On The Definition Of Damage In Time-Dependent Healing Models For Salt Rock

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Abstract. This research is motivated by the increasing need for geostorage facilities, mainly: nuclear waste disposals, high-pressure gas reservoirs and carbon dioxide sequestration systems. Salt rock has favourable creep properties, enabling crack healing in relatively low pressure and low temperature conditions. Contrary to models proposed in Continuum Damage Mechanics, creep models can predict damage increase and decrease. However, the formulation does not allow modelling time independent crack opening and the resulting anisotropy of stiffness and deformation. Moreover, a distinction needs to be made between reversible and irreversible crack-induced deformation. A compression test including a healing phase has been simulated using a model in which healing of deformation and stiffness recovery are not clearly distinguished. The results show an inconsistency between deformation and healing evolutions. To overcome this problem, an alternative modelling framework is proposed to predict anisotropic healing deformation and stiffness recovery.

damage; healing; salt rock; inelastic deformation; stiffness recovery; internal variable

INTRODUCTION

Inelastic deformation in salt at conditions of geo-storage systems is dominated by isochoric dislocation processes, Diffusive Mass Transfer (DMT), and dilatant micro-cracking. During transient creep, dislocation glide induces strain hardening. During steady creep, dislocation slip is balanced by recovery processes involving cross-slip, diffusion and recrystallization (Senseny et al., 1992). The discussion presented in this paper is restricted to low temperatures (typically, below 50°C) and short test periods, for which steady state creep strains have a negligible softening effect. Glide-induced hardening produces crystal pile-ups and stress concentrations, resulting in micro-cracking. This study aims to predict crack-induced damage and consequent healing processes occurring after transient creep.
Pouya (2000) proposed a micro-macro approach to model the impact of glide on elastoplastic deformation at low temperature. Such a multi-scale approach does not exist to predict the recovery of micro-cracking effects. Rock “healing” is defined from the way it is measured in the laboratory, at the macro-scale of a Representative Elementary Volume (REV). Because deformation induced by dislocation creep is isochoric, crack damage in salt has often been associated to inelastic dilatant deformation (Chan et al., 2001). Damage grows in stress states above the “dilatancy boundary”, whereas below this boundary, inelastic contractant strains compensate damage deformation (Hou, 2003). Within the dilatancy boundary, damage cannot grow nor decrease (Hunsche & Hampel, 1999). Salt healing has been assessed from experimental measures of permeability (e.g. Schulze, 2007) and/or inelastic strains (e.g. Lux et al., 2000). The reduction of connected porosity does not imply mechanical strength recovery (Fuenkajorn et al., 2003).

In the framework proposed by Chan et al. (1998; 2001), the inelastic strain rate is the sum of viscoplastic, damage and healing deformation rates. Each inelastic strain rate is derived from a work-conjugate stress variable. A scalar damage variable is introduced in creep kinetic equations to model material softening. The damage rate is governed by two evolution functions (one for crack opening, one for crack closure and healing). Chan et al. subjected salt samples to an axial compression to generate cracks parallel to the axis, and studied creep “healing” under hydrostatic compression. They proved experimentally that crack “healing” occurred in two stages:

1. A rapid crack closure phase, during which the closure of cracks (parallel to the axis) causes positive lateral deformation (shrinkage) and negative axial deformation (extension),

2. A slow healing phase, during which closed cracks (parallel to the axis) effectively heal, causing positive lateral deformation (shrinkage).

Two independent evolution laws are introduced:

1. The healing strain rate depends on the first stress invariant (confining effect),

2. The increment of damage is the difference between a damage function and a healing function, with the following healing rate:

\[
\dot{h} = \frac{\omega (\sigma_{eq}^h - \sigma_b)}{\tau \mu} H (\sigma_{eq}^h - \sigma_b)
\]  

(1)

in which \( H \) is Heaviside function, \( \mu \) is the shear modulus, \( \tau \) is a creep characteristic time, \( \omega \) is the scalar damage variable, \( \sigma_{eq}^h \) is the power-conjugate equivalent stress measure associated to creep healing, and \( \sigma_b \) is the compression stress defining the dilatancy boundary.

Chan et al. used the classical relationship of Continuum Damage Mechanics to correlate the bulk modulus of cracked salt to damage:

\[
K(\omega) = (1 - \omega) K_0
\]

(2)

in which \( K_0 \) stands for the undamaged bulk modulus. However, the relationship between the healing strain rate and the increment of damage was not proven theoretically. This paper aims to:

1. Determine whether it is necessary to introduce two (or more) internal variables to model the effects of healing on deformation and stiffness, or if these effects are interdependent,

2. Model the anisotropy induced by damage and healing on deformation and stiffness.
CONTINUUM DAMAGE MECHANICS FRAMEWORK

Effects of Crack Opening on Deformation and Stiffness

Two phenomenological approaches may be used to model the effects of cracking on deformation and stiffness: associated plasticity with softening, or Continuum Damage Mechanics with a scalar damage variable linked to volumetric strains (Thorel & Ghoreychi, 1996). Another option consists in coupling Continuum Damage Mechanics (to capture softening) to the framework of elasto-plasticity (to predict irreversible strains). Damage models proposed for rock generally assume that irreversible strains are purely plastic (Chiarelli et al., 2003; Conil et al., 2004; Maleki, 2004; Zhou et al., 2008), and ignore crack-induced irreversible strains \( \varepsilon_{id} \). Two independent dissipation potentials are thus required to close the formulation (one for damage, one for plasticity). If damage (often defined as the second-order density tensor \( \Omega \)) and crack-induced irreversible strains \( \varepsilon_{id} \) are independent, two evolution laws are also needed (Abu Al Rub & Voyiadjis, 2003): a damage criterion (for stiffness degradation), and a yield function (for irreversible strains). In Halm and Dragon’s model (1998), \( \varepsilon_{id} \) and \( \Omega \) are coupled. The deformation tensor is decomposed as:

\[
\varepsilon = \varepsilon^e + \varepsilon^{ed} + \varepsilon_{id} = \varepsilon^e + \varepsilon_{id}
\]

in which \( \varepsilon^e \) is the purely elastic deformation (obtained in the absence of damage), and \( \varepsilon^{ed} \) is the reversible crack-induced deformation (caused by the degradation of the stiffness tensor, cf. Figure 1).

Crack-induced deformation components can be computed as (Xu et al., 2012):

\[
\varepsilon^{ed} = \left( C(\Omega)^{-1} - C_0^{-1} \right) : \sigma
\]

\[
\varepsilon_{id} = g C(\Omega)^{-1} : \Omega
\]

in which \( C_0 \) and \( C(\Omega) \) are respectively the intact (undamaged) and damaged stiffness tensors, and \( g \) is a material parameter.

Unilateral Effects of Crack Closure

The recovery of compression strength by the closure of tensile cracks is known as “unilateral effects” in Continuum Damage Mechanics. Modelling the anisotropy induced by crack opening and closure is challenging, because the possible rotation of the principal base of damage makes classical constitutive models inconsistent (Chaboche, 1992). Indeed, most models for mechanical crack closure are either isotropic (Mazars, 1984), or restricted to mode I failure (Ortiz, 1985). Halm and Dragon’s model (1998) accounts for the recovery of the shear modulus due to friction. Bargellini et al. (2006; 2007) developed a discrete formulation, in which damage and healing directions are fixed, and only crack density can vary. In general, the unilateral recovery of stiffness induced by crack closure writes
(Chaboche, 1993):

\[
C_{\text{eff}} = C(\Omega) + \eta \sum_{k=1}^{3} H(-\varepsilon_{ik}) P^k : \left[ C_0 - C(\Omega) \right] : P^k
\]  

(6)

in which \( C_{\text{eff}} \) is the partially recovered stiffness tensor, and \( P^k \) is the fourth-order projection tensor for the direction normal to the k-th crack plane \( \left( n^k \right) : P^k = n^k \otimes n^k \otimes n^k \otimes n^k \). When \( 0 < \eta < 1 \), only a partial stiffness recovery is permitted. If \( \eta = 1 \), the diagonal coefficients of the stiffness tensor \( \left( C_{\text{dam}}(\Omega) \right) \) are equal to the diagonal coefficients of the undamaged stiffness tensor \( \left( C_{\text{int}} \right) \). This corresponds to the maximum stiffness recovery allowed by unilateral crack-closure.

**HEALING MODEL FOR ANISOTROPIC DAMAGE**

**Number Of Internal Variables**

Miao et al. (1995) modelled the anisotropic effects of damage and healing on deformation and stiffness, for both rate-dependent and rate-independent processes. Three internal variables are used: the inelastic deformation tensor, the damaged mechanical stiffness tensor, and a scalar healing variable (measuring material surface energy). The reduction of stiffness is modelled by Continuum Damage Mechanics, whereas inelastic deformation is considered plastic (or viscoplastic). In damage and healing models proposed for salt rock (Hunsche & Hampel, 1999; Hou, 2003; Chan et al., 1998), only isotropic cracking effects are accounted for, by means of volumetric viscoplastic damage and healing deformation components. The absence of “yield function” \( f_y \) (or damage/healing criterion) avoids requiring Kuhn-Tucker optimality conditions, that is to say (Hansen & Schreyer, 1994): \( df_y \leq 0 \); \( d\lambda_y \geq 0 \); \( df_y \cdot d\lambda_y = 0 \), where \( d\lambda_y \) stands for the increment of plastic (or damage/healing) multiplier. Damage and inelastic strains can increase or decrease, and there is no formal distinction between “irreversible” and “reversible” crack-induced deformation (Figure 1). If creep damage deformation is reversible (i.e. if the deformation of a damaged sample after unloading is zero), damage and crack-induced deformation are related by Equation 4 for a stress-controlled test, and by:

\[
\varepsilon^{\text{ed}} = \left( \mathbf{1} - C_0^{-1} : C(\Omega) \right) : \varepsilon
\]

(7)

for a strain-controlled test. Assuming that crack-induced deformation is reversible results in a dependence between the healing variable \( H \) used to model stiffness recovery, and the (reversible) healing deformation \( \varepsilon^{\text{he}} \):

\[
\sigma = C_e(\Omega - H) : \left( \varepsilon^{\text{ed}} + \varepsilon^{\text{ed}} + \varepsilon^{\text{he}} \right)
\]

(8)

If, as proven by Chan et al.’s experiments (1998), crack closure and healing are two different processes (“mechanism 1” and “mechanism 2”, respectively), it becomes necessary to distinguish “irreversible” crack-deformation (due to residual crack openings) from “reversible” crack deformation (due to stiffness degradation). Two independent internal variables are thus required to model rapid deformation
healing that can compensate “irreversible” crack opening, and slow healing, that can induce shrinkage in the directions perpendicular to the closed cracks. In salt rock, crack closure can occur at constant stress, even if the tensile stress at cracks tips is not entirely released. However, the characteristic time for this creep process is much less than the healing characteristic time, which, according to Chan et al. (1998), may be attributed to Diffusive Mass Transfer (DMT). Chan et al. (1998) introduced two constitutive laws to model healing effects:

1. A volumetric healing deformation component:

\[ \varepsilon_h^v = \varepsilon_v \exp \left[ -\frac{t}{\frac{3}{2} \tau} \left( I_1 - x_{10} \sigma_I \right) H \left( I_1 - x_{10} \sigma_I \right) \right] \]

in which \( \tau \) is the healing characteristic time, \( \varepsilon_v \) is the total volumetric deformation, \( I_1 \) is the first stress invariant, \( \sigma_I \) is the major principal stress, and \( x_{10} \) is a material parameter accounting for healing anisotropy. Under hydrostatic compression, a sample containing cracks parallel to direction 1 is such that \( x_{10} = 1 - \frac{\varepsilon_1}{\varepsilon_2} \).

2. The healing rate given in Equation 1, with the characteristic time:

\[ \tau = \tau_0 \exp \left[ k \varepsilon_h H \left( -\varepsilon_v \right) \right] + \tau_1 \]

in which \( k \) is a positive material parameter. When the material is in compression (\( \varepsilon_v \) positive), DMT healing time is \( \tau_0 + \tau_1 \). For highly tensile states (\( \varepsilon_v \) negative), DMT healing cannot occur and the characteristic time \( \tau \) tends to the closure characteristic time \( \tau_1 \).

A triaxial test was simulated to analyse the healing model governed by Equations 1, 8, 9 and 10. An isotropic confining phase (AB) was followed by a compression phase in direction 1 at constant confining pressure (BC), causing the development of cracks parallel to direction 1. The state of stress was maintained for various periods of time (CD), and the compression was then released (DE). The expected response is illustrated in Figure 2: the elastic compression phase ends at \( C_1 \), and the inelastic compression phase ends at \( C_2 \). At \( t = \tau = \tau_1 \) (point \( D_1 \)), irreversible crack-induced deformation is compensated by crack-closure, but stiffness is not recovered. Deformation can be negative (extension) when cracks are closed, due to the mismatch between crystals at the lips of the cracks. Between \( t = \tau = \tau_1 \) and \( t = \tau = \tau_1 + \tau_0 \) (point \( D_2 \)), stiffness is progressively recovered: the slope of the stress-strain curve during the unloading phase increases. If healing allows total recovery, the slope at \( t = \tau = \tau_1 + \tau_0 \) is equal to the slope of the undamaged compression phase (point \( D_3 \)). The test simulated was strain-controlled. It was assumed that the creep stage (CD) occurred with open cracks. Large tensile strains imposed: \( \tau = \tau_1 \) (Equation 10). For this test: \( \tau_1 = 700s \) (in Chan et al.’s experiments, the order of magnitude is \( 10s < \tau_1 < 100s \)). Figure 3 shows that the healing variable \( \varepsilon_h \) remained equal to zero during the confining (AB) and compression (BC) phases, increased linearly with time during the creep phase (CD), and remained constant during the unloading phase (DE). According to Equation 9 and Figure 2, the healing deformation should increase as the creep period (CD) increases. Figure 4 shows that the predicted lateral deformation was the same for all the creep periods tested. The
modelling framework described by equations 1, 8, 9 and 10 cannot capture the difference between deformation healing and stiffness recovery.

Proposed Modelling Framework for Anisotropic Damage and Healing

An alternative framework is proposed to predict the effects of crack closure and healing on both deformation and stiffness. Crack closure is modelled as a creep process that can fully compensate the irreversible damage deformation caused by crack opening, but cannot originate stiffness recovery. The concept is thus different from Continuum Damage Mechanics “unilateral effects”. Crack healing is defined as the mechanical recovery resulting from slow DMT processes, and implies that some damage deformation is reversible. The increment of deformation is decomposed as:

\[
\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{id} + \dot{\varepsilon}^{ed} + \dot{\varepsilon}^{eh} + \dot{\varepsilon}^{ih}
\]  

(11)

The expression of the isochoric strain rate associated to transient creep \( \dot{\varepsilon}^{eh} \) is well-known (Günther & Salzer, 2007), and will not be discussed herein. \( \dot{\varepsilon}^{ih} \) is the healing strain rate associated to stiffness recovery. As explained above, bilateral stiffness recovery is only possible if what is meant by “healing” involves “sintering”. In this paper, it is hypothesized that bilateral stiffness recovery is the consequence of what Chan et al. called “mechanism 2”. Accordingly, it is proposed to model stiffness recovery by using a DMT equation (Senseny et al., 1992) for each principal stress direction \( k=1,2,3 \):

\[
\dot{\varepsilon}^{eh}_k = C \left( \frac{\sigma_k}{E} \right)^n \exp \left( -\frac{Q}{RT} \right)
\]  

(12)

in which \( C \) and \( n \) are material parameters, \( Q \) is the diffusion activation energy, \( R \) is the constant of perfects gases, and \( T \) is temperature. The increment of healing \( \dot{H} \) can be deduced by combining Equations 8 and 12.

\( \dot{\varepsilon}^{eh} \) is the healing strain rate associated to crack closure, depending on the intensity of hydrostatic compressive stress (Chan et al., 1998). Let us suppose that a laboratory test is performed under controlled boundary conditions. The increment of total volumetric work is known and writes:

\[
\dot{W}_{vt}^{tot} = \sigma(t) : \dot{\delta}
\]  

(13)

in which \( \dot{\delta} \) is a displacement increment. The increment of elastic volumetric work writes:

\[
\dot{W}_{vt}^{E} = J \sigma(t) : (\dot{\varepsilon}^{el} + \dot{\varepsilon}^{id} + \dot{\varepsilon}^{eh})
\]  

(14)

in which \( J \) is the Jacobian of the geometric transform associated to the sample coordinate system. At this modelling stage, the evolution laws of the deformation components involved in Equation 14 are known (cf. Equations 7 & 12), which allows determining the increment of irreversible volumetric work (Thorel & Ghoreychi, 1996):

\[
\dot{W}_{vt}^{irr} = \dot{W}_{vt}^{E} - \dot{W}_{vt}^{tot} = J \sigma(t) : (\dot{\varepsilon}^{el} + \dot{\varepsilon}^{id} + \dot{\varepsilon}^{eh})
\]  

(15)

in which \( \dot{\varepsilon}^{eh} \) can be deduced by combining Equations 13-15 to a constitutive law similar to the one stated in Equation 5 for \( \varepsilon^{id} \), and to the evolution published in (Günther & Salzer, 2007) for \( \dot{\varepsilon}^{eh} \).
CONCLUSIONS

In most of the constitutive models proposed for salt rock, damage and healing are considered as isotropic creep processes. The thermodynamic framework allows damage to increase with crack opening and to decrease with crack closure and healing. However, the formulation does not make it possible to model time independent crack opening and the resulting anisotropy of stiffness and deformation. Moreover, the formal dependence between the healing rate and the increment of healing deformation is not clearly explained. To illustrate the need to distinguish between reversible and irreversible crack-induced deformation, a triaxial compression test including a creep period was simulated with a model in which healing of deformation and stiffness recovery are not clearly distinguished. The results show an inconsistency between deformation and healing evolutions. To overcome this problem, an alternative modelling framework is proposed to predict anisotropic healing deformation and stiffness recovery. The model is expected to bring new insights on the prediction of the Excavation Damaged Zone around salt caverns used for the storage of compressed air. The modelling framework is currently being extended to the coupling of damage to creep processes occurring at high temperature, in order to improve the design of nuclear waste disposals in salt rock.
REFERENCES


FIGURE CAPTIONS

Fig. 1. Decomposition of the deformation tensor: $\varepsilon^{el}$ is the purely elastic deformation, $\varepsilon^{ed}$ is the additional reversible strain induced by stiffness degradation, and $\varepsilon^{id}$ represents residual crack openings.

Fig. 2. Expected healing behaviour during the compression phase (BC) and the creep phase (CD). Elastic compression stops at $C_1$, and the compression phase ends at $C_2$. Crack closure is achieved at $D_1$, and crack healing is complete at $D_3$. At $D_2$, cracks are closed and healing is processing (partial stiffness recovery and total compensation of irreversible damage-induced deformation).

Fig. 3. Evolution of the healing variable with lateral strains for various creep periods: isotropic compression (AB), axial compression at constant lateral confining pressure (BC), creep phase at constant stress (CD), axial unloading phase at constant lateral confining pressure (DE).

Fig. 4. Evolution of lateral strains with axial deformation for various creep periods: isotropic compression (AB), axial compression at constant lateral confining pressure (BC), creep phase at constant stress (CD), axial unloading phase at constant lateral confining pressure (DE).