Expectation-Oriented Framework for Automating Approximate Programming

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Abstract

This paper describes ExpAX, a framework for automating approximate programming based on programmer-specified error expectations. Three components constitute ExpAX: (1) a programming model based on a new kind of program specification, which we refer to as *expectations*. Our programming model enables programmers to implicitly relax the accuracy constraints without explicitly marking operations approximate; (2) a novel approximation safety analysis that automatically identifies a safe-to-approximate subset of the program operations; and (3) an optimization that automatically marks a subset of the safe-to-approximate operations as approximate while considering the error expectation. Further, we formulate the process of automatically marking operations as approximate as an optimization problem and provide a genetic algorithm to solve it. We evaluate ExpAX using a diverse set of applications and show that it can provide significant energy savings while improving the quality-of-result degradation. ExpAX automatically excludes the safe-to-approximate operations that if approximated lead to significant quality degradation.

1. Introduction

Energy efficiency is a first-class design constraint in computer systems. Its potential benefits go beyond reduced power demands in servers and longer battery life in mobile devices. Notably, improving energy efficiency has become a requirement due to limits of device scaling in what is termed the *dark silicon problem* [9]. As per-transistor speed and efficiency improvements diminish, radical departures from conventional approaches are necessary to improve the performance and efficiency of general-purpose computing. One such departure is general-purpose approximate computing.

Conventional techniques in energy-efficient computing navigate a design space defined by the two dimensions of performance and energy, and traditionally trade one for the other. General-purpose approximate computing explores a third dimension—that of error. This error dimension concerns relaxing the robust digital abstraction of full accuracy in general-purpose computing, and trades the accuracy of computation for gains in both energy and performance.

Approximation can only be beneficial if a large body of applications can tolerate inexact computation without incurring severe degradation of output quality. Recent research [1, 2, 5–8, 10–18, 20, 21] has in fact shown that many emerging applications in both cloud and mobile services inherently have such a tolerance. These applications span a wide range of domains including web search, big-data analytics, machine learning, multimedia, cyber-physical systems, speech and pattern recognition, and many more. In fact, there is an opportunity due to the current emergence of approximation-tolerant applications and the growing unreliability of transistors as technology scales down to atomic levels [9]. For these diverse domains of applications, providing programming models and compiler optimizations for approximation can provide significant opportunities to improve performance and energy efficiency at the architecture and circuit level by eliminating the high tax of providing full accuracy [6–8, 17].

State-of-the-art systems for approximate computing such as EnerJ [18] and Rely [4] require programmers to *manually and explicitly* declare low-level details, such as the specific variables and operations to be approximated, and provide safety [18] or quality of result guarantees [4]. In contrast, we focus on providing an automated framework that allows programmers to express concise, high-level, and intuitive *error expectation specifications*, and automatically finds the approximable subset of operations in the program. In doing so, our framework enables programmers to implicitly declare *which* parts of the program are safely approximable while explicitly expressing *how much* approximation is preferable.

Figure 1 shows our automated expectation-oriented framework, called ExpAX. It constitutes three phases: (1) programming, (2) analysis, and (3) optimization.

First, ExpAX allows programmers to *implicitly* relax the accuracy constraints on low-level program data and operations by *explicitly* specifying error expectations on program outputs. Our programming model provides the syntax and semantics for specifying such high-level error expectations. In this model, the program itself without the specific expectation carries the most strict semantics in which approximation is not allowed. Programmers add the expectations to implicitly relax the accuracy requirements without explicitly specifying where the approximation is allowed.

Second, ExpAX includes a novel *approximation safety analysis* that automatically finds a candidate subset of program operations that can be approximated without violating program safety guarantees. The program outputs on which the programmer has specified error expectations are inputs to this analysis.

Third, ExpAX includes an optimization framework that selectively marks a number of the candidate operations as approximate while considering the specified error expectations. We formulate the problem of selecting these operations as a general optimization procedure that minimizes error and energy with respect to the specified expectations. Thus, in our formulation, the error expectations guide the optimization procedure to strike a balance between quality-of-result degradation and energy savings, and focuses approximation to satisfy programmer expectations.

The optimization procedure is parameterized by a system specification that models error and energy characteristics of the underlying hardware on which the program will execute. While the programmer specifies the error expectations in a high-level, hardware-independent manner, our optimization formulation automatically considers the low-level hardware parameters without exposing them to the programmer. ExpAX thereby enhances the portability of the approximate programs. We implement an instance of the optimization procedure using genetic algorithms and evaluate the effectiveness of the resulting framework using a diverse set of benchmarks. Our empirical results show that for many applications, there is a subset of safe-to-approximate operations that if approximated will result in significant quality-of-result degradation. This insight and the results confirms that automating approximate programming is of significant value. Further, our optimization formulation and its genetic implementation can automatically find and filter this subset while providing significant energy savings.

In summary, our approach provides a system-independent framework that automates approximate programming while enabling programmers to implicitly specify safe-to-approximate parts of the program and express their error expectations using a novel

$$
\begin{array}{rccrccl}
(\textit{constant}) & r & \in & \mathbb{R} & (\textit{assignment label}) & l & \in \ \mathbb{L} \\
(\textit{variable}) & v & \in & \mathbb{V} & (\textit{expectation label}) & k & \in \ \mathbb{K} \\
(\textit{operation}) & o & \in & \mathbb{O} & (\textit{expression}) & e & ::= \ v \mid r
\end{array}
$$

$$
\begin{array}{rll}
(\textit{program}) & s ::= & v :=^{l} o(e_1, e_2) \\
& \mid & \phi^{k} \\
& \mid & s_1 \ ; \ s_2 \\
& \mid & \textsf{if } (v > 0) \textsf{ then } s_1 \textsf{ else } s_2 \\
& \mid & \textsf{while } (v > 0) \textsf{ do } s
\end{array}
$$

Figure 2: Language syntax.

$$
\begin{array}{rlll}
(\textit{error}) & c & \in & \mathbb{R}_{0,1} = [0,1] \\
(\textit{error magnitude}) & f & \in & (\mathbb{R} \times \mathbb{R}) \rightarrow \mathbb{R}_{0,1} \\
(\textit{expectation}) & \phi & ::= & \textsf{rate}(v) < c \\
& & \mid & \textsf{magnitude}(v) < c \textsf{ using } f \\
& & \mid & \textsf{magnitude}(v) > c \textsf{ using } f \textsf{ with rate} < c'
\end{array}
$$

Figure 3: Expectation syntax.

$$
\begin{array}{rlllll}
(\textit{error model}) & \epsilon & \in & \mathbb{E} & = & (\mathbb{R}_{0,1} \times \mathbb{R}) \\
(\textit{energy model}) & j & \in & \mathbb{J} & = & (\mathbb{R}_{0,1} \times \mathbb{R}) \\
(\textit{system spec}) & \psi & \in & \mathbb{O} \rightarrow (\mathbb{E} \times \mathbb{J}) & &
\end{array}
$$

Figure 4: System specification.

approximate programming model. Our approach does not provide quality-of-result guarantees. But it represents a best effort-solution to reduce the programmer effort, enhance the portability of the approximate programs across different hardware systems, strike a balance between quality-of-result degradation and energy efficiency, and focus approximation based on programmer preferences.

2. Expectation-Oriented Programming Model

In this section, we present a core calculus that distills the essence of our expectation-oriented programming model. We first describe the abstract syntax of programs (Section 2.1). We then formalize a system specification, which provides both the error models and the energy models of approximable program operations under the system that executes the programs (Section 2.2). Finally, we provide an instrumented semantics of these programs under a given system (Section 2.3). The semantics induces lightweight runtime instrumentation that allows our optimization framework (described later in Section 4) to automatically tune which program operations to approximate on the given system in a manner that optimizes energy while taking into account all the programmer-specified expectations on error bounds.

2.1 Language Syntax

We assume a simple imperative language shown in Figure 2. A program s in the language is a single procedure with the usual control-flow constructs (sequential composition, branching, and loops). The language supports only real-valued data, only binary operations o on them, and only expressions of the form $v > 0$ in conditionals. We limit our formalism to this simplified setting for clarity of exposition. It is straightforward to incorporate procedure calls and structured data types such as arrays, records, and pointers. These extensions are supported in our implementation for the full Java language and we describe their handling in Section 5.

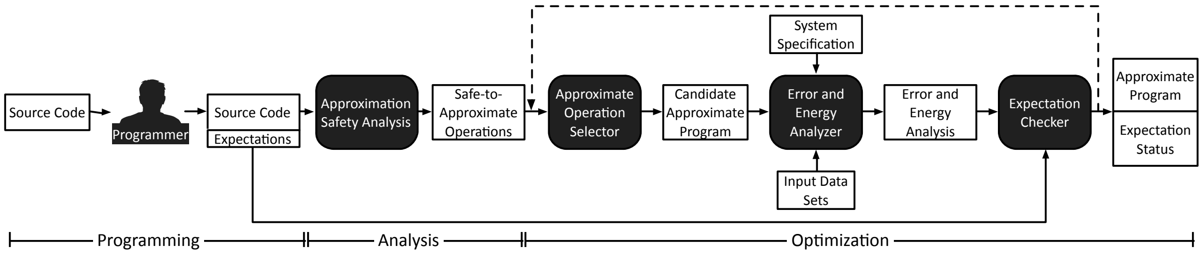
Our simple language supports two kinds of primitive statements, both of which play a special role in our approximation framework: *assignments* and *expectations*. Assignments are the only places in the program where approximations may occur, providing opportunities to save energy at the cost of introducing computation errors. We assume that each assignment has a unique label $l \in \mathbb{L}$. Conversely, expectations ϕ are the only places in the program where the programmer may specify acceptable bounds on such errors. We assume that each expectation has a unique label $k \in \mathbb{K}$.

The syntax of expectations is presented in Figure 3. An expectation allows the programmer to express, at a certain program point, a bound on the error in the data value of a certain program variable. We allow three kinds of expectations that differ in the aspect of the error that they bound: the error *rate*, the error *magnitude*, or both. We first informally explain the meaning of these expectations and then give real-world examples illustrating their usefulness. Section 2.3 provides the formal semantics of these expectations.

- Expectation $\textsf{rate}(v) < c$ states that the rate at which an error is incurred on variable v should be bounded by c. Specifically, suppose this expectation is executed n_2 times in an execution, and suppose the value of v each time this expectation is executed deviates from its exact value n_1 times. Then, the ratio n_1/n_2 should be bounded by c.
- Expectation $\textsf{magnitude}(v) < c \textsf{ using } f$ states that the normalized magnitude of the error incurred on variable v should be bounded by c. Unlike the error rate, which can be computed universally for all variables, the error magnitude is application-specific: each application domain may use a different metric for quantifying the magnitude of error, such as signal-to-noise ratio (SNR), root mean squared error, relative error, etc. Hence, this expectation asks the programmer to specify how to compute this metric, via a function f that takes two arguments—the potentially erroneous value of v and the exact value of v—and returns the normalized magnitude of that error.
- Expectation $\textsf{magnitude}(v) > c \textsf{ using } f \textsf{ with rate} < c'$ allows to bound both the error rate and the error magnitude: it states that the rate at which the error incurred on variable v exceeds normalized magnitude c is bounded by c'.

We illustrate the above three kinds of expectations on a real-world program shown in Figure 5 that performs edge detection on an image. The edge detection algorithm first converts the image to grayscale (lines 6–8). Then, it slides a 3×3 window over the pixels of the grayscale image and calculates the gradient of the window's center pixel to its eight neighboring pixels (lines 9–16). For brevity, we omit showing the body of the `build_window` function. Since the precise gradient calculation is compute intensive, image processing algorithms use a Sobel filter (lines 25–35), which gives an estimation of the gradient. Thus, the application is inherently approximate.

We envision the programmer specifying acceptable bounds on errors resulting from approximations in the edge detection application, by means of three expectations indicated by the *accept* keyword in the figure. The first expectation is specified on the entirety of the `output_image` (line 17). It states that less than 0.1 (10%) magnitude of error (root-mean-squared difference of pixels of the exact and approximate output) is acceptable. The second expectation specifies that on less than 35% of the grayscale pixel conversions, the error magnitude (relative error) can exceed 0.9 (90%). The third expectation specifies that upto 25% of the times `gradient` is calculated, any amount of error is acceptable. These specifications capture the domain knowledge of the programmer about the application and their expectations of approximations. Further, the specified expectations serve to implicitly identify any operations contributing to the computation of data that can be potentially approximated. In practice, only the first expectation specification on the `output_image` (line 17) suffices for our approximation safety analysis (Section 3) to identify the approximable subset of operations. But our optimization framework (Section 4) consumes all three expectations, using them together to guide the optimization process that ultimately determines which of the approximable operations to approximate in order to yield the best accuracy/energy



Source Code
Programmer
Source Code
Expectations
Approximation Safety Analysis
Safe-to-Approximate Operations
Approximate Operation Selector
Candidate Approximate Program
System Specification
Error and Energy Analyzer
Input Data Sets
Error and Energy Analysis
Expectation Checker
Approximate Program
Expectation Status
Programming
Analysis
Optimization

Figure 1: Overview of our expectation-oriented framework, ExpAX, that automates approximate programming.

```
void detect_edges(Pixel[][] input_image,
                  Pixel[][] output_image) {
  float[WIDTH][HEIGHT] gray_image;
  float[3][3] window;
  float gradient;
  for(int y = 0; y < HEIGHT; ++y)
    for(int x = 0; x < WIDTH; ++x)
      gray_image[x][y] = to_grayscale(input_image[x][y]);
  for(int y = 0; y < HEIGHT; ++y)
    for(int x = 0; x < WIDTH; ++x) {
      build_window(gray_image, x, y, window);
      gradient = sobel(p);
      output_image[x][y].r = gradient;
      output_image[x][y].g = gradient;
      output_image[x][y].b = gradient;
    }
  accept magnitude[output_image] < 0.1;
}
float to_grayscale(Pixel p) {
  float luminance;
  luminance = p.r * 0.30 + p.g * 0.59 + p.b * 0.11;
  accept magnitude[luminance] > 0.9 with rate < 0.35;
  return luminance;
}
float sobel(float[3][3] p) {
  float x, y, gradient;
  x =  (p[0][0] + 2 * p[0][1] + p[0][2]);
  x += (p[2][0] + 2 * p[2][1] + p[2][2]);
  y =  (p[0][2] + 2 * p[1][2] + p[2][2]);
  y += (p[0][0] + 2 * p[1][1] + p[2][0]);
  gradient = sqrt(x * x + y * y);
  gradient = (gradient > 0.7070) ? 0.7070 : gradient;
  accept rate[gradient] < 0.25;
  return gradient;
}
```

Figure 5: Program illustrating expectation-oriented programming.

tradeoff on a given system. We next describe the system specification that is needed for this purpose.

2.2 System Specification

The system specification provides the error model and the energy model of the system on which our programs execute. Our approximation framework is parametric in the system specification to allow tuning the accuracy/energy tradeoff of the same program in a manner that is optimal for the given system.

We adopt system specifications of the form shown in Figure 4. Such a specification ψ specifies an error model ϵ and an energy model j for each approximable operation. In our formalism, this is every operation $o \in \mathbb{O}$ on real-valued data. Error models and energy models are specified in our framework as follows:

- An error model ϵ for a given operation is a pair (c, r) such that c is the rate at which the operation, if run approximately, is expected to compute its result inaccurately; moreover, the magnitude of the error in this case is $\pm r$.
- An energy model j for a given operation is also a pair (c, r) such that r is the energy that the operation costs, if run exactly, while c is the fraction of energy that is saved if the same operation is run approximately.

$$\begin{array}{rccl} (\textit{approximated assignments}) & L & \subseteq & \mathbb{L} \\ (\textit{program state}) & \rho, \rho^* & \in & \mathbb{V} \to \mathbb{R} \\ (\textit{error expectation values}) & \theta & \in & \mathbb{K} \to (\mathbb{Z}^2 \cup \mathbb{R}) \end{array}$$

Figure 6: Semantic domains of instrumented program.

In practice, system specifications may be even more expressive than the form described above. For instance, they may provide error rates and energy savings as functions of not only the kind of operation but even the operand values. Our optimization framework, described in Section 4, can easily be extended to support richer system specifications by virtue of being black-box.

2.3 Instrumented Program Semantics

We now provide an instrumented semantics of programs under a given system specification. The goal of this semantics is two-fold: first, it precisely specifies the meaning of expectations; and second, it specifies the runtime instrumentation that our optimization framework needs in order to measure the impact on accuracy and energy of approximating a given set of assignments in the program.

Figure 6 shows the domains of the instrumented semantics. We use L to denote the set of labels of assignments in the program that must be approximated. We use ρ, a valuation of real-valued data to all program variables, to denote both the input to the program and the runtime state of the program at any instant of execution. Finally, we use θ to denote a valuation to all expectations in the program at any instant of execution. The value of expectation labeled k, denoted $\theta(k)$, is either a pair of integers (n_1, n_2) or a real value c, depending upon whether the expectation tracks the error rate or the error magnitude, respectively. In particular, n_1/n_2 denotes the error rate thus far in the execution, and c denotes the largest error magnitude witnessed thus far. Tracking these data suffices to determine, at any instant of execution, whether or not each expectation in the program meets its specified error bound.

We define an instrumented semantics of programs using the above semantic domains. Figure 7 shows the rules of the semantics for the most interesting cases: assignments and expectations. For brevity, we omit the rules for the remaining kinds of statements, as they are relatively straightforward. Each rule is of the form:

$$L \models_{\psi} \langle s, \rho_1, \rho_1^*, \theta_1 \rangle \overset{r}{\rightsquigarrow} \langle \rho_2, \rho_2^*, \theta_2 \rangle$$

and describes a possible execution of program s under the assumption that the set of approximated assignments in the program is L, the start state is ρ_1 with expectation valuation θ_1, and the system is specified by ψ. The execution ends in state ρ_2 with expectation valuation θ_2, and the energy cost of executing all assignments (approximated as well as exact) in the execution is r. Note that ρ_1 and ρ_2 track the actual (i.e., potentially erroneous) values of variables in the approximated program. We call these *actual states*, in contrast to corresponding *shadow states* ρ_1^* and ρ_2^* that track the exact val-

$$L \models_\psi \langle v :=^l o(e_1, e_2), \rho, \rho^*, \theta\rangle \overset{r_j}{\rightsquigarrow} \langle \rho[v \mapsto r], \rho^*[v \mapsto r^*], \theta\rangle \qquad [\text{if } l \notin L] \qquad (\text{ASGN-EXACT})$$

$$\text{where } \left[\ \psi(o) = ((\_,\_),(\_, r_j)) \text{ and } r = [\![o(e_1,e_2)]\!](\rho) \text{ and } r^* = [\![o(e_1,e_2)]\!](\rho^*)\ \right]$$

$$L \models_\psi \langle v :=^l o(e_1, e_2), \rho, \rho^*, \theta\rangle \overset{r_1}{\rightsquigarrow} \langle \rho[v \mapsto r_2], \rho^*[v \mapsto r^*], \theta\rangle \qquad [\text{if } l \in L] \qquad (\text{ASGN-APPROX})$$

$$\text{where } \left[\begin{array}{c} \psi(o) = ((c_\epsilon, r_\epsilon),(c_j, r_j)) \text{ and } r = [\![o(e_1,e_2)]\!](\rho) \text{ and } r^* = [\![o(e_1,e_2)]\!](\rho^*) \\ \text{and } r_1 = r_j(1-c_j) \text{ and } r_2 = \left\{\begin{array}{ll} r & \text{with probability } 1-c_\epsilon \\ r \pm r_\epsilon & \text{with probability } c_\epsilon \end{array}\right. \end{array}\right]$$

$$L \models_\psi \langle (\mathsf{rate}(v) < c)^k, \rho, \rho^*, \theta\rangle \overset{0}{\rightsquigarrow} \langle \rho, \rho^*, \theta[k \mapsto (n_1', n_2+1)]\rangle \qquad (\text{EXP-RATE})$$

$$\text{where } \left[\theta(k) = (n_1, n_2) \text{ and } n_1' = \left\{\begin{array}{ll} n_1 + 1 & \text{if } \rho(v) \neq \rho^*(v) \\ n_1 & \text{otherwise} \end{array}\right.\right]$$

$$L \models_\psi \langle (\mathsf{magnitude}(v) < c \text{ using } f)^k, \rho, \rho^*, \theta\rangle \overset{0}{\rightsquigarrow} \langle \rho, \rho^*, \theta[k \mapsto max(\theta(k), f(\rho(v), \rho^*(v)))]\rangle \qquad (\text{EXP-MAG})$$

$$L \models_\psi \langle (\mathsf{magnitude}(v) > c \text{ using } f \text{ with rate} < c')^k, \rho, \rho^*, \theta\rangle \overset{0}{\rightsquigarrow} \langle \rho, \rho^*, \theta[k \mapsto (n_1', n_2+1)]\rangle \qquad (\text{EXP-BOTH})$$

$$\text{where } \left[\theta(k) = (n_1, n_2) \text{ and } n_1' = \left\{\begin{array}{ll} n_1 + 1 & \text{if } f(\rho(v), \rho^*(v)) > c \\ n_1 & \text{otherwise} \end{array}\right.\right]$$

Figure 7: Instrumented program semantics. Rules for compound statements (sequential composition, branching, loops) are omitted for brevity.

ues of variables. We require shadow states to compute expectation valuations θ. For instance, to determine the valuation of expectation $\mathsf{rate}(v) < c$ at the end of an execution, we need to know the fraction of times that this expectation was executed in which an error was incurred on v, which in turn needs determining whether or not v had an exact value each time the expectation was reached in the execution.

To summarize, an instrumented program execution maintains the following extra information at any instant of execution:

- a shadow state ρ^*, a vector of real-valued data of length $|\mathbb{V}|$ that tracks the exact current value of each program variable;
- a real-valued data r tracking the cumulative energy cost of all assignments executed thus far; and
- the expectation valuation θ, a vector of integer pairs or real values of length $|\mathbb{K}|$ that tracks the current error incurred at each expectation in the program.

We next explain the semantics of assignments and expectations.

Semantics of Assignments. Rules (ASGN-EXACT) and (ASGN-APPROX) in Figure 7 show the execution of an exact and an approximate assignment, respectively. The assignment $v :=^l o(e_1, e_2)$ is approximate if and only if its label l is contained in set L. We use $[\![o(e_1, e_2)]\!](\rho)$ to denote the result of expression $o(e_1, e_2)$ in state ρ. We need to determine i) the energy cost of this assignment, and ii) the value of variable v after the assignment executes, in the actual state (determining its value in the shadow state is straightforward). To determine these two quantities, we use the system specification ψ to get the error model of operation o as (c_ϵ, r_ϵ) and its energy model as (c_j, r_j). Then, in the exact case, the assignment costs energy r_j, whereas in the approximate case, it costs lesser energy $r_j(1 - c_j)$. Moreover, in the approximate case, the assignment executes erroneously with probability c_ϵ and accurately with probability $1 - c_\epsilon$; in the erroneous case, the assignment yields a potentially erroneous value for variable v, namely $r \pm r_\epsilon$ instead of value r that would result if the assignment executed accurately.

Semantics of Expectations. Rules (EXP-RATE), (EXP-MAG), and (EXP-BOTH) in Figure 7 show the execution of the three kinds of expectations. The only thing that these rules modify is the error expectation value of $\theta(k)$, where k is the label of the expectation. We explain each of these three rules separately.

Rule (EXP-RATE) handles the execution of the expectation $\mathsf{rate}(v) < c$, updating incoming value $\theta(k) = (n_1, n_2)$ to either $(n_1, n_2{+}1)$ or $(n_1{+}1, n_2{+}1)$, depending upon whether the actual and shadow values of variable v are equal or not equal, respectively. In both cases, we increment n_2—the number of times this expectation has been executed thus far. But we increment n_1—the number of times this expectation has incurred an error thus far—only in the latter case. At the end of the entire program's execution, we can determine whether or not the error rate of this expectation—over all instances it was reached during that execution—was under the programmer-defined bound c; dividing n_1 by n_2 in the final value of $\theta(k)$ provides this error rate.

Rule (EXP-MAG) handles the execution of the expectation $\mathsf{magnitude}(v) < c$ using f, updating incoming value $\theta(k)$ to the greater of $\theta(k)$ and the new magnitude of error incurred by this expectation, as determined by programmer-defined function f. The reason it suffices to keep only the maximum value is, at the end of the entire program's execution, we only require knowing whether or not the maximum error magnitude of this expectation—over all instances it was reached during that execution—was under the programmer-specified bound c.

Finally, Rule (EXP-BOTH) handles the execution of the expectation $\mathsf{magnitude}(v) > c$ using f with rate $< c'$, updating incoming value $\theta(k) = (n_1, n_2)$ as in Rule (EXP-RATE), except that the condition under which n_1 is incremented is not that the most recent execution of this expectation incurred an error, but instead that it incurred an error whose magnitude exceeded the programmer-specified bound c (according to programmer-defined function f).

3. Approximation Safety Analysis

In this section, we present an approximation safety analysis that determines a subset of assignments $\mathcal{L} \subseteq \mathbb{L}$ that is safe to approximate. In practice, there are two kinds of assignments that are unsafe to approximate. The first kind is those that might violate memory safety, e.g., cause null pointer dereferences or index arrays out of bounds. The second kind are those that might violate functional correctness (i.e., application-specific) properties of the program.

To allow us to reason about approximation safety, we extend our simple language with two kinds of assertions, $\mathsf{assert}(v)$ and $\mathsf{assert\_all}$. The former checks that v has the exact value, while the latter checks that all *live* variables that are not *endorsed* by some expectation have exact value. We explain the meaning of these assertions by an example before providing their formal semantics.

```
(float, float) m(p : { float x, float y }) {
    assert(p);
    v1 := p.x * 0.5;    // l1
    v2 := p.y * 0.7;    // l2
    v3 := o(v1, v2);    // l3
    v4 := o(v2, v2);    // l4
    rate(v3) < 0.1;
    assert_all;
    return(v3, v4);
}
```

Consider function m above that takes a pointer p to a pair of real numbers and returns another pair of real numbers. Our framework generates an assertion assert(p) at every deference of variable p. For brevity, we only show it once, at the dominating dereference. This assertion says that it is unsafe to approximate any assignment whose result propagates to p at the entry of m; otherwise, memory safety could be violated at the dereference of p in the body of m.

Our framework also generates assertion assert_all at the end of m. This assertion says that it is unsafe to approximate any assignment whose result propagates to variable v4 at the end of m. The reason is that v4 is live at the end of m and the programmer has not specified any expectation on the error in the value of v4. Our framework therefore assumes that the programmer expects the value of v4 to be exact (e.g., in order to satisfy an implicit functional correctness property). This in turn prevents our framework from approximating assignments l2 and l4, since they both contribute to the value of v4 at the end of m (note that l2 assigns to v2, which is used at l4 to compute v4).

On the other hand, it is safe to approximate any remaining assignment whose result propagates to variable v3 at the end of m. The reason is that the programmer expects errors in the value of v3, via the expectation rate(v3) < 0.1 just before the end of m. This in turn leads our framework to conclude that it is safe to approximate assignments l3 and l1 (note that l2 also contributes to the value of v3 but it was deemed unsafe to approximate above because it also contributes to the value of v4). Thus, for this example, only assignments in { l1, l3 } are safe to approximate.

A semantics formalizing the meaning of our assertions is shown in Figure 8. Each rule is of the form:

$$L \models \langle s, \rho_1, T_1, E_1 \rangle \rightsquigarrow \langle \rho_2, T_2, E_2 \rangle \mid \textbf{error}$$

It describes whether an execution of the program s starting with input state ρ_1 will violate any assertions in s if the set of assignments L is approximated. If any assertion is violated, the final state is designated **error**; otherwise, it is the normal output state ρ_2.

To determine whether any assertion will be violated, the semantics tracks two sets of variables T and E, which we call the *tainted* and *endorsed* sets, respectively. Intuitively, a variable gets tainted if its value is affected by some assignment in the set L, and a variable gets endorsed if an expectation on that variable is executed.

We explain the most interesting rules in Figure 8. Rule (ASGN) states that the target variable v of an assignment $v :=^l o(e_1, e_2)$ gets tainted if l is an approximated assignment (i.e., $l \in L$) or if a variable used in e_1 or e_2 is already tainted. Rule (VAR-FAIL) states that assert(v) fails if v is tainted (i.e., $v \in T$). Rule (ALL-FAIL) states that assert_all fails if any tainted variable is not endorsed (i.e., $T \not\subseteq E$). Strictly, we require only *live* tainted variables to be endorsed, but for simplicity the semantics assumes all variables are live, though our implementation does a liveness analysis.

Given the semantics of assertions, we can characterize the sets of assignments that are safe to approximate, as follows:

DEFINITION 3.1. ***(Approximation Safety Problem)*** *A set of assignments L is safe to approximate if for every ρ, there exists ρ', T, E such that $L \models \langle s, \rho, \emptyset, \emptyset \rangle \rightsquigarrow \langle \rho', T, E \rangle$.*

We now present a static analysis that conservatively estimates a set of assignments that are safe to approximate. The analysis is shown in Figure 9 using type rules. It states that a set of assignments L is safe to approximate in a program s if there exists tainted sets of variables T_1, T_2 and endorsed sets of variables E_1, E_2 such that the type judgment $L, \langle T_1, E_1 \rangle \vdash s \triangleright \langle T_2, E_2 \rangle$ holds. The rules are self-explanatory and mirror those of the semantics of assertions in Figure 8. The most interesting one is rule (IF) which states that outgoing tainted sets T_1 and T_2 of the true and false branches of a conditional statement are unioned, whereas the outgoing endorsed sets E_1 and E_2 are intersected. The reason is that it is safe to grow tainted sets and shrink endorsed sets, as evident in the checks of rules (ASSERT-VAR) and (ASSERT-ALL).

$$L, \langle T, E \rangle \vdash v :=^l o(e_1, e_2) \triangleright \langle (T \setminus \{v\}) \cup T', E \setminus \{v\} \rangle \quad \text{(ASGN)}$$

$$\text{where } T' = \begin{cases} \{v\} & \text{if } l \in L \text{ or } uses(e_1, e_2) \cap T \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

$$L, \langle T, E \rangle \vdash \mathsf{rate}(v) < c \triangleright \langle T, E \cup \{v\} \rangle \quad \text{(EXP-RATE)}$$

$$L, \langle T, E \rangle \vdash \mathsf{assert}(v) \triangleright \langle T, E \rangle \quad [\text{if } v \notin T] \quad \text{(ASSERT-VAR)}$$

$$L, \langle T, E \rangle \vdash \mathsf{assert\_all} \triangleright \langle T, E \rangle \quad [\text{if } T \subseteq E] \quad \text{(ASSERT-ALL)}$$

$$\frac{L, \langle T_1, E_1 \rangle \vdash s_1 \triangleright \langle T_2, E_2 \rangle \qquad L, \langle T_2, E_2 \rangle \vdash s_2 \triangleright \langle T_3, E_3 \rangle}{L, \langle T_1, E_1 \rangle \vdash s_1 \; ; \; s_2 \triangleright \langle T_3, E_3 \rangle} \quad \text{(SEQ)}$$

$$\frac{L, \langle T, E \rangle \vdash s_1 \triangleright \langle T_1, E_1 \rangle \qquad L, \langle T, E \rangle \vdash s_2 \triangleright \langle T_2, E_2 \rangle}{L, \langle T, E \rangle \vdash \mathsf{if}\ (v > 0)\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2 \triangleright \langle T_1 \cup T_2, E_1 \cap E_2 \rangle} \quad \text{(IF)}$$

$$\frac{L, \langle T, E \rangle \vdash s \triangleright \langle T, E \rangle}{L, \langle T, E \rangle \vdash \mathsf{while}\ (v > 0)\ \mathsf{do}\ s \triangleright \langle T, E \rangle} \quad \text{(WHILE)}$$

Figure 9: Approximation safety analysis. Rules for the two kinds of expectations not shown for brevity are similar to (EXP-RATE).

The following theorem states that our static analysis is sound:

THEOREM 3.2. ***(Soundness)*** *If $L, \langle T_1, E_1 \rangle \vdash s \triangleright \langle T_1', E_1' \rangle$ and $T_2 \subseteq T_1$ and $E_1 \subseteq E_2$ then $\exists \rho'$ such that $L \models \langle s, T_2, E_2 \rangle \rightsquigarrow \langle \rho', T_2', E_2' \rangle$ and $T_2' \subseteq T_1'$ and $E_1' \subseteq E_2'$.*

We seek to find as large a set of assignments as possible that is safe to approximate, so as to maximize the opportunities for energy savings. One can use techniques for finding optimal abstractions (e.g., [22]) to efficiently identify the largest set of safe approximable assignments. Hereafter, we denote the set of safe approximable assignments as $\mathcal{L}$. They serve as an input to our optimization framework, which we present next.

4. Optimization Framework

We first formulate the optimization problem that we seek to solve (Section 4.1) and then describe a concrete genetic-based algorithm that we have implemented to solve it (Section 4.2).

4.1 Optimization Problem Formulation

Figure 10 presents the formulation of our optimization problem to automatically select a subset of the safe-to-approximate assignments. The objective is to approximate as much of the program to reduce energy while minimizing error, considering the specified error expectations. To this end, the optimization procedure aims to minimize the weighted normalized sum of three components:

1. Number of static assignments in the program that are not approximate ($1 - |\mathbf{L}|/|\mathcal{L}|$).
2. Excess error (rates and magnitudes) incurred due to approximations, over and above the bounds specified in expectations by programmers ($error(\rho)$).
3. Energy cost incurred due to lack of approximation ($energy(\rho)$).

In our optimization framework, we assume that error and energy are computed with respect to given program input datasets ($\rho_1, ..., \rho_n$).

$$L \models \langle v :=^{l} o(e_1,e_2), \rho, T, E\rangle \rightsquigarrow \langle \rho[v \mapsto [\![o(e_1,e_2)]\!](\rho)], (T\setminus\{v\})\cup T', E\setminus\{v\}\rangle \left[T' = \begin{cases}\{v\} & \text{if } l \in L \text{ or } uses(e_1,e_2)\cap T \neq \emptyset \\ \emptyset & \text{otherwise}\end{cases}\right] \quad (\textsc{Asgn})$$

$$L \models \langle \mathsf{rate}(v) < c, \rho, T, E\rangle \rightsquigarrow \langle T, E \cup \{v\}\rangle \quad (\textsc{Exp-Rate})$$

$$L \models \langle \mathsf{assert}(v), \rho, T, E\rangle \rightsquigarrow \langle \rho, T, E\rangle \quad [\text{if } v \notin T] \quad (\textsc{Var-Pass}) \qquad L \models \langle \mathsf{assert\_all}, \rho, T, E\rangle \rightsquigarrow \langle \rho, T, E\rangle \quad [\text{if } T \subseteq E] \quad (\textsc{All-Pass})$$

$$L \models \langle \mathsf{assert}(v), \rho, T, E\rangle \rightsquigarrow \mathbf{error} \quad [\text{if } v \in T] \quad (\textsc{Var-Fail}) \qquad L \models \langle \mathsf{assert\_all}, \rho, T, E\rangle \rightsquigarrow \mathbf{error} \quad [\text{if } T \not\subseteq E] \quad (\textsc{All-Fall})$$

$$\frac{L \models \langle s_1, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_2, T_2, E_2\rangle \qquad L \models \langle s_2, \rho_2, T_2, E_2\rangle \rightsquigarrow \langle \rho_3, T_3, E_3\rangle}{L \models \langle s_1 \; ; \; s_2, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_3, T_3, E_3\rangle} \quad (\textsc{Seq})$$

$$L \models \langle \mathsf{if}\ (v > 0)\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2, \rho, T, E\rangle \rightsquigarrow \mathbf{error} \quad [\text{if } v \in T] \quad (\textsc{If-Fail})$$

$$\frac{L \models \langle s_1, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_2, T_2, E_2\rangle}{L \models \langle \mathsf{if}\ (v > 0)\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_2, T_2, E_2\rangle} \quad [\text{if } v \notin T_1 \ \wedge\ \rho_1(v) > 0] \quad (\textsc{If-True})$$

$$\frac{L \models \langle s_2, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_2, T_2, E_2\rangle}{L \models \langle \mathsf{if}\ (v > 0)\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2, \rho_1, T_1, E_1\rangle \rightsquigarrow \langle \rho_2, T_2, E_2\rangle} \quad [\text{if } v \notin T_1 \ \wedge\ \rho_1(v) \leq 0] \quad (\textsc{If-False})$$

Figure 8: Semantics of approximation safety. Rules for other kinds of expectations, loops, and rules propagating **error** are elided for brevity.

Inputs:

(1) Program $\Pi = \langle s, \mathcal{L}, \mathbb{K}\rangle$
(2) Program input datasets $\Delta = \{\rho_1, ..., \rho_n\}$
(3) System specification ψ

Output:

Best set of approximable assignments $\mathbf{L} \subseteq \mathcal{L}$.

Minimize:

$\sum_{\rho\in\Delta} \Big(\alpha(1 - \frac{|\mathbf{L}|}{|\mathcal{L}|}) + \beta\, error(\rho) + \gamma\, energy(\rho)\Big)$

where $\alpha, \beta, \gamma \in \mathbb{R}_{0,1}$ and $\alpha + \beta + \gamma = 1$ and:

(1) $error(\rho) = \dfrac{\sum_{k\in\mathbb{K}} error(\theta(k), bound(k))}{|\mathbb{K}|}$ where:

- θ *s.t.* $\exists \rho_1, \rho_2, r : \ \mathbf{L} \models_{\psi} \langle s, \rho, \rho, \lambda k.0\rangle \overset{r}{\rightsquigarrow} \langle \rho_1, \rho_2, \theta\rangle$
- $bound(k) = c$ where expectation labeled k is of form:
  - $\mathsf{rate}(v) < c$ *or*
  - $\mathsf{magnitude}(v) < c$ using f *or*
  - $\mathsf{magnitude}(v) > c'$ using f $\mathsf{with\ rate} < c$
- $error(c_1, c_2) = \begin{cases} c_1 - c_2 & \text{if } c_1 > c_2 \\ 0 & \text{otherwise}\end{cases}$

(2) $energy(\rho) = \dfrac{r}{r^*}$ where:

- r *s.t.* $\exists \rho_1, \rho_2, \theta : \ \mathbf{L} \models_{\psi} \langle s, \rho, \rho, \lambda k.0\rangle \overset{r}{\rightsquigarrow} \langle \rho_1, \rho_2, \theta\rangle$
- r^* *s.t.* $\exists \rho_1, \rho_2, \theta : \ \emptyset \models_{\psi} \langle s, \rho, \rho, \lambda k.0\rangle \overset{r^*}{\rightsquigarrow} \langle \rho_1, \rho_2, \theta\rangle$

Figure 10: Optimization problem formulation.

Minimizing $energy(\rho)$ and the number of non-approximate static assignments reduces energy but can lead to significant quality-of-result degradation. The $error(\rho)$ term in the minimization objective functions strikes a balance between saving energy and preserving the quality-of-results, while considering the programmer expectations. The $energy(\rho)$ term in the formulation is the energy dissipation when a subset of the approximable assignments are approximated, normalized to the energy dissipation of the program when executed on the same input dataset with no approximation.

Incorporating expectation in the optimization. The $error(\rho)$ term is the normalized sum of error (rate or magnitude) at the site of all expectations when the program runs on the ρ dataset. We incorporate the expectations in the optimization through $error(\rho)$. We assume error on an expectation site is 0, if the observed error is less than the expectation bound. Otherwise, we assume the error is the difference of the observed error and the expectation bound. Intuitively, when the observed error is less than the specified bound, the output is acceptable, same as the case where error is zero. The optimization objective is to push the error below the specified bound since the programmer has specified that error below the expectation bound is acceptable.

4.2 Genetic Optimization Framework

We develop a genetic algorithm that explores the space of possible solutions for the formulated optimization problem and finds a best-effort answer. However, there may be other algorithms for solving the formulated optimization problem.

Genetic algorithms. Genetic algorithms are *guided* random algorithms that iteratively explore the solution space of a problem and find a best-fitting solution that maximizes an objective. The first step in developing a genetic algorithm is defining an encoding for the possible solutions, namely phenotypes. Then, the algorithm randomly populates the first generation of the candidate solutions. The algorithm assigns scores to each individual solution based on a fitness function. The fitness function is derived from the optimization objective. Defining this fitness function is another important component of the genetic algorithm. Each genetic algorithm also includes two operators, crossover and mutation. These operators pseudo randomly generate the next generation of the solutions based on the scores of the current generation of solutions. Below we describe all the components of our genetic algorithm, which is presented in Algorithm 1.

Phenotype encoding. In our framework, a solution is a program in which a subset of the assignments is marked approximate. The approximation safety analysis provides a candidate subset of assignments that are safe to approximate. We assign a vector to this subset whose elements indicate whether the corresponding assignment is approximate (`'0'`) or precise (`'1'`). For simplicity, in this paper, we only assume a binary state for each assignment. However, a generalization of our approach can assign more than two levels to each element, allowing multiple levels of approximation for each assignment. The bit-vector is the template for encoding phenotypes and its bit pattern is used to generate an approximate version of the program. We associate the bit-vector with the subset resulting from the approximation safety analysis and not the entire set of assignments, to avoid generating unsafe solutions.

Fitness function. The fitness function assigns scores to each phenotype. The higher the score, the higher the chance that the phenotype is selected for producing the next generation. The fitness

Algorithm 1 Genetic algorithm to generate approximate program.

```
1:  INPUT: Program $\Pi = \langle s, \mathcal{L}, \mathbb{K} \rangle$:
        - $s$ is program body
        - $\mathcal{L}$ is set of safe approximable assignments in $s$
        - $\mathbb{K}$ is set of all expectations in $s$
2:  INPUT: Program input datasets $\Delta = \{\rho_1, ..., \rho_n\}$
3:  INPUT: System specification $\psi$
4:  INPUT: Genetic algorithm parameters $\langle N, M, P \rangle$:
        - $N$ is population size
        - $M$ is number of generations
        - $P$ is probability of mutation
5:  OUTPUT: Bit-vector with the highest global fitness elite_phenotype
6:  $\alpha$ := 0.2, $\beta$ := 0.4, $\gamma$ := 0.4
7:  for i in 1..$N$ do
8:      phenotype[i] := random bit vector of size $|\mathcal{L}|$
9:  end for
10: for k in 1..$M$ do
11:     var f, g of type $\mathbb{R}_{0,1}[N]$
12:     for [parallel] i in 1..$N$ do
13:         error[i], energy[i] := execute($\Pi$, phenotype[i], $\Delta$, $\psi$)
14:         f[i] := $\left(\alpha \frac{\sum \text{phenotype[i]}}{|\mathcal{L}|} + \beta \text{ error[i]} + \gamma \text{ energy[i]}\right)^{-1}$
15:     end for
16:     for i in 1..$N$ do
17:         g[i] := f[i]/ $\sum$ f[i]
18:         if f[i] $>$ f_elite_phenotype then
19:             f_elite_phenotype := f[i]
20:             elite_phenotype := phenotype[i]
21:         end if
22:     end for
23:     for i in 1..$N$ do
24:         x := roulette_wheel(g)
25:         y := roulette_wheel(g)
26:         phenotype[x], phenotype[y] :=
27:             crossover(phenotype[x], phenotype[y])
28:         phenotype[x] := mutate(phenotype[x], $P$)
29:         phenotype[y] := mutate(phenotype[y], $P$)
30:     end for
31: end for
```

function encapsulates the optimization objective and guides the genetic algorithm in producing better solutions. Our fitness function is defined as follows based on the objective function in Figure 10:

$$f(phenotype) = \left(\alpha \frac{\sum_j phenotype_j}{|\mathcal{L}|} + \beta\, error + \gamma\, energy\right)^{-1}$$

We use the inverse of minimization object in Figure 10 to calculate fitness scores. That is, a phenotype with more approximate instructions ($\frac{\sum_j phenotype_j}{|\mathcal{L}|}$), less $error$, and less $energy$ has a higher score and higher chance of reproducing better solutions. Since we use `'0'` to mark an assignment as approximate, the lower the $\sum_j phenotype_j$, the more instructions are approximated, which is in correspondence with the formulation in Figure 10. As discussed, the $error$ incorporates the programmer-specified expectations. Since not all the static assignments contribute equally to the dynamic error and energy dissipation of the program, we set $\alpha = 0.2$, $\beta = 0.4$, and $\gamma = 0.4$. This setting equally weighs the dynamic information captured in $error$ and $energy$ while giving less importance to the number of static assignments that are approximated. We generate approximate versions of the program based on the phenotypes and run them with the input datasets. We make each version of the program an independent thread and run them in parallel. Executing different input datasets can also be parallel.

Roulette wheel gene selection. We use the roulette wheel selection strategy for generating the next generation. The probability of selecting the i^{th} phenotype is g[i] $= \frac{f_i}{\sum f_i}$. That is, phenotype with higher fitness have a better chance of producing offsprings. Nonetheless, the low score phenotypes will still have the chance to reproduce sine they may contain part of a better solution that will manifest several generation later.

Crossover and mutation. The crossover operator takes two selected phenotypes and recombines them to generate a new solution. All the phenotypes are of the same size. Therefore, crossover randomly picks a location on the bit-vector, cuts the two bit-vectors, and swaps the cut parts. Since crossover does not improve the diversity of the solutions, with a small probability, we mutate the phenotypes after each crossover. The mutation operator randomly flips a bit.

Elitism. We record the best global solution across all the generations. This approach is referred to as elitism.

Table 1: Benchmarks, quality metric (Mag=Magnitude), and result of approximation safety analysis: # approximable operations.

	Description	Quality Metric	# Lines	# Approximable Operations
fft		Mag: Avg entry diff	168	123
sor	SciMark2	Mag: Avg entry diff	36	23
mc	benchmark:	Mag: Normalized diff	59	11
smm	scientific kernels	Mag: Avg normalized diff	38	7
lu		Mag: Avg entry diff	283	46
zxing	Bar code decoder for mobile phones	Rate of incorrect reads	26171	756
jmeint	jMonkeyEngine game: triangle intersection kernel	Rate of incorrect decisions	5962	1046
imagefill	ImageJ image processing: flood-filling kernel	Mag: Avg pixel diff	156	130
raytracer	3D image renderer	Mag: Avg pixel diff	174	199

5. Evaluation

5.1 Methodology

Benchmarks. We examine nine benchmarks written in Java, which are the same programs used in [18]. Five of them come from the SciMark2 suite. ZXing is a multi-format bar code recognizer developed for Android phones. jMonkeyEngine is a game development engine; the benchmark is the engine's 3D triangle-intersection algorithm used for collision detection. ImageJ is a library for image manipulation; the application from this library is the flood-filler algorithm. Finally, the suite includes a simple 3D raytracer. Table 1 summarizes the benchmarks and shows their quality metric.

Application-specific quality metrics. For jmeint and zxing, the quality metric is a rate. For jmeint, the quality metric is the rate of incorrect intersection decisions. Similarly, for the zxing bar code recognizer, the quality metric is the rate of unsuccessful decodings of a sample QR code image. For the rest of the benchmarks, the quality metric is defined on the magnitude of error, which is calculated based on an *application-specific* quality-of-result metric. For most of the applications, the metric is the root-mean-squared error of the output vector, matrix, or pixel array.

Genetic optimization framework. We implement the genetic framework described earlier to automatically find the approximate version of the program considering the specified expectations. We

Table 2: Summary of the (*error rate*, *energy saving*)'s with our four system specifications derived from [18].

Operation	Mild	Medium	High	Aggressive
Instruction	$(10^{-6}, 12\%)$	$(10^{-4}, 22\%)$	$(10^{-3}, 26\%)$	$(10^{-2}, 30\%)$
On-chip read	$(10^{-16.7}, 70\%)$	$(10^{-7.4}, 80\%)$	$(10^{-5.2}, 85\%)$	$(10^{-3}, 90\%)$
On-chip write	$(10^{-5.6}, 70\%)$	$(10^{-4.9}, 80\%)$	$(10^{-4}, 85\%)$	$(10^{-3}, 90\%)$
Off-chip	$(10^{-9}, 17\%)$	$(10^{-5}, 22\%)$	$(10^{-4}, 23\%)$	$(10^{-3}, 24\%)$

use a fixed-size population of 30 phenotypes across all the generations. We run the genetic algorithm for 10 generations. We use a low probability of 0.02 for mutation. Using low probability for mutation is a common practice in using genetic algorithms and prevents the genetic optimization from random oscillations. We did not explore genetic optimization beyond 10 generations and with populations larger than 30. Running the genetic algorithm for more than 10 generations and/or with larger populations can only lead to potentially better solutions. However, the reported experiments provide the grounds to show the correctness and effectiveness of our genetic optimization framework. We also made our genetic algorithm parallel. The programs generated based on phenotypes of a generation run as independent threads in parallel.

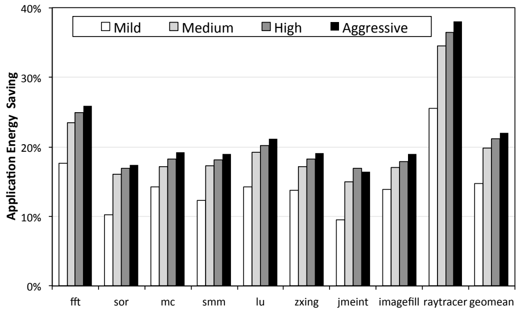
Simulator. We modified the open-source simulator provided in [18] and built our genetic optimization framework on top of the modified simulator. The simulator allows the instrumentation of the method calls, object creation and destruction, arithmetic operators, and memory accesses to collect statistics and inject errors based on the system specifications. The runtime system of the simulator, which is a Java library and is invoked by the instrumentation, records memory-footprint and arithmetic-operation statistics while simultaneously injecting error to emulate approximate execution. The modified simulator at the end records the error rate or error magnitude at the expectation site. The simulator also calculates the energy savings associated with the approximate version of the program, which is generated based on phenotypes of the genetic algorithm. We run each application five times to compensate for the randomness of the error injection and average the results. The results from the simulation are fed back to the genetic algorithm to calculate the fitness and score of each phenotype.

System specifications. We derive four system specifications from the energy models provided in [18] as shown in Table 2. We consider a simplified processor/memory system energy dissipation model that comprises three components: instruction execution, on-chip storage (SRAM), and off-chip storage (DRAM). While these simplified models provide good-enough energy estimates, our framework can easily replace these models with more accurate system specifications. In fact, one of the advantages of our optimization framework is its modularity and system-independence.

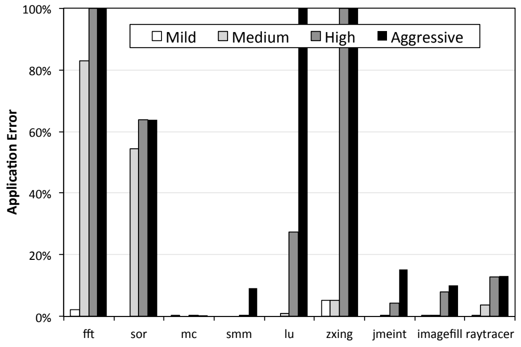
Architecture model. We assume an architecture similar to Truffle [8] that can execute interleaving of precise and approximate instructions and does not incur overhead when switching between approximate and precise instructions. In our architecture model, register files, functional units, caches, and memory can switch between approximate and precise state. However, the instruction fetch, decode, and commit are always precise. Our system specifications consider the overhead of keeping these part of the system precise.

5.2 Experimental Results

Approximating all safe-to-approximate operations. Figure 11 shows the energy and error when approximating all the safe-to-approximate operations. As depicted, the geometric mean energy savings ranges from 15% with the Mild system specification to 22% with the Aggressive system specification. The sor shows the least energy savings (10% with Mild) while raytracer shows the highest



(a) Application energy savings.



(b) Application error.

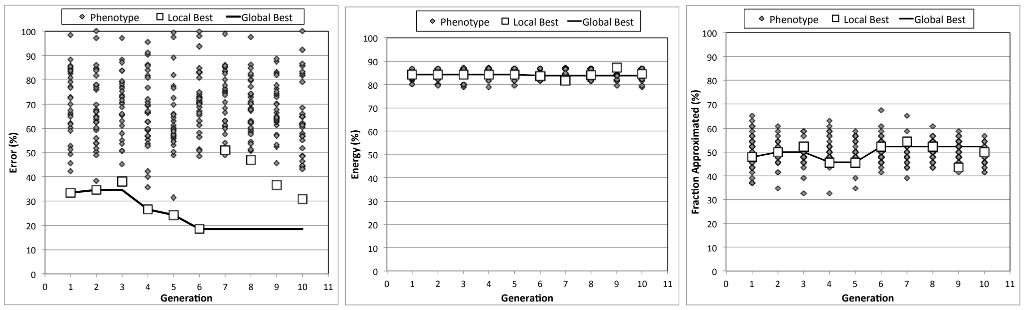
Figure 11: Energy savings and error when approximating all safe-to-approximate operations.

energy savings (38% with Aggressive). All the applications show acceptable error levels with the Mild system. However, there is a jump in error when the system specification changes from Mild to Medium. These results show the upper bound on energy savings.

Genetic optimization. To understand the effectiveness of the genetic optimization, we take a close look at lu when it undergoes the genetic optimization under the Aggressive system specification with the output expectation set to 0.00. We will also present the energy savings and error levels for all the applications. Figure 12a depicts the distribution of error for lu in each generation of the genetic optimization. As shown, the application shows a wide spectrum of error, 18%–100%. *The result shows that there is a subset of safe-to-approximate operations that if approximated can lead to significant quality-of-result degradation.*

In contrast to error, as Figure 12b depicts, lu's energy profile has a narrow spectrum. That is, carefully selecting which operation to approximate can lead to significant energy savings, while delivering significantly improved quality-of-results. Compared to approximating all safe-to-approximate operations, our automated genetic framework improves the error level from 100% to 18%, while only reducing the energy savings from 21% to 16%. That is, $5.5\times$ improvement in quality-of-results with only 31% reduction in energy benefits. Finally, Figure 12c shows the fraction of candidate static operations that are approximated across generations. The results show that if only a little more than half of the candidate operations are approximated in lu, significant energy saving is achievable with significantly improved quality-of-results. These results show that our optimization framework strikes a balance between energy savings and error. Theses results also suggest that formulating the selection of approximate operations as an optimization can enable effective approximation even on highly unreliable hardware.

Effect of error bounds on the genetic optimization. Figure 13 shows (a) error, (b) energy savings, and (c) fraction of approximated safe-to-approximate static operations under the Medium sys-



(a) Error. (b) Energy. (c) Fraction approximated.

Figure 12: The lu benchmark under the genetic optimization with the Aggressive system specification and the output expectation set to `0.00`.

tem specification. The output expectation bounds are set to `0.00`, `0.03`, `0.05`, `0.10`, `0.50`, and `0.75`. The last bar in Figures 13(a-c) represents the case when all the safe-to-approximate operations are approximated. In general, the larger the expectation bounds, the more relaxed the error requirements. Almost consistently, as the error requirements become more strict, the genetic algorithm finds lower-error versions of the approximate programs, while maintaining significant energy benefits (Figure 13b). The genetic optimization also filters out the subset of the static operations that lead to significant quality-of-result degradation (Figure 13c). Since the genetic algorithm is a guided random algorithm, there are some expected minor fluctuations in the results.

As Figure 13a shows, the correlation of error reduction with tighter error bounds is evident in the case of fft. The automated genetic optimization has been able to find approximate versions of fft that exhibit significantly better quality-of-results while delivering significant energy savings. This case shows that the bounds guide the genetic optimization to find and filter out safe-to-approximate operations that lead to significant quality degradation, while maintaining significant energy savings (Figure 13b). For sor, the genetic algorithm has been able to satisfy all the expectations and reduce the error to negligible levels when the bounds are strict. For mc, smm, lu, and zxing, imagefill, raytracer, the genetic algorithm has been able to find a near-to-optimal solution. However, in case of jmeint the genetic algorithm has failed to find the optimal solution, which is approximating all the operations. We can always consider the fully approximated solution as a candidate solution and bypass genetic optimization if it satisfies the expectations. As Figure 13c shows, for fft, sor, lu, zxing, and raytracer the genetic optimization filters out the subset of safe-to-approximate operations that when approximated lead to quality degradations. We performed similar experiments with the Aggressive system specification and found similar trends. However, due to space limitations we did not include those results. *These results confirm that automating the process of selecting approximate operations is of significant value and confirms the effectiveness of our optimization formulation and our concrete genetic algorithm solution.*

5.3 Limitations

Large programs. For larger programs, the framework can divide the application into smaller kernels and apply the optimization step-by-step. However, the order in which the kernels go under the optimization can have an effect on the optimality of the results.

Online versus offline optimization. In this paper, we only focused on offline optimization. However, it is possible to further optimize the program after deployment as part of a just-in-time (JIT) compiler. Nonetheless, there is tradeoff between allocated compute and energy resources to the optimization and the benefits from extended online optimization. One option is to offload the online optimization to the cloud where the compute resources are abundant and the parallelism in our optimization can be effectively exploited.

Symbolic approximate optimization. Our current system is a data-driven optimization framework which relies on profiling information. However, future research can incorporate probabilistic symbolic execution into our framework.

Quality-of-result guarantees. Our framework is a best-effort optimization that does not provide quality-of-result guarantees. Future research may investigate using a programming language such as Rely [4] as our back-end, which provides static guarantees with explicit approximate annotations.

6. Related Work

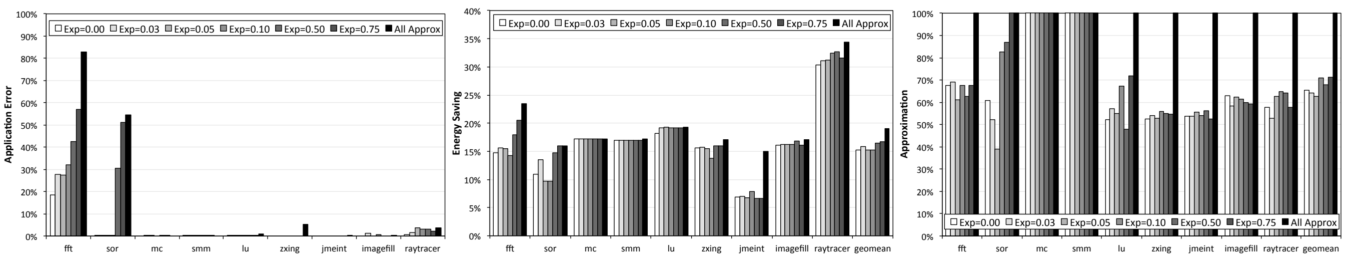
There is a large body of work on languages, reasoning, analyses, transformations, and synthesis for approximate programs. We discuss each of these in more detail below.

EnerJ [18] and Rely [4] are imperative programming languages for approximate computing. EnerJ allows programmers to specify, via type qualifiers, which data in their programs can be approximate and which data must be exact. Their type system statically guarantees isolation of the exact and approximate parts of the program, and guides the compiler to generate code that achieves high energy savings at little accuracy cost. Rely allows programmers to specify quantitative reliability requirements in explicitly annotated approximate programs, and provides a static analysis that reasons symbolically about reliability in order to prove that programs meet those requirements on a given unreliable hardware specification.

The programming model of ExpAX differs from that in EnerJ and Rely in crucial ways. Unlike in EnerJ, expectations in ExpAX specify error bounds, not just isolation of exact data from approximate data. Moreover, expectations implicitly dictate such isolation, as our approximation safety analysis demonstrates. Unlike in Rely, expectations specify programmer preferences, not requirements. Besides, expectations implicitly determine which program operations are approximable, where in Rely they are annotated explicitly. Finally, ExpAX provides an optimization framework lacking in those languages, to automatically determine the subset of the safe-to-approximate operations to be approximated in a manner that optimizes energy savings and error expectations under a given system specification.

Carbin et al. [3] propose a relational Hoare-like logic for reasoning about the correctness of approximate programs. They provide stronger guarantees than we do but also require more programmer involvement than in our work.

Transformations have been proposed to approximate programs, including substitution-based and sampling-based ones (e.g., loop perforation [16, 20]), more recently with probabilistic correctness guarantees [19] and optimization procedures [23] that automatically pick different versions of a function's transformations. These



(a) Error. (b) Energy. (c) Fraction of static instructions approximated.

Figure 13: Results of the genetic optimization under the `Medium` system specification with expectation bounds set to `0.00`, `0.03`, `0.05`, `0.10`, `0.50`, and `0.75` along with the results when all the safe-to-approximate operations are approximated.

techniques operate at the granularity of functions. We provide an analysis and an optimization that operates on the fine granularity of single operations. The randomized algorithm in [23] does optimization at the granularity of functions which is not scalable to the fine-grained approximation model that we investigate. Furthermore, we provide a novel programming model that is implicit and requires less involvement from the programmer.

Green [2] is a framework that monitors the quality-of-service (QoS) requirement as programs run and automatically adjusts the level of approximation, which in their case is either early termination of loops or choosing an approximate version of hot functions. Green provides a code-centric programming model for annotating approximable loops and functions that is more suited for coarse grain software-based approximation. In contrast, we provide an implicit programming model, a static analysis, and a profile-driven optimization that automatically finds approximate operations that are in the granularity of single instructions and execute on unreliable hardware.

7. Conclusions

We described ExpAX, an expectation-oriented framework for automating approximate programming, and its three components: programming, analysis, and optimization. We developed a programming model and a new program specification, referred to as expectations. Our programming model enables programmers to implicitly relax the accuracy constraints on low-level program data and operations without explicitly marking them as approximate. Further, the expectations allow programmers to quantitatively express their error preferences. Then, we developed a approximation safety analysis that using the high-level expectation specifications automatically finds the candidate safe-to-approximate subset of the operations. Then, we described a system-independent optimization formulation for selectively marking a number of the candidate operations as approximate. Further, we provided a concrete instantiation of the optimization framework using a genetic algorithm and evaluated its effectiveness using a diverse set of applications. The results show that in many cases, there is a subset of the safe-to-approximate operations that if approximated can lead to significant quality-of-result degradations. We show that our genetic optimization framework effectively excludes these operations from approximation. The results confirm that automating this process is of significant value and can in many cases enable effective approximation even on highly unreliable hardware. Our approach is a best-effort solution to automate approximate programming, improve their portability, automatically balance quality-of-result degradation and efficiency, and guide approximation based on programmers' expectations.

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