Simulation of the unsaturated Excavation Damage Zone around a tunnel using a fully coupled damage-plasticity model

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ABSTRACT

During tunnel excavation, stress redistribution produces plastic deformation and damage around the opening. Moreover the surrounding soil can be either saturated or unsaturated. Suction has a significant influence on the mechanical behaviour of geomaterials.

Depending on their stress state and on their moisture content, clay-based materials can exhibit either a ductile or a brittle behaviour. Plasticity leads to permanent strains and damage causes the deterioration of the soil elastic and hydraulic properties.

The damage-plasticity model proposed in this work is formulated in terms of a damaged constitutive stress, defined from the principle of Bishop’s hydro-mechanical stress (for unsaturated conditions), and from the principle of damaged effective stress used in Continuum Damage Mechanics. The evolution laws are obtained by using the principle of strain equivalence.

This hydro-mechanical damage-plasticity model was implemented in a Finite Element code. The excavation of a tunnel is simulated at different constant suctions. The results obtained illustrate the influence of suction on the development of plastic and damaged zones.

INTRODUCTION

Experimental evidence show that clays can exhibit either a brittle or a ductile behaviour (Dehandschutter et al., 2004). Under deviatoric loading, clays can undergo large permanent strains, which are taken into account in classical elastoplastic models such as the ones based on Cam-Clay (Roscoe and Burland, 1968). But some micro-cracks can also develop and lead to a decrease of clay rigidity and to a modification of its hydraulic properties. This dissipative phenomenon is described by damage mechanics. The transition between both behaviours depends on multiple factors including clay moisture content (Al-Shayea, 2001).
Some models have been developed for argillites considering damage and plasticity couplings (Chiarelli et al., 2003, Conil et al., 2004) and even considering unsaturated states of this rock (Hoxha et al., 2007, Jia et al., 2007). These models, initially formulated for rock, ignore some specific important features of clay soil behaviour, such as the dependence of elastic moduli to pressure. Moreover, damage-plasticity models proposed for rock so far fail at predicting the transition between ductile and brittle behaviour associated with suction increase.

The proposed work aims to develop a framework to couple damage and plasticity in unsaturated porous media, for which clay minerals in the solid matrix are expected to play an important role.

The main assumptions on which the formulation of the model is based are first presented. Then a tunnel excavation is simulated at different suctions. The results illustrate how suction affect the ductile/brittle behaviour.

**OUTLINE OF THE CONSTITUTIVE MODEL**

The model is derived under the assumption that permanent strains are purely plastic, and that damage only affects stiffness properties of the geomaterial.

Under small strain assumption, the total strain, \( \varepsilon \), can be decomposed into an elastic (\( \varepsilon^e \)) and a plastic (\( \varepsilon^p \)) part:

\[
\varepsilon = \varepsilon^e + \varepsilon^p
\]

**General framework**

**Mechanical state variable**  The proposed model is based on a hyper-elastic framework, i.e. the stress used in the constitutive equations is work-conjugate to elastic strains through an elastic potential \( \psi^e(\varepsilon^e) \).

In an intact unsaturated material, the capillary tension in inter-particles water meniscus acts like an isotropic compression on the solid matrix. Houlsby (1997) showed that the quantity which is work-conjugate to elastic strains, that we will call the constitutive stress, is defined as

\[
\sigma^* = \sigma - p_a I_d + s S_l(s) I_d
\]

in which \( p_a, p_w, s = p_a - p_w, S_l \) and \( I_d \) are respectively the air pressure, the water pressure, suction, the degree of saturation and the identity matrix.

This constitutive stress is thermodynamically conjugate to elastic strains through an elastic-damage potential \( \psi^e(\varepsilon^e, d) \).

\[
\sigma^* = \frac{\partial \psi^e(\varepsilon^e, d)}{\partial \varepsilon^e}
\]

The average effect of micro-cracks and micro voids on soil properties is measured by a scalar damage variable \( d \). \( d \) ranges from \( d = 0 \) for an intact material to \( d = 1 \) for a totally damaged material with no residual resistance.
As cracks open, the effective material cross section resisting internal forces decreases. This induces an increase of internal stress in the solid matrix, even under constant far-field stress conditions. This phenomenon is usually captured by defining an effective stress (as opposed to total, far-field stress). A usual expression used in Continuum Damage Mechanics is (Kachanov, 1958):

\[
\tilde{\sigma}^* = \frac{\sigma^*}{1 - d} = \frac{\sigma - p_d I_d + s S_1(s) I_d}{1 - d}
\]  

(1)

In this paper we will call this stress the *damaged constitutive stress*. Applying the Principle of Strain Equivalence (Lemaitre, 1996) within a hyper-elastic framework leads to:

\[
\tilde{\sigma}^* = \frac{\partial \psi_0^e}{\partial \varepsilon^e}
\]  

(2)

\[
\psi_0^e(\varepsilon^e) = \psi^e(\varepsilon^e, d) (1 - d)
\]

Equation 2 implies that, in a damaged material, the damaged constitutive stress increment is linked to the elastic strain increment by the same equation that the constitutive stress in an intact material:

\[
\dot{\tilde{\sigma}}^* = D_e (d = 0) \dot{\varepsilon}^e
\]

The stress defined in equation 1 is introduced in the plastic potentials, which results in a plastic model expressed in “damaged constitutive stress” instead of total stress.

**Plasticity framework**  The model is formulated within the framework of hardening plasticity. The hardening variable \(\chi(\chi_0, s)\) is a function of suction and it also depends on \(\chi_0\), characterising hardening in the saturated state, i.e. when \(s = 0\). The yield criterion, \(f_p\), and the plastic potential, \(g_p\), are functions of the stress state and the hardening parameter.

\[
f_p = f_p(\tilde{\sigma}^*, \chi(\chi_0, s))
\]

\[
g_p = g_p(\tilde{\sigma}^*, \chi(\chi_0, s))
\]

Flow rule and hardening law take the following form:

\[
\dot{\varepsilon}^p = \Lambda_p \frac{\partial g_p}{\partial \tilde{\sigma}^*}
\]

\[
\dot{\chi}_0 = f(\dot{\varepsilon}^p)
\]

**Damage**  Damage criterion, \(f_d\), is a function of the stress state and of the damage variable which acts as a hardening parameter.

\[
f_d = f_d(\tilde{\sigma}^*, d)
\]

Damage evolution law is given by:

\[
\dot{d} = \Lambda_d \frac{\partial f_d}{\partial \tilde{\sigma}^*}
\]
Specific functions for clays

We consider the following variables:

\[ \tilde{p}^* = \frac{1}{3} \text{tr}(\tilde{\sigma}^*), \quad \tilde{\sigma}^*_d = \tilde{\sigma}^* - \tilde{p}^* \mathbf{I}_d, \quad \tilde{q}^* = \sqrt{\frac{3}{2}} \tilde{\sigma}^*_d : \tilde{\sigma}^*_d, \quad \varepsilon_v = \text{tr}(\varepsilon), \quad \varepsilon_d = \varepsilon - \frac{1}{3} \varepsilon \mathbf{I}_d \]

The specific equations chosen to represent clay behaviour are summarised in Table 1.

**Table 1. Specific functions for clays**

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Linear elasticity (^a)</th>
<th>(\psi_0 = \frac{K}{2} (\varepsilon_v)^2 + G \varepsilon_d : \varepsilon_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retention properties</td>
<td>Van Genuchten (1980)</td>
<td>(S_l(s) = \left( \frac{1}{1 + (\alpha s)^m} \right)^n )</td>
</tr>
<tr>
<td>Plasticity</td>
<td>Barcelona Basic Model (Alonso et al., 1990) (^b)</td>
<td>(f_p(\tilde{\sigma}^<em>, \tilde{p}_0, s) = \tilde{q}^</em> - M^2 \tilde{p}^<em>(\tilde{p}_0, s) - \tilde{p}^</em> ) (g_p(\tilde{\sigma}^<em>, \tilde{p}_0, s) = \zeta \tilde{q}^</em> - M^2 \tilde{p}^<em>(\tilde{p}_0, s) - \tilde{p}^</em> )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\tilde{p}_c = \tilde{p}_r \left( \frac{\tilde{p}_0}{\tilde{p}_r} \right)^{\frac{\lambda_0 - \kappa}{\lambda_0 + \kappa}} + S_l s ) (\lambda(s) = \lambda_0 [1 - r \exp(-\beta s) + r] )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\dot{\tilde{P}}_0 = \frac{\tilde{p}_0}{\lambda_0 - \kappa} \dot{\varepsilon}_v )</td>
</tr>
<tr>
<td>Damage</td>
<td></td>
<td>(f_d(\tilde{\sigma}^<em>, d) = \tilde{q}^</em> - C_2 \tilde{p}^* - C_0 - C_1 d ) (\dot{d} = \frac{1}{C_1} \left[ -\frac{C_2}{3} \mathbf{I}_d + \frac{3\tilde{\sigma}^<em>_d}{2\tilde{q}^</em>} \right] : \dot{\tilde{\sigma}^*} )</td>
</tr>
</tbody>
</table>

\(^a\)Since this work focuses on plastic and damage evolution, for the sake of simplicity, elasticity is supposed to be linear. To account for pressure dependency, a hyperelastic potential like the one of Houlsby et al. (2005) could be used. 

\(^b\)The model is modified to be written in terms of the damaged constitutive stress

**EXCAVATION MODELLING**

The above model as been implemented into the Finite Element code \(\theta\)-Stock (Gatmiri and Arson, 2008).

To illustrate the capabilities of the model, we simulate the excavation of a tunnel of radius \(R\) in Boom clay (note that in this conceptual study, the numerical value of \(R\) does not need to be specified). Excavation is simulated at different constant suctions, under the assumption that fluid transfers are fast enough to consider drained conditions.
Initial conditions and material parameters are taken of the same order of magnitude as the ones found in the literature for Boom clay at the HADES underground laboratory at Mol, Belgium (Forge report, 2010; Romero, 1999; Della Vecchia et al., 2011).

The goal of the numerical work presented below is not to simulate a real experiment but to illustrate qualitatively the capabilities of the model.

**Boom clay parameters**

Retention and mechanical parameters are given in Table 2 and 3.

**Table 2. Retention curve parameters**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$n$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.28</td>
<td>2.3</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Table 3. Material mechanical parameters**

<table>
<thead>
<tr>
<th>Elast</th>
<th>Plasticity</th>
<th>Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$G$</td>
<td>$M$</td>
</tr>
<tr>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
</tr>
<tr>
<td>250</td>
<td>115</td>
<td>1</td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

Stress path at the tunnel wall

To understand better how our model handle the transition between plastic and brittle behaviour, let see the stress paths followed during excavation at tunnel wall. For two different values of suction (s=0 and s=1MPa), Figures 2 and 3 show the damaged constitutive stress path (thick solid line), the total stress path (thick dashed line) and plasticity (thin solid line) and damage (thin dashed line) criteria.

Figure 2. Total and damaged constitutive stress path for saturated state

Figure 3. Total and damaged constitutive stress path for suction = 1 MPa

These stress paths illustrate the main mechanism allowing for transition from
ductile to brittle behaviour, which is the competition between the two dissipative phenomena. Indeed, for a saturated soil, the stress path reaches the plastic yield surface before reaching the damage criterion. The occurrence of plasticity modifies the stress path and prevents the development of high damage. For high suctions, the elastic domain is much larger, due to the dependence of the yield surface on suction, and the stress path will reach the damage criterion first, which leads to a more brittle behaviour.

Both the damaged constitutive stress path ($A^*B^*C^*$) and the total stress path (ABC) are represented in Figure 3. Before damage is initiated (AB) the difference between the two stresses is only due to suction, which increases the confining pressure on the soil skeleton. After damage initiation (BC), the stress acting on soil skeleton increases due to the appearance of microcracks, while total stress tends to decrease.

**Stress spatial evolution**

Figures 4 and 5 below show the evolution of these stresses around the tunnel, for two states of suction: $s=0$ and $s=1$ MPa ($R$ denotes the radius of the tunnel).

![Figure 4. Total and damaged constitutive stress for saturated state](image1.png)

**Figure 4. Total and damaged constitutive stress for saturated state**

![Figure 5. Total and damaged constitutive stress for suction =1 MPa](image2.png)

**Figure 5. Total and damaged constitutive stress for suction =1 MPa**
For the saturated case, plasticity is the main dissipative phenomenon and leads to a decompression of tangential stress around the opening. In the unsaturated case, there is also a decompression of total stress but the value of damaged constitutive stress is much higher.

**Development of plastic and damage zones**

The spatial evolution of damage and plasticity (represented by the norm of the plastic strain matrix) are given in Figures 6 and 7.

![Figure 6](image_url)

**Figure 6.** Plastic strain distribution at various constant suctions.

![Figure 7](image_url)

**Figure 7.** Damage distribution at various constant suctions.

The amplitude of plastic strains and the extent of the plastic zone both decrease with suction.

Damage at the tunnel wall increases when suction is higher, which is consistent with the model formulation, which was aimed to capture the shift from ductile to brittle behaviour associated with suction changes.

Damage develops further away from the excavation as suction is lower and as the plastic zone is wider. This observation is assumed to be due to the choice of consti-
tutive models used for plastic and damage criteria, as explained from the stress paths illustrated in Figures 2 and 3. Indeed, Figure 3 show that the occurrence of plasticity substantially modifies the stress path which contributes to the development of damage in the plastic zone.

CONCLUSION

A fully coupled damage-plasticity model is proposed for unsaturated geomaterials. The model is formulated with a "damaged constitutive stress", which accounts for the effects of suction and damage on stress undergone by the soil skeleton. This stress is used in the plastic yield criterion, which depends on suction. These assumptions allow capturing the transition from a ductile behaviour (for saturated states of soil) to a brittle behaviour (for unsaturated states).

The proposed model has been used to simulate the excavation of a tunnel with the Finite Element Method. Numerical results predict a wide plastic zone and high values of plastic strains when the ambient suction applied is low. For low suctions, damage remains low, but develops in the whole plastic zone, as opposed to states of high suction, in which damage is higher but localised in a narrower zone.

In future studies we plan to consider transient fluid flows and to study the effect of desaturation and resaturation on the mechanical state of the host geomaterial.

REFERENCES


