Set-Valued Protocols for Almost Consensus of Multiagent Systems with Uncertain Interagent Communication

Teymur Sadikhov, Wassim M. Haddad, Rafal Goebel, and Magnus Egerstedt

***Abstract*— One of the main challenges in robotics applications is dealing with inaccurate sensor data. Specifically, for a group of mobile robots the measurement of the exact location of the other robots relative to a particular robot is often inaccurate due to sensor uncertainty or detrimental environmental conditions. In this paper, we address the consensus problem for a group of agent robots with uncertain interagent communication. Measurement uncertainty is characterized by balls of radius r centered at the neighboring agents exact locations. We show that the agents reach an almost consensus state and converge to a time-varying ball of radius r and include an analysis approach to the problem based on set-valued analysis. Finally, several illustrative numerical examples are provided to demonstrate the efficacy of the proposed set-valued consensus protocol framework.**

I. Introduction

In this paper, we consider a multiagent consensus problem in which agents have sensors with limited accuracy. Specifically, in numerous network system applications agents can detect the location of the neighboring agents only approximately. This problem arises in robotics applications involving low sensor quality or detrimental environmental conditions. In such a setting, it is desirable that the agents reach consensus approximately. We develop a set-valued consensus protocol that guarantees that the agents converge to a time-varying set of diameter $2r$ when the agents have sensors that can detect the location of the neighboring agents with accuracy up to a ball of radius r centered at the actual location of the neighboring agents. This set is shown to be time-varying, in the sense that only the differences between agents positions are, in the limit, small. Due to the uncertainty in interagent communication, we use difference inclusions and set-valued analysis to describe the problem formulation.

Set-valued analysis has been previously used for consensus control. In [1], the author uses set-valued Lyapunov functions to study convergence of multiagent dynamical systems. The approach involves constructing set-valued Lyapunov functions from convex sets that depend on the agent states. In [1]–[3], the authors address stability of each equilibrium point in the sense that the system solutions approach an equilibrium from a neighborhood of equilibria. Reference [3] considers barycentric coordinate maps, whereas [1] and [2] consider difference equations and difference inclusions, respectively. Necessary and sufficient conditions for pointwise asymptotic stability for multiagent consensus problems using set-valued Lyapunov analysis are presented in [4]. More recently, the authors in [5] consider an asynchronous rendezvous problem using set-valued consensus theory. Specifically, a design strategy for multiagent consensus is developed by requiring two consecutive way-points to be included within a minimum convex region covering the two associated anticipated-way-point sets.

In this paper, we build on the framework of [1], [4] and [6] to develop almost consensus protocols for multiagent systems with uncertain interagent communication. Specifically, the proposed protocol algorithm modifies the set-valued consensus update maps of the agents by assuming that the locations of all agents, including the agents calculating the update map, are within a ball of radius r. However, since the update sets of our design protocol do not satisfy a strict convexity assumption, our results go beyond the results of [1] by employing a set-valued invariance principle.

This research was supported in part by the Air Force Office of Scientific Research under Grant FA9550-12-1-0192

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II. Notation and Mathematical Preliminaries

A. Notation

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\overline{\mathbb{Z}}_+$ denotes the set of nonnegative integers, and $(\cdot)^{\mathrm{T}}$ denotes transpose. We write $\partial \mathcal{S}$, $\overline{\mathcal{S}}$, $|\mathcal{S}|$, and $\operatorname{co} \mathcal{S}$ to denote the boundary, closure, cardinality, and convex hull of the subset $\mathcal{S} \subset \mathbb{R}^n$, respectively. Furthermore, we write $\|\cdot\|$ for the Euclidean vector norm on $\mathbb{R}^n$, $\mathcal{B}_\varepsilon(\alpha)$, $\alpha \in \mathbb{R}^n$, $\varepsilon > 0$, for the *open ball centered* at α with *radius* ε, $\operatorname{dist}(p, \mathcal{M})$ for the distance from a point p to the set $\mathcal{M}$, that is, $\operatorname{dist}(p, \mathcal{M}) \triangleq \inf_{x \in \mathcal{M}} \|p - x\|$, and $x(k) \to \mathcal{M}$ as $k \to \infty$, where $k \in \overline{\mathbb{Z}}_+$, to denote that the trajectory $x(k)$ approaches the set $\mathcal{M}$, that is, for every $\varepsilon > 0$ there exists $N_0 > 0$ such that $\operatorname{dist}(x(k), \mathcal{M}) < \varepsilon$ for all $k > N_0$. Finally, the notions of openness, convergence, continuity, and compactness that we use throughout the paper refer to the topology generated on $\mathbb{R}^n$ by the norm $\|\cdot\|$.

In this paper, we consider difference inclusions of the form

$$x(k+1) \in \mathcal{F}(x(k)), \quad x(0) = x_0, \quad k \in \overline{\mathbb{Z}}_+, \tag{1}$$

where, for every $k \in \overline{\mathbb{Z}}_+$, $x(k) \in \mathbb{R}^n$, $\mathcal{F} : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is a *set-valued map* that assigns sets to points, and $2^{\mathbb{R}^n}$ denotes the collection of all subsets of $\mathbb{R}^n$. The set-valued map $\mathcal{F}$

has a *nonempty value at* x if $\mathcal{F}(x) \neq \varnothing$. It is assumed that $\mathcal{F}$ has nonempty values for ever $x \in \mathbb{R}^n$. Hence, maximal solutions to (1) are complete, and consequently, by a *solution* of (1) with initial condition $x(0) = x_0$ we mean a function $x : \overline{\mathbb{Z}}_+ \to \mathbb{R}^n$ that satisfies (1).

The set-valued map $\mathcal{F} : \mathbb{R}^n \to 2^{\mathbb{R}^n}$ is *outer semicontinuous at* x if, for every sequence $\{x_i\}_{i=0}^{\infty}$ such that $\lim_{i\to\infty} x_i = x$, every convergent sequence $\{y_i\}_{i=0}^{\infty}$ with $y_i \in \mathcal{F}(x_i)$ satisfies $\lim_{i\to\infty} y_i \in \mathcal{F}(x)$. $\mathcal{F}$ is *continuous at* x if $\mathcal{F}$ is outer semicontinuous at x and, for every $y \in \mathcal{F}(x)$ and every convergent sequence $\{x_i\}_{i=0}^{\infty}$, there exists $y_i \in \mathcal{F}(x_i)$ such that $\lim_{i\to\infty} y_i = y$. $\mathcal{F}(x)$ is *locally bounded at* x if there exists a neighborhood $\mathcal{N}$ of x such that $\mathcal{F}(\mathcal{N}) = \cup_{z\in\mathcal{N}}\mathcal{F}(z)$ is bounded. If $\mathcal{F}$ has compact values and is locally bounded at x, then $\mathcal{F}$ is *upper semicontinuous at* x, that is, for every $\varepsilon > 0$, there exists δ such that, for all $z \in \mathbb{R}^n$ satisfying $\|z - x\| < \delta$, $\mathcal{F}(z) \subseteq \mathcal{F}(x) + \overline{\mathcal{B}}_\varepsilon(0)$.

Given the function $\gamma : \overline{\mathbb{Z}}_+ \to \mathbb{R}^n$, the *positive limit set of* γ is the set $\Omega(\gamma)$ of points $y \in \mathbb{R}^n$ for which there exists an increasing divergent sequence $\{k_n\}_{n=0}^{\infty}$ satisfying $\lim_{n\to\infty} \gamma(k_n) = y$. We denote the positive limit set of a solution $\psi(\cdot)$ of (1) by $\Omega(\psi)$. The positive limit set of a bounded solution of (1) is nonempty, compact, and weakly forward invariant with respect to (1) [7]. Furthermore, in this paper we distinguish between the set inclusions $\subset$ and $\subseteq$; namely, $\subset$ denotes a strict inclusion, whereas $\subseteq$ denotes a nonstrict inclusion. Finally, we use the Minkowski sum for summation of sets with an analogous definition for set subtraction. Namely, for the sets $\mathcal{X}, \mathcal{Y} \subset \mathbb{R}^n$, $\mathcal{X} + \mathcal{Y}$ and $\mathcal{X} - \mathcal{Y}$ denote, respectively, the set of all vectors $z \in \mathbb{R}^n$ such that $z = x + y$ and $z = x - y$, where $x \in \mathcal{X}$ and $y \in \mathcal{Y}$.

B. *Consensus over networks*

The consensus problem appears frequently in coordination of multiagent systems and involves finding an algorithm that enables a group of agents in a network to agree upon certain quantities of interest with undirected and directed information flow [8]–[10]. In this paper, we use undirected graphs with all-to-all graph connectivity to represent a network. A graph $\mathfrak{G}$ is *all-to-all connected* if every node of $\mathfrak{G}$ is connected to every other node of $\mathfrak{G}$. Furthermore, we denote the value of the node $i \in \{1, \ldots, N\}$ at time step k by $x_i(k) \in \mathbb{R}^n$.

The consensus problem involves the design of a dynamic algorithm that guarantees system state *equipartition* [8], [10], that is, $\lim_{k\to\infty} x_i(k) = q \in \mathbb{R}^n$ for $i = 1, \ldots, N$. In this paper, we consider a variant of the distributed consensus algorithms of the form ([8])

$$x_i(k+1) = x_i(k) + \frac{1}{N}\sum_{j=1}^{N}(x_j(k) - x_i(k)),$$
$$x_i(0) = x_{i0}, \quad k \in \overline{\mathbb{Z}}_+, \quad i = 1, \ldots, N. \quad (2)$$

III. Problem Formulation

In this paper, we consider a multiagent network in which N agents reach an almost consensus state and we use the terminology *agent state* and *agent location* interchangibly.

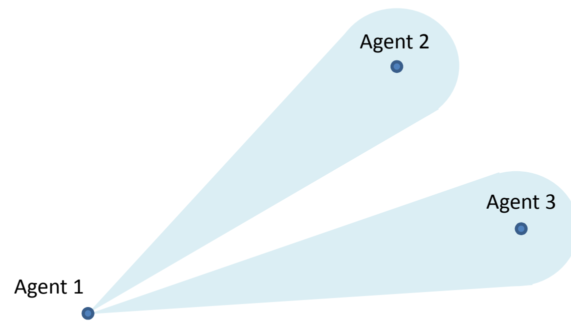

Fig. 1. Visualization of sets $\mathcal{X}_2 - x_1$ and $\mathcal{X}_3 - x_1$ used in agent's 1 update map.

Each agent $i \in \{1, \ldots, N\}$ has a sensor with accuracy r, that is, each agent i can detect the location of other agents with accuracy of up to a ball of radius r centered at the actual location of other agents. Specifically, the approximate location of agent i as measured by agent j is given by the set

$$\mathcal{X}_i = \{p \in \mathbb{R}^n : \|p - x_i\|_2 \leq r\}, \quad i = 1, \ldots, N.$$

The network consensus problem considered in this paper involves the design of a dynamic protocol that guarantees almost system state equipartition, that is, the difference between any two agent states decreases to below a certain threshold dependent on the sensor accuracy r. Specifically, each agent i uses an update protocol similar to (2). However, since only approximate information of the location of other agents is available at any given instant of time, the update protocol is constructed using approximate location information only. Thus, the update protocol, for $i = 1, \ldots, N$, has the form

$$x_i(k+1) \in \mathcal{F}_i(x(k)) \triangleq x_i(k) + \frac{1}{N}\sum_{j=1}^{N}(\mathcal{X}_j(k) - x_i(k)),$$
$$x_i(0) = x_{i0}, \quad k \in \overline{\mathbb{Z}}_+, \quad (3)$$

where $x \triangleq [x_1^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathcal{X}_j - x_i$ denotes the set of all vectors $z \in \mathbb{R}^n$ such that $z = y - x_i$ with $y \in \mathcal{X}_j$. Note that for the protocol given by (2) every agent has information of the exact location of other agents, whereas for the protocol given by (3) only approximate location information of other agents is available.

To further elucidate the protocol architecture given by (3), consider a connected network consisting of three agents. In this case, the update protocol for agent 1 is given by

$$\begin{aligned} x_1(k+1) &\in \mathcal{F}_1(x(k)) \\ &= x_1(k) + \frac{1}{3}(\mathcal{X}_1(k) - x_1(k) \\ &\quad + \mathcal{X}_2(k) - x_1(k) + \mathcal{X}_3(k) - x_1(k)), \\ &\qquad x_1(0) = x_{10}, \quad k \in \overline{\mathbb{Z}}_+, \end{aligned}$$

where the sets $\mathcal{X}_2 - x_1$ and $\mathcal{X}_3 - x_1$ are depicted in Figure 1.

IV. Discrete-Time Consensus with All-to-All Graph Connectivity

In this section, we consider a discrete-time consensus protocol for multiagent systems involving an all-to-all graph connectivity network given by

$$\begin{aligned} x_i(k+1) &\in \alpha\frac{1}{N}\sum_{j=1}^{N}\mathcal{X}_j(k)+(1-\alpha)x_i(k) \\ &= \mathcal{B}_{\alpha r}(\alpha x_{\rm ave}(k))+(1-\alpha)x_i(k), \\ & x_i(0)=x_{i0},\quad k\in\overline{\mathbb{Z}}_+,\quad i=1,\ldots,N, \quad (4) \end{aligned}$$

where $\alpha\in(0,1]$ and $x_{\rm ave}(k)\triangleq\frac{1}{N}\sum_{i=1}^{N}x_i(k)$. The result below shows that, for large enough k, $\|x_i(k+1)-x_j(k+1)\|_2$ is close to or less than or equal to $2r$.

Proposition 4.1: Consider a network of N agents with an all-to-all graph connectivity given by (4). Then, $\limsup_{k\to\infty}\|x_i(k)-x_j(k)\|_2 \le 2r$ for every $i,j=1,\ldots,N$.

Proof: For $i,j=1,\ldots,N$, it follows from (4) that

$$\begin{aligned} x_i(k+1)-x_j(k+1) \in{}& \mathcal{B}_{\alpha r}(\alpha x_{\rm ave}(k))-\mathcal{B}_{\alpha r}(\alpha x_{\rm ave}(k)) \\ &+(1-\alpha)(x_i(k)-x_j(k)), \\ & k\in\overline{\mathbb{Z}}_+, \end{aligned}$$

which implies

$$\begin{aligned} \|x_i(k+1)-x_j(k+1)\|_2 \le{}& (1-\alpha)\|x_i(k)-x_j(k)\|_2 \\ &+2r\alpha. \end{aligned}$$

Hence, since $\|x_i(k+1)-x_j(k+1)\|_2\le\|x_i(k)-x_j(k)\|_2$ for $\|x_i(k)-x_j(k)\|_2\ge 2r$, it follows that $\|x_i(k)-x_j(k)\|_2\le 2r$ as $k\to\infty$ for all $i,j=1,\ldots,N$. ∎

V. Discrete-Time Consensus with All-to-All Graph Connectivity: A Set-Valued Analysis Approach

In this section, we present a set-valued approach for the discrete-time consensus protocol considered in Section IV. The following theorem gives a general set-valued invariance principle using the set-valued analysis tools developed in [4].

Theorem 5.1: Consider the difference inclusion (1). Assume that $\mathcal{F}:\mathbb{R}^n\to 2^{\mathbb{R}^n}$ is outer semicontinuous and locally bounded with nonempty values for all $x\in\mathbb{R}^n$. Let $V:\mathbb{R}^n\to 2^{\mathbb{R}^n}$ be a continuous set-valued map and let $\mathcal{M}\subset\mathbb{R}^n$ be a closed set such that the following statements hold.

i) $V(\mathcal{F}(x))\subseteq V(x)$ for every $x\in\mathbb{R}^n$.

ii) If $V(y)=V(x)$ for some $y\in\mathcal{F}(x)$, then $x\in\mathcal{M}$.

Then every bounded solution $x:\overline{\mathbb{Z}}_+\to\mathbb{R}^n$ of (1) converges to $\mathcal{M}$, that is, $\lim_{k\to\infty}{\rm dist}(x(k),\mathcal{M})=0$.

Proof: It follows from *i)* that $V(\psi(k+1))\subseteq V(\psi(k))$ for every solution $\psi(k)$, $k\in\overline{\mathbb{Z}}_+$, of (1). Hence, the sequence of closed sets $\{V(\psi(k)\}_{k=0}^{\infty}$ is nonincreasing and hence $\lim_{k\to\infty}V(\psi(k))=\cap_{k=0}^{\infty}V(\psi(k))\triangleq\mathcal{V}$ [7]. Next, note that since $\psi(k)$, $k\in\overline{\mathbb{Z}}_+$, is bounded, $\Omega(\psi)$ is nonempty. Now, for all $x\in\Omega(\psi)$, it follows from the definition of $\Omega(\psi)$ and the continuity of V that $V(x)=\mathcal{V}$. Moreover, the outer semicontinuity of $\mathcal{F}$ ensures that $\Omega(\psi)$ is weakly positively (and negatively) invariant. Specifically, for every $x\in\Omega(\psi)$, there exists $y\in\mathcal{F}(x)$ such that $y\in\Omega(\psi)$. Thus, for every $x\in\Omega(\psi)$, there exists $y\in\mathcal{F}(x)$ such that $V(x)=V(y)=\mathcal{V}$, and hence, $\Omega(\psi)\subseteq\mathcal{M}$. Finally, since ${\rm dist}(\psi(k),\omega(\psi))\to 0$ as $k\to 0$, it follows that $\psi(k)\to\mathcal{M}$ as $k\to\infty$. ∎

Next, we illustrate Theorem 5.1 by applying it to the network system given by (4). The conclusions of the proposition below are weaker than those obtained directly in the previous section. However, the approach can prove beneficial in nonlinear settings where direct computation relying on linear structure is not possible as well as for partial graph connectivity structures with directed information flow.

Proposition 5.1: Consider a network of N agents with an all-to-all graph connectivity given by (4) and let x be a bounded solution of (4). Then, $\limsup_{k\to\infty}\|x_i(k)-x_j(k)\|_2\le 4r$ for every $i,j=1,\ldots,N$.

Proof: Let the set-valued map $V:\mathbb{R}^n\to 2^{\mathbb{R}^n}$ be given by

$$V(x)=\mathcal{B}_{\delta_1(x)}(x_{\rm ave})\times\cdots\times\mathcal{B}_{\delta_N(x)}(x_{\rm ave}),$$

where, for $i\in\{1,\ldots,N\}$,

$$\delta_i(x)=\begin{cases}\|x_i-x_{\rm ave}\|_2, & \|x_i-x_{\rm ave}\|_2\ge 2r,\\ 2r, & \|x_i-x_{\rm ave}\|_2\le 2r,\end{cases}$$

and "×" denotes Cartesian product. Note that V is continuous and has closed and bounded values. Next, it can be shown using a similar argument as in the proof of Proposition 4.1 that

$$\begin{aligned} x_i(k+1)-x_{\rm ave}(k+1) \in{}& \mathcal{B}_{\alpha r}(\alpha x_{\rm ave}(k))-\mathcal{B}_{\alpha r}(x_{\rm ave}(k)) \\ &+(1-\alpha)x_i(k),\quad k\in\overline{\mathbb{Z}}_+, \end{aligned}$$

which implies

$$\begin{aligned} \|x_i(k+1)-x_{\rm ave}(k+1)\|_2 \le{}& (1-\alpha)\|x_i(k)-x_{\rm ave}(k)\|_2 \\ &+2r\alpha. \end{aligned}$$

Hence, the function $\delta_i(\cdot)$ decreases for $\|x_i-x_{\rm ave}\|_2>2r$ and remains constant for $\|x_i-x_{\rm ave}\|_2\le 2r$, $i\in\{1,\ldots,N\}$, and hence, Conditions *i)* and *ii)* of Theorem 5.1 are satisfied. Now, it follows from Theorem 5.1 that every bounded solution $x_i(\cdot)$, $i\in\{1,\ldots,N\}$, converges to $\mathcal{B}_{2r}(x_{\rm ave})$. Hence, $\|x_i(k)-x_j(k)\|_2\le 4r$ as $k\to\infty$ for all $i,j=1,\ldots,N$. ∎

VI. Continuous-Time Consensus with All-to-All Graph Connectivity

In this section, we consider the continuous-time analogue of the consensus problem presented in Section IV. The continuous-time consensus problem involves the design of an update map that guarantees system state equipartition, that is, $\lim_{t\to\infty}x_i(t)=q\in\mathbb{R}^n$ for $i=1,\ldots,N$. Here we consider a variant of the classical distributed consensus algorithm involving an all-to-all graph topology given by

$$\dot{x}_i(t)=\frac{1}{N}\sum_{i=1}^{N}(x_j(t)-x_i(t)),\quad x_i(0)=x_{i0},$$

$$t \geq 0, \quad i = 1, \ldots, N.$$

Specifically, we assume that only approximate information of the location of neighboring agents is available at any given instant of time with (i, j) agent uncertainty $\|d_{ij}(t)\|_2 \leq r$, $t \geq 0$. In this case, the update protocol for agent i is given by

$$\begin{aligned} \dot{x}_i(t) &= \frac{1}{N}\sum_{j=1}^{N}[x_j(t) + d_{ij}(t) - x_i(t)] \\ &= x_{\text{ave}}(t) + d_i(t) - x_i(t), \quad x_i(0) = x_{i0}, \quad t \geq 0, \\ & \qquad i = 1, \ldots, N, \quad (5) \end{aligned}$$

where $x_{\text{ave}}(t) \triangleq \frac{1}{N}\sum_{j=1}^{N} x_j(t)$ and $d_i(t) \triangleq \frac{1}{N}\sum_{j=1}^{N} d_{ij}(t)$.

Proposition 6.1: Consider a network of N agents with all-to-all graph connectivity given by (5). Then, $\limsup_{t\to\infty} \|x_i(t) - x_j(t)\|_2 \leq 2r$ for every $i, j = 1, \ldots, N$.

Proof: Using (5), for $i, j = 1, \ldots, N$, it follows that

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\|x_i(t) - x_j(t)\|_2^2\right) \\ &= (x_i(t) - x_j(t))^{\mathrm{T}}\frac{\mathrm{d}}{\mathrm{d}t}(x_i(t) - x_j(t)) \\ &= (x_i(t) - x_j(t))^{\mathrm{T}}[x_{\text{ave}}(t) + d_i(t) - x_i(t) \\ &\quad -(x_{\text{ave}}(t) + d_j(t) - x_j(t))] \\ &= -\|x_i(t) - x_j(t)\|_2^2 + (x_i(t) - x_j(t))^{\mathrm{T}}(d_i(t) - d_j(t)) \\ &\leq -\|x_i(t) - x_j(t)\|_2^2 + 2r\|x_i(t) - x_j(t)\|_2, \\ &\qquad x_i(0) - x_j(0) = x_{i0} - x_{j0}, \quad t \geq 0, \end{aligned}$$

where the last inequality follows from the fact that

$$\|d_i(t) - d_j(t)\|_2 \leq \|d_i(t)\|_2 + \|d_j(t)\|_2 \leq 2r, \quad t \geq 0.$$

Hence, $\|x_i(t) - x_j(t)\|_2$ is decreasing function of time as long as $\|x_i(t) - x_{\text{ave}}(t)\|_2 > 2r$, $t \geq 0$. Now, it follows that $\|x_i(t) - x_j(t)\|_2 \leq 2r$ as $t \to \infty$ for all $i, j = 1, \ldots, N$. ■

VII. Illustrative Numerical Examples

In this section, we present two numerical examples to demonstrate the efficacy of the proposed framework. Specifically, we consider a random network of 10 agents with an all-to-all connectivity and agent dynamics given by (3). Figures 2, 3, 4, and 5 show the initial, intermediate, and final configurations, respectively, of the network of agents when agents have sensor accuracy of radius 1. The simulation shows that the agents reach a time-varying consensus set with diameter less than $2r = 2$. Similarly, Figures 6, 7, 8, and 9 show the initial, intermediate, and final configurations, respectively, of the network of 10 agents when agents have sensor accuracy of radius 0.5. The simulation shows that the agents reach a time-varying consensus set with diameter less than $2r = 1$.

VIII. Conclusion

In this paper, we considered the problem of set-valued protocols for almost consensus for multiagent systems with uncertain interagent communications, wherein the agents can detect the location of the neighboring agents only up to an

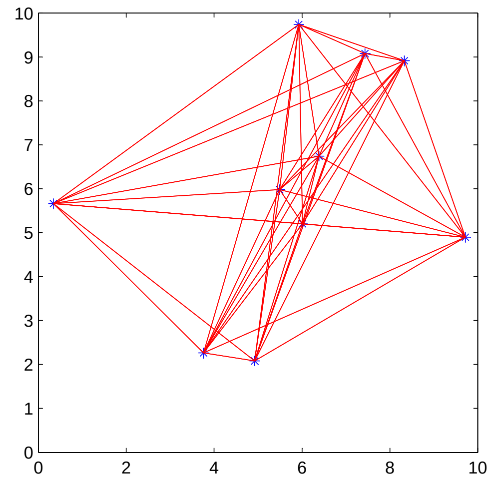

Fig. 2. Initial network configuration of 10 agents with sensor accuracy of radius $r = 1$.

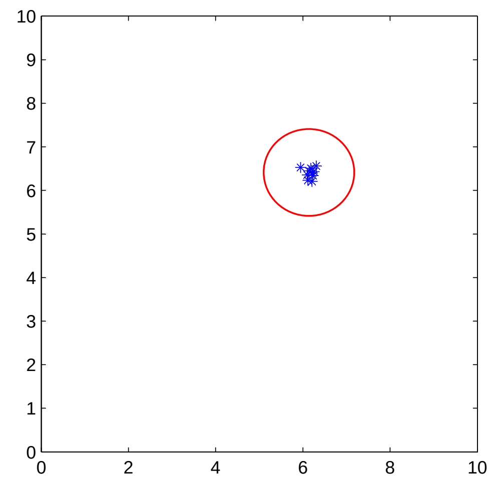

Fig. 3. Network configuration of 10 agents with sensor accuracy of radius $r = 1$ at 50th time step.

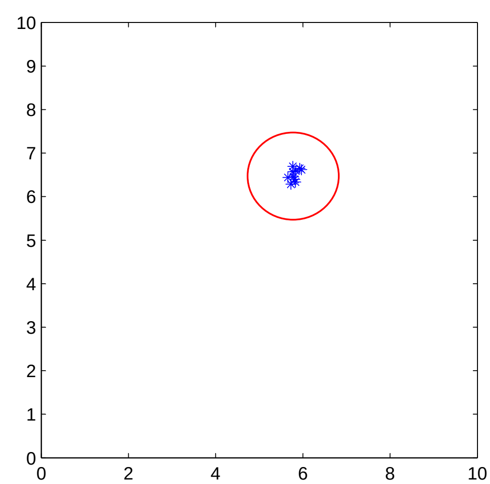

Fig. 4. Network configuration of 10 agents with sensor accuracy of radius $r = 1$ at 100th time step.

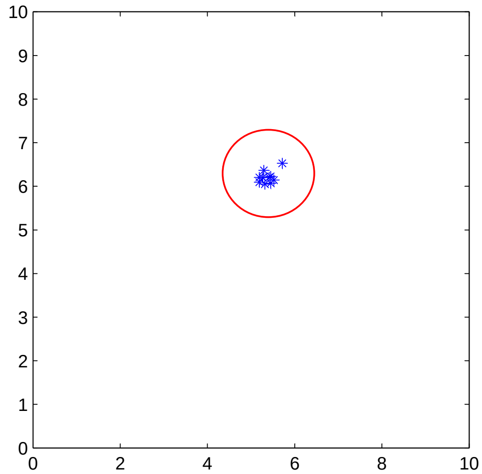

Fig. 5. Network configuration of 10 agents with sensor accuracy of radius $r = 1$ at 150th time step.

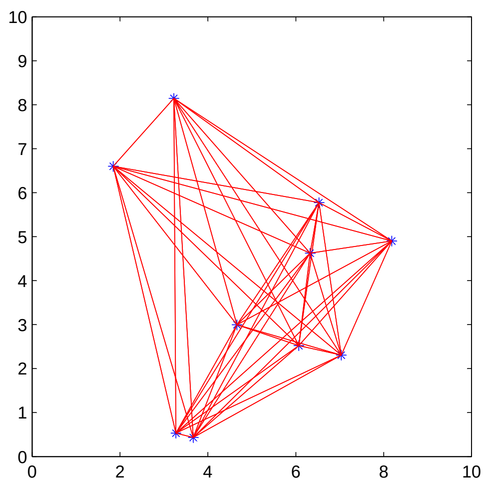

Fig. 6. Initial network configuration of 10 agents with sensor accuracy of radius $r = 0.5$.

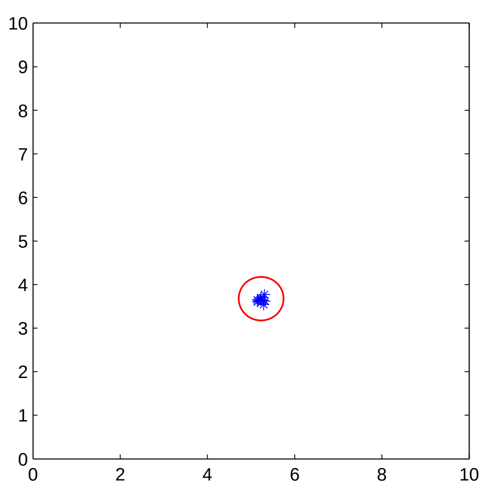

Fig. 7. Network configuration of 10 agents with sensor accuracy of radius $r = 0.5$ at 50th time step.

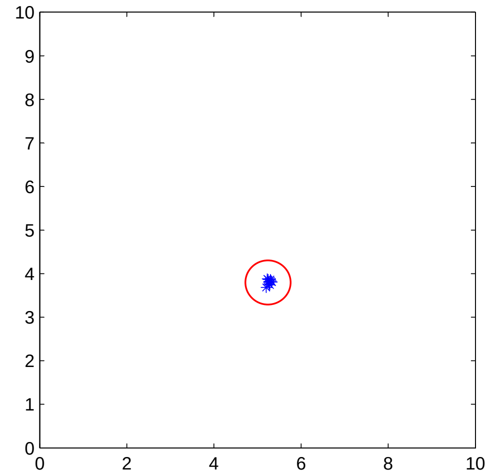

Fig. 8. Network configuration of 10 agents with sensor accuracy of radius $r = 0.5$ at 100th time step.

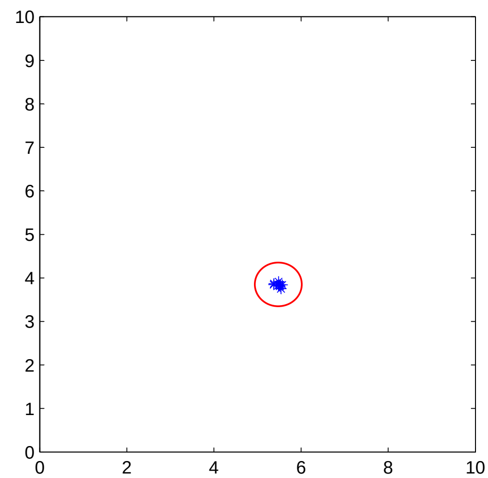

Fig. 9. Network configuration of 10 agents with sensor accuracy of radius $r = 0.5$ at 150th time step.

accuracy of a ball of radius r. Using set-valued maps and a set-valued invariance principle we showed that the agents converge to a time-varying set of diameter $4r$. Since the agent dynamics are an element of a set-valued convex map, the set to which the agents converge is time-varying unless a collision avoidance strategy or a stopping criteria is enforced. Future extensions will focus on using the proposed set-valued framework to develop control design protocols for static and dynamic networks with partial directed uncertain interagent communication.

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