CFD Simulation of Multi-Dimensional Effects in Inertance Tube
Pulse Tube Cryocoolers

Approval

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# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ................................................................................................................... III

LIST OF TABLES .......................................................................................................................... VIII

LIST OF FIGURES ......................................................................................................................... IX

SUMMARY ................................................................................................................................... XVIII

INTRODUCTION AND BACKGROUND .................................................................................. 1

1.1 INTRODUCTION ................................................................................................................... 1

1.2 BACKGROUND ..................................................................................................................... 7

1.2.1 History of Cryogenic Refrigerators ............................................................................. 7

1.2.2 Pulse Tube Cryocoolers ............................................................................................. 9

1.2.3 Application of PTCs .................................................................................................. 10

1.3 REVIEW OF RECENT LITERATURE ............................................................................. 11

EXPERIMENTAL SETUP AND OPERATIONS ........................................................................ 17

2.1 EXPERIMENTAL SETUP ............................................................................................... 17

2.2 COMPONENT DESCRIPTIONS ....................................................................................... 21

2.2.1 Compressor .................................................................................................................. 21

2.2.2 Warm Heat Exchanger 1 (WHX1) ........................................................................... 21
LIST OF TABLES

Table 1: GIT OPTC Test Data .................................................................................. 20
Table 2: Component Radiuses ............................................................................... 50
Table 3: Boundary Conditions and Initial Conditions for MOD1 and MOD2 ....... 53
Table 4: Cycle Averaged Energy Balance Results ................................................. 74
LIST OF FIGURES

Figure 1: Schematic Configuration of the Basic Pulse Tube Cryocoolers (BPTC) ...... 2

Figure 2: Schematic Configuration of the Orifice Pulse Tube Cryocooler (OPTC) ..... 4

Figure 3: Schematic Configuration of the Inertance Tube Pulse Tube Cryocooler (ITP TC) ................................................................................................................................. 5

Figure 4: GIT OPTC Experimental Apparatus View 1 .............................................. 18

Figure 5: GIT OPTC Experimental Apparatus View 2 .............................................. 18

Figure 6: GIT OPTC Experimental Apparatus View 3 .............................................. 19

Figure 7: Schematic of the Simple Vapor Compression Cycle ................................. 25

Figure 8: Macroscopic Energy Balance .................................................................. 27

Figure 9: Energy Balance of System Components ................................................. 28

Figure 10: Gas Temperature and Mass Flow Rate Phase Shift Relationships ......... 30

Figure 11: 3-Dimensional View of the ITPTC (L/D=12)$_{pt}$ ...................................... 33

Figure 12: 3-Dimensional View of the ITPTC (L/D=2)$_{pt}$ ..................................... 33

Figure 13: Meshed 2-D Axisymmetric Large Diameter Model Generated by the Gambit Software ................................................................. 35

Figure 14: 2-Dimensional Axisymmetric Mesh for MOD1 ..................................... 38

Figure 15: 2-Dimensional Axisymmetric Mesh for MOD2 ..................................... 39
Figure 16: 2-Dimensional Axi-symmetric Coordinate System

Figure 17: Dimensions of the PTC System MOD1. (All dimensions are in m.)

Figure 18: Dimensions of the PTC System MOD2. (All dimensions are in m.)

Figure 19: CHX Temperature for Case 1 (Adiabatic Wall B.C.) and Case 2 (1W cooling load)

Figure 20: Cycle-average Temperature Distribution for Case 1 (Adiabatic B.C.)

Figure 21: Snapshot of Density Distribution for Case 1 (Adiabatic Wall B.C.)

Figure 22: Temperature Contours for Case 1 (Adiabatic Wall B.C.) [K]

Figure 23: Snapshot of Density Contours for Case 1 (Adiabatic Wall B.C.) [kg/m3]

Figure 24: Temperature Contour for Case 2 (1W Cooling Load) [K]

Figure 25: Density Contours for Case 2 (1W Cooling Load) [kg/m3]

Figure 26: Refrigeration Load for Case 3 (Isothermal Wall B.C.)

Figure 27: Refrigeration Load for Case 3 (Isothermal Wall B.C.) with Zoomed Load Coordinate

Figure 28: Cycle-average Temperature Distribution for Case 3 (Isothermal Wall B.C.)

Figure 29: Local Density Distribution for Case 3 (Isothermal Wall B.C.)

Figure 30: Temperature Contours for Case 3 (Isothermal Wall B.C.) [K]

Figure 31: Density Contours for Case 3 (Isothermal Wall B.C.) [kg/m3]
Figure 32: CHX Temperature for Case 4 (Adiabatic Wall B.C.) and Case 5 (1W Cooling Load) ............................................................................................................ 67

Figure 33: Temperature Contours for Case 4 (Adiabatic Wall B.C.) [K]. .................. 68

Figure 34: Density Contours for Case 4 (Adiabatic Wall B.C.) [kg/m3]. ................. 68

Figure 35: Temperature Contours for Case 5 (1W Cooling Load) [K]. ..................... 69

Figure 36: Density Contours for Case 5 (1W Cooling Load) [kg/m3]. ....................... 69

Figure 37: Refrigeration Load for Case 6 (Isothermal Wall B.C.) ............................ 70

Figure 38: Temperature Contours for Case 6 (Isothermal Wall B.C.) [K]. ............. 71

Figure 39: Density Contours for Case 6 (Isothermal Wall B.C.) [kg/m3]. ............... 71

Figure 40: Energy Balance for the ITPTC ................................................................. 73

Figure 41: Snapshot of Velocity Vectors in the Regenerator with L/D = 7.25 [m/s]. 77

Figure 42: Snapshot of Velocity Vectors at the vicinity of the Regenerator Outlet with

L/D = 7.5 [m/s]. .................................................................................................... 77

Figure 43: Snapshot of Velocity Vectors at the vicinity of the Pulse Tube Inlet with L/ D = 12, in [m/s]. ........................................................................... 78

Figure 44: Snapshot of Velocity Vectors and Temperatures [K] in the Regenerator with

L/D = 1.25. ....................................................................................................... 78

Figure 45: Snapshot of Velocity Vectors and Temperatures [K] in the Pulse Tube L/D = 2. .................................................................................................... 79
Figure 46: Snapshot of Velocity Vectors and Temperatures [K] at the vicinity of the Pulse Tube Inlet with L/D = 2. ................................................................. 80

Figure 47: Snapshot of Velocity Vectors and Temperatures [K] in the Pulse Tube with L/D = 2. ........................................................................... 81

Figure 48: Snapshot of Velocity Vectors and Temperatures [K] at the vicinity of the Surge Volume Inlet. ................................................................. 82

Figure A. 1: Temperature Distribution of Stirling Cryocoolers [3] ......................... 86
## NOMENCLATURE

### Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>$C$</td>
<td>Inertial Resistance Factor</td>
<td>$[m^{-1}]$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific Gas Constant</td>
<td>$[J/kg-k]$</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$E$</td>
<td>Total Energy</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>External body forces or source terms</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity acceleration</td>
<td>$[m/s^2]$</td>
</tr>
<tr>
<td>$\dot{H}$</td>
<td>Enthalpy</td>
<td>$[W]$</td>
</tr>
<tr>
<td>$I$</td>
<td>Unit (Identity) tensor</td>
<td></td>
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<tr>
<td>$\hat{k}$</td>
<td>Thermal Conductivity</td>
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<tr>
<td>$L$</td>
<td>Length</td>
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<tr>
<td>$\dot{m}$</td>
<td>Mass Flow Rate</td>
<td>$[kg/s]$</td>
</tr>
<tr>
<td>MOD1</td>
<td>Model One</td>
<td></td>
</tr>
<tr>
<td>MOD2</td>
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<tr>
<td>$P$</td>
<td>Pressure</td>
<td>$[N/m^2]$</td>
</tr>
<tr>
<td>Period</td>
<td>Period of the Cycle</td>
<td>$[sec]$</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Heat Rate</td>
<td>$[W]$</td>
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</table>
r           Radial Coordinate, [m]
R           Gas constant
S           Source term
t           Time, [sec]
T           Temperature, [K]
W           Power, [W]
v           Velocity, [m/s]
V           Volume, [m³]
x           Axial Coordinate, [m]
X           Piston Displacement, [m]

Greek Letters

β           Permeability, [m²]
ε           Porosity of Medium
μ           Fluid molecular Viscosity, [kg/m-s]
ρ           Density, [kg/m³]
$\theta$  Rotating Swirl Coordinate

$\tau$  Stress Tensor

$\omega$  Angular frequency, [rad/s]

$\psi$  Diffusion flux

Subscripts

amplitude  Amplitude of Signal

chx  Cold Heat Exchanger

coeff  Coefficient

comp  Compressor

eff  Effective

f  Fluid

in  incoming

inert  Inertance Tube

j  Summations

loss  Losses

m  Mass
<table>
<thead>
<tr>
<th>out</th>
<th>outgoing</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Constant Pressure</td>
</tr>
<tr>
<td>porous</td>
<td>Porous Medium</td>
</tr>
<tr>
<td>pt</td>
<td>Pulse Tube</td>
</tr>
<tr>
<td>pv</td>
<td>PV Work</td>
</tr>
<tr>
<td>r</td>
<td>Radial Direction</td>
</tr>
<tr>
<td>refrig</td>
<td>Refrigeration</td>
</tr>
<tr>
<td>regen</td>
<td>Regenerator</td>
</tr>
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<td>reject</td>
<td>Rejection</td>
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<tr>
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<td>Solid</td>
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</tr>
<tr>
<td>tl</td>
<td>Transfer Line</td>
</tr>
<tr>
<td>whx1</td>
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</tr>
<tr>
<td>whx2</td>
<td>Warm Heat Exchanger 2</td>
</tr>
<tr>
<td>x</td>
<td>Axial Direction</td>
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**Superscripts**

<p>| TP | Transpose |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>B.C.</td>
<td>Boundary Condition</td>
</tr>
<tr>
<td>C.V.</td>
<td>Control Volume</td>
</tr>
<tr>
<td>dt</td>
<td>Differential time</td>
</tr>
<tr>
<td>dT</td>
<td>Differential Temperature</td>
</tr>
<tr>
<td>dX</td>
<td>Differential Piston Displacement</td>
</tr>
<tr>
<td>dV</td>
<td>Differential Volume</td>
</tr>
<tr>
<td>∂</td>
<td>Differential Operator</td>
</tr>
<tr>
<td>∇</td>
<td>Gradient Operator</td>
</tr>
<tr>
<td>®</td>
<td>Vector Form</td>
</tr>
<tr>
<td>&lt;&gt;</td>
<td>Cyclic Average Quantity</td>
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SUMMARY

Inertance Tube Pulse Tube Cryocoolers (ITPTC) are a class of rugged and high-endurance refrigeration systems that operate without a moving part at their low temperature end, and are capable of reaching 4 K or lower. ITPTCs are suitable for application in space vehicles, and attempts are underway worldwide to improve their performance and miniaturize their size. The thermo-fluidic processes in ITPTCs are complicated, however, and the details of the mechanisms underlying their performance are not well understood. Elucidation of these underlying processes is the objective of this investigation.

In this study, the commercial computational fluid dynamic (CFD) package Fluent® was utilized for modeling the entire ITPTC system that includes a compressor, an after cooler, a regenerator that is represented as a porous medium, a pulse tube, cold and warm heat exchangers, an inertance tube, and a reservoir. The simulations represent a fully-coupled system operating in steady-periodic mode, without any arbitrary assumptions. The objective was to examine the extent of multi-dimensional flow effects in an inertance tube pulse tube cryocoolers, and their impact on the performance of these cryocoolers.

Computer simulations were performed for two complete ITPTC systems that
were geometrically similar except for the length-to-diameter ratios of their regenerators, pulse tubes, and inertance tubes. For each ITPTC system three separate simulations were performed, one with an adiabatic cold-end heat exchanger (CHX), one with a known cooling heat load, and one with a pre-specified CHX temperature. Each simulation would start with an assumed uniform system temperature and continue until steady periodic conditions were achieved.

The results indicate that CFD simulations are capable of elucidating the complex periodic processes in PTCs very well. The simulation results also show that a one-dimensional modeling of PTCs is appropriate only when all the components of the PTC have very large aspect ratios (i.e., L/D >>1). Significant multi-dimensional flow effects occur at the vicinity of component-to-component junctions, and secondary-flow recirculation patterns develop when one or more components of the PTC system have small aspect ratios. The simulation results, although limited in scope, also suggest that ITPTCs will have a better overall performance if they are made of components with large aspect ratios.
CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

Cryogenics is the branch of physics that studies the phenomena that occur at very low temperatures, close to the lowest theoretically attainable temperature (absolute zero, 0 K equivalent to -273.15 °C or -459.67 °F). In engineering, cryogenics can be best described as an application which operates in the temperature range from absolute zero to 120 K.

One system that embodies the cryogenics phenomena in the engineering applications is the Pulse Tube Cryocooler (PTC). PTCs are refrigerators capable of cooling to temperatures below 120 Kelvin. Unlike the ordinary refrigeration cycles which utilize the vapor compression cycle as described in classical thermodynamics, a PTC implements the theory of oscillatory compression and expansion of the gas within a closed volume to achieve desire refrigeration. Being oscillatory, a PTC is a non-steady system that requires time dependent solutions. However, like many other periodic systems, PTCs attain quasi-steady periodic state (steady-periodic mode).
meaning that any property of the system at any point in a cycle will reach the same state in the next cycle, and so on. When the system reaches this type of periodic mode we declare that the thermodynamic properties within the system have reached their own steady-periodic state. This is analogous to the well-known Stoke’s Second Problem in fluid mechanics.

PTCs are classified into three main different groups. First, and the oldest, is the Basic Pulse Tube Cryocooler (BPTC). Figure 1 is a schematic of a BPTC. The BPTC concept was first proposed by Gifford and Longsworth in 1963.

Figure 1: Schematic Configuration of the Basic Pulse Tube Cryocooler (BPTC)
BPTC is composed of six main components: compressor, warm heat exchanger 1 (WHX1), regenerator, cold heat exchanger (CHX), pulse tube, and warm heat exchanger 2 (WHX2). In a BPTC, the oscillatory pressure waves impose a shuttling effect to the working fluid in the pulse tube. This shuttling effect creates an energy interaction between the pulse tube wall and the working fluid. This is commonly known as a surface heat pumping process [1]. Thus, the BPTC achieves refrigeration through the surface heat pumping process between the working fluid and the pulse tube walls. BPTCs have relatively low coefficients of performance, and can typically reach a cold-end temperature of only 124 K [2].

The second type of PTC is the Orifice Pulse Tube Cryocooler (OPTC), shown in Figure 2. OPTCs are significantly better than BPTCs, and were among the most widely-used cryocoolers until the mid 1990s, along with Stirling Cryocoolers. Stirling Cryocoolers will be discussed later.
The schematic configuration of an OPTC can be viewed as a modification of the BPTC. This modification is made by including an orifice valve and a surge volume at the warm end of the BPTC, as depicted in Figure 2. These additional components create an advantageous in-phase relationship between the mass flow and temperature within the pulse tube to enhance the heat transfer mechanism.

The third, and the most recently invented type of the PTC is the Inertance Tube Pulse Tube Cryocooler (ITPTC). In this type of PTC, the orifice valve is replaced by a long inertance tube (Figure 3). The implementation of the inductance phenomenon generates an advantageous phase shift in the pulse tube and produces an improved enthalpy flow [7].
As noted in the above figures, all the major PTC models operate using fully-closed systems, meaning that no mass is exchanged between the cryocooler and the environment. Most importantly, the only moving component is the piston face which oscillates back and forth, leading to periodic pressurization and depressurization of the working fluid. In most PTCs, helium is chosen as the working fluid, primarily because it offers the lowest critical temperature compared to other available gases and helium has high volumetric heat capacity and high thermal conductivity.

The main objective of the PTC engineer is to accurately model the entire
system in order to predict its performance, and thereby perform design calculations and improve the operation performance of these cryocoolers. At the current stage of worldwide research, the exact mechanisms underlying PTC’s performance are not well understood. Further, the complexity of the periodic flows in the PTC makes the system analysis more obscure. The recent availability of powerful computational fluid dynamics (CFD) software that is capable of rigorously modeling transient and multi-dimensional flow and heat transfer processes in complex geometries provides a good opportunity for detailed modeling of PTCs. A successful demonstration of the suitability of CFD-type analyses for PTCs is why this research is important. Evidently, in order to accurately predict and improve the performance of the entire cooler a reasonably thorough understanding of the thermo-fluidic processes in the system is required. One way to understand the detailed thermo-fluidic processes is by numerically solving the continuum governing equations based on the fundamental principles, without making arbitrary simplifying assumptions. CFD tools and methods make this possible. It should, however, be emphasized that CFD-type predictions, like all other model-generated results, are reliable only when they are verified against experimental data. The importance of experiments should therefore be emphasized.

Thus, in this study, multi-dimensional continuum governing equations are numerically solved to investigate the details of the thermo-fluidic processes in
ITPTCs, and to predict their overall performance. An issue of particular interest is the relevance of the predictions of a one-dimensional model to the performance of an ITPTC. This is important because the most advanced current design methods are based on one-dimensional flow models. Thus, one would like to know when a one-dimensional model is valid, and when it is necessary to use a multi-dimensional model. These issues will be addressed along the way.

To accurately model the systems of interest and to solve the governing continuum equations without making any arbitrary assumptions, the commercial Computational Fluid Dynamics (CFD) package FLUENT is utilized. The entire ITPTC system that includes a compressor, a regenerator that is represented as porous medium, a pulse tube, cold and warm heat exchangers that are also represented as porous media, an inertia tube, and a surge volume (reservoir) will be modeled. The simulation results are then presented and validated. Recommendations for future work will then be provided.

1.2 Background

1.2.1 History of Cryogenic Refrigerators

Although the primary focus of this research is on PTCs, another highly efficient cryogenic refrigeration system will be briefly described here because of its
worldwide acceptance by the cryogenics industry and operational similarities to PTC. This refrigeration system is known as the Stirling Cryocooler (SC), and is described in some detail in Appendix A. This cooler uses the well known Stirling thermodynamic cycle, named after Robert Stirling, who in 1816 introduced the use of a regenerative heat exchanger in his hot air engine [3]. The principal reason that these cryocoolers achieve refrigeration is by implementing the regenerative heat exchanger instead of conventional recuperative heat exchanger. The main difference between the SC and the PTC is the expander device. Unlike the PTC, the SC does not include the pulse tube, orifice or inertance tube, and the surge volume. Instead, SC imposes a physical expansion piston to implement the expansion process. The foremost disadvantage of the SC is the existence of a moving piston at the cold end which can lead to reliability difficulties and generate potential vibration problems in many applications. For this reason, many space or military applications prefer PTCs over the SCs since PTCs have no moving parts at the cold end which offers better reliability for long life applications. Nonetheless, currently SCs operate at higher efficiencies than the PTCs, particularly for low temperature (< 50 K) operation. The reason for the higher efficiency of SCs is that the aforementioned mechanically-driven piston that provides expansion can easily be adjusted in order to provide an optimum phase lag between mass flow and pressure. Thus, the achievable
efficiency of the SCs have typically surpassed the efficiency of the PTCs. A significant improvement in the efficiency of PTCs is thus needed.

1.2.2 Pulse Tube Cryocoolers

Gifford and Longsworth from Syracuse University in 1963 pioneered the introduction of pulse tube cryocoolers [4]. Their first design was based on a hollow cylindrical tube with one end open and the other end closed. The closed end is exposed to an ambient temperature heat exchanger, while the open end represents the cold end. As a result of the oscillatory flow field caused by the piston, the open end was subjected to an oscillating pressure from the regenerator, causing the open end to cool [5]. This refrigerator is commonly known as BTPC and is shown in Figure 1.

In 1984, a Russian researcher by the name Mikulin added an orifice valve and a surge volume to the closed end of the BPTC, as shown in Figure 2 [6]. The purpose behind these additional components is to create a more appropriate phase relationship between the oscillatory pressure and the mass flow rate. By this advantageous phase difference, the OPTC achieved higher performance efficiency than the BPTC. This type of cryocooler could lead to temperatures as low as 60-120 K [2], and later became the modern PTC, commonly known as OPTC, as shown in
Figure 2. A further improvement in the performance of OPTCs can be achieved by using two orifice valves, one between the regenerator and the pulse tube, the other between the pulse tube and the reservoir (Double Inlet OPTC). With this type, temperatures as low as 42 K have been reported [2]. Furthermore, multi-stage coolers and the GM (Gifford-McMahon) style coolers have reached temperatures below 10 K [3].

In the mid 1990’s, the orifice valve of the OPTC was replaced by the long inertia tube. In its most basic form, an inertia tube is simply a long and narrow tube that imposes a hydraulic resistance and causes a basically adjustable delay between the pressure responses of the pulse tube and the reservoir. In fact, by employing an electrical analogy, Roach and Kashani have shown why the inductance added by the inertia tube allows for an improved power transfer in the pulse tube [7].

1.2.3 Application of PTCs

Currently, PTCs are primarily used to refrigerate systems with small heat loads, in the range of a few watts, at very low temperatures. Nonetheless, PTCs have the capability of refrigerating tens of watts by means of inputting large compressor work. There are many other applications that could potentially benefit from PTCs,
however, only if the efficiency of PTCs can be improved to make them competitive with more conventional refrigeration systems such as classical simple vapor compression refrigerators. Magnetic resonance imaging (MRI), infrared focal plane arrays and detectors, gas liquefaction, maser amplifiers, mine sweeping magnets, space instruments, and weapon systems are examples of current applications [5].

1.3 Review of Recent Literature

From a thermodynamic point of view, in almost every engineering application, there exists a net entropy generation, meaning that irreversibility always exists and deteriorates the application’s performances. Minimizing these irreversibilities is in fact the impetus behind much of regenerator and pulse tube design efforts. PTCs, being a typical engineering application, also encounter the issue of irreversibility. Pressure drop across the orifice, viscous dissipation, undesirable heat transfer caused by conduction, convection, radiation and dispersion, can all be viewed as typical loss mechanisms. Nevertheless, it is believed that the bulk of the losses occur in regenerators [5]. Due to the structure of the porous medium in the regenerator, often consisting of wire mesh screens, numerous loss mechanisms are encountered. These loss mechanisms include enthalpy flow, wall-thickness conduction, gas conduction, porous media matrix conduction, and dispersion. Needless to say, a
highly efficient cryocooler can be obtained if these losses can be minimized. Therefore, since the birth of PTCs, extensive research efforts have been underway to improve the regenerator performance.

Flow and heat transfer in porous media are mature branches of science, and a vast literature related to porous media exists. However, most of the existing literature is of little relevance to PTC regenerators due to the periodic working conditions of the latter. Mathematical and theoretical models dealing with periodic flow and heat transfer in porous media are relatively few. Among the early published studies, Siegel analyzed oscillatory flow in a porous material, and showed analytically that the flow oscillation of a fluid within a channel can enhance the axial transport of energy [8]. In a related study, Siegel investigated the forced convection in slow laminar flow in a channel with uniform wall heat flux, and showed that flow oscillations enhanced energy transfer in the axial direction [9]. Earlier, an analytical solution to the diffusion in oscillatory pipe flow had been published by Watson [10].

Kaviany investigated the steady laminar flow through the space bounded between two parallel plates, where he neglected the inertial term in the momentum equation and the axial conduction term in the fluid energy equation [11]. His results indicated that the Nusselt number increased with the porous media shape parameter, a parameter that is directly proportional to the porosity of the medium and the width
of the channel [11]. No oscillations were considered by Kaviany. The same physical system was subsequently studied by Khodadadi, who considered an oscillatory (pulsating) flow, and showed that a phase lag occurred between the pressure and velocity, and this phase lag diminished for highly viscous fluids [12].

Pressure drop in the regenerator is believed to be an important factor in affecting the flow and heat transfer between a gas and a solid porous matrix [5]. The pressure drop across the regenerator must be estimated with reasonable accuracy in order to correctly predict the heat transfer coefficient. Little is known about pressure drop in oscillatory flow in porous media, however. Consequently, many investigators use correlations that represent steady flow pressure drop, for modeling the flow in the regenerators of PTCs [5, 13]. The inadequacy of this method is of course relatively well understood. For example, Organ [14] argues that the pressure drop across the regenerators cannot be correlated with steady flow pressure drop correlations in Stirling Cycle Machines, since they operate in oscillatory manner. Organ also states that the discrepancy between experimental measurements and theoretical predictions can primarily be attributed to the fact that steady-flow correlations do not take into account the unsteady effects which arise from the cyclic nature of the flow processes in the Stirling machines [14].

There are also computational models of regenerators available that better
represent the actual thermo-fluidic processes. Kashani and Roach, from NASA Ames Research Center, have developed a computer program known as ARCOPTTR [15]. This program utilizes the 1-D fluid mass, momentum, and energy conservation equations to simulate the thermo-fluidic processes in the regenerator. One-dimensional computational models have also been developed at NIST (REGEN 3.1 [16]) and the Los Alamos National Laboratory (LANL) (DeltaE [16]). All these models ignored the possible effect of turbulence [16]. In a recent 1-D numerical simulation, Ju et al. [17] modeled entire orifice and double-inlet PTCs, by using a common set of fluid conservation equations everywhere. The friction factor and heat transfer coefficients were chosen to be the larger among relevant laminar and turbulent correlations everywhere, however. A common feature of the above models is that they all assume that the working fluid and the solid porous matrix in the regenerator are everywhere at thermal equilibrium.

Harvey has recently developed a 1-D model based on the volume averaged method using the numerical technique of method of lines to solve the regenerator equations [18]. The model by Harvey uses the best available closure relations for the 1-D conservation equations. A major distinction between the model by Harvey and other models published in the open literature is that Harvey’s model is capable of accounting for thermal non-equilibrium between the working fluid and the solid
structure in the regenerator. The selected correlations, however, are primarily based on steady state literature.

Gedeon has also developed a 1-D model that simulates not only the regenerator but the entire pulse tube cryocooler system [19]. Gedeon's work was implemented into a program known as Sage. An important attribute of Sage that distinguishes it from other published models is its embedded optimization routine, whereby the program can actually optimize any user-selected geometrical parameter to achieve the system's optimum performance [19]. However, Harvey et al have shown that Sage predictions can be rather inaccurate when the code is used with its default parameters, and requires adjusting the coefficient values of the losses in the pulse tube in order to match the experiment results [20]. The results of Harvey et al. show that Sage may generally over predict the coefficient of performance of pulse tube coolers.

Although most of the efforts in the past have been conducted specifically to understand and improve the regenerators, other components of the PTC systems have also been investigated for better understanding the loss phenomena. Among such components is the pulse tube. Kittel et al. [21] performed a detailed study of the flow field in a pulse tube, and showed that flow circulation, commonly known as a secondary enthalpy streaming effect, creates a convective enthalpy flow from the hot
end to the cold end that acts as a loss mechanism. Kirkconnell [22] developed a 1-
Dimensional model to simulate the thermo-fluid interactions in the pulse tube. In a follow-up study, Kirkconnell et al. [23] experimentally studied the effect of the pulse tube aspect ratio (Length/Diameter ratio) on the overall performance of a PTC. They found that pulse tubes with different aspect ratios but identical volumes performed identically for the range of aspect ratios considered (aspect ratios from 10 to 16). Lastly, the contribution of the aforementioned Kashani and Roach should be reiterated; they analyzed the use of the inertance tube instead of an orifice valve and showed that this substitution resulted in a higher overall system performance due to an improved enthalpy transfer [7].

PTCs have also recently been investigated from the First and Second Law points of view by some investigators, including Richardson and Evan [24] and Kornhauser and Smith [25]. The latter authors examined the compression and expansion processes in the piston-cylinder system of a PTC. The major contribution of Kornhauser and Smith [25] was the modeling of the heat transfer coefficient in complex form. Since PTC’s operation is likely to depend strongly on the phase lag associated with heat-up and cool-down between the working fluid and the solid porous structure, modeling the heat transfer coefficient in terms of complex numbers can be a better technique for some components of a PTC systems.
CHAPTER 2

EXPERIMENTAL SETUP AND OPERATIONS

2.1 Experimental Setup

Although the numerical and computational studies are the main focus of this research, a description of the test apparatus at the Cryogenic Research Laboratory at Georgia Institute of Technology should be provided here. This apparatus offers a working OPTC prototype which has been extensively used by Jeremy Harvey and Carl Kirkconnell of Raytheon [5]. Furthermore, some experiments were performed with this test apparatus as part of this investigation. Photographs of this apparatus are shown in Figures 4, 5 and 6.

In order to achieve its minimum cold tip temperature during the experiments, the CHX was exposed to a vacuum environment, as it would be in a typical application. The Dewar, shown in the above Figures, is utilized to provide the vacuum environment for the elimination of any condensation effects during the tests. The regenerator, cold heat exchanger (CHX), and the pulse tube are all housed inside the Dewar.
Figure 4 : GIT OPTC Experimental Apparatus View 1

Figure 5 : GIT OPTC Experimental Apparatus View 2
Detailed discussion of the main components of the test facility will be provided in the forthcoming subsections. The geometric characteristics of the experimental apparatus are the same as those for MOD1 system listed in the forthcoming Table 2.

Table 1 is a summary of previous test data obtained by Harvey [5], using the Georgia Institute of Technology OPTC test facility. This table thus illustrates the performance history of the aforementioned test facility.
According to Table 1, the lowest temperature, 76 K, was achieved with 50 W of an electrical input power (provided to the compressor), using the 400 mesh stainless steel porous material in the regenerator, in a test performed in February, 1999.

Recently, as part of this investigation, a new test was performed in 11/2003 using the 30 micron porous material. These 30 micron screens were packed individually in the regenerator tube at the Class-100 clean room available in the Micro Electro-Mechanical Systems (MEMS) research laboratory at GIT. The operating parameters were: pressure = 3.1 MPa, frequency = 37 Hz, working fluid = research grade helium and thermal load = 0. At periodic-steady state, a low-end temperature of 155 K could be achieved. The low-end temperature (No load) was obtained upon optimization of the frequency and the orifice diameter.
2.2 Component Descriptions

2.2.1 Compressor

The main function of the compressor is to supply gas pressurization and depressurization in the closed chamber. Electrical power is applied to the compressor where this electrical work is converted into the mechanical energy associated with sinusoidal pressure waves, resulting in gas motion kinetic energy, and viscous dissipation. In an ideal compressor, the electrical power provided to the compressor must be equal to \( f \int PdV \), where the integration is performed over an entire cycle, \( P \) is the compressor pressure, and \( f \) is the compressor frequency. In an actual system, however, the above-mentioned power (the PV power) is always less than the actual measured electrical power due to the irreversibility associated with the friction losses at the interface of the piston head. The compressor used is a reciprocating dual-opposed-piston design. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.

2.2.2 Warm Heat Exchanger 1 (WHX1)

The function of the ideal WHX1 is to extract and dispose all the heat that is generated in the compressor volume during the gas compression. This minimizes the warm end temperature so that the regenerator can work more efficiently by
minimizing enthalpy flow from the warm end to the cold end. Typically, these types of heat exchangers are assembled using copper wire mesh screens that are directly in contact with the housing wall. In our housing apparatus, this housing wall exchanges heat with a water-loop that removes the heat that is generated in the process. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.

2.2.3 Regenerator

The regenerator is the most important component in our test apparatus. Its function is to absorb the heat from the incoming gas during the forward stroke, and deliver that heat back to the gas during the return stroke. Ideally, PTC regenerators with no pressure drop and a heat exchanger effectiveness of 1 are desired, in order to achieve the maximum enthalpy flow in the pulse tube. The performances of the real regenerators are of course far from ideal. Stainless steel wire screens are usually selected as the regenerator packing material, since they offer high heat transfer surface areas, low pressure drop, high heat capacity, and low thermal conductivity. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.
2.2.4 Cold Heat Exchanger (CHX)

CHX can be best viewed as the equivalent of the evaporator in the vapor compression refrigeration cycle. This is where the refrigeration load is absorbed by the system. The construction details of the CHX in our apparatus is exactly the same as the aforementioned WHX1 where copper wire mesh screens are used to exchange heat with the housing wall, and thereby receive the applied heat load. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.

2.2.5 Pulse Tube

The main objective of the pulse tube is to carry the heat from the cold end to the warm end by an enthalpy flow. By imposing a correct phase difference between pressure and mass flow in the pulse tube by phase shifting mechanisms, heat load is carried away from the CHX to the WHX2. Physically, the pulse tube is simply a thin-walled, hollow cylindrical tube made up of stainless steel. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.

2.2.6 Warm Heat Exchanger 2 (WHX2)

Upon receiving the enthalpy flow from the pulse tube, the heat load is rejected
to the environment in the same manner as described for the WHX1. The physical geometry and the assembly details of WHX 2 in our test facility are identical to the aforementioned WHX1. The geometric characteristics are the same as those for MOD1 system in the forthcoming Table 2.

2.2.7 Orifice Valve, Inertance Tube and Surge Volume

The role of either the inertance tube or the orifice valve is to appropriately adjust the phase difference between the mass flow rate and the pressure. By controlling the orifice diameter or the inertance tube diameter and length, the desired phase relationship can be obtained. In our apparatus, the orifice valve is a needle valve, and the inertance tube will be an open cylindrical stainless tube. The diameter and length of the inertance tube will be specified later in the paper. In comparison with the aforementioned pulse tube, the inertance tube is much longer, and its diameter is much smaller.

The Surge Volume can be viewed as a closed buffer reservoir of sufficient volume to allow for small pressure variations resulting from the oscillating mass flow. This is required because any pressure oscillation in phase with the surge volume inlet mass flow serves to retard the phase shift and mass flow amplitude, which reduces performance.
2.3 Pulse Tube Cryocooler Operation Principles

The operation principles of PTCs are rather different than conventional refrigeration systems. The major difference is between the regenerative heat exchanger process in the PTC and the recuperative heat exchanger process in most conventional refrigeration systems. The methods of removing heat from the cold environment to the warm environment are somewhat different as well. The vapor compression cycle (shown in Figure 7) operates in a steady flow fashion where heat is transported from the evaporator to the condenser by a constant and steady mass flow rate. The PTC relies on an oscillatory pressure wave in the system for transporting heat from the CHX to the WHX2.

![Figure 7: Schematic of the Simple Vapor Compression Cycle](image)
How does the cooling actually occur in the oscillating pressure environment in the PTC? Where exactly is the heat absorbed and rejected? Before answering these questions, the concept of cyclic properties must be understood first. Because PTC operates in steady-periodic mode, the thermodynamic properties such as enthalpy flow, \( <H> \), heat flow, \( <Q> \), and power \( <W> \) are evaluated in the form of cyclic integrals. The appropriate instantaneous thermodynamic properties are integrated over the entire cycle and divided by the period of that cycle to obtain the cyclic averaged quantity of interest. For example, the compressor power is evaluated from the following integration.

\[
\langle \dot{W} \rangle_{PV} = \int \frac{P}{\dot{V}} \, dt = \frac{1}{\text{Period}} \int P(t) \dot{V}(t) \, dt
\]  

(1)

where \( P \) and \( V \), are instantaneous pressure and volume. Average enthalpy flow rate, \( <H> \), and average heat flow rate, \( <Q> \), are calculated similarly.
Figure 8, a schematic of a PTC, demonstrates that the PTC’s heat absorption and rejection occur at the cold heat exchanger, CHX, and the two warm heat exchangers WHX1 and WHX2, respectively. Clearly, WHX2 is equivalent to a condenser in a conventional vapor compression cycle, and CHX is equivalent to an evaporator. During the PTC operation, most of the work done by the compressor is rejected through the WHX1. The rest of the energy that is not rejected through WHX1 is carried through by the enthalpy flow \( <\dot{H}_\text{rejen} > \) in the regenerator. This can be seen in the component energy balance schematics shown in Figure 9.
Figure 9: Energy Balance of System Components

The regenerator enthalpy flow $\dot{H}_{\text{regen}}$, the additional refrigeration load $\dot{Q}_{\text{refig}}$, and the heat flow representing all the losses, $\dot{Q}_{\text{loss}}$ (such as gas conduction, solid matrix conduction, and dispersion), are all absorbed at the CHX, therefore:
\( \langle H \rangle_{\text{chx}} = \langle Q \rangle_{\text{refg}} + \langle H \rangle_{\text{regen}} + \langle Q \rangle_{\text{loss}} \)  

(2)

This enthalpy flow enters the pulse tube, and travels down the tube, reaches WHX2, and is then rejected to the environment. Assuming that the working fluid is an ideal gas, any enthalpy flow term can be cast as:

\[
\langle H \rangle = \frac{1}{\text{Period}} \int \dot{m} C_p T \, dt ,
\]

(3)

where:

\( \dot{m} \) = Mass flow rate,

\( C_p \) = Gas specific heat,

\( T \) = Temperature of the gas.

According to the equation (3), if an oscillating mass flow rate \( \dot{m} \) is in phase with the oscillating gas temperature \( T \) (see Figure 10b), then a net enthalpy flow exists in the pulse tube flowing from the cold end to the warm end (i.e., \( \langle H \rangle_{\text{chx}} > 0 \)).

(Note that \( \dot{m} > 0 \) when flow is from left to right in Figure 8.) On the other hand, if an oscillating mass flow rate \( \dot{m} \) is out of phase with oscillating gas temperature
T (Figure 10a), then little or no enthalpy flow will exist in the pulse tube, which results in minimum cooling. The definition of in phase and out of phase relationship can be shown explicitly through Figure 10.

Figure 10: Gas Temperature and Mass Flow Rate Phase Shift Relationships

(a): Out-of-Phase by $\frac{\pi}{2}$

(b): In-Phase

Figure 10 depicts two examples of phase shift between gas temperature and mass flux. The first example (Figure 10a) demonstrates a case where the mass flow rate and the temperature oscillations are about 90 degrees apart. In this circumstance, little or no enthalpy flow takes place. In fact, with temperature and the
mass flow rate being 90 degrees out of phase, one will always be zero when the other one is at its peak. Thus, out of phase relationships tend to produce poor refrigeration due to minimum enthalpy flow in the pulse tube. On the other hand, if the mass flow rate and the temperature oscillations are in phase as illustrated in the second example (Figure 10b), good enthalpy flow can exist in the pulse tube.

Therefore, in PTC design, achieving the optimum phase relationship is a crucial step towards obtaining the best overall performance.
CHAPTER 3

MODELING AND GOVERNING EQUATIONS

3.1 Computational Fluid Dynamics (CFD)

Central Processing Unit (CPU) technologies are growing extremely fast today. Consequently, standard computer processing time has decreased by orders of magnitude compared to only a decade ago, and high performance computers are now available. Engineers today have better tools for solving complex engineering problems in a short time. The availability of fast computers equipped with very large memories also allow for remarkably precise numerical simulations. As a result, computational fluid dynamic (CFD) tools, both commercial and research, are now available. One of the most highly respected CFD codes is Fluent [26]. Fluent is a state-of-the-art computer program for modeling fluid flow and heat transfer process in complex engineering problems. Fluent offers the flexibility of meshing any complex geometry and solving complicated 2-Dimensional and 3-Dimensional problems. Transient flow and transport phenomena in porous media, two-phase flow, and
volumetrically-generating sources can all be modeled by Fluent. Fluent numerically solves the entire continuum fluid and energy equations with no arbitrary assumptions.

Figures 11 and 12 illustrate the physical geometry of the ITPTC systems of interest in the forthcoming simulations. The two systems are identical, with the exception of different regenerator and pulse tube aspect ratios (Length/Diameter).

![Figure 11 : 3-Dimensional View of the ITPTC (L/D=12) pt.](image)

![Figure 12 : 3-Dimensional View of the ITPTC (L/D=2) pt.](image)
Figures 11 and 12 show that every component of the ITPTC is in fact cylindrical in shape, and all the components are aligned in series and form an axi-symmetric system. The ITPTC is therefore modeled in a 2-dimensional axi-symmetric coordinate system. The potential asymmetry caused by gravity is thus neglected. This gravity term however will be very important if the order of magnitude of the acceleration becomes comparable with other terms (such as temporal acceleration, convective acceleration, etc.) in the momentum equation.

Fluent 6.1.22 has a dynamic meshing function. This function allows the user to create deforming mesh volumes such that applications involving volume compression and expansion can be modeled. Thus, in view of Fluent’s versatility, its capability for solving the compression and expansion volumes, and modeling capability for porous media, it was selected for the simulation of the entire ITPTC in this study.

3.2 Modeling of Inertance Tube Pulse Tube Cryocoolers

The first step in CFD simulation is the nodalization of the physical system of interest. Nodalization was done by the Gambit [27] code. First, an actual physical drawing of the problem was created in the 2-dimensional axi-symmetric coordinate system using the Gambit software. Once the drawings were generated, meshing was
Figure 13: Meshed 2-D Axi-symmetric Large Diameter Model Generated by the Gambit Software

applied to the drawing, as shown in Figure 13.

Upon a successful meshing process, the drawing is then imported into the Fluent software. At this stage, the material properties and the boundary conditions are assigned to the system based on the actual problem definitions. Fluent offers a variety of boundary conditions to satisfy typical engineering problems such as an isothermal wall, constant heat flux, velocity inlet, and adiabatic surfaces. In addition, the control volumes can be modeled as a homogeneous structure, source generating homogeneous structure, and a porous medium. Once the appropriate volumes containing continuum structures have been defined, the desired numerical schemes such as first order upwind and second order upwind, are chosen based on user’s desired numerical accuracy. Lastly, error requirements are also set to achieve the user’s desired convergence tolerance.
The two physical systems depicted in Figures 11 and 12 were simulated in this study. The two systems, as noted earlier, are identical in shape, volume, and geometry with the exception of their different pulse tube and regenerator length-to-diameter ratios and their inertance tube length. These two simulations will be referred to from this point on as MOD1 and MOD2, respectively. MOD1 dimensions were derived from the actual experiment apparatus at GIT while the dimensions of the regenerator and the pulse tube for MOD2 were arbitrarily chosen to decrease the aspect ratios by factor of 6. Schematics showing their detailed nodalization schemes are displayed in Figures 14 and 15, respectively.

In these simulations the thermal resistance of all the walls is neglected since these resistances are negligibly small. However, in BPTC, the wall thickness of the pulse tube is extremely important since the heat is transported by the thickness walls using the theory of the surface heat pumping [1]. Hence thickness of the walls must be modeled if BPTC is to be correctly modeled by Fluent.

In order to model the piston and cylinder, Fluent’s dynamic meshing function must be used. A User Defined Function (UDF) was developed in C programming language to simulate the piston cylinder effect. The piston head motion is accordingly found from:
Piston Displacement = \( X = X_{\text{amplitude}} \sin(\omega \cdot t) \) \hspace{1cm} (4)

Piston Head Velocity = \( \frac{dX}{dt} = \omega \cdot X_{\text{amplitude}} \cos(\omega \cdot t) \) \hspace{1cm} (5)

where \( \omega = 213.62 \text{ [rad/s]} \), \( X_{\text{amplitude}} = 4.511 \times 10^{-3} \text{ [m]} \), and time increment of 7.3529e-4 second were assumed.

3.3 Governing Equations

Continuum-based conservation equations can be applied everywhere in the ITPTC system. This is appropriate since the mean free path of gas molecules is typically much smaller than the characteristic dimension of the ITPTC components. The regenerator and heat exchangers can in some circumstances be exceptions to the aforementioned statement, however, when the characteristic dimensions of their microporous structure are comparable, or even smaller than the gas mean free path. For this reason, these components are modeled using porous media methods, as described later. Thus, continuum based conservation of mass, momentum, energy equations along with the equation of state of the working fluid are used for all components, except the regenerator and the heat exchangers. The general governing equations used by the Fluent code \[26\] are as follows
Figure 14 : 2-Dimensional Axi-symmetric Mesh for MOD1
Figure 15: 2-Dimensional Axi-symmetric Mesh for MOD2
Conservation of Mass Equation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = S_m
\]  

(6)

where:

\(\nabla\) = Gradient operator

\(\rho\) = Density of the gas

\(\mathbf{v}\) = Velocity in vector form

\(S_m\) = Source term

\(t\) = Time

(Note that \(S_m=0\) for our case.)

Conservation of Momentum Equation:

\[
\frac{\partial}{\partial t} \left( \rho \, \mathbf{v} \right) + \nabla \cdot \left( \rho \, \mathbf{v} \, \mathbf{v} \right) = -\nabla \cdot \mathbf{p} + \nabla \cdot (\mathbf{\tau}) + \rho \, \mathbf{g} + \mathbf{F}
\]  

(7)

where:

\(\mathbf{p}\) = Static Pressure

\(\mathbf{\tau}\) = Stress Tensors
\( \ddot{g} = \text{Gravity acceleration} \)

\( \ddot{F} = \text{External body forces or source terms, e.g., terms associated with porous media.} \)

Assuming that the working fluid is Newtonian, the constitutive relation for shear stress-strain rate is:

\[
\tau = \mu \left[ \left( \nabla \ddot{v} + \nabla \ddot{v}^{T} \right) - \frac{2}{3} \nabla \cdot \ddot{v} \ I \right] \tag{8}
\]

where:

\( \mu = \text{Fluid molecular viscosity} \)

\( I = \text{Unit (Identity) tensor} \)

\( TP = \text{Transpose} \)

Conservation of Energy:

\[
\frac{\partial}{\partial t} \left( \rho \, E \right) + \nabla \cdot (\dot{\ddot{v}} \, (\rho \, E + \rho \, p)) = \nabla \cdot \left( k_{\text{eff}} \nabla \, T - \sum_{j} \hat{h}_{j} \ddot{v}_{j} + (\tau \cdot \ddot{v}) \right) + S \tag{9}
\]

where:

\[
E = \hat{h} - \frac{p}{\rho} + \frac{\dot{v}^{2}}{2} \tag{10}
\]

\[
\hat{h} = \int_{t_{1}}^{t} \! \left( \frac{\rho}{c_{p}} \right) \, dT \tag{11}
\]
\[ \dot{k}_{\text{eff}} = k + k_1 \]  

(12)

\( k \) = Gas thermal conductivity

\( k_1 \) = Turbulence thermal conductivity

\( c_p \) = Specific heat of the gas

\( \dot{h} \) = Local enthalpy

\( T \) = Temperature of the gas

\( v \) = Local velocity

\( \psi_j \) = Diffusion flux of species

\( S \) = Source term which can be caused by chemical reactions or volumetric heat generation.

The diffusion flux is only important in multi-component flows. Evidently, \( \psi_j = 0 \) for our case.

Equation of State assuming ideal gas behavior:

\[ p = \rho RT \]  

(13)

where:
\( R = \text{Gas constant} \)

Note: For simplicity, Ideal gas equation of state was implemented in this analysis; however, the real gas equation of state should be utilized.

The above equations can be simplified. First, given the axi-symmetric configuration of the modeled system, and assuming a negligible asymmetry caused by gravity, the aforementioned equations can be cast in 2-dimensional cylindrical polar coordinate systems as:

Continuity Equation:

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho v_r \right) + \frac{\partial}{\partial x} \left( \rho v_x \right) = 0 \tag{14}
\]

where:

\( r \) = Radial coordinate

\( x \) = Axial coordinate

\( v_r \) = Radial Velocity

\( v_x \) = Axial Velocity
Figure 16: 2-Dimensional Axi-symmetric Coordinate System

Momentum Equation in Axial Direction:

\[
\frac{\partial}{\partial t}(\rho v_x) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_x v_r) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r v_x) = -\frac{\partial p}{\partial x} \\
+ \frac{1}{r} \frac{\partial}{\partial x} \left[ r \mu \left( 2 \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot \vec{v}) \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \mu \left( \frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x} \right) \right]
\]

(15)

Momentum Equation in Radial Direction:

\[
\frac{\partial}{\partial t}(\rho v_r) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_x v_r) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r v_r) = -\frac{\partial p}{\partial r} \\
+ \frac{1}{r} \frac{\partial}{\partial x} \left[ r \mu \left( \frac{\partial v_r}{\partial x} + \frac{\partial v_x}{\partial r} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \mu \left( 2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot \vec{v}) \right) \right] \\
- 2 \mu \frac{v_r}{r^2} + \frac{2}{3} \mu \left( \nabla \cdot \vec{v} \right)
\]

(16)
where \( v_r \) and \( v_x \) are the radial and axial components of the velocity vector.

Note that:

\[
\nabla \cdot \vec{v} = \frac{\partial}{\partial x} v_x + \frac{\partial}{\partial r} v_r + \frac{v_r}{r} 
\]

(17)

Also, note that the body forces (gravity forces) and any other external forces have been neglected in the above equations.

Energy Equation:

\[
\frac{\partial}{\partial t} (\rho E) + \nabla \cdot (\vec{v} (\rho E + p)) = \nabla \cdot \left( k_{\text{eff}} \nabla T + (\tau \cdot \vec{v}) \right) 
\]

(18)

Equations (13), (14), (15), (16), and (18) are to be numerically solved for the compressor, transfer line, pulse tube, inertance tube, and the surge volume.

The regenerator, WHX1, WHX2, and CHX, as mentioned before, can be more appropriately modeled using porous-media methods. The porous-media will be modeled using the volume-averaged conservation equations for mass and momentum. The volume-averaged mass conservation equation follows:

\[
\frac{\partial}{\partial t} (\varepsilon \rho) + \nabla \cdot (\varepsilon \rho \vec{v}) = 0
\]

(19)
where $\varepsilon$ is the porosity of the porous medium.

With respect to momentum, porous media are modeled by introducing two new terms to the volume-averaged momentum equation: the Darcy term, which represents a pressure drop term directly proportional to the velocity, and an inertial term which is proportional to the velocity square. Assuming a homogeneous and isotropic solid matrix, the following force terms are included in the $x$ and $r$ volume–averaged momentum equations.

\[
F_{\text{porous}_x} = -\left( \frac{\mu}{\beta} v_x + \frac{1}{2} C \rho \left| \vec{\nu} \right| v_x \right) \quad (20)
\]

\[
F_{\text{porous}_r} = -\left( \frac{\mu}{\beta} v_r + \frac{1}{2} C \rho \left| \vec{\nu} \right| v_r \right) \quad (21)
\]

where:

$\mu$ = Fluid molecular viscosity

$\beta$ = Permeability

$C$ = Inertial resistance factor

$v$ = Velocity
In the aforementioned equations, the first term represents the Darcy term. The second term is often referred to as the Forchheimer term, and represents the fluid inertia. Fluent requires a user-input inertial resistance factor $C$ and the permeability parameter $\beta$ for the porous medium. The inertial resistance factor and the permeability must be specified based on relevant correlations or experimental data.

In summary, the porous-medium momentum equation, in generic vectorial form, can be represented as:

$$\frac{\partial}{\partial t} \left( \epsilon \rho \bar{v} \right) + \nabla \cdot \left( \epsilon \rho \bar{v} \bar{v} \right) = -\epsilon \nabla p + \nabla \cdot \left( \epsilon \tau \right) - \left( \frac{\mu}{\beta} \bar{v} + \frac{1}{2} C \rho |\bar{v}| \bar{v} \right)$$

(22)

where $\bar{v}$ is the physical velocity, and is related to superficial velocity $\bar{u}$ according to

$$\bar{u} = \epsilon \bar{v}$$
The axial and radial components of the above equation are:

Axial Direction:

\[
\frac{\partial}{\partial t} (\varepsilon \rho v_x) + \frac{1}{r} \frac{\partial}{\partial x} (\varepsilon r \rho v_x v_x) + \frac{1}{r} \frac{\partial}{\partial r} (\varepsilon r \rho v_r v_x) = - \frac{\partial (\varepsilon p)}{\partial x} \\
+ \frac{1}{r} \frac{\partial}{\partial x} \left[ r \mu \left( 2 \frac{\partial (\varepsilon v_x)}{\partial x} - \frac{2}{3} (\nabla \cdot (\varepsilon \vec{v})) \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \mu \left( \frac{\partial (\varepsilon v_r)}{\partial r} + \frac{\partial (\varepsilon v_r)}{\partial x} \right) \right] \\
+ - \left( \frac{\mu}{\beta} v_x + \frac{1}{2} C \rho |\vec{v}| v_x \right)_{\text{porous medium}}
\]

(23)

Radial Direction:

\[
\frac{\partial}{\partial t} (\varepsilon \rho v_r) + \frac{1}{r} \frac{\partial}{\partial x} (\varepsilon r \rho v_x v_r) + \frac{1}{r} \frac{\partial}{\partial r} (\varepsilon r \rho v_r v_r) = - \frac{\partial (\varepsilon p)}{\partial r} \\
+ \frac{1}{r} \frac{\partial}{\partial x} \left[ r \mu \left( \frac{\partial (\varepsilon v_r)}{\partial x} + \frac{\partial (\varepsilon v_x)}{\partial r} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \mu \left( 2 \frac{\partial (\varepsilon v_r)}{\partial r} - \frac{2}{3} (\nabla \cdot (\varepsilon \vec{v})) \right) \right] \\
- 2 \mu \varepsilon \frac{v_r}{r^2} + \frac{2}{3} \mu \frac{v_r}{r} (\nabla \cdot (\varepsilon \vec{v})) + - \left( \frac{\mu}{\beta} v_r + \frac{1}{2} C \rho |\vec{v}| v_r \right)_{\text{porous medium}}
\]

(24)

The porous-medium energy equation must account for the fluid-porous structure interactions. When thermodynamic non-equilibrium between the fluid and the porous structure is accounted for, separate energy conservation equations must be solved for the fluid and the solid structure. The aforementioned thermodynamic non-equilibrium is usually small, however, and often a single energy equation
representing both the solid and gas phases is used. Accordingly, in this study, local thermal equilibrium assumption is applied. Thus, the single energy equation is used:

\[
\frac{\partial}{\partial t} \left( \varepsilon \rho_f E_f + (1 - \varepsilon) \rho_s E_s \right) + \nabla \cdot \left( \bar{v} \left( \rho_f E_f + P \right) \right) = \nabla \cdot \left[ \hat{K} \nabla T + (\tau \cdot \bar{v}) \right]
\]  

where

\[
\hat{K} = \varepsilon \hat{K}_f + (1 - \varepsilon) \hat{K}_s
\]

\(\varepsilon\) = Porosity of medium

\(\hat{K}_s\) = Solid medium thermal conductivity

\(\hat{K}_f\) = Fluid thermal conductivity

\(E_f\) = Total fluid energy

\(E_s\) = Total solid energy

3.4 Dimensions, Boundary and Initial Conditions of the Simulated Systems

The physical dimensions of the two perceived systems MOD1 and MOD2 are summarized in Table 2 and Figures 17 and 18. The dimensions of the inertance tube for MOD1 and MOD2 were obtained from the Sage [19] program analysis.
### Table 2: Component Radiiues

<table>
<thead>
<tr>
<th>SI units [m]</th>
<th>MOD1</th>
<th>MOD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Compressor)</td>
<td>9.54E-03</td>
<td>9.54E-03</td>
</tr>
<tr>
<td>B (Transfer Line)</td>
<td>1.55E-03</td>
<td>1.55E-03</td>
</tr>
<tr>
<td>C (WHX1)</td>
<td>4.00E-03</td>
<td>4.00E-03</td>
</tr>
<tr>
<td>D (Regenerator)</td>
<td>4.00E-03</td>
<td>0.01</td>
</tr>
<tr>
<td>E (CHX)</td>
<td>3.00E-03</td>
<td>3.00E-03</td>
</tr>
<tr>
<td>F (Pulse Tube)</td>
<td>2.50E-03</td>
<td>7.50E-03</td>
</tr>
<tr>
<td>G (WHX2)</td>
<td>4.00E-03</td>
<td>4.00E-03</td>
</tr>
<tr>
<td>H (Inert. Tube)</td>
<td>4.25E-04</td>
<td>5.955E-04</td>
</tr>
<tr>
<td>I (Surge Volume)</td>
<td>0.013</td>
<td>0.013</td>
</tr>
</tbody>
</table>

**Figure 17**: Dimensions of the PTC System MOD1. (All dimensions are in m.)
The most common method for CFD simulation of a steady-state process is to perform a transient analysis using appropriate boundary conditions, and continue with the transient simulation until a steady-state numerical solution is obtained. Accordingly, although system’s steady periodic operational results are only of interest both models MOD1 and MOD2 were simulated using a transient analysis. One of the advantages of transient analysis is the capability of predicting the actual cooling time. Periodic-steady conditions are assumed when all system parameters are repeated from one cycle to the next within acceptable margins. The boundary conditions and initial temperatures of the aforementioned systems MOD1 and MOD2 are summarized in Table 3. The entries in this table that are in bold characters and are designated by an
asterisk are in fact calculated results from the simulations. As noted in Table 3, three
different cases are simulated for each of the assumed MOD1 and MOD2 systems.

The three cases address operation under adiabatic CHX wall (Case 1, Zero cooling
load), a CHX cooling load of 1 W (Case 2), and a constant known CHX surface
temperature (Case 3). These boundary conditions were chosen to simulate the cold-
end load curve results.

As noted in Table 3, all solid surfaces are either isothermal, or adiabatic.

For all the simulated cases, the initial temperature of gas in the entire system was
assumed to be at 300K for MOD1, and 293 K for MOD2.
<table>
<thead>
<tr>
<th>Study Cases</th>
<th>MOD1</th>
<th>MOD2</th>
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</thead>
<tbody>
<tr>
<td>Case 1</td>
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<td>Adiabatic</td>
</tr>
<tr>
<td>Case 2</td>
<td>Adiabatic</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>Case 3</td>
<td>Adiabatic</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>Case 4</td>
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<td>Case 5</td>
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<td>Case 6</td>
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<table>
<thead>
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<th>Compressor Wall</th>
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<td>Adiabatic</td>
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<table>
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<td>Adiabatic</td>
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<table>
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<table>
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<td>293 K</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>CHX LOAD (W)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0 W</td>
<td>1 W</td>
<td>6.39* W</td>
</tr>
<tr>
<td>0 W</td>
<td>1 W</td>
<td>2.57* W</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHX Temp</th>
<th>MOD1</th>
<th>MOD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>87* K</td>
<td>100* K</td>
<td>278* K</td>
</tr>
<tr>
<td>150 K</td>
<td>280* K</td>
<td>282 K</td>
</tr>
</tbody>
</table>

* These values are obtained from the experiment data [5]. + These are CHX, WHX1, WHX2, and Regenerator parameters.
RESULTS AND DISCUSSIONS

Steady periodic CFD simulation results will be presented and discussed in this chapter. The simulations correspond to cases 1-6, the conditions of which are summarized in Table 3. Cases 1-3 correspond to the ITPTC system MOD1, while cases 4-6 are related to the ITPTC system MOD2. The geometric characteristics of these ITPTC systems are depicted in Figure 17 and Table 2.

4.1 Case 1

The first case to be analyzed is Case 1 corresponding to the MOD1 system. Case 1 utilized an adiabatic wall boundary condition at the CHX, equivalent to no refrigeration load applied to the overall system. To simulate this system, a total of 4344 mesh points was used for MOD1, and 4385 mesh points were used for MOD2.
Figure 19 depicts the variation of the cold tip temperature as a function of time. Evidently, the system has achieved a steady-periodic state after about 250 seconds. The ultimate CHX temperature of 87 K was obtained approximately after 4 minutes of simulation time. In fact, the simulation took 100 seconds of transient time to reach within 10% of the steady periodic temperature. However, it should be emphasized that in the actual system the cooling time will be higher than what was predicted in the result since the thermal mass were not implemented in the simulated systems.

Figures 20 and 21 display the calculated cross section-average temperature and density distributions along the entire simulated system, respectively. The
temperature profile depicts the cycle-average quantity, and represents the steady-periodic conditions. However, the density profile depicts a local instantaneous snapshot of the system. The parts of these profiles that correspond to the regenerator and the pulse tube are interesting. The regenerator temperature distributions are approximately linear along the axial direction, while density distributions and temperature distributions of the pulse tube tend to be parabolic in shape.

Figure 20: Cycle-average Temperature Distribution for Case 1 (Adiabatic B.C.)
These density distribution trends are consistent with the ideal gas equation of state. In the simulations, the entire system was initially at 300 K. Once numerical convergence is achieved, the CHX temperature stabilized at 87 K. Evidently, Fluent can reach an expected steady-periodic state, starting from an arbitrary initial condition. Fluent’s way of solving the problem thus resembles typical experimental testing processes. In the depicted simulations, to verify that the system has reached steady-periodic state, Fluent examines the CHX temperature (just as one would do in an experiment), to see if the temperature of the CHX is identically repeated from a cycle to the next.
Figure 22: Temperature Contours for Case 1 (Adiabatic Wall B.C.) [K].

Figure 23: Snapshot of Density Contours for Case 1 (Adiabatic Wall B.C.) [kg/m³].
Figure 22 depicts the cycle-average temperature contours in the system and Figure 23 depicts the local instantaneous density contours in the system, under steady-periodic conditions. The contours are of course consistent with Figures 20 and 21, where cross-section averaged profiles were displayed. The distributions in Figures 22 and 23 suggest that, overall, the profiles are approximately one-dimensional (i.e., little variations of temperature or density over the system cross-section) in most of the ITPTC system. Relatively significant lateral non-uniformity in temperature can be noticed in the pulse tube near the CHX, however.

4.2 Case 2

This case represents the same physical system (MOD1) as case 1. A constant wall heat flux is assumed for the CHX, however. This is equivalent to an actual system undergoing a refrigeration process with a refrigeration load of 1W. The simulation is started from an assumed temperature of 300 K, and continues until steady-periodic conditions are reached. Figure 19 displays the temporal variation of the cycle-average temperature of the CHX surface. As noted, the simulation predicts an average temperature of 100 K.

The system took approximately 180 seconds of transient time to reach steady-periodic state. In comparison with Case 1, which represented the same
physical system and boundary conditions but with zero cooling load, the CHX has stabilized to a significantly higher average temperature. This is of course expected because when a refrigeration load is applied to the system, the operating cold tip temperature should increase.

Figure 24: Temperature Contour for Case 2 (1W Cooling Load) [K].
The Contours of temperature and density are shown in Figures 24 and 25, respectively. These contours are qualitatively similar to the contours depicted in Figures 22 and 23 and indicate approximately one-dimensional conditions in most of the system.

4.3 Case 3

This case represents the same physical system and boundary conditions as Case 1 and 2, with the exception of the boundary condition for CHX. In this case, a constant surface temperature (i.e., a target refrigerated surface temperature) of 150
K was imposed on CHX. In this case, because of the imposed CHX isothermal wall boundary condition, the heat rate coming into the CHX was actually calculated. Accordingly, steady-periodic condition should in principle be assumed when the CHX wall heat flux is identically repeated from a cycle to the next (See Figures 26-29). This, however, appears not feasible with Fluent. As noted in Figure 27, cycle-to-cycle oscillations occur even after quasi steady-periodic state has been reached. The source of these oscillations is believed to be numerical, and associated with the difficulty of imposing a constant wall temperature boundary condition when the boundary temperature is a function of time. For this simulation case quasi steady periodic condition implies time-invariant properties when these properties are averaged over several cycles. According to the simulation results, steady-periodic state was reached with a cooling load of 6.39 W. In other words, the system is disposing 6.39 W of refrigeration load at an operating cold end temperature of 150 K.
Figure 26: Refrigeration Load for Case 3 (Isothermal Wall B.C.)

Figure 27: Refrigeration Load for Case 3 (Isothermal Wall B.C.) with Zoomed Load Coordinate
Figure 28: Cycle-average Temperature Distribution for Case 3 (Isothermal Wall B.C.)

Figure 29: Local Density Distribution for Case 3 (Isothermal Wall B.C.)
Figure 30: Temperature Contours for Case 3 (Isothermal Wall B.C.) [K].

Figure 31: Density Contours for Case 3 (Isothermal Wall B.C.) [kg/m³].
Temperature and density contours are depicted in Figures 30 and 31, and once again display approximately one-dimensional distributions along most of the simulated systems.

4.4 Cases 4-6

These cases are based on the physical system represented by the MOD2 ITPTC (see Table 2, and Figure 17). The main differences between MOD1 and MOD2 systems are the large diameters of the regenerator and pulse tube in the latter and the large step changes in diameter between components. A large diameter of these components implies smaller aspect ratios for them. Smaller aspect ratios, as will be noted later, may cause considerable multi-dimensional flow effects. An adiabatic surface wall condition (zero cooling load) was imposed on the CHX for Case 4, and an average cooling load of 1 W was imposed on CHX for Case 5. Finally, an isothermal wall temperature of 282 K was assumed for the CHX component in Case 6. The steady-periodic state for Cases 4 and 5 means the identical repetition of all properties from cycle to cycle. For Case 6, however, quasi steady-periodic conditions are feasible, as discussed in section 4.3.
Figure 32, depicts the temporal variation of average temperature of CHX for Cases 4 and 5. It indicates that cold tip temperature stabilized at 278 K for Case 4 when steady-periodic state is achieved. The steady-periodic CHX surface temperature was approximately 280 K for Case 5, where cooling load of 1W was imposed on CHX. Both simulations were started assuming an initial system temperature of 293 K. These steady-periodic temperatures are considerably higher than the steady-periodic CHX temperature obtained for Cases 1 and 2. The MOD2 system is evidently quite inferior to MOD1. The cold tip temperatures of 278 K and 280 K are in fact too high to be considered as cryogenic.
Figure 33: Temperature Contours for Case 4 (Adiabatic Wall B.C.) [K].

Figure 34: Density Contours for Case 4 (Adiabatic Wall B.C.) [kg/m³].
Figure 35: Temperature Contours for Case 5 (1W Cooling Load) [K].

Figure 36: Density Contours for Case 5 (1W Cooling Load) [kg/m³].
The temperature and density distribution contours for Cases 4 and 5 are depicted in Figures 33-36. These figures show that, although MOD2 is performing inefficiently in comparison with the MOD1 system, the CHX is indeed getting slightly colder from its initial temperature.

Lastly, the system generated 2.57 W of refrigeration based on an isothermal boundary temperature of 282 K.

![Figure 37: Refrigeration Load for Case 6 (Isothermal Wall B.C.)](image)
Figure 38: Temperature Contours for Case 6 (Isothermal Wall B.C.) [K].

Figure 39: Density Contours for Case 6 (Isothermal Wall B.C.) [kg/m³].
Figures 37-39 display the simulation results for Case 6, where an isothermal wall surface temperature of 282 K was imposed on the CHX component of the MOD2 system. The average heat flux reached 2.57 W once steady-periodic conditions were obtained. Figure 37 shows that steady-periodic state of the overall system was reached after approximately 20 seconds of simulated operation time.

4.5 Convergence

The periodic nature of the flow and boundary conditions in the aforementioned simulations renders the issue of convergence and the conditions necessary for steady-periodic conditions complicated. There are many ways to verify the convergence of the entire system. One way is by using the volume-averaged temperature and the heat transfer rate at the CHX as indicators. One can argue that when these parameters are repeated identically from cycle to cycle then the steady-periodic state has been achieved. However, the first law of thermodynamics can also be utilized to validate the steady-periodic state. Evidently, steady-periodic state can be assumed in principle only when the input and output amounts of energy to the entire system during a cycle are equal. To perform a first-law analysis of the entire system, consider the macroscopic control volume depicted in Figure 40. Steady-periodic state requires that:
Figure 40: Energy Balance for the ITPTC

\[
\langle \dot{W} \rangle_{PV} + \langle \dot{Q} \rangle_{\text{refrig}} = \langle \dot{Q} \rangle_{\text{comp}} + \langle \dot{Q} \rangle_{\text{reject}}
\]  

(25)

Where, for any property \( \xi \) :

\[
\left\langle \xi \right\rangle = \int \dot{\xi} \, dt
\]

Although equation (25) should be strictly satisfied at steady-periodic and fully-converged conditions, a more realistic representation of first law would be:

\[
\left| \langle \dot{W} \rangle_{PV} - \langle \dot{Q} \rangle_{\text{comp}} \right| + \left| \langle \dot{Q} \rangle_{\text{refrig}} \right| - \left| \langle \dot{Q} \rangle_{\text{reject}} \right| = \text{Error}
\]  

(26)

In the numerical simulations, Error should evidently be minimized. Table 4 is a summary of the results of the first-law analysis for all the aforementioned cases. The
Table 4 includes all the energy exchange terms between the system and the environment, and the overall error calculated from equations (26).

Table 4: Cycle Averaged Energy Balance Results

<table>
<thead>
<tr>
<th></th>
<th>CASE 1</th>
<th>CASE 2</th>
<th>CASE 3</th>
<th>CASE 4</th>
<th>CASE 5</th>
<th>CASE 6</th>
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</thead>
<tbody>
<tr>
<td>CHX TEMP. (K)</td>
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<td>150</td>
<td>278</td>
<td>280</td>
<td>282</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{refrig}}$ (W)</td>
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</tbody>
</table>

Table 4 indicates that overall energy balance is not strictly satisfied for any of the simulation cases. This is despite the fact that, in all these simulations, the system had reached steady periodic state, as noted in Figures 19, 27, 32, and 37. This apparent lack of energy balance is believed to be primarily caused by numerical errors and the inaccuracy of the method used for evaluating the cycled averaged compressor power $\langle \dot{W}_{pv} \rangle$. The latter term has been calculated using equation (1), and equation (1) evidently represents a reversible process. The impact of numerical errors associated with the computations must also be emphasized. Although Fluent
solves the entire set of governing equations with relatively tight convergence criteria, the numerical scheme always exhibits some errors due to its many inherent approximations.

4.6 Multi-Dimensional Flow Effects

Arguments for multi-dimensional modeling of large PTC systems were presented in Chapter 1. The significance of flow multi-dimensional effects, and conditions that enhance them, will be addressed in this section. Intuition suggests that multi-dimensional flow effects become more significant for any component as that components’ aspect ratio (length to diameter ratio) is reduced. Also, one would expect to see the most significant flow multi-dimensionalities at the vicinity of component-to-component junctions. Our investigation of the details of the flow field at the critical component-to-component junctions of the system MOD2 in fact support the above intuitive observations. Figures 41-47 represent typical snapshots of the fluid velocities at the component-to-component junctions and their vicinity. Clearly, multi-dimensional effects are large for components that have smaller aspect ratios. Secondary flows in the form of vortices and recirculation patterns can be observed, in the short pulse tubes depicted in Figures 45-47. Equally important are the 2-dimensional velocity vectors at the vicinity of the short regenerator inlet and outlet
depicted in Figure 44. These multi-dimensional flows undoubtedly impact pressure drop, dissipation, and heat transfer processes in the porous regenerator. The aforementioned observation regarding multi-dimensional flow effects, and comparison among the simulation cases for the perceived PTC systems MOD1 and MOD2 as summarized in Table 2, allow us to make two important observations. First, one-dimensional analysis is likely to be inadequate for PTC systems that include one or more component that have a small aspect ratio. A one-dimensional analysis could be quite adequate for a PTC system if all of its components have large aspect ratios, on the other hand. Second, the overall performance of an ITPTC is likely to deteriorate when some of its main components have small aspect ratios. A strong emphasis should be noted here regarding the step change in diameter of MOD2 from MOD1. Although small aspect ratio components demonstrated higher multi-dimensionalities it is not clear from these simulations that how much of these multi-dimensionalities are caused by the step change in diameter or the reduced aspect ratios.

The above observations are based on a limited number of CFD simulations, it must be emphasized. More simulation studies and experiments are evidently needed before conclusive observations can be made.
Figure 41: Snapshot of Velocity Vectors in the Regenerator with L/D = 7.25 [m/s].

Figure 42: Snapshot of Velocity Vectors at the vicinity of the Regenerator Outlet with L/D = 7.5 [m/s].
Figure 43: Snapshot of Velocity Vectors at the vicinity of the Pulse Tube Inlet with L/D = 12, in [m/s].

Figure 44: Snapshot of Velocity Vectors and Temperatures [K] in the Regenerator with L/D = 1.25.
Figure 45: Snapshot of Velocity Vectors and Temperatures [K] in the Pulse Tube L/D = 2.
Figure 46: Snapshot of Velocity Vectors and Temperatures [K] at the vicinity of the Pulse Tube Inlet with $L/D = 2$. 
Figure 47: Snapshot of Velocity Vectors and Temperatures [K] in the Pulse Tube with L/D = 2.
Figure 48: Snapshot of Velocity Vectors and Temperatures [K] at the vicinity of the Surge Volume Inlet.
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

5.1 Concluding Remarks

In this study, the commercial computational fluid dynamic (CFD) package Fluent® was utilized for modeling the entire large ITPTC system that includes a compressor, an after cooler, a regenerator that is represented as a porous medium, a pulse tube, cold and warm heat exchangers, an inertance tube, and a reservoir. The simulations represented a fully-coupled system operating in steady periodic mode, without any arbitrary assumptions other than ideal gas and no gravity effects. The objective was to examine the extent of multi-dimensional flow effects in inertance tube pulse tube cryocoolers, and their impact on the performance of these cryocoolers.

Computer simulations have been performed and presented for two complete ITPTC systems that were geometrically similar except for the length-to-diameter ratios of their regenerators, pulse tubes, and the inertance tube lengths. For each ITPTC system three separate simulations were analyzed. One simulation assumed
an adiabatic cold-end heat exchanger (CHX), another assumed a known cooling heat load, and the last assumed a pre-specified CHX temperature. The simulations were performed using the Fluent ® CFD software. Each simulation started with an assumed uniform system temperature, and continued until steady periodic conditions were achieved.

The results showed that CFD simulations are capable of elucidating the complex periodic processes in PTCs very well. The simulation results also showed that a one-dimensional modeling of PTCs is appropriate only when all the components of the PTC have very large aspect ratios (i.e., L/D >>1). Significant multi-dimensional flow and temperature effects occurred at the vicinity of component-to-component junctions, and secondary-flow recirculation patterns developed, when one or more components of the PTC system had a small aspect ratio. Our simulation results, although limited in scope, also suggested that ITPTCs will have a better overall performance if they are made of components with large aspect ratios.

5.2 Recommendations for Future Work

The Fluent simulation results presented and discussed in this thesis provided reasonable predictions that had expected trends everywhere. Such CFD simulations provide very valuable details about local and instantaneous processes.
However, simulation results of this type need to be verified against experiments in order to ascertain their correctness. To this end, careful experiments with measurements that are suitable for the validation of CFD models are recommended.

The simulations performed in this study also indicated that components with small aspect ratios support multi-dimensional flow processes, and may lead to an overall system performance that is inferior to a similar system that is made of components with large aspect ratios only. These observations were based on a limited set of simulations, however, and more simulations covering a wider range of geometric and operational parameters are recommended.

The flow phenomena in PTC are oscillatory, and in certain circumstances multi-dimensional. The empirical closure relations and parameters that are often used in CFD models are mostly based on steady-state data, however. The accuracy and relevance of these closure parameters to oscillatory flow conditions are questionable. Experiments aimed at measurement and correlation of friction factor and heat transfer coefficient under oscillatory flow conditions, in flow and geometric conditions relevant to those encountered in PTCs, are recommended.
A1. Stirling Crycooler

Stirling Cryocoolers (SC) use a secondary expansion piston instead of the pulse tube, the orifice valve and the inertance tube. SCs are therefore simpler in comparison with PTCs. The existence of a secondary piston, however, is its main disadvantage.

Figure A. 1: Temperature Distribution of Stirling Cryocoolers [3]
The principles of operations of SC are described by Ackermann [3]. Figure A.1 depicts a schematic of a SC system, and provides a temperature vs. position profile for the working fluid. During a cycle, the following sequences of processes take place:

- Process 1-2: Working fluid cools down to cold environment temperature by flowing through the regenerator and experiencing a heat transfer process from the gas to the regenerator matrix.
- Process 2-3: Additional heat transfer takes place with the CHX surface wall.
- Process 3-4: An adiabatic expansion process takes place due to the expansion of the piston.
- Process 4-5: Gas inside the piston returns back to the CHX and absorbs the refrigeration load ($\dot{Q}_{\text{retrig}}$)
- Process 5-6: Gas is warmed up due to the heat transferred from the regenerator matrix to the gas.
- Process 6-7: Gas is further heated by the WHX surface wall.
- Process 7-8: Gas is compressed adiabatically in the compressor
- Process 8-1: Heat ($\dot{Q}_{\text{reject}}$) is rejected through the WHX
A2. Fluent User Defined Functions

A2.1 UDF Compressor Code

#include "udf.h"

DEFINE_CG_MOTION(vel_comp, dt, vel, omega, time, dtime)
{
    real freq=34.0;

    real w=2.0*M_PI*freq;

    real Xcomp=0.004511;

    /* reset velocities */

    NV_S (vel, =, 0.0);

    NV_S (omega, =, 0.0);

    vel[0] = w*Xcomp*cos(w*time);
}

REFERENCES


