Stiffness and Deformation of Salt Rock Subject to Anisotropic Damage and Temperature-Dependent Healing

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ABSTRACT: This research is motivated by the increasing need for geostorage facilities, mainly: nuclear waste disposals, high-pressure gas reservoirs and carbon dioxide sequestration systems. A new constitutive model is formulated to account for anisotropic damage due to tensile cracks and healing due to Diffusive Mass Transfer (DMT). The damage variable is the difference between the second-order crack density tensor and a scalar viscoplastic healing variable. The viscoplastic healing variable is needed to model the effects of DMT on the reduction of damage-induced deformation. Contrary to the damage and healing models previously proposed for salt rock, the proposed framework accounts for crack-induced anisotropy, and anisotropy is treated in both damage and healing evolution laws. Compression, extension and compression loading and unloading cycles have been simulated with Theta-Stock Finite Element code. The results illustrate well the influence of anisotropic damage on stiffness degradation and residual strain development. An algorithm has also been written to study the trends of the coupled damage and healing model for a stress path comprising an isotropic compression, and axial compression, a healing period and an unloading phase. The results match the theoretical expectations, and show that the proposed model can predict anisotropic damage and healing.

1. INTRODUCTION

Saving energy resources and reducing carbon emissions can be achieved by resorting to geo-storage, a technique consisting in injecting fluids or disposing solids in the bedrock [1]. Salt cavities are often used to store high-pressure gas that can be used to activate turbines at peak hours. Rock salt is also an attractive host rock for deep waste disposals, due to favorable creep characteristics and low gas permeability, which helps ensure containment. However, these same creep properties can induce crack damage in the vicinity of deep cavities and repositories and significantly compromise security. In general, inelastic deformation in salt at conditions of geo-storage systems is dominated by isochoric dislocation processes, fluid-assisted Diffusive Mass Transfer (DMT), and dilatant micro-cracking. Rock “healing” is generally defined from the way it is measured in the laboratory, at the macro-scale of a Representative Elementary Volume (REV). Because deformation induced by dislocation creep is isochoric, crack damage in salt is often associated to inelastic dilatant deformation [2]. Damage grows in stress states above the “dilatancy boundary”, whereas below this boundary, inelastic contractant strains compensate damage deformation [3-4]. Within the dilatancy boundary, damage cannot grow or decrease [5].

In the framework proposed by Chan et al. [2, 6-7], the inelastic strain rate is the sum of viscoplastic, damage and healing deformation rates. Each inelastic strain rate is derived from a work-conjugate stress variable, which plays the role of a dissipation potential. Damage and healing are thus rate-dependent. Each creep rate is controlled by an equivalent measure of strain rate. Four stress-like dissipation potentials and four deformation kinetic equations are needed. In addition, a scalar crack density is introduced in creep kinetic equations to model material softening. The rate of damage is controlled by two evolution functions (one for crack opening, one for crack closure and healing). Chan et al.’s model gives a precise account of the influence of microscopic creep and cracking processes on deformation, but cannot predict damage-induced anisotropy on strains and stiffness. The goal of this research work is to account for crack-induced anisotropy in a consistent damage and healing model for salt rock.

The general thermodynamic framework is presented in the first part of this paper. The proposed model combines a mechanical anisotropic damage model of Continuum Damage Mechanics with a time-dependent healing model that has been validated for salt rock under various stress and temperature conditions. The mechanical damage model has been implemented in a Finite Element code. Several laboratory tests have been simulated to illustrate the development of anisotropic damage under axial compression and axial tension. Coupled damage and healing processes have been simulated at the Gauss point. Several stress paths are shown in part 3 of this paper.
2. OUTLINE OF THE DAMAGE AND HEALING MODEL

Damage and healing are modeled as two different dissipation processes, with two different internal variables, governed by different evolution laws. Damage is defined as the second-order crack density tensor [8], and accumulates if tensile strains exceed a certain threshold; constitutive equations are derived in a way very similar to plasticity. Kuhn-Tucker consistency equations impose that damage cannot decrease. As a result, another variable needs to be introduced in order to model healing and the consequent recovery effects on stiffness. Following the approach of Chan et al. [7], a scalar healing variable is used to account for creep processes. The difference between the second-order crack density tensor and the scalar creep healing variable defines a new damage variable that can be used to predict the effects of cracking and healing on both stiffness and deformation.

2.1. Mechanical Model of Anisotropic Damage

The mechanical damage model is adapted from the work of Halm & Dragon [9]. The formulation is modified in order to conjugate Cauchy’s stress to elastic strains (ε) instead of total strain (ε), in accordance with the thermodynamic framework advocated by Lemaître & Desmorat [10]. In Halm & Dragon’s model, it is assumed that the elastic deformation energy contains linear terms in ε and quadratic terms in D:

\[ \sigma = \lambda (\text{tr} \epsilon) + 2 \mu \epsilon + g D + \alpha [\text{tr} (\epsilon \cdot D)] I + (\text{tr} \epsilon) D + 2 \beta (\epsilon \cdot D + D \cdot \epsilon) \]  

(1)

\( \lambda \) and \( \mu \) are Lamé coefficients. \( g \), \( \alpha \) and \( \beta \) are damaged material parameters. Due to thermodynamic conjugation relationships:

\[ \sigma = D \epsilon (D) : \epsilon \]  

(2)

in which \( \epsilon \) is the total elastic deformation. As a result:

\[ d \sigma = D \epsilon (D) d \epsilon + \left( \frac{\partial D \epsilon (D)}{\partial D} \epsilon \right) d D \]  

(3)

Two types of damage strains are modeled (Fig.1). Elastic damage-induced strains \( \epsilon^{\text{id}} \) are due to the degradation of mechanical stiffness. For an increment of stress \( d \sigma \):

\[ d \epsilon^{\text{id}} = [D^{\text{id}} (D) - D^{\text{id}}_0] : d \sigma = D^{\text{id}}_0 (D) : d \sigma - d \epsilon \]  

(4)

in which \( D_0 \) is the undamaged stiffness tensor and \( \epsilon \) is the purely elastic deformation, that would be obtained in the absence of damage. The term \( g D \) contributes to residual stress due to remaining crack opening after unloading (i.e. stress that remains when deformation is released to zero). The resulting irreversible damage-induced deformation is thus equal to:

\[ \epsilon^{\text{id}} = D^{\text{id}} (D)^{-1} : g D \]  

(5)

Total deformation is thus decomposed as [11]:

\[ \epsilon = \epsilon^{\text{id}} + \epsilon^{\text{id}} + \epsilon^{\text{id}} = \epsilon^{\text{id}} + \epsilon^{\text{id}} \]  

(6)

Fig.1. Deformation decomposition for a typical loading/unloading cycle: the total elastic deformation comprises a purely elastic component \( \epsilon^{\text{id}} \) and a reversible damage-induced component \( \epsilon^{\text{id}} \).

The damaged stiffness tensor is obtained by combining Eq. (1), (3), (5) and (6):

\[ d \sigma = (D \epsilon (D) - D_0) : d \epsilon = D^{\text{id}} (D) : d \epsilon \]  

(7)

Computation details are provided in [12]. The force conjugate to damage (Y) is obtained from Helmholtz free energy:

\[ Y = - \frac{\partial W}{\partial D} = - g \epsilon - \alpha (\text{tr} \epsilon) \epsilon - 2 \beta (\epsilon \cdot \epsilon) \]  

(8)

Y is decomposed into two strain energy release terms: the residual effects term \( Y_1 \) and the recoverable energy term \( Y_2 \). The former one is further decomposed into a splitting part \( Y_1^f \) (which contributes to damage) and a non-splitting part \( Y_1^\perp \):

\[ Y = Y_1 + Y_2, Y_1 = Y_1^f + Y_1^\perp = - g \epsilon, Y_2 = - \alpha (\text{tr} \epsilon) \epsilon - 2 \beta (\epsilon \cdot \epsilon) \]

(9)

Damage is assumed to grow with tensile strains, according to the following damage criterion:

\[ f_d (Y_1^f, D) = \sqrt{\text{tr} (Y_1^f \cdot Y_1^f) + B \text{tr} (Y_1^f \cdot D) - C_0 - C_1 \text{tr} (D)} \]  

(10)

Applying the consistency conditions, \( f_d = 0 \) and \( \dot{f}_d = 0 \), we can calculate the Lagrange multiplier and the damage increment:
For instance, in a triaxial compression test, damage is characteristic time. The expression given in Chan et al.’s model of healing strain [7]. The expression (this assumption is strengthened relative to the previous damaged stiffness tensor. This means that healing can result in a partial recovery of rock strength. The frameworks of Continuum Damage Mechanics and viscoplastic heating are unified by decomposing the increment (or rate) of total deformation as:

\[
\dot{\varepsilon} = \dot{\varepsilon}_v + \dot{\varepsilon}_w + (\delta, \dot{\varepsilon})\delta
\]
3. NUMERICAL RESULTS

3.1. Finite Element Simulation of Anisotropic Damage due to Monotonic and Cyclic Loading

The mechanical anisotropic damage model described in 2.1 has been implemented in Theta-Stock Finite Element program [14]. Three laboratory tests have been simulated in order to study damage evolution in axial compression and axial tension. Cylindrical rock samples and axis-symmetric conditions are assumed, horizontal displacements are fixed on the axis of symmetry, and vertical displacements are fixed at the base of the sample. Material parameters are taken from the calibration work of Halm & Dragon [9] for Fontainebleau sandstone. Specific salt rock parameters would have to be determined in the laboratory, but the test principles would be the same. Damage trends are discussed for a confined triaxial compression test, for an axial tension test and for cyclic compression loading and unloading phases.

Table 1. Material Parameters Used in the FEM Simulations

<table>
<thead>
<tr>
<th>$\lambda$ (Pa)</th>
<th>$\mu$ (Pa)</th>
<th>$\alpha$ (Pa)</th>
<th>$\beta$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.63 \times 10^9$</td>
<td>$1.75 \times 10^9$</td>
<td>$1.9 \times 10^8$</td>
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</tr>
<tr>
<td>$g$ (Pa)</td>
<td>$C_0$ (Pa)</td>
<td>$C_1$ (Pa)</td>
<td>$B$</td>
</tr>
<tr>
<td>$1.1 \times 10^8$</td>
<td>1000</td>
<td>$5.5 \times 10^7$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Triaxial Compression Test.** An isotropic compression of 15 MPa is followed by an axial compression phase at constant radial confining pressure, in order to reach a deviatoric stress of 200MPa at the top of the sample. This is followed by an unloading stage, at the end of which the sample is subjected to a 15MPa isotropic compression (Fig.2a). During the axial compression phase, deviatoric stress generates lateral tensile strains, resulting in lateral damage. Fig.2.b. shows the amount of deviatoric stress necessary to pass the tensile strain threshold and trigger damage. Once initiated, lateral damage keeps on increasing until the unloading phase starts. Unloading decreases deviatoric stress (to zero) while damage remains constant. Fig.2a shows the damage-induced residual strain in the axial direction (irreversible sample shortening induced by crack opening).

**Uniaxial tension test.** This test is performed in one phase: a tensile stress of 14.5MPa is applied at the top of the sample (with no confining pressure). In this test, tensile deformation develops in the axial direction, and correspondingly, it is expected to produce crack planes perpendicular to the axis, i.e. axial damage and no lateral damage. Fig.3.b. shows the evolution of axial damage: initially, tension is elastic, and cracks start to appear for a vertical tension of -60kPa (tension counted negative with the soil mechanics sign convention). Fig.3.a. shows that the stress/strain curve ceases to be linear when damage is initiated.

**Axial Loading and Unloading Cycles.** A compression stress of 8MPa is applied at the top of the sample during an unconfined compression phase. This is followed by an unloading phase, at the end of which the top boundary is free of stress. The third load step consists of applying a 16MPa compression at the top of the sample – with no confining pressure. Compression is released during the fourth load step. Fig.4 shows that the axial stress/strain curve ceases to be linear for very low deviatoric stress (less than 500kPa), which means that damage initiates shortly after the beginning of the first compression stage. The curve is linear during both unloading phases, which corresponds to the expected elastic response of the material when tensile deformation is released. During the reloading phase from a 0MPa to a 16MPa compression stress, the material stays in the elastic domain until 8MPa (reversible path). Then the behavior becomes non-linear because new cracks develop due to additional compression. The slopes of the stress/strain curve during the unloading phases gives the...
damaged Young’s modulus obtained at the end of each compression stage. A refined result analysis shows that the Young’s modulus after the first compression is less than the initial (undamaged) Young’s modulus and more than the Young’s modulus after the second compression phase. This is in agreement with the observed growth of damage.

(a) Axial Compression Phase
The sample is loaded by increasing the axial strain (direction 1) with a constant strain rate. Therefore, in the incremental stress-strain relationship, \( d\sigma_2 = d\sigma_3 = 0 \) and \( d\varepsilon_1 \) are known. Because \( d\varepsilon_2 = d\varepsilon_3 \) (axis-symmetric configuration), there are only two independent scalar equations to solve in relation 19. An iterative procedure similar to Picard’s method is used to solve for \( d\sigma_1 \) and \( d\varepsilon_2 \) (and \( d\varepsilon_3 \)). The total strain for the loading increment \( k \) (noted \( \varepsilon_1^k \)) can then be updated.

(iii) Checking the Damage Condition
With the updated strain \( \varepsilon_1^k \) and the amount of damage obtained at the end of the preceding loading increment \( (D_{k-1}) \), the damage yield function can be tested:

\[
 f_D \left( \left( \varepsilon_D^k \right)_k, \left( D_{k-1} \right)_k \right) \\
 = \sqrt{\text{tr} \left( \left( Y_D^k \right)_k \cdot \left( Y_D^k \right)_k \right)} + B\text{tr} \left( \left( Y_D^k \right)_k \cdot \left( D_{k-1} \right)_k \right) - C_0 - C_1 \text{tr} \left( \left( D_{k-1} \right)_k \right)
\]

(20)

Where \( \left( Y_D^k \right)_k \) can be updated with \( \varepsilon_1^k \) from Eq. (8). The following procedure is followed:

(a) If \( f_D < 0 \), there is no damage, go back to step 2 to update stress with the damaged elastic stiffness tensor \( (D_e(D)) \),

(b) If \( f_D \geq 0 \), go to next step to calculate the damage increment by an iterative procedure.

(iv) Iterative Computation of the Damage Increment
Eq. (11) and (12) provide a way to compute the damage increment. At the first iteration, damage is computed with the deformation obtained at the end of the preceding load step. Because the stiffness at the preceding increment is expected to be less damaged than at the current increment, strains computed from the stiffness obtained at the preceding increment are

\[
d\sigma = D_e : d\varepsilon
\]
expected to be less than the strains that should be obtained at the current increment. Thus, some iterations are executed until the difference between updated variables between two iterations falls below a certain tolerance. Iterations stop when convergence is reached. In the latter case, the next loading increment (step 2) can be applied.

Fig. 5. Loading path for the simulation of damage and healing

1) Isotropic compression (A→B)  
2) Axial compression loading (B→C)  
3) Healing (waiting time, C→D)  
4) Axial unloading (D→E)

Fig. 6. Simulation of a triaxial compression test with damage and healing (soil mechanics sign convention). (a) Deviatoric stress versus axial deformation (used for test control). In the proposed model, healing can only reduce crack-induced deformation in the directions perpendicular to the crack planes – radial strains in this particular simulation. This explains why axial deformation does not evolve during the creep phase (CD). (b) Deviatoric stress versus lateral strain. Due to healing, the slope of the stress-strain curve during unloading (DE) is larger than the slope of the stress-strain curve at the end of the loading phase (BC).

Stress/strain Curves. Fig.6 shows the stress/strain curves obtained for a confining pressure of 15MPa and a maximum axial deformation of 0.526%. There is no stress difference during the isotropic compression phase (AB). The stress/strain curves show a non-linear behavior almost as soon as the axial compression phase is initiated, which corresponds to the damage growth plotted in Fig.8 (BC). In the model proposed in this paper, healing processes are assumed to compensate damaged deformation in the lateral directions only (Section 2). This explains why axial deformation does not change during the “waiting/healing stage” (CD), while some of the damage-induced deformation is compensated by healing in the lateral direction (plateau CD in Fig.6.b). A quantitative analysis of the numerical results shows that the Young’s modulus (slope of the stress/strain plot) during the unloading stage (DE) is less than the initial (undamaged) Young’s modulus, and more than the Young’s modulus that would be obtained if no waiting/healing time were simulated.

Evolution of the Lateral Strain Components (Fig.7). During the isotropic compression phase (AB), the material is elastic, and as a result, the relationship between total axial strains and total lateral strains is linear. This relationship stops to be linear during the axial compression phase (BC), when damage (and lateral damage-induced deformation) starts to develop (Fig.7.b&c). During the waiting/healing time, healing deformation compensates damaged-induced deformation in the lateral direction only, which explains the vertical line (CD) observed in the plots of the total and damaged lateral strains (Fig.7.b&c). Pure elastic deformation is not influenced by healing (Fig.7.d). During the unloading phase (DE), the material’s behavior is linear elastic, which explains the proportionality observed
between axial and lateral deformation. Note that the relationship between lateral damage-induced deformation and total axial deformation is linear, because on the unloading path, the variation of damage-induced deformation is equal to the variation of elastic damage-induced deformation: \( \dot{\varepsilon}^{el} = \dot{\varepsilon}^{ed} = 0 \). The trends of the proposed model differ from the predictions of isotropic damage models including unilateral effects (Frémont & Nedjar, 1998) or creep healing (Chan et al., 1998).

**Evolution of the internal variables** (Fig.8). During the isotropic compression phase (AB), the material is elastic, and the damage and healing variables are equal to zero. With the soil mechanics sign convention, lateral deformation becomes positive, and point B is shifted from A to the right. During the axial compression phase (BC), lateral damage develops and there is no healing. Therefore, \( A = D, \ h = 0 \). During the waiting/healing phase, healing increases and the crack density tensor ceases to grow. Due to Kuhn-Tucker consistency conditions, the eigenvalues of the crack density tensor cannot decrease, but the eigenvalues of the new damage variable (A), do decrease. During the unloading phase (DE), the material is elastic, and as a result, none of the internal variables evolves. In particular, the plots of A and D are both horizontal lines.

**Fig. 7.** Evolution of the components of total (b), inelastic (c) and purely elastic (d) lateral deformation during the simulation of a triaxial compression test with damage and healing. Plot (a) compares the orders of magnitude of the various components.

4. CONCLUSION

Damage is defined as the second-order crack density tensor and grows if tensile strains exceed a certain threshold: constitutive equations are derived in a way very similar to plasticity. Kuhn-Tucker consistency equations impose that damage cannot decrease. That is the reason why another internal variable is introduced in order to model healing and the consequent recovery effects on stiffness. The difference between the second-order crack density tensor and a scalar creep healing variable defines a new damage variable that replaces the second-order crack density tensor in the expression of the damaged stiffness tensor. In addition, a viscoplastic healing deformation is introduced in the formulation, in order to model the effects of Diffusive Mass Transfer on the reduction of damage-induced deformation. Although the healing variable is a scalar, the model can predict stiffness recovery in case of anisotropic damage. In addition, healing deformation is parallel to the crack density tensor. Contrary to existing damage and healing models, the proposed framework accounts for crack-induced anisotropy, and anisotropy is included in both damage and healing evolution laws.
Computation, extension and compression loading and loading cycles have been simulated with Theta-Stock Finite Element code. The results illustrate well the influence of anisotropic damage on stiffness degradation and residual strain development. An algorithm has also been written to study the trends of the coupled damage and healing model for a stress path comprising an isotropic compression, and axial compression, a healing period and an unloading phase. The results match the theoretical expectations, and show that the proposed model can predict anisotropic damage and healing.

The next step is to validate and calibrate the model against real experimental data. Future modeling work will also be dedicated to the coupled effects of crack opening, closure and healing, including unilateral effects and healing kinetics. Damage and healing models of rock are expected to provide a formal framework to assess the reliability of geotorage – in particular: nuclear waste disposals and carbon dioxide sequestration.

Fig. 8. Evolution of the internal variables during the simulation of a triaxial compression test with damage and healing. (a) Healing scalar variable. (b) Lateral component of the crack density tensor and of the new damage variable.

REFERENCES