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Active

Project #: E-20-644
Center # : 10/24-6-R7129-0A0
Cost share #:
Center shr #:
Rev #: 0
OCA file #:
Work type : RES
Document : AGR
Contract entity: GTRC

Contract#: AGREEMENT DATED 1/8/91
Mod #:
Prime #:

Subprojects ? : N
Main project #:

Project unit: CIVIL ENGR
Unit code: 02.010.116
Project director(s):
RIX G J CIVIL ENGR (404)894-2292

Sponsor/division names: ISMES, SPA / SERIATY (BERGAMO), ITALY
Sponsor/division codes: 707 / 004

Award period: 910108 to 910707 (performance) 910707 (reports)

Sponsor amount	New this change	Total to date
Contract value	9,877.00	9,877.00
Funded	9,877.00	9,877.00
Cost sharing amount		0.00

Does subcontracting plan apply ?: N

Title: ACCURACY AND RESOLUTION OF SURFACE WAVE INVERSION

PROJECT ADMINISTRATION DATA

OCA contact: E. Faith Gleason 894-4820

Sponsor technical contact	Sponsor issuing office
DR. FRANZ SMITS	DR. DOMENICO BRUZZI
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ISMES S.P.A.	ISMES S.P.A.
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24068 SERIATE (BERGAMO)	24100 BERGAMO BG
ITALIA	ITALY

Security class (U,C,S,TS) : U
ONR resident rep. is ACO (Y/N): N
Defense priority rating : NA
NA supplemental sheet
Equipment title vests with: Sponsor GIT
NOT APPLICABLE/ NONE PROPOSED.
Administrative comments -
INITIATION.
* NOTE: ENCLOSE EXPORT DOCUMENTS WITH PROJECT DELIVERABLE(S).

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GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 03/07/95

Project No. E-20-644 Center No. 10/24-6-R7129-0A0

Project Director RIX G J School/Lab CIVIL ENGR

Sponsor ISMES, SPA/SERIATY (BERGAMO), ITALY

Contract/Grant No. AGREEMENT DATED 1/8/91 Contract Entity GTRC

Prime Contract No.

Title ACCURACY AND RESOLUTION OF SURFACE WAVE INVERSION

Effective Completion Date 910707 (Performance) 910707 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	N	
Final Report of Inventions and/or Subcontracts	N	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

E-20-644
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Influence of the Distribution of Dispersion Data on the Accuracy and Resolution of Surface Wave Inversion

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Abstract

Shear wave velocity profiles of soils and pavements may be evaluated nondestructively using surface wave tests such as the Spectral Analysis of Surface Waves (SASW) method. In these tests, dispersion data are measured in situ and inverted using least squares techniques to obtain the shear wave velocity profile. The resolution and accuracy of the shear wave velocity profile depend on the data acquisition and processing techniques used to determined the dispersion curve. This paper examines the influence of the number of dispersion points, the maximum wavelength, and the distribution of dispersion data with wavelength on the accuracy and resolution of the shear wave velocity profile. The approach adopted is to calculate theoretical dispersion curves that contain different distribution of points for an assumed profile. Each of the dispersion curves is inverted to investigate the influence of the various distributions on the resulting shear wave velocity profile. The results indicated that the best overall accuracy and resolution is obtained when the dispersion data is evenly distributed between the minimum and maximum wavelengths and the maximum wavelength is one to two times the maximum desired depth of the shear wave velocity profile. The number of points did not appear to significantly influence the inverted profile as long as the number of points remains greater than the number of layers in the inversion profile.

Introduction

The Spectral Analysis of Surface Waves (SASW) method is a relatively new in situ seismic technique that may be used to determine the initial tangent shear modulus or shear wave velocity profile of a site. One of the principal advantages of the SASW method over seismic methods such as crosshole or downhole tests is that the source and receivers are on the ground surface and boreholes may be avoided. This feature has made the SASW method useful for a variety of applications including site investigations involving hard-to-sample soils

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(Stokoe et al., 1988; Stokoe et al., 1989) and nondestructive pavement evaluations (Nazarian and Stokoe, 1989; Rix and Stokoe, 1990)

Many elements of the SASW method are based on its predecessor, the Steady-State Rayleigh Wave method (Richart et al., 1970), but two important elements of SASW testing are significantly different from the procedures used in steady-state testing. The first difference is the way in which experimental dispersion curves are determined. Spectral analysis techniques are employed in SASW testing to measure the surface wave phase velocity at several hundred frequencies simultaneously rather than individually as in the steady-state method. This reduces the time required for field testing substantially.

The second important difference between the steady-state and SASW methods is in the inversion algorithms used to backcalculate the shear wave velocity or shear modulus profile from the experimental dispersion curve data. In the steady-state method, empirical inversion algorithms based on simple approximations were used and often resulted in erroneous shear wave velocity profiles. The SASW method makes use of theoretically-based inversion algorithms such as those employed by Nazarian (1984) or Hossain and Drnevich (1989) that have improved the accuracy of the resulting shear wave velocity profiles.

Since inversion is an integral part of surface wave testing, it is important that it be well understood. Two aspects of inversion that have not been fully investigated are accuracy and resolution. An accurate profile is one in which the calculated shear wave velocity agrees closely with the actual velocity. Resolution is related to the degree of uncertainty in the calculated velocity. The objective of this paper is to evaluate the influence on the accuracy and resolution of surface wave inversion of (1) the maximum wavelength contained in the experimental dispersion curve, (2) the number of experimental dispersion points, and (3) the relative amounts of high-frequency (short-wavelength) and low-frequency (long-wavelength) data in the experimental dispersion curve. Knowledge of the effects of the number and distribution of experimental dispersion data on accuracy and resolution can provide valuable information on how to select receiver spacings and data collection procedures (e.g. linear versus logarithmic frequency scaling) to improve the SASW method.

Least Squares Inversion Algorithm

A stiffness matrix formulation similar to that often used in structural engineering is used to determine a characteristic equation at each frequency contained in the experimental dispersion curve (Kausel and Roesset, 1981). The characteristic equation has the following form:

$$f\,(\omega, V_R, V_{s1}, h_1, \rho_1, \nu_1, \ldots, V_{sn}, \rho_n, \nu_n) = 0 \tag{1}$$

where ω is the circular frequency ($2\pi f$), V_R is the surface wave phase velocity, V_s is the shear wave velocity, h is the layer thickness, ρ is the mass density, ν is the Poisson's ratio, and n is the number of layers used to model the site. Equation 1 forms the basis of a solution to the problem of backcalculating the velocities of the layers using observed values of ω and V_R (a dispersion curve). If the experimental dispersion curve consists of m data points (ω-V_R pairs), a set of m equations of the form of Eq. 1 can be written in which the unknowns are the properties of the n layers in the profile. To reduce the number of unknowns, the mass density and Poisson's ratio of each layer are generally fixed. Values of these two parameters usually vary within a small range and errors in the assumed values do not have a significant influence (Nazarian, 1984). If a sufficiently large number of layers has been chosen to reproduce the variation of properties in the actual profile accurately, the thicknesses of the layers can also be fixed. The remaining unknowns are the n shear wave velocities of the layers. The result is m nonlinear, implicit equations in n unknowns:

$$\begin{aligned}
&f_1\,(\omega_1, V_{R1}, V_{s1}, h_1, \rho_1, \nu_1, \ldots, V_{sn}, \rho_n, \nu_n) = 0 \\
&f_2\,(\omega_2, V_{R2}, V_{s1}, h_1, \rho_1, \nu_1, \ldots, V_{sn}, \rho_n, \nu_n) = 0 \\
&\ldots \\
&f_m\,(\omega_m, V_{Rm}, V_{s1}, h_1, \rho_1, \nu_1, \ldots, V_{sn}, \rho_n, \nu_n) = 0
\end{aligned} \tag{2}$$

or in abbreviated notation:

$$\mathbf{f}(\mathbf{x}) = 0 \tag{3}$$

where $\mathbf{x}$ represents both the known (surface wave velocities) and unknown (layer velocities) quantities. Approximately 60 dispersion points (m) and 10 to 15 layers (n) are used in a typical backcalculation. Since the number of data points contained in the dispersion curve is generally much larger than the number of layers used to model the pavement (m>n), least squares solution techniques must be used to evaluate the unknowns in this system of equations.

The objective of the backcalculation algorithm is to minimize the residuals of the m equations given in Eq. 2. More explicitly, the algorithm minimizes the objective function:

$$S(\mathbf{x}) = \frac{1}{2}\,[\mathbf{f}(\mathbf{x})^H\, C_T^{-1}\, \mathbf{f}(\mathbf{x}) + (\mathbf{x} - \mathbf{x}_{prior})^H\, C_X^{-1}\, (\mathbf{x} - \mathbf{x}_{prior})] \tag{4}$$

where

$$\mathbf{x} = \{V_{s1}, V_{s2}, \ldots, V_{sn}, V_{R1}, V_{R2}, \ldots, V_{Rm}\}^T \tag{5}$$

is a vector containing both the knowns and unknowns (surface wave velocities and layer shear wave velocities, respectively); $\mathbf{x}_{prior}$ is a vector containing the a priori (initial) values of the knowns and unknowns; C_T is a diagonal matrix with terms representing modelling errors; and C_X is a diagonal matrix containing the variances of the initial values of surface wave velocity and layer shear wave velocity. The superscript H denotes the Hermitian (complex conjugate) transpose. The first term on the right hand side of Eq. 4 is associated with the residuals of the m equations; the second term is associated with the differences between the current values of the knowns and unknowns and their initial values. The second term is intended to impose a constraint on a solution which strays too far from "reasonable" values of the surface wave velocity and layer shear wave velocity.

To minimize S(x), Newton-Raphson procedure is used (Tarantola, 1987). An iterative solution is necessary because of the nonlinear nature of the problem. The initial value for the shear wave velocity of each layer can be chosen using a number of methods. One approach is to use the simple empirical equation developed by Heukelom and Foster (1960) to calculate the initial profile. More accurate initial values for the surface layer and half space can often be selected by examining the values of the surface wave velocity for very short and very long wavelengths, respectively. Another approach is to use simple, closed-form methods of calculating a theoretical dispersion curve for a Gibson half space (Vardoulakis and Vrettos, 1988) to determine the initial shear wave velocity profile. For each iteration the updated values of the surface wave velocity and shear wave velocity are calculated:

$$\mathbf{x}_{j+1} = \mathbf{x}_{prior} - \mu_j C_X \mathbf{F}_j^H [C_T + \mathbf{F}_j C_X \mathbf{F}_j^H]^{-1} [\mathbf{f}(\mathbf{x}_j) - \mathbf{F}_j (\mathbf{x}_j - \mathbf{x}_{prior})] \tag{6}$$

where j is the iteration number and

$$\mathbf{F}_j = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_{m+n}} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\mathbf{x})}{\partial x_{m+n}} \\ \cdots & & & \\ \frac{\partial f_m(\mathbf{x})}{\partial x_1} & \frac{\partial f_m(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f_m(\mathbf{x})}{\partial x_{m+n}} \end{bmatrix}_{\mathbf{x}_j} \tag{7}$$

is the matrix of derivatives of f(x) with respect to the surface wave velocities and layer shear wave velocities. Values of **x** for the first iteration, $\mathbf{x}_1$, are set equal to the initial values, $\mathbf{x}_{prior}$. Iterations continue until the difference in shear wave velocities for two consecutive iterations is small or until the root mean square (rms) error becomes less than one.

In addition to the final values of shear wave velocity, the algorithm also provides information concerning the uncertainty (variance) in the backcalculated velocity of each layer. In this study, the resolution of the backcalculated profile is evaluated using the ratio of the final variance to the initial variance for each layer. This ratio varies from zero (well resolved) to one (poorly resolved).

Theoretical Dispersion Curves

The approach adopted in this study to evaluate the accuracy and resolution of surface wave inversion was to generate a series of theoretical dispersion curves containing different maximum wavelengths, number of points, and distributions of points with wavelength for an assumed ("true") shear wave velocity profile. The stiffness matrix method that is the basis of the inversion algorithm described in the previous section is also the basis of the program that was used to generate the theoretical dispersion curves. The curves were then inverted to obtain a shear wave velocity profile that could be compared to the true profile to evaluate the accuracy and resolution of the inversion algorithm.

The assumed profile used in this study was that of a site near Austin, Texas at which an extensive series of crosshole tests were used to establish the shear wave velocity profile (Rix, 1988). The shear wave velocity profile of the site is shown in Fig. 1. One of the reasons that this site was chosen is that the low-velocity sand layer between 36 and 42 ft (11.0 and 12.8 m) provides an opportunity to evaluate the influence of various distributions of dispersion data on the inversion algorithm's ability to resolve this layer.

Using the assumed profile shown in Fig. 1, a "master" theoretical dispersion curve was generated containing 1000 points with a maximum wavelength of 250 ft (76 m). From this master curve, subsets were selected to study the influence of the distribution of points with wavelength, the number of points, and the maximum wavelength. To study the effect of the distribution of dispersion data with wavelength, three curves were chosen with one curve containing predominantly short-wavelength data, a second curve containing data that was evenly distributed between the minimum and maximum wavelength, and a third curve containing predominantly long-wavelength data. These three dispersion curves are shown in Fig. 2. The maximum wavelength (100 ft or 30.5 m) and the number of points (60) was held constant. The influence of the maximum wavelength was investigated using four curves with maximum wavelengths of 20, 50, 100, and 250 ft (6.1, 15.3, 30.5, and 76.3 m). In each of these four curves, the number of points (60) and the distribution of points (evenly distributed) was fixed. Finally, to evaluate the influence of the number of points, three dispersion curves consisting of 20, 40, and 60 points each were selected from the master curve. Each curve had a maximum wavelength of 100 ft (30.5 m) and the points were evenly distributed between the minimum and maximum wavelengths.

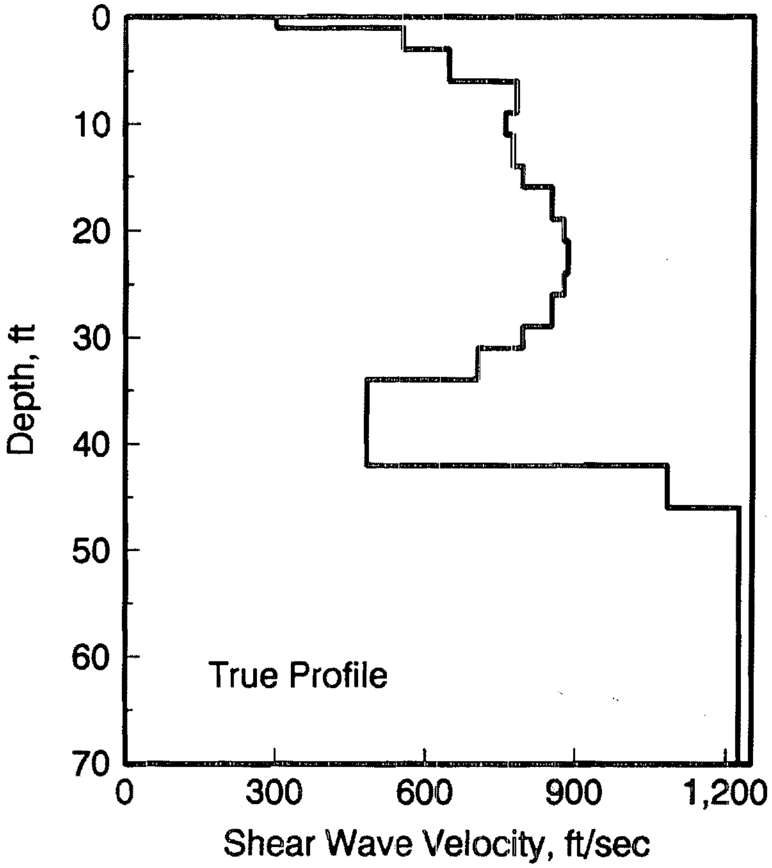

Figure 1 Assumed Profile Used to Generate Theoretical Dispersion Curves with Various Distributions of Data

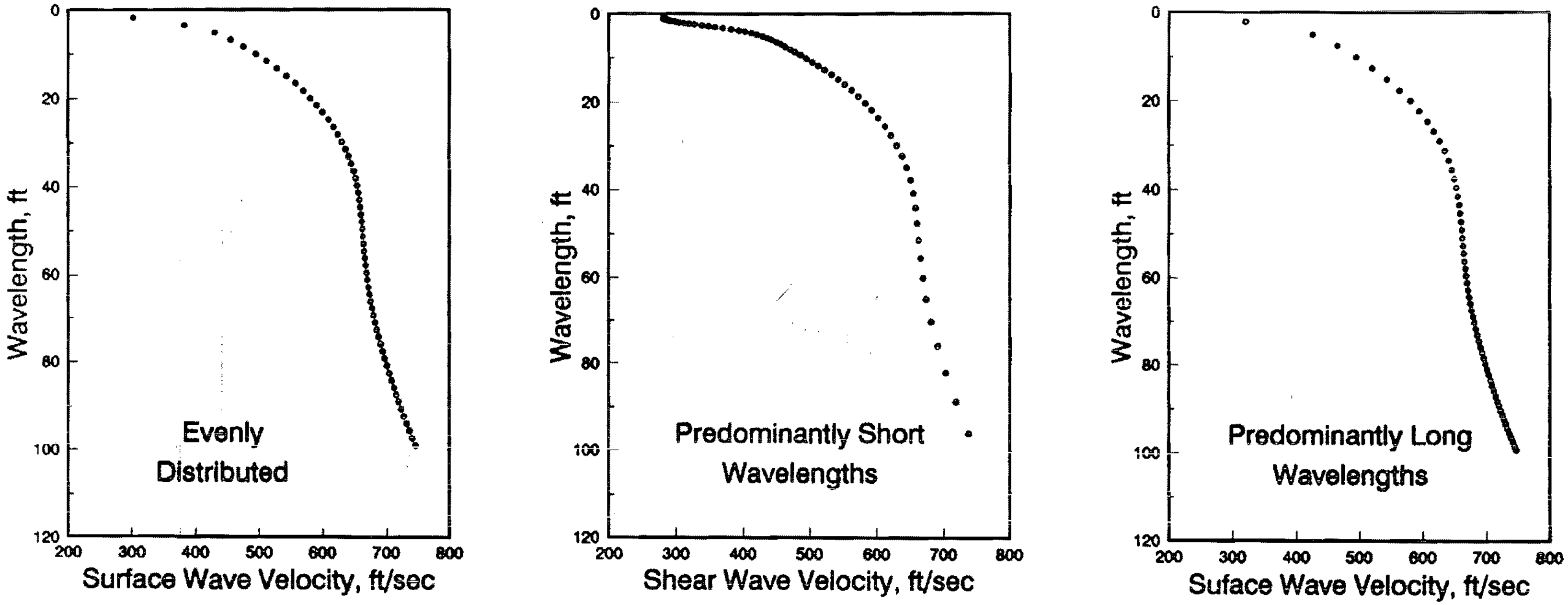

Figure 2 Dispersion Curves Used to Study the Influence of the Distribution of Dispersion Data with Wavelength

Results

To establish a reference for comparison, an inversion analysis was performed using a profile in which the layer thicknesses were nearly identical to those in the "true" profile. (Some adjacent layers with similar velocities were combined into a single layer for simplicity.) The theoretical dispersion curve used for this analysis contained 60 points and a maximum wavelength of 250 ft (76.3 m) with the dispersion data evenly distributed between the minimum and maximum wavelengths. The initial shear wave velocities for inversion were chosen using a procedure based on the Gibson half space model mentioned previously. The inverted profile reproduces the true profile accurately as shown in Fig. 3. The resolution of the inverted profile is indicated by the ratios of the final variances to the initial variances for each layer shown in Table 1. One interpretation of these variances is they represent the uncertainty in the inverted velocity profile. This uncertainty can be expressed in the form of "error bars" as shown in Fig. 3. For layers that are relatively thin such as the two layers from 30 to 34 ft (9.2 to 10.4 m), the range of uncertainty is large because large changes in the velocities of these layers would not increase the rms error significantly. For layers that are thicker, the resolution is much better because large changes in the velocities of these layers would degrade the solution noticeably (Leipski, 1991). This analysis using exact layering indicates that the inversion algorithm is capable of yielding an accurate, well-resolved profile under ideal conditions.

Table 1 Final and Initial Variances for Inversion Analysis Using Exact Layering

Layer No.	Initial Variance (ft/sec)^2	Final Variance (ft/sec)^2	$\frac{\text{Final Variance}}{\text{Initial Variance}}$
1	10000	7621	0.76
2	10000	4200	0.42
3	10000	5329	0.53
4	40000	3025	0.08
5	40000	8082	0.20
6	90000	81340	0.90
7	90000	74693	0.83
8	90000	13995	0.16
9	90000	78793	0.88
Half Space	90000	289	0.01

Since it is unlikely that the exact layering at a site would be known without the aid of boreholes, the remaining inversion analyses were performed using layers that did not coincide exactly with the layers in the true profile.

We believe that this approach provides a more objective evaluation of the accuracy and resolution of the inversion algorithm.

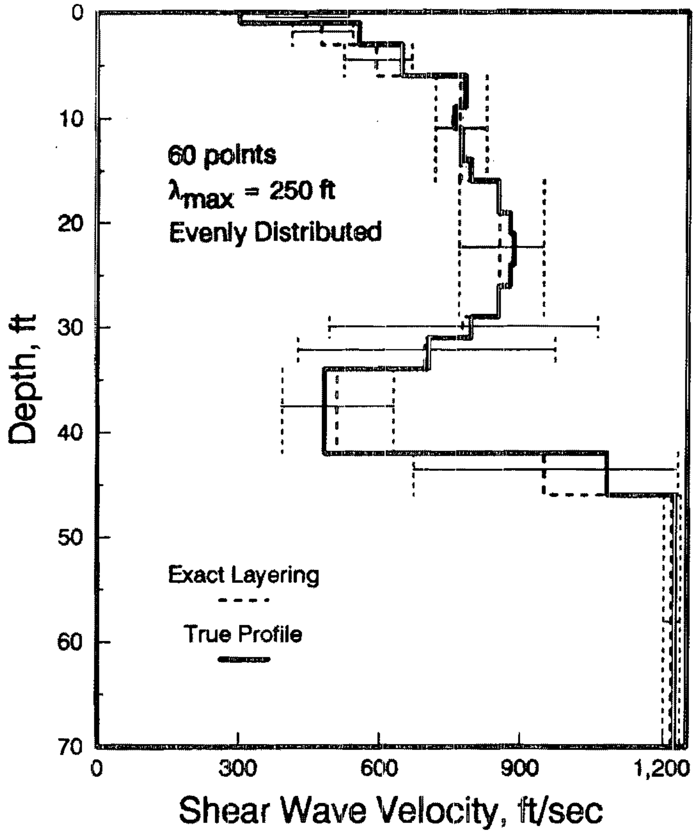

Figure 3 Results of Inversion Analysis Using Exact Layering

Influence of the Distribution of Dispersion Data with Wavelength

The results of the inversion analyses performed to evaluate the effect of the distribution of dispersion data with wavelength are shown in Fig. 4. All three distributions yield roughly the same profile with a few minor exceptions. As expected, the dispersion curve containing predominantly long wavelengths provides a slightly more accurate profile at depths greater than 46 ft (14.0 m). All three analyses indicate that there is a low-velocity layer in the profile but do not yield a minimum velocity that closely matches the true velocity of the layer between 34 and 42 ft (10.4 and 12.8 m). The reason for this apparent failure of the inversion algorithm

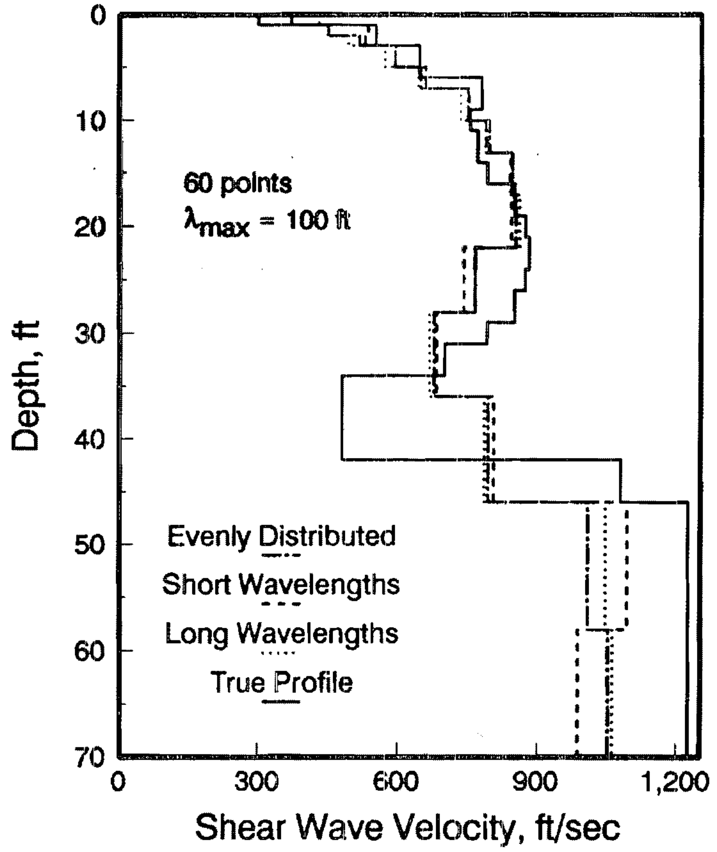

Figure 4 Influence of the Distribution of Dispersion Data with Wavelength on the Inverted Shear Wave Velocity Profile

to identify the low-velocity layer accurately is that the layers in the inversion profile span the boundary between the low-velocity layer and the higher velocity layers above and beneath. Because the layers span these boundaries, the inversion algorithm yields the average value of the low and high-velocity layers.

The differences in the resolution of each of the three different distributions of data with wavelength are given in Table 2. As expected, the dispersion curve containing predominantly short wavelengths best resolves the near-surface layers as indicated by the low ratios of final to initial variance. The tradeoff is that the deeper layers are not as well resolved. The dispersion curve containing mainly long wavelengths provides the best resolution at depth but does not resolve near-surface layers as well. The dispersion curve in which the points are evenly distributed appears to provide best balance between adequate resolution for both near-surface and deeper layers.

Table 2 Ratios of Final Variance to Initial Variance for Various Distributions of Dispersion Data with Wavelength

Layer No.	Predominantly Short Wavelengths	Evenly Distributed Wavelengths	Predominantly Long Wavelengths
1	0.06	0.72	0.86
2	0.45	0.77	0.82
3	0.67	0.83	0.85
4	0.66	0.75	0.77
5	0.86	0.88	0.90
6	0.64	0.65	0.66
7	0.83	0.83	0.84
8	0.87	0.86	0.87
9	0.77	0.76	0.77
10	0.80	0.79	0.80
11	0.76	0.71	0.70
12	0.78	0.73	0.71
13	0.90	0.85	0.86
Half Space	0.85	0.80	0.77

Influence of the Maximum Wavelength

The influence of the maximum wavelength contained in the dispersion curve is illustrated in Fig. 5. The dispersion curve containing a maximum wavelength of 20 ft (6.1 m) yields the poorest accuracy of the four dispersion curves. This is apparent in the poor match between the inverted profile and the true profile in the range of depths from 17 to 28 ft (5.2 to 8.5 m). The remaining curves with maximum wavelengths ranging from 50 to 250 ft (15.3 to 76.3 m) provide reasonably accurate results in this range of depths. As expected, the curve with a maximum wavelength of 250 ft (76.3 m) produces the most accurate results at large depths; the value of the shear wave velocity in the inverted profile almost exactly matches that of the true profile. As the length of the maximum wavelength increases, the accuracy of the inverted profile in the vicinity of the low-velocity layer also increases. This is particularly apparent between 36 and 46 ft (11.0 and 14.0 m).

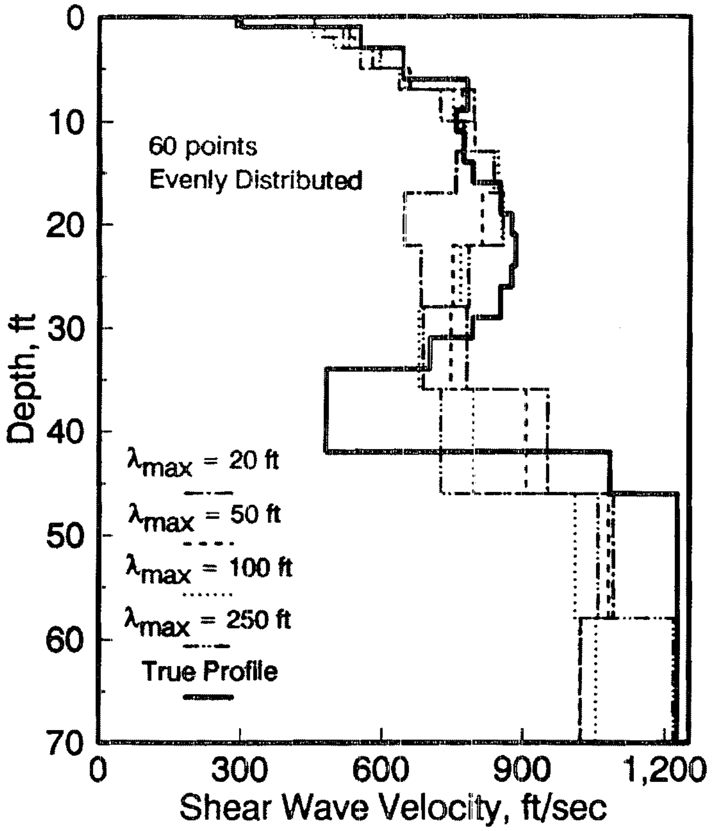

Figure 5 Influence of the Maximum Wavelength on the Inverted Shear Wave Velocity Profile

The effect of different maximum wavelengths on the resolution of the inverted profile is presented in Table 3. The dispersion curves with maximum wavelengths of 20 and 50 ft (6.1 and 15.3 m) provide the best resolution near the surface as expected. Because these curves lack long-wavelength data, the resolution is poor at depth. For the dispersion curve with a 250-ft (76.3-m) maximum wavelength, the resolution at depth is best, but the near-surface resolution is poor because there are relatively few, short-wavelength points. The curve with a maximum wavelength of 100 ft (30.5 m) appears to provide the best overall resolution since the ratios of final to initial variance are fairly consistent with depth.

Table 3 Ratios of Final to Initial Variance for Different Maximum Wavelengths

Layer No.	λ_{max} = 20 ft	λ_{max} = 50 ft	λ_{max} = 100 ft	λ_{max} = 250 ft
1	0.43	0.42	0.72	0.90
2	0.64	0.67	0.77	0.90
3	0.80	0.74	0.83	0.87
4	0.80	0.67	0.75	0.81
5	0.88	0.85	0.88	0.92
6	0.66	0.62	0.65	0.68
7	0.89	0.81	0.82	0.86
8	0.95	0.84	0.86	0.88
9	0.93	0.75	0.76	0.78
10	0.93	0.79	0.79	0.82
11	0.93	0.81	0.71	0.67
12	0.97	0.93	0.73	0.62
13	0.97	0.96	0.85	0.67
Half Space	0.99	0.99	0.80	0.08

Influence of the Number of Dispersion Points

Results of the inversion analyses for dispersion curves containing 20, 40, and 60 points are shown in Fig. 6. All three dispersion curves yield inverted profiles that are virtually the same. Although the curves containing 40 and 60 points provide slightly more accurate results in the vicinity of the low-velocity layer and at depth, the differences are very small. The influence of the number of points on the resolution is shown in Table 4. Increasing the number of points increases the resolution of each of the layers, but, again, the differences between the three curves are slight.

Conclusions

The Spectral Analysis of Surface Waves (SASW) method is an in situ seismic method that utilizes surface (Rayleigh) waves to determine the shear wave velocity profile of a site nondestructively. An important part of surface wave testing is inversion in which the shear wave velocity profile is backcalculated from an experimental dispersion curve. The accuracy and resolution of the inversion algorithm has thus far not been fully investigated. The intent of the work described in this paper was to study the influence of the distribution of dispersion data wavelength, the maximum wavelength, and the number of points contained in the dispersion curve on the accuracy and resolution of the inverted profiles.

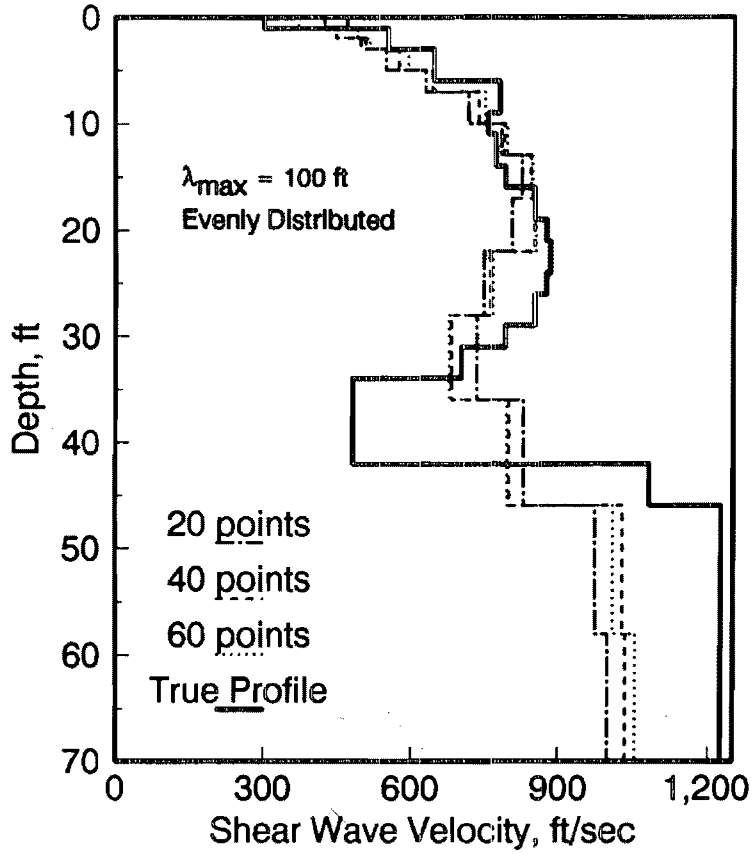

Figure 6 Influence of the Number of Points on the Inverted Shear Wave Velocity Profile

Under ideal conditions when the layering used in the inversion algorithm coincides with the "true" layering at the site, the inversion algorithm produces accurate, well-resolved profiles. An important conclusion from this study is that when a layer used in the inversion analysis spans the boundary between two or more layers in the true profile, the velocity of the layer in the inversion analysis will tend toward the average of the velocities of the true profile. We do not view this as a weakness in the inversion algorithm, but believe that it highlights the benefits of obtaining as much information about the site from other sources as possible before performing the inversion analysis. The best means of avoiding the tendency to average the velocities of two or more layers is to use layers in the inversion analysis that are thin enough to reproduce variations in the true profile.

Table 4 Ratios of Final Variance to Initial Variance for Different Number of Points in the Dispersion Curve

Layer No.	20 points	40 points	60 points
1	0.92	0.84	0.72
2	0.93	0.81	0.77
3	0.89	0.85	0.83
4	0.83	0.78	0.75
5	0.93	0.90	0.88
6	0.70	0.67	0.65
7	0.87	0.84	0.83
8	0.89	0.87	0.86
9	0.79	0.77	0.76
10	0.80	0.80	0.79
11	0.76	0.73	0.71
12	0.77	0.74	0.73
13	0.89	0.87	0.85
Half Space	0.83	0.81	0.80

Different distributions of dispersion data influence the accuracy and resolution of the inverted profile in a predictable manner. When data is concentrated at either short or long wavelengths, the accuracy and resolution are improved for the near-surface or deeper layers, respectively. Using a dispersion curve that contains data evenly distributed between the minimum and maximum wavelengths appears to offer the best balance of accuracy and resolution over the entire range of depths.

The effect of the maximum wavelength on the inverted profile is also as expected. To obtain a shear wave velocity profile to a given depth, it appears that it is desirable to have a maximum wavelength on the order of one to two times the depth to yield reasonably accurate and well-resolved velocities. Clearly it would be desirable to have wavelengths that are several times the depth, but the slight gain in accuracy and resolution are offset by the decreased depth of penetration for a given maximum wavelength.

The influence of the number of points on the inverted profile is somewhat surprising. We expected the increase in accuracy and resolution to be significant when the number of dispersion points increased from 20 to 60, but the differences in both accuracy and resolution are small.

The results of this study have several implications for gathering experimental dispersion data in the field. Most surface wave tests to date have been performed using source-receiver spacings based on the common receiver mid-point (CRMP) geometry (Nazarian, 1984). This progression of receiver spacings tends to result in curves with large number of predominantly short-wavelength dispersion points. This study indicates that it may be wiser to focus on obtaining relatively fewer dispersion points that are more equally distributed over the range of measured wavelengths. The results also point out the importance of trying to obtain as much long-wavelength data as possible to increase both the depth of penetration and the accuracy and resolution.

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