ANALYSING TIME SERIES DATA

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ABSTRACT

This paper investigates the use of auditory perceptualisation for analysing the statistical properties of time series data. We introduce the problem domain and provide basic background on higher order statistics like skewness and kurtosis. The chosen approach was direct audification because of the inherent time line and the high number of data points usually available for time series. We present the tools we developed to investigate this problem domain and elaborate on a listening test we conducted to find perceptual dimensions that would correlate with the statistical properties. The results indicate that there is evidence that kurtosis correlates with roughness or sharpness and that participants were able to distinguish signals with increasing difference of the kurtosis. For the setting in the experiment the just noticeable difference was found to be 5. The collected data did not show any similar evidence for skewness and it remains unclear whether this is perceivable in direct audification at all.

[Keywords: Audification, Time series data, Higher-order statistics]

1. INTRODUCTION

The analysis of time series data is key to many scientific disciplines. Time series may be the result of measurements, unknown processes or simply digitised signals. Although usually visualised and analysed through statistics, the inherent relationship to time makes them particularly suitable for a representation through sound.

The following work was conducted as part of the interdisciplinary research project SonEnvir\(^1\) [1]. SonEnvir investigates the use of sonification in a number of different scientific fields, highlighting research problems that can potentially benefit from auditory perceptualisation as an alternative form of analysis to visualisation or statistics. By drawing upon many different sonifications and research problems, SonEnvir aims at the development of a generalised sonification environment for scientific data. Target sciences involved include neurology, theoretical physics, sociology and signal processing and speech communication. In all of the target sciences promising problem statements were selected for the development of sonification prototypes. Through the unique diversity of backgrounds and questions, the approach demanded highest flexibility and an open-minded attitude of all researchers involved. As expected, the scepticism in the communities of the target sciences was substantial and for a lot of problems we only were able to scratch the surface without answering real-world questions.

Through our work, however, we highlighted the benefits of auditory perceptualisation and raised awareness for sonification as an alternative method to analyse and explore data in the target sciences and various other scientific fields.

The Signal Processing and Speech Communication Laboratory focuses on research in the area of non-linear signal processing methods, algorithm engineering and applications thereof in speech communication and telecommunication. After investigating sonification approaches to the analysis of stochastic processes and wave propagation in ultra-wide-band communication [2], the focus for the last phase in SonEnvir was on the analysis of time series data. In signal processing and speech communication, most of the data to handle are sequences of values over time - time series. There are many properties of time series data that interest the researcher. Besides the analysis in the frequency domain, the statistical distribution of values provides important information about the data at hand. With this work we aim at investigating the use of sonification in analysing the statistical properties of amplitude distributions in time series data. From the target science’s point of view this can be used as a method for the classification of signals of unknown origin or for the classification of surrogate data to be used in experiments in telecommunication systems.

The following will provide some background to the mathematics of statistical analysis of time series data. In section 3 we will present the sonification tools that we developed before section 4 will discuss a psychoacoustic experiment conducted. Finally, we conclude the paper and elaborate on future work in section 5.

2. BACKGROUND

The statistical analysis of time series data is concerned about the distribution of values without taking into account their sequence in time. As we will see later, changing the sequence of values in a time series, completely destroys the frequency information while keeping the statistical properties intact. The most well known statistical properties of time series data is the arithmetic mean (1) and the variance (2).

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]  

(1)

\[
\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2
\]  

(2)

However, higher order statistics provides more properties of time series data describing more detailed the shape of the underlying probability function. They all derive from the statistical moments

\(^1\)http://sonenvir.at
of a distribution defined by

$$\mu'_n = \sum_{i=1}^{n} (x_i - \alpha)^n P(x)$$

where $n$ is the order of the moment, $\alpha$ the value around which the moment is taken and $P(x)$ the probability function. The moments are most commonly taken around the mean, which is equivalent to the first moment $\mu_1$. The second moment around the mean (or second central moment) is equivalent with the variance $\sigma^2$ and hence, the squared standard deviation $\sigma$.

Higher order moments define the skewness and kurtosis of the distribution. The skewness is a measurement for the asymmetry of the probability function, meaning a distribution has high skewness if its probability function has a more pronounced tail to one end than to the other. The skew is defined by

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}$$

with $\mu_i$ being the $i$-th central moment. The kurtosis describes the "peakedness" of a probability function; the more pronounced peaks there are in the probability function, the higher the kurtosis of the distribution. It is defined by

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

Both values distinguish time series data and are significant properties in signal processing.

The inherent time line and the often high numbers of data values in time series data suggest the use of the most direct approach to auditory perceptualisation - audification. When interpreted as sonic waveform the statistical properties of time series data become acoustical dimensions of perception. The variance corresponds directly to the power of the signal, hence (non-linearly) to its perceived loudness. The mean, however, is nothing more than an offset and is not perceivable. The question is whether the skewness and the kurtosis of signals can be assigned to perceptional dimensions too.

### 3. SONIFICATION TOOLS

In order to investigate the statistical properties of time series data through audification we first developed a simple tool that allows for defining arbitrary probability functions for noise. Subsequently, we built a more generic analysing tool that makes it possible to analyse any kind of signal which was also used as the underlying framework for the experiment in section 4.

#### 3.1. PDFShaper

The PDFShaper is an interactive audification tool that allows users to draw probability functions and hear the resulting distribution as audification in real-time. Figure 1 shows the user interface. PDFShaper provides four graphs (top down): the probability function, the mapping function, the measured histogram and the frequency spectrum. The tool allows the user to interactively draw in the first graph to create any kind of distribution. It then calculates a mapping function which is defined by

$$C(x) = \frac{1}{g(x)} = \int_0^x P(t)dt$$

where $C(x)$ is the cumulative probability function and $g(x)$ is a mapping function that if applied to a uniform distribution $y$ produces values according to the probability function $P(t)$. This mapping function essentially shapes values from a uniform distribution to any desired probability function $P(t)$.

In the screenshot shown, the probability function is drawn into the top graph as shifted exponential function. After applying the mapping function shown in the second graph to white noise, the third graph shows the real-time histogram of the result. It approximately resembles the target probability function. Note that both, skew and kurtosis are relatively high in this example as the probability function is shifted to the right and has a sharp peak.

#### 3.2. TSAnalyser

The TSAnalyser is a tool to load any time series data and analyse its statistical properties. Figure 2 shows the user interface. Besides providing statistical information about the file loaded (aiff format) it shows a histogram and a spectrum. Its main feature is to be able to "scramble" the signal. By that, it randomly re-orders the values in the time series and hence, destroys all spectral information. If analysing the amplitude distribution, the spectral information is often distracting. Scrambling a signal will result in a noise-like sound with the same statistical properties as the original. In the screenshot the loaded file is a speech sample that comes with ev-
probability function
\[ f(x; \alpha, \beta, c, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(t)e^{-itx}dt \quad (7) \]
\[ \phi(t) = e^{it\mu - |ct|^n(1 - c\text{sign}(t)\Phi)} \quad (8) \]
\[ \Phi = \tan\left(\frac{\pi\alpha}{2}\right) \quad (9) \]

Where \( \alpha \) is an exponent, \( \beta \) directly controls the skewness and \( c \) and \( \mu \) are scaling parameters. There is no analytic solution to the integral, but there are special cases in which the distribution behaves in specific ways. For example, for \( \alpha = 2 \) the distribution reduces to a Gaussian distribution. Fortunately, the Levi distribution was implemented as a number generator in the GNU Scientific Library (GLS) [6]. By providing the \( \alpha \) and \( \beta \) it allows for generating sequences of numbers for that distribution of any length.

For the experiment we generated 24 signals with skew ranging from -0.19 to 0.25 and kurtosis ranging from 0.17 to 14. It seemed to be impossible to completely de-couple skew from kurtosis. So, we decided to generate two sets, one that has insignificant changes in skew, but a range in kurtosis of 0.16 to 14. While the other covered the full range for skew and 0.15 to 5 for kurtosis. All signals were normalised to have a variance of 0.001 and were 3 seconds long with a 0.2 seconds fade-in and fade-out.

4.2. Experiment

The experiment was designed as a similarity listening test. Participants were listening to sequences of three signals and had to select two which they perceived as most similar. A sequence would be randomly arranged out of the signal under investigation (each of the 24), another, randomly chosen signal out of the 24 and the first signal scrambled. It was pointed out to participants that they will not hear two exactly similar sounds within the sequence, but they were asked to select the two that sounded most similar. The signal under investigation and its scrambled counterpart were essential different signals, but shared the same statistical properties. It was not specified which quality of the sound they should listen for to make this decision. This and the scrambling was done to make sure that participants focus on a generic quality of the noise rather than specific events within the signals.

After a brief written introduction into the problem domain and the nature of the experiment, participants started off with a training phase of three sequences to learn the user interface. For this training phase, the signals with the largest differences in skew and kurtosis were chosen to give people an idea about what to expect. Subsequently, each of the sets were played: Set one with 9 sequences, Set two with 15. The sequence of the sets was altered with each participant. Participants were also able to replay the sequence as often as they wished and adjust the volume to their taste. Figure 3 shows the user interface used. A post-questionnaire probed for the sound quality participants used to distinguish the quality. Further questions were if participants could tell any difference between the sets and if they had the feeling that there was any learning effect i.e. whether it got easier during the experiment.

4.3. Results

Eleven participants took part in the experiment, most of them working colleagues or students at the institute. Four participants were
members of the SonEnvir team and had more substantial background on the topic which, however, did not seem to have had any impact on their results.

The collected data shows that there is a significant increase in the probability of choosing the correct signals as the difference in kurtosis and skew increased. Figure 4 shows the average probabilities in four different ranges of $\Delta$ kurtosis. The skew in this set was nearly constant ($\pm 0.001$), so the resulting difference in correctness is related to the change in $\Delta$ kurtosis. While up to the difference of 5 in kurtosis the probability is only insignificantly higher than 0.33, the probability of random selection - and even decreases, there is a considerable increase thereafter, topping at over 70% at differences of around 11. This indicates that 5 is the threshold for just noticeable differences for kurtosis. This is also supported by the results from set 2 as shown in figure 5.

For skewness the matter is more difficult as we had no independent control over it. Although the data from set 2 suggests that there is an increase in probability with increasing difference in skew (as shown in figure 6), this might also be related to the difference in kurtosis. Looking at the probability of correctness over both, the difference in kurtosis and the difference in skew (as in figure 7) reveals that it is unlikely that the increase is related to the change in $\Delta$ skewness. While in every spine in which $\Delta$ skew is constant the probability increases with increasing $\Delta$ kurtosis, this is not the case vice versa. Summarising, we found evidence that participants could reliably detect changes in kurtosis greater than 5, but we did not find enough prove in the case of skewness. This might indicate that we need to use a different dataset so that we have bigger differences in skew while having small values for the kurtosis. However, for this another family of distributions must be found.

The number of times participants used the replay option seemed to have no impact on their performance. Figure 8 shows the number of replays of all data points over $\Delta$ kurtosis. Red crosses indicate correct answers, black dots incorrect answers. Although participants replayed the sequence more often when the difference in kurtosis was small, there is no evidence that they were more successful when using more replays.

The answers to the post-questionnaire must be seen in the light of the data analysis above. The quality participants assessed to drive their decisions must be linked to the kurtosis rather than skewness in the signal. The most common answers for this quality were crackling and the frequency of events. Others included roughness and spikes. However, some participants also stated that they heard different colours of noise and other artefacts related to the frequency spectrum. This is a common effect when being exposed to noise signals for a longer period of time. Even if the spectrum of noise is not changing at all (as in our case), humans often start to imagine hearing tones and other frequency related patterns. Asked for adjectives to describe the quality the partici-
participants provided *cracking*, *clicking*, *sizzling*, *annoying*, *rhythmic*, *sharp*, *rough* and *bright - dark*. Which, in retrospect, correlates nicely with the kurtosis being the “peakedness” of the probability function.

There was no agreement over which set was easier. Most participants said there was hardly any difference while some would state the one or the other. Also people in average felt that there was no learning curve involved and the examples were short enough for them not to get too tired over listening to them.

5. CONCLUSION

In this paper we presented an approach to analyse statistical properties of time series data by auditory means. We provided some background on the mathematics involved and presented the tools for audification of time series data that were developed. Subsequently, we described a listening test in which we intended to investigate the perceptual dimensions that would correlate with higher order statistical properties like skew and kurtosis. We discussed the data chosen and the design of the experiment. The results show that there is evidence that participants improved in distinguishing noise signals as the difference in kurtosis increased. The data suggests that in this setting the just noticeable difference was 5. However, for skew we were not able to find similar evidence. In a post-questionnaire we probed for the qualities that participants used to distinguish the signals and presented a set of related adjectives.

Future work has to investigate why there was nothing to be found for skewness in the signals. It might have been the case that our range of values did not allow for segregation and a different source for data must be found to have independent control over skew. However, it might also be the case that skew is not perceivable in direct audification and a different sonification approach has to be chosen to pronounce this property.

6. REFERENCES


