UNDERSTANDING THE CENTRALIZED-DECENTRALIZED ELECTRIFICATION PARADIGM

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UNDERSTANDING THE CENTRALIZED-DECENTRALIZED ELEC TRIFICATION PARADIGM

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SUMMARY

Two methodologies are presented for analyzing the choice between centralized and decentralized energy infrastructures from a least-cost perspective. The first of these develops a novel minimum spanning tree network algorithm to approximate the shortest-length network that connects a given fraction of total system population. This algorithm is used to identify high priority locations for decentralized electrification in 150 countries. The second methodology utilizes a mixed-integer programming framework to determine the least-cost combination of centralized and decentralized electricity infrastructure that is capable of serving demand throughout a given system. This methodology is demonstrated through a case study of Rwanda. The centralized-decentralized electrification paradigm is also approached from an energy security perspective, incorporating stochastic events and probabilistic parameters into a simulation model that is used to compare different development paths. The impact of explicitly modeling stochastic events as opposed to utilizing a conventional formulation is also considered. Finally, a subsidy-free lighting cost curve is developed and a model is presented to compare the costs and benefits of three different financial mechanisms that can be employed to make capital intensive energy systems more accessible to rural populations. The optimal contract is determined on the basis of utility-maximization for a range of costs to the providing agency and a comprehensive single and multi-factor sensitivity analysis is performed.
CHAPTER I
INTRODUCTION

It is currently estimated that approximately 1.2 billion people in the world do not have access to electricity and of these some 85% reside in rural areas [1]. The problem is especially pronounced in sub-Saharan Africa, where the overall electrification rate falls to 30%. The presence of electricity in a region can directly improve a wide range of services such as health care, sanitation, education and communication, greatly improving quality of life. As such, enabling effective, large-scale electrification may provide significant humanitarian benefit.

While the benefits of electricity are widely accepted, there is significant debate over the best means to carry out the electrification process [2]. Most developed countries rely on a centralized electricity generation and distribution system, however it has been asserted that developing countries face fundamentally different energy problems than those in the industrialized world [3]. In developed countries, electricity is generated at scale in large central plants and then distributed to end users through a transmission network. These networks can be expensive and in most cases take many years or decades to fully develop. Additionally, the development of a centralized infrastructure requires significant upfront investment that may commit a region to a certain development path for years to come. In developing nations where energy infrastructure is less developed, populations are more dispersed and access to capital is limited it is worth considering the conditions under which the energy needs could be met more effectively through decentralized means.

Nations or regions seeking electrification have more options today than they had 20 or 50 years ago, as decentralized generation technologies are now more cost-effective. Electricity can be generated in the backyards of remote end users through solar photovoltaic cells or wind turbines. Small scale plants can burn locally produced biomass and deliver electricity through village sized micro grids. Some regions may be able to bypass centralized technologies entirely, a concept known as leapfrogging [4].
An example of this phenomenon can be seen in today’s telecommunications industry, where traditional landline telephone infrastructure is being leapfrogged in developing countries in favor of cellular technologies.

Different electricity generation options are often compared individually on a cost basis; however, these analyses do not fully account for the specifics of a given situation in which new generation capacity is required. Such specifics may include population distribution, resource availability, geography and the energy services demanded by new consumers. It is also possible that decentralized systems may be implemented and scaled more effectively in the absence of a competent central authority. Additionally, traditional cost analysis often assumes the existence of a sophisticated electricity transmission infrastructure or includes a simple variable cost of transmission. This is generally appropriate in developed regions where such infrastructure exists, but in developing countries the required investment in transmission and distribution infrastructure may account for a large fraction of total system costs.

Chapters 2 and 3 present two methodologies for analyzing the choice between centralized and decentralized energy infrastructures from a least-cost perspective. Chapter 4 approaches the centralized-decentralized paradigm from an energy security perspective, developing a model that simulates stochastic events and probabilistic parameters to compare cost and service level of different centralized and decentralized development paths. Chapter 5 develops a subsidy-free cost of lighting curve and presents a model for determining the utility-maximizing financial contract that can be used to make capital intensive energy systems more accessible to the rural poor.

The methodology in chapter 2 utilizes a novel network expansion algorithm to closely approximate the shortest-length network that connects a given fraction of total system population. It is shown that the choice between centralized and decentralized infrastructure is largely dependent on four primary parameters: the marginal increase in network length required to serve an additional unit of population, the marginal increase in electricity consumption from connecting an additional unit of population to the centralized network, the unit cost of transmission infrastructure and the cost difference between decentralized and centralized electricity generation. A metric based on these parameters is introduced to quantify the extent to which a given population node could be
cost-effectively served by decentralized energy infrastructure and the model is applied to 150 countries around the world.

The methodology in chapter 3 takes a more deterministic approach to understanding the choice between centralized and decentralized infrastructures from a least-cost perspective. A mixed-integer programming framework is formulated and used to determine the least-cost combination of centralized and decentralized systems that is capable of serving a given level of electricity demand. This methodology is demonstrated through a case study of Rwanda. The model is also executed using various levels of data resolution and the tradeoff between increasingly detailed results and increasing computational requirements is analyzed.

Chapter 4 presents a methodology that builds on chapter 3 by analyzing the impacts of several important energy security factors that are faced in the developing world. This methodology simulates various stochastic events, including infrastructure outages and disruptions to development budgets, as well as probabilistic parameters, including electricity demand and commodity prices. The model is computationally inexpensive and is executed to compare the impact of different proposed development paths on the basis of the distribution of their outcomes, instead of the mean outcome. The effect of implementing a stochastic, as opposed to conventional, modeling formulation is also analyzed.

In chapter 5, a subsidy-free lighting cost curve for various electric and non-electric technologies is presented to provide a means of comparing these technologies on common ground. This analysis is used to better understand how subsidies that are targeted to enable rural energy service consumption can be allocated most effectively. In addition, a model is developed to compare the three different financial mechanisms that can be employed to make capital intensive energy systems more accessible to rural populations. An analysis is performed to determine the optimal contract for a given cost to the providing agency on the basis of consumer utility-maximization. A comprehensive single and multi-factor sensitivity analysis is also conducted in order to understand how variations in key parameters impact the effectiveness of each financial mechanism.
1.1 Background

1.1.1 Centralized vs. Decentralized Electrification

Developing countries seeking to increase electrification in rural areas are faced with a wide range of different technological options. These options generally fall into two broad categories, centralized systems, which rely on electricity generation at scale in large centralized facilities and subsequent distribution through a transmission network, and decentralized systems, which generate electricity at or near demand sites and do not require significant distribution networks. Typical first-order analyses of these two development paths are focused on determining the least-cost option in various situations and geographic regions. Consequently, there exists a significant body of literature that discusses the costs of various centralized and decentralized technologies for rural electrification. A subset of this work makes direct comparisons between these options while considering geographic factors such as the spatial distribution of population or demand sites. A broad overview of literature addressing the costs of centralized and decentralized electrification options is first provided before offering more detailed summaries of work that is particularly relevant to this thesis.

Many studies focus on comparisons between centralized and decentralized options that are based on the levelized costs of generation from various technologies. Singh and Singh provide a methodology for determining realistic levelized costs of electricity from solar PV systems under various financing options and draw comparisons with the cost of electricity from the grid [5]. Nouni et al. estimate the levelized cost of delivering electricity that is generated at centralized coal plants to rural areas in India and make a direct comparison to the estimated levelized costs of several distributed technologies [6]. A study in Cameroon similarly estimates the levelized costs of several distributed technologies and uses these results to calculate breakeven grid extension distances [7], while another study in Rwanda compares the costs of small solar PV systems to grid extension in three rural regions [8]. The optimal design of decentralized rural electrification systems has also been studied for general applications in developing countries [9], as well as for specific locations, such as Senegal [10, 11] and Vietnam [12]. Rural electrification has also been modeled at the country-level in India while
considering potential environmental impacts [13]. A survey based methodology in Bangladesh finds that small solar home systems are financially attractive for small rural business and households that demand lighting and entertainment services [14]. Another study in Bangladesh proposes a metric to measure the financial viability of solar home systems while also considering environmental impacts and finds that solar home systems can be attractive options when used for income generating activities [15].

Several specific studies that are of particular relevance to chapter 2 of this thesis are now discussed in more detail. Sinha and Kandpal developed an early approach to analyzing the decision between rural electrification through decentralized systems or centralized grid expansion [16]. They propose a set of equations that can be used to estimate the cost of grid expansion in India as a function of distance from nearest 33 kV line and peak demand levels. These costs are then compared to various decentralized options and it is determined that decentralized technologies could be cost-effectively implemented in isolated villages with low load factors. Their cost data are likely somewhat outdated but the methodology provides excellent guidance and inspiration for future work.

Lambert et al. present a computational tool that employs a combinatorial optimization routine to determine a near optimal autonomous power system given a set of demand points [17]. They do not perform a direct comparison of grid extension and off-grid power systems, but rather provide a means for estimating the cost of autonomous systems. They use a two level optimization procedure and two different optimization algorithms: a minimum spanning tree algorithm based on Prim [18] and a simulated annealing algorithm in which random changes are made to the optimization variables. Their analysis is focused on village level systems where each node must be supplied with power from either an isolated power source, such as a solar PV unit, or connection to a centralized grid. They proceed to compare the efficacy and computational efficiency of several different algorithms and determine that the simulated annealing algorithm has several advantages over the others used.

Kaijuka discusses the use of GIS for rural electricity planning and tries to identify patterns of demand areas for priority investment [19]. The author uses a GIS database to locate residences and health, education and government centers that require
electrification in Uganda. These locations are then allocated “benefit points” to rank their importance in terms of social value, willingness to pay for electricity, future consumption and long term sustainability. These structures are aggregated on the sub-county level and the total benefit points in each sub-county are used to provide an approximation of the importance of providing electricity to that area. The results are largely qualitative in nature and no optimization algorithms are executed to determine optimal electrification paths.

Parshall et al. develop a spatial planning model to facilitate grid expansion in areas with low existing coverage [20]. They apply their methodology to a case study of Kenya and determine that, in most circumstances, grid connection is less costly than off-grid options. They consider two categories of grid expansion, extension of the medium voltage “backbone” and connection to the medium voltage backbone with low voltage lines. They find that in regions with a medium voltage backbone, grid extension is often cheaper than off-grid options. In regions without an existing medium voltage backbone, they calculate the critical length of medium voltage line such that the overall cost of grid extension equals the cost of decentralized options. They utilize a combinatorial algorithm to find the least-cost electricity network given existing lines and demand points. They compare their algorithm to that presented by Lambert et al. [17] and conclude that the Lambert approach is more detailed, but computationally expensive, and therefore better suited for smaller, sub-national areas. The method is applied to a 10km by10km test region and a case study of Kenya. An average cost of $1900 per household connected is estimated under the realistic penetration scenario in Kenya.

In a companion paper to Parshall et al. [20], Zvoleff et al. [21] present a quantitative model to address the question of how population settlement patterns affect the cost of electricity transmission infrastructure. The model is implemented at the village level and applied to nine villages in Africa based on structure data obtained from satellite imagery. Two comparative indices are developed, a homogeneity index and a near neighbor index. An algorithm is developed to minimize the mean interconnection distance subject to the requirement that a given penetration rate is achieved. In the case of 100% penetration there is a unique solution to this problem that can be found relatively easily with Prim’s algorithm [18]. For a connection rate of less than 100% however, the
solution becomes dependent on the starting node and no computationally efficient algorithm exists to find a unique solution. The authors develop a composite algorithm to address this optimization problem and identify a near-optimal solution. A comparison of the composite Prim’s algorithm (CPA) with a more rigorous algorithm that has been used to address a problem known as the k-MST problem in computer science literature is also presented [22]. Results show that the CPA is much more easily implemented and the results match closely. The CPA is then applied to the nine villages and the mean interconnection distance is plotted as a function of penetration rate for each.

Deichmann et al. present a spatial modeling framework that is used to determine where “stand-alone renewable generation is a cost-effective alternative to centralized grid supply” [23]. A network algorithm is developed to determine optimal locations for substations or bulk supply points (BSPs) on the HV transmission grid. These BSPs are assumed to serve all demand points within 120km, which is the typical range of an 11kV or 33kV distribution line. Existing power stations are considered by the model, but the existing grid is not incorporated. The costs of grid expansion are compared to the costs of providing electricity through single-household and micro-grid systems considering technologies such as, wind, solar PV, diesel and bio-diesel generators. The levelized costs of electricity are estimated for each of these technologies and the methodology is applied to case studies of Ethiopia, Ghana and Kenya. They find that decentralized renewable technologies will likely play a significant role in rural electrification efforts in Sub-Saharan Africa, but they will not provide a universal solution. Even with anticipated decreases in the cost of distributed systems, electricity from the centralized grid will still provide the most cost-effective option in populated areas.

Chapter 2 of this thesis builds upon previous work by proposing a new algorithm for network expansion that considers weighted-nodes and has the objective of determining the shortest length network that connects a given fraction of the total weight in the system.

1.1.1.1 Generation expansion planning

Chapter 3 of this thesis complements chapter 2 by developing a mixed-integer programming framework to determine optimal electricity infrastructure development
strategies, while explicitly considering existing transmission and generation infrastructure. This approach is an example of the widely researched Generation Expansion Planning (GEP) problem, which can refer to electricity infrastructure planning for a range of geographic and temporal scales and with the application of various secondary objectives and constraints, e.g. environmental or regulatory. The GEP literature is too extensive to list comprehensively, but some relevant examples include: the application of a two-stage stochastic mixed-integer programming framework by Jin et al. to model GEP by optimizing both cost and risk of unexpected operational costs [24], a decomposition of the GEP problem faced by utilities into hourly sub-problems [25], a game theoretic analysis of GEP in markets with unbundled electricity generation and transmission [26], an application of GEP that considers power plant location selection [27] and an application of GEP specific to distributed generation and clean technologies [28]. Several studies have modeled stochastic events to explicitly consider system reliability as a secondary objective in GEP [29–31]. Variations on well-known algorithms have been proposed by Park et al. to solve the GEP problem through a hybrid approach [32] and by Villumsen and Philpott in the case of switchable transmission elements [33]. Mixed-integer programming frameworks have also been applied by Smith and Mesa to analyze the tradeoff between various rural electrification objectives in Colombia [34] and by Fan et al. to analyze power grid islanding [35]. Foley et al. review several proprietary energy modeling software tools that are being used to analyze increasingly liberalized markets and new renewable and emissions policy targets [36].

The methodology presented in Chapter 3 balances the problem of providing a country-level analysis of electricity infrastructure development while maintaining a computationally manageable problem space. To this end, the model does not explicitly manage time-dependent power flow through the system, but rather focuses on ensuring that sufficient infrastructure capacity is developed to meet both average and peak electricity demand levels. A mixed-integer program is formulated and solved for a range of scenarios to broadly examine how the choice between centralized and decentralized infrastructures is impacted by changes to sensitive parameters.

Chapter 4 provides a further extension of this framework that is used to simulate stochastic events and explore the potential outcomes of different development paths. It
quickly becomes computationally difficult to obtain solutions to the GEP when stochastic parameters are considered. Previous work addressing stochastic events in GEP has generally focused on developing algorithms or methodologies to solve dynamic programming problems explicitly. These formulations may have one or more objectives, which generally include minimizing cost, power outages or various negative environmental impacts. Some examples include: a formulation of GEP as an optimal control problem where load fluctuations and plant outages are random parameters that follow a Gaussian distribution [37], a new heuristic for stochastic programming in GEP over a 20-year time horizon [38], a stochastic programming model for generation expansion with random demand, generation availability, transmission capacity and probabilistic constraints [39], a GEP methodology that incorporates components of interval linear programming and fuzzy linear programming [40], a dynamic programming framework that balances minimization of cost and environmental impact [41] and a model that combines stochastic dynamic programming and game theory to understand the impacts of regulatory uncertainty on wind power expansion [42]. Tekiner et al. propose a multi-period model that applies Monte Carlo simulation to balance multiple objectives in GEP [43], while Pereira and Saraiva use genetic algorithms to model actions of several generation agents in the face of uncertain demand and costs [44]. Numerous optimization frameworks have been proposed to incorporate parameter uncertainty with applications to energy systems management [45–50].

The analysis in chapter 4 focuses on understanding how the energy security and system reliability of different infrastructure development paths are affected by stochastic events and parameters. Of primary interest are the impacts of long construction horizons faced by centralized infrastructures, variable commodity prices, random transmission generation and transmission outages and budget instability. In contrast to many other studies that develop optimization frameworks to endogenously determine optimal development paths, this methodology simulates exogenously defined development plans to generate a distribution of results for various comparative metrics, such as cost and level of served electricity demand. Rather than using aggregated metrics and indicators to quantify the various different dimensions of grid security, we focus directly on the level of enabled electricity consumption and the cost of providing these services.
1.1.2 Energy Services and Benefits

When analyzing the potential impact of different development plans, it is important to remember that energy itself is not inherently beneficial. Rather, benefit is derived from the energy services (lighting, cooking, entertainment, refrigeration etc.) that are enabled by energy access. This concept was first pioneered by Goldemberg and Johansson [4] and continues to receive significant attention today as researchers attempt to quantify the impact of various potential development strategies. A study of the relationship between electricity access and socio-economic development in Mozambique found that rural electrification can be a powerful tool in enabling structural transformation [51]. Similar studies find electrification rate to be an explanatory variable of literacy rate in the Indian state of Assam [52], evaluate the effectiveness of rural electrification in reducing energy poverty in Brazil [53], find that solar home systems provide significant direct and indirect benefits to users in rural Bangladesh [54] and determine that access to electricity enables significant increases in productivity and income for micro-enterprises in rural Kenya [55]. In Namibia it was determined that the socio-economic impacts of electrification through the grid and through solar home systems were comparable and the authors propose that subsidies for solar home systems are made more equitable with subsidies for electricity from the grid [56]. A broader study finds a positive correlation between the UN human development index and per capita energy consumption for 120 different countries and determines that small increases in energy access can enable large gains in development for the world’s poor [57]. Other work reviews the role of infrastructure in economic growth and assesses the impact of electrification on income generation in rural areas [58], suggests that electrification is a necessary but insufficient condition for economic development [59] and that in practice solar home systems often fail to live up to their promises of cost-effectiveness and poverty alleviation [60].

The perspective of understanding the full range of benefits enabled by various technologies and development strategies provides motivation for chapters 4 and 5 of this thesis, where development strategies are evaluated on factors beyond least-cost.
1.1.3 Energy Security

The concept of energy security is gaining popularity as a means of identifying and quantifying the diverse technological, economic and political factors that facilitate the provision of energy services. In industrialized nations, energy security usually refers to ensuring a stable supply of imported fuels for energy generation in the face of political embargos, fuel price and supply shocks, natural disasters and terrorist attacks. In developing countries a number of other factors may influence the ability to maintain a reliable supply of energy services, such as unstable political transitions, corrupt government officials, technical limitations, natural resource constraints and system sabotage due to civil unrest or war.

There is much debate as to the precise definition of energy security and numerous methodologies and indicators for assessing energy security have been proposed. It has been argued that energy security can take on different dimensions depending on geography, timeframe and the energy source being considered [61]. Kruyt et al. provide an overview of available energy security indicators, which are classified into four categories: availability, accessibility, affordability and acceptability [62]. Vivoda argues that a more comprehensive definition of energy security is necessary and proposes 11 dimensions and 44 attributes to evaluate energy security in the Asia-Pacific region [63]. Sovacool provides a response to Vivoda, supporting his work, but also proposing additional energy security dimensions [64]. Sovacool and Mukherjee have also proposed five main dimensions of energy security availability: affordability, technology development, sustainability and regulation and offer 320 simple indicators that policy makers can use to evaluate energy security at the national level [65]. In a separate paper this framework is used to develop a comprehensive index that is applied to evaluate energy security in 18 countries [66]. Cherp provides a critique of Sovacool, citing the lack of context in his analysis and stressing a need to discriminate between primary and secondary energy security concerns [67]. Sovacool and Brown also calculate an original energy security index for 22 countries and conduct a more detailed analysis of energy security performance in Denmark, Japan, the United States and Spain [68]. Other studies suggest classifying energy security indicators as either ex-post or ex-ante [69], propose two new approaches for measuring long-term energy services security [70] and measure
the cost of energy security for four energy sources in South Korea [71]. A broad review of different characterizations of energy security provides a summary definition as “the continuity of energy supplies relative to demand” [72]. Additional work has focused on understanding perceptions of and attitudes towards energy security issues through global surveys [73, 74].

Power outages are common throughout the developing world and supply lines distributing fuel for cooking and lighting to unelectrified rural areas are often arduous and can easily become disrupted. The unreliability of centralized electricity networks in many developing nations leads consumers to rely heavily on personal diesel generators, which are costly, noisy, inefficient and have negative environmental impacts [75]. A survey of small enterprises in sub-Saharan Africa found that 49% of respondents identified electricity reliability to be a major business constraint and 44% owned onsite diesel generators [75]. Across the region, generators account for 13% of surveyed firm’s electricity consumption, with this proportion exceeding 50% in Chad, Guinea-Bissau and Liberia.

Many efforts have been made to estimate the economic impact of power outages, with results varying significantly in different locations or applications. Beenstock provides a theoretical framework for treating generators as insurance against power outages and inferring the cost of outages from revealed preferences [76]. Tishler identifies four factors that contribute to outage costs in the commercial and industrial sectors: forgone profit, reduction in productivity, damage and labor payments [77]. Baarsma and Hop find that outage costs are generally higher in developed countries, as there are fewer switching options, and they estimate the cost of a one hour outage in the Netherlands to be €5.00 for residential customers and €52.30 for industrial customers [78]. Nooij et al. also estimate outage costs for different sectors, times of day and regions in the Netherlands, with averaged results ranging from €2.70/kWh on weekday nights to €12.50/kWh on weekend evenings [79]. In a complementary paper, these data are used to determine efficient electricity rationing strategies to be applied when demand exceeds supply [80]. A review of 21 studies estimating outage costs in private households, primarily in the developed world found data to vary wildly, with a mean cost of €9.39/kWh and a standard deviation of €14.72/kWh [81]. In the developing world,
outages costs in the industrial sector in Pakistan have been estimated to be approximately 6.67 Rs./kWh for planned outages and 11.73 Rs./kWh for unplanned outages in 1986 [82]. This corresponds to $0.54/kWh and $0.96/kWh respectively in present dollars when adjusted for inflation. Bose et al. propose three different methodologies for estimating outage costs in the agricultural and industrial sectors in India, with results ranging from 2.63 Rs./kWh to 22.10 Rs./kWh, or $.08/kWh and $0.68/kWh respectively in present dollars when adjusted for inflation [83]. A methodology has also been proposed to consider outage costs in setting electricity tariffs in Chile [84].

Chapter 4 presents a model to understand how energy security is enabled or inhibited by various infrastructure development paths in the developing world.

1.1.4 Energy Service Delivery Mechanisms

There are two primary dimensions of increasing access to energy services, technical and institutional. The technical dimension refers to the specific technologies that are used to provide energy services, for example lighting can be provided by kerosene lanterns, LED bulbs powered by a small rooftop SHS or incandescent bulbs powered by a central electric grid. The institutional dimension refers to the specific institutional policies, business practices and financial mechanisms that are employed to increase access to energy services. Examples include direct donations, subsidies, equipment rental or leasing programs and microfinance programs.

1.1.4.1 Solar Home Systems

Small solar home systems (SHS) are rapidly becoming the most popular technology for distributed rural electrification in many parts of the world because they forego reliance on electricity provision from a central authority and are easily scalable. The distributed generation technology considered in chapters 3 and 4 is modeled after a small solar home system with peak generation capacity of approximately 50W. Chapter 5 focuses specifically on understanding the economics of solar home systems and the institutional mechanisms that can be employed to aid their penetration.

A number of studies have analyzed the financial viability or socio-economic impacts of SHS in various parts of the world, including Bangladesh [14, 15, 54, 85, 86]
and India and Sri Lanka [59, 87, 88]. Other studies examine factors that have impeded the penetration of SHS [58, 89–92], analyze electricity consumption levels for SHS users [93, 94] or discuss user expectations, education and experiences regarding new SHS installations [60, 95–97]. Additional studies analyze the institutional dynamics of SHS by comparing the relative effectiveness of market versus donor-based SHS programs in El Salvador [98] and examining problems faced by public-private partnerships in Africa [99]. A set of quantitative quality of life indicators has been proposed to better understand the socio-economic impacts of SHS development [100]. Studies have also examined the effectiveness of solar electrification in increasing economic productivity of rural micro-enterprises [101] and alleviating poverty [102] in Ghana.

### 1.1.4.2 Financial Delivery Mechanisms

While solar home systems are an increasingly popular option for rural electrification in developing countries, the cost of these systems has prevented them from reaching higher levels of penetration. A problem is particularly posed by the fact that SHS require significant upfront capital investment, whereas electricity from centralized sources can generally be paid for on a marginal basis. In many regions, poor populations simply do not have access to the capital required to obtain a SHS. Chapter 5 considers three specific financial mechanisms that can be used to overcome the large capital requirement for obtaining an SHS, thereby increasing access to the poor. These are technology subsidies, rental contracts and microloan programs. For overviews of different institutional strategies that have been employed to promote rural electrification around the world see Palit and Chaurey [103] and Zerreffi [104]. Miller also discusses lessons learnt from early attempts by the World Bank to provide large-scale loans to support SHS dissemination in India, Sri Lanka and Indonesia [105].

**Subsidies**

Many governments provide electricity and fuel subsidies to increase energy access for low-income populations. There is evidence that these subsidies are ineffective [106–108], negatively distort markets [109] and in many cases disproportionately benefit wealthier populations [110, 111]. Solomon and Georgianna present a theorem for
determining the optimal subsidy level for new energy sources, find strong evidence to support offsetting subsidies on future energy sources [112].

Some studies discuss the option of replacing inefficient subsidies with other welfare enhancing programs in Iran [113] and India [114]. Other studies examine the role of energy subsidies in technology selection for irrigation pumping in rural India [115], or bridging the gap between cost and affordability of rural electrification in Nepal [116], the impact of energy subsidies on demand and emissions in China [117] and the effect of electricity industry reform on cross-subsidization in liberalizing markets around the world [118]. It has also been found that subsidies in the form of foreign aid are only useful in countries with functional institutional support and sound policies [119].

**Rental**

Rental, or fee-for-service, programs have been employed in a number of countries around the world to increase rural electricity access. These programs can be implemented either by enabling rural entrepreneurs to operate as small-scale utilities and sell fixed quantities of electricity to consumers, or by providing systems directly to end users in exchange for a periodic payment.

To date, most research of fee-for-service contracts involves analyses of empirical observations and results. Studies have asserted the general need for increased rural cooperatives [120] and innovative financing models [121] for rural energy development. Lemaire provides two analyses of fee-for-service programs in South Africa [122] and Zambia [123]. The first contrasts benefits and limitations of the South African program with micro-credit models that have been instituted in other parts of the world, while the second focuses on the financial performance of three particular fee-for-service enterprises. Other studies suggest that the fee-for-service model has great promise for implementation in Laos [124] and discuss the relative success of small electricity cooperatives in rural Tanzania [125] and Bangladesh [126] and the major flaws encountered by similar efforts in Fiji [127].

**Microfinance**

The growing popularity of microfinance has seen a similar growth in academic literature devoted to understanding the mechanics of various microfinance strategies and
its effectiveness as a tool for poverty alleviation. This literature often seeks to determine how effective microfinance is at increasing the wealth or standard of living of participants, generally finding positive impacts [128–132].

Stiglitz examines how peer monitoring transfers risk from the bank to the cosigner and improves borrower welfare [133], while Morduch challenges the often promised win-win nature of microlending [134]. Other studies examine the market dynamics of pro-poor versus pro-profit microfinance institutions [135], the role of subsidies in the early success of microfinance institutions [136] and the trade-off between outreach and financial sustainability faced by microfinance institutions (MFIs) [137]. Hoff and Stiglitz develop a theoretical economic model of microfinance market dynamics and conclude that subsidies may have the perverse effect of increasing prices due to increased competition and the costs of information sharing [138]. Morduch argues that poorly designed subsidies can both hurt the performance of microlenders and undermine social objectives [139]. Alternatively, Hudon and Traca find that subsidies up to a certain amount can positively impact MFI efficiency, however subsidies in excess of this threshold begin to have negative effects [140]. Srinivasan argues that interest subsidies offered for loans to purchase solar systems in India are superior to direct government technology subsidies [141]. Stiglitz and Weiss also discuss the market inefficiencies that can arise as a consequence of market interventions in credit markets, such as subsidization and credit rationing [142].

A number of studies analyze the interaction between formal and informal credit finding that formal lending may be less suitable for the agricultural sector [143], the availability of cheap formal credit can worsen the terms of informal lending contracts [144], government subsidies to formal lenders may reduce profits for both formal and informal lenders [145], and that competition between formal and informal lenders can lead to information asymmetries and may result in poor clients being worse off [146]. Aleem finds that the rates offered by informal lenders in Pakistan are less than their average costs, but exceed their marginal costs, which supports the premise that the informal credit market operates under monopolistic competition and imperfect information [147]. Besley outlines a number of key factors that are specific to, or more pronounced in, rural credit markets and may cause market failures. These include lack of
collateral, under developed complementary markets, costly enforcement, asymmetrical information and correlated default risks [148]. For a comprehensive overview of the economic theory and real-world practice of microfinance see Armendariz and Morduch [149].

Chapter 5 presents a methodology to quantitatively compare the cost-effectiveness of these three institutional mechanisms in promoting energy access and increasing social welfare.
CHAPTER II
LEAST COST NETWORK EVALUATION OF CENTRALIZED AND DECENTRALIZED CONTRIBUTIONS TO GLOBAL ELECTRIFICATION

2.1 Introduction
In the energy sector, it may be that some combination of centralized and decentralized technologies could best serve a previously un-electrified country or region. Some work has been done to analyze optimal electrification efforts at a local or regional scale [17, 150], including a methodology to estimate the cost of a local-level electricity distribution network based on network expansion algorithms [21]. Work at the national scale tends to focus on policy implementation to aid development [151, 152], though a methodology based on network algorithms has also been applied to case study analyses of Ethiopia, Ghana and Kenya [23]. Some studies attempt to identify priority locations for new electrification [19] and specifically decentralized electrification [6], while others attempt to analyze the decision paradigm between centralized and decentralized generation infrastructure more generally [20].

We present an electricity grid expansion model based on algorithms that solve the well-known minimum spanning tree (MST) problem found in computer science, graph theory and other network applications [153]. Our model builds upon such algorithms to determine a near minimum-length network that connects a desired percentage of the total population in a given region. We apply our methodology to the development and planning of high-voltage electricity transmission infrastructure at the country level. However, the model could also easily be applied to smaller scale systems if appropriate cost and population data were obtained.

In this study we develop a general methodology to estimate the costs of centralized electricity generation infrastructure, which can be compared to the costs of a comparable decentralized electricity generation infrastructure. We then analyze the cost-benefit tradeoff between the low cost of centralized generation and the high cost of its distribution and the relatively higher cost of decentralized generation but lower cost of distribution. This information is used to determine the conditions under which a region
might be most cost-effectively served by centralized or decentralized electrification options. Our model is applied to 150 countries around the world and the results are aggregated and presented along with three case study analyses of the choices between centralized and decentralized electrification in Botswana, Uganda and Bangladesh.

2.2 Methodology

Given a region with discrete weighted nodes, it is desirable to determine the shortest length network that can connect a given percentage of the total weight. We present a two-step algorithm for determining a near minimum-length network that meets this criterion.

2.2.1 Graph Theory

In the context of mathematics and computer science, graph theory is the study of the pair-wise relations between objects in different sets or groups. More formally, a graph G is defined as a collection of vertices V (also called nodes) and edges E, where an edge connects two vertices in G. A fundamental problem in graph theory is determining the minimum spanning tree (MST) of a given graph, or the subset of edges that connect all of the vertices in the graph into a single continuous network with the shortest possible length.

2.2.2 MST Algorithms

Prim’s algorithm is a well known computational algorithm that can be used to solve the MST problem [18]. Prim’s algorithm is useful for a wide range of applications, but it only guarantees an optimal solution for a tree spanning all of the nodes in a given set. In some cases it may be desirable to find the shortest tree that spans only a certain number of the nodes in a set. This is referred to as the k-MST problem, which is known to be NP-hard [154] and there is no known computationally efficient algorithm capable of solving the problem. An alternative approximation heuristic has been presented specifically for applications in structure level electrical grid expansion [21]. This is referred to as the Composite Prim’s Algorithm (CPA). In the CPA the original Prim’s Algorithm is executed until a desired percentage of the nodes have been connected, at which point the algorithm is truncated.
It is also interesting to consider situations in which nodes have associated weights, so that the inclusion of some nodes in the connected network is valued more highly than the inclusion of others. This class of problems was introduced as the Node Weighted Steiner Tree Problem (NWST) [155] and has been expanded as the Prize Collecting Steiner Tree Problem (PCST) [156]. A PCST is the profit-maximizing connected sub-graph of an edge- and node-weighted graph, where each edge has an associated cost and each node has an associated benefit or weight.

### 2.2.3 Weighted Composite Prim’s Algorithm

We address a problem that varies slightly from the traditional PCST and is applicable to the development of electricity transmission networks. In this problem the weight or ‘prize’ associated with each node represents its population and our goal is to find the minimum length network that connects a given portion of the total population of the entire graph or region. Whereas the traditional PCST determines the total connected weight endogenously as a consequence of profit maximization, we exogenously provide a target connected fraction and seek to minimize the cost of achieving that connection level. Additionally, while a traditional Steiner Tree allows for intermediate edges and nodes to be added to the graph, we do not consider such connections. This reduces the computational intensity of the algorithm and, given the relatively dense nature of the node data used, does not significantly affect the results.

A two-phase algorithm for a near-minimum length network connecting a given percentage of a set of weighted nodes is now presented. This will be referred to as the Weighted Composite Prim’s Algorithm (WCPA).

In the first phase, a starting node is chosen and the weight-to-distance ratio is calculated between the starting node and all remaining unconnected nodes. In practice the algorithm generally performs most effectively when the initial node is chosen to be one of the most heavily weighted. A connection is then made between the starting node and the node for which the greatest weight-to-distance ratio exists and this node is brought into the connected set. In the context of grid expansion applications, this provides the most efficient means of providing grid access on a cost per person basis. This process is continued similarly to Prim’s Algorithm; each time the weight-to-distance
ratio is calculated between all nodes in the connected set and all nodes in the unconnected set. When a desired percentage of the total weight has entered the connected set, the first phase of the algorithm is terminated.

In the second phase, the nodes that were connected by the network in the first phase are isolated and Prim’s Algorithm is executed on only these connected nodes. This process eliminates inefficient ‘doubling back’ effects that often result from the first phase of the algorithm. The result of the two-phase algorithm is the MST which is guaranteed to optimally span the nodes selected in the first phase. As the generated network depends on the choice of starting node in the first phase, the algorithm can be executed multiple times from different initial nodes, and the minimum length network from all these executions can be selected. This algorithm can be expressed as follows.

1. Given a non-empty graph G with vertices V and edges E, where V’ and E’ are the sets of connected vertices and edges, $D_{ij}$ is the length of edge (i,j) and W(k) is the weight of node k.
2. Choose desired percentage of total system weight to be connected, r.
3. Choose an initial node x so that $V’=\{x\}$ $E’=\{\}$
4. Let $\{i’,j’\}$ be the pair that solves $\{i’,j’\} = \text{argmax}\{\frac{W(j)}{D_{ij}}, i’ \in V’, j’ \notin V’\}$
5. Add $j’$ to $V’$ and $\{i’,j’\}$ to $E’$
6. Repeat steps 4-5 until $\sum_{k \in V'} W(k) \geq r \cdot \sum_{k \in V} W(k)$
7. Execute Prim’s Algorithm to completion over the set of vertices $V_p=\{V’\}$
Figure 2.1: The first phase of the WCPA is applied to a new network which is illustrated with its associated node weights and intermodal distances in a. In b. an initial node is chosen and the weight to distance ratios between it and all adjacent nodes are calculated. In c. the connection with the largest such ratio is made and a new node is brought into the connected set making the total connected population 16. This process is repeated in d. at which point the total connected population reaches the target of 20 and the first phase is terminated with a network length of 8.
Figure 2.2: In the second phase of the WCPA, Prim’s Algorithm is applied to the isolated set of nodes that were connected in the first phase, which are depicted in a. In b, an initial node is chosen and the distances between it and all remaining nodes are calculated. In c, the shortest of these connections is made and a new node is brought into the connected set. This process is repeated in d, at which point the isolated nodes are spanned by a network of length 6.

2.3 Cost Calculation

We now develop a metric to assess the relative feasibility of decentralized electrification in a given population node. The metric, which is referred to as $y^*$, is the breakeven electricity consumption level at which a node is economically indifferent between centralized and decentralized electrification. A high $y^*$ value implies that a node is relatively well-suited for decentralized infrastructure. We assume that decentralized generation is more costly than centralized generation before transmission costs are considered. In situations where the cost of centralized generation without transmission surpasses the cost of decentralized generation, it will of course always be optimal to provide decentralized infrastructure. The $y^*$ value in such regions can be thought of as being infinite and specific examples of this possibility will be discussed later in more detail.

Our algorithm estimates the minimum network length required to connect a set of nodes containing a given percentage of the total system weight, but it does not consider connections that may be made on a scale smaller than the node resolution. Therefore these networks should be understood to represent ‘large’ transmission or distribution lines, where ‘large’ is relative to node resolution. Decentralized electricity generation technologies typically require local transmission infrastructure to deliver electricity from
the generation source to each consumer. A similar local transmission infrastructure is necessary for centralized generation to deliver electricity from high voltage lines to end users. For the purpose of a general analysis, we assume that local transmission infrastructure required within a node will be the same for either centralized or decentralized generation; site specific applications would require refined data for these and other parameters. Here, the costs of centralized generation plus the transmission costs determined by the WCPA will be compared to the costs of decentralized generation without transmission costs.

2.3.1 Levelized Cost of Electricity

The levelized cost of electricity (LCOE) is a metric that is commonly used to encompass the full costs of electricity generation into a unit price. It is the ratio of the present value of all costs associated with electricity generation to the time-discounted, lifetime output of a generation system, as shown in Equation 2.1. For an overview of LCOE literature and a more detailed account of the levelized cost calculation used in this analysis see Borin et al. [157].

\[
    LCOE = \frac{PV[ Costs ]}{Discounted\ Electricity\ Generation} \tag{2.1}
\]

Borin et al. present a range of levelized costs for various generation technologies based on costs of recent projects in the United States. Costs range from about 6 cents/kWh for natural gas combined cycle to 11 cents/kWh for biomass. Because many of the factors that influence the cost of electricity, such as the costs and availability of fuel, labor and materials, are location dependent the LCOE of different forms of electricity generation may vary significantly across regions. Deichmann et al. [23] report estimates of the LCOE of centralized generation in Kenya, 10.70 cents/kWh, Ghana, 7.23 cents/kWh and Ethiopia, 5.80 cents/kWh. They also present estimates of the LCOE of decentralized generation options in Ethiopia, including 15.5 cents/kWh for larger wind, 25.5 cents/kWh for small wind, 25 cents/kWh for diesel and biodiesel, and about 75 cents/kWh for combined PV and wind.
Buchholz and Da Silva [158] estimated that small-scale wood-based biopower could be provided in rural Uganda for 11 cents/kWh with grid based hydropower costing 5 cents/kWh, individual solar panels costing 19 cents/kWh and distributed fossil generators costing 39 cents/kWh.

In India, the LCOE of centralized electricity generation before transmission from coal/gas thermal, hydro and nuclear have been estimated to be approximately 4 cents/kWh, 4.5 cents/kWh and 6 cents/kWh respectively [6]. LCOE values from several decentralized sources are also estimated to be approximately, 30-58 cents/kWh for small biomass, 31-50 cents/kWh for diesel generators, 10-20 cents/kWh for small hydro 63-110 cents/kWh for solar photovoltaic and 15-103 cents/kWh for small wind generators.

The choice between centralized and decentralized electricity generation does not depend on the absolute costs of electricity generation options, but rather on the difference in cost between decentralized and centralized electricity generation. We explore cost difference values ranging from 0 to 25 cents/kWh, with some examples at a cost difference of 10 cents/kWh.

### 2.3.2 Electricity Transmission

Most electricity is transmitted by alternating current (AC), though direct current (DC) is used for some transmission systems, particularly over long distances. The presence of economies of scale is evident by the decreasing average cost on a $/MW-km basis of a transmission line as capacity increases [159]. It is generally more cost efficient to utilize a smaller number of high capacity lines than a greater number of low capacity lines.

Transmission lines over 230kV are typically considered to be extra-high voltage (EHV) while lines between 33kV and 230kV are considered high voltage (HV). Medium (1kV to 33kV) and low voltage lines (<1kV) may also be used for more localized distribution.

#### 2.3.2.1 Levelized Cost of Transmission

To express grid expansion costs on the same basis as distributed electricity, we formulate the annualized levelized cost of transmission, \( C_T \), as shown in Equation 2.2.
Here one unit of output is considered to be one year of operation, so $C_T$ can be thought of as time discounted cost of operation for one year when accounting for the full lifecycle costs of the transmission infrastructure. This cost can then be compared on common ground to the cost of generating enough electricity to meet one year of demand. We use the financial parameters listed in Table 2.1, which are on par with those typically used to determine levelized costs of large scale electricity generation infrastructure development (Borin et al. 2011). Countries are divided into three categories based on their level of development and discount rates of 5%, 10% and 15% are assumed for the most developed group, middle group and least developed group respectively.

### Table 2.1: Financial parameters used for calculation of transmission costs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrying Charge</td>
<td>15%</td>
</tr>
<tr>
<td>Usable Lifetime</td>
<td>50 years</td>
</tr>
<tr>
<td>Book Life</td>
<td>30 years</td>
</tr>
<tr>
<td>Annual O&amp;M</td>
<td>1% of overnight cost</td>
</tr>
<tr>
<td>Real Discount Rate</td>
<td>5-15%</td>
</tr>
</tbody>
</table>

Depending on the discount rate, these parameters imply a $C_T$ of $136-158/km$-year for every $1000/km of overnight transmission cost, or an annual levelized cost of 13.6%-15.8% of the total overnight transmission cost. Data on transmission costs, reviewed in section 5.3, may range from roughly $50,000/km to $500,000/km and a cost of $200,000/km is used in our baseline scenario. A transmission line with an overnight cost of $200,000/km would translate to a $C_T$ of $27,000-32,000/km-year for discount rates of 5 to 15%. 

\[
C_T = \frac{PV[\text{Costs}]}{Discounted \text{ Years of Operation}} 
\]  

(2.2)
2.3.3 Total Infrastructure Costs

The WCPA is first executed for a country or region and the minimum generated network length is determined as a function of centralized fraction. These data are then supplied to the following economic analysis.

2.3.3.1 Centralized

There are two components to the cost of meeting electricity demand through centralized generation: the infrastructure cost of transmission lines, $C_T$, and the variable cost of generating electricity from centralized generation technologies, $C_C$. These can be compared as levelized costs, which are expressed in terms of cents/meter-year of transmission line and cents/kWh respectively. The total annual levelized cost of centralized generation ($L_C$) is calculated as follows. Let $X(r_C)$ be the minimum network length as calculated by the WCPA, and let $Y_C(r_C)$ be the total annual electricity consumption of the connected population. Both are functions of the fraction of the population served by centralized electricity ($r_C$).

$$L_C(r_C, C_T, C_C) = X(r_C) \cdot C_T + Y_C(r_C) \cdot C_C \quad (2.3)$$

2.3.3.2 Decentralized

The total annual levelized cost of decentralized generation ($L_D$) is calculated similarly. As discussed previously we do not consider small scale transmission costs for decentralized or centralized electricity; therefore, the total decentralized levelized cost is a function only of demand for decentralized electricity ($Y_D(r_D)$) and the levelized cost of decentralized electricity ($C_D$).

$$L_D(r_D, C_D) = Y_D(r_D) \cdot C_D \quad (2.4)$$
2.3.3.3 System Cost Minimization

Given a total electricity consumption level, \( Y_T = Y_C(r_C) + Y_D(r_C) \), the minimum system cost of an electricity infrastructure that consists of a centralized network serving \( r_C \) percent of the total population is given by Equation 2.5.

\[
\min_{0 \leq r_C \leq 1} \ [X(r_C) \cdot C_T + Y_C(r_C) \cdot C_c] + [(Y_T - Y_C(r_C)) \cdot C_D]
\]  

Equation 2.6 shows that after differentiation the problem reduces to equating the marginal costs of centralized and decentralized infrastructure at a given electrification rate.

\[
[X'(r_C) \cdot C_T + Y_C'(r_C) \cdot C_c] = [Y_C'(r_C) \cdot C_D]
\]  

The result depends on four input variables: the marginal increase in the network length \( X'(r) \) required to serve an additional unit of weight that is generated by the WCPA, the marginal increase in centralized electricity consumption from the network addition \( Y'(r_C) \), the levelized transmission overnight cost, and the cost difference between decentralized and centralized electricity generation, which we will call \( \Delta C \).

\( Y'(r_C) \) can also be expressed as the average per capita consumption of the newly connected population, \( y(r_C) \), multiplied by the number of people added to the centralized network, \( p \).

\[
[X'(r_C) \cdot C_T + y(r_C) \cdot p \cdot C_c] = [y(r_C) \cdot p \cdot C_D]
\]  

We can define the breakeven per capita electricity generation level, \( y^* \), for each centralized fraction. Solving Equation 2.7 for \( y^* \) yields the following relation.

\[
y^*(r_C) = \frac{x'(r_C)C_T}{p \Delta C}
\]
For any node, \( y^* \) is the maximum per capita electricity generation level for which a decentralized distribution network is less expensive than a centralized network. Take for example a \( y^* \) of 1000 kWh/person. This implies that if the actual per capita electricity consumption level in a node is greater than 1000 kWh/year, a centralized infrastructure will be cheaper overall. If the actual consumption level is less than 1000 kWh/year, a decentralized infrastructure will be cheaper. A higher \( y^* \) implies that a region is better-suited for decentralized electricity infrastructure.

Network connections are made in the order of their efficiency on the basis of weight added per unit length so the incremental network length required to connect an additional unit of weight will increase as the network expands. As a result, \( X(r_C) \) will be a convex function when considered on the macro scale. We also find this condition to hold in general for the empirical data generated by the WCPA and later present a method for smoothing the network length data to generate a close fitting function that is convex throughout the whole domain.

The second order condition guarantees an optimal minimum value for the system cost provided that Equation 2.9 holds.

\[
y'(r_C) < \frac{X''(r_C) \cdot c_T}{\rho \cdot \Delta c} \quad (2.9)
\]

As \( X''(r_C) > 0 \), Equation 2.9 will always hold if \( y'(r_C) \) is negative, or if per capita electricity consumption levels decrease as the network expands to less densely populated areas.

The convexity of \( X(r_C) \) implies that as the centralized network expands the incremental cost of connecting an additional person to the network will increase and decentralized electrification will become increasingly cost-effective.

### 2.3.4 Approximation of Marginal Network Length

Equation 2.8 shows that the network length function, \( X(r_C) \), is not a direct input into the economic analysis; only its derivative, \( X'(r_C) \), is required. The following curve-fitting methodology is used to smooth out small-scale fluctuations in the calculated
network length and develop a suitable approximation of $X'(r_C)$ that is positive and monotonically increasing. This serves as an input to the economic analyses. A smooth approximation of $X'(r_C)$ is found by first taking the convex hull, $H$, of $X(r_C)$, which is the minimal convex set containing $X(r_C)$.

$$H = \left\{ \sum_{i=1}^{k} \lambda_i \cdot x_i \mid x_i \in X(r_C), \lambda_i \in \mathbb{R}, \lambda_i \geq 0, \sum_{i=1}^{k} \lambda_i = 1, k = 1, 2, \ldots \right\} \quad (2.10)$$

An approximation of the derivative of the length function, $\bar{X}'(r_C)$, is then calculated at the $k$ hull points, $x_i$, directly from the piecewise linear function defined by the hull.

$$\bar{X}'(x_i) = \frac{X(x_{i+1}) - X(x_i)}{x_{i+1} - x_i} \quad i = 1, 2, \ldots, k - 1 \quad (2.11)$$

Derivative approximations for the rest of the domain are determined by linearly interpolating between hull points.

$$\bar{X}'(\alpha \cdot x_i + (1 - \alpha) \cdot x_{i+1}) = \alpha \cdot \bar{X}'(x_i) + (1 - \alpha) \cdot \bar{X}'(x_{i+1}) \quad \forall \alpha \in [0,1], \quad i = 1, 2, \ldots, k - 1 \quad (2.12)$$

We found this methodology to provide a more consistent fit to the generated network lengths across the full range of centralized fractions than a least squares approach. The process smoothes out small fluctuations in the generated network lengths and provides a continuous, positive and increasing function for $X'(r_C)$ from which $y^*$ can be calculated.

2.4 Data

2.4.1 Population

All the country specific population data used in this analysis are from the Gridded World Population Project [160]. This project maintains two separate databases; one consists of over 50,000 known population settlements while the other divides the world into regular grids and contains information on the population residing in each. Our model first uses the settlement data, with each known settlement treated as a node with
weight equal to its recorded population. All settlements in the database have a population greater than 1000 people and may be classified as either urban or rural. For grids that do not contain a recorded settlement, we use the gridded database that has a resolution of 15 arc minutes on a side, corresponding to an area of roughly $770 \text{ km}^2$ at the equator and $550 \text{ km}^2$ at 45 degrees latitude. The result is a set of nodes that are highly concentrated in populated regions but also adequately represent the distribution of population in less populated regions.

### 2.4.2 Electricity Consumption

The potential demand for electricity in areas that currently are not electrified may be influenced by a number of factors such as electricity prices, consumer income, electricity availability and reliability, as well as the presence of services that require electricity. We do not attempt to explicitly model future electricity demand throughout currently unelectrified regions. Rather, we first briefly discuss select literature relevant to the estimation of electricity demand in developing regions. We then present results covering a wide range of potential consumption levels and provide some specific results under the assumption that newly electrified regions will consume electricity at the same rate as the currently electrified population in each country. We will henceforth refer to the levels of electricity consumption per capita (ECPC) as well as electricity consumption per capita electrified (ECPCe). The ECPCe is the per capita electricity consumption level of only the population that has access to electricity, or a country’s ECPC divided by its electrification rate. Data for country level electrification rates are taken from the International Energy Agency Electricity Access Database [161]. For countries for which such data are not available, electrification rates are assumed to be equal to the regional average.

It has been estimated that 10 W of power can provide a small household with sufficient lighting for reading and other simple tasks, as well as enough electricity to power basic communication services, such as a cell phone or a radio [162]. A 10 W power output used for four hours per day corresponds to about 15 kWh of electricity consumption per year. Another estimate suggests that domestic household electricity
consumption of 75 kWh per year (8.6 W) is necessary to meet the United Nations Millennium Development Goals [162].

An analysis of rural electrification options in Uganda has assumed domestic household electricity demand of 30 kWh/month (40 W) [158] as, in 2003, electricity consumption under 30 kWh/month was subject to a significantly lower tariff [163]. Combined with the electricity needs of shared community services, such as schools and health centers, as well as commercial usage, Buchholz and De Silve estimate that this corresponds with per capita consumption of 55 kWh/year.

A summary of recent annual reports from the Zimbabwe Electricity Supply Authority (ZESA) finds that low-income consumers use approximately 50 kWh of electricity per household per month, primarily for lighting and radio usage [164]. In some rural communities in South Africa 50kWh/month of electricity are provided to low-income households free of charge [165]. Participants in a rural electrification program in Tanzania consumed approximately 35 kWh per capita in 2002 [125].

In 2008, the per capita electricity consumption in most of the developed world was greater than 5000 kWh/year and exceeded 50,000 kWh/year in Iceland [166]. Per capita electricity consumption (ECPC and ECPCe) values for all considered countries are presented in the appendix.

Our analysis is framed to calculate the level of electricity consumption for which the costs of centralized and decentralized infrastructures are equal for a given population node; as such we do not make explicit estimations of electricity demand in different regions. Rather, for each country we present the fraction of population residing in nodes which could be cost-effectively served wholly by decentralized infrastructure for a range of different consumption levels from 25 kWh/year to 25,000 kWh/year. These results do not require that consumption is constant throughout each considered country, but instead identify nodes that would be cost-effectively served by decentralized infrastructure for given consumption rates.

2.4.3 Transmission Costs

The costs of electricity transmission infrastructure depend on a number of external factors and reported or estimated prices can vary significantly on a case by case
basis. Transmission line capacity is also a function of many factors, but generally increases with increasing voltage. Capacity limitations for a specific transmission line may be dictated by thermal capacity (MW) for shorter segments or voltage drop (MW-km) for longer segments.

A regression analysis of publically available data filed to the U.S. Federal Energy Regulatory Commission (FERC) between 1994 and 2000 concluded that transmission costs per unit capacity decrease with the total nominal capacity in a roughly logarithmic fashion [159]. These results indicate that a typical new 138kV line with 100-200MW of transmission capacity would cost on average roughly $125,000-$200,000/km. Similarly, lower voltage lines with capacities of 10-25MW would cost on average roughly $15,000-$40,000/km. However, this data set also demonstrates a high level of variability between the costs of different projects with the same nominal capacity. Other data suggest the cost of a new 132kV transmission line to be $435/MW-km, or $43,500-$87,000/km, in New South Wales Australia, including land but not substation cost [167]. Data from Colombia suggest a new 138kV transmission cost of $310/MW-km, or $31,000-$62,000/km and a new 220kV transmission cost of $160/MW-km, or $48,000-$80,000/km, including land and substation costs [167].

A study that combines data from recent transmission expansion projects in Europe estimates the baseline cost of new transmission over flat land for a single 220kV line to be $242,200/km and for a single 380kV line to be $361,440/km [168]. Another study that analyzes the costs of grid expansion in the United States present cost assumptions of $63,000/km to $250,000/km for 115kV lines and $187,000/km to $1,000,000/km for 230kV lines [169].

Deichmann et al. [23] report an estimated cost of $90,000/km for 132kV transmission lines and $192,000/km for 220kV lines. These estimates are for implementation in sub-Saharan Africa and do not include the costs required for compensators, capacitors and transformers. An analysis of transmission infrastructure in West Africa presents cost estimates of several grid expansion projects that vary from approximately $90,000/km to $500,000/km, with most falling between $150,000/km and $250,000/km [170].
In the analysis that follows, we consider transmission costs ranging from $50,000 to $500,000 per km, with a number of examples at a cost of $200,000/km. This range is intended to model the potential costs of high voltage lines in the range of 66kV to 230kV.

### 2.5 Results

The WCPA is applied to analyze centralized electricity development in 150 countries. In each case the existing electricity transmission infrastructure is ignored and the algorithm is allowed to develop near minimum-length networks in 1% intervals from 0% to 100% centralized electricity infrastructure. This approach provides a data set that can be used to isolate the effect of population distribution on network expansion and the choice between centralized and decentralized generation infrastructures.

The WCPA is executed starting from each of the five most densely populated nodes in a given country; the shortest length network generated from these five iterations is used for each centralized fraction. The WCPA does not consider the possibility of using of multiple disaggregated networks with individual generation sources to connect a population. This could be achieved by dividing a region into disconnected sub-regions. Our analysis considers each country as a whole and finds a single low-length network to span each centralized fraction.

#### 2.5.1 Case Studies

We present case study analyses of centralized grid development in three countries, Botswana, Uganda and Bangladesh. These countries are chosen because of their relatively low current electrification rates and their differing population distribution profiles.

For these case studies we establish ranges of potential values for the overnight cost of transmission infrastructure and the extra cost of decentralized generation. We make assumptions for transmission costs and the extra cost of decentralized generation in the middle of these potential ranges and analyze the sensitivities of our results around these key parameters. The transmission cost parameter is taken to be constant throughout each individual network and can be thought of as the average transmission cost per kilometer.
Each analysis is presented as a 3x3 color plot matrix of the $y^*$ value in each node in a given country. Three values of the difference between centralized and decentralized generation cost, $\Delta C$, are varied along the x-axis, while three values of transmission overnight cost are varied along the y-axis. The resultant figure depicts nine potential scenarios for each combination of these two parameters.

### 2.5.1.1 Botswana

Botswana is a country in southern Africa with a population of approximately 1.5 million people and 560,000 km$^2$ of total land area, for a population density of 2.6 people/km$^2$; one of the least densely populated countries of the 150 considered by this analysis. Per capita electricity consumption is 1,282 kWh/year and currently 45% of households have regular access to electricity, implying an ECPCe of 2849 kWh/year [161, 171]. Both of these values are significantly higher than average for sub-Saharan Africa. Botswana imports large quantities of electricity from South Africa; however the country has significant coal resources and is expanding its domestic centralized generation infrastructure. Rural electrification is also underway to meet expected increases in rural demand. Diesel generators and solar photovoltaics are the most
common decentralized technologies considered for these applications.

Figure 2.3: Matrix plot of breakeven electricity consumption per capita for different overnight transmission costs and extra costs of decentralized generation in Botswana.

Figure 2.3 demonstrates how the low population density of Botswana makes much of the country relatively well suited for decentralized electrification. It can be seen that in the baseline scenario, at the center of figure 2.3, much of the western part of the country would be cost-effectively served wholly by decentralized infrastructure even for electricity consumption levels of 10,000kWh/year or greater. Exceptions can be seen in several settlements located in otherwise sparsely populated regions of the country. The most notable of these is small city of about 50,000 people called Maun, located in the north central part of the country. If this settlement consumes electricity at the same rate as the rest of the electrified population in Botswana, it would be most cost-effectively served by a centralized infrastructure even in the cases of a transmission overnight cost of $200,000/km and ΔC of 5 cents/kWh (right-most map on middle row) or a transmission overnight cost of $500,000/km and a ΔC of 10 cents/kWh (central map on top row).
2.5.1.2 Uganda

Uganda is a country in East Africa with a population of approximately 32 million people and 213,000 km\(^2\) of total land area, for a population density of 155 people/km\(^2\). The per capita electricity consumption is 60 kWh/year and currently only 9% of households have regular access to electricity, one of the lowest electrification rates in the world [161, 171]. This implies an ECPCe of roughly 667 kWh/year. Currently, only 1% of total energy consumption in Uganda is from electricity, while over 90% is derived from burning traditional wood-based fuel [172]. Growth in electricity demand is outpacing supply, which is being largely met by electricity imports. There are also significant untapped hydro resources in the country and several large hydroelectric projects have been proposed which could provide up to 1GW or more of generation capacity [173].

Figure 2.4: Matrix plot of breakeven electricity consumption per capita for different overnight transmission costs and extra costs of decentralized generation in Uganda.
Figure 2.4 shows how the relatively low electricity consumption makes much of Uganda well-suited for decentralized electrification. Under the baseline scenario in the center of figure 2.4, there are a number of regions with y* values that exceed the 689 kWh/year ECPCe level currently consumed in Uganda. However, the higher population density makes Uganda much less suited for decentralized electrification than Botswana for similar levels of electricity consumption. It can be seen that in the baseline (central) figure, there are no nodes with a y* value greater than roughly 2000 kWh/year. This implies that there are no regions that would be served wholly by decentralized technologies under the baseline scenario if per capita electricity consumption grew to levels greater than roughly 2000 kWh/year.

### 2.5.1.3 Bangladesh

Bangladesh is a country in south Asia with a population of approximately 163 million people and 144,000 km$^2$ of total land area. This makes Bangladesh the most densely populated sizable country in the world with a population density of 1136 people/km$^2$, almost three times greater than the second most densely populated, South Korea (421 people/km$^2$). Average per capita electricity consumption is 151 kWh/year and currently 41% of households have regular access to electricity, resulting in an ECPCe of 368 kWh/year [161, 171]. Bangladesh has minimal reserves of oil and coal, but significant natural gas resources that are not fully utilized. State owned electricity generation currently provides 4700 MW of capacity [174]. Many households in Bangladesh are not connected to the centralized grid and the country has seen a recent surge in small-scale decentralized generation. As of 2011, over one million homes are
powered by rooftop solar systems [175].

Figure 2.5: Matrix plot of levelized cost differential for different overnight transmission costs and electricity demand levels in Bangladesh.

Figure 2.5 demonstrates how the high population density of Bangladesh makes it less well-suited for decentralized electrification than either Botswana or Uganda. Under the baseline scenario there are few nodes that would be served by decentralized electricity for average electricity consumption levels greater than 300 kWh/year. Even though the economic analysis indicates that centralized electricity would be more cost effective than decentralized electricity, it is decentralized electricity that is growing rapidly. The expansion of rooftop solar technologies in Bangladesh has created economies of scale that allow small-scale systems to be installed in a period of days instead of years and enable these products to be financed at relatively low interest rates through the backing of microfinance organizations. The relative merits of decentralized electricity in terms of accessibility, ease of installation, and reliability are outweighing
the cost advantages of unrealized centralized generation. This situation may be repeated in other countries.

### 2.5.2 Aggregated Data

We now aggregate data from over 150,000 nodes in all 150 considered countries. Figure 2.6 depicts the $y^*$ value of each node as a function of its population under the baseline assumptions of a $200,000/km$ average transmission infrastructure overnight cost and 10 cents/kWh extra cost of decentralized generation.

![Figure 2.6: Plot of maximum per capita electricity consumption for which decentralized electricity generation would be less expensive than a centralized system, as a function of population for nodes worldwide, in the case of $200,000/km$ average transmission overnight cost of and 10 cents/kWh extra cost of decentralized generation. The relative suitability of decentralized electrification in a given node increases with increasing values of $y^*$.

Figure 2.6 shows an underlying linear behavior with negative slope in this log-log representation. This reflects the formulation of Equation 2.8, which can be rewritten as
In this example, with $C_T = 30,000/km$-yr, $\Delta C = 0.10$/kWh and, considering plausible inter-settlement distances, an incremental network length $X'$ on the order of 10-100 km, the right side of Equation 2.13 is approximately constant in the range of $3 \times 10^6 - 10^7$ kWh/yr. This basic line forms the backdrop of figure 2.6, mediated by the details of population distribution and geography that is incorporated in the network analysis of transmission lengths.

Figure 2.7 shows the percentage of population and area of each country that can be cost-effectively served wholly by decentralized systems, if those areas were to consume electricity at the same rate as the currently electrified population in each country. In reality, consumption levels in newly electrified areas will not necessarily mirror those in areas that are already electrified, and as such results for each individual country should be interpreted in context. However this assumption provides a reasonable baseline for analyzing the broader relationship between decentralized fractions and electricity consumption density. These results show a clear correlation between potential electricity consumption density, which is calculated by multiplying current ECPCe in each country by the population and dividing by the total area of the country. The plot assumes the baseline values of $200,000/km$ average transmission line costs and a 10 cents/kWh extra cost of decentralized generation.
**Figure 2.7:** The percentage of population and area that can be cost-effectively served wholly by decentralized electrification for 150 countries as a function potential electricity consumption density. Potential electricity consumption density is calculated by assuming countrywide electricity consumption levels equal to the average consumption of currently electrified populations and dividing by the country’s total area.

These plots show that there is a ‘saturation point’ of potential consumption density around 200,000 kWh/year-km$^2$ at which point decentralized percentages of both population and area drop to near zero in most cases. This reflects Equation 2.13; the metric of kWh/yr-km$^2$ can be interpreted as per-person consumption kWh/person-yr times the population density people/km$^2$. A table with decentralized electrification percentages and potential population densities for each considered country is given in appendix A.

Table 2.2 shows the 11 countries for which more than 50% of the population can be cost-effectively served by decentralized infrastructure, while table 2.3 shows the 13 countries for which more than 90% of the nodes can be cost-effectively served by decentralized infrastructure. Both tables assume a $200,000/km transmission overnight cost, 10 cents/kWh extra cost of decentralized generation and electricity consumption equal to the current consumption of the electrified population.
Table 2.2: Countries for which greater than 50% of the population can be cost-effectively served wholly by decentralized infrastructure.

<table>
<thead>
<tr>
<th>Country</th>
<th>% of Population Decentralized</th>
<th>Population Density (people/km²)</th>
<th>Electricity Consumption Per Capita (kWh/year)</th>
<th>Electrification Rate</th>
<th>Electricity Consumption Per Capita Electrified (kWh/year)</th>
<th>Potential Electricity Consumption Density (kWh/km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chad</td>
<td>91.7%</td>
<td>9</td>
<td>9</td>
<td>29%</td>
<td>31</td>
<td>268</td>
</tr>
<tr>
<td>Equatorial Guinea</td>
<td>76.3%</td>
<td>16</td>
<td>39</td>
<td>29%</td>
<td>134</td>
<td>2,139</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>76.1%</td>
<td>79</td>
<td>14</td>
<td>29%</td>
<td>48</td>
<td>3,833</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>75.8%</td>
<td>49</td>
<td>8</td>
<td>14%</td>
<td>57</td>
<td>2,807</td>
</tr>
<tr>
<td>Central African Republic</td>
<td>75.5%</td>
<td>7</td>
<td>22</td>
<td>29%</td>
<td>76</td>
<td>556</td>
</tr>
<tr>
<td>Mali</td>
<td>72.9%</td>
<td>12</td>
<td>34</td>
<td>29%</td>
<td>117</td>
<td>1,435</td>
</tr>
<tr>
<td>Guinea-Bissau</td>
<td>70.9%</td>
<td>38</td>
<td>38</td>
<td>29%</td>
<td>131</td>
<td>4,963</td>
</tr>
<tr>
<td>Somalia</td>
<td>64.9%</td>
<td>20</td>
<td>26</td>
<td>29%</td>
<td>90</td>
<td>1,828</td>
</tr>
<tr>
<td>Sudan</td>
<td>56.0%</td>
<td>15</td>
<td>76</td>
<td>31%</td>
<td>245</td>
<td>3,765</td>
</tr>
<tr>
<td>Eritrea</td>
<td>55.5%</td>
<td>44</td>
<td>38</td>
<td>32%</td>
<td>119</td>
<td>5,257</td>
</tr>
<tr>
<td>Niger</td>
<td>50.0%</td>
<td>14</td>
<td>36</td>
<td>29%</td>
<td>124</td>
<td>1,776</td>
</tr>
</tbody>
</table>

Table 2.3: Countries for which greater than 90% of the nodes can be cost-effectively served wholly by decentralized infrastructure.

<table>
<thead>
<tr>
<th>Country</th>
<th>% of Nodes Decentralized</th>
<th>Population Density (people/km²)</th>
<th>Electricity Consumption Per Capita (kWh/year)</th>
<th>Electrification Rate</th>
<th>Electricity Consumption Per Capita Electrified (kWh/year)</th>
<th>Potential Electricity Consumption Density (kWh/km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central African Republic</td>
<td>99.8%</td>
<td>7</td>
<td>22</td>
<td>29%</td>
<td>76</td>
<td>556</td>
</tr>
<tr>
<td>Chad</td>
<td>99.5%</td>
<td>9</td>
<td>9</td>
<td>29%</td>
<td>31</td>
<td>268</td>
</tr>
<tr>
<td>Mongolia</td>
<td>98.7%</td>
<td>2</td>
<td>965</td>
<td>67%</td>
<td>1,440</td>
<td>2,712</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>98.0%</td>
<td>49</td>
<td>8</td>
<td>14%</td>
<td>57</td>
<td>2,807</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>97.7%</td>
<td>79</td>
<td>14</td>
<td>29%</td>
<td>48</td>
<td>3,833</td>
</tr>
<tr>
<td>Mali</td>
<td>97.4%</td>
<td>12</td>
<td>34</td>
<td>29%</td>
<td>117</td>
<td>1,435</td>
</tr>
<tr>
<td>Mauritania</td>
<td>96.4%</td>
<td>4</td>
<td>118</td>
<td>29%</td>
<td>407</td>
<td>1,453</td>
</tr>
<tr>
<td>Guinea-Bissau</td>
<td>96.2%</td>
<td>38</td>
<td>38</td>
<td>29%</td>
<td>131</td>
<td>4,963</td>
</tr>
<tr>
<td>Somalia</td>
<td>95.1%</td>
<td>20</td>
<td>26</td>
<td>29%</td>
<td>90</td>
<td>1,828</td>
</tr>
<tr>
<td>Niger</td>
<td>95.1%</td>
<td>14</td>
<td>36</td>
<td>29%</td>
<td>124</td>
<td>1,776</td>
</tr>
<tr>
<td>Guyana</td>
<td>93.6%</td>
<td>3</td>
<td>896</td>
<td>93%</td>
<td>963</td>
<td>2,832</td>
</tr>
<tr>
<td>Sudan</td>
<td>93.5%</td>
<td>15</td>
<td>76</td>
<td>31%</td>
<td>245</td>
<td>3,765</td>
</tr>
<tr>
<td>Equatorial Guinea</td>
<td>92.5%</td>
<td>16</td>
<td>39</td>
<td>29%</td>
<td>134</td>
<td>2,139</td>
</tr>
</tbody>
</table>

Figure 2.8 shows the cumulative fraction of world population that resides in a node with a $y^*$ value at or above a certain level. For example, if the extra cost of decentralized generation were 10 cents/kWh, roughly 3-4% of world population could be cost-effectively served by decentralized infrastructure if they were to consume electricity
at a rate of 1000 kWh/person-year.

Figure 2.8: The cumulative world population residing in a node with a y* value at or above each value on the x-axis. This figure can be used to interpret the fraction of the world population that can be cost-effectively served wholly by decentralized infrastructure for a given rate of electricity consumption per capita.

Figure 2.9 shows the percentage of population that can be cost-effectively served by decentralized infrastructure in each country, if that population were to consume electricity at the country’s ECPCe. It can be seen that in most of the developed world 5% or less of the population lives in regions that would be classified as decentralized by our analysis. There is more potential for decentralized infrastructure in the developing world, particularly in Africa. This is largely due to the relatively low current levels of electricity consumption in these regions.
2.6 Conclusions

Global electrification can use both centralized and decentralized electricity generation technologies. We have shown that there are a number of countries, particularly in Africa, for which decentralized electricity may be more cost effective than centralized for a substantial fraction of the population and land area. The variables that favor decentralized electricity - low population densities, low electricity consumption, and low cost increment compared to centralized electricity – are well known. The basic structure of the relationship can be seen in Figure 2.6: as population increases there is a hyperbolic decrease in the consumption rates that are suitable for decentralized generation. However, this basic relationship is mediated by variations in population distribution and geography. Application of the network algorithm developed here provides a more detailed assessment of the regions of a country in which decentralized generation is likely to be more cost effective. This assessment is not at the level of...
detailed project planning, but does indicate the specific regions and populations for which decentralized generation may be more cost-effective. The case studies of Botswana, Uganda, and Bangladesh illustrate both the basic characteristics of low population density and low consumption favoring decentralized generation, and also the algorithm-based result of the optimal network for centralized electricity and corresponding regions for decentralized electricity.

The choice of centralized or decentralized electricity may not be decided on the basis of cost alone. Centralized generation may be developed even when more expensive than decentralized generation, potentially to ensure universal access or to promote substantial increases in electricity consumption. Decentralized generation may be developed if the central utility authority does not provide adequate or reliable electricity. Of the three case study countries, Bangladesh was the one for which decentralized electricity was least cost effective, due to the very high population densities. Yet, with centralized electricity not meeting demand, there is considerable decentralized electricity development in Bangladesh. This analysis considers only system costs and assumes competent, reliable electricity development and management, but when centralized electricity is not available or reliable, decentralized electricity systems may be preferred.

The general analysis shows that for the majority of the world’s population and area, centralized electricity, at typical current prices, is the most cost effective approach. The analysis also shows that decentralized electricity can make a substantial contribution to global electrification. There are many regions, particularly in Africa, where decentralized generation could provide electricity at lower cost than centralized approaches.
CHAPTER III
A MIXED INTEGER PROGRAMMING MODEL FOR ELECTRICITY INFRASTRUCTURE DEVELOPMENT

3.1 Introduction
The costs of different electricity generation technologies are often compared on the basis of levelized cost of electricity (LCOE), which incorporates the full lifecycle costs of a generation technology into a single metric. The literature is replete with different methodologies for performing these calculations and comparisons [157, 176–180]. Such calculations have also been specifically applied to determine the levelized costs of decentralized generation technologies, such as solar photovoltaics, under a variety of different financial assumptions [5, 181]. However, it has been argued that the levelized cost metric may be inappropriate for comparing intermittent generation technologies with varying dispatch profiles, such as wind or solar [182]. Therefore, analyses beyond simple levelized cost comparisons are likely necessary to fully evaluate the choice between generation technologies, particularly when intermittent technologies are being considered.

In developing countries with low electrification rates, it is important to consider the cost per household of new connections to a centralized electric grid [20, 21]. The cost of new transmission and distribution infrastructure can also be combined with a levelized cost of generation analysis to identify regions that may be more or less suitable for decentralized infrastructures [23, 183].

We present a methodology for determining the lowest cost electricity generation and transmission infrastructure that can serve electricity demand in a given region. Our work focuses on developing the specific details of such an infrastructure system (transmission line voltages, generation facility locations), as opposed to examining the economic feasibility of different options from a broader perspective. In contrast to some earlier studies, we apply our methodology to large-scale, country level planning involving both centralized and decentralized generation facilities and relatively high voltage transmission lines (30+ kV). However, the model could easily be used to analyze the development of smaller-scale networks as well.
A case study of electricity infrastructure development in Rwanda is presented to illustrate an application of the methodology at the country level. The infrastructure data informing the model are based on existing electricity generation and transmission facilities in Rwanda as well as potential or proposed projects. We present sensitivities around a range of different levels of average power demand and costs of decentralized technologies. Additionally, we examine how the generated network is affected by changes in the resolution of the population data, and analyze the tradeoff between more precise results and faster computation time.

Centralized electricity offers the advantage of a generally lower unit cost of generation; however centralized facilities also require relatively large generation capacities to achieve such an economy of scale. Decentralized generation generally has a greater unit cost of electricity generation, but offers the flexibility of smaller facilities that can be located close to demand sites. As such, costly high-voltage transmission networks are not required to deliver decentralized electricity to the end user. Therefore, despite the generally higher unit cost of decentralized electricity, decentralized systems may provide a cheaper overall option in some locations when considering the combined costs of generation and transmission.

The case study is primarily meant to illustrate our methodology and provide a high-level analysis of the choice between centralized and decentralized electricity infrastructure development in Rwanda. The results can be used by planners to inform high-level development strategies and policies. An analysis to be used for specific project implementation would benefit from more detailed data, specifically regarding the capacities and costs of transmission lines in Rwanda, as well as explicit modeling of time-dependent grid load.

### 3.2 Methodology

We formulate a mixed-integer program (MIP) for planning an electricity generation, transmission and distribution infrastructure. The primary objective is to determine the lowest-cost combined centralized and decentralized infrastructure that is capable of serving electricity demand in a region. The power flow in this formulation is dictated by constraints that ensure there is an energy balance at each demand node. As
such, the model does not explicitly model time-dependent demand, but rather finds the minimum-cost infrastructure that ensures there is sufficient generation and transmission capacity throughout the system to meet both average and peak demand levels. This formulation was chosen to minimize computational requirements while also ensuring that sufficient infrastructure is available to serve demand. A linear direct current (DC) approximation of alternating current (AC) power flow could also be employed at the cost of some additional computational requirements.

Power generation facilities are characterized by peak generation capacity as well as capacity factors that dictate average generation levels. Constraints require that total generation capacity is sufficient to satisfy both peak power demand and system-wide yearly energy demand. The required transmission infrastructure is largely independent of load shape, and is rather dictated primarily by the peak load level. The binding transmission constraint requires that there is sufficient capacity to transmit peak levels of power demand. The locations of centralized generation facilities are also considered.

The economics of centralized and decentralized options are compared on the basis of the present value of the full lifecycle costs of each technology including overnight cost, financing, operations and maintenance, and fuel. These various cost streams associated with the construction and operation of a generation facility are discounted over the plant lifetime to determine the present value of lifetime operation. Our calculations are applied over a 50 year time horizon, which is the assumed operational lifetime for centralized technologies and transmission infrastructure. Solar home systems are assumed to have a 25 year lifetime and therefore must be replaced once over a 50 year horizon. A real annual discount rate of 5% is used and the MIP is solved to minimize the total discounted lifetime cost of meeting system-wide peak and average power demand.

### 3.2.1 Mixed-Integer Program

A mixed-integer program is formulated to find the combination of centralized and decentralized infrastructure that minimizes the total cost of meeting system-wide electricity demand in a region. The region is modeled as a set of nodes or demand points, each of which may be connected to any of its neighbors by a transmission line. The formulation below includes the option of developing either a high or low voltage...
transmission line on each edge; however, arbitrarily many classes of transmission lines
could be included at the cost of additional computational resources.

3.2.1.1 Decision variables

Decentralized electricity generation may be implemented at any node without
restriction, while new centralized generation facilities are restricted to pre-identified
projects. The decision variables are described and classified according to their type
below. Continuous variables are represented by \( x \) while binary variables are represented
by \( y \), \( m \) is the number of possible edges in the system, \( n \) is the number of nodes and \( r \) is
the number of potential new centralized generation facilities.

- \( x_{1,1} \ldots x_{1,n} \): Average transmission power level in each edge
- \( x_{2,1} \ldots x_{2,m} \): Peak transmission power level in each edge
- \( y_{3,1} \ldots y_{3,m} \): Indicator for a high voltage transmission line on each edge
- \( y_{4,1} \ldots y_{4,m} \): Indicator for a low voltage transmission on each edge
- \( x_{5,1} \ldots x_{5,n} \): Average centralized consumption in each node
- \( x_{6,1} \ldots x_{6,n} \): Peak centralized consumption in each node
- \( y_{7,1} \ldots y_{7,n} \): Indicator for a local centralized distribution network in each node
- \( x_{8,1} \ldots x_{8,n} \): Decentralized capacity in each node
- \( y_{9,1} \ldots y_{9,r} \): Indicator for each centralized generation facility

Additional parameters used in the formulation are outlined below; the index \( i \) refers to
edges while the index \( j \) refers to nodes and \( k \) refers to centralized facilities.

- \( d_i \): Length of edge \( i \) in kilometers
- \( T_h, T_l \): Power capacity of high and low voltage transmission lines
- \( C_h, C_l \): Cost per kilometer of high and low voltage transmission lines
- \( C_D \): Cost of decentralized generation technology, per kW capacity
- \( C_k \): Present value of lifecycle cost of centralized facility \( k \)
- \( C_p \): Cost of connecting one person to the transmission line within a node
- \( RR \): Reserve requirement for transmission lines, includes the effects of
  transmission losses
• $CF_D$: Capacity factor of decentralized generation technology
• $CF_k$: Capacity factor of centralized generation facility $k$
• $I_j$: The set of all edges feeding into node $j$
• $O_j$: The set of all edges feeding out of node $j$
• $DA_j$: Average electricity power demand in node $j$
• $DP_j$: Peak electricity power demand in node $j$
• $TC_k$: Power generation capacity of centralized facility $k$
• $P_j$: The population of node $j$

### 3.2.1.2 Objective function

The objective of this optimization problem is to minimize the total cost of meeting electricity demand in a given system. The objective function can be expressed as follows,

$$
\min \left\{ \sum_{i=1}^{m} d_i \cdot C_h \cdot y_{3,i} + \sum_{i=1}^{m} d_i \cdot C_t \cdot y_{4,i} + \sum_{j=1}^{n} C_p \cdot P_j \cdot y_{7,j} + \sum_{j=1}^{n} C_d \cdot x_{8,j} + \sum_{k=1}^{r} C_k \cdot y_{9,k} \right\}
$$

(3.1)

Here the first and second terms respectively represent the cost of new high and low voltage transmission lines. The third term represents the cost of distribution line infilling within each demand node that is connected to the centralized grid. The fourth and fifth terms respectively represent the costs of new decentralized and centralized generation facilities. Recurring cost streams are discounted and encapsulated into a present value equivalent over a 50 year time horizon.

### 3.2.1.3 Constraints

The objective function is minimized subject to the following constraints.
A first constraint ensures that the flow of power into and out of each node is consistent. That is, the flow of power into a node plus the generation in that node must be greater than consumption plus the power flow out; this holds for both average and peak power. Decentralized generation is not considered to be connected to the grid and is therefore not included.

\[
\sum_{i \in I_j} x_{1,i} - \sum_{i \in O_j} x_{1,i} - x_{5,j} + \sum_{k \in Gen_j} CF_k \cdot T_{ck} \cdot y_{9,k} \geq 0 \quad \forall j \quad (3.2)
\]

\[
\sum_{i \in I_j} x_{2,i} - \sum_{i \in O_j} x_{2,i} - x_{6,j} + \sum_{k \in Gen_j} T_{ck} \cdot y_{9,k} \geq 0 \quad \forall j \quad (3.3)
\]

The next constraint ensures that centralized and decentralized consumption in each node combine to meet both average and peak demand. The reserve requirement applies to centralized consumption to ensure proper resiliency in the transmission lines and also includes the effects of transmission losses. The purpose of the reserve requirement is to provide a buffer of additional generation and transmission capacity to ensure that service is still provided in the event that demand marginally exceeds anticipated peak levels. The reserve requirement can also provide resiliency in the event that a generation facility goes offline. In developing countries electricity generation is often unreliable and plant outages may be fairly common. This effect can be accounted for by including a relatively high reserve requirement to ensure that service is provided even if certain infrastructure components temporarily cease to function. Decentralized solar home systems include a controller that strictly limits the power being drawn from the system. As the load of each individual system cannot exceed this level, a reserve requirement is not necessary to ensure resiliency due to unanticipated demand. Decentralized systems may also go offline due to malfunction or required maintenance; however, this analysis does not consider a backup option for such systems.

\[
\frac{1}{RR} \cdot x_{5,j} + CF_P \cdot x_{8,j} \geq D_{A_j} \quad \forall j \quad (3.4)
\]

\[
\frac{1}{RR} \cdot x_{6,j} + x_{8,j} \geq D_{P_j} \quad \forall j \quad (3.5)
\]
A third constraint requires that a node must be connected to the transmission backbone by a local distribution network in order for centralized power to be consumed throughout that node.

\[ x_{5,j} - D_{A_j} \cdot y_{7,j} \leq 0 ~ \forall j \] (3.6)
\[ x_{6,j} - D_{P_j} \cdot y_{7,j} \leq 0 ~ \forall j \] (3.7)

An additional constraint ensures that no more than one transmission line is constructed on each edge.

\[ y_{3,i} + y_{4,i} \leq 1 ~ \forall i \] (3.8)

A final set of constraints ensures that the capacity of each line is greater than the absolute values of both the average and peak power levels.

\[ |x_{1,i}| - y_{3,i} \cdot T_h - y_{4,i} \cdot T_l \leq 0 ~ \forall i \] (3.9)
\[ |x_{2,i}| - y_{3,i} \cdot T_h - y_{4,i} \cdot T_l \leq 0 ~ \forall i \] (3.10)

All variables are non-negative with the exception of \( x_{1,i} \) and \( x_{2,i} \) where the direction of power flow is defined by their sign.

### 3.3 Rwanda Case Study

Rwanda is a landlocked country in East Africa with a population of approximately 10 million people that has one of the least developed electricity infrastructures of any country in the world. The country is represented as a set of demand centers, or nodes, which are derived from population data available from The Gridded World Population Project [160]. In this case study, we utilize population data with a node resolution of 2.5 arcminutes as our original data set, and consolidate this set as desired to form geographical representations of population dispersion in Rwanda with node resolution of 3, 6, 12 and 24 arcminutes. Each node has associated average and peak power demands,
which must each be satisfied through a combination of centralized and decentralized electricity.

### 3.3.1 Electricity Services

In rural communities, small quantities of electricity are often first used to replace kerosene lighting with electric lights. Kerosene can be costly and provide poor quality light. Additional electricity may be demanded to provide power for a radio, television or a cellular telephone.

A case study of rural village energy use in Kenya found that widely used kerosene lamps produced an average output of about 20 lumens, which generally enables only basic activities and is not enough for comfortable reading [162]. These lamps can be replaced by 7 W compact fluorescent bulbs that provide 300 lumens, enough for reading and the illumination of a small room. Additional electricity may be used to power a radio or cellular telephone, which can be accomplished with about 3 W of power. This 10 W peak demand level might be considered the minimal practical electric load for a single household in a rural setting. As a household may include five or more inhabitants, as little as 2 W of power per capita could be sufficient to significantly affect quality of life in many un-electrified regions. In 2005 it was estimated that a 20 W solar system in Kenya would cost $150 to $200 [162].

The growth of small-scale solar electrification has perhaps been most rapid in Bangladesh over the past decade, which now has over one million homes powered by solar systems and is on pace to pass 2.5 million by 2014 [175]. This effort has been led by Grammen Shakti, an organization that provides individual solar home systems ranging from 10-135 W in output capacity. These systems include a battery, all necessary controllers and cables and CFL bulbs. The systems cost approximately $6/W to $7/W and are designed to provide electricity for four hours per day. A typical 50 W system provides a 12 volt DC power output that can operate four 7 watt CFL bulbs and a 17 inch black and white television and includes a battery with 80 amp-hours of storage capacity; it is sold in Bangladesh for 29,500 taka ($350) [184]. The battery provides a maximum energy storage capacity of .96 kWh, which is sufficient to service a 50 W load for 19.2 hours on a single full charge. In rural settings, a significant portion of energy services are
consumed in the form of evening lighting after the sun has set. This battery helps ensure that the solar home system can provide a consistent power output after sunset when the photovoltaic panel is no longer able to directly generate electricity.

### 3.3.1.1 Average and Peak Demand

Our model finds the minimum cost infrastructure that satisfies both average and peak power demand levels. We define the peak factor of any given node to be the ratio of peak power demand to average power demand. The peak factor of the centralized electricity network in Rwanda was 1.75 in 2003 and this has been projected to drop to 1.67 in 2011 [185]. In order to avoid outages, enough generation capacity must be available in order to serve peak demand. Therefore systems with higher peak factors will incur greater costs due to lower average utilization. We assume a peak factor of 1.7 for loads that are served by new centralized infrastructure as well as for those served by new decentralized systems. This model does not consider temporal load curve resolution beyond the average and peak demand levels as these are the two parameters that will primarily influence infrastructure investment decisions. More detailed load curve information could be included at the cost of additional computational requirements. Variations in the peak factor over a reasonable range of values have a modest impact on results.

We apply our methodology to understand how the variations in electricity demand influence the decision between different generation technologies. Electricity demand of the currently electrified population is based on recent consumption data, while demand levels for newly electrified populations will be varied to determine how these changes affect the optimal development strategy.

### 3.3.1.2 Current Infrastructure

As of 2009, Rwanda has approximately 55 MW of available centralized electricity generation capacity, spread fairly evenly between hydroelectric and oil thermal plants [186]. There is also a small amount of microhyrdro generation as well as a 250 kW solar array near the capital, Kigali. The electric grid in Rwanda has two high voltage backbones, with approximately 285 km of 110 kV and 64 km of 70 kV lines, and there is
also a significant 30 kV network as well as 15 kV and 6.6 kV distribution lines [187].

The current level of generation capacity is not sufficient to adequately serve demand in Rwanda and the shortages have led to dramatic increases in electricity prices and utility scale implementation of diesel generators. Currently only 6% of the population has access to electricity and the Rwandan government has laid out an aggressive target to increase the electrification rate to 35% by 2020, with an emphasis placed on rural areas. As a result, electricity demand per capita is also expected to increase from 30 kWh/year to 100 kWh/year over that time [186]. Combined with an expected 30% growth in population, total countrywide electricity demand could increase by a factor of four or five in the next decade.

Our case study of Rwanda includes nine existing centralized generation facilities with a total operating capacity of approximately 55 MW.

3.3.1.3 Future Infrastructure

Centralized Generation

It has been estimated that unutilized power generation resources in Rwanda could amount to 1,200 MW, more than enough to meet increasing demand [186]. These include significant untapped hydroelectric resources and large methane gas deposits that are known to exist at the bottom of Lake Kivu on the western border of the country. Solar irradiation in Rwanda is relatively high at an average of 5.2 kWh/m$^2$day [187]. Rwanda also has an estimated 170-320 MW of geothermal power potential as well as wind resources, but further investigations are necessary before these technologies are implemented [187].

Methane deposits in Lake Kivu have been discovered which may be able to provide as much as 700 MW of power and discussions have targeted realizing 350 MW of this in the next decade [187]. An initial 25 MW plant has been announced with future planned expansions to 100 MW [188]. The additional, targeted but unplanned, capacity of 250 MW is also included in our model at a comparable unit cost. As the proposed large hydroelectric projects at Rusizi and Rusumo would each be shared between three countries, these figures represent the Rwandan share of power and costs.
Rwanda is geographically situated on the East Africa Rift, a region with significant potential for electricity generation from underground geothermal heat. It has been estimated that more than 300 MW of geothermal power could be generated in the western part of the country near Lake Kivu. Initial feasibility explorations are now underway, and the Rwandan Energy Ministry is targeting 310 MW of new geothermal power capacity over the next seven years at a cost of $935 million; however a third-party cost analysis has not been performed. We include an additional 20% in the cost to account for the required feasibility studies and potential optimism in the government estimate. We consider 13 potential new generation facilities with total capacity of approximately 840 MW for future construction. The capacities and costs for these potential generation sites are listed in table 3.1 and represent best estimates based on a variety of information sources. Cost calculations for centralized technologies assume a 15% carrying charge applied over a 30 year book life and annual operations and maintenance costs equal to 3% of the overnight cost. The Lake Kivu project includes the costs of infrastructure to extract methane from the lake and it is assumed that no additional fuel costs will be incurred.

Table 3.1: Potential centralized generation facilities in Rwanda [186–191].

<table>
<thead>
<tr>
<th>Location</th>
<th>Capacity (MW)</th>
<th>Overnight Cost (M$)</th>
<th>Generation Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Kivu I</td>
<td>25</td>
<td>140</td>
<td>Methane Thermal</td>
</tr>
<tr>
<td>Lake Kivu II</td>
<td>75</td>
<td>185</td>
<td>Methane Thermal</td>
</tr>
<tr>
<td>Lake Kivu III</td>
<td>250</td>
<td>620</td>
<td>Methane Thermal</td>
</tr>
<tr>
<td>Mukungwa II</td>
<td>2.5</td>
<td>7.5</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Mukungwa III</td>
<td>2.2</td>
<td>6.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Nyabarongo</td>
<td>27.5</td>
<td>97.7</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rukarara</td>
<td>9.5</td>
<td>20</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rubavu</td>
<td>3.2</td>
<td>9.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rugezi</td>
<td>2.2</td>
<td>6.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rusizi III</td>
<td>48</td>
<td>150</td>
<td>Hydroelectric</td>
</tr>
</tbody>
</table>
Rusizi IV & 68 & 212 & Hydroelectric
Rusumo & 20 & 57 & Hydroelectric
Lake Kivu & 310 & 1120 & Geothermal

As our model only includes requirements to meet average and peak demand, the time-dependent output profiles and each generation technology are not explicitly modeled. The ability of each generation technology to meet peak demand is defined by its rated generation capacity, while its ability to meet average demand is defined by its capacity factor. Thermal technologies are assumed to operate at an 85% capacity factor and hydroelectric technologies at a 40% capacity factor. These effects are accounted for through constraints in the MIP formulation.

**Decentralized Generation**

The decentralized technology considered by the model is a small, solar home system, which offers peak generation capacity of 50 W and includes a 12 volt battery with 80 amp-hours of storage capacity. The maximum load on each system is assumed to be equal to the peak generation capacity, or approximately 4 amps for a 50 W system. This implies that a fully charged battery can provide roughly 20 hours of uninterrupted power to a system operating at the maximum load level. Solar home systems are gaining popularity throughout the developing world as a viable alternative to centralized options that are easily scalable to meet peak demand levels greater than 50 W and can be implemented relatively quickly.

We analyze sensitivity around the cost of this technology, with scenarios assuming installed costs of $5, $6 and $7 per watt. In each scenario the systems are assumed to be financed at 8% interest over three years with annual maintenance costs equal to 1% of the overnight cost and require one system replacement after 25 years. It is assumed that these systems operate at a 16% capacity factor and are located at demand sites; therefore they do not require distribution infrastructure. For the purposes of this analysis, electricity generated from decentralized technologies is assumed to be a perfect substitute for electricity generated from centralized technologies.

**Transmission and Distribution**
In the case study, the term ‘transmission’ will be used to refer to lines of 30 kV or greater while ‘distribution’ will be used for lines of less than 30 kV. The current electricity transmission infrastructure in Rwanda is made up of 30 kV, 70 kV and 110 kV lines. Our grid extension model only considers adding new 30 kV and 110 kV lines so as to limit computational complexity.

Our model explicitly characterizes the expansion of transmission infrastructure (30 kV and 110 kV), while the costs of extending distribution infrastructure (<30 kV) to individual demand centers within each node are considered on the basis of a fixed cost per household. An analysis of electric grid expansion in Kenya estimated the cost of connecting households to a nearby medium voltage transmission line within the same demand node to be $1246/household [20]. We assume a comparable cost of $1250/household in our analysis, or $250/person assuming five people per household. In this analysis, the entire population of a node is considered to be centralized if such a distribution network is developed.

The capacity and overnight cost of each transmission line class are consistent with values used in other similar studies of electric grid expansion in Sub-Saharan Africa [20, 23] and an empirical study of the costs of transmission infrastructure [159]. The costs of transmission lines are further augmented to account for the costs of transformers and other transmission infrastructure, which are not explicitly modeled by our methodology. These costs typically account for roughly 60% of total transmission costs. We do not have access to data that would allow us to account for geographic factors and terrain features, which may affect the unit cost of each individual transmission segment. Therefore a homogenous, average cost of transmission is applied to all potential segments throughout the region.

The present cost of transmission infrastructure is calculated based on the assumption of a 15% carrying charge over a 30 year book life and annual operations and maintenance costs equal to 1% of overnight cost. Capacity and cost of transmission and distribution lines are shown in table 3.2.
Table 3.2: Transmission line parameters

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Maximum Capacity</th>
<th>Overnight Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 kV</td>
<td>100 MW</td>
<td>$150,000/km</td>
</tr>
<tr>
<td>30 kV</td>
<td>7.5MW</td>
<td>$15,000/km</td>
</tr>
</tbody>
</table>

3.4 Results

In applying our model to a case study analysis of Rwanda, we seek to understand how the decision between centralized and decentralized infrastructures is affected by variations in the potential electricity demand of new consumers and the cost of decentralized generation technologies. Sensitivities are analyzed around two key parameters; average per capita electricity consumption for newly electrified populations, and the cost of decentralized generation technologies. We also explore the effect of scaling the model formulation to include more or less detailed population data, in terms of both results and computation time. Results are presented for 15 scenarios encompassing combinations of five different demand levels and three different decentralized generation costs. These 15 scenarios are also executed at four different node resolutions. All results were generated by solving the MIP with a target optimality gap of 1% and a maximum runtime of 25,000 seconds.

3.4.1 Electricity Development in Rwanda

Figure 3.1 shows the fraction of the population that would be served by centralized electricity infrastructure under each scenario. A node that is served by centralized infrastructure may also produce decentralized generation, but this is not observed to occur significantly in our results. A single star denotes a solution that failed to converge within 1% of optimality in the 25,000 second time limit, while a double star denotes a scenario that failed to converge within 5% of optimality.

It can be seen that at a cost of $5/W or $6/W, Rwanda could be largely served by decentralized generation for average consumption levels at or below 4W per capita. At a cost of $7/W Rwanda could be largely served by decentralized generation for consumption levels at or below 2W/capita. The results for scenarios executed at each node resolution are largely consistent, with the exception of two 3’ scenarios. However,
neither of these solutions ($5/W decentralized cost with 6W/capita consumption and
$7/W decentralized cost with 4W/capita consumption) converged to the within 1% of
optimality within the time limit. Early termination will generally result in a partially
developed transmission infrastructure. This is likely the cause of the discrepancy
between these scenarios at 3’ resolution and the other resolutions.

These results show that only the capital of Kigali, which contains approximately
7% of total countrywide population, would be served by centralized infrastructure when
consumption is below these thresholds. When using population data with 24 arcminute
resolution, the single largest node contains roughly 14% of the total population, which
accounts for the difference that is observed. Other slight differences are present between
the scenarios executed with different node resolution, however the same broad trend
generally holds in each case.

It’s also notable that there are very few combinations of parameters that lead to a
combined network with significant portions of both centralized and decentralized
electrification. In each cost scenario there are certain tipping points, where the optimal
countrywide electrification plan shifts from almost an entirely centralized infrastructure
to an almost entirely decentralized infrastructure. The tipping point is also generally
consistent across scenarios executed at each node resolution. This suggests that less
detailed population data may be used to identify these tipping points just as effectively as
more detailed population data, while requiring far fewer computational resources.

Figure 3.1: Percent of population served by decentralized electricity infrastructure in the
Rwanda case study. * >1% optimality gap ** >5% optimality gap.

Figure 3.2 shows the present value of total system wide cost of the combined
centralized and decentralized infrastructure that serves the given demand level. It can be
seen that total costs tend to decrease somewhat as the distance between nodes increases, however the results are fairly consistent. The differences are likely explained by the lower total network length required to serve a set of fewer nodes, combined with the fact that the per household cost of local distribution infrastructure is assumed to be the same for scenarios executed at each resolution.

Table 3.2: Total system cost for electricity development in the Rwanda case study (billion $).

<table>
<thead>
<tr>
<th>Average Power (W/capita)</th>
<th>3' resolution</th>
<th>6' resolution</th>
<th>12' resolution</th>
<th>24' resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4.9 5.0 5.1</td>
<td>4.8 4.8 4.9</td>
<td>4.7 4.7 4.7</td>
<td>4.4 4.4 4.5</td>
</tr>
<tr>
<td>8</td>
<td>4.3 4.3 4.3</td>
<td>4.1 4.1 4.1</td>
<td>4.1 4.1 4.1</td>
<td>3.8 3.9 3.9</td>
</tr>
<tr>
<td>6</td>
<td>3.9 3.9 3.9</td>
<td>3.6 3.7 3.7</td>
<td>3.6 3.6 3.6</td>
<td>3.4 3.4 3.5</td>
</tr>
<tr>
<td>4</td>
<td>2.7 3.2 3.6</td>
<td>2.7 3.2 3.5</td>
<td>2.7 3.2 3.4</td>
<td>2.7 3.1 3.4</td>
</tr>
<tr>
<td>2</td>
<td>1.5 1.7 2.0</td>
<td>1.4 1.7 2.0</td>
<td>1.5 1.7 2.0</td>
<td>1.5 1.8 2.0</td>
</tr>
</tbody>
</table>

Figures 3.3 and 3.4 show the new centralized and decentralized generation capacity that is required in each scenario. As might be expected, scenarios with low levels of new centralized capacity generally have high levels of decentralized capacity and vice versa. The values in these figures correspond with the tipping point between the country being largely powered by centralized and decentralized systems as was observed in figure 3.1. The 10W per capita scenarios generally call for 100 MW of new methane thermal capacity at Lake Kivu, as well new hydroelectric capacity at Nyabarongo, Rusizi and Rusumo as well as a number of other small new developments; though some variations are present between scenarios and resolutions. The 8 W per capita scenarios call for the 100MW addition at Lake Kivu and new capacity at Rusizi, while the 6 W per capita scenarios call for only 100 MW at Lake Kivu and several other smaller facilities. These results are generally quite consistent across the different node resolutions, with the exception of the two 3’ resolution scenarios that were previously discussed.

These results are similar to the results of Levin and Thomas [183], in which a network algorithm was used for 150 countries to find the optimal fraction of the population for centralized and decentralized electricity. For Rwanda, at 6 W/capita (50 kWh/person-year) the network algorithm finds 7% decentralized, and at 8 W/capita (69 kWh/person-year) it finds 3% decentralized. This study differs from that previous study.
not only in algorithm, but also in having more detailed transmission and centralized generation cost data and in taking into account the existing transmission and generation infrastructure.

Figure 3.3: New centralized generation capacity (MW) for the Rwanda case study

Figure 3.4: New decentralized generation capacity (MW) for the Rwanda case study

3.4.2 Computational Requirements

We also analyze the computational resources required to solve the MIP when different data resolutions are used. The MIP is formulated in MATLAB and solved using CPLEX 12.2 and Concert 2.6 technology on 2.4 to 2.8 GHz processors.

Table 3.3 shows the number of nodes, edges and variables present in the MIP formulation for each node resolution. Figure 3.5 shows the runtime of each scenario as a function of the fraction of population decentralized in the optimal network for that scenario. All scenarios were executed with a target 1% optimality gap and a maximum run time of 25,000 seconds. It can be seen that scenarios with optimal networks that are primarily centralized require more computational resources to solve, as the specifics of the entire transmission network must be determined. Formulations using population data with 12 or 24 arcminute resolution were able to determine a solution within 1% of
optimality in a few seconds, where as those with 3 or 6 arcminute resolution often could not achieve the same optimality after the maximum allowed 25,000 seconds.

Table 3.3: Number of nodes, edges, and variables for each resolution in the Rwanda case study

<table>
<thead>
<tr>
<th>Node Resolution (arcminutes)</th>
<th>Nodes</th>
<th>Edges</th>
<th>Binary Variables</th>
<th>Continuous Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>19</td>
<td>51</td>
<td>134</td>
<td>159</td>
</tr>
<tr>
<td>12</td>
<td>65</td>
<td>212</td>
<td>502</td>
<td>619</td>
</tr>
<tr>
<td>6</td>
<td>224</td>
<td>789</td>
<td>1815</td>
<td>2250</td>
</tr>
<tr>
<td>3</td>
<td>820</td>
<td>3057</td>
<td>6947</td>
<td>8574</td>
</tr>
</tbody>
</table>

Figure 3.5: Run time required to achieve a 1% optimality gap as a function of percent decentralized, for spatial resolution of 3, 6, 9, and 12 arcminutes. All runs were limited to 25,000 seconds.

Figures 3.6-3.9 depict the minimum-cost infrastructure under various decentralized generation cost and average consumption level scenarios. In the figures
dashed lines are used to denote new transmission infrastructure, while solid lines denote existing infrastructure.

Figures 3.6 and 3.7 illustrate how the model may generate optimal networks that differ slightly but are largely consistent when executed with population data at different resolutions. Figure 3.6 shows the optimal network under the assumption of $6/W decentralized generation overnight cost and 4 W per capita average power consumption for newly electrified populations when executed with 6 arcminute node resolution. No new centralized generation capacity is developed, and the existing transmission network is expanded to serve only one additional node. Many of the nodes that are currently connected by a transmission line do not develop local distribution networks and 86% of the population is served by decentralized generation. Figure 3.7 shows similar results for the same scenario when executed at 12' node resolution. No new centralized generation capacity is developed, no extensions are made to the existing transmission grid and 85% of the population is served by decentralized generation.

**Figure 3.6**: Electricity system for the Rwanda case study, for decentralized generation cost of $6/W and average electricity consumption of 4 W/capita for newly connected consumers. Spatial resolution is 6 arcminutes.
Figure 3.7: Electricity system for the Rwanda case study, for decentralized generation cost of $6/W and average electricity consumption of 4 W/capita for newly connected consumers. Spatial resolution is 12 arcminutes.

Comparison of figure 3.6 and 3.7 with figures 3.8 and 3.9 show how a small increase in average power demand for new consumers can cause the optimal network to tip from being primarily decentralized to almost entirely centralized. The increase in demand leads to investment in new centralized generation and transmission infrastructure, most notably, 100 MW of methane thermal generation capacity at Lake Kivu. Only a small number of nodes, encompassing 1% of total countrywide population, are served by decentralized systems. The results are largely consistent when the same scenario is executed at 12’ resolution as opposed to 6’ resolution. The same centralized generation facilities are developed and in this case less than 1% of the total population is served by decentralized generation.
Figure 3.8: Electricity system for the Rwanda case study, for decentralized generation cost of $6/W and average electricity consumption of 6 W/capita for newly connected consumers. Spatial resolution is 6 arcminutes.
**Figure 3.9:** Electricity system for the Rwanda case study, for decentralized generation cost of $6/W and average electricity consumption of 6 W/capita for newly connected consumers. Spatial resolution is 12 arcminutes.

### 3.5 Discussion

We have presented a methodology that can be easily implemented to model the decision between centralized and decentralized electricity infrastructure development and help understand how this decision is affected by the sensitivity of key parameters. To demonstrate this methodology, we have applied it to a case study of electricity infrastructure development in Rwanda. We wish to be clear that results specific to the Rwandan case should be interpreted in the context of the various assumptions that were made.

It is shown that in the case of Rwanda, there are few combinations of parameters that result in optimal networks with significant contributions from both centralized and decentralized infrastructures. Rather, there appear to be tipping points where the optimal infrastructure shifts from being largely centralized to largely decentralized. For reasonable cost of decentralized systems, this tipping point appears to occur at an average electricity consumption level of approximately 4 W per capita, or 20 W for an average five person household. Given a typical solar home system that operates for four hours per day, this consumption level corresponds to a household system with a peak capacity of approximately 120 W. Our results suggest that broadly speaking, higher demand levels would be more cost-effectively served by centralized infrastructures. However, our model only optimizes the total anticipated lifetime cost of a given countrywide electricity network, and there are a number of other factors that may influence the decision between different infrastructures and technologies. Decentralized infrastructures may be preferred despite higher costs for a variety of reasons including lack of a competent central authority, ease and speed of installation, improved system reliability and increased autonomy and independence. The current rapid penetration of new solar home systems in Bangladesh is an example of this phenomenon.

We also demonstrate that using population data of different resolutions may affect the specific details of the optimal network. However the broad trends of either
decentralization or centralization generally remain consistent across the considered node resolutions. We also show how the data resolution affects computational time. Our methodology may be applied to better understand how changes in key parameters can lead to potential tipping points from primarily centralized to primarily decentralized infrastructures.

As rigorous sensitivity analysis requires the formulated MIP to be solved numerous times to consider all potential combinations of parameters, it is important to formulate the MIP so that it may be solved in a reasonable timeframe. We find that a formulation with 65 nodes can generally be solved in 10 seconds or less given our computational resources. Formulating the MIP with 224 nodes significantly increases the computational requirements, with some scenarios unable to converge to a solution within 1% of optimal in 25,000 seconds. However, both formulations identify the same tipping points in our case study of Rwanda, and other results are also largely consistent between scenarios executed at these resolutions. Therefore, we find that in the case of Rwanda, any potential benefits resulting from increasing data resolution from 12’ to 6’ do not justify the increased computational requirements.

The number of nodes in the formulation will of course also be affected by the size of the region being analyzed. Rwanda is a relatively small country; an analysis of larger countries would have to be formulated with less dense node resolution in order to be solved in a similar timeframe. We recommend that for expedient analysis, the network should be limited to approximately 200 nodes. However, larger formulations can also be analyzed if more computational resources are available or larger optimality gaps are tolerable. Additionally, formulations that allow for more classes of new transmission lines can be considered; however this increases the number of binary variables in the MIP and will significantly affect the computational requirements. This tradeoff should therefore be considered when formulating the system that will be analyzed.

We chose to demonstrate our methodology by analyzing the sensitivity of the optimal network to changes in two key parameters, decentralized generation cost and average new electricity consumption. However, the methodology could also be used to analyze the effect of changing a number of other parameters, such as cost of transmission and
distribution infrastructure and the cost or availability of new centralized generation facilities.
CHAPTER IV
SIMULATION OF ENERGY SECURITY AND RESILIENCY

4.1 Introduction

Chapters 2 and 3 present two methodologies for determining the least-cost combination of centralized and decentralized electricity infrastructure that is capable of serving demand in a given region. In chapter 2 it was shown that Bangladesh could be served most cost-effectively by a primarily centralized infrastructure, largely due to the high population density of the country. However, according to some experts, Bangladesh has been experiencing “the fastest deployment of solar energy anywhere in the world” and the country is expected to reach 2.5 million system installations by 2014 [175]. This evidence strongly suggests that there are factors beyond those modeled in chapter 2 that influence development paths, and chapter 4 presents a new model for electricity infrastructure planning that addresses some of these issues.

There are many possible explanations for this discrepancy between modeling results and observed data. Large scale centralized development plans require significant capital to execute, and even with the benefit of private investment or foreign aid many countries do not have access to sufficient funds. Even in cases where adequate funding is made available, development plans are often executed poorly or not at all due to ineffectual central authorities or corrupt governments. Additionally, many developing countries are constrained by limited availability of natural resources. Many development models, such as those outlined in chapters 2 and 3, also assume that centralized infrastructures will provide reliable service to those who are connected, however the reality in developing countries often is quite different. A World Bank sponsored survey of small enterprises found that 49% of respondents in sub-Saharan Africa identified electricity reliability to be a major business constraint, with 13% total of their electricity consumption coming from onsite diesel generators [75].

Another factor that is often overlooked in quantifying the potential impact of different development strategies, is the lead time required for a proposed infrastructure to be implemented. Centralized electricity generation and transmission systems may take years or even decades to fully develop. In developing countries this lead time can be
further amplified by limited access to capital, conflicting political goals, poor central management and corruption.

In the developing world, delayed consumption of goods or services may also significantly impact the consumer’s health, education or income generation, whereas a similar delay in the developed world may simply cause a minor inconvenience. As a result, poor populations in developing countries are often observed to have higher rates than their counterparts in developed countries. This as can be seen through their relatively low savings rates and willingness to accept high interest rates on loans [143, 192]. This implies that these populations highly value the ability to consume services in the present, as compared to potential future consumption. As such, long lead times that delay consumption will have a greater relative impact on poor populations.

Large, centrally planned electrification networks generally benefit from economies of scale to produce electricity at a lower unit cost. However, these cost benefits may not be justified if the centralized systems are inefficiently planned or slowly implemented. Additionally, poorly planned or underdeveloped centralized networks are subject to frequent power outages which can impose a considerable cost to end users. Distributed systems do not require significant centralized planning or coordination, can be disseminated and scaled more readily through small private enterprises and are generally reasonably reliable. A shift towards distributed systems transfers decision making away from central authorities and puts it in hands of consumers, enabling free market forces to more quickly determine the optimal allocation of systems. On the other hand, distributed systems may have limited expansion capacity, may be more expensive, and may receive less subsidization than centralized systems.

Chapter 4 presents a methodology for analyzing the impacts of infrastructure lead time, unreliability and other important energy security factors that are faced in the developing world. This methodology incorporates the impacts of stochastic events including random electricity demand and commodity prices, generation and transmission outages and rare events such as political instability. The model is computationally inexpensive and can be executed quickly to generate a distribution of possible outcomes for each proposed development path, including the total lifetime cost of a system and its ability to serve demand. In contrast to many other studies that develop optimization
frameworks to endogenously determine optimal development paths, this methodology simulates exogenously defined development plans to generate a distribution of results for various comparative metrics, such as cost and level of served electricity demand. Rather than using aggregated metrics and indicators to quantify the various different dimensions of grid security, this analysis focuses directly on the level of enabled electricity demand and the cost of providing these services.

The impact of explicitly simulating stochastic events and drawing probabilistic parameters from a distribution, as opposed to assuming constant mean parameter values, is also analyzed. These stochastic events include outages to generation facilities or transmission line segments, disruptions to available development budgets and variations in commodity prices. Through simulation of these parameters and events the impacts of long construction horizons and budget overruns, which are often faced by large-scale infrastructure development projects, are also examined.

4.2 Methodology

As discussed in chapter 1, there are numerous methodologies and algorithms that can be used to formulate and solve stochastic programming problems related to network expansion in the face of uncertainty. Unfortunately, it is computationally expensive to explicitly solve complex stochastic programs. Additionally, in order to generate an optimal development strategy, a modeler must first define an objective function. In some situations this is a simple task, as often the goal is simply to minimize the cost of reaching a certain development target. However, when multiple competing objectives are present, the modeler must create a composite objective function or develop a multi-objective programming formulation. It is also not always clear how the various objectives should be balanced or weighted and this can lead to a somewhat arbitrary definition of what constitutes an optimal solution. Furthermore, the solution is generally based on the expected mean value of the objective function, which makes it difficult to consider the full range of potential outcomes.

We present an alternative approach where exogenously defined development plans are simulated through a periodic model that incorporates stochastic variables. As such, computationally expensive stochastic programming techniques are not required and it
becomes possible to execute repeated simulations and generate a distribution of potential results for several comparative metrics, namely cost and service level.

The structure of this model is similar to the one presented in chapter 3; a region is represented as a set of demand nodes, each of which may be connected to any of its neighbors by a transmission line. Electricity generation facilities are present at a number of predefined locations and electricity is delivered from these generation sites to demand nodes through the transmission network. The rules governing the flow of electricity through the network are the same as were employed in chapter 3. At any given node the flow of power into the node plus the generation in that node exceed demand in that node plus the flow of power out of the node. There are also key differences between this model and the one presented in chapter 3. Whereas the methodology in chapter 3 found the minimum cost network capable of serving different demand levels, this methodology makes no assumption that demand is met. Rather, it compares exogenously defined development plans on the basis of their cost, ability to effectively serve demand and other comparative metrics. A linear programming (LP) framework is employed to minimize the amount of unserved electricity demand each period and to ensure that the flow of power through the system is consistent. However, decisions regarding the construction of new transmission and generation infrastructure are provided as inputs by the user and are not included in the LP formulation.

The model is executed over a 20-year time horizon with four periods in each year and two timesteps in each period to simulate base and peak demand levels. In each timestep a linear program is solved in order to determine the flow of power through the network and subsequent servicing of electricity demand. As was discussed in chapter 1, many efforts have been made to estimate the economic cost of unserved electricity, however results vary widely based on the environment and situation [81]. We choose to not make explicit assumptions about this cost, and instead simply minimize the quantity of unserved energy in each period. This keeps the cost and reliability metrics segregated rather than attempting to combine them into a potentially ambiguous composite function.
4.2.1 Decision Variables

The decision variables, which are all continuous in this formulation, are the electricity consumption and generation in each node as well as the power being transmitted on each network edge. Note that capital investments, such as new generation facilities or transmission lines, are not decision variables as these are provided by the modeler.

- \( x_j \): electricity consumption level in node \( j \)
- \( g_j \): electricity generation level in node \( j \)
- \( y_i \): transmission power level in edge \( i \)

Other parameters used in the formulation are outlined below.

- \( D_j \): electricity demand in node \( j \)
- \( C_i \): transmission capacity of edge \( i \)

4.2.2 Objective Function and Constraints

The objective function is formulated to minimize the quantity of unserved electricity demand across all nodes in the system.

\[
\min \left\{ \sum_{j \in N} (D_j - x_j) \right\} \quad (4.1)
\]

The following constraints are also imposed. First, electricity consumption in each node must be less than demand in that node and greater than zero.

\[
0 \leq x_j \leq D_j \quad \forall \ j \in I_c \quad (4.2)
\]

Second, the flow of power throughout the system must be consistent. Electricity can be generated by large centralized plants or decentralized technologies such as solar home
systems or diesel generators. Here $I_j$ represents the set of all edges leading into node $j$ and $O_j$ represents the set of all edges leading out of node $j$.

$$\sum_{i \in I_j} y_i - \sum_{i \in O_j} y_i - x_j + g_j \geq 0 \quad \forall j \quad (4.3)$$

Finally, the capacity of each line must be greater than the quantity of power being transmitted by that line.

$$|y_i| - C_i \leq 0 \quad \forall i \quad (4)$$

Electricity demand in nodes that are not connected to the grid can be met through individual decentralized technologies, such as solar home systems, or left unserved.

### 4.2.3 Stochastic Events and Parameters

The stochastic events and parameters that are considered by this analysis are listed in Tables 4.1 and 4.2. Each stochastic parameter is drawn from a normal distribution with the defined mean and standard deviation, and stochastic events occur with the defined probability each quarterly period. The price of fuel oil is randomly updated each period in accordance with geometric Brownian motion, as in Equation 4.5, where $p_t$ is the price in period $t$, $\mu$ is the expected mean annual growth rate, $\sigma$ is the standard deviation of this growth rate and $n$ is the number of periods per year. $N(0,1)$ represents a random variable generated from the standard normal distribution with a mean of 0 and a standard deviation of 1.
Table 4.1: Stochastic parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 on-Grid Electricity Demand (kWh/person)</td>
<td>600</td>
<td>60</td>
</tr>
<tr>
<td>2011 off-Grid Electricity Demand (kWh/person)</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>2030 on-Grid Electricity Demand (kWh/person)</td>
<td>1200</td>
<td>120</td>
</tr>
<tr>
<td>2030 off-Grid Electricity Demand (kWh/person)</td>
<td>500</td>
<td>50</td>
</tr>
<tr>
<td>Annual Development Budget (million $)</td>
<td>250</td>
<td>62.5</td>
</tr>
<tr>
<td>Annual Oil Price Increase</td>
<td>2.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Transmission Loss Rate</td>
<td>20%</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 4.2: Stochastic event probabilities.

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation Facility Outage</td>
<td>10%</td>
</tr>
<tr>
<td>Transmission Line Outage</td>
<td>10%</td>
</tr>
<tr>
<td>Political Unrest</td>
<td>5%</td>
</tr>
</tbody>
</table>

\[ p_{t+1} = p_t + p_t \cdot \left( \frac{\mu}{n} + \frac{\sigma}{\sqrt{n}} \cdot N(0,1) \right) \]  \hspace{1cm} (4.5)

The modeler also defines a set of development objectives which can include new centralized generation facilities, transmission lines, local distribution lines and distributed solar or diesel generation systems. These are placed in a development queue and contributions are made towards their construction each period. Each period, there is a limited budget that can be applied to constructing new projects and repairing existing infrastructure that requires maintenance. In each period, this development budget is generated as a normal random variable with predefined mean and standard deviation. This reflects the natural variability in the length of time it takes to complete different infrastructure projects. In addition, a number of stochastic events have a modeler-defined probability of occurring in each period. These include outages at each generation site, outages on each transmission edge, and instances of government political instability. Outages require repairs to be made, diverting development capital from other projects in
the queue and delaying their construction. Instances of political instability result in no
development capital being available for that period, also adversely impacting the
expedient construction of new infrastructure.

4.2.4 Model Simulation

Each period is simulated according the following steps,

1. Probabilistic parameters are generated, including
   a. Available development capital
   b. Changes in commodity prices
   c. Demand in each node during each time step
2. Stochastic events are simulated, including
   a. Disruption to generation facilities
   b. Disruption to transmission lines
   c. Disruption to available development budget
3. Repairs are performed on infrastructure that is out of service
4. Construction is performed on new projects in the development queue
5. Demand is served for each node and each timestep

This model is driven by random variable generation and a simple linear program,
which require relatively few computational resources compared to the MIP methodology
presented in chapter 3. As a result, it is possible to perform a number of simulations and
obtain distributions of the resulting total system cost and quantity of unserved electricity
over the entire 20-year time horizon for each proposed development path. This provides
insight into the impact of stochastic events on the cost and availability of energy services
through analysis of the full range of potential outcomes, rather than focusing on
estimating the mean result.
4.3 Rwanda Case Study

A case study analysis of Rwanda is now presented to demonstrate an application of the model. This analysis complements previous work, which provides a deterministic MIP formulation for determining the least-cost combination of centralized and decentralized electrification infrastructure that is capable of meeting demand [193]. We build upon this work by adding time periodic and stochastic elements in order to understand how these factors influence the cost of a given infrastructure and its ability to serve demand.

Rwanda is a small, landlocked country in East Africa with a population of roughly 10 million people. In our model, the country is represented as a set of 224 demand nodes in an evenly spaced grid with an intermodal distance of six arcminutes or approximately 11 km. Each node has an associated electricity demand that evolves over time and can be met through electricity from the grid, or decentralized technologies such as a solar home system (SHS) or diesel generator.

In 2004, Rwanda had an electrification rate of 6%, which grew slightly to 7% by 2009. However, most of the electrified population is located in the capital of Kigali, with less than 1% of the rural population having access. An aggressive electrification campaign combined with significant external funding increased this rate to 10% by early 2012 and is on track to reach 16% in 2013 [194]. We use 2010 as the base year of our analysis and do not directly incorporate any more recent development. A total of 55 MW of centralized generating capacity is considered, consisting mostly of hydroelectric and oil thermal generation facilities. There is also a 250 kW solar array near Kigali and an assortment of micro-hydro generation throughout the country [186]. The grid in Rwanda has two high voltage transmission backbones of 100 kV and 70 kV and a network of 30 kV, 15 kV and 6.6 kV distribution lines [187]. Eleven potential new generation facilities are considered by this analysis, which are outlined in Table 4.3. Construction on some of these facilities may have recently commenced; however, we do not include such recent activity in our baseline scenario in order to maintain consistency with earlier work.
Table 4.3: Potential centralized generation sites considered by this analysis [186–191].

<table>
<thead>
<tr>
<th>Location</th>
<th>Capacity (MW)</th>
<th>Overnight Cost (M$)</th>
<th>Generation Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Kivu I</td>
<td>25</td>
<td>140</td>
<td>Methane Thermal</td>
</tr>
<tr>
<td>Lake Kivu II</td>
<td>75</td>
<td>185</td>
<td>Methane Thermal</td>
</tr>
<tr>
<td>Mukungwa II</td>
<td>2.5</td>
<td>7.5</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Mukungwa III</td>
<td>2.2</td>
<td>6.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Nyabarongo</td>
<td>27.5</td>
<td>97.7</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rukarara</td>
<td>9.5</td>
<td>20</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rubavu</td>
<td>3.2</td>
<td>9.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rugezi</td>
<td>2.2</td>
<td>6.6</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rusizi III</td>
<td>48</td>
<td>150</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rusizi IV</td>
<td>68</td>
<td>212</td>
<td>Hydroelectric</td>
</tr>
<tr>
<td>Rusumo</td>
<td>20</td>
<td>57</td>
<td>Hydroelectric</td>
</tr>
</tbody>
</table>

4.3.1 Baseline Parameters

The baseline values of transmission line parameters are shown in Table 4.4.

Table 4.4: Baseline parameter values for transmission infrastructure [193]

<table>
<thead>
<tr>
<th>30kV</th>
<th>70kV</th>
<th>110kV</th>
<th>220kV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Cost ($/km)</td>
<td>15,000</td>
<td>75,000</td>
<td>150,000</td>
</tr>
<tr>
<td>Transmission Capacity (MW)</td>
<td>7.5</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Repair Cost ($/km)</td>
<td>1,500</td>
<td>75,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Construction Time (years)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Repair Time (months)</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

In addition to relatively high voltage transmission lines, low voltage distribution networks are also required within each node to connect individual demand points to the centralized grid; a process referred to as distribution infilling. A fixed cost of $2000 per household throughout the country is assumed, which is based on previous estimates in Kenya [20], though in practice this figure may vary regionally based on local geography.
and other factors. Solar home systems are also considered as a potential decentralized generation technology at a cost of $7 per Wp (peak watt), including a controller, battery and all wiring. This price is comparable with that of a 50 Wp SHS that is currently widely available in Bangladesh ($320 USD) [195].

Under baseline conditions, roughly 53% of the population lives in a node that is connected to the grid, but only 7% of the population actually has immediate grid access. Most of the electrified population in Rwanda is located in the capital of Kigali, where it is assumed that 50% of the population has immediate access to the grid. Initial electricity demand in grid-connected nodes is based on historical data that recorded 240 GWh of total consumption in 2009, 80% of which was in Kigali [171]. Assuming a 50% electrification rate in Kigali and 8% in other grid connected nodes, this corresponds with a per capita demand of roughly 600 kWh for consumers in Kigali who currently have electricity access, and 100 kWh for consumers outside of Kigali who have electricity access. We assume that all un-electrified populations have equivalent potential demands for electricity, and that these demand levels grow linearly to 1200 kWh and 500 kWh per capita by 2030 in Kigali and outside of Kigali respectively. Population is also assumed to grow at 3% per year, consistent with historical trends.

The model is executed over a 20-year time horizon, with four periods each year. Each period is also subdivided into two time steps to represent base and peak demand levels. The ratio of peak power demand to base levels in Rwanda was 1.75 in 2003 and this has been projected to drop to 1.67 in 2011 [185]. We assume that peak power demand is 1.70 times the base level and that the peak period lasts for 10% of total time.

### 4.3.2 Development Scenarios

Two development scenarios are considered in addition to a baseline scenario where no new development is made.

1. **NONE**

   In this scenario no new centralized generation and transmission or decentralized generation capacity is added to the current infrastructure. Current facilities are maintained and repairs are made when outages occur.
2. **SHS**
   In this scenario, solar home systems are developed with the goal of providing a 250 Wp SHS to 75% of households that do not currently have immediate grid access. No new centralized generation facilities, transmission lines or distribution infilling lines are constructed.

3. **GRID**
   In this scenario, the centralized grid is expanded with the goal of connecting all nodes in Rwanda. In addition, local distribution infilling is developed to provide 75% of the population in Kigali and 50% of the population in all other nodes with grid access. No solar home systems are developed, and all proposed centralized generation facilities (Table 4.3) are queued for development.

These two scenarios were specifically selected to explore the differences between centralized and decentralized development paths. This modeling framework could also be used to compare numerous variations on these two scenarios, by adjusting the penetration rate and average unit size of the SHS scenario, or the considered centralized generation facilities and extent of grid expansion in the GRID scenario. Hybrid scenarios that implement a combination both centralized and decentralized technologies could also be examined. The two chosen scenarios are analyzed to demonstrate a potential application of the model. However they are by no means intended to represent to full range of development options available to a country such as Rwanda.

**4.4 Results**

**4.4.1 Conventional**

The model is first executed for all three scenarios over a 20-year time horizon, with four annual periods and baseline parameter values. The model is executed conventionally, meaning that there are no random events and the standard deviation of all probabilistic parameters is set to zero. All generation facilities operate consistently at a predetermined generating capacity, transmission lines have a constant transmission
capacity and the development budget is also constant each period. Oil prices also fluctuate conventionally, increasing by 2.5% each year. The values of several parameters are adjusted from their baseline values so that they are equal to the mean expected result when stochastic events are considered. For example, the capacity factor of each centralized generation facility is reduced by 10% in the conventional formulation to reflect the 10% chance of an outage each period when stochastic events are considered. Similarly, the transmission capacity of each line is reduced by 10% and the annual development budget is reduced by 5% to reflect the possibility of political unrest. Fixed annual repair costs are also added, which are equal to the mean expected costs when outages are explicitly modeled.

![Figure 4.1: The annual costs of each development scenario over the 20-year time horizon under conventional conditions](image)

Figure 4.1 shows the annual expenditure for each development scenario under conventional formulation. The SHS scenario requires nine years of capital expenditures equal to the full annual budget of $250 million in order to install new solar home systems for in regions without electricity access. This period is followed by lower annual capital expenditures to keep pace with population growth. The GRID scenario requires a longer period of sustained capital investment to construct new transmission infrastructure and generation facilities. Once new centralized infrastructure is constructed there is also a need for additional repairs and maintenance. The total capital expenditures required by the grid scenario are greater than they are for the SHS scenario. However, the GRID scenario also serves more electricity demand over the 20-year time horizon, so these extra costs do not necessarily imply that the SHS development path would be preferred option. Lifetime costs of generation will be compared on a levelized basis later in this analysis.
Figure 4.2: Served demand for each scenario over the 20 year time horizon. Dashed lines represent fixed per capita consumption levels and reflect the fact that, for each fixed level, total consumption increases over time in accordance with population growth.

Figure 4.2 shows the average level of served demand for each scenario over the 20-year time horizon, dashed lines represent fixed per capita consumption levels. When there is no development, the service level remains essentially constant, as no new consumers are provided with electricity access and no new generation capacity is constructed. In the SHS scenario, new generation capacity is added immediately, reaching initial electrification targets and 85 MW of average served demand by 2019. At this point SHS development continues at a slower pace through 2030 to keep pace with increasing population, reaching an average consumption level of 120 MW. Under the GRID scenario there is a three year delay before new generation capacity comes online, which represents the minimum anticipated lead time of these new facilities. New generation begins to come online in 2014 and continues through 2016, at which point
capital investments are directed towards new transmission infrastructure. Average served demand grows to 195 MW in 2023 and increases further to 210 MW in 2030.

Figure 4.3: Discounted served demand and discounted cost of each scenario under conventional conditions.

Figure 4.3 depicts the present cost and corresponding present quantity of served electricity demand for each scenario. The ratio of these two terms gives the levelized cost of electricity, several fixed values of which are indicated with dashed lines. The NONE scenario is able to provide electricity at the lowest levelized cost of the three scenarios. However, as no new infrastructure is developed, only a relatively small quantity of discounted demand, 1850 GWh, is served. The SHS scenario serves more discounted demand, 5259 GWh, at a greater unit cost than the NONE scenario, $0.38 per kWh compared to $0.19 per kWh. The GRID scenario serves still more discounted demand, 7450 GWh, at a lower unit cost than the SHS scenario, $0.32 per kWh.

It is perhaps more useful to consider the cost and service level of the SHS and GRID scenarios that occur in addition to what is experienced under the NONE scenario.
The new served demand and new system cost are calculated by subtracting the average service level and cost experienced under the no development scenario. This isolates the cost of new electricity generation and provides a constructive metric for comparing the costs of new generation under each scenario. Taking the ratio of these two values gives the levelized cost of new electricity (LCOE\textsubscript{new}), which reflects the unit cost of any additional service that is provided by these two scenarios.

\[ LCOE_{\text{new}} = \frac{PV(Y_i) - PV(Y_{\text{NONE}})}{PV(C_i) - PV(C_{\text{NONE}})} \quad (4.7) \]

When considering only new costs and generation, the relative cost-effectiveness of the GRID scenario increases slightly relative to the SHS scenario. The LCOE\textsubscript{new} of the SHS scenario is $0.48 per kWh, while the LCOE\textsubscript{new} of the GRID scenario is $0.37 per kWh. This occurs because the relatively cheap electricity generated under the NONE scenario is no longer included and as a result the average generation cost of the SHS scenario increases more significantly.

The levelized carbon emissions of each development scenario are shown in Figure 4.4. These are calculated similarly to the levelized cost of electricity by taking the ratio of discounted lifetime carbon emissions and discounted lifetime electricity generation. Under baseline conditions, .52 kg of CO\textsubscript{2} are emitted for each kWh of electricity that is generated, equivalent to total annual emissions of roughly 27,000 tonnes. Under the SHS scenario, no new carbon emitting electricity generation is developed and, due to the new solar generation, the levelized emissions rate falls to .18 kg per kWh. The GRID scenario adds a number of hydroelectric generation facilities that do not generate emissions, but also adds new methane plants at Lake Kivu that do result in additional emissions. The levelized emissions rate under the GRID scenario adds a number of hydroelectric generation facilities that do not generate emissions, but also adds new methane plants at Lake Kivu that do result in additional emissions. The levelized emissions rate under the GRID scenario is .46 kg per kWh, with total annual emissions growing more than 4-fold versus the SHS scenario to approximately 120,000 tonnes of CO\textsubscript{2} in 2016 and beyond. The same methodology that was used to calculate the LCOE\textsubscript{new} values can be applied to yield .32 kg of CO\textsubscript{2} emissions for each new kWh of generation under the GRID scenario, versus zero new emissions under the SHS scenario.
4.4.2 Stochastic

We now analyze the effect of explicitly modeling stochastic events as opposed to considering fixed generation and transmission capacities, commodity prices and annual development budgets. Under this formulation, each generation facility and transmission line has 10% chance of going offline in any period, at which point a repair cost is incurred. Parameters are chosen so that the expected mean capacity factor, transmission capacity and repair cost is the same under these stochastic conditions as they were under the conventional formulation. Commodity prices also fluctuate according to Geometric Brownian Motion, with mean annual cost change equal to the constant rate applied under conventional conditions. We are, therefore, able to explore how well a conventional simulation using mean values is able to replicate a more explicit stochastic simulation.

Each scenario is executed over 50 iterations and, for each iteration, random events are held consistent across all three scenarios. For example, if during the first iteration of the NONE scenario a particular generation facility experiences and outage in year 2018, then the same outage will occur during the first iteration of the SHS and GRID scenarios.
Therefore it is possible to analyze how each scenario manages the same set of random occurrences.

Figures 4.5 and 4.6 show the range of outcomes across the 50 iterations for each scenario. The lifetime discounted cost varies from $1.90 billion to $2.30 billion for the SHS scenario, and from $2.25 billion to $2.75 billion for the GRID scenario. The level of discounted demand varies from 4650 GWh to 5150 GWh and from 5000 GWh to 6700 GWh for the SHS and GRID scenarios respectively.

Figure 4.7 shows the LCOE\textsubscript{new} values for each iteration of the SHS and GRID scenarios under stochastic simulation. The horizontal black line represents the LCOE\textsubscript{new} values obtained under conventional modeling, $0.48 per kWh and $0.37 per kWh for the SHS and GRID scenarios respectively. The LCOE\textsubscript{new} values of the SHS scenario are essentially identical to the deterministic results for all 50 iterations. This is explained by the fact that new solar home systems are not affected by random generation outages or transmission disturbances. All stochastic events affecting the grid that are experienced under the SHS scenario are also experienced during the same iteration of the NONE scenario. These effects therefore cancel out in the calculation of LCOE\textsubscript{new} for each SHS iteration.

The GRID scenario shows more variable results, with LCOE\textsubscript{new} values ranging from $0.44 per kWh to $0.53 per kWh, and a mean result of $0.49 per kWh. Perhaps more significant is the fact that for all 50 iterations, the LCOE\textsubscript{new} of the GRID scenario is greater under stochastic simulation than under conventional modeling. In 30 out of 50 iterations the LCOE\textsubscript{new} of the GRID scenario also exceeds the LCOE\textsubscript{new} of the SHS scenario, when stochastic events are explicitly modeled.
Figure 4.5: Discounted new served demand and discounted new cost of each model iteration under stochastic conditions. Dashed lines indicate fixed levelized costs of generation and the box indicates the region that is highlighted in Figure 4.6.
Figure 4.6: The boxed region of Figure 4.5 is shown in more detail to highlight the outcomes of all 50 iterations for the SHS and GRID scenarios.
Figure 4.7: LCOE\textsubscript{new} for each iteration of the solar home systems and grid expansion scenarios. The horizontal black lines represent the LCOE\textsubscript{new} values from the conventional formulation.

4.4.3 Sensitivity Analysis

The model is also executed with variations in two key parameters to understand the sensitivity of each development scenario to changes in these parameters. These are the annual development budget and the discount rate. We consider both the stochastic and conventional results for the GRID scenario, and omit the conventional SHS result because of its similarity to the stochastic result.

4.4.3.1 Discount Rate

The choice of a discount rate can heavily influence the results of a long-term cost-benefit analysis. A discount rate can be thought of as the time value of money, or as the opportunity cost of tying up funds and being forced to passing on other potential investments. A higher discount rate places more relative weight on consumption and expenditure in early periods. Discount rates are often based on the interest rate paid by an investment that is perceived to be risk-free, such as a U.S. Treasury Bill or other high-quality corporate bonds [196]. A social discount rate can also be applied to account for the relative value of present social benefit as compared to future social benefit. A common application of social discount rates can be found in environmental impact analyses, which must place a quantitative value on present and future environmental well-being [197, 198]. This same method can also be applied to the consumption of energy services, which may in turn improve health and generate economic output [199]. The choice of a social discount rate may be based on the appropriate financial discount rate [200]; however, there is more latitude in this choice as social benefit is a more subjective concept than financial performance.

In the developed world a discount rate of 3-7% is commonly applied, while in the developing world, where investment uncertainty is often higher and credit markets are less liquid, a higher discount rate of 8-15% may be used [192]. However, discount rates
can also vary significantly within countries or between different consumers. In this analysis a baseline discount rate of 10% is applied and the sensitivity of results to a discount rate ranging from 0% to 25% is also analyzed.

Figure 4.8 shows the average LCOE\textsubscript{new} across all 50 iterations for each development scenario, while varying the consumer discount rate from 0% to 25% and holding all other parameters at their baseline values. As was seen previously, under the baseline consumer discount rate of 10%, the average LCOE\textsubscript{new} of the SHS and stochastic GRID scenarios are fairly similar, $0.48$ per kWh and $0.49$ per kWh respectively. While, the LCOE\textsubscript{new} of the convention GRID result is significantly less, $0.37$ per kWh.

As the consumer discount rate increases, the LCOE\textsubscript{new} of both GRID results increase relative to the SHS scenario; the opposite holds for decreasing discount rates. This is explained by the fact that the GRID scenario requires upfront capital investment to, but delays the provision of additional generation for several years until new generation
infrastructure is constructed. Alternatively, the initial capital investments in the SHS scenario enable additional demand to be served immediately. When the discount rate is high, this early consumption is weighted relatively more heavily, leading to a lower levelized cost. Therefore, capital investments that do not immediately produce tangible benefits will generally be relatively more attractive for consumers and investors with lower discount rates. When the consumer discount reaches 25\%, the LCOE_{new} of the SHS scenario ($1.57) is significantly less than that of the stochastic GRID result ($2.03) and comparable to that of the conventional GRID result ($1.49). It can be difficult to precisely determine the actual discount rate of different consumers, and as such, estimations or approximations are often made when conducting economic analyses. These results illustrate how significantly the choice of a discount rate can influence the findings of such analyses, while also highlighting the differences between the conventional and stochastic formulations of the GRID scenario.

### 4.4.3.2 Development Budget

Large-scale infrastructure development projects in developing countries may experience delays, run over budget or be restricted by limited availability of capital. These variations in lead time can significantly impact the realized cost and benefit of a given development plan, particularly when consumer discount rates are high. We now analyze how variations in the average annual development budget affect the cost and service level of both the SHS and GRID development scenarios. The development budget analysis is conducted by varying the mean annual budget from $50 million to $500 million while holding all other parameters constant. For each fixed development budget, five simulations are conducted and the mean result is calculated.
Figure 4.9: Discounted new cost and new served demand for the SHS and stochastic GRID and conventional GRID scenarios as the average annual development budget is varied from $50 million to $500 million. The numerical labels denote the development budget in millions for the corresponding data point.

Figure 4.9 shows how the discounted new cost and level of new served demand changes as the average annual development budget is varied from $50 million to $500 billion. The GRID scenarios are more affected than the SHS scenario by changes in the annual development budget. With an annual development budget of $50 million the stochastic GRID scenario serves 460 GWh of discounted new demand, compared to the 770 GWh that are served under the SHS scenario for a similar cost. Under conventional formulation the GRID scenario serves significantly more discounted demand, 2500 GWh, at a greater discounted cost ($750 million vs. $400 million). The stochastic GRID scenario continues to serve less demand than the SHS scenario when the annual development budget is less than $200 million. For budgets in excess of $200 million the discounted service level of the stochastic GRID scenario exceeds that of the SHS scenario.
These results are explained by the fact that centralized infrastructure development requires large, sustained investments in order to provide future generation several years in the future. As a result, a limited development budget can significantly delay project completion so no electricity is generated in early periods. Additionally, centralized generation facilities involve so-called ‘lumpy investments’. This means that the entire benefit of a new generation facility is only realized after it is completely developed, while no benefit is realized for partial development. Alternatively, the size of each development unit is much smaller for SHS scenarios (a 50 Wp system), and therefore a partial level of development still results in a proportional level of generation and benefit. Specifically, assuming a three year minimum construction time for each facility, the annual development cost of simultaneously constructing all of the considered centralized generation plants (Table 4.3) is approximately $300 million. Therefore, the baseline $250 million annual budget is roughly sufficient to enable completion of all new generation plants by 2015, when also accounting for some immediate transmission upgrades and required repairs. At this point the entire budget then becomes available for transmission expansion and further capacity upgrades to ensure that the new generation capacity is able to reach consumers. A $500 million annual budget enables all the new generation sites to be constructed in their minimum three year construction times. This allows new centralized generation to reach consumers more rapidly, and this early consumption is therefore discounted less heavily in the calculation of total discounted served demand. Increases to the annual budget beyond $250 million do not significantly increase the level of discounted demand served under the SHS scenario. This is explained because there is not sufficient electricity demand in early periods to justify such a significant level of investment.

The additional benefit of a large budget is more pronounced under the conventional formulation of the GRID scenario, than the stochastic formulation. This can be partially explained by the fact that a consistent and predictable annual budget is generally preferable over an annual budget with the same mean that varies unexpectedly from year to year. Most development plans are designed with a specific budget in mind, and therefore unexpected budget reductions can cause delays and lead to inefficient spending. On the other hand, resources may not be available to efficiently take advantage
of unexpected budget windfalls, and in our analysis these funds are not carried over to future periods. This reflects political reality, where departments or organizations with excess funds may see their future budgets reassigned to other needier areas.

Figure 4.10: LCOE\textsubscript{new} for the SHS scenario and both formulations of the GRID scenario as the average annual development budget is varied from $100 million to $1 billion.

Figure 4.10 shows how the LCOE\textsubscript{new} values for the SHS scenario and both formulations of the GRID scenario change as the average annual development budget increases from $50 million to $500 million. When the annual development budget is $50 million, the LCOE\textsubscript{new} of the SHS scenario ($0.52) is less than the LCOE\textsubscript{new} of both the stochastic ($0.89) and conventional ($0.58) formulations of the GRID scenario. As the development budget increases, the LCOE\textsubscript{new} of the both GRID scenario formulations gradually decrease, and for a budget of $250 or greater the LCOE\textsubscript{new} of the stochastic result is equal to, or marginally less than, that of the SHS scenario. The LCOE\textsubscript{new} of the SHS scenario is largely unaffected by changes in the development budget. This is explained by the fact that every unit of expenditure under the SHS scenario results in a proportional, and immediate, amount of new electricity generation.
4.5 Conclusions

We have demonstrated a stochastic model for simulating the service of electricity demand when considering various stochastic events and parameters, through a case study of Rwanda over a 20-year time horizon. This model provides a basis for comparing the cost of different electricity infrastructure development paths and their ability to electrify new populations and serve increasing electricity demand. We present an analysis of two specific development paths to demonstrate a potential application of the model. We also compare the impact of explicitly simulating stochastic events and parameters as opposed to assuming constant mean parameter values. Finally, we examine sensitivities around the discount rate and annual development budget.

The purpose of our analysis is not to suggest specific development plans or budget levels for Rwanda, but rather to use this case study as a tool for demonstrating our methodology and understanding the impacts of utilizing different modeling frameworks. It would, therefore, be prudent to explore more options and utilize more detailed input data before using the results to drive long term policy implementation.

Our results suggest that the SHS scenario enables more immediate electrification of new populations, but the GRID scenario provides greater quantities of electricity over the long term. Given our baseline assumptions, the levelized cost of a new unit of electricity generation is similar for both the SHS scenario and the stochastic formulation of the GRID scenario. It is also shown that the GRID scenario presented in this analysis results in increased CO\textsubscript{2} emissions of .32 kg per new kWh of generation as compared to the SHS scenario. We do not attempt to directly value the environmental cost of these extra emissions (and other externalities associated with electricity generation from fossil fuels), instead allowing the reader to interpret these results in the context of their own valuation. As a result, the optimal development path of these two options may fluctuate depending on various chosen parameter values including the development budget, discount rate, infrastructure costs, outage probabilities, demand levels and the value placed on environmental degradation. It is also possible that some combination of these two development paths could serve demand more efficiently than either individually.

We find that reductions in the annual development budget and increases in the discount rate hinder the effectiveness of the GRID scenario more significantly than the
SHS scenario. We therefore suggest that additional caution should be taken when implementing a large-scale centralized infrastructure development project in favor of a decentralized counterpart that may appear to be more expensive or have comparable cost. While the centralized plan may seem to be the cheaper option, it may also be more susceptible to unforeseen random events, such as political transitions and budget disruptions that are often characteristic of developing countries. This is partially explained by the fact that a distributed infrastructure can generally be implemented more quickly than a centralized infrastructure that requires significant quantities of coordinated capital investment, planning and organization. As a result, the benefit of an investment in distributed technologies is realized more immediately, whereas with a centralized infrastructure, a sustained long term investment is required to realize the promise of future benefits. Therefore, the disruption, alteration or cancellation of a long-term, large-scale centralized project, may result in large expenditures that do not create any added benefit to investors and potential consumers.

Additionally, we find that conventional modeling of centralized development may underestimate the true costs of electricity provision, when stochastic events and probabilistic parameters are present. As an example of this effect, consider a high voltage transmission line with 100 MW of capacity that typically transmits no more than 80 MW of power and is assumed to require 10% downtime for maintenance and repairs. If the conventional approach is taken, and it is assumed that 10% downtime results in a consistent capacity of 90 MW, there will be no reduction in the amount of power that is transmitted. However, if outages are directly simulated, periods of downtime will occur when the line is not capable of transmitting any power, resulting in disruptions and reduced service levels. Another example can be seen in the case of a large generation facility that periodically requires repairs or maintenance at significant cost. Under conventional conditions it may be the case that there is always a sufficient budget to fund the assumed average cost of such repairs. However, when probabilistic parameters are modeled, a service disruption may occur during a period of limited budget availability. As a consequence, the generation facility may experience a longer than expected period of downtime. An effort can be made to account for this effect by adjusting the expected
repair time of a given facility. However, it can be difficult to accurately estimate the
effects of such interactions without the benefit of explicit stochastic modeling.

We also show that the choice of consumer discount rate heavily impacts the
optimal choice of development path, with high discount rates favoring decentralized
development plans that are less ‘lumpy’ and can be implemented more quickly. Access
to electricity provides rural populations with new economic opportunities, increased work
hours, and improved education, health, security and entertainment. Many studies have
attempted to quantify the social and economic benefits of rural electrification, and there is
significant evidence to suggest that rural electrification has a positive impact on enabling
structural transformation and development [51, 57, 201], reducing poverty [53] and
enabling other social and economic benefits [55, 56, 58]. In developing countries, where
consumer discount rates are high, the extra time required to develop a comprehensive
centralized network can result in an opportunity cost to these consumers, which would be
avoided by more quickly implemented distributed solutions.
5.1 Introduction

Access to electricity is a widely cited development goal, but opinions often differ on the optimal means of extending electrification to regions that are not currently served. It has been seen in previous chapters that a number of parameters can affect this decision, both from least-cost and other tangential perspectives. However, electricity on its own does not have an intrinsic value: electricity creates value to the consumer through the energy services that it enables [201]. As such, it has been suggested that development plans focus first on determining which energy services support development goals, before then working to understand which energy carriers can provide these services [162].

Previous chapters have focused primarily on comparing various options for rural electrification, but do not directly address non-electric alternatives for providing similar energy services. Evidence from numerous empirical studies suggest that lighting is one of the first energy services enabled by rural electrification [51, 54, 56, 86, 89, 93]. Lighting can be provided through a variety of non-electric means, including paraffin candles, kerosene lanterns and battery powered flashlights all of which are commonly used in un-electrified areas today. These various technologies can be directly compared on the basis of their cost of providing 1000-lumen-hours of lighting service [202].

Chapter 5 addresses the costs of providing lighting services considering electric lighting through grid-based centralized systems, distributed electricity systems and non-electric lighting systems. Section 5.2 evaluates the cost of electricity from grid expansion and from decentralized systems as a function of annual per capita electricity consumption and the per-household cost of grid connection. It also evaluates the subsidies for grid expansion by comparing the cost of electrification with the price charged to consumers. Section 5.3 evaluates the cost of subsidy-free lighting, comparing LED lighting powered by grid electricity and distributed electricity with lighting from solar lanterns, battery-operated flashlights and kerosene lamps. Section 5.4 evaluates financing mechanisms for providing distributed electricity on the basis of their ability to increase consumer utility
for a given cost to the providing agency. The considered financial mechanisms are direct subsidies, rental programs, and microloans.

5.2 Decentralized Electricity vs. Grid Expansion

The costs of different electricity generation technologies are often compared on the basis of their levelized cost, a metric that encapsulates the total lifecycle costs of electricity generation into a single cost per unit of generation. While levelized cost is a useful comparative metric, it also has limitations. For example, it has been argued that levelized cost is not appropriate for comparing centralized generation technologies with those that have intermittent output profiles, such as solar or wind, as levelized cost analyses do not account for the time-dependent value of electricity [182]. In addition, levelized cost calculations do not generally include the costs of transmission infrastructure that are required by centralized generation technologies. These costs may represent a significant portion of the total cost of electricity, particularly in regions with low levels of electricity consumption. As such, centralized technologies, which offer a small variable cost but require a large fixed transmission cost, may be cheaper in high consumption regions, while decentralized technologies may be cheaper in low consumption regions. Instead of comparing the levelized costs of centralized and decentralized technologies in absolute terms, it may be appropriate to consider the consumption level at which the least-cost option switches from decentralized to centralized [183]. For low electricity consumption levels, a distributed technology may provide the most cost effective option even if generation costs are relatively high on a per unit basis.

The costs of grid extension depend on a number of factors, including the geographical distribution of the target population. Previous studies have looked at how various factors impact the decision between developing new centralized or decentralized infrastructures [9, 21, 23, 183, 203, 204]. As a basis for examination of electrification services and financing, we model the decision for new electrification by analyzing a range of potential costs of both grid extension and solar home systems.
A previous analysis of grid expansion costs in Kenya has estimated a cost of $10,600/km in 2007 dollars for low voltage (LV) transmission lines used to connect rural households with peak power demand less than 1 kW to the nearest MV backbone, in addition to a fixed $150 connection cost per household [20]. Applying a spatial planning model to a randomly distributed population with density of 250 people/km$^2$, the average cost of connecting a new household is estimated to be $1780 under 100% penetration and $1250 under a realistic penetration scenario.

Equation 5.1 can be used to calculate the cost per household of electricity distribution infilling in various different situations. Here $D_I$ represents the distance of the initial connection required to connect a community to a transmission backbone, $D_{HH}$ represents the average distance between households, $N$ represents the number of households in the community, $x$ represents the per kilometer cost of LV lines and $F$ represents the fixed connection cost per household.

$$C_{HH} = \left( \frac{D_I}{N} + D_{HH} \right) \cdot x + F \quad (5.1)$$

For example, assuming a transmission line cost of $10,600/km and a fixed connection cost of $150 per household, a community 5km from the MV backbone with 20 households that are on average 50m apart could be connected to the grid for a cost of $3,330 per household.

For rural electrification, the unit cost of electricity delivered from the grid is highly dependent on consumption levels, or the quantity of electricity over which the fixed costs of transmission infrastructure can be spread. In Figure 5.1 the levelized cost of electricity from a SHS and the grid are shown as a function of average annual household consumption for a range of different marginal household connection costs. The fixed annual costs of grid expansion and a SHS are calculated based on financial parameters and SHS capital costs that are typical of Ghana; these are summarized in Tables 5.1 and 5.2 [205]. It is assumed that solar home systems are indefinitely scalable at the same unit cost as a 100 W system.
Table 5.1: Parameters for calculation of fixed annual costs from a SHS and the grid [205].

<table>
<thead>
<tr>
<th></th>
<th>SHS</th>
<th>Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan Term (years)</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>28%</td>
<td>15%</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>Annual Maintenance</td>
<td>$5/year</td>
<td>1% of capital cost</td>
</tr>
<tr>
<td>Lifetime (years)</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>Battery Replacement Frequency (years)</td>
<td>5</td>
<td>---</td>
</tr>
<tr>
<td>Marginal Generation Cost ($/kWh)</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.2: Full and subsidized capital costs of a 15 W, 50 W and 100W SHS. Solar home systems are often packaged with appliances, such as lighting, televisions and fans that are specifically designed for use with the system. These prices represent best estimates of the cost of a system including a solar panel, controller, battery and wiring, but not including any accompanying appliances [205].

<table>
<thead>
<tr>
<th>Power Capacity</th>
<th>List Price</th>
<th>Subsidized Price</th>
<th>Battery Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 W</td>
<td>$450</td>
<td>$300</td>
<td>$68</td>
</tr>
<tr>
<td>50 W</td>
<td>$725</td>
<td>$425</td>
<td>$109</td>
</tr>
<tr>
<td>100 W</td>
<td>$900</td>
<td>$550</td>
<td>$135</td>
</tr>
</tbody>
</table>
Figure 5.1: The levelized cost of electricity from an unsubsidized SHS is compared to the levelized cost of centralized electricity when accounting for varying marginal household grid connection costs.

It can be seen from Figure 5.1 that a 15 W or 50 W SHS is a cheaper option for corresponding levels of electricity consumption if the marginal household grid connection costs exceed $1000 per household. A fully utilized 100 Wp system is the cheaper option if marginal connection costs exceed roughly $1200 per household. For higher levels of consumption electricity from the grid becomes increasingly cost effective and for consumption of 1000 kWh per year a SHS is only the cheapest option if marginal connection costs exceed $10,000 per household. These results indicate that for low consumption levels, solar home systems may provide a more cost-effective option in targeted communities than grid expansion.

In rural, low-consumption, grid-connected regions, it is unlikely that the full cost of grid expansion would be passed on to the consumers through grid tariffs. Figure 5.1 indicates that the true cost of grid electricity in rural regions can easily exceed $1.00 per kWh if consumption levels are low or connections costs are high. Governments often consider electricity access to be a basic service that should be universally provided. Therefore direct subsidies or cross-subsidies are commonly offered for centralized electricity consumption in rural areas, and this full cost of electricity provision is often not passed on to consumers. For example, in Brazil [206], Cambodia [207], Ghana [208] and South Africa [209], rural grid tariffs do not generally exceed $0.30 per kWh. These rates imply a level of subsidization in grid connected regions where connection costs are high or consumption levels are low. Figure 5.2 illustrates the magnitude of this subsidy as a function of household electricity consumption, based on the rural grid tariff of roughly $0.30 cents per kWh in South Africa. This subsidy would be greater in countries such as Brazil and Ghana where the rural grid tariff is equal to or less than the tariff in more urban regions. If the government would otherwise largely finance the costs of grid expansion, it may be more cost-effective to provide subsidized or even free SHS for areas with high connection costs or small anticipated consumption levels.
Figure 5.2: The subsidy of electricity from the grid to rural customers in South Africa is shown as a function of annual electricity consumption for a range of marginal household connection costs.

5.3 Cost Comparison of Lighting Technologies

The value of electricity derives from its ability to provide energy services. Lighting is one of the most important energy services enabled by rural electrification, and can be provided by electricity from either the grid or a SHS, a solar lantern, a flashlight with standard alkaline batteries or other non-electric technologies such as kerosene or paraffin lamps.

High-efficiency electric lighting technologies include light emitting diodes (LEDs) and compact fluorescent lights (CFLs). We consider a small three-watt LED light source as our benchmark electric lighting technology for rural applications. Commercially available warm white LED packages had an estimated efficiency of 70 lumens per watt (lm/W) in 2009, and a corresponding first cost of $36 per kilolumen (klm). These figures were anticipated to improve dramatically to 128 lm/W and 11 $/klm by 2012 and 234 lm/W and 1.10 $/klm by 2020 [210]. We conservatively assume 2009 values of 70 lm/W and $36/klm, which may more accurately reflect technologies that are currently widely available in developing countries. Therefore a typical 3 W LED light
source would provide a light output of 210 lumens, roughly equivalent to a 15 W incandescent bulb.

In addition to grid connection and three different SHS capacities, we consider a small solar lantern that is powered by a 1.5 Wp solar panel, produces a maximum light output of 40 lumens and costs $40. Lanterns such as this one are becoming widely available in developing countries, and these specifications represent a typical product that is currently available in Ghana [205]. We also consider two additional lighting technologies: battery powered incandescent flashlights and simple kerosene lamps. The LED flashlights are assumed to run on AA batteries that cost $0.25 each and have 3 Wh capacity. The kerosene lamp is assumed to cost $2.00 and provide eight lumens of light output. The parameter assumptions for these lighting technologies are outlined in Table 5.3 and Table 5.4.

**Table 5.3: Fuel and efficiency assumptions for considered lighting technologies**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LED Efficiency</td>
<td>70 lm/W</td>
</tr>
<tr>
<td>LED Cost</td>
<td>$35/klm</td>
</tr>
<tr>
<td>AA Battery Capacity</td>
<td>2 Ah</td>
</tr>
<tr>
<td>AA Battery Cost</td>
<td>$0.25</td>
</tr>
<tr>
<td>Kerosene Fuel Cost</td>
<td>$0.60 / L</td>
</tr>
</tbody>
</table>

**Table 5.4: Cost of lighting calculation before cost of electricity is applied**

<table>
<thead>
<tr>
<th></th>
<th>LED</th>
<th>LED</th>
<th>Kerosene</th>
<th>Solar Lantern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Cost ($)</td>
<td>$7.35</td>
<td></td>
<td>$2.00</td>
<td>$40.00</td>
</tr>
<tr>
<td>Lifetime (years)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Electric Power (W)</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>Electricity Used (kWh/year)</td>
<td>4.4</td>
<td>1.5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Fuel Used (L/year)</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Annual Fuel Cost ($)</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$7.80</td>
<td>$0.00</td>
</tr>
<tr>
<td>Annual Replacement Cost ($)</td>
<td>$1.47</td>
<td>$0.69</td>
<td>$0.40</td>
<td>$8.00</td>
</tr>
<tr>
<td>Total Annual Cost ($)</td>
<td>$1.47</td>
<td>$0.69</td>
<td>$8.20</td>
<td>$8.00</td>
</tr>
<tr>
<td>Light Output (lm)</td>
<td>210</td>
<td>70</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>Annual light output (1000 lm-hours)</td>
<td>307</td>
<td>102</td>
<td>11</td>
<td>58.4</td>
</tr>
<tr>
<td>Cost w/o electricity ($/1000 lm-h)</td>
<td>$0.005</td>
<td>$0.007</td>
<td>$0.720</td>
<td>$0.137</td>
</tr>
</tbody>
</table>
Each technology has an associated variable cost of operation that comes from required part replacements, fuel, batteries or electricity. This cost is the same for each unit of generated light output. In addition, each technology requires an upfront fixed investment such as the cost of a light bulb, lantern, solar home system or grid connection, which is spread over the entire lifetime of usage. Therefore, in general, the unit cost of light output decreases as consumption levels increase. Rather than applying a fixed cost to each kWh of centralized electricity consumption, we consider the true cost of electricity provision as a function of consumption level and household connection cost, as outlined in Figure 5.1. Our economic analysis is framed in the context of electric delivery systems (SHS or grid) being used exclusively to provide lighting. These systems can also be used to provide other energy services, in which case the consumer would be substituting potential light consumption for another more highly desired service such as television, radio or cell phone charging. However, in order to compare electric and non-electric lighting technologies on common ground we frame our discussion and analysis in terms of light consumption.

Figure 5.3 shows the cost of light output from various technologies as a function of annual light output. Vertical lines indicate the annual light output from a single kerosene lantern and one or ten 3 W LED sources operating for 4 hours per day, as well as the average household light consumption in India and throughout the entire world, assuming five people per household [211].
Figure 5.3: The total cost of light output is shown for a various lighting technologies as a function of annual light production. Data on the costs of electricity from SHS the grid are from Tables 5.1 and 5.2.

Simple kerosene lanterns provide the cheapest lighting for very small consumption levels, less than 10 klh per year, due to their low capital requirement. LED flashlights are a marginally more expensive lighting source than kerosene lanterns for all consumption levels, though this relationship could easily reverse if kerosene prices or LED efficiencies increase. Solar lanterns are the cheapest off-grid lighting option for annual light production less than 700 klh, at which point a 15 W SHS becomes the cheapest option. A $500 per household grid connection also has roughly the same cost for this level of light production. As annual lighting production approaches 15,000 klh, the average level in India, a $2,000 per household grid connection becomes a lower-cost option than solar technologies. For production levels equal to the world average of 101,000 klh, a grid connection is the least-cost option as long as the connection cost is less than $10,000 per household.

As has been discussed previously, grid tariffs in rural, low-consumption regions often do not reflect the true cost of electricity provision. Electricity consumption for these consumers is therefore subsidized, whether consciously or otherwise. Figure 5.4 depicts the level of this grid subsidy as a percentage of the cheapest off-grid lighting
option. Cost assumptions for electricity from the grid are based on the tariff structure in South Africa; however, this calculation could be similarly applied to other countries. The resultant subsidy levels will be somewhat conservative compared to potential calculations in other regions as South Africa has a relatively high grid tariff compared to other countries that have been discussed, such as Brazil and Ghana. As an example, according to Figure 5.3, a household that costs $2,000 to connect to the grid and consumes 2,100 klh of light output (or equivalent other electric services) annually could have that service provided most cost-effectively by a 50 W SHS for about $147. A similar service could be provided through grid connection for a true cost of roughly $286 per year; however the realized cost to consumer in rural South Africa would only be about $9. Therefore, if this consumer is provided with grid access they are receiving an annual subsidy of $277, which is equal to roughly 188% of what it would cost to provide the same service with a SHS.

This analysis does not consider potential changes in demand over time. If demand were expected to increase significantly, an investment in grid expansion would become more cost-effective. However, this analysis also ignores some of the drawbacks of grid expansion, such as the required lead time. In many cases rural consumers must wait years for a grid connection to be provided by a central authority, whereas an independent and autonomously operated SHS could be obtained and installed in a month or less.
Figure 5.4: The subsidy provided for grid electricity is shown as a percentage of the cost of providing the same service from the least-cost distributed technology.

5.4 Financial Delivery Mechanisms

5.4.1 Overview

We have discussed how distributed technologies can often be less expensive than grid expansion; as shown above, small solar home systems may provide the lowest cost lighting services for lighting levels between about two LED lights and the average lighting level in India, depending on grid connection costs. However, for a variety of reasons low cost distributed electricity technologies are beyond the reach of many potential consumers. Technologies such as a SHS or solar lantern require a relatively large upfront capital investment in exchange for long-term provision of energy services at a low variable rate. The cost savings over the entire usage period may justify the investment, however, many rural consumers simply do not have access to the capital or credit required to make the initial purchase. This suggests that innovative credit mechanisms and business models could help consumers overcome the capital-intensive nature of these products and thereby increase access to electricity.
Here we present a model to compare three specific financial mechanisms that can be used to overcome the large capital requirement for obtaining a SHS on the basis of their ability to increase consumer net utility. These are technology subsidies, rental contracts and microloan programs. This model can be applied to determine the utility maximizing financial contract given a desired level of subsidization or cost to the providing agency – e.g. government, NGO or aid organization. We calculate the utility gain realized by a consumer under each delivery mechanism for a range of expenditures by the providing agency to determine the most cost-effective means of increasing consumer utility. We make no attempt to determine the optimal level of subsidization, and include situations where the desired subsidy level may be zero or even negative in our analysis. We provide a framework for determining the optimal delivery mechanism for a given cost (or profit) to those providing electricity services to consumers. The decision of whether or not to subsidize electricity systems and the extent of this potential involvement is left to each organization or government.

Previous work comparing different contracts for financing energy development includes a study by Srinivasan that compares the direct capital subsidy offered by the Indian government for solar PV systems with the interest subsidy offered for solar water heating systems [141]. Srinivasan finds the interest subsidy to be the superior option as, contrary to a direct subsidy, it does not have to be customized for a specific product offering and can be disseminated through existing capital markets. Chandrasekar and Kandpal similarly develop a mathematical framework for comparing the effective capital cost of solar technologies when different financial incentives are provided; such incentives include capital subsidies, low interest loans and accelerated tax depreciation [212]. They present threshold values below which the loan is preferred over the capital subsidy, finding that a loan offered at a 2% interest rate is equivalent to a 14.32% capital subsidy.

Much additional research in the context of financing for rural energy development is focused on analyzing empirical findings from programs that have been implemented in recent years. There is also a large body of literature discussing the general economic theory of subsidies, and a growing literature discussing the theory of subsidized and unsubsidized microloans. However much of this work focuses on one particular contract
or mechanism, as opposed to directly comparing multiple different options. An overview of relevant previous work specific to each of the three considered financial mechanisms was provided in chapter 1. A discussion of the experiences with real-world use of SHS and implementations of these financial mechanisms in several different countries is also provided in section 5.5.

5.4.2 Methodology

Consumers are allocated a fixed monthly income and may apply their money either to the purchase of a SHS or to other expenditures. Consumers receive utility for each unit of electricity they generate and also for each unit of income they apply to cash expenditures. The utility from each opportunity is a concave function with diminishing marginal returns, so that the utility gained from each successive unit of expenditure is marginally less than from the previous unit. Here $x$ is the monetary expenditure in a given period and $y$ is the electricity consumption level. All of the variables appearing in this section are listed in Table 5.5.

$$U = a \cdot x^b + c \cdot y^d$$  \hspace{1cm} (5.2)

In this analysis, the value of $a$ is fixed at one, meaning that the first dollar of expenditure each month provides one unit of utility. Because the system is assumed to produce a fixed quantity $y$ of electricity in each period that it is operational, it is more convenient to directly calculate the utility gained from this level of consumption. This can be expressed in terms of the monetary expenditure that would provide the same level of utility,

$$U = x^b + x^b$$  \hspace{1cm} (5.3)

where $x'$ is such that,

$$x^b = c \cdot y^d$$  \hspace{1cm} (5.4)

A sensitivity analysis will later consider a range of possible values for $b$ and $x'$.
5.4.2.1 Baseline

Two client scenarios are considered as reference points. In the first, the client
does not purchase a SHS but rather spends their entire monthly income on other
expenditures.

\[
U_{BASE_1} = \sum_{t=1}^{T} \frac{I^b}{(1 + d_c)^t} \tag{5.5}
\]

In the second, the client saves the entirety of their monthly income until they are
able to afford a SHS at full list price, at which point a purchase is made. The purchase
period is represented by \( t^* \) and disposable income during the SHS lifetime is equal to
total income less maintenance and battery replacement costs. The client also generates
electricity throughout the lifetime of the SHS. All equations presented here assume that
the lifetime of the SHS is greater than the time horizon, \( T \).

\[
U_{BASE_2} = \sum_{t=t^*}^{T} \frac{x^b}{(1 + d_c)^t} + \frac{I_{D_\beta}^b}{(1 + d_c)^t} \tag{5.6}
\]

where,

\[
t^* = \left\lfloor \frac{cost}{I} \right\rfloor \tag{5.7}
\]

\[
I_{D_\beta} = I - (maint + bat) \tag{5.8}
\]

The reference scenario is taken to be the utility-maximizing choice of these two
decision paths. There is no cost to the agency in the reference scenario. Our analysis
assumes that a consumer always has the option to immediately purchase a SHS at its full
price.
### 5.4.2.2 Subsidy

A basic technology subsidy is perhaps the simplest option for reducing the initial investment required to obtain an individual SHS. By reducing the required initial cost, clients who choose to save their income are able to obtain systems sooner and apply more of their future income to other expenditures.

The calculation of client utility under a subsidy is essentially the same as under the reference scenario where a SHS is purchased, except that the subsidy enables the client to purchase the system sooner. The disposable income under a subsidy contract \( I_{DS} \) is equal to the disposable income under the second baseline scenario as expressed in Equation 5.8.

\[
U_{SUB} = \sum_{t=t^*}^{T} \frac{x^b}{(1 + d_c)^t} + \frac{I_{DS}^b}{(1 + d_c)^t} \quad (5.9)
\]

\[
t^* = \left\lfloor \frac{\text{cost} - s}{l} \right\rfloor \quad (5.10)
\]

The cost incurred by the agency is simply the time-discounted value of the subsidy allocated in period \( t^* \).

\[
C_{SUB} = \frac{s}{(1 + d_a)^{t^*}} \quad (5.11)
\]

### 5.4.2.3 Rental

Under a rental contract, consumers pay a monthly fee for a SHS that is independent of their consumption level. The rental contract allows the consumer to obtain a SHS in the first period. The consumer is no longer responsible for maintenance and battery replacement, so their disposable income during the rental contract is simply their total income less the rental fee. After the expiration of the rental contract, the consumer applies their entire income to other expenditures.
Under a rental contract, the agency is responsible for battery replacements, monthly maintenance and administrative costs for operating the program. There is a fixed cost for each rental contract that is realized in the first period and a variable cost that is realized during each period that the SHS is being rented. The agency also purchases the SHS in the first period and receives a salvage value when each SHS is returned. The salvage value assumes flat depreciation over the lifetime of the SHS and is further adjusted by the salvage ratio.

\[
U_{RENTAL} = \sum_{t=1}^{t_{rent}} x^b + I_{D_r}^b \frac{1}{(1 + d_c)^t} + \sum_{t=t_{rent}+1}^{T} \frac{I^b}{(1 + d_c)^t} \tag{5.12}
\]

\[
I_{D_r} = I - r \tag{5.13}
\]

Here is the SHS salvage value at the termination of the rental contract.

\[
C_{RENTAL} = \frac{\text{cost} + f_r}{(1 + d_a)} + \sum_{t=1}^{t_{rent}} \left[ \frac{\text{maint} + \text{bat} + v_r - r}{(1 + d_a)^t} - \frac{SV(t_{rent})}{(1 + d_a)^{t_{rent}}} \right]\tag{5.14}
\]

\[
SV(t) = \frac{\text{life} - t}{\text{life}} \cdot sr \cdot \text{cost} \tag{5.15}
\]

### 5.4.2.4 Microloan

A microloan contract provides clients with capital for an immediate SHS purchase. Clients are then responsible for making regular monthly loan repayments which include interest. Similar to the rental contract, the agency faces fixed and variable management costs to administer the loan and collect monthly payments. The contract differs from the rental contract in that clients own their system outright and are therefore responsible for any maintenance and battery replacements once the system warranty expires. Clients can also default on their loan, causing the agency to lose out on future payments and forcing them to repossess the SHS, obtaining a salvage value in return. After clients default they apply their full income to other expenditures.
Calculations for client utility and cost to the agency under the microloan contract are similar to the rental contract. The client makes a monthly payment \( p \) during each period until the loan is paid off in full at the conclusion of the loan term.

\[
p = \frac{\text{cost} \cdot i}{1 - (1 + i)^{-t_{loan}}} \tag{5.16}
\]

The client’s utility can be calculated as follows,

\[
U_{\text{loan}} = \sum_{t=1}^{dt} \frac{x^b + I_{Dt}^b}{(1 + d_c)^t} + \sum_{t=dt+1}^{T} \left[ (1 - dr) \cdot \frac{x^b + I_{Dt}^b}{(1 + d_c)^t} + dr \cdot \frac{l^b}{(1 + d_c)^t} \right] \tag{5.17}
\]

The client is responsible for system maintenance and battery replacement throughout the lifetime of the SHS, but they only make a loan payment through the loan term. Their disposable income is therefore defined as,

\[
I_{Dt} = \begin{cases} 
I - (\text{maint} + \text{bat} + p): & t \leq t_{\text{loan}} \\
I - (\text{maint} + \text{bat}): & t > t_{\text{loan}}
\end{cases} \tag{5.18}
\]

Under a microloan contract, the agency is responsible for administrative costs of operating the program. There is a fixed cost for each microloan contract that is realized in the first period and a variable cost that is realized during each period that loan payments are being collected. The agency must also purchase the SHS in the first period.

\[
C_{\text{loan}} = \frac{\text{cost} + f_i}{1 + d_a} + \sum_{t=1}^{dt} \frac{v_r - p}{(1 + d_a)^t} + \sum_{t=dt}^{t_{\text{loan}}} \left[ (1 - dr) \cdot \frac{v_r - p}{(1 + d_a)^t} \right] + dr \cdot \frac{SV(dt)}{(1 + d_a)^{dt}} \tag{5.19}
\]

### 5.4.3 Contract Comparison

We now directly compare the utility levels and costs of each contract to analyze the conditions under which one contract will be preferred to another. A direct
A comparison of Equations 5.9 and 5.12 shows that the subsidy contract provides greater utility than the rental contract when,

$$\sum_{t=1}^{t^*} -\frac{(I_{D_r} - x^b)}{(1 + d_c)^t} + \sum_{t=t^*+1}^{t_{rent}} \frac{I_{D_s} - I_{D_r}}{(1 + d_c)^t} + \sum_{t=t_{rent}+1}^{T} \frac{x^b + I_{D_s} - I^b}{(1 + d_c)^t} > 0 \quad (5.20)$$

The first term of Equation 5.20 represents the utility lost under the subsidy contract by requiring the consumer to save income for several periods in order to purchase the SHS. Thereby, under a subsidy contract, consumption and expenditures are delayed until period $t^*$, whereas under the rental contract they begin immediately. The second term represents the period during which the SHS is operational under both contracts. The net utility here comes from the difference between disposable income under the subsidy and rental contracts, $I_{D_s}$ and $I_{D_r}$ respectively. The subsidy contract will provide greater disposable income when monthly maintenance and battery replacement costs are less than the monthly rental fee. This will usually be the case as the rental contract has to cover maintenance and battery replacement expenses in addition to any administration costs of the rental program. The third term represents the period after the rental term has expired but while the subsidized SHS purchase is still operational. The second and third terms will generally result in positive net utility from the subsidy contract compared to the rental contract. Therefore, the subsidy contract will provide greater utility when the additional utility gained in these periods exceeds the utility lost between periods one and $t^*$. This will be the case when the consumer discount rate is low and utility in later periods is therefore valued more heavily. The net utility difference between the subsidy and rental contracts will also increase as the difference between the monthly rental fee and the costs of maintenance and battery replacement increases.

A similar comparison of Equations 5.9 and 5.17 shows that the subsidy contract generates greater utility than the loan contract when

$$\sum_{t=1}^{t^*} -\frac{(x^b + I_{D_l})}{(1 + d_c)^t} + \sum_{t=t^*+1}^{dt} \frac{I_{D_s} - I_{D_l}}{(1 + d_c)^t} + \sum_{t=dt+1}^{T} \frac{I_{D_s} - (1 - dr) \cdot I_{D_l} + dr \cdot (x^b - I^b)}{(1 + d_c)^t} > 0 \quad (5.21)$$
The first two terms are largely analogous to the first two terms of Equation 5.20, accounting for the early period consumption and expenditure enabled by the loan contract, and the difference in disposable incomes after the subsidized SHS has been purchased. The third term accounts for the risk of loan default. When the default rate is 0%, this term is identical to the second term. The second and third terms will generally result in a net positive utility for the subsidy contract compared to the loan contract. The subsidy contract will generate greater total utility than the loan contract when then the positive contributions from these two terms outweigh the utility lost under the subsidy contract in periods one through $t^*$. Therefore a low consumer discount rate will benefit the subsidy contract more than the loan contract, as will an increasing loan default rate.

A comparison of Equations 5.12 and 5.17 shows that the rental contract will provide more utility than the loan contract when

$$
\sum_{t=1}^{dt} \frac{I_{Dr}^b - I_{Dl}^b}{(1 + d_c)^t} + \sum_{t=dt}^{t_{rent}} \frac{I_{Dr}^b - (1 - dr) \cdot I_{Dl}^b + dr \cdot (x^b - I^b)}{(1 + d_c)^t} \\
+ \sum_{t=t_{rent}}^{T} \frac{(1 - dr) \cdot (I^b - I_{Dl}^b - x^b)}{(1 + d_c)^t} > 0 \quad \text{(5.22)}
$$

The rental and loan contracts are analogous in many ways, as both involve a monthly payment in exchange for SHS service. However, there may be differences in the monthly payment amount, the payback period, the consumption period and administrative costs. When the loan term and rental term are equal and both greater than $T$ and the loan default rate is zero then the two contracts are in fact functionally identical; both provide equal utility for any given cost to the agency. However, variations around these parameters lead to differences between the two contracts. The preferred contract will be the one that encourages longer periods of usage and can be administered more cheaply. For example, an increased average rental term or increased loan default rate will result in the rental contract being preferred over the loan contract.

Additionally, as the loan term decreases, the corresponding monthly payment will increase. If the consumer has a higher discount rate than the agency, this compression of
payments into a shorter period will adversely affect the consumer more than it will benefit the agency. If the required monthly payment exceeds a consumer’s monthly income, they will not be able to participate in the loan program.

The fact that a loan cannot be offered with an interest rate below 0% also makes the loan contract impractical when high levels of subsidization are desired. In practice a loan with a negative interest rate could be offered, however as this does not lead to full cost recovery, such an arrangement would more closely resemble a rental contract.

Unlike the loan contract, the rental contract requires the agency to pay for maintenance and battery replacements. However, in general this simply shifts both client expenditure and agency cost upwards without fundamentally affecting the utility that can be provided for a given cost. Another key factor in distinguishing the rental and microloan contracts is the potential difference in administrative costs between them. If one contract is cheaper to administer then the net utility that can be provided for a given cost will increase.

5.4.4 Numerical Analysis

The above methodology is designed to calculate the total cost and net consumer utility gain of each contract for a given set of static parameter values. Each delivery mechanism has one decision parameter that heavily influences the cost to the agency; these are the subsidy amount, the monthly service fee (rental) and the microloan interest rate. In practice, these parameter values would generally be chosen by the agency in order to achieve their desired level of subsidization or profit. We now present an analysis that calculates the net utility gain realized by a consumer and the cost to the agency across a range of decision parameters values, while holding all other parameters constant. The values of the parameters required for this calculation will vary greatly based on macroeconomic conditions and other market dynamics. The following application of this model uses the baseline parameter values outlined in Table 5.5; we stress that discretion should be exercised when attempting to broadly apply these results to specific situations. This analysis is framed by the baseline assumptions that lead to near indifference between the loan and rental contracts. As discussed previously, if the average consumption period under a rental contract exceeds the average consumption
period under a loan contract then the rental contract will likely be preferred in all situations, and vice versa. Starting from this perspective of near indifference allows us to more closely examine the parameter combinations that cause the optimal contract to tip from a loan to a rental contract. Figure 5.5 shows the increase in utility and cost as the decision parameter is varied for each delivery mechanism; other sensitive parameters are fixed at baseline values. Net utility is calculated as the difference between the utility from each contract and the baseline scenario. The net utility for each contract is a mostly-linear function of cost to the agency; these lines will be referred to as cost-benefit frontiers. This figure demonstrates an example application of this methodology with fixed parameters and also provides a baseline for further analysis. A direct comparison of these functions then yields the optimal (utility-maximizing) contract for each cost.

The dynamics of these contracts are highly sensitive to variations in the input parameters, and it is difficult to draw broad conclusions from one static analysis. Fixed baseline values are assumed for all other financial and technical parameters and multivariate sensitivity analysis and Monte Carlo simulation are later performed to analyze the effect of variations in 16 of these parameters.

Table 5.5 shows the baseline values that are assumed for all other parameters, as well as the range of values for each parameter that will be assumed during sensitivity analysis and Monte Carlo simulations. The decision parameters do not have baseline values as they are varied in order to create the cost-benefit function. The baseline technology in this analysis is a 50 W peak capacity SHS operating at a 16% capacity factor, which costs $725 and provides 5.84 kWh of electricity generation each month. All calculations are made over a period of 120 monthly timesteps, corresponding to a ten year horizon.
Table 5.5: Parameters of financial service mechanisms for electrification programs.
Baseline values for all parameters are shown along with the symbols used to represent them. The given discount and interest rates are in annual terms, but the variables used in equations correspond to the monthly equivalent rate. The variable $a$ is omitted because its value is normalized to one.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Baseline</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Subsidy</td>
<td>$s$</td>
<td>-300</td>
<td>725</td>
<td></td>
</tr>
<tr>
<td>Monthly Rental Fee ($)</td>
<td>$r$</td>
<td>0</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Microloan Interest Rate</td>
<td>$i$</td>
<td>0</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

**General**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Income ($)</td>
<td>$I$</td>
<td>50</td>
</tr>
<tr>
<td>NGO Discount Rate</td>
<td>$d_n$</td>
<td>10%</td>
</tr>
<tr>
<td>Client Discount Rate</td>
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<td>20%</td>
</tr>
<tr>
<td>$a$</td>
<td>$a_{exp}$</td>
<td>1</td>
</tr>
<tr>
<td>$b$</td>
<td>$b_{exp}$</td>
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</tr>
<tr>
<td>$x'$</td>
<td>$x'$</td>
<td>15</td>
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</tbody>
</table>

**SHS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS Cost</td>
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<td>725</td>
</tr>
<tr>
<td>SHS Life (months)</td>
<td>$t_{life}$</td>
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<tr>
<td>Maintenance ($/month)</td>
<td>$main$</td>
<td>0.50</td>
</tr>
<tr>
<td>Battery Cost ($)</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Battery Replacement Frequency (months)</td>
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<td>48</td>
</tr>
<tr>
<td>Monthly Battery Cost ($)</td>
<td>$bat$</td>
<td>2.08</td>
</tr>
<tr>
<td>Capacity (Wp)</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Capacity Factor</td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>Monthly SHS Generation (kWh)</td>
<td>$y$</td>
<td>5.84</td>
</tr>
</tbody>
</table>

**Loan**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan Term (months)</td>
<td>$t_{loan}$</td>
<td>36</td>
</tr>
<tr>
<td>Fixed Loan Cost ($/loan)</td>
<td>$f_l$</td>
<td>10</td>
</tr>
</tbody>
</table>
Variable Loan Cost ($/month) \( v_t \) 0 5 2
Default Rate \( d_r \) 20% 0% 40%
Average Time Before Default (months) \( d_t \) 18 18 18

<table>
<thead>
<tr>
<th>Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Rental Term (months) ( t_{rent} ) 60 12 120</td>
</tr>
<tr>
<td>Salvage Ratio ( s_r ) 50% 0% 100%</td>
</tr>
<tr>
<td>Fixed Rental Cost ($/rental) ( f_r ) 5 0 20</td>
</tr>
<tr>
<td>Variable Rental Cost ($/month) ( v_r ) 1 0 5</td>
</tr>
</tbody>
</table>

**Figure 5.5:** The increase in utility realized by a client is shown as a function of cost to the agency under baseline parameter assumptions. The numbers marking data points represent the corresponding subsidy amount ($), monthly rental fee ($) and annual interest rate for the subsidy, rental and microloan contract respectively. The cost to the agency can also be thought of as the level of subsidization to the consumer.
Figure 5.5 shows the cost-benefit functions for each contract that result from varying the decision parameters while holding all other parameters at their baseline values. The numerical data labels denote the values of each decision parameters that result in the corresponding agency cost. For example, these labels indicate that an agency seeking to break even on each transaction could do so by offering no direct subsidy, a rental contract with a monthly fee of $16.50 or a loan contract with a 19% annual interest rate. Similarly, an agency looking to offer a subsidy with a present value of $150 could do so by offering a direct subsidy of $165, a rental contract with a monthly fee of $13 or a zero-interest loan. The amount of the direct subsidy differs from the level of subsidization because the subsidy is allocated at the time of purchase and is, therefore, time-discounted accordingly. A negative agency cost implies that the agency is making a profit from the transaction in addition to any profit margins that are built into the market price of a system. A negative net utility implies that the consumer receives more utility by either purchasing a SHS at full market price or not obtaining a SHS at all.

It can be seen that for a cost to the agency of -$260 or less, the net utilities of all three contracts are less than zero, which implies that the consumer would be better off not entering a contract. It may still be optimal for the consumer to purchase a SHS, however, if such is the case they would best off doing so through a direct purchase at the full list price. Both the rental contract and the loan contract generate positive net utility for costs greater than -$260.

The rental contract and the loan contract provide fairly comparable net utility for any cost to the agency with the loan contract being slightly preferred until the cost-benefit frontiers intersect at a cost of $-55. This point corresponds with an annual interest rate of roughly 25% and a monthly rental fee of roughly $17. The loan contract is then preferred until the total cost to the agency reaches $150, at which point the annual interest rate becomes 0% and further subsidization is not possible through a loan contract. The utility from the subsidy contract exceeds the utility from the rental contract at a present cost of about $380, or a direct subsidy of $400.

The utility increase enabled by each contract is roughly a linear function of the cost to the agency, however the slope of this linear relationship is different in all three cases. Under the direct subsidy, the client gains approximately .55 units of utility (utils)
for each additional dollar contributed by the agency. The gain is approximately .39 utils per dollar for the loan contract and .32 utils per dollar for the rental contract. The differences in these values can be explained by the dynamics of each contract, which are now discussed to provide further insight into the situations where one contract may be preferred to another.

Under a rental contract, the client is able to obtain a system immediately. Decreasing the rental fee allows the client to allocate more of their income to other expenditures, thereby marginally increasing their total realized utility in each period. Rental contracts would be preferred over subsidies by clients with minimal savings and a high discount rate, as it enables these clients to obtain a system immediately whereas a subsidy would require them to save for several periods before making the purchase. This effect is also seen in Equation 5.20.

Under a loan contract, a reduction in the interest rate lowers monthly payments, increasing client disposable income and decreasing agency income, similar to a rental contract. However, these effects only take place over the duration of the loan as opposed to being spread out across the entire rental contract. As the client discount rate is generally greater than the agency discount rate, this consolidation of payments into earlier time periods affects the client more heavily than the agency. Therefore, under a microloan contract, a reduction in income for the agency enables a gain in client utility that is relatively larger (less discounted) than it is under the rental contract. As a result, the slope of the microloan cost-benefit frontier is greater than the slope of the rental cost-benefit frontier.

Increasing the technology subsidy allows the client to purchase a system earlier and use the system during time periods that were previously unutilized. This provides the client with utility from both system usage and other expenditure in an early time period that is relatively undiscounted; this has a significant impact on total realized utility. Therefore, the slope of the subsidy cost-benefit frontier is greater than it is for the other two contracts.
5.4.5 Single-factor Sensitivity Analysis

We now perform a single-factor sensitivity analysis in order to understand how changes in key parameters influence the optimal choice of financial contract. Figures 5.6-5.11 show the optimal contract for a given cost to the agency while varying a single sensitive parameter. The ‘direct purchase’ outcome indicates that a consumer will prefer to purchase a SHS outright at the full market cost, while the ‘no SHS’ outcome indicates that the consumer will not obtain a SHS, preferring instead to apply their income to other expenditures.

For example, Figure 5.6 illustrates the effect of varying the consumer discount rate between 0 and 30%. For a cost of $100 per SHS to the agency, a direct subsidy contract provides the consumer with the greatest utility when the consumer discount rate is below approximately 5%. A loan contract becomes optimal for consumer discount rates between 5% and 22%, and the rental contract is optimal for consumer discount rates in excess of 22%. It can also be seen if the agency were to profit on each system, the consumer would prefer to directly purchase a SHS at the list price when their discount rate is below 5%. The subsidy contract can never generate additional profit for the agency as it is assumed that a system can always be purchased at list price on the open market. For consumer discount rates in excess of 5% the agency can generate a profit that increases from $0 to $300 by offering a loan or rental contract. High consumer discount rates favor the rental and loan contracts over a subsidy contract because they allow consumers to obtain a SHS immediately. A high consumer discount rate favors the rental contract over the loan contract because payments for the rental contract are made over a longer period than for the loan contract, and these future payments are discounted more heavily. The vertical right edge of the loan optimality region that occurs at an agency cost of approximately $150 is due to the assumed inability of the loan contract to offer a negative interest rate.
Figure 5.6: The effect of varying the consumer discount rate on the optimal financial contract for a given cost to the agency is shown.

Figure 5.7 similarly shows the effect of varying the agency discount rate between 0 and 20%. It is seen that an agency with a discount rate below 10% can provide a SHS to increase consumer utility while also making a profit of up to $300 through a rental contract. As the agency discount rate increases from 10% to 20% this maximum profit decreases to about $200. An agency with a low discount rate places a relatively high value on future income. They are, therefore, able to offer a low monthly rental fee, which benefits the consumer, and still consider the exchange to be profitable. As the agency discount rate increases, the loan contract becomes more attractive since the loan payments are compressed into a shorter, less discounted timeframe.
Figure 5.7: The effect of varying the agency discount rate on the optimal financial contract for a given cost to the agency is shown.

Figure 5.8 shows the effect of varying the length of the average rental term from 12 to 120 months. The rental contract becomes more favorable for a longer average rental term, as the fixed costs of administering the program are spread over a longer usage period. Additionally, consumers obtain utility from electricity consumption over a longer period of time. If the average rental term drops below approximately 50 months, then the loan contract and subsidy contracts are generally more effective than the rental contract.
Figure 5.8: The effect of varying the rental term on the optimal financial contract for a given cost to the agency is shown.

Figure 5.9 shows the effect of varying the length of the loan term from 12 to 60 months. The loan contract becomes more favorable over a longer loan term as this reduces monthly payments and extends the payback period for consumers. Under baseline assumptions, the consumer discount rate is greater than the agency discount rate; this delay in repayment benefits the consumer more than it costs the agency. A shorter loan term also increases monthly payments and may prevent consumers from participating in the market. This limits the range of interest rates that a agency is able to offer its customers, while still ensuring that they can afford the monthly payments. For example, when the loan term is shorter than 14 months, the required monthly payment exceeds the consumer’s $50/month income even in the case of a 0% interest rate. The right edge of the region where the loan contract is optimal is defined by the inability of the loan contract to offer an interest rate below 0%, thereby limiting the degree of potential subsidization.
Figure 5.9: The effect of varying the loan term on the optimal financial contract for a given cost to the agency is shown.

Figures 5.10 and 5.11 look at the effect of varying how consumer utility is calculated. In Figure 5.10, the coefficient that dictates the decreasing marginal returns for monetary expenditures is varied from 0.6 to 1.0. This parameter has a significant impact on the optimal contract. For values approaching 1.0, the consumer has less incentive to obtain a SHS, preferring instead to apply their income to other expenditures. For values approaching 0.6 the consumer prefers to obtain a SHS even in situations where the agency is making a profit on the transaction. A high value also favors the rental contract over the loan contract, as consumers have more disposable income while making rental payments than they do while making loan payments.
Figure 5.10: The effect of varying a consumer’s utility from monetary expenditure on the optimal financial contract for a given cost to the agency is shown.

Figure 5.11 similarly looks at the impact of varying the utility obtained by a consumer for one month of electricity consumption from a SHS. This parameter represents the quantity of monetary expenditure that provides the same level of utility to a consumer as one month (5.84 kWh) of electricity consumption. As this value drops, a consumer becomes less incentivized to obtain SHS, preferring instead to apply their income to other expenditures. Higher values favor the loan contract because it enables a longer period of electricity consumption than the rental contract under baseline assumptions. For values of 14 or less, the consumer prefers to not obtain a SHS when the level of total subsidization is too low. For values greater than 15 they prefer to obtain a SHS through a direct purchase when the agency profit from offering a contract exceeds $275 per system.
Figure 5.11: The effect of varying a consumer’s utility from electricity consumption on the optimal financial contract for a given cost to the agency is shown.

5.4.6 Multi-factor Sensitivity Analysis

The single-factor sensitivity analysis provides some insight into the relative importance of variations in several key parameters. However, the input parameters that impact the financial dynamics of these three contracts may be interdependent. Therefore, the practice of single-factor sensitivity analysis may be uninformative or even misleading when there are a large number of interacting sensitive parameters.

Saltelli and Annoni discuss the reasons that modelers have been hesitant to abandon single-factor sensitivity analysis and construct a geometric proof to show the shortcomings of this method [213]. They advocate several alternative multi-factor sensitivity analysis techniques including regression analysis, an elementary effects method and variance based indicators. Our analysis utilizes the following variance based sensitivity analysis as illustrated in Saltelli and Annoni. This method is also discussed in more detail and further demonstrated with several examples by Saltelli et al. [214].
A variance based sensitivity index is now used to measure the relative impact of variations in each of 16 sensitive input factors on the optimal contract. Three output factors are considered for each given cost to the agency, a binary variable for each contract to represent whether or not that contract is optimal. The sensitivity index, $S_i$, is represented as follows,

$$S_i = \frac{V_{X_i} \left( E_{X_{-i}}(Y|X_i) \right)}{V(Y)} \quad (5.22)$$

Here $X_i$ represents a particular input factor, $Y$ represents the output factor and $X_{-i}$ represents the set of all other input factors while $X_i$ is held fixed. Therefore, $S_i$ is calculated by picking a specific input factor, fixing the value of that input factor and varying all other input factors through Monte Carlo simulation. These factor values are randomly chosen from a uniform distribution in the range of +/- 20% of the baseline parameter value. A constant range is chosen to maintain consistency in evaluating the relative sensitivity of each input parameter. The output factor is determined for each of these parameter combinations and the mean is calculated. The value of $X_i$ is then adjusted, and the process is repeated. A mean output value will result for each fixed value of $X_i$, and the variance of these means is taken as representative index of the relative impact of input factor $X_i$. This analysis is performed for all input factors and the indices for each factor are divided by the total variance of all the resultant output factor values to obtain normalized values of $S_i$ that fall between 0 and 1. This index measures the effect that fixing input factor $X_i$ would have on reducing the variance of the outputs.

This analysis is applied to the 16 sensitive parameters listed in Table 5.5. Each parameter is varied in the range of +/- 20% of its baseline value and the $S_i$ metric is calculated. Therefore the $S_i$ value represents the relative impact of the same fractional shift in each parameter. Because of its exponential nature, the expenditure coefficient ($b$) is varied in the range of +/- 6% to maintain the common scale of variations between parameters. This corresponds with approximately a +/- 20% shift in utility for a monthly expenditure of $25.
Figures 5.12-5.14 show the $S_i$ values for each contract and across a range of costs to the agency; only the five most sensitive parameters across the three contracts are depicted. Figure 5.12 indicates that the loan contract is primarily sensitive to changes in parameters in the range of a -$300 to $100 cost to the agency. The most significant contributions are due to the default rate and length of the rental term. While the loan contract is not directly affected by the rental term length, an increased rental term will generally cause a shift in optimality from the loan contract to the rental contract in this cost range. The loan contract is largely insensitive for greater costs, as it is generally not viable for levels of subsidization in excess of $300 per system. Figure 5.13 shows that the rental contract is primarily sensitive to variations in the rental term and the consumer discount rate. These effects are seen across the entire cost range of -$300 to $700 per system. The rental contract is also sensitive to changes in the SHS cost for higher desired levels of subsidization, as reductions in cost cause a shift from the rental contract to the subsidy contract in this range. Figure 5.14 similarly shows that the subsidy contract is mostly sensitive to changes in the consumer discount rate, the rental term length and the SHS cost. The subsidy contract is not sensitive to variations in parameters for lower costs to the agency, as it is never the optimal contract in cases where agency costs are negative.
Figure 5.12: Multi-factor sensitivity metrics for five parameters that affect the optimality of the loan contract are shown as a function of cost to the agency.
Figure 5.13: Multi-factor sensitivity metrics for five parameters that affect the optimality of the rental contract are shown as a function of cost to the agency.
Figure 5.14: Multi-factor sensitivity metrics for five parameters that affect the optimality of the subsidy contract are shown as a function of cost to the agency.

5.4.7 Monte Carlo Simulation

A Monte Carlo simulation is also executed to understand the range of potential results while varying all 16 sensitive input parameters. For each iteration, a random value of each sensitive parameter is drawn from a uniform distribution between the low and high values that are outlined in Table 5.5. Figure 5.15 shows the results of 10,000 determinations of the optimal contract for a cost to the agency ranging from -$300 to $700. It is seen that for negative costs (profits) to the agency, consumers are often better off either not participating in the program or not obtaining a SHS. As the cost to the agency increases the direct subsidy contract becomes increasingly attractive. The loan contract is optimal more frequently for relatively small costs or profits to the agency, while the rental contract is frequently optimal over a wider range of costs. As the desired level of subsidization increases the subsidy contract becomes optimal in an increasing number of iterations, exceeding 80% for a $700 cost to the agency.
Figure 5.15: Monte Carlo simulation is conducted over 10,000 iterations to determine the probably of each contract being optimal for a given cost to the agency. Values for the 16 sensitive parameters are drawn from a uniform distribution between the values shown in Table 5.5.

5.5 Discussion

Small solar home systems (SHS) are rapidly becoming a popular technology for distributed rural electrification in many parts of the world because of their relatively small unit size, ease of installation and scalability. A number of studies have analyzed the financial viability or socio-economic impacts of SHS in various parts of the world, including Bangladesh [14, 15, 54, 85, 86] and India and Sri Lanka [59, 87, 88]. Other studies examine factors that have impeded the penetration of SHS [58, 89–92], analyze electricity consumption levels for SHS users [93, 94] or discuss user expectations, education and experiences regarding new SHS installations [60, 95–97]. Additional studies analyze the institutional dynamics of SHS by comparing the relative effectiveness of market versus donor-based SHS programs in El Salvador [98] and examining problems faced by public-private partnerships in Africa [99]. A set of quantitative quality of life
indicators has been proposed to better understand the socio-economic impacts of SHS development [100]. Studies have also examined the effectiveness of solar electrification in increasing economic productivity of rural micro-enterprises [101] and alleviating poverty [102] in Ghana.

The cost of SHS systems has prevented them from reaching higher levels of penetration. SHS require significant upfront capital investment, and while electricity from centralized sources require even greater upfront capital, consumers can generally pay for grid electricity on a marginal basis. In many regions, poor populations simply do not have access to the capital required to obtain a SHS. We have considered three specific financial mechanisms that can be used to overcome the large capital requirement for obtaining a SHS, thereby increasing access to the poor. These are technology subsidies, rental programs and microloans. For overviews of different institutional strategies that have been employed to promote rural electrification around the world see Palit and Chaurey [103] and Zerreffi [104]. Miller also discusses lessons learnt from early attempts by the World Bank to provide large-scale loans to support SHS dissemination in India, Sri Lanka and Indonesia [105].

5.5.1 Subsidies

One example of a subsidy approach to decentralized electrification is Brazil, which in 2002 initiated an ambitious plan to provide electricity to the roughly 8 million Brazilians who did not have electricity access at that time. Most of these people reside in rural regions that are difficult to serve cost-effectively, and therefore electricity access had eluded them. Brazil has taken an organizationally centralized approach to distributed electricity provision. The plan primarily relied on a concession system, where a company was given a regional monopoly on electricity distribution under the requirement that they meet certain universal service provisions. These concessionaires are generally government-owned or heavily regulated utilities that provide electricity through diesel mini-grids and solar home systems. As many rural residents have a minimal ability to pay for electrification, rural grid tariffs are cross-subsidized by other consumers. In 2007 the average residential grid tariff across Brazil was about $0.15 per kWh in present U.S. dollars, while rural consumers paid about $0.09 cents per kWh on average [206]. Results
so far have been largely positive, though development has been slower than was hoped and the program is still heavily reliant on the government for subsidies and organizational support [104].

Rural electrification can also proceed without subsidies, as has been the case in Cambodia. It has been estimated that the lone official Cambodian electric utility provided electricity access to only 15% of the population as of 2004. However, in 2006 other estimates suggested that as much as 90-95% of the population had access to some basic level of electricity service. This difference is primarily explained by a number of small, privately operated, rural electricity providers that are unlicensed and unregulated by the government. These providers generally operate diesel powered mini-grids or battery charging stations that provide very small quantities of electricity at relatively high rates to local customers. Official grid tariffs are structured on the basis of full-cost recovery, and vary from as low as $0.15 per kWh in Phnom Penh to $0.30 in more rural regions. However, many rural consumers pay as much as $1.00 for electricity from unregulated sources [207]. The Cambodian experience provides an excellent example of how unregulated, free market forces can generate organizationally decentralized solutions when a competent centralized option is not available [104].

5.5.2 Rental

South Africa has taken a fee-for-service approach to decentralized electrification. In 1999 South Africa launched a rural electrification program that follows a fee-for-service concession model, with the goal of providing 300,000 new solar home systems to un-electrified populations. The government provided a significant portion (as much as 88%) of the required upfront capital cost for the SHS and customers then pay a monthly fee to a utility company for their service. Some regions also subsidize portions of the monthly fee. While subsidies represent a large percentage of the total system cost, the absolute cost is often lower than the amount that the government might otherwise contribute towards grid expansion for each new connection.

Eskom, the primary electric utility in South Africa, offers a range of grid tariff plans for different classes of urban or rural and industrial or residential customers. Low-usage urban consumers are entitled to the Home Light plan, which offers tiered pricing
rates that range from $0.075 to $0.15 per kWh depending on consumption level. In rural regions, low-usage consumers are charged a fixed rate of approximately $.30 per kWh [209]. This policy contrasts the case of Brazil, where subsidized rural tariffs fall below urban rates for similar consumers. The lower level of rural grid subsidization increases the opportunity for decentralized technologies to be implemented cost-effectively. The SHS fee-for-service program has demonstrated that small, well-managed, publicly supported companies can effectively complement the more traditional large utility model in providing rural electricity services at reasonable costs [122].

5.5.3 Microfinance

Taking a micro-loan approach, the Energy Ministry of Ghana initiated a program in cooperation with The World Bank in early 2011 that was designed to help improve rural electricity access through the installation of new small-scale, solar home systems. The project has specifically targeted communities that are not anticipated to receive a connection to the national electric grid in the next 5-10 years and are located in the largely rural Northern, Upper East and Upper West regions of Ghana. Members of these communities are eligible to receive loans from local rural banks that are used to purchase subsidized solar home systems. Ghana currently provides subsidized electricity from the grid by offering a tiered pricing structure based on consumption level. This includes a so-called lifeline tariff that charges all customers who consume less than 50 kWh in a given month a flat fee of approximately $1.25. The tariff changes to a variable rate for higher consumption levels, gradually increasing from $.10 to $.13 per kWh [208].

5.6 Conclusions

Our results indicate that distributed technologies such as solar lanterns and solar home systems may provide a more cost-effective option than grid extension for providing electricity services to sparsely-populated regions where the demand for energy services is relatively small. We have also shown that rental and loan contracts can be implemented to make SHS more accessible to rural consumers, and in some cases also generate a profit for the provider.
A key benefit provided by electric systems is access to lighting; a service that can also be provided by non-electric technologies. We showed that a single solar lantern provides lighting more cost-effectively than flashlights or kerosene lamps if annual light production exceeds roughly 10klh per year. At higher levels, enough to power up to ten light sources, solar home systems are more cost-effective than electricity from the grid, unless grid connection costs are less than approximately $1000 per household. For annual lighting consumption equal to the average consumption level in India, solar home systems are more cost-effective than electricity from the grid when marginal grid connection costs exceed $2000 per household.

The grid tariff structure in many countries implies a level of subsidization, particularly in regions that have low electricity consumption and where grid connection is expensive. In some cases this implicit subsidy may exceed the entire cost of distributed technologies. In these cases welfare could be enhanced by providing large targeted subsidies that make it possible for the rural poor to purchase solar home systems for a nominal cost. Examples, such as the subsidized fee-for-service SHS program in South Africa and the microloan program in Ghana, suggest that some governments are aware of this trend and have accordingly begun to channel subsidies towards decentralized technologies.

The required upfront cost of solar home systems is beyond the means of many of the rural poor. To address this issue, some governments and development agencies have provided subsidies and developed micro-loan programs to reduce the cost to the level at which many of the rural power can pay. Nevertheless, these subsidized systems are still very expensive for many of the rural poor with the result that adoption rates are low, default or non-payment rates are high, and program costs are high as well. These program costs are effectively an additional subsidy to the SHS program; however, the subsidies for grid-based rural electrification are often significantly higher. Overall the result is that in solar home system programs, the rural poor pay a high price for each unit of electricity consumption and do not receive the same government assistance as those who rely on electricity from the grid.

By considering overall system costs of providing lighting to the rural poor, we have shown the cost of lighting for a range of lighting technologies and both solar home
systems and grid systems. By reducing system costs to the level that can be paid upfront without financing similar levels of service could be achieved with significantly higher levels of subsidies to the rural poor and corresponding reduction in overhead costs of the program.

We also directly compare the effectiveness of three different financial contracts that have been used to make solar home systems more accessible to the rural poor. We show that rental and loan contracts are largely analogous and may provide similar net utility increases for certain parameter combinations. A loan contract generally condenses higher consumer payments into a shorter period, which may prevent some low income consumers from participating. However, this contract may be preferred to a rental contract if consumption continues for a longer period than it does under the rental contract. Loan contracts may also be relatively preferred to rental contracts if the client discount rate is low or the agency discount rate is high. Subsidy contracts may be a more effective option if the agency is able to subsidize a significant portion of the total system cost. Subsidy contracts also become more attractive when client discount rates are low and agency discount rates are high. They also benefit from minimal administrative expenses.

Multi-factor sensitivity analysis suggests that optimality of each contract is more sensitive to changes in the consumer discount rate than it is to changes in the agency discount rate. It is also shows that the length or the rental term has a significant impact on whether or not the rental contract is optimal. A longer rental term resulting in less turnover reduces administrative costs and allows for more extended periods of consumption.

Monte Carlo simulation across a reasonable range of values for all 16 of the sensitive parameters illustrates that an agency can generate a profit from a rental or loan program when conditions are favorable. It is also seen that the loan contract is feasible across more parameter values when costs or profits to the agency are small and the subsidy contract tends to be optimal in the majority of situations when the desired level of subsidization is high.
CHAPTER VI
CONCLUSIONS

This thesis is focused on understanding the choice between centralized and decentralized electricity infrastructure development strategies, with particular application to developing countries where millions of people lack regular access to reliable electricity and other energy services. Several complementary analyses of this decision paradigm are presented implementing novel tools that span a range of different technical fields including network algorithm design, linear and mixed-integer programming and stochastic simulation. The main contribution of this thesis is to develop energy modeling frameworks that consider the full range of parameters affecting the determination of optimal development strategies, many of which are often overlooked by traditional analyses. These parameters include the true costs of transmission infrastructure as well as the implications of outages and system unreliability, infrastructure lead times, uncertain development budgets, high discount rates and the unique dynamics of rural credit markets.

Chapter 2 presents a methodology for identifying priority locations for distributed electricity generation technologies, such as solar home systems. An original network algorithm is developed to determine a near-minimum length network that is capable of spanning a given fraction of population in a region. These data are then used to calculate a metric that quantifies the relative cost-effectiveness of decentralized electricity infrastructure development in a given region. Several case studies are presented to demonstrate potential applications of this methodology, and aggregated results are presented for 150 countries around the world. The analysis suggests that the majority of the world’s population can be served most cost-effectively by centralized electricity networks. However, it also identifies a number of key regions where distributed technologies may be implemented cost-effectively when electricity demand levels are below certain threshold values. The new algorithm presented in this chapter is demonstrated through an application to electricity networks, but it could easily be applied to similar analysis and planning of numerous other types of country-level network infrastructure including water or fuel distribution, transportation and communication.
Chapter 3 presents another least-cost methodology that approaches the centralized-decentralized electrification paradigm from a different perspective. A mixed-integer programming framework is developed to determine the minimum-cost combination of centralized and decentralized electrification infrastructure that is capable of serving electricity demand in a specified region. A case study of Rwanda is presented to demonstrate the model, and a sensitivity analysis is conducted around the cost of decentralized generation technology and electricity demand levels. A tipping point effect is found to exist in the case of Rwanda, where the least-cost infrastructure tips directly from mostly centralized to mostly decentralized as these two parameters are varied. The impact of varying the geographic resolution of this model is also analyzed, and the computational requirements of several different formulations are discussed. The same general tipping points are identified across all four considered geographic resolutions. This finding suggests that lower-resolution data may be considered when computational resources are limited and only high-level results are desired. As was the case in chapter 2, the methodologies presented in this chapter are also broadly applicable to other network applications where centralized and decentralized infrastructure development options may be considered. Examples may include the choice between centralized water treatment and subsequent distribution versus localized treatment, landline versus cellular telecommunications or land-based versus air-based transportation networks. The model is also scalable and could easily be applied to evaluate least-cost infrastructure at the regional or village level.

Chapter 4 builds upon the work presented in chapter 3, by introducing stochastic events and probabilistic parameters into a model for serving electricity demand through a combined centralized and decentralized electricity infrastructure. A temporal dimension is also introduced to analyze the lead time requirement of different infrastructure development paths. Additionally, results obtained when explicitly modeling stochastic events are compared to results obtained through conventional modeling where constant parameter values are assumed. This work is motivated by the fact that the penetration of solar home systems is growing rapidly in the densely populated country of Bangladesh, which contrasts indications in chapters 2 and 3 that densely populated regions can often be served more cost-effectively by centralized infrastructures. Such empirical evidence
suggests that there may be factors beyond those considered in chapters 2 and 3 that influence energy related decision making in the developing world. In contrast to chapter 3, the model in chapter 4 does not directly determine the optimal infrastructure that is capable of serving demand in a region. Rather, it provides a simulation-based tool that can quickly compare the cost and service level provided by various user defined development plans. A distributed infrastructure development scenario is shown to enable more immediate electrification, while a centralized scenario provides a higher level of service over the considered 20-year time horizon. It is also shown that the levelized costs of generation from the centralized scenario are more heavily influenced by explicit stochastic modeling and variations in the available development budget and discount rate. These results suggest that caution should be taken to ensure that such factors are adequately considered when a first-order, cost-based analysis indicates that a centralized infrastructure might be implemented more cheaply than a decentralized infrastructure.

The simulation model developed in this chapter was demonstrated through an analysis of two development scenarios that represent drastically different development paths. However, this model could also be applied to understand different outcomes resulting from development paths that vary more subtly. For example, it would be possible to analyze the impact of commencing construction on a new generation facility immediately as opposed to several years in the future. It would also be possible to explore how a specific development scenario responds to variations in key variables. Examples might include analyses of different demand growth scenarios or a technology learning scenario where the cost of a solar home system reduces over time as more units are installed.

The methodologies presented in chapters 2 and 3 are formulated with the explicit objective of determining the least-cost electricity infrastructure, while the work in chapter 4 also frames results in terms of the cost of electricity provision. However, no assumptions are made as to how the electricity provider – utility, government or private organization – will recover costs through tariffs to their consumers. An implicit consequence of this formulation is the assumption that an electricity provider has a primary objective of minimizing their costs. In reality, electricity generators and providers may be heavily regulated, subsidized or government-owned and rarely operate under true free-market conditions. As a result, the revenue of an electricity provider may
be decoupled from their service level or tied directly to their costs of provision. Due to various regulations and mandated social objectives providers may also be restricted in their ability to freely vary tariffs based on consumer class and generation technology. Therefore, an electricity provider may not be incentivized to pursue a particular development strategy even if it offers the lowest-cost means of electricity provision. The dynamics of different electricity markets are not explicitly modeled by the work in chapters 2, 3 and 4. However, it is important to consider how these methodologies and results may be affected by different political environments and market structures.

Chapter 5 addresses issues that differ slightly from those analyzed in chapters 2, 3 and 4. A subsidy-free cost of lighting curve is first developed for a range of different electric and non-electric technologies. The costs of electric lighting in this calculation incorporate the true cost of electricity provision as a function of electricity consumption and grid connection cost, rather than assuming fixed, subsidized grid tariffs as is common in other analyses. It is shown that kerosene based lighting is the lowest-cost option only for very low levels of lighting consumption. Solar technologies provide a promising option for lighting consumption equivalent to roughly one to ten LED lights that are operated for four hours per day. For higher levels of consumption, electricity from the grid becomes an increasingly cost-effective option, particularly when grid connection costs are low. However, solar options may still be cost-effective when grid connection costs are high. It is also shown that electricity from the grid is often heavily subsidized in low-consumption regions of developing countries, and it is suggested that some of these subsidies be allocated to instead support increased penetration distributed generation technologies. Finally, a theoretical model is formulated to quantify the ability of three different financial delivery mechanisms, direct subsidies, rental programs and microloans, to provide solar home systems to low-income populations. These mechanisms are compared on the basis of their ability to increase consumer utility for a given cost to the agency that is administering the financial contract. A numerical analysis based on a realistic range of parameter values suggests that direct subsidies may be preferred when a high level of subsidization is desired, while microloan contracts becoming increasingly effective when small levels of subsidization or profit are desired.
The results presented by this thesis build upon existing literature in the fields of energy policy, energy economics and international development. They provide a resource for international organizations, governments and NGOs to inform decision making in formulation of global energy policy.
APPENDICES

Appendix A: The calculated fractions of both population and nodes that are cost-effectively served by decentralized electricity in 150 countries are shown for a range of consumption levels, pertaining to the methodology and analysis presented in chapter 2. Electrification statistics are also presented for each country. *Electricity Consumption Per Capita Electrified* (ECPCe) is equal to *Electricity Consumption Per Capita* (ECPC) divided by *Electrification Rate*. *Potential Electricity Consumption Density* is calculated by multiplying *Population Density* and *ECPCe*. The right-most columns of the table indicate the fractions of population and nodes in each country that are cost-effectively served by decentralized electricity for each given level of electricity consumption. The column labeled *Current ECPCe* presents the population and node fractions for electricity consumption levels equal to the *ECPCe* in each country. These results do not require that per capita electricity consumption be constant throughout each country. They instead correspond to the fraction of nodes, and the fraction of people who reside in nodes, that would be cost-effectively served by decentralized infrastructure for given consumption levels.

Table A1: Electrification statistics for the 150 countries that were analyzed in chapter 2.

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<th>Electricity Consumption Per Capita (kWh/year)</th>
<th>Electrification Rate</th>
<th>Electricity Consumption Per Capita Electrified (kWh/year)</th>
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Appendix B: Results of the single-factor sensitivity analysis performed in section 5.4.5 are shown for 10 input parameters that were not previously presented.

**Figure B1:** The effect of varying the salvage ratio on the optimal financial contract for a given cost to the agency is shown.

**Figure B2:** The effect of varying the fixed loan cost on the optimal financial contract for a given cost to the agency is shown.
Figure B3: The effect of varying the variable loan cost on the optimal financial contract for a given cost to the agency is shown.

Figure B4: The effect of varying the battery replacement frequency on the optimal financial contract for a given cost to the agency is shown.
**Figure B5:** The effect of varying the monthly maintenance cost on the optimal financial contract for a given cost to the agency is shown.

**Figure B6:** The effect of varying the monthly consumer income on the optimal financial contract for a given cost to the agency is shown.
Figure B7: The effect of varying the loan contract default rate on the optimal financial contract for a given cost to the agency is shown.

Figure B8: The effect of varying the fixed rental cost on the optimal financial contract for a given cost to the agency is shown.
Figure B9: The effect of varying the variable rental cost on the optimal financial contract for a given cost to the agency is shown.

Figure B10: The effect of varying the SHS cost on the optimal financial contract for a given cost to the agency is shown.
REFERENCES


