

## TWO TOOLS FOR INTEGRATING SONIFICATION INTO CALCULUS INSTRUCTION<sup>1</sup>

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### ABSTRACT

Two sonification tools are presented for use in calculus instruction. The first is a web-based tool for teaching students to interpret sonifications. The other is a spreadsheet-based tool that uses sonification to support and reinforce graphical and numeric representations of functions. We also illustrate how the tools could be used, and present data on the usability of the tools and the ability of students to interpret our sonifications.

### 1. INTRODUCTION

A significant number of students in the United States are not successful in their attempt to learn Calculus. This creates a stumbling block to the education of future engineers, scientists, and mathematicians.

Sonification has been investigated as a possible mathematics instructional tool [1, 2, 3]. This seems reasonable, since Gardner's theory of multiple intelligences suggests students learn better when multiple senses are engaged [7], and one example of this approach is to use vision and audition to reinforce each other. There is some new work integrating sonification into precalculus instruction [6]; however, it has not been widely integrated into calculus instruction.

Use of multiple simultaneous representations is a widely accepted method for improving student learning and success rate in calculus, facilitating construction of a conceptual basis for the content [4] and the development of problem-solving strategies [5]. The Calculus Reform Movement initially emphasized representing mathematics numerically, graphically, and analytically, and referred to this approach as the Rule of Three. Soon, this Rule expanded to a Rule of Four which added oral or written representations of mathematics [4]. We expect that dovetailing sonification with the Rule of Four to create a Rule of Five [8] would be a successful pedagogical approach.

This poster presents two tools we have developed as part of our project to integrate sonification into calculus instruction through laboratory activities and frequent classroom activities and to measure the impact these activities have on student learning of calculus. Byproducts of this project include teaching students how to interpret and use sonifications, identifying design principles for mathematical sonifications, and engaging mathematics, mathematics education, and computer science students in undergraduate research in sonification. We will demonstrate these tools using a laptop and headphones placed in front of the poster.

### 2. TRAINING ACTIVITY

We use a web-based activity to teach students to interpret our sonifications. The activity is a series of multiple-choice questions, with feedback hyperlinked to each response. There are three primary learning goals for this activity. The first is to estimate location on a number scale. The second is to hear qualitative properties of functions such as increasing/decreasing or concavity and match these properties with visible properties of graphs. The last is to use the sonification of a function to estimate locations of its zeroes and extreme values. It is assumed that students will use headphones during the activity, as it is intended to be used during a class period.

#### 2.1. Results with the First Version

In the first section of the activity, we sonified values on a discrete number scale, logarithmically mapping each value to the fundamental frequency of a chord. Major chords were used to represent non-negative values, and minor chords to represent negative values. A student listened to three sounds in succession for each exercise: the smallest value in the scale, followed by a rest, the largest value in the scale followed by a rest, and finally the target sound. The student was asked to estimate the target value. For example, students were asked to estimate which whole number between 0 and 10 corresponds to the target sound in [Exercise1.mp3]. Although this procedure differs somewhat from related work [9], the modification was justified by use of "training tones" in musical ear-training [10].

Students had significant success on this section of the activity, supporting earlier findings of Sándor and Lane [9]. Forty-four students in a Calculus course for non-mathematics majors completed the activity in October 2005. Each student recorded his initial response to each question, although he received feedback upon making a choice and was allowed to return to try the exercise repeatedly to gain understanding. The students averaged fourteen correct responses to the twenty questions on this section; when we allow a margin of error of one position on the scale, the percentage correct approaches 90%.

In the second section of the activity, students were expected to match a graph to a sonification. In these sonifications, we linearly mapped the horizontal axis to time, and we logarithmically mapped the vertical axis to fundamental frequencies of chords. Note that the y-values of our functions vary continuously, so the frequency of the chords' component waveforms must also vary continuously. To generate a

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<sup>1</sup> Support for this work was provided by the National Science Foundation, under grant DUE-0442450.

waveform with continuously varying frequency  $f(t)$ , we integrate  $f$  to get the waveform:

$$\omega(t) = \sin(2\pi F(t)) \tag{1}$$

where  $F(t)$  is any anti-derivative of  $f(t)$ . Major and minor chords were again used to distinguish between non-negative and negative  $y$ -values.

Consider Figure 1, two sample exercises for teaching a student how to match a graph to a sound. To listen to the sonification, [Ex21.mp3], a student clicks on the link, Sound 21. When a student floats the mouse over one of the graphs, she sees an enlargement of it. Graph 3, which corresponds to Sound 21, is shown enlarged in Figure 1. The lower panel of Figure 1 is the display for Sound 23, [Ex23.mp3].

Students performed well on matching graphs to sonifications, averaging over 70% correct on these ten questions. Performance was mixed, however. For example, for Sound 21 nearly 90% of the responses were correct, while only 50% were correct for Sound 23. The task for Sound 23 is more complicated because a student needs to discriminate among different types of increasing, while on Sound 21, the student just needs to identify increasing and decreasing behavior. Further research on this will be required to better understand what factors account for such variation in performance.

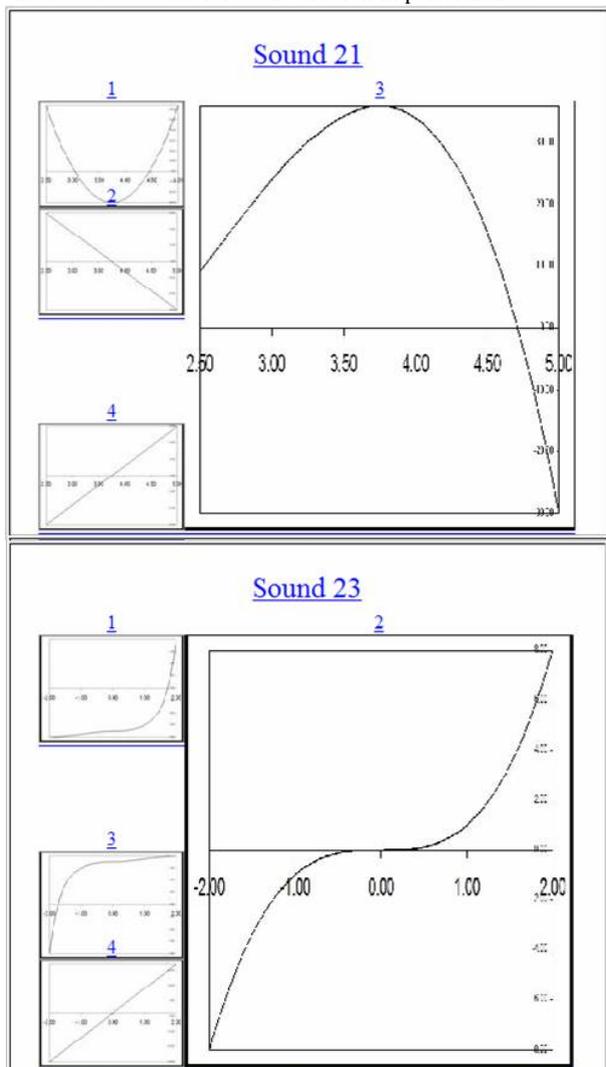


Figure 1. Exercises for matching a graph to a sound.

In the third section of the activity, we sonified functions as in the second section, but also provided a cue to allow students to keep track of position on the horizontal axis. We did this by increasing the amplitude of the waveform fourfold for 0.4 of a second three times, thus dividing the sonification into four equally spaced intervals. An illustration of this process is converting [Line.mp3] into [LinePulsed.mp3].

We expected students to identify which interval contained the extreme value or the  $x$ -intercept of the function. For example, we asked students to identify the interval which contains the maximum value of the function sonified by [Ex31.mp3], and we asked students to identify the interval which contains the  $x$ -intercept of the function sonified by [Ex36.mp3].

Students were significantly less successful on exercises in this section than on exercises in the first two sections. On average each student answered fewer than four of these ten questions correctly. In fact, only four of the ten questions were answered correctly by 40% or more of the students.

## 2.2. Analysis of Results

One explanation for the poor performance on Part 3 is that the minor chord cue for negative values was less effective than expected. This is serious because students must identify sign changes to connect extreme values of a function to its derivative. We have since developed a stronger cue for negative values. Now, all values are sonified with major chords, and negative values have random noise added to the chord. Thus, the sonification for the graph in Figure 2 becomes [NewNegative.mp3], instead of [OldNegative.mp3].

A review of the data for Part 3 might indicate that this is strong evidence that students cannot perform these tasks proficiently. However, we discovered there was a problem with the website. Students did not receive the feedback we had intended when they clicked on their choice. This caused confusion for the students that increased as they worked through the exercises. This suggests the data is inconclusive.

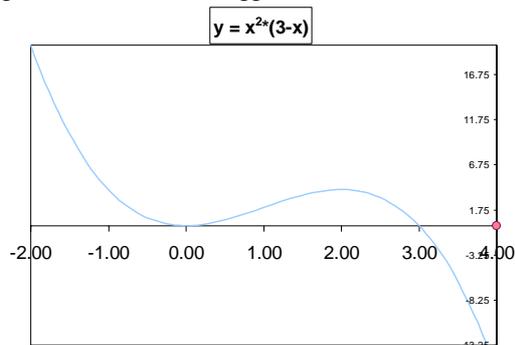


Figure 2. Two sonifications of this graph demonstrate an improved cue for negative & positive values.

## 3. EXCEL SONIFICATION TOOL

While the web-based activity is designed to teach students to interpret sonifications, our second tool is designed for students to use sonifications to learn calculus. We wanted a tool that students could use to create auditory graphs, to explore and learn about limits, derivatives, concavity, and extreme values. We embedded this tool in a Microsoft Excel spreadsheet for three reasons. First, many of our students are familiar with

spreadsheets already. Second, for those students who are not familiar with spreadsheets, learning to use spreadsheets is a valuable learning goal in its own right. Finally, spreadsheets are powerful tools for learning and applying calculus.

Since Excel does not have functions for generating or playing sounds, we developed sonification software to interface with the spreadsheet. We developed Visual Basic code and an ActiveX Control for the sonification piece. Other pieces of our tool, including orchestration of the package, are managed by Excel; this is why we say our tool is *embedded* in Excel.

### 3.1. Components of the Spreadsheet Tool

Our spreadsheet tool consists of four components: three that are visible in Figure 3, and the sonification itself. In Figure 3, on the left side, we have a table of x-values, y-values, estimated derivative values, and estimated second derivative values. The user specifies the function here, usually by entering a formula in the Y column. Slopes are estimated using ratios of first differences of the y- and x-values.

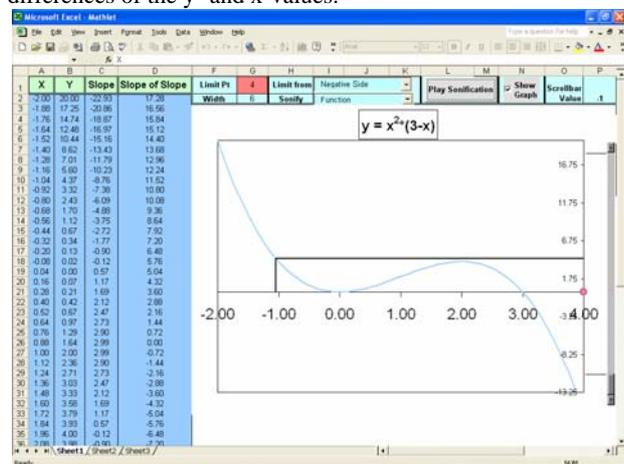


Figure 3. Three components of the spreadsheet tool: a table of values, a graph, and a control area.

Most of the screen is devoted to a graph of the function. An important feature of the graph is an animated cursor, synchronized with the sonification by the ActiveX control. The cursor is shown here as two lines intersecting the graph, one horizontal and one vertical. We use other cursor styles when sonifying the derivative or the integral.

Above the graph, we have an area for control of parameters. The user can choose what function characteristic to sonify, what domain to use, or to hide the graph to focus exclusively on the sonification. We will discuss these features in more detail later.

The fourth component is the .wav file sonification that the software generates. This sonification is generated by the software, rather than being retrieved from a library of sounds, which allows users to sonify a wide variety of functions on a wide range of domains. The file is stored on the user's PC when it is created, and can be recalled at the user's request.

### 3.2. What the Software Does

Figure 4 shows a portion of the parameter control area, where the user can choose which aspect of the function to sonify.

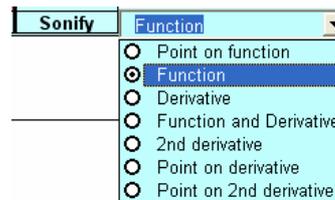


Figure 4. Aspects of the function which can be sonified.

The user could choose to sonify a point on the function, on the derivative, or on the second derivative. In this case, the user would click an x-value in the table, and the software would play a chord corresponding to the corresponding value in the Y column, the Slope column, or the Slope of Slope column.

Alternatively, the user could choose to sonify the function itself. When the user clicks the "Play Sonification" button, the software creates and stores a .wav file sonification, unless one already exists for this function. Then the file is immediately played, and the cursor is animated on the graph, synchronized with the sonification.

The user could also choose to sonify the derivative of the function, and then the animated cursor would be a short tangent line segment. If the user chooses to sonify the function and the derivative, the function sonification plays on the left channel, while the derivative plays on the right.

When calculus students study limits, it is important that they be able to focus exclusively on the negative side, exclusively on the positive side, or on both sides of the limit point simultaneously. We have incorporated this in an option box, illustrated in Figure 5.

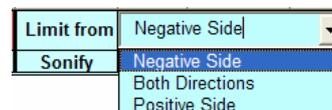


Figure 5. Choosing limit direction.

When the user chooses negative side, the animated cursor moves from left to right, and the sound is synchronized with it. When the user chooses positive side, the animated cursor moves from right to left and the sound is synchronized with it. From both sides, two animated cursors are visible, one moving from the left and one from the right. The sound synchronized with these cursors is stereo, with the left channel playing the sonification for the negative side and the right channel for the positive side. Figure 6 is a snapshot of the cursor when sonifying a function from both the positive and negative sides; here, the limit point of interest is  $x = 1$ , and the sonification is [BothSides.mp3].

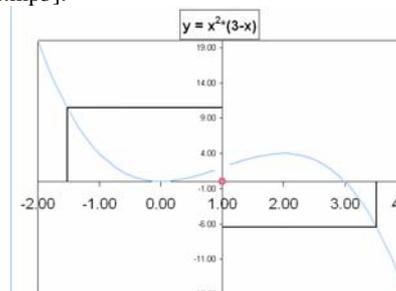


Figure 6. Two cursors for limits from both sides.

### 3.3. Future Plans

We will sonify directional derivatives and contour maps. We plan to investigate the role surround sound might play in calculus instruction. We also plan to simplify the spreadsheet tool's interface by creating an Excel Add-In and make it more accessible to visually impaired users. We hope to make the spreadsheet tool available in other operating systems and in spreadsheets other than Excel.

## 4. ACTIVITIES USING THE SPREADSHEET

The spreadsheet tool can be used by an instructor to demonstrate calculus skills and concepts or by students interactively. Here we present illustrations of how the tool can be used in these contexts.

### 4.1. Instructor Applications

We believe that sonification, like computer or calculator technology, will be beneficial in instruction only if it is integrated regularly, even daily, into the teaching process. Also, students, even those proficient at using spreadsheets, will need instruction in using this sonification tool. Therefore, an instructor using this tool should do so frequently class. For instance, when introducing families of functions like the quadratic, it would be natural to sonify examples in class to demonstrate their properties. Or, when discussing solutions to equations, it would be instructive to use sonification to reinforce the visual representation of intersecting graphs.

### 4.2. The Limit Concept

Students can use this tool to explore the limit concept. We used an activity on limits as the first activity where students used the spreadsheet tool. We began the activity with a concrete example, grounded in profit and quantity, a context to which students could relate. We started by asking questions such as "If quantity increases to almost 250, is profit getting bigger or smaller? Is there a limit to how big (or small) the profit gets?" Using these questions, we developed the concept of limit, including limit from negative or positive side. We then asked students to explain how sonification reinforced what they saw in the graph. Then, we asked students to hide the graph and use only the sonification to find a limit. Finally, students tested their skills with other functions, including some with asymptotes.

## 5. PROJECT EVALUATION

The purpose of our project is to investigate if students are more successful at performing standard symbolic, graphical, or numeric tasks involving limits, derivatives, and integrals, when sonification is used as a standard tool in both instruction and exercise. We plan to address several student learning objectives with sonification, including: finding limits, such as  $\lim_{x \rightarrow 3} x^2 - 9$ ,

from a graph; explaining the limit concept in writing, using appropriate mathematical symbols and terms; identifying critical points of a function; and identifying where a function increases or decreases, and is concave up or down.

In Spring 2007, we will collect data from two sections of calculus, each taught by Hetzler. We will use four sonification

laboratory activities in addition to the training activity. Each of these activities will be assigned to only one section of the course; at the same time, the other section will complete an equivalent activity that does not incorporate sonification. In this way, we will be able to compare each student's performance on learning objectives addressed with sonification to his performance on learning objectives not addressed with sonification, and we will be able to compare the sonification group's performance to the non-sonification group's performance on the same learning objective.

## 6. CONCLUSIONS

Preliminary data indicates our web-based tool and spreadsheet tool have potential to become effective instructional tools. Students seem to learn to interpret auditory graphs quickly. Anecdotal evidence gathered suggests these tools help them learn. Students quickly engage with the spreadsheet and use it to generate a variety of auditory graphs. We intend to develop new activities and evaluate and refine the ones presented here.

## 7. REFERENCES

- [1] T. L. Bonebright, M. A. Nees, T. T. Connerly, and G. R. McCain, "Testing the Effectiveness of Sonified Graphs for Education: A Programmatic Research Project," in *Proc. Seventh Int. Conf. on Aud. Disp. (ICAD)*, Espoo, Finland, July 2001.
- [2] J. H. Flowers, D. C. Buhman, and K. D. Turnage, "Data Sonification from the Desktop: Should Sound be part of Standard Data Analysis Software?" in *Proc. Third Int. Conf. on Auditory Display (ICAD)*, Palo Alto, CA, U.S.A., November 1996.
- [3] S. C. Peres and D. M. Lane, "Sonification of Statistical Graphs," in *Proc. Ninth Int. Conf. on Aud. Disp. (ICAD)*, Boston, MA, U.S.A., July 2003.
- [4] K. G. Snook, "A Continuum of Choice: Instructional Techniques in Undergraduate Mathematics," in *MAA Notes*, Vol. 61, *Changing Core Mathematics*, C. Arney and D. Small, Eds. Washington, D.C.: Mathematical Association of America, 2002, pp. 143-149.
- [5] D. Cohen, ed., *Crossroads in Mathematics*. Memphis: American Mathematical Association of Two-Year Colleges, 1995.
- [6] F. Van Scoy, D. McLaughlin, and A. Fullmer, "Auditory Augmentation of Haptic Graphs: Developing a Graphic Tool for Teaching Precalculus Skill to Blind Students," in *Proc. Eleventh Int. Conf. on Aud. Disp. (ICAD)*, Limerick, Ireland, July 2005.
- [7] H. Gardner, *The Unschooled Mind: How Children Think and How Schools Should Teach*. New York: Perseus Books, L.L.C. 1993.
- [8] S. M. Hetzler and R. M. Tardiff, "A Rule of Five," Maryland-District of Columbia-Virginia Mathematical Association of America Section Meeting, Williamsburg, VA, U.S.A., April 1997, <http://faculty.salisbury.edu/~smhetzler/ruleof5>.
- [9] A. Sándor and D. M. Lane, "Sonification of Absolute Values with Single and Multiple Dimensions," in *Proc. Ninth Int. Conf. on Aud. Disp. (ICAD)*, Boston, MA, U.S.A., July 2003.
- [10] M. T. Dvorak, private communication, May 2004.