Wiener processes [60],

35
and to include dynamics where both position and velocity uncertainty are modeled by superposed Wiener processes [15]. Computational simulation have included hybrid-state Markov processes with switching coefficients and or Petri nets [17, 18]. By introducing uncertainty in the position and trajectory evolution, research showed that the minimum relative displacement between conflicting aircraft can follow a normal distribution, and that the minimum distance between them has a folded normal distribution [71]. While the Wiener process, or Brownian motion, is a good approximation over short time intervals, longer time frames may lead to divergence and inconsistent results. To better model behavior over longer intervals, the Ornstein-Uhlenbeck model - also called mean-reverting - introduced in [122] can be applied.

3.1.4 Conflict detection and resolution algorithms

Probabilistic models of trajectory variability have focused on understanding the occurrence of conflicts. ICAO has defined the acceptable levels of fatal accident risk at one mid-air collision, or physical incrossing, per $10^9$ flight hours [67]. The classic model used to predict risk values for air traffic management, known as the Reich collision model, was developed in 1964 [137]. Over the last twenty years, the Reich model has been generalized to express collision risk based on first hitting times of Markov processes [6, 19].

The application of conflict detection research is usually the design of resolution algorithms, with varying approaches to state information, state propagation, detection threshold, resolution method, maneuvering dimensions, and multiple conflict handling [87]. Various approaches include time-inhomogeneous Markov chains to describe the relative position between two aircraft [62], spatially correlated wind perturbations [61], and decentralized algorithms and used reachability analysis [165] to solve the resulting stochastic differential equations through potential fields methods [127] or through switching diffusion models approximated by Markov chains [126]. In more
recent research, the conflict resolution algorithms have used mixed integer programming [116]. This work builds on the fuel-optimal mixed integer linear program with parametric workload constraints presented in [161].

3.1.5 Air traffic flow models

Despite all the aforementioned research, little is available on stochastic traffic flow management and route structure design. Poisson process models for traffic flows have been postulated in the past, and research has developed management strategies by postulating such models of Poisson process flows [106, 162]. Furthermore, through simulations and flight tests, feasible flow throughput bounds have been shown to be approximatively Erlang-distributed [139, 140]. However, evidence concerning inconsistent variance levels indicates limitations of this model [147]. This work uses the Poisson model but also tests the sensitivity to variations in the distribution.

The Poisson flow model has nevertheless raised some doubts, as conflicts at route intersections were shown to be expressible as a sum of correlated random variables with a variance larger than that obtained by the Poisson flow model [147]. Research has also shown that the circadian variations of the aircraft arrival process do not correspond to a homogeneous Poisson process, but that the arrivals can nevertheless be modeled as a non-homogeneous process over that time frame [145].

3.2 Method

This section details the method used in the control cost demonstration. The different models are introduced, starting with the autonomous navigation model which is an original development of this research. The section continues with other simulation modules which were developed elsewhere but integrated for the purpose of this research.
3.2.1 Autonomous navigation model

A diagram of the autonomous aircraft navigation module called by the simulator is shown in Figure 10. A detailed explanation for each step of the process follows in this section.

3.2.1.1 Ornstein-Uhlenbeck stochastic aircraft model

This section explains the model used to simulate aircraft autonomous navigation. The model attempts to capture the uncertainty with which aircraft follow trajectories, also known as Total System Error (TSE). The ICAO Performance-based Navigation (PBN) manual [69] defines TSE using Equation (1) as the root sum of squares of Path Definition Error (PDE), Flight Technical Error (FTE), and Navigation System Error (NSE), for which it assumes independent, zero-mean Gaussian distributions.

PDE occurs when the path defined in the RNAV system does not correspond to the path expected to be flown over the ground, and is assumed negligible due to the multiple pilot checks and read-backs that prevent erroneous waypoint programming; FTE relates to the air crew or autopilot’s ability to follow the defined path or track; NSE refers to the difference between the aircraft’s estimated position and actual position. Further ICAO assumptions are that FTE is an ergodic stochastic process within a given flight control mode but that nothing can be said of the NSE due to
sensor errors, relative position from navaids, and inertial errors.

\[ TSE^2 = FTE^2 + NSE^2 + PDE^2 \]  \hspace{1cm} (1)

To convert the static description of error into a dynamic model that can be integrated to the simulation based on an explicit stochastic differential equation, TSE oscillations around a deterministic trajectory are modeled as an Ornstein-Uhlenbeck mean reverting process. The Ornstein-Uhlenbeck process is a stationary Gaussian process with bounded variance and is governed by the stochastic differential equation shown in Equation (2).

The mean reverting model is meant to reproduce the behavior of inaccurate navigation (modeled in the volatility and the Brownian oscillation \( \sigma dW_t \)), with a controlling cockpit (pilot and guidance equipment jointly functioning as a complex socio-technical system \([65]\)). The cockpit attempts to prevent major deviations (modeled in the elasticity \( \kappa \)) from the deterministic trajectory (the mean \( \mu \), also referred to as process drift). To our knowledge, this model was first introduced in \([122]\), but has not thus far been used in a complex air traffic simulation.

\[ dX_t = \kappa (\mu - X_t) dt + \sigma dW_t \]  \hspace{1cm} (2)

Here \( \mu \) is the mean vector, \( \kappa \) the elasticity matrix, \( \sigma \) the volatility matrix, and \( W_t \) a Wiener process (standard Brownian motion). \( X_t \) represents the actual position of the aircraft relative to its prescribed position in the 4-D trajectory. This corresponds to a change of variable \( X = P_{4D} - Vt \), where \( P_{4D} \) is the time parametrization of the 4-D trajectory, \( V \) is the velocity vector, and \( t \) is time.

The Ornstein-Uhlenbeck model is an order increase in complexity over the Brownian model, but more applicable to the longer times it takes an aircraft to cross a sector, circa 20 minutes. Under the Brownian model for longer characteristic times,
the position of the aircraft would be an indifferent diffusion. This makes the model unrealistic, while under the Ornstein-Uhlenbeck model the elasticity term more strongly bounds the probability of higher deviations. For this model, the expected total delay and spatial deviation summed over the trajectory length is 0. Nevertheless, the amplitude and uncertainty of local occurrences is critical to study as these will prompt a controller intervention. In other words, the Ornstein-Uhlenbeck process provides a more realistic model of aircraft adherence to a prescribed trajectory for longer time frames, and is better applicable in the frame of analyzing controller oversight roles.

Mathematically, the future position of a Brownian realization has a normal distribution with variance that grows linearly with time. As variance grows, the normal distributions flattens and widens, leading the realization of the process to almost surely hit every point in space given enough time. While the Brownian model is satisfactory for short-time analyses of fleeting aircraft encounters [60], its use for longer times - on the order of magnitude of an aircraft crossing a sector, circa 20 minutes - is more difficult to justify.

This research seeks to obtain results on the aggregated effect through a Monte Carlo simulation where the Ornstein-Uhlenbeck elementary process is repeatedly used for autonomous aircraft within a full-scale simulator of air traffic within a sector. Some closed-form expressions exist which can help verify the result, such as the conditional probability of an aircraft reaching the allowed precision bounds (specified in the RNP standard) over a defined time frame. These derivations are carried out in Appendix A.

For autonomous flows, the control cost is calculated as the number of interventions needed to prevent an aircraft from trespassing the allowed precision bounds. The oscillation dimensions of an aircraft are calculated longitudinally, laterally, and vertically.

Furthermore, an important assumption has been made regarding the probability
Table 1: Ornstein-Uhlenbeck aircraft model parameters

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<th>Lateral</th>
<th>Vertical</th>
<th>Longitudinal</th>
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<tbody>
<tr>
<td>$\kappa$</td>
<td>$3.492 \text{ min}^{-1}$</td>
<td>$1.841 \text{ min}^{-1}$</td>
<td>$2.1662 \text{ min}^{-1}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$2.79 \cdot 10^{-2} \text{ NM}$</td>
<td>$8.034 \text{ ft}$</td>
<td>$9.965 \cdot 10^{-2} \text{ NM}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$7.27 \cdot 10^{-2} \text{ NM}$</td>
<td>$8.683 \text{ ft}$</td>
<td>$0.2774 \text{ NM}$</td>
</tr>
</tbody>
</table>

of pilot reaction. PBN requires that the airborne navigation equipment has the function of self performance monitoring and alert, such as receiver autonomous integrity monitoring (RAIM) [69], which enables the crew to monitor the TSE. In that case, should the pilot become aware of the deviation and react in time, we assume the controller would still be required to provide a certain amount of control cost. Control cost would be manifested either through a redundant intervention, a confirmation of pilot correction, or simply as the observation of the aircraft and readiness to intervene. However this control cost should provide a reasonable estimate on the differential control cost composed by different roles. To justify this we assume the volatility of the process captures all shortcomings in the capabilities of the cockpit - understood as a socio-technical system [65].

3.2.1.2 Model calibration

To calibrate the Ornstein-Uhlenbeck stochastic process model to match navigation uncertainty, the Gaussian and Markovian properties of the Ornstein-Uhlenbeck process are used to formulate the log-likelihood function [158]. The log-likelihood function is explicitly given in Appendix A and can be maximized numerically. The resulting parameters $\mu, \kappa, \sigma$ of the Ornstein-Uhlenbeck model used in the simulation - for spatial units expressed in nautical miles (lateral and longitudinal) or feet (vertical) and temporal units expressed in minutes - are shown in Table 1.

Due to insufficient experimental data required to calibrate the stochastic model, a random number generator was used to generate fictitious samples of error tracking
data. These samples conserve the statistical properties noted in the literature, and as such provide a good baseline representation of navigation error.

### 3.2.1.3 Generated empirical navigation data

A Johnson $S_U$ “unbounded system” [76] is identified in the literature [96] as the best fit probability density function for the lateral FTE of a Boeing 747, under RNP-0.3 sensitivity $^1$.

A note on the chosen level of accuracy: RNP 0.3 is the typical navigation accuracy during straight final approach, inside terminal airspace, where any aircraft will not only change horizontal positions continuously, but also lower its altitude simultaneously. In enroute flight, RNP-5 or RNP-2 may be more relevant in terms of requirements, while the actual FTE will depend on factors such as flight control laws and fight operating modes, leg types, flight conditions and aircraft performance. To stay away from arbitrary corrections of the parameter values, we choose to rely on the available data and restrict assumptions to suppose that aircraft in 4-D flows would fly using their highest available accuracy level. Section 3.3.2 addresses this assumption by testing under double FTE variance.

Equations (3) and (4) represent the analytical expressions of the probability density function for the FTE model used in this paper. The Johnson $S_U$ distribution is a transformation of the standard normal distribution, and is better adapted to model the observed heavy-tailed skewed data. If $X_{SNV}$ is a standard normal variate then its transformed $X_{SU}$ is of the type given in Equation (3), with $\gamma$, $\delta$, $\lambda$ and $\xi$ different scaling parameters for the family of distributions. The transformed density function is shown in Equation (4).

---

$^1$RNP is a type of performance-based navigation that defines the probabilistic tolerance bounds for a trajectory between two points in space. Thus, RNP-0.3 sensitivity means the cross-track error will remain less or equal to the RNP level (0.3 NM in this case) with 95% probability, and has a probability of less than 0.001% to exceed twice the RNP level (0.6 NM in this case)
The required Johnson $S_U$ parameters $\gamma$, $\delta$, $\lambda$, $\xi$ can be identified by numerically solving the Winterbon equations which are 24th order algebraic equations [171]. The equations rely on statistical moments to fit a corresponding probability density function. We in turn use the parametrized density function to define a random number generator. The random number generator uses the parameter values in Table 2 to generate samples that are used in the calibration of the Ornstein-Uhlenbeck process which in turn drives the autonomous navigation module of the simulator, as shown in Figure 10. The transformed standard normal quartile values shown in Table 3 can then be deduced from the resulting cumulative density function.

Due to the practical difficulty in obtaining experimental FTE recordings, the numerical calibration of the Ornstein-Uhlenbeck model (2) for the aircraft was performed based on simulated data. In conformance to the findings of Levy et al., the Johnson unbound system $S_U$ given at (3) was used to generate fictitious FTE data (see Figure

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<tr>
<td>$\gamma$</td>
<td>0.4566</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.897</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0443 NM</td>
<td>7.2907 ft</td>
<td>0.2145 NM</td>
</tr>
<tr>
<td>$\xi$</td>
<td>-0.01567 NM</td>
<td>10.0362 ft</td>
<td>-0.0401 NM</td>
</tr>
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</table>
Table 3: Johnson $S_U$ quartiles

<table>
<thead>
<tr>
<th></th>
<th>Lateral (NM)</th>
<th>Vertical (ft)</th>
<th>Longitudinal (NM)</th>
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<tbody>
<tr>
<td>$Q_1$</td>
<td>$-6.98 \cdot 10^{-2}$</td>
<td>1.147</td>
<td>-0.302</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>$-3.89 \cdot 10^{-2}$</td>
<td>6.215</td>
<td>-0.152</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>$-1.46 \cdot 10^{-2}$</td>
<td>10.2</td>
<td>$-3.52 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$Q_4$</td>
<td>$9.98 \cdot 10^{-3}$</td>
<td>14.27</td>
<td>$8.42 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

The properly parametrized random number generator provided data adhering to the moments in Table 4, with a 1-minute sampling time step which corresponds to the sampling rate of trajectory data used in the CD&R module elsewhere in the simulator.

Figure 11: Simulated FTE histograms and theoretical density functions
3.2.1.4 Statistical parameters

Some literature exists which can help quantify the various statistical parameters representative of navigation error. While [96] has significant information relevant to this research, little other numerical and statistical parameters for navigation error can be found in the literature, which mostly deals with small aircraft and general aviation [64,112,166].

Related work is focused on precision approaches, often in the context of continuous descent arrivals (CDA) [23,30,53], although some work on performance benefits from improved glass cockpits (synthetic vision) presents data on observed flight technical performance and error [148]. In one study, especially relevant to the present paper, prototype flight management capabilities under 4-D trajectories use temporal RNP baseline tolerance ranges [7]. Among older research, studies focused on GPS performance provide accuracy information for automated landing procedures [25], and RNAV navigation [113], while a noisy closed loop model of cross-track error during precision approach is found in [164].

Current ICAO standards for Precision-based Navigation demand that the Total System Error remain equal to or less than the required navigation accuracy (i.e. the RNP level) with 95% probability, and that the TSE has a probability of less than $10^{-5}$ in exceeding twice the required navigation accuracy. Typically, the $10^{-5}$ requirement provides a greater constraint: assuming a normally distributed cross-track TSE, this bounds the standard deviation to be $\sigma \leq 0.45\cdot\text{RNP}$, while the 95% requirement sets a bound at $\sigma \leq 0.51\cdot\text{RNP}$. These however are statistical constraints relying on unbiased sampling, which is difficult to achieve in practice. The only dynamic constraint taken into account by onboard equipment is an alert if there is a greater than $10^{-7}$ probability that the error exceed twice the RNP level within one hour. A full stochastic process characterization such as the one introduced in this paper is more rigorous and allows for a better assessment of risk.
Table 4: FTE statistical moments [96]

<table>
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<tr>
<th>Moment</th>
<th>Lateral</th>
<th>Vertical</th>
<th>Longitudinal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>-0.028 NM</td>
<td>8 ft</td>
<td>-0.1 NM</td>
</tr>
<tr>
<td>$\mu_2 = \sigma^2$</td>
<td>$9 \cdot 10^{-4}$ NM$^2$</td>
<td>26 ft$^2$</td>
<td>$2.25 \cdot 10^{-2}$ NM$^2$</td>
</tr>
<tr>
<td>$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$</td>
<td></td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>$\beta_2 = \frac{\mu_4}{\mu_2^2}$</td>
<td></td>
<td>5.107</td>
<td></td>
</tr>
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</table>

Indeed, according to [96] assuming a static normal distribution of the FTE significantly underestimates the risk of transgressing a containment area. Because of the heavy-tailed occurrences, the ICAO regulatory $10^{-5}$ probability extreme value boundary is a hundred times more likely to be hit under a Johnson $S_U$ distribution (i.e. with a $10^{-3}$ probability) than under a normal distribution. The first four moments (mean, variance, relative skewness, relative kurtosis) used to identify the parameters of the Johnson $S_U$ system are shown in Table 4.

### 3.2.2 Other simulation models

As shown in Figure 9, the simulation relies on four different modules that describe air traffic, which are linked together by a model of the airspace. Taskload calculation for autonomous flows uses the autonomous navigation model introduced previously. The autonomous navigation model, however, depicts individual aircraft. The taskload for one or multiple flows aggregates such probabilities from individual aircraft by using a flow scheduling model. The other type of airspace is controlled airspace. In controlled airspace, a controller is expected to perform conflict detection and resolution activities, represented by the CD&R model. CD&R is applied to non-autonomous traffic, provided by the controlled traffic model. The relative allocation of the two types of airspace inputs into the simulation and is governed by the airspace network model.
3.2.2.1 Flow scheduling model

A diagram of the autonomous flow scheduling module called by the simulator is shown in Figure 12. A detailed explanation for each step of the process follows.

![Diagram of Autonomous Flow Scheduling Module](image)

**Figure 12:** Autonomous flow scheduling module diagram

For the autonomous flows we adopt a widely accepted model of interarrival times distributed according to an invariant Poisson process; this is equivalent to modeling passage times along the flow by an exponential distribution, and is a fair approximation of the system behavior over a short-time horizon (hourly time frame). The present research performs Monte Carlo simulations to obtain the distribution of events over a two hour period. This duration was chosen as the typical length of work uninterrupted by breaks encountered in operational contexts. Furthermore, that duration allows the stationary assumptions to hold on this time scale.

Equations (5) and (6) are the analytical expression of the flow model. If $N(t)$ is a cumulative count of all aircraft having entered the sector, then the probability distribution for the number of aircraft entering a flow between times $t$ and $t + \tau$ is
given by (5) and the probability density for the values of interarrival times $x$ is given by (6).

\[
P[N(t + \tau) - N(t) = k] = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!} \quad (5)
\]

\[
f_\lambda(x) = \lambda \exp^{-\lambda x}, x \geq 0 \quad (6)
\]

Most rigorously, the distribution of velocities inside a flow is also a random variable [144]. In this simulation, the schedule is matched with randomized aircraft information based on historical data (see Section 3.2.2.3). The velocities are chosen as constant to represent optimal average speeds of each aircraft type, and present some small variation around 480 knots. Obviously such a model neglects weather effects.

We justify this assumption in the context of 4-D trajectories by constraints added to the along-track performance of the aircraft, which is not the case in current procedures. For a more realistic case, the flow would need to be modeled as a birth-death process with two different Poisson processes, on entry and on exit from the controlled region. This adds some difficulty to the problem, without making it prohibitive.

The statistical study of the typical traffic entering Cleveland center conducted in [145] provides mean aircraft spacing between 50 and 200 NM for the ten busiest flows in the sector, and an hourly Poisson intensity parameter $\lambda_{center}$ between 10 and 100 (aircraft per hour) depending on time of day, with values averaging 80 for most of the busy times (6 AM to 8 PM EST). For the ten busiest flows in the center, the velocity and spacing values provide average intensity parameters $\lambda_{flow}$ from 10 to 2.5 (aircraft per hour). A reasonable estimate for the time an aircraft spends in a sector is 20 minutes.

For each of the flows in the simulation, its Poisson process parameter is set to reflect historical traffic densities along the routes it replaces. The traffic load is also
controlled as an input through a traffic multiplier variable which allows the evaluation of various future scenarios. The flow scheduling module integrates data from the sector airspace network model presented in Section 3.2.2.4.

### 3.2.2.2 Conflict detection and resolution model

A diagram of the CD&R module called by the simulator is shown in Figure 13.

![CD&R module diagram](image)

**Figure 13:** CD&R module diagram

The module provides conflict detection and resolution by solving a mixed-integer linear program following the formulation presented in [161]. The optimization minimizes fuel costs, with constraints for separation and by accounting for controller workload limitations through a penalty function. In line with the nomenclature of [161], the present simulation uses a \( CRP - \lambda = 10^{-8} \) parameter. This parameter is the Lagrange multiplier used to include workload constraints in the optimization. We refer to the paper in question for further details. While fuel burn costs as a function of airspeed typically result in nonlinear problems and nonconvex spaces, special ordered sets of type two (SOS2) are used to partition the solution space and obtain a tight linear convex approximation.
Each MILP is solved to deconflict a traffic scenario. The traffic scenario is produced by the traffic module presented in Section 3.2.2.3. Controller taskload is consequently deduced from the scenario solution as the number of required speed changes given to aircraft present in the airspace. Validation of this model is difficult, since in practice distinguishing which controller interventions are due to a conflict resolution and which are not is very hard to measure on real traffic. Specifically, it is not obvious at any given time which maneuvers are objectively necessary to avoid conflict, which are more subjective preventive measures, and at which time horizon these occur respectively. Results from the model appear nonetheless to match experimental measures. The activity assessment of en route air traffic control has shown that controllers request an average of 34 trajectory modifications per hour when handling traffic over a typical sector [103]. This number is very close to the results produced by the model (around 38 interventions required per hour for conflict resolution on a fully controlled network).

The computing time for this module varies greatly on the traffic volume and route network complexity (see Section 3.2.2.4 for an explanation of the airspace network model). At present-day traffic volume and over a subset of all routes in the sector, the algorithm runs in near real-time. A normal-traffic scenario is solved in 0.2 seconds on a quad-core, 2.5 GHz Intel Xeon E5420 CPU with 8 GB RAM. With higher traffic load (10X) and more available routes (nearing 100% of the network), processing times can increase to around 1 minute 45 seconds. Nevertheless, it should be emphasized that such traffic volumes are totally unrealistic under any circumstances, since forecasts of traffic growth are capped at an 85% increase over the next 20 years [47].

3.2.2.3 Controlled traffic model

A diagram of the controlled traffic module called by the simulator to generate traffic scenarios is shown in Figure 14.
The traffic scenario referred to in Section 3.2.2.2 is extrapolated from historical trajectories stored as Enhanced Traffic Management System (ETMS) data. The database contains one-minute radar tracking along with aircraft type and filed flight-plan, as well as detailed temporal information on the flight.

As with the autonomous flows presented in Section 3.2.2.1, the traffic load of the scenario is considered a simulation input, controlled through a traffic multiplier variable. The routes where traffic is simulated are selected by the sector airspace network model which allocates autonomous and controlled routes (see Section 3.2.2.4).

For the given controlled routes, a search is performed in the ETMS database to extract all concerned aircraft. The method uses the k-nearest neighbor search algorithm to identify aircraft. Aircraft are then randomly extracted from this reference pool until the desired traffic levels are reached. For realism, some further spatial variability is created by allowing a Gaussian process $\mathcal{N}(\mu = 0, \sigma = 5NM)$ to act on the entry and exit locations. Thus no two identical aircraft trajectories are used in the traffic scenario. Equivalently, the linear program presented in Section 3.2.2.2 is not
degenerate (there are no zeros among the objective function coefficients) and thus the simplex method used for solving is guaranteed to be finite and reach optimality. As before with the autonomous flows in Section 3.2.2.1, the schedule of sector entry for controlled routes is also generated by a Poisson process where the mean parameter reflects traffic intensity. A more detailed explanation on the traffic scenarios can be found in Section 5.2, where this example is used to justify the benefits in airspace design that can be derived from data mining.

3.2.2.4 Airspace network model

A methodology to define flows by clustering aircraft trajectories has been demonstrated in [52]. Radar tracking data is first subject to Principal Component Analysis (PCA) which reduces its dimensionality. Flow clustering is then performed on the processed data using the DBSCAN algorithm [39].

This approach was applied to the Enhanced Traffic Management System (ETMS) data for Cleveland center (labeled ZOB). ZOB was selected because of its significance within the continental United States. A majority of flights originating in New York or Chicago airports traverse this center, as do many coast-to-coast routes. The enroute traffic is therefore comprised of multiple high volume routes. This makes ZOB center well suited for the purpose of the present research to evaluate the allocation of autonomous and controlled airspace in enroute sectors.

While trajectory clusters are a step toward dimensionality reduction, the probabilistic nature of the concept in terms of spatial extent does not sufficiently describe a system of flow corridors. Further building from the clustered flows, a network graph representing the airspace was introduced in [99]. Nodes of the network are locations where aircraft may switch flows. The edges of the oriented network are defined based on origin and destination. For the purpose of this research, a smaller scale network was extracted, presented in Figure 15. The network represents routes restricted to
only one sector, ZOB49, with new entry - exit points identified and positioned on the sector boundary. The use of the network for only one sector is meant to better capture a single controller’s workspace.

![Figure 15: Route network limited to sector ZOB49](image)

Figure 15: Route network limited to sector ZOB49

Figure 16 shows a density plot of trajectories in the center as well as the complete network graph.

The simulator uses an autonomy ratio to extract and redefine subnetworks as either autonomous or controlled. The ratio refers to number of flows, ordered from busiest to least busy. Figure 17 show different ratios of extracted subnetworks. A 25% autonomy ratio means 25% of the busiest flows in the sector are defined to support 4-D self-separating navigation. These flows make up an autonomous subnetwork. It is important to note that traffic distribution is heavily skewed toward the most important flows: Figure 18 shows that the 25% busiest flows carry around 85% of the traffic in the sector.

The extraction of subnetworks from the initial sector network, as represented in Figure 17, is performed so as to favor straight paths and ensure entry-exit connectivity.
Figure 16: Trajectory density plot and graph representation of Cleveland center [99].

This means that from any node in the autonomous subnetwork, the majority of aircraft will be able to exit the sector without leaving autonomous flows, and without performing extreme heading changes. These properties are essential to deliver desired benefits in terms of time gained and/or fuel burned [77]. On those flows, taskload is created by aircraft exceeding their alloted precision bounds. The other 75% of flows in the network comprise the controlled subnetwork. The controlled subnetwork relies on traditional ATC, with controller guidance to ensure conflict detection and resolution.

Figure 17: Extracted subnetworks
3.3 Results

This section presents the results of the control cost simulation. After presenting the predicted control costs, a sensitivity analysis is performed to test the effect of the model assumptions. The section continues with a discussion of the application to airspace design, and ends with a proposed stochastic model for control cost.

3.3.1 Control cost predicted by simulation

In autonomous airspace, control cost is calculated as the number of interventions needed to prevent an aircraft from trespassing the allowed precision bounds. In controlled airspace, control cost is calculated as the number of speed changes that were required to prevent a conflict and were given to aircraft in the airspace.

A Monte Carlo simulation for sector ZOB49 (Cleveland center) is performed. Two independent variables are of interest: percent autonomy allocations over the network.
model (ranging from 1% to 100% of number of flows, starting with the busiest) and traffic level (1 or 2 times present-day traffic, maintaining proportions between flows). For each autonomy ratio and traffic volume, the control cost is sampled multiple times to generate a probability distribution. Each test generates traffic over a 2 hour period and calculates the resulting control cost. For each autonomy/traffic parameter value pair, 5,000 samples were generated on the autonomous subnetwork and 3,000 samples on the controlled subnetwork.

Figure 19 shows the mean control cost values - with 95% confidence intervals - for the autonomous and controlled subnetworks. As the autonomy ratio increases, the autonomous subnetwork increases and the controlled subnetwork becomes smaller. Consequently, the main source of control cost moves from conflict detection and resolution (on the controlled subnetwork) to deviation supervision (on the autonomous subnetwork). Conversely, with less autonomy, the controlled subnetwork is more complex, and thus more control cost is required to deconflict possible trajectories from more different routes. As might be expected, a higher level of traffic increases the control cost, but the relation is not linear.

Figure 20 shows the mean duration between interventions for the autonomous and controlled subnetworks. As the autonomy ratio increases, the task rate periodicity becomes smaller for the autonomous subnetwork, and conversely becomes larger for the controlled subnetwork.

3.3.2 Sensitivity analysis

Two types of sensitivity analyses were performed on the autonomous subnetwork. The effect of flow scheduling models is shown by comparing Poisson to bounded uniform distributions. The Poisson model (with exponential interarrivals) is described in Section 3.2.2.1. The bounded uniform model is a random process where for a flow with mean interarrival time \( \mu \), each observation is randomly drawn from the equiprobable
Figure 19: Mean control cost required for different autonomy allocations and traffic levels

interval $\left[\frac{\mu}{2}, \frac{3\mu}{2}\right]$. The effect of navigation accuracy is shown by comparing the error described in Section 3.2.1 with an FTE model where variance is doubled, but RNP standard is maintained.
Figure 20: Mean task rate required for different autonomy allocations and traffic levels

Figure 21 shows the sensitivity of control cost (number of interventions per two hours shift) to autonomy ratio, flow scheduling model, and aircraft accuracy (FTE).
Figure 21: Sensitivity analysis

It can be seen that FTE has a significant impact, while autonomy ratio and flow scheduling model are not as important. Using uniformly distributed flows instead of Poisson flows has a slight effect by increasing mean control cost. It has been shown in Figure 19 that traffic volume also has an increasing effect, less than proportional however.
3.4 Airspace design insights

3.4.1 Discussion

The effect of autonomy ratio and traffic volume are shown in Figure 19 and Figure 20. The fastest increase in control cost is due to the top 25% busiest flows - containing over 80% of the traffic in the sector. Adding more or less routes to the autonomous subnetwork continues to increase control cost at a slower rate. This is because most of the traffic is concentrated on the busiest flows, as shown in Figure 18. The average control cost confirms that with low requirements for controller intervention, allocating airspace autonomy to most of the high-traffic routes is not likely to pose safety concerns.

In the case of controlled routes, however, control cost mean and standard deviation grow exponentially as the number of controlled flows increases (i.e. the autonomy ratio decreases). Despite much lower traffic volumes, on aggregate, in the controlled subnetwork, as more flows are added, controllers must account for aircraft trajectories coming from different directions. Deconflicting more variable trajectories is the source of both a higher average number of interventions, as well as higher variance.

Increasing traffic volumes impact control cost resulting from both the autonomous and the controlled network. With increasing traffic numbers a dual effect is reflected in control cost, as both the mean and the standard deviation increase. The required control cost is dependent on four main drivers: total number of aircraft in the sector, number of aircraft with autonomous versus controlled navigation, aircraft navigation accuracy on the autonomous subnetwork, and route complexity with points of intersection on the controlled subnetwork.

The average number of interventions increases proportionally with traffic on the controlled subnetwork, which is not true on the autonomous subnetwork. On the controlled routes variance also grows proportionally, which does not happen on the
autonomous network. Such a result is significant when considering future traffic demands, as a higher variance in control cost signals more uncertainty and irregularity in event occurrences. Therefore, while capacity limits may be extended and average control cost requirements may remain tolerable, considerable effort will have to be directed at reducing risk through mitigating the induced variability by ensuring more consistent and regular operations. High control cost variance is a symptom of unpredictable and potentially unsafe airspace.

When comparing the effects of traffic volume and autonomy ratio, we come to four conclusions. First, the autonomy ratio affects control cost from the controlled subnetwork in an exponential manner, but has a more linear effect on control cost from the autonomous subnetwork. Second, traffic volume affects control cost from both subnetworks in an approximately linear fashion. On the controlled subnetwork the effect is proportional, while on the autonomous subnetwork the multiplying effect is less than one. The linearity is true both in terms of mean but also regarding standard deviation. This indicates consistently higher control cost, but also larger variability which is even more costly in cognitive resources. Third, reducing the autonomy ratio has a less drastic effect on the controlled network than increasing the traffic. Thus, some level of autonomy can serve as a safeguard against heavy control cost due to traffic volume increases. By going from 2.5% to 0% autonomy, which translates in a doubling of total controlled traffic (albeit over more routes), control cost on the controlled subnetwork increases slightly from 30 to 35. However, doubling the traffic volumes on each of the controlled routes takes control cost from 35 to 105. Fourth, while autonomy is not necessary at current traffic levels, with increasing traffic a suitably chosen level of autonomy can provide relief to the controller.

An interesting application of these results is on the right policy for autonomous routing implementation. By designing autonomy allocation starting with the more heavily circulated flows, it has been shown here that safe control cost numbers can
be maintained at double today’s traffic. Furthermore, adding some autonomy (covering 20% or less of most circulated flows) can have a net positive effect of reducing controller control cost and allowing him or her to cope with traffic volumes much higher than encountered today. By directly controlling less circulated routes, where the aggregate traffic is only a fraction of the overall traffic, the controller would have more time to directly focus on those aircraft requiring unusual routing conditions, without being overwhelmed. This results holds even with increasing traffic, as much as double today’s volume. The downside is poor robustness in terms of directly controlled routes. With an increase in the complexity of controlled routes, control cost increases exponentially.

The other option is to start by providing autonomy to less circulated routes. Two benefits are apparent. First, less density means more slack for error, as the aircraft navigation imprecision is not likely to lead to unsafe situations. Second, the controller would primarily focus his or her attention on the high density routes, where direct control can be maintained during the transition. But since these routes already maintain a sense of spatial order, the effect of increasing traffic on control cost would not be as severe.

The relative effect of traffic volume on different flows is a determining factor for both controlled and autonomous routes. In the case of autonomous routes where the controller provides oversight, it is the traffic density that determines control cost; in the case of controlled routes traffic volume is important, but the geometric multiplicity of possible entry and exit points in the sector has a significant impact.

By transforming the multiple, highly variable low-traffic routes into autonomous 4-D flows, the aggregate controller control cost coming from solving complex conflicts is reduced. Thus, such a policy would allow throughput increase, free up controller cognitive resources, and provide an opportunity for a seamless transition from current operations. Yet the main benefits of autonomous 4D scheduling - not requiring
routine separation assurance from the controller in high density traffic - would not be capitalized on.

The question of sensitivity of these results to model parameters must also be raised before final conclusions may be drawn. The sensitivity analysis in Figure 21 points to two important points. First, the flow scheduling model (Poisson or uniform) only has a marginal effect. Second, navigation accuracy as modeled by FTE plays a very significant role. Very similar results were obtained for task rate (interval between interventions).

In this sensitivity analysis lies an essential conclusion for autonomous navigation: the validation of high precision in aircraft navigation is the most significant driver of real-world applicability of aircraft autonomy. Without precise navigation (low FTE), autonomy allocation will result in drastic control cost requirements due to the frequent breach of safety bounds. A caveat to this statement is the assumptions made throughout the paper that autonomous flow deconfliction will continue to be feasible from a computing standpoint. In terms of scheduling, uniformly distributed flows will contribute to reduce variance even if they do not reduce the average required control cost.

3.4.2 Proposed model

Based on the results of the simulation, models can be fitted to the control cost and rate over the autonomous subnetwork. Figure 22 shows probability functions for these two variables. A maximum likelihood estimation is carried out to identify the best model. The four plots in Figure 23 and Figure 24 show the goodness of fit of several distributions to control cost data, for two traffic levels (1X and 2X), and for the two different subnetworks (controlled and autonomous). Each plot estimates the fit for different autonomy ratios in the network. The y-axis values are the signal-to-noise ratios of the fit. Signal-to-noise ratio for an estimated parameter is defined as the
maximum likelihood value of the parameter divided by the standard deviation of the estimation (which is also half the spread of the 95% confidence interval). Therefore, the higher the signal-to-noise ratio, the better the fit.

For distributions where several characteristic parameters must be estimated, the distribution signal-to-noise ratio is taken as the mean of the multiple parameter signal-to-noise ratios. An example of a distribution characterized by two parameters is the normal distribution, defined by its mean and standard distribution. For distributions where only one characteristic parameter must be estimated, the signal-to-noise ratio of that parameter is the signal-to-noise ratio of the distribution. An example of a distribution characterized by one parameter is the Poisson distribution, defined by only its mean.

The full list of distributions tested is given in Table 5. Figure 23 and Figure 24 identify the Poisson process to be the best fit for control cost in all four cases. As can be expected, the signal-to-noise ratio improves when there are more data samples which can be used by the maximum likelihood estimator. This explains why the signal-to-noise ratio for control cost over the autonomous subnetwork increases when the autonomy ratio increases, and why the signal-to-noise ratio over the controlled subnetwork increases when the autonomy ratio decreases.

With control cost fitted to a Poisson process, by definition task rate is fitted to an exponential process. If \( N(t) \) is a cumulative count of interventions required of the controller up to a time \( t \), then the probability distribution for control cost between times \( t \) and \( t + \tau \) is given by Equation (7) and the probability density for task rate (times between interventions) is given by Equation (8). The relation between parameter values and autonomy ratio or traffic multiplier is highly nonlinear, as represented in Figure 25 for taskload, and Figure 26 for task rate.
Table 5: List of distributions used in maximum likelihood estimation

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<td>Exponential</td>
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<td>ev</td>
<td>Extreme Value</td>
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<td>gam</td>
<td>Gamma</td>
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<tr>
<td>gev</td>
<td>Generalized Extreme Value</td>
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<td>gp</td>
<td>Generalized Pareto</td>
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<td>wbl</td>
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Figure 22: Control cost and task rate probabilities required for different autonomy allocations in the airspace at current traffic level

\[ \mathcal{P}[N(t + \tau) - N(t) = k] = \frac{e^{-\lambda \tau} (\lambda \tau)^k}{k!} \]  \hspace{1cm} (7)

\[ f_\mu(x) = \frac{\exp\left(-\frac{x}{\mu}\right)}{\mu}, x \geq 0 \]  \hspace{1cm} (8)
Figure 23: Maximum likelihood estimation goodness of fit for control cost, 1X traffic

The main insight from the Poisson model is the dimensionality reduction sufficient of describing the system. Complex interactions between many different stochastic components, shown in Figure 10, can be simplified to a single parameter description. The fact that a Poisson process characterizes the system is not surprising. Poisson
Figure 24: Maximum likelihood estimation goodness of fit for control cost, 2X traffic processes are also referred to as obeying the law of small numbers or rare events: these are improbable events that have many opportunities to happen. Moreover, the occurrence is quasi-independent of time since last event. For both the model of aircraft deviation or conflict occurrence, these properties hold on the system level.
Therefore, the use of a Poisson process could also have been postulated with some conceptual backing.

The Poisson model adheres to the definition of a lean model of control cost. Lean
Figure 26: Task rate: Exponential process parameter fit

models are defined by four characteristics. First, a lean model is a compact descriptor - the Poisson process is a well-known stochastic process defined by a single parameter. Second, the lean model has forecasting ability - the use of a Poisson model in a
simulation provides expected control cost results. Third, the lean model can guide systematic airspace design - the Poisson process allows the mapping of design variables (specifically, autonomy allocation) to design evaluation metrics (specifically, control cost). Fourth, the lean model requires low computational cost for use online - a random number generator can be used to replicate the characteristics of the Poisson model.

3.5 Summary

This research presents a simulator infrastructure to identify the effect of navigation autonomy on airspace risk and control control cost through Monte Carlo methods. By balancing directly controlled routes with autonomous 4-D trajectory flows in the airspace, the control cost required to maintain system performance and safety is drastically impacted.

An air traffic network model based on [99] is used as a baseline for allocating the two types of airspace, autonomous and controlled, within one sector managed by a controller. The autonomy ratio of the accessible network and the traffic multiplier which inflates aircraft numbers are the two parameters used to define several scenarios.

For the autonomous flows, fundamental stochastic models of the aircraft navigation error (an Ornstein-Uhlenbeck process) and of the flow scheduling (a Poisson process) are introduced, along with reasonable numerical values of the model parameters. The Ornstein-Uhlenbeck aircraft model is calibrated - by a maximum likelihood estimation - to match simulated data, obtained from a random number generator defined by a Johnson unbounded system $S_U$. The use of simulated data to calibrate the model is a requirement in the absence of sufficient experimental data; the random number generator provides fictitious data consistent with statistical characteristics of the aircraft motion. The Poisson flow model uses intensity parameters to meter each flow schedule consistently with traffic information extracted from the existing
network model. The control cost from overseeing autonomous flows is calculated as the number of interventions required to reset aircraft on the verge of trespassing the allowed imprecision bounds of their 4-D trajectories.

For the controlled airspace, fuel-optimal trajectories of individual aircraft are calculated and the traffic scenario is solved using a mixed integer linear program based on [161]. The linear program maintains minimal safety separation between aircraft at all times while minimizing deviations from their optimal trajectories. The control cost is calculated as the required total number of maneuvers aircraft must perform. Historical radar data is used to seed the simulation and generate aircraft trajectory samples according to the traffic multiplier and autonomy ratio inputs.

This research shows several things: First, the autonomy ratio affects control cost from the controlled subnetwork in an exponential manner, but has an almost linear effect on control cost from the autonomous subnetwork. Second, traffic volume affects control cost from both subnetworks in approximately linear fashion, both in terms of mean but also regarding standard deviation. While the higher expected control cost remains manageable, the higher standard deviation indicates a large variability. Variability is more costly in cognitive terms, and signals an unpredictable and potentially unsafe system. Third, reducing the autonomy ratio has a less drastic effect on the controlled network than increasing the traffic. Thus, some level of autonomy can serve as a safeguard against heavy control cost due to traffic volume increases. Fourth, the flow scheduling model only has marginal effect. Fifth, the autonomous navigation error plays a very significant role, as identified by the sensitivity analysis. Sixth, a Poisson process can be used to model control cost resulting from both the autonomous and control subnetworks.

An application for these results is policy design of autonomy allocation in the airspace. Furthermore, the research implies that future efforts must be directed at
further reducing navigation errors and route complexity, and ensuring a smoothly-operating airspace through robust self-deconflicting algorithms.

Some additional factors which will need to be considered in future research include interactions between the two types of navigation. Thus, in the case of navigation downgrading, additional control cost would be caused by aircraft exiting autonomous flows to join the controlled subnetwork, or vice-versa. This different source of control cost would then be related with the separate sources from autonomous and controlled subnetworks.
CHAPTER IV

MODEL AND APPLICATION OF INHERENT SAFETY

This chapter demonstrates airspace design centered on inherent safety and is based on research published by the author [123]. The method is to identify inherent characteristics of route geometries that may lead to the occurrence of TCAS advisories, and use that information to redesign airspace. Two steps are therefore required. First, identifying which trajectory geometries lead to TCAS advisories, and second, extrapolating that information to inform systemic airspace design.

A lean model of inherent safety is constructed in the form of multidimensional convex polyhedra that allow the mapping of design variables (specifically, route geometry) to design evaluation metrics (specifically, inherent safety). Lean models are defined by four characteristics: they are compact descriptors, have forecasting ability, can guide systematic airspace design, and require low computational cost for use online.

Short of being able to experiment with real trajectories, the research demonstrates a novel method to identify trajectories that generate collision avoidance advisories. The TCAS advisory issued to a pilot is highly sensitive to the trajectory of an intruder aircraft relative to the ownship flown by the pilot. Further, the complexity of the TCAS logic requires a novel method for mapping trajectories to the range of possible advisories. These challenges are overcome by applying a Rapidly-exploring Random Tree (RRT) algorithm in large-scale fast-time simulations to establish the mapping between the space of relative trajectories and TCAS advisories.

The trajectory definition and guidance used in the simulator are presented in Section 4.2. For the purpose of generalizability, the intruder trajectories are defined
in a frame of reference relative to the ownship. Section 4.3 elaborates on the RRT method used to search trajectories creating the desired TCAS advisories, explains the methodology behind scenario development, and validates the accuracy of the RRT method through experimental testing.

The RRT method therefore accurately predicts unsafe trajectories. By identifying unsafe trajectories and relating them to flow geometries, informed decisions can be made in route design. Section 4.4 presents the application to route design. Trajectories are clustered, analyzed, and a compact descriptor (a multidimensional polyhedra) is created, which can then be used to define route properties.

### 4.1 Background

This section presents a review of the literature relevant to the safety application. The traditional view of safety is explained, and two main themes are then evoked: a presentation of TCAS and human-in-the-loop interactions with TCAS.

#### 4.1.1 Traditional view of safety

Defining what exactly is meant by the safety of airspace is not an easy problem. Fault tree analysis has been proposed as a comprehensive way to evaluate air traffic safety. Fault trees are used in system reliability and certification for analyzing hazards and their underlying causes [94]. To construct a fault tree, a failure or an error is first identified. An expert then establishes possible causes which make up the branches of the tree.

While this method allows the verification of equipment reliability, reliability does not necessarily imply safety. Reliability is an adequate measure of safety for systems that are inherently dangerous and must be regulated, such as nuclear plants. For complex systems where equipment must provide a positive contribution to fulfill a purpose, which is the case of ATM handling traffic, reliable but poorly designed equipment may cause an operationally unsafe situation [50]. Studies have mentioned
circumstances where air traffic safety is emergent and issues may arise due to complex and unexpected interactions between components, even when no failure occurs [16]. Approaches to modeling emergent systemic risks mainly rely on agent-based simulations [131, 154].

Air traffic safety is usually understood to mean a lack of conflicts between aircraft. Conflict detection and resolution (CD&R) research has therefore had explicit the ambition to improve the safety of air traffic. Most of this research has developed and improved algorithms to automate CD&R [87]. Attempts have also been made to formalize CD&R and generate provably-safe maneuvers [156]. These formal proofs are nevertheless only true within the bounds of the postulated models.

A widely accepted model is the gaussian definition of the uncertainty with which aircraft follow trajectories, also known as Total System Error (TSE). The ICAO PBN manual [69] defines TSE as a zero-mean Gaussian distribution. The acceptable probability of accident is defined by ICAO to be one mid-air collision per $10^9$ flight hours [67]. Thus, the separation distance is defined as a consequence of overlapping the tails of two gaussian distributions [138], such that the joint probability respects the safety standard. The model is important in this case, since if TSE is assumed to be fat-tailed instead of gaussian, the same separation distance would in fact carry a risk a hundred times greater [96]. Static constraints are defined by imposing a no-fly hazard zone around aircraft, thus constraining allowable states.

However, as new technologies are developed and improved accuracy is enabled, these metrics cease to make sense at a strategic level, and in a dynamic context. Current ICAO standards for PBN demand that the TSE remain equal to or less than the required navigation accuracy (i.e. the RNP level) with 95% probability, and that the TSE has a probability of less than $10^{-5}$ in exceeding twice the required navigation accuracy [69]. Typically, the $10^{-5}$ requirement provides a greater constraint. Assuming a normally distributed cross-track TSE, properties of the normal distribution
bound the standard deviation to be $\sigma \leq 0.45\cdot\text{RNP}$, while the 95% requirement sets a bound at $\sigma \leq 0.51\cdot\text{RNP}$. These inequalities are independent on specific RNP values. Therefore, risk of deviations becomes so small that probabilistic treatment is limited to a tactical level.

At the strategic level, safety-driven airspace design has been established for terminal approach patterns. The FAA Terminal Instrument Procedures (TERPS) [41] and the ICAO PANS-OPS [68] prescribe the criteria for the formulation, review, approval and the publishing of procedures for instrument flight rules (IFR) operations on terminal approach and departure. TERPS and PANS-OPS criteria specify the minimum measure of obstacle clearance that provides a satisfactory level of vertical protection from obstructions on normal aircraft operations.

TERPS and PANS-OPS standardize the design of terminal airspace such that a pilot of an aircraft under instrument meteorological conditions transfers from the beginning of the initial approach to landing by means of radio, GPS, or inertial navigation system with no assistance from air traffic control. The pilot should be able to navigate to the airport, hold in the vicinity of the airport if required, fly to a position from where a safe visual landing can be made, or execute a missed approach if the visibility is insufficient to execute a safe landing. The procedures must be defined and published so that aircraft can land even with radio failure. Yet significant differences exist between TERPS and PANS-OPS, for example regarding circling approaches where the assumed radius of turn and minimum obstacle clearance are different [152].

While TERPS and PANS-OPS are a first step toward safety-driven airspace design, they only apply to terminal areas. As in the case of taskload, the literature for enroute air traffic uses metrics of safety in a descriptive manner, without attempting to use it proactively. Only very recently has conflict risk been proposed as a potential metric for airspace planning and design, but thus far the conflict risk model has been
published without its actual applications to airspace planning [111]. Metrics which rely on conflict probability must analyze specific trajectories and maintain a tactical focus. For airspace design, surrogates such as TCAS hazard-zones are therefore required in order to manipulate the inherent safety of a route geometry at a strategic level.

4.1.2 Traffic alert and collision avoidance system (TCAS)

This thesis characterizes the inherent safety of the airspace by TCAS advisories. TCAS has been developed under responsibility of the Federal Aviation Administration (FAA) since 1981 and has been required for operation in the U.S. since 1993 [46]. TCAS is currently required on all commercial turbine-powered transport aircraft with more than 30 passenger seats, or Maximum Takeoff Weight above 33,000 lbs; other aircraft have also been installing it voluntarily. The system displays the location and altitude of aircraft within a selected range and provides an alert based on time to conflict. The initial alert is a Traffic Advisory (TA), followed by an RA when a maneuver is required within 5 seconds [20]. TCAS has been monitored and updated since its inception; the most recent update is TCAS II version 7.1.

The RA determines an allowable range of vertical speeds which may require vertical maneuvering by the ownship aircraft, or may constrain the aircraft from increasing or decreasing vertical speed. Furthermore, if both aircraft are equipped with TCAS, their maneuvers are coordinated to operate in opposing climb/descend senses. Figure 27 represents the set of possible initial RAs issued by the TCAS system. Throughout the duration of the RA, the maneuver may strengthen (increase required vertical speed), weaken (decrease required vertical speed), or reverse (switch to opposite vertical speed). Figure 28 represents the full range of possible RA.

While conflict resolution is meant to ensure separation and thus maintain the first-order property of safety, some research has shown that typical avoidance maneuvers,
when coupled to certain route geometry structure, may lead to cascading conflicts. With only a subset of trajectory geometries guaranteed to be stable [98], the problem of air structure degradation remains a concern. Furthermore, in some rare cases when a conflict is not resolved, the traffic situation may degrade itself to the verge of a mid-air collision. The ICAO has defined the acceptable levels of fatal accident risk at one mid-air collision (physical incrossing) per $10^9$ flight hours [67]. In that case, TCAS is the last-resort solution to prevent a catastrophe. By understanding the relation

**Figure 27:** Tree of possible initial TCAS RAs
between trajectory geometry and conflict risk, better informed choices can be made in airspace design.

4.2 Method

This section presents the algorithmic method used to construct the inherent safety demonstration. The simulated intruder trajectory is first explained, before the mapping between trajectories and TCAS advisories is presented.
4.2.1 Simulated intruder trajectory

4.2.1.1 Trajectory waypoints

The intruder aircraft dynamic model is designed for, and applied in, ATC simulations [75]. Trajectories are defined by six-dimensional waypoints: three dimensions are used to define the spatial position of the waypoint, and three dimensions are used to define the velocity vector at that location. The definition of trajectories via waypoints implicitly creates abstractions of the trajectory space. Each waypoint adds six dimensions to the subspace and provides a closer approximation of the span of possible trajectories.

Figure 29 shows the trajectory implementation [24]. The intruder uses two kinds of waypoints in its trajectory. After initialization, the intruder flies along absolute 4D waypoints (location + time) which follow a standard terminal arrival route (STAR). The resulting trajectory of the intruder is indistinguishable from those of surrounding traffic. When the ownship passes a synchronization waypoint, the intruder starts following relative 6D waypoints (with time constraints to ensure convergence). The intruder thus adapts to the ownship’s trajectory in order to generate an RA.

As we rely on this parametrization when referring to advisories inverse images, it is important to note that waypoints are considered in the ownship reference frame. Such a definition allows generalization of the results. For simplicity and efficiency, we wish to identify the lowest dimension where consistent and repeatable resolution advisories can be obtained.

4.2.1.2 Intruder guidance

The intruder guidance must not only generate desired RAs, but also create feasible trajectories subject to realistic flight dynamics. Among the six dimensions for each waypoint, some dimensions take precedence where they are more important within the TCAS logic generating RAs. For example, vertical velocity is more significant
Figure 29: Intruder trajectory relative to the ownship in a typical scenario

than vertical position in the generation of an RA. Thus, only one trajectory solution is available in most cases.

The guidance logic has two steps. First, a commanded trajectory is interpolated between the given waypoints. If only one waypoint is given, then the trajectory is a linear extrapolation with steady altitude, heading, and speed. If two or more waypoints are given, the interpolated trajectory will be piecewise parabolic between the two waypoints. A parabola (a second order polynomial) is the simplest interpolating solution which passes between two points and verifies specified tangency constraints, particularly the ability to simultaneously control both position and velocity. Such a commanded trajectory is shown in Figure 30.

Among infinitely many possible solutions, the benefit of choosing a second order polynomial is that of relying on an explicit simple formulation which can be easily
used with the controller in the simulated aircraft dynamics.

However, it is important to note that intruder aircraft will not effectively follow a parabola. Rather, the parabolas will be functions that are fed as commanded trajectories for the outer loop physical model to track. The aircraft dynamics result from the integration of the differential equations in the aircraft model [75] and verify realistic saturation and response limitations on achieving the commanded trajectory. The resulting simulated dynamics issue a trajectory between prescribed waypoints, with set velocities in those points.

The corresponding guidance equations are given in Table 6, and some are represented in Figure 30. The target speed $V_T(t)$ is calculated from a moving average to avoid large oscillations in response to variations in the ownship trajectory. The heading $\psi_T$ and vertical rate $\dot{z}_T$ are set such that the commanded trajectory is a parabola. $\psi_{wp}$, $\dot{z}_{wp}$, and $V_{wp}$ are the constraints given by the waypoint. $d_{wp}$ and $cd_{wp}$ are the straight-line and the curvilinear distances to the waypoint respectively. $\psi_{direct}$ is the bearing of the waypoint from the current position and heading. $\alpha_i$ and $\beta$ are constants governing the moving-average-smoothing of the velocity.

The second step is the execution of the intruder trajectories. These trajectories serve as commands to the aircraft dynamic model, which is subject to limitations on aircraft performance. The resulting trajectory is thus realistic with respect to the aircraft performance specifications, which may not achieve the full parabola.
**Table 6: Intruder commanded equations**

\[
\begin{align*}
\psi_T &= 2\psi_{\text{direct}} - \psi_{wp} \\
\dot{z}_T &= 2\frac{z_{wp} - z}{t_{wp} - t} - \dot{z}_{wp} \\
V_{T,t} &= \sum_{i=0}^{3} \alpha_{t-i}V*_{T,t-i} \\
V*_{T,t} &= V_{wp} \pm \frac{cd_{wp}}{d_{wp}} \beta \sqrt{|cd_{wp} - V_{wp}(t_{wp} - t)|} \\
\alpha_t &= 0.5; \quad \alpha_{t-1} = 0.25; \quad \alpha_{t-2} = 0.15; \quad \alpha_{t-3} = 0.1 \\
\beta &\approx 1.4
\end{align*}
\]

In effect, the saturation bounds on the states of the aircraft model prevent the aircraft from reaching all arbitrarily chosen waypoints and from exactly tracking the parabolas between them. While theoretically limiting, this poses no concern in practice. While the random sampling which determines the waypoint coordinates spreads over large bounds, using the aircraft guidance inherently provides realistic trajectories.

The differential equations defining the aircraft dynamic model are given in Table 7 [74]. We use standard aerospace conventions for the state space vector (\(\phi\) designates roll, \(\theta\) designates pitch, \(\psi\) designates yaw, \(T\) is thrust, \(V\) is the velocity and \(a\) is a scaling parameter). The controller model and the controller saturation constraints are also given in Table 7. The commanded controller states are \(z_T, V_T\) and \(\psi_T\), and are set to follow the commanded parabola. These are defined in accordance with the waypoint constraints, as discussed previously. A diagram of the control loops is shown in Figure 31.

The waypoints which define intruder trajectory solutions are relative to the ownship trajectory. Thus, the guidance module is adaptive. The intruder dynamically
Several four inner-loop controllers takes in the commanded each dimension, roll, pitch, and airspeed. The set of developed as a model of pilot guidance behavior in three outer-loop controllers. Each controller was in Figure 1 mimic pilot control behavior in many tasks. The following sections will review the current state of element to achieve a specified velocity and altitude. The specific linkages between aircraft, inner-loop control behavior for a manual control task is the crossover requirement that

\[ \dot{x} = V \cos \gamma \cos \psi \quad \dot{y} = V \cos \gamma \sin \psi \quad \dot{z} = V \sin \gamma \quad \psi = \frac{g \tan \phi}{\nu} \]

A simple but effective representation of this closed-loop behavior for a manual control task is the crossover loop element that tracks attitude through commands to aircraft control surfaces, and an outer-loop element that generates the attitude commands to the inner-loop knowledge of inner- and outer-loop control behavior.

Table 7: Intruder aircraft guidance dynamics

<table>
<thead>
<tr>
<th>Plant model</th>
<th>Controller model</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{x} = V \cos \gamma \cos \psi )</td>
<td>( \dot{V} = KV(V_T - V) )</td>
<td>( V_{\min} \leq V_T \leq V_{\max} )</td>
</tr>
<tr>
<td>( \dot{y} = V \cos \gamma \sin \psi )</td>
<td>( \dot{\phi} = K_{\psi}(\psi_T - \psi) )</td>
<td>(</td>
</tr>
<tr>
<td>( \dot{z} = V \sin \gamma )</td>
<td>( \dot{\gamma} = K_{\dot{\psi}}(\dot{z}_T - \dot{z}) )</td>
<td>(</td>
</tr>
<tr>
<td>( \psi = \frac{g \tan \phi}{\nu} )</td>
<td>( \dot{z}_T = K_z(z_T - z) )</td>
<td>( z_{\min} \leq \dot{z}<em>T \leq z</em>{\max} )</td>
</tr>
</tbody>
</table>

**Figure 31:** Generic control model [74]

adjust its trajectory to account for the ownship movement. The TCAS logic continues to output the advisory that is desired by the experimenter in a specific scenario. The generated solution is robust with respect to input from the human pilot on the ownship trajectory.
4.2.2 Mapping trajectories to resolutions

This section illustrates how RRT can be used to map intruder trajectories to the TCAS advisories they create. The inverse mapping can then be used to identify the relative trajectories creating each type of RA.

4.2.2.1 Inverse problem formulation

In the traditional conflict detection problem, the direct question usually asked is “Will this specific trajectory result in a conflict?”, or, more precisely in the case of TCAS, “Which, if any, Resolution Advisory will this specific trajectory result in?” This characterizes the direct mapping of a trajectory onto its image in the advisory space, $\mathcal{TCAS}: \text{Trajectory} \rightarrow \text{Advisory}$.

For this research however, the question we are attempting to answer is “What kind of trajectories result in a specific advisory being issued?” The semantics here are essential, for the question verbalizes an inverse problem to the one mentioned above. Abstractly, we are characterizing the inverse mapping from the advisory space back into the trajectory space, $\mathcal{TCAS}^{-1}: \text{Advisory} \rightarrow \text{Trajectory}$. With the inverse mapping, the space of trajectories is partitioned according to trajectories which result in RAs being issued. This allows for a more refined measure of safety, and distinguishes

![Figure 32: Trajectory to RA direct and inverse mappings](image)
conflicts which result in collision avoidance maneuvers and those which do not. Figure 32 shows a representation of the direct and inverse mappings between the trajectory space and the resolution advisory space.

The trajectory and RA spaces are extremely different in their topologies. While the RA space is a discrete finite space (see Figure 28), the intruder trajectory space is an infinitely-dimensional, continuous, functional space.

Through the inverse mapping method, the trajectory space can essentially be partitioned into inverse images of the respective RAs. However, the full infinite trajectory space is not of interest for two reasons. First, the projections of RA inverse images not null only in a restricted subspace of the whole trajectory space. Second, corresponding to the model of intruder guidance given in Section 4.2.1, we are only partitioning the space of approximations of real trajectories using the least number of waypoints sufficient to achieve desired RAs.

4.2.2.2 The curse of dimensionality

Through the inverse mapping method, the trajectory space can essentially be partitioned into inverse images of the respective RAs. Even if the intruder trajectory is defined by only a single or pair of waypoints, the dimensionality of the problem makes a complete search intractable: $3 \cdot 10^8$ possibilities for a single waypoint (corresponding to 15–20 variations in each dimension of the waypoint) become $9 \cdot 10^{16}$ permutations for two waypoints. These numbers are impractical in terms of the processing times required to fully map the space. Fortunately, the full infinite trajectory space is not of interest for two reasons. First, only trajectories from a restricted subspace can lead to RAs. Second, corresponding to the model of intruder guidance given in Section 4.2.1, we are only using the least number of waypoints sufficient to achieve desired RAs (two in this case).
Thus, the RRT algorithm [92] provides a more efficient method as a compromise between an exhaustive space search and a pure random walk. Furthermore, building a tree structure to connect the random samples adds information and provides a more meaningful description, as relations of proximity and boundary become explicit. Most importantly, the method returns approximate results on the space partition from the early stages of computation, and these results can be further refined as desired through more computation.

4.2.2.3 Rapidly-exploring random tree algorithm

RRT is an algorithm to explore and fill an open space by randomly constructing a tree. The tree is constructed incrementally to quickly reduce the expected distance of a randomly-chosen point to the tree branches. In graph theory, a tree is an undirected graph in which any two vertices (or nodes) are connected by exactly one simple path. In other words, any connected graph without cycles is a tree. With this algorithm, the tree is constructed in such a way that every new sample in the space is added by connecting it to the closest sample already in the tree. The sampling is done incrementally within certain bounds of the existing nodes. The more samples are generated, the finer the sampling becomes and the shorter the tree edges are. This heuristic goes a long way to accelerate the computational process by providing a rough initial estimate, which is then improved with additional iterations.

The algorithm is especially well suited to the mapping of complex (nonconvex and high-dimensional) spaces, and it is frequently used for path planning problems that involve obstacles and differential constraints. The conventional application of RRT is robotic path planning [79]. However, the above description of space exploration may also be understood abstractly. This has led us to use this method to map the RA space. By abstracting the search space to the trajectory space, we are effectively searching for paths within that space (a 12 dimensional space if two waypoints are
used) with the purpose of identifying the boundaries of goal regions describing trajecto-
ries that create different RAs, as defined in our analysis by two 6-D waypoints. From the model of aircraft dynamics and the guidance, a collection of waypoints implictly represents a trajectory that can be flown by the intruder aircraft relative to the ownship. Thus, each node of the tree is an abstraction of a trajectory represented by a collection of waypoints.

The pseudocode for constructing an RRT is given in Algorithm 1. For a precise description of the RRT algorithm in its original form, please see [79]. The behavior of the algorithm as applied here is as follows:

1. At every iteration, a 12-dimensional sample is defined. The sample corresponds to a trajectory with two six-dimensional waypoints. The selection of the sample is biased toward the largest unexplored region of the search space.

2. The node in the tree that is closest to the new sample is identified. The distance used is the euclidean norm (in normalized coordinates).

3. The sample is projected toward the closest node, so that its distance to the tree does not exceed a certain length. The distance is inversely related to the density of points in the tree. Nodes will be far apart and spread over the space when the search begins, and get closer together at every iteration.

4. The projected sample is linked to the tree with a new edge and becomes a new node. The TCAS algorithm evaluates the corresponding trajectory for a possible RA.

5. The newly created node is tagged with what RA, if any, is generated.

6. The tree is then ready for a new sample.

7. A search is deemed complete when the RRT space has been searched to an
adequate level of granularity (the authors typically constrained some of the search variables and allowed the tree to grow to as much as 10 million nodes).

8. The nodes situated as close to the center of each RA region are selected as the intruder trajectories in the simulator. Well-centered nodes are the most likely to generate the desired RAs, since small changes that can occur in the relative conditions between ownship and intruder still result in a node within the RA region. The boundaries of the RA regions are defined by the edges which connect two nodes labeled with two different RAs.

Algorithm 1 Pseudocode for constructing an RRT [92]

\begin{verbatim}
procedure BUILD_RRT(x_init)
    \( T \).init(x_init);
    for \( k = 1 \) to \( K \) do
        \( x_{rand} \) ← RANDOM_STATE();
        Extend(\( T, x_{rand} \))
    end for
    return \( T \)
end procedure

procedure Extend(\( T, x \))
    \( x_{near} \) ← NEAREST_NEIGHBOR(\( x, T \));
    if NEW_STATE(\( x, x_{near}, x_{new}, u_{new} \)) then
        \( T \).add_vertex(\( x_{new} \));
        \( T \).add_edge(\( x_{near}, x_{new}, u_{new} \));
        if \( x_{new} = x \) then
            return Reached
        else
            return Advanced
        end if
    end if
end if
return Trapped
end procedure
\end{verbatim}

4.2.2.4 Trajectory space

The RRT method maps the boundaries that partition the TCAS logic output by what class of trajectories result in a specific RA. As an example of the insight provided
by this mapping, Figure 33 shows a tree constructed by varying only two states in the twelve-dimensional trajectory space. The ownship conditions are set at 7000 ft altitude, level flight, 225 knots speed. The intruder starts with a head-on trajectory, level. Only two of the intruder states are varied in this example: relative altitudes at the first and the second waypoint. All other 10 states would normally be varied, but are fixed in this example - as given in Table 8 - so that the mapping can be accurately portrayed in two dimensions. Figure 34 represents the traffic situation in the horizontal plane, relative to the ownship.

The growth of the tree reveals an obvious partition of the waypoint space in terms of corresponding RAs. For clarity, the specific resolutions are grouped in terms of their high-level categories of Up/Down and Corrective/Preventive.

Figure 33: Construction of the RRT in the trajectory waypoint space
Table 8: Intruder states in RRT search example

<table>
<thead>
<tr>
<th>States</th>
<th>1st Waypoint</th>
<th>2nd Waypoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Range (NM)</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Relative Bearing (deg)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Relative Altitude (ft)</td>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>Speed (kts)</td>
<td>260</td>
<td>260</td>
</tr>
<tr>
<td>Relative Heading</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>Altitude Rate</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 34: Horizontal plane traffic situation

Because the problem becomes over-constrained, some slack is required for feasibility. We choose to allow some tolerance in the altitude rates. In the guidance logic, the commanded relative altitude has priority over commanded altitude rate. Therefore, the actual intruder trajectory does not always adhere to the altitude rate prescribed in the waypoints constraints. Figure 35 shows the actual intruder altitude and altitude rate recorded at time of RA. While waypoint constraints in Table 8 are fixed at 0 altitude rate, a wider span is observed in the achieved intruder trajectories.

The mapping between the waypoint space and the realized intruder trajectory is
not transparent, nor is it bijective. But this mapping has the advantage of encapsulating both the TCAS logic and the intruder guidance logic, and thus provides a true black box approach with accessible controllability of the simulation outcome. It is important to note the distinction between the waypoints and the commanded trajectory which are static definitions produced by the RRT search, and the actual trajectory which is dynamically simulated with changing conditions.

4.3 Results

This section presents the results of the inherent safety application. The piloted simulator study is first explained, and serves the purpose of validating the accuracy of the method. The section ends by presenting the application of the method to airspace design.

4.3.1 Piloted simulator study

The HITL experiment examines pilot responses to TCAS in the context of an integrated air traffic control and flight simulator facility [133]. The purpose of the study is to identify how pilot interaction with ATC influences pilot responses to RAs, and
conversely how features of the RA influence subsequent pilot communication with ATC. The results of this study are reported in several papers [132–134].

Airline pilots fly a medium-fidelity simulated B747-400 in instrument meteorological conditions [70] while interacting with an experimenter acting as air traffic controller. The surrounding air traffic appears to the pilot on the TCAS traffic situation display.

Nominal background air traffic is provided by the FAA Target Generation Facility software (TGF), which connects to the simulator infrastructure over the High Level Architecture connection (HLA). TGF replays traffic data recorded from airport operations around the Dallas-Fort Worth airport. TGF also enables a human air traffic controller to interact with the traffic by providing displays of the traffic environment.

Typically, piloted flights begin around an altitude of 10,000 to 20,000 feet and lasted 15 minutes. Each pilot performs flights, and in each flight pilots encounter two traffic scenarios. Some of scenarios result only in TAs and require no maneuvering, while other scenarios result in TAs followed by RAs.

The pilots are tasked with flying one of three standard arrival route into DFW airport. Figure 36 shows the fixes and the three arrival routes used in the experiment. Typically, piloted flights began around an altitude of 10,000 to 20,000 feet and lasted 15 minutes. The flights ended during the approach intercept, i.e. when the aircraft came within ‘one dot’ of the localizer beam indicating the approach course. Each pilot flew eight scenarios, and in each scenario pilots encountered two traffic events. Some of events resulted only in TAs and required no maneuvering, while other events resulted in TAs followed by RAs. The RA locations for the first study are shown approximatively in Figure 36. In the first study, the intruder trajectories for scenarios C2, D2 and G were manually designed. All other scenarios used output from the RRT search method.
4.3.2 Scenario creation

Scenarios are first sketched out on paper to identify key variables predicted to influence the type of TCAS RA they generate. These variables are then mapped to intruder trajectories that are scripted into the simulation. However, none of these first estimates of intruder trajectories are sufficient and they are iterated through repeated runs of the flight simulator in real-time with experimenters playing the role of an actual pilot. This averaged roughly a day of flight simulator flights per scenario for those scenarios that are configured manually, until an exact specification of intruder trajectory is found that appears to be robust with respect to RA occurrence, albeit
not always RA success - i.e. generating an RA but not always returning the desired RA type.

In contrast, the scenarios developed using RRT start with the same paper sketch to identify those initial segments of the intruder trajectory that would appear to be realistic. This step identifies the intruder origination points in actual traffic streams, such that trajectories would make sense to the pilot. The RRT method is then used to identify the relative trajectory for this traffic following its origination from a traffic stream. Once this trajectory is identified, it is entered in the simulator and a single simulator flight is generally performed to verify it.

Therefore, in all studies involving pilot responses to TCAS, including the manual scenarios, the best intent was made to create robust scenarios. The primary purpose of these scenarios is not to serve as a baseline for the RRT method, but to setup test conditions involving human subjects in an expensive and difficult simulator test [133].

Relative to a manual parametrization, which is a laborious and unrepeatable process of trial and error, the RRT method is found to provide fast and generalizable output that specifies the commanded intruder trajectory exactly.

### 4.3.3 Experimental validation

Four studies have been performed up to the time of writing this thesis. This section analyzes the accuracy of the RRT method in generating RAs. These experiments serve as validation for the trajectory analysis which can be then applied to airspace design.

Figure 37 shows the expected and observed RAs from the four studies. The plot also distinguished RRT and manually defined scenarios. These are only taken from studies 1 and 4. The x-axis categorizes scenarios according to the expected RA. The y-axis corresponds to the observed RA in each case. The data points are slightly translated to show frequency. In a perfectly accurate experiment, where all observed
RAs corresponded to expected RAs, clusters of data points would only appear along the first diagonal.

Two metrics of accuracy can be defined: RA occurrence designates cases when any RA was triggered, and RA success designates cases when the specific desired RA was triggered.

Figure 38 quantifies the reliability of the RRT method under these two metrics. Once again, results for the RRT and manual scenarios are only taken from studies 1 and 4, while the overall statistics encompass all studies. Table 9 shows the same ratios for study 1. The reliability of the RRT method is apparent, in terms of RA occurrence and RA success.

![Figure 37: Expected vs Observed RAs in all 4 studies](image)

In the first study, the RRT method was extremely effective in generating an RA, with 100% RA existence in all but one scenario where it was used. Scenario D1 is the exception, having an 88% RA occurrence rate, although when an RA did occur, it was of the expected type. Crossing descents such as those used in traffic scenarios E and F are the RAs most sensitive to variation in the ownship trajectory, and are only possible within a small range of relative altitudes and altitude rates. The precision requirements thus explain the imperfect generation of crossing RAs in a
HITL simulation, indicated by the lesser RA success ratios. Furthermore, scenario B involved contradictory information communicated to the pilot over the radio, which further exacerbated variability in the trajectory flown by the pilot. Scenarios C2 and D2 were scripted “manually”, to represent conflicts in particular traffic instances such as during parallel landing approach, and do not use output from RRT. Scenario G did not contain any RAs, and thus the RRT method was not used. These scenarios are mentioned here for comparison purposes.

The robustness of the RRT system is illustrated by these high ratios. Furthermore, another essential advantage of the RRT method is its ease of use. Relative to a manual parametrization, which is a laborious and unrepeatable process of trial and error, the RRT method was found to provide fast and generalizable output that specified the commanded intruder trajectory exactly. This method, here applied to generating RAs, can be used to establish traffic events in a general sense. Such means of testing are key to the development of ACAS X [81] and other related systems which are in turn vital to the development of the Next Generation Air Transportation System (NextGen).
<table>
<thead>
<tr>
<th>Study</th>
<th>Scenario</th>
<th>RA Situation</th>
<th>RA Occurrence</th>
<th>RA Success</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 A</td>
<td>Climb after HIKAY</td>
<td>100%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>1 B</td>
<td>Climb after KARLA, conflicting radio information</td>
<td>100%</td>
<td>57%</td>
<td></td>
</tr>
<tr>
<td>1 C1</td>
<td>Descend between HOWDY and TACKE</td>
<td>100%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>1 D1</td>
<td>Descend after UKW</td>
<td>88%</td>
<td>88%</td>
<td></td>
</tr>
<tr>
<td>1 E</td>
<td>Crossing Descent after COVIE</td>
<td>100%</td>
<td>88%</td>
<td></td>
</tr>
<tr>
<td>1 F</td>
<td>Crossing Descent after TACKE</td>
<td>100%</td>
<td>94%</td>
<td></td>
</tr>
<tr>
<td>1 H</td>
<td>Descend after LEMYN</td>
<td>100%</td>
<td>69%</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Manual</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 C2</td>
<td>Climb after DIETZ, VFR traffic</td>
<td>100%</td>
<td>82%</td>
<td></td>
</tr>
<tr>
<td>1 D2</td>
<td>Monitor Vertical Speed after HIKAY, intercept approach</td>
<td>69%</td>
<td>63%</td>
<td></td>
</tr>
<tr>
<td>1 G</td>
<td>TA only</td>
<td>n/a</td>
<td>n/a</td>
<td></td>
</tr>
</tbody>
</table>
4.4 Airspace design insights

4.4.1 Route clustering

The accurate characterization of trajectory risk through the RRT method is validated in the experimental context. To serve the purpose of this thesis, the next step is applying this method to support airspace design.

The application starts with the compilation of trajectory clusters. Figure 39 shows a plot of 5,000 possible trajectories which result in a climb RA being issued. The plot is in relative coordinates in the ownship body frame (the x axis points toward the front of the ownship). The two plots show the three-dimensional and longitudinal sections of the plot.

However, the spatial distribution only tells half the story: the velocities are essential for determining the risk of collision and thus triggering an RA. Figure 40 shows the phase plots (position and velocity) for these same trajectories along the longitudinal and vertical directions. In these, the distribution of trajectories is more apparent. For example, the vertical phase plot clearly shows that trajectories coming from below and climbing are predominantly responsible for generating Climb RAs. While this result in general is intuitive, a precise quantification of the concerned trajectories is not.

![Figure 39: Trajectories resulting in Climb RA](image)
From the trajectories, the convex hull of the clusters can be extracted. The hull delimits the set of trajectories resulting in a specific RA, and is defined in the phase space which has six dimensions: three dimensions for the physical position, and three dimensions for the velocity of the intruder aircraft relative to the ownship. Figure 41 shows the hull of the Climb RA cluster. The plots represent the cluster in three-dimensional space and its longitudinal section.

These clusters are defined in relative coordinates. The application to airspace design requires them to be overlaid on defined flows. The full design process must
account for both position and velocity characteristics and the different kinds of possible RAs. Representing all six-dimensional constraints is not possible visually, but such information can be included in flow design and autonomous routing algorithms. If only spatial constraints are represented, Figure 42 shows a planar projection of a flow and the corresponding unsafe zone for Climb RA. This figure illustrates how TCAS mapping can be applied to airspace design. For each flow, areas surrounding it are characterized as unsafe for different kinds of resolutions. Other flows or trajectories which would pass through these areas pose a safety risk, which can be handled through velocity and spatial trajectory redesign.

![Flow plot](attachment:image1.png)  ![Unsafe zone](attachment:image2.png)

**Figure 42:** Unsafe areas in the airspace

The unsafe trajectory cluster adheres to the definition of a lean model.

First, a lean model is a compact descriptor - the multidimensional convex polyhedra that define the cluster surfaces are simple, multilinear, geometric shapes. Second, the lean model has forecasting ability - experimental validation has shown that trajectories inside the cluster lead to TCAS advisories. Third, the lean model can guide systematic airspace design - the cluster allows the mapping of design variables (specifically, route geometry) to design evaluation metrics (specifically, inherent safety). Fourth, the lean model requires low computational cost for use online - the cluster envelopes are polyhedra that define linear constraints in a computer program.
4.4.2 Flow characterization

This section illustrates an application of the unsafe clusters to airspace design. The merit of the method is that clusters are able to systematically capture route characterization by using the relatively simple geometrical formulation of a polyhedron. Since the clusters are six-dimensional, the constraints put on route geometries are complex: restrictions are not simply where routes go, but also how. Specifically, velocity caps can be introduced, and the explicit relation to desired spatial flow separation is accessible.

Currently, flow velocity constraints and spatial positioning are determined and parametrized ad-hoc. Letters of agreement at sector hand-off points define altitude and velocity bounds based on intuition and experience, rather than a systematic analysis. By providing a quantitative way to relate the allowed vertical rate in a climbing flow and the vertical separation to a level flow above, the clusters described in this chapter have several applications.

The geometric descriptors can predict cases where routes will need to be redesigned based on TCAS alerts. A noteworthy such case is the Dallas Bump geometry [167]. The name is fairly descriptive: a departure path existed at Dallas Fort Worth airport where aircraft climbing at the prescribed rate of 3,500 feet per minute toward FL 170 conflicted with steady aircraft at FL 180. The traffic geometry is shown in Figure 43.

![Figure 43: Dallas Bump geometry](image)

From an intent-based perspective, the traffic situation in nominally safe: if the level-off occurs as it should, there would be no problem. However, the situation is
not safe from a state-based perspective. Given the existing positions and velocities of
the aircraft, any deviation from the prescribed clearance would present a near-miss
risk. Therefore, TCAS “Climb” RAs were frequently issued to the latter, resulting in
a trajectory that appears to go over a bump. At the same time, the climbing aircraft
would receive a “Monitor Vertical Speed” advisory. Because of the persistence of
these events, a cap was introduced on the maximum vertical rate of 1,000 feet per
minute above FL 150, which in turn solved the problem.

An example of the benefits given by the cluster method in such a traffic geometry
include the ability to explicitly predict the trade-off between flow vertical separation
and climb rate. This relation is nothing more than the outline of the cluster poly-
hedron exterior surface over the two dimensions of interest, altitude difference and
altitude rate, respectively. Figure 44 represents a zoomed perspective on the cluster
outer surface. The plot represents the trade-off between maximum climb rate allowed
and vertical separation between flows. Trajectories with a vertical rate that remains
below the boundary are outside the cluster and therefore safe. Trajectories with a
vertical rate above the boundary are inside the unsafe cluster. To better illustrate
the meaning of the separation frontier, a sample of trajectories are represented inside
the cluster: each dot represents a trajectory that results in a “Climb” RA.

4.5 Summary
The research applies a Rapidly-exploring Random Tree algorithm to establish such
a mapping between the space of relative trajectories and the space of TCAS RAs.
The algorithm explores the space of intruder trajectories as defined by multiple 6-D
waypoints (three dimensions for position and three dimensions for velocity). The
TCAS logic evaluates the different intruder trajectories relative to the ownship and
issues the relevant RA, should any be required. The mapping is used to select the
trajectory waypoint most likely to result in specific RAs.
Figure 44: Trade-off between climb rate and flow separation

Such waypoints corresponding to a desired advisory can be converted into trajectories flown by intruder aircraft during the HITL simulations. Experimental testing shows that the RRT method provides faster, easier, more generalizable, and more robust results relative to manual parametrization, which is slow, tedious, and comes after a trial and error process specific to each traffic circumstance.

By demonstrating robustness, the results from piloted simulations highlight the potential of this method for pilot training and for research and development.

Having validated the RRT’s ability to map trajectories to TCAS advisories in the
HITL experiment, this method can now be applied to airspace design by defining clusters of risky trajectories. These clusters are defined in six dimensions, encompassing spatial and kinematic information. The multidimensional extent of these clusters constitutes unsafe zones, in spatial and kinematic terms. Flow design must account for these dangerous geometries and thus avoid risky trajectories by constraining distance, velocities, or both. Such unsafe zones can be proactively eliminated from designed airspace through the use of explicit trade-off relations.
CHAPTER V

UNIFYING PRINCIPLES AND PROPOSED METHOD

This chapter presents the methodological contribution of the thesis. In this methodology, data mining and aggregation are used offline to synthesize lean models of the system. Lean models describe system behavior at a high-level. These compact descriptors can then be used to design the airspace and rapidly iterate. Their main benefit is requiring scarce computational resources for use online. The offline generation can be heavy but only needs to be performed once.

A discussion of the current approach to airspace design is conducted in Section 5.1. The potential of big data is highlighted in Section 5.2, where the data sources used in the thesis are also explained. A comparison of approaches to simulation is presented in Section 5.3, and the contributed methodology - data aggregation and lean model synthesis - is explained in Section 5.4.

5.1 The problem of current airspace design

Airspace design is a complex matter which has not been systematically undertaken. Airspace has historically been redesigned by operational personnel and is a mixture of art and science. Local airspace experts identify problems such as heavy traffic, and propose solutions. Few quantitative techniques are used to evaluate potential designs. The entire process from problem definition to design evaluation is iterative [42]. While able to refine and improve proposed solutions, this process is heavily anchored in past experience and requires significant time and resources.

Airspace design is usually limited to resectorization and flow management. This localized focus may create propagating phenomena through the NAS which are not anticipated or understood. Efforts have focused on capacity improvements through
weather prediction [84], routing optimization [151], airport arrival and departure queuing [4], flow metering and ground holding.

Reducing controller workload has been a constraint leading to resectorization, without being used in more systematic airspace design which includes routes and flows. For example, the FAA Sector Design Analysis Tool (SDAT) uses several metrics such as required intervention rate to evaluate the effect of airspace and traffic changes on sector capacity and traffic complexity [54]. In some cases, newly redesigned sectors have been created smaller and with fewer traffic flows. For example, sectors ZMP15 and ZMP16 in Minneapolis center are dominated by aircraft arrivals into Minneapolis - St. Paul International Airport and are significantly smaller than other sectors in the center. But a limitation of smaller sectors is the need for additional controller staff, the reduced ability to perform efficient conflict avoidance maneuvers, and increased coordination between adjacent sectors to handle aircraft transitions [160].

NASA has recognized the importance of novel airspace design methods, and has created the Dynamic Airspace Configuration (DAC) technical area to group projects on this topic [82]. Initial concepts applicable to DAC focus on three types of airspace: restructured, adaptable, and generic. Restructured airspace is designed to get the most efficiency through technologies such as self-separation and 4D-trajectories. Adaptable airspace is fluidly reorganized to accommodate fluctuating demand. Generic airspace is designed in a way that promotes interchangeability among facilities and controllers.

The concepts are also aimed at different time horizons. The mid-term concepts distinguish between high-altitude airspace with user-preferred routings - involving tubes or corridors in the sky [59] - and low-altitude airspace with super density and metroplex areas around the busiest airports. The long-term airspace concepts distinguish four types of regions: airspace with automated separation assurance, high-altitude airspace, super density and metroplex airspace, and traditional structured airspace.
More systematic processes are now being developed. Some of these use quantitative analysis to define target concepts, which are then shaped by subject matter experts to achieve operational feasibility [27]. Nevertheless, these methods remain anchored in past or present operational paradigms and are unsuited for future concepts of operation for which more comprehensive and less empirical approaches are needed.

The idea of using conflict risk assessment for airspace planning and design has only been recently mentioned in the literature [111], but so far only the conflict risk model has been published. The model is intended to compare different airspace designs and organizational scenarios under different traffic flow levels, but no applications or design considerations resulting from it are available at this time.

The problem of airspace design requires a formulation which must address several key points resulting from the primary function of the airspace. Designing airspace means operationalizing a concept. Formally, a concept is a definition of function allocation and associated performance requirements. Since the airspace is meant to safely allow the movement of aircraft, considerations must include safety, routing, and priorities [12]. While humans maintain an active involved role, concern for their capabilities and limitations is a fourth consideration.

5.2 Making sense of big data sources

Airspace design is presently driven mainly by an empirical iterative process reliant on subject matter expertise with a geographically localized focus [42]. Considerable research has been put into devising analytic metrics that evaluate air traffic health, starting from the simplistic Monitor Alert Parameter (MAP) which is directly related to average sector flight time per aircraft [48], to measures of complexity [105] and dynamic density [101].

Analytic models, however, require fitting with empirical data before they can be
parametrized and become directly applicable. Aviation, like so many other fields, has now entered the age of big data. A multitude of possible data sources exists which could potentially be used for a more data-driven approach to airspace design. Examples of relevant data include air traffic scenarios, airspace configurations and airport layouts, aircraft performance characteristics, and airline economics. Yet reports have identified a lack of concertation in database assembly and maintenance, thus failing to effectively use the potential of this wealth of information [114].

A noteworthy effort in this direction is the deployment by the FAA of the Performance Data Analysis and Reporting System (PDARS) since 1999 [31]. PDARS contains detailed aircraft tracking information along with a range of performance measures such as traffic counts, travel times, travel distances, traffic flows, and intrail separations. A more basic database is the Enhanced Traffic Management System (ETMS). ETMS contains aircraft trajectories from 1 minute radar tracking and some metadata such as filed flight plan and transponder information (aircraft type, ICAO 24-bit address,...) for each recorded aircraft.

Making sense of this wealth of data is a difficult endeavor that has yet to be fully exploited. PDARS shows how data mining has been gradually put to use over the past decade in air traffic management, but its applications remain limited to descriptive performance diagnosis and anomaly detection. Approaches include identifying loss of separation [110] or more general controller operational errors [109], estimating weather-caused delays [108], and airspace complexity estimation [145]. Almost no research seeks to extract models which could be used proactively in airspace design under future concepts of operation, such as applying current conflict detection to forecasting free flight conditions [2].

The gap in research corresponds to a gap from analysis to design. Data is currently used on a descriptive level, while airspace design requires it to be applied in a
prescriptive way, such that insights can be systematically extracted. For the demonstrations to airspace design shown in Chapters 3 and 4, this thesis uses data generated from several various sources dealing with aircraft guidance, navigation, and surveillance. In each case, data is aggregated into a database whose specificity depends on the intended application. After aggregation, the data is analyzed, and a model is synthesized. Design insights can be obtained from the prescriptive use of the model.

For the control cost CD&R module presented in Chapter 3, an uncontrolled open-loop air traffic model is used [160]. The uncontrolled air traffic model attempts to simulate traffic without controller intervention. The overlay of a conflict detection and resolution algorithm [161] thus allows an estimate of required controller interventions in the airspace by replicating potential conflicts which need to be resolved. The traffic model is based on a resampling of historical aircraft radar data taken from ETMS, with added variance corresponding to the observed probability distributions of parameters such as aircraft types, exact routing relative to filed flight plan, sector entry and exit points.

Using this method for generating traffic conditions is an easy alternative to software such as the Future ATM Concepts Evaluation Tool (FACET) developed at NASA Ames Research Center [14], which is ITAR-restricted and was unavailable to the author for security reasons. FACET is a sophisticated simulation engine which uses aircraft performance profiles, airspace models, weather data, and flight schedules to individually model trajectories for the climb, cruise, and descent phases of flight for each type of aircraft. Like FACET, the sampling method used in the taskload CD&R module presented in Chapter 3 is capable of quickly generating and analyzing thousands of aircraft trajectories.

For the control cost autonomous module, also presented in Chapter 3, trajectory data was generated through a stochastic process in a Monte Carlo simulation. The
stochastic process represents the variable trajectories of aircraft which follow prescribed 4D flows. The 4D flows correspond to routes that are identified by clustering observed traffic [52]. The centroids of the clusters are pieced together into a network graph model of the airspace [99].

For the inherent safety application shown in Chapter 4, a database was generated by simulating pairs of aircraft trajectories using a flight control model designed for ATC simulations. The model uses an adaptive control architecture for stability, and allows the specification of the closed-loop behavior of the aircraft [75]. The relative trajectory pairs are defined by a series of waypoints with position and velocity constraints. These waypoints and their corresponding TCAS resolution advisories are stored into a tree structure. Further post-processing allows a reduction of the tree to multidimensional convex polyhedra, which are much simpler to represent.

5.3 The gap between analysis and design

The essential question that is related to the available data sources is its use within a coherent and reasoned level of detail. Therefore, it is vital to understand what the appropriate level of detail is. Indeed, more detail in a model or simulation seldom directly implies greater accuracy of the results.

Most research directions have directly applied the wealth of available data to seed complex large-scale simulations. The modern approach enabled by distributed computing and accessible processing power is based on multi-agent simulations. The benefits of agent-based simulations are high fidelity and the ability to discover emergent properties, as well as the relatively straightforward validation. While discrete-event simulations have been successfully used for established operations [157], agent-based simulations are a promising path toward evaluating and comparing new concepts that can be explicitly parametrized by locus of control [131] or function allocation [80]. A recent application of agent-based simulations is assessing air traffic risk. Risk has
traditionally been modeled by fault trees and other sequential and epidemiological accident models. Systemic accident models consider accidents to occur because of complex and variable interactions which are suited for simulation using agents [154].

Agent-based models have been applied to many aspects of traffic and transportation systems such as dynamic routing, congestion management, and intelligent traffic control, on different modes including road, rail, and air [21]. Air traffic management systems such as the NAS are believed to be based on complex emergent behavior [150]. In such systems, phenomena emerge from interactions between individuals and cannot be predicted from examining individual behavior. Agent-based simulations which integrate cognitive models of human performance, physical models of technology behavior and descriptions of the operating environment are therefore a plausible method of forecasting the impact of new concepts of operation.

Conversely, traditional optimizations using utility functions are inadequate for the design of a cooperative distributed air traffic control system. Where utility functions do not account for sophisticated social behavior, multi-agent simulations may use game theory to design such a complex system [57]. Integrated gate-to-gate modeling tools which capture the interactions of participants in the NAS through large-scale, distributed agent-based simulations are meant to develop and evaluate system-wide candidate operational concepts for air traffic control [155].

A wide review of agent-based models and equation-based models has concluded that “agent-based modeling is most appropriate for domains characterized by a high degree of localization and distribution and dominated by discrete decisions. Equation-based modeling is most naturally applied to systems that can be modeled centrally, and in which the dynamics are dominated by physical laws rather than information processing” [118].
Agent-based simulations are not the ideal choice under all circumstances. Emergence is not always a requirement, or primary consideration, and can even be a downside as a low-level to high-level mapping is not achieved in the process. The weakness of wide-scale simulations is their cost in resources, time, and the heavy framework required to simulate complex systems. Agent-based simulations that involve complex systems require significant effort in their development, as detailed interactions must be identified and modeled. Lighter equation-based simulations can and should be used in the initial stages to identify key configurations. A risk which could lead to waste of resources by simulation designers and users is the selection of models inappropriate for their needs [114]. An example of such inappropriate use is creating a highly-detailed model for planning infrastructure developments such as airport capacity expansion. For such broad long-term policy decisions, lean approximate models are more adequate than a detailed simulation which requires exact layouts and precise but speculative and uncertain flight schedules far into the future. Therefore, it is important to recognize the relation between the design problem and the model choice.

5.4 Proposed methodology

The methodological contribution of the thesis is an approach to airspace design using lean models. Rules of a general methodology can be defined based on the demonstrations for control cost, in Chapter 3, and for inherent safety, in Chapter 4, respectively. The methodology has the purpose of adding control cost and inherent safety as metrics of airspace design, where traditionally these have been constraints on the objective of adding capacity and minimizing delays. The importance of inherent safety and control cost as functional objectives in a reframed problem of airspace design has been discussed in greater depth in Chapter 2.

The use of lean models is a result of a natural flowdown from the airspace view to the requirements. The perspective of the airspace is given in Chapter 2, and
poses two main questions. The first question that comes from the perspective on the airspace and is on the appropriate level of detail. The research seeks to identify the macroscopic effects of operational changes on the strategic level. The second question that comes from the perspective on the airspace is on the appropriate computational technique. The objective of the research is to map low-level changes to high-level properties, and provide predictive results that can inform airspace design decisions.

Choosing the appropriate level of detail allows a discussion of design insights. The scope of the lean model is situated at a level of granularity that identifies the macroscopic effects of operational changes on the strategic level. For example, Chapter 3 shows that complex interactions between many different stochastic components that determine the control cost of a system can be simplified to a single parameter description. The lean model also provides design insights, such as the fact that the flow scheduling has marginal effect on control cost, while autonomous navigation error and tolerance play a very significant role. Finally, the model shows that reducing the autonomy ratio has a less drastic effect on the controlled network than increasing the traffic. Thus, some level of autonomy can serve as a safeguard against heavy control cost due to traffic volume increases. Likewise, Chapter 4 shows that the inherent safety of routes can be characterized, determined, and predicted by a relatively simple convex polyhedra (albeit multi-dimensional and involving spatial and kinematic information).

Choosing the appropriate computational technique allows an analysis of the effect of design variables. The lean model technique maps low-level changes to high-level properties and provides predictive results. For example, Chapter 3 shows how the two design variables, route geometry and autonomy allocation, influence control cost. Thus, the autonomy ratio affects control cost from the controlled subnetwork in an exponential manner, but has an almost linear effect on control cost from the autonomous
subnetwork. On the other hand, traffic volume affects control cost from both subnetworks in approximately linear fashion, both in terms of mean but also regarding standard deviation. The higher standard deviation indicates a large variability, which is more costly in cognitive terms, and signals an unpredictable and potentially unsafe system. Likewise, Chapter 4 provides direct trade-off relations between spatial and kinematic constraints on route geometries that preserve safety. The demonstration quantifies an intuitive result, specifically that maximum tolerable altitude rate on a climbing flow is quasi-inversely related to the vertical separation with a flow some distance above the level-off altitude.

Lean models are a natural outcome of these two requirements. Lean models are defined by four characteristics: they are compact descriptors, have forecasting ability, can guide systematic airspace design, and require low computational cost for use online. The use of lean models allows the mapping of design variables (route geometry, autonomy allocation) to design evaluation metrics (inherent safety, control cost).

In the methodology presented by this thesis, data mining and aggregation are used offline to synthesize lean models of the system. These models can be used to design the airspace and rapidly iterate with many parameter combinations. The goal of the methodology is to support systematic airspace design with additional new metrics. Through the methodology, detailed operational and airspace characteristics can be defined *ex post* rather than fed into the models *ex ante*. Thus, the methodology is normative where more conventional approaches are evaluative.

Compact descriptors are not a requirement for systematic airspace design, and a different approach has used granular agent-based simulations, as discussed in Section 5.3. A practical benefit of the lean model methodology relative to the high-definition simulations is requiring scarce computational resources for use online. The offline generation is computationally intensive, but only needs to be performed once. The
resulting models which are synthesized from data aggregation are lean, but also sufficiently precise and usable in a deductive manner to identify design recommendations. The two approaches may also be used in conjunction, by identifying promising or critical configurations through the aggregated models, and then refining design decisions and precisely evaluating circumstances through a narrowly-scoped, higher-resolution, targeted simulation.

With this methodology, airspace can be redesigned to solve identified limitations. The design process is scoped and guided by performance considerations, as it has long been advocated [55]. The performance effects can be output from the lean models, and refined by ulterior high-definition simulations. Such an approach is preferable to a random search where metrics to allow comparisons or rankings between possible configurations are doubtful, or an exhaustive search which is an intractable and futile endeavor in a system as complex as the airspace.
CHAPTER VI

CONCLUSION

The research presented in this thesis is motivated by the fact that air traffic demand is growing. Air traffic operations must change to ensure performance, rather than focus on integrating technology. The approach to airspace design must account for such new concepts of operations. As operations change, the approach to airspace design must also be radically altered. Thus, there is a need for methods that can enable new designs, do not depend on the current concept of operations, and can also support quantifiable performance goals.

First, models that can integrate additional design objectives differ from past approaches. In the past, airspace planning has focused on capacity and delay improvements through flow management, with the inclusion of taskload and safety as constraints. Second, models that support a range of concepts of operations must not be pinned to current operations. No such systematic design methods exist, which are not conceptually reliant on current operations. Third, predictive models are needed to guide airspace design or enable new design methods. For this, predictive and parsimonious models of airspace under the new paradigm are a necessary step.

The perspective of the airspace used in this thesis is a specific one. The focus is on the enroute portion of flight, and is concerned with strategic decisions. Two design variables parametrize this vision of the airspace: the geometry of the flows, and the delegated autonomy and authority for separation. Three metrics are relevant to the perspective taken in the thesis: control cost, inherent safety, and capacity.

The first contribution of the thesis is to demonstrate two applications of airspace analysis and design: assessing the inherent safety and control cost of the airspace.
Two results are shown, a model which estimates control cost depending on autonomy allocation and traffic volume, and the characterization of inherent safety conditions which prevent unsafe trajectories. The second contribution of the thesis is a set of guiding principles that unifies the cases of application and proposes a general methodology. In this approach, data mining and aggregation are used offline to synthesize lean models of the system. These compact descriptors can then be used to design the airspace and rapidly iterate.

The first application is focused on control cost. The research uses a Monte Carlo simulator framework which allocates directly controlled routes and autonomous self-deconflicting 4-D trajectory flows in the airspace. Several stochastic models of aircraft scheduling, navigation precision, and conflict detection and resolution are interconnected in the simulation.

Results show that complex interactions between many different stochastic components that determine the control cost of a system can be simplified to a single parameter description. The lean model also provides design insights, such as the fact that the flow scheduling has marginal effect on control cost, while autonomous navigation error and tolerance play a very significant role. Finally, the model shows that reducing the autonomy ratio has a less drastic effect on the controlled network than increasing the traffic. Thus, some level of autonomy can serve as a safeguard against heavy control cost due to traffic volume increases. Results also show how the two design variables, route geometry and autonomy allocation, influence control cost. Thus, the autonomy ratio affects control cost from the controlled subnetwork in an exponential manner, but has an almost linear effect on control cost from the autonomous subnetwork. On the other hand, traffic volume affects control cost from both subnetworks in approximately linear fashion, both in terms of mean but also regarding standard deviation. The higher standard deviation indicates a large variability, which is more costly in cognitive terms, and signals an unpredictable and
potentially unsafe system.

The second application is focused on inherent safety. The research demonstrates a novel method to reliably generate collision avoidance advisories, in piloted simulations, by the widely-used TCAS. The TCAS advisory issued to a pilot is highly sensitive to the trajectory of an intruder aircraft relative to the ownship flown by the pilot. In realistic piloted simulations, a pre-scripted intruder trajectory will not reliably result in the relative dynamics that lead to a desired TCAS advisory. Further, the complexity of the TCAS logic requires a novel method for mapping trajectories to the range of possible advisories. The research uses a Rapidly-exploring Random Tree algorithm in large-scale fast-time simulations to establish the mapping between the space of relative trajectories and TCAS advisories. These trajectories are then created in piloted simulations through guidance algorithms. Experimental piloted simulations results demonstrate the ease of use and robustness of this method, and validate the characterization of trajectories according to their safety risk. The unsafe trajectory clusters are described by convex surfaces. These convex surfaces, when overlapped on several flows from the route network, indicate hazard zones. These hazard zones are the areas which lead to unsafe trajectories and must be avoided by all other routes in the network, in addition to nominal separation requirements. The method supports safety-centered airspace design.

Results show that the inherent safety of routes can be characterized, determined, and predicted by relatively simple convex polyhedra (albeit multi-dimensional and involving spatial and kinematic information). Results also provide direct trade-off relations between spatial and kinematic constraints on route geometries that preserve safety. The demonstration quantifies an intuitive result, specifically that maximum tolerable altitude rate on a climbing flow is quasi-inversely related to the vertical separation with a flow some distance above the level-off altitude.

The thesis generalizes unifying principles from these two demonstrations to define
a new approach to airspace design. Systematic analytic modeling is a desirable direction, however such models require fitting with empirical data before they can be parametrized and become directly applicable. A higher level of detail is not always better, and bridging the gap between analysis and design is another challenge.

The second contribution of the thesis is the approach to airspace design using lean models. The use of lean models is a result of a natural flowdown from the airspace view to the requirements. The first question posed by the perspective on the airspace is the appropriate level of detail. Choosing the appropriate level of detail allows a discussion of design insights. The scope of the lean model is situated at a level of granularity that identifies the macroscopic effects of operational changes on the strategic level. Choosing the appropriate computational technique allows an analysis of the effect of design variables. The lean model technique maps low-level changes to high-level properties and provides predictive results.

The proposed methodology has three steps: aggregate data, synthesize lean model, guide design. In the first step, a database is built and mined. In the second step, a lean model, as defined above, is synthesized. The third step is the extraction of insights in design choices and a focus on promising aspects for further localized and detailed simulations.

6.1 Suggested extension to capacity

This thesis has not focused on designing airspace for capacity because of the large body of knowledge surrounding it [63]. However, the methods developed in this thesis can be extended such that capacity can be addressed in a similar manner to the two other topics discussed in the thesis, safety and control cost.

A possible approach to designing airspace for capacity has been presented by the author elsewhere [100, 124]. The research focuses on the potential of mean field
The approach is in line with the methodology of the thesis, which advocates airspace design by using lean models constructed from aggregated data.

The purpose of the mean field games model is to model routing decisions which would be made by autonomous aircraft, thereby predicting congestion and demand propagation. The mean field games model therefore fits as a complement to the other empirical applications by addressing capacity issues. The model is meant to be used for capacity-centered airspace design, in relation to the taskload-centered and safety-centered approaches discussed in the thesis.

From a computational perspective, two possible approaches exist: continuous infinite state-space and discrete finite state-space. The first approach is the numerical resolution of the coupled partial differential equations with states defined continuously in both time and space. Aircraft are modeled by a density distribution over the entire possible space. This model is closest to the formal application of mean field games but unsuitable for complex systems because of the processing requirements.

For a complex graph used in a route network, the connectivity matrix alone is a few hundred lines, and expressing costs or state transitions quickly becomes impractical in a numerical environment. Foreseeable difficulties in the continuous approach could arise because of processing cost and stability of the equation discretization schemes.

The second approach is the numerical simulation using discrete state definitions. In this approach, individual agents are considered. These agents have a finite number of possible options. Agents define their strategy based on a mean distribution of the other agents. This approach is better suited to the graph model of possible trajectories.

Foreseeable difficulties in the discrete approach could arise in the expression of this complex routing problem in a way which avoids infinite recursive calls. Furthermore, it is not clear if implicit calculation of all individual strategies can be avoided while
the optimal strategies are being elaborated. Such a full computation which would
defy the point of the mean field games model and likely produce as much processing
overload as the continuous, infinite-state approach.

6.2 Suggested future directions

The work decomposition can be expressed in the generic systems engineering nomen-
clature represented in Figure 2, in Chapter 1: the proposed concept of operations
and triple objectives of design discussed in Chapters 1 and 2, respectively, reframe
the airspace design problem, thus essentially establishing new requirements. The two
contributions of the thesis follow at a more applied level. The demonstrations in
Chapters 3 and 4 make up the detailed design, while the unifying methodology in
Chapter 5 is a formalization of the architecture.

The work presented in the thesis has thus followed a top-down approach going from
concept (most abstract) to detailed design (least abstract). The systems engineering
model suggests future steps which can take the bottom-up approach. Such a reverse
approach would place this thesis in a continuity and close the loop. Future follow-up
work can build up from the detailed design exhibited here to integrated functions,
validation, and validation [135]. These future steps would address questions
such as:

Integration: How to connect the different models and applications?

Verification: Does the system meet specifications?

Validation: Does the system address the needs?

To implement a bottom-up approach, a common thread must be identified for
integration. A suggested common perspective could be the micro/macro duality of
scale which has been used in this thesis. The lean model methodology establishes
a mapping going from microscopic causes (individual trajectory deviations and uncertainties) to macroscopic effects (aggregated control cost, and intrinsic safety of a route design). The suggested extension to capacity mentioned in Section 6.1 could also adhere to this schema, since routing strategies are established at the micro level of individual aircraft, resulting in traffic flow dynamics and congestion at the macro level. However, the spatial scale of the different applications remains somewhat different: the application to control cost and taskload deals with cross-sector trajectories, while the application to safety is based on results over a few nautical miles. Nevertheless, the approach shown in this thesis uses that safety information to provide more macroscopic flow safety information.

In future work, the low-level to high-level mappings and models constructed in the thesis can thus be integrated to guide design of the airspace, but also to test concepts for verification and validation purposes. This research can thus bring value to efforts meant to design or identify superior and inferior airspace structures. For design, the safety and control cost models can be applied to create representative airspace structures by varying concept parameters. For testing, the models can evaluate the airspace characteristics for these metrics.
APPENDIX A

THEORETICAL DERIVATIONS OF CONTROL COST PROBABILITY

This section details theoretical derivations supporting the taskload application. Formulas related to the autonomous navigation model and taskload probabilities for several basic elements of a route structure are shown.

A.1 Aircraft model

Boundary hitting times The probability density function for the first hitting time of a boundary $\tau_1^{(k)} = \inf\{t : X_t \geq k\}$ for an origin at $X_0$ and a level $k$ by an unidimensional Ornstein-Uhlenbeck process has the closed-form solution shown in Equation (9), as given by [93]

$$
P[\tau^{(k)} \in dt] = f_{\tau}(t) = \frac{k - X_0}{\sigma^2} \left( \frac{\kappa}{\sigma^2 \sinh \kappa t} \right)^{\frac{3}{2}} \times \ldots$$

$$\times \exp \left[ \frac{\kappa}{2\sigma^2} \left( \frac{X_0}{\sigma^2} - \mu \right)^2 - (k - \mu)^2 + \sigma^2 t - \left( \frac{X_0}{\sigma^2} \right)^2 \coth \kappa t \right]$$

(9)

In the multidimensional case, no closed-form solution is known for the first hitting time density function of a correlated Brownian motion with drift [102]. Such an n-dimensional stochastic process $X_t$ is a solution to (10). But through a change of variables and by using the scalability property of the Wiener process, the Ornstein-Uhlenbeck process given in (2) can be re-written as (11).

$$dX_t = \mu dt + \sigma dW_t$$

(10)

$$dX_{\frac{t}{\sigma^2}} = -\frac{\kappa}{\sigma^2} X_{\frac{t}{\sigma^2}} dt + dW_t$$

(11)
From comparing (11) to (10), for which no closed-form solution of the first hitting time density is known, it becomes apparent that attempting to express hitting time density for a multidimensional correlated Ornstein-Uhlenbeck process is a daunting task. An assumption of uncorrelated dimensions must therefore be adopted. By assuming that the along-track and cross-track deviations are decoupled (an assumption previously used in [115]), then implicitly the subjacent unidimensional Ornstein-Uhlenbeck processes can be considered to be uncorrelated.

**Taskload probability** From this, a measure of the taskload required for controlling an individual aircraft is obtained as the probability of the number of interventions over a given period $T_{\text{max}}$. Assuming all realizations of the aircraft trajectory (after each corrective intervention from the controller) are independent, the (discrete) probability (12) is obtained from autoconvoluting the hitting times density:

$$P[N(T) \geq n] = P[\tau_1 + \cdots + \tau_n \leq T_{\text{max}}]$$

$$\cdots = \int_0^{T_{\text{max}}} \{n-1\} f_\tau(t) dt$$

(12)

Here $f_\tau(t)$ is the hitting time density function, $N(T)$ is the number of corrections over time $T_{\text{max}}$, $\{\tau_1 \cdots \tau_n\}$ are the successive hitting times, and $\{n-1\}$ represents $n-1$ (continuous) autoconvolutions of the density function.

$$\{0\} f_\tau(t) = f_\tau(t)$$

$$\{1\} f_\tau(t) = \int_0^t f_\tau(x) \cdot f_\tau(t-x) dx$$

$$\{k+1\} f_\tau(t) = \int_0^t \{k\} f_\tau(x) \cdot f_\tau(t-x) dx$$

Therefore, for $n \geq 1$ the probability of the number of interventions is (13).
\[ P[N(T) = n] = \int_0^{T_{\text{max}}} \left\{ \binom{n-1}{\star} - \binom{n}{\star} \right\} f_\tau(t) dt \quad (13) \]

\[ P[N(T) = 0] = 1 - P[N(T) \geq 1] \]

\[ \cdots = 1 - \int_0^{T_{\text{max}}} f_\tau(t) dt \quad (14) \]

**FTE stochastic model calibration** By applying Itô’s lemma, it can be shown that for any fixed \( s \) and \( t \), \( 0 \leq s \leq t \), the random variable \( X_t \) conditional upon \( X_s \) of the Ornstein-Uhlenbeck process has the form (15), where \( N(0, 1) \) denotes a standard normal distribution. The relationship (16) between consecutive observations \( X_i \) and \( X_{i+1} \) with a timestep \( \delta t \) is therefore affine with an independent and identically distributed random noise \( \epsilon \).

\[ X_t = X_s e^{-\kappa(t-s)} + \mu(1 - e^{-\kappa(t-s)}) \ldots \]

\[ \ldots + \sigma \left( \frac{1 - e^{-2\kappa(t-s)}}{2\kappa} \right)^{\frac{1}{2}} \cdot N(0, 1) \quad (15) \]

\[ X_{i+1} = aX_i + b + \epsilon \quad (16) \]

A least squares linear regression is used to identify the recursion parameters \( a, b, \) and the standard deviation of the noise \( \sigma_\epsilon \), from which the parameters (17) of the Ornstein-Uhlenbeck stochastic differential equation (2) can be deduced.

\[ \kappa = -\frac{\ln a}{\delta t} \]

\[ \mu = \frac{b}{1-a} \]

\[ \sigma = \sigma_\epsilon \left( \frac{-2 \ln a}{\delta t(1-a^2)} \right)^{\frac{1}{2}} \quad (17) \]

A maximum likelihood estimate method was also conducted. From (16), \( \{X_{i+1} - aX_i - b = \epsilon\} \) is a normal random variable, and so the conditional probability density function of \( X_{i+1} \) given \( X_i \) with a time step \( \delta t \) is shown in (18), while the log-likelihood
function of \( n + 1 \) observations \( \{X_0, \ldots X_n\} \) is given in (19).

\[
f_{[X_{i+1}|X_i]}(x) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \ldots \times \exp \left[ -\frac{(x - X_i e^{-\kappa \delta t} - \mu(1 - e^{-\kappa \delta t}))^2}{2\hat{\sigma}^2} \right] \quad (18)
\]

\[
\hat{\sigma} = \sigma \left[ \frac{1 - e^{-2\kappa \delta t}}{2\kappa} \right]^{\frac{1}{2}}
\]

\[
\mathcal{L}(\mu, \kappa, \hat{\sigma}) = \sum_{i=0}^{n-1} \ln f_{[X_{i+1}|X_i]}(X_{i+1})
\]

\[
\ldots = -\frac{n}{2} \ln 2\pi - n \ln \hat{\sigma} \ldots
\]

\[
\ldots = -\frac{1}{2\hat{\sigma}^2} \sum_{i=0}^{n-1} [X_{i+1} - X_i e^{-\kappa \delta t} - \mu(1 - e^{-\kappa \delta t})]^2 \quad (19)
\]

The argument of the maximum of \( \mathcal{L} \) found from the three partial derivatives \( \frac{\partial \mathcal{L}}{\partial \kappa} \), \( \frac{\partial \mathcal{L}}{\partial \mu} \), \( \frac{\partial \mathcal{L}}{\partial \hat{\sigma}} \) gives the system in (20).

\[
\kappa = -\frac{1}{\delta t} \ln \frac{\sum_{i=0}^{n-1} (X_{i+1} - \mu)(X_i - \mu)}{\sum_{i=0}^{n-1} (X_{i+1} - \mu)^2}
\]

\[
\mu = \frac{\sum_{i=0}^{n-1} X_{i+1} - X_i e^{-\kappa \delta t}}{n(1 - e^{-\kappa \delta t})}
\]

\[
\hat{\sigma}^2 = \frac{1}{n} \sum_{i=0}^{n-1} [X_{i+1} - \mu e^{-\kappa \delta t}(X_i - \mu)]^2 \quad (20)
\]

### A.2 Analytic taskload probabilities

**Single lane** From the Poisson process flow model in (5), the probability of the number of aircraft \( M \) simultaneously controlled (a random variable) can be deduced:
if it takes an aircraft $T_{\text{cross}}$ to cross the airspace, then the probability that at any
given time there are $k$ aircraft present is given in (21); if there are $M = k \geq 1$ aircraft
present, and the aircraft $i$ requires $N_i^{(T)}$ interventions over time $T$, then the total
taskload the controller is subject to has a probability given in (22).

$$\mathcal{P}[M = k] = \frac{e^{-\lambda T_{\text{cross}}} (\lambda T_{\text{cross}})^k}{k!} \quad (21)$$

$$\mathcal{P}[N_{1}^{(T)} + \cdots + N_{k}^{(T)} = n] = \left[ k^{k-1} \right] \mathcal{P}\{N^{(T)}\}[n] \quad (22)$$

Here $\left[ k^{k-1} \right] \mathcal{P}\{N^{(T)}\}$ represents $k - 1$ discrete autoconvolutions of the single aircraft
taskload probability $\mathcal{P}\{N^{(T)}\}$ from (13). Obviously for $M = 0$, there will be no
intervention with probability 1.

$$\left[ 0^{0} \right] \mathcal{P}\{N^{(T)}\}[n] = \mathcal{P}[N^{(T)} = n]$$

$$\left[ 1^{1} \right] \mathcal{P}\{N^{(T)}\}[n] = \sum_{i=0}^{n} \mathcal{P}[N^{(T)} = i] \cdot \mathcal{P}[N^{(T)} = n - i]$$

$$\left[ m+1^{m+1} \right] \mathcal{P}\{N^{(T)}\}[n] = \sum_{i=0}^{n} \left[ m^{m} \right] \mathcal{P}\{N^{(T)}\}[i] \cdot \mathcal{P}[N^{(T)} = n - i]$$

Since the number of aircraft is a random variable, combining (21) and (22) gives
the probability (23) of the overall taskload $N_{\lambda}^{(T)}$ for the flow with intensity $\lambda$ over
time $T$ for $n \geq 1$; $\mathcal{P}[N^{(T)} = 0]$ is the probability that no intervention is required for
one aircraft, given by (14).

$$\mathcal{P}[N_{\lambda}^{(T)} = n] = \sum_{i=1}^{+\infty} \mathcal{P}[M = i] \cdot \left[ i^{-1} \right] \mathcal{P}\{N^{(T)}\}[n] \quad (23)$$

$$\mathcal{P}[N_{\lambda}^{(T)} = 0] = \sum_{i=0}^{+\infty} \mathcal{P}[M = i] \cdot (\mathcal{P}[N^{(T)} = 0])^i$$
Multiple parallel lanes  If the spatial extents are identical, and assuming independence of the flows, the whole system is equivalent to a single Poisson process with cumulative intensities $\lambda_{tot} = \sum \lambda_i$. Single lane results can thus be simply generalized to (24).

$$\mathcal{P}\{N_{\cup \lambda_i}^{(T)}\} = \mathcal{P}\{N_{\sum \lambda_i}^{(T)}\}$$  \hspace{1cm} (24)

If the spatial extents of the flows are different, the problem is slightly more complex. The taskload probability (25) for $j \geq 2$ different flows with $\{e_1, \cdots, e_j\}$ spatial extents, $\{\lambda_1, \cdots, \lambda_j\}$ intensities, and $\{N_{\lambda_1, e_1}^{(T)}, \cdots, N_{\lambda_j, e_j}^{(T)}\}$ interventions per flow is

$$\mathcal{P}[N_{\cup \lambda_i}^{(T)} = n] = \mathcal{P}[N_{\lambda_1, e_1}^{(T)} + \cdots + N_{\lambda_j, e_j}^{(T)} = n]$$

$$\cdots = \left[ \bigotimes_{i=1}^{j} \right] \mathcal{P}\{N_{\lambda_i, e_i}^{(T)}\}[n]$$  \hspace{1cm} (25)

where $\left[ \bigotimes_{i=1}^{j} \right]$ designates the successive convolutions of the $\mathcal{P}\{N_{\lambda_i, e_i}^{(T)}\}$ taskload probabilities given in (23) for flows 1 to $j$. This operator is well defined since convolution is associative.

Crossings and mergings  For a minimum possible approach distance $D_{min}$ (taken to be for example 5 NM, i.e. the conflict separation standard), the symmetrical safe-zone boundaries $x_1$ and $x_2$ in each of the flows are defined by (26).

$$(x_1 + x_2 \cos \alpha - \frac{e_2}{2} \cos \alpha) \ldots$$

$$\ldots + (- \frac{e_1}{2} + x_2 \sin \alpha + \frac{e_2}{2} \sin \alpha)^2 = D_{min}^2$$

$$(x_1 + x_2 \cos \alpha - \frac{e_2}{2} \sin \alpha)^2 \ldots$$

$$\ldots + (- \frac{e_1}{2} + x_2 \sin \alpha + \frac{e_2}{2} \cos \alpha)^2 = D_{min}^2$$  \hspace{1cm} (26)
A discrete convolution $[\ast]$ in (27) gives the total taskload over time $T$ for $n \geq 1$ accounting for $A - 1$ conflicts at the flow crossing and for $N_{\lambda_1 \cup \lambda_2}^{(T)}$ interventions to maintain the structure.

\[
\mathcal{P}[N_{\lambda_1 \times \lambda_2}^{(T)} = n] = \mathcal{P}[A - 1 + N_{\lambda_1 \cup \lambda_2}^{(T)} = n] \\
\cdots = \mathcal{P}\{A - 1\}[\ast]\mathcal{P}\{N_{\lambda_1 \cup \lambda_2}^{(T)}\} \\
\cdots = \sum_{i=0}^{n} \mathcal{P}[A = i + 1] \cdot \mathcal{P}[N_{\lambda_1 \cup \lambda_2}^{(T)} = n - i] \quad (27)
\]

\[
\mathcal{P}[N_{\lambda_1 \times \lambda_2}^{(T)} = 0] = \mathcal{P}[A = 0] + \mathcal{P}[A = 1] \cdot \mathcal{P}[N_{\lambda_1 \cup \lambda_2}^{(T)} = 0]
\]
REFERENCES


