Style-based Abstractions for Human Motion Classification

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ABSTRACT

This paper presents an approach to motion analysis for robotics in which a quantitative definition of "style of motion" is used to classify movements. In particular, we present a method for generating a "best match" signal for empirical data via a two stage optimal control formulation. The first stage consists of the generation of trajectories that mimic empirical data. In the second stage, an inverse problem is solved in order to obtain the "stylistic parameters" that best recreate the empirical data. This method is amenable to human motion analysis in that it not only produces a matching trajectory but, in doing so, classifies its *quality*. This classification allows for the production of additional trajectories, between any two endpoints, in the same style as the empirical reference data. The method not only enables robotic mimicry of human style but can also provide insights into genres of stylized movement, equipping cyberphysical systems with a deeper interpretation of human movement.

1. INTRODUCTION

In order to manage the complexity associated with cyberphysical systems, abstractions are typically needed to express and analyze characteristics of these systems. Such an abstraction-based approach is often taken when describing human motion, e.g., by segmenting the motion into primitive movements or so-called movemes. Indeed, there are several definitions of what the appropriate abstraction for robotic motion generation might be. We aim to build cyberphysical systems where these abstractions are defined *kinesthetically* – through movement theory and an individual's actual movement as a trackable reference. In order to do this, tools that account for human motion style in a principled way are needed, and we present one here.

In previous publications [1, 2, 3, 4], we have made a case for a precise definition of stylistic movement that is rooted in dance theory. The method outlined here applies this definition to real data and extracts style parameters that can then be used to automatically generate robotic movements that are stylistically similar. Similarly, others have proposed data-driven approaches to human-inspired robotic control [5, 6, 7], and some use an inverse optimal control framework [8, 9, 10, 11].

We are also influenced by the body of work that has attempted to segment dynamic motion primitives (*movemes* [12]) which may be combined to create full-fledged movement sequences [13, 14, 15, 16] and used to generate commands for robots [17, 18, 19]. Previous work in style analysis [15, 20, 17], typically learn statistical models from real data. These methods have a boon in their lack of assumptions on the data they analyze, but in this paper, we pose an alternate method that uses a customizable model for how the data may vary with the hopes of producing a more corporally meaningful analysis that allows for the generation of novel trajectories in the same style.

On the part of finding a set of abstractions that resonate with the physical experience of human movement, we turn to dance scholar Rudolf Laban whose set of codified *motion factors* describe *quality*, a quantity observed by mover and viewer that describes the nature of any given movement. His eight basic *efforts* illustrate the variety of possibly qualities. In [4] simple (e.g., linear) trajectories between two poses were endowed with various qualities via a linear-quadratic (LQ) optimal control framework. The weights in the quadratic cost function were linked with the effort system laid out by Laban in order to produce trajectories of different qualities. Thus, we think of these weights as stylistic parameters.

This paper extends that work and introduces a method for approximating an empirical signal using optimal control. As a seeming by-product of the signal recreation process, we extract optimal parameters from the template cost function that can be used to make comparisons between signals and recreate new trajectories that are somehow of the same ilk, or style. Our goal is to create a system output – via the same four effort-producing weights as in [4] – that matches an empirical reference trajectory with the idea that this reference will be a recording of human movement, e.g., motion capture data.

The outline of this paper follows. The specifics of our mapping from Laban's movement qualities to an optimal control problem are outlined in Sec. 2 to provide some intuition for this framework. Section 3 sets up our two stage approach where the output of an LQ system is compared to empirical data and optimal weighting parameters are found via an inverse optimal control problem. We then pose that these weighting parameters define the quality – in the sense of Laban – of the empirical data. We outline this problem's application to human motion analysis in Sec. 4 where our example demonstrates the double, mirrored Hamiltonian dynamics that emerge from this paired optimal control set up. We discuss how this technique will extend the capabilities of cyberphysical systems in Sec. 5.

2. TEMPLATE FOR MOVEMENT QUALITY

This section reviews the method presented in [4]. A key detail of this method is a mapping between kinesthetic notions of style and optimal control. Namely, we made the following association:

$$Q \sim \textit{direct} \tag{1}$$
$$R \sim \textit{light} \tag{2}$$
$$P \sim \textit{sustained} \tag{3}$$
$$S \sim \textit{bound}. \tag{4}$$

where Q, R, P, and S are weight matrices used in an LQ optimal control problem that are associated with words describing specific aspects of dance scholar Rudolf Laban's concept of movement *quality*.

In [4], we used this mapping in a method for generating a family of trajectories between two known poses. The distinguishing feature of each trajectory was its use of Laban's *motion factors* as generated according to an LQ optimal control problem. Here, on the other hand, we will think of this generative method as simulating a trajectory with which to compare raw data and extract the four weighting parameters – thus classifying raw data according to a kinesthetic notion of movement quality. We now provide a review of this theory and our corresponding control-theoretic interpretation.

The four motion factors, space, weight, time, and flow describe how movements which have the same end points may vary in quality as follows: a movement's use of space may be direct or flexible; its use of weight may be light or strong; its use of time may be sustained or sudden; and its use of flow may be bound or free. Here we will generalize these binary quantities to a continuous scale and generate trajectories corresponding to combinations of these various qualities.

Consider a system with an input $u = [u_1, u_2, ..., u_m]'$, a state $x = [x_1, x_2, ...x_n]'$, and an output $y = [y_1, y_2, ...y_l]'$ which tracks a reference signal $\sigma = [\sigma_1, \sigma_2, ...\sigma_l]'$. This reference is thought of as the nominal movement between the start and end poses; in other words, it is the most basic way to get from point A to point B upon which variations may be added.

In [4] we established the quadratic cost function

$$J_u = \frac{1}{2}\int_0^T [(y-\sigma)'Q(y-\sigma) + u'Ru + \dot{x}'P\dot{x}]\,dt + \frac{1}{2}(y-\sigma)'S(y-\sigma)\Big|_T \tag{5}$$

in order to find an input u principled on the matrices $Q \in \mathbb{R}^{l\times l}$, $R \in \mathbb{R}^{m\times m}$, $P \in \mathbb{R}^{n\times n}$, and $S \in \mathbb{R}^{l\times l}$. By construction, each of these matrices are positive definite and symmetric.

Using these weights as the parameters for varying the *style* of the resulting trajectory, we solve the optimal control problem

$$\min_u J_u \tag{6}$$
$$s.t. \begin{cases} \dot{x} = Ax + Bu & x(0) = x_0 \\ y = Cx \end{cases}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{l\times n}$. The optimal x (and thus y) are elected by the weights in Eq. 5.

Our interpretation of Laban's experiential analysis as parameters in a technical problem enables us to vary trajectories according to the different ways in which humans move. As each of the entries in the weight matrices in Eq. 5 increases relative to each other motions take on a quality that is more direct, light, sustained, and bound. Conversely, for relatively small weights, a motion may be flexible, strong, sudden, and free. Thus, Q deals with a movement's use of *space*, R with its use of *weight*, P with its use of *time*, and S with the *flow* between movements.

The dynamosphere, Fig. 1, shows the relationship between space, weight, and time. The fourth factor, flow, deals with the way movements are connected together: bound series of movements connect with rigid precision. Around the dynamosphere are the so-called eight basic *efforts*. Moving among these efforts corresponds to changing the use of motion factors and, thus, changing

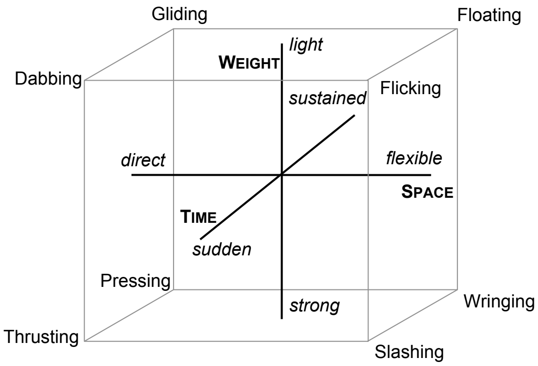

Figure 1: The dynamosphere. Laban's arrangement of eight basic efforts according to the axes of space, weight, and time. In bold font are the three Laban *motion factors* which deal with single movements; in italics are the two qualities Laban associates with each factor; and in plain font are the eight basic efforts which result from the pairwise combination of each quality.

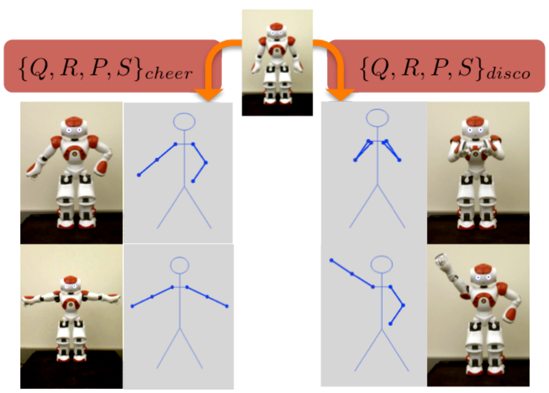

Figure 2: This figure is meant to illustrate how the parameters extracted from real human data in this paper could be implemented on a robot.

the quality of movement. These eight efforts have the names of common movements like dab, flick, and wring. However, their definition is precise in Laban's framework: they epitomize the use of each motion factor's two extremes. [21, 22]

Dab has a direct use of space, a light sense of weight, and sudden sense of time. What differentiates 'dab' from 'flick'? According to their arrangement in the dynamosphere, just one motion factor: space. A movement classified as a 'flick' has a flexible (or indirect) use of space. On the other hand, 'wring' differs from 'dab' in all three factors: it has a flexible use of space, a strong sense of weight, and a sustained use of time. Try it for yourself. These factors make intuitive sense to the body and can be used to implement motion quality in cyberphysical systems as in Fig. 2.

3. THE INVERSE PROBLEM

Here, we set up the optimal control framework that will enable the application to human movement analysis we aim to realize. In Sec. 4 we will demonstrate this for the specific style template discussed in the previous section. In Sec. 4 the two reference trajectories we will employ, $\rho \in \mathbb{R}^l$ and $\sigma \in \mathbb{R}^l$, correspond to real, empirical data of human movement and the nominal movement between the same endpoints, respectively.

In general, consider the optimal control problem

$$\min_u J_u = \int_0^T F(x, \sigma, \pi)dt + \psi(x(T), \sigma(T), \pi) \tag{7}$$

$$s.t. \begin{cases} \dot{x} = f(x, u) & x(0) = x_0 \\ y = h(x) \end{cases} \tag{8}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^l$ is the output, σ is a reference, and π is a vector of weighting parameters.

The maximum principle states that the optimizer, u^*, can be expressed as a function of x, ξ, σ, and π, where ξ is the costate satisfying

$$\dot{\xi} = -\frac{\partial F}{\partial x}^T - \frac{\partial f}{\partial x}^T \xi$$
$$\xi(T) = \frac{\partial \psi}{\partial x}(x(T), \sigma(T), \pi). \tag{9}$$

Plugging in the optimal u^* into the equations for x and ξ gives the expression for the Hamiltonian dynamics:

$$\dot{x} = f_x(x, u^*(x, \xi, \sigma, \pi)) \tag{10}$$
$$\dot{\xi} = f_\xi(\xi, u^*(x, \xi, \sigma, \pi)), \tag{11}$$

which we denote with

$$\dot{x} = f_x(x, \xi, \sigma, \pi) \tag{12}$$
$$\dot{\xi} = f_\xi(x, \xi, \sigma, \pi). \tag{13}$$

We now define a second cost function which we wish to minimize with respect to the weighting parameters π under the constraints imposed by the problem just outlined. That is,

$$\min_\pi J_\pi = \int_0^T L(x, \rho)dt + \Psi(x(T), \rho(T)) \tag{14}$$

$$s.t. \begin{cases} \dot{x} = f_x(x, \xi, \sigma, \pi) & x(0) = x_0 \\ \dot{\xi} = f_\xi(x, \xi, \sigma, \pi) & \xi(T) = \frac{\partial \psi}{\partial x}(x(T), \sigma(T), \pi) \end{cases} \tag{15}$$

where ρ is the empirical data we wish to mimic and classify.

Theorem 3.1. *The first order optimality conditions on π with respect to the cost given in Eq. 14, under the constraints in Eq. 15, are*

$$\mu(0) = 0 \tag{16}$$

where

$$\dot{\mu} = -\lambda_x \frac{\partial f_x}{\partial \pi} - \lambda_\xi \frac{\partial f_\xi}{\partial \pi}$$
$$\mu(T) = 0 \quad (17)$$

and with

$$\dot{x} = f_x(x, \xi, \sigma, \pi) \qquad x(0) = x_0 \quad (18)$$
$$\dot{\xi} = f_\xi(x, \xi, \sigma, \pi) \qquad \xi(T) = \frac{\partial \psi}{\partial x}(x(T), \sigma(T), \pi) \quad (19)$$
$$\dot{\lambda}_x = -\frac{\partial L}{\partial x} - \lambda_x \frac{\partial f_x}{\partial x} - \lambda_\xi \frac{\partial f_\xi}{\partial x}$$
$$\lambda_x(T) = \frac{\partial \Psi}{\partial x}(x(T)) - \lambda_\xi(T) \frac{\partial^2 \psi}{\partial x^2}(x(T)) \quad (20)$$
$$\dot{\lambda}_\xi = -\lambda_x \frac{\partial f_x}{\partial \xi} - \lambda_\xi \frac{\partial f_\xi}{\partial \xi} \qquad \lambda_\xi(0) = 0. \quad (21)$$

Proof. By including the constraints in the cost we get the augmented cost, which is given by

$$\hat{J}_\pi = \int_0^T \left[L(x, \rho) + \lambda_x (f_x - \dot{x}) + \lambda_\xi (f_\xi - \dot{\xi}) \right] dt + \Psi(x(T), \rho(T)).$$

Now consider a variation in π such that

$$\pi \mapsto \pi + \epsilon\theta.$$

Such a variation also causes a variation in our states

$$\Rightarrow \begin{cases} x \mapsto x + \epsilon\eta \\ \xi \mapsto \xi + \epsilon\nu \end{cases}$$

and also disturbs the costate's boundary condition, $\xi(T)$.

$$\xi(T) \mapsto \xi(T) + \epsilon\nu(T)$$

Now, computing the directional derivative of the augmented cost:

$$\delta\hat{J}_\pi(\pi; \theta) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[\hat{J}_\pi(\pi + \epsilon\theta) - \hat{J}_\pi(\pi) \right]$$
$$= \int_0^T \left[\frac{\partial L}{\partial x}\eta + \lambda_x \left(\frac{\partial f_x}{\partial x}\eta + \frac{\partial f_x}{\partial \xi}\nu + \frac{\partial f_x}{\partial \pi}\theta \right) - \lambda_x \dot{\eta} \right.$$
$$\left. + \lambda_\xi \left(\frac{\partial f_\xi}{\partial x}\eta + \frac{\partial f_\xi}{\partial \xi}\nu + \frac{\partial f_\xi}{\partial \pi}\theta \right) - \lambda_\xi \dot{\nu} \right] dt + \frac{\partial \Psi}{\partial x}(x(T))\eta(T).$$

Integrating by parts and collecting terms by variation we get:

$$= \int_0^T \left(\frac{\partial L}{\partial x} + \lambda_x \frac{\partial f_x}{\partial x} + \lambda_\xi \frac{\partial f_\xi}{\partial x} + \dot{\lambda}_x \right) \eta \, dt + \lambda_x(0)\eta(0) - \lambda_x(T)\eta(T)$$
$$+ \int_0^T \left(\lambda_x \frac{\partial f_x}{\partial \xi} + \lambda_\xi \frac{\partial f_\xi}{\partial \xi} + \dot{\lambda}_\xi \right) \nu \, dt + \lambda_\xi(0)\nu(0) - \lambda_\xi(T)\nu(T)$$
$$+ \int_0^T \left(\lambda_x \frac{\partial f_x}{\partial \pi} + \lambda_\xi \frac{\partial f_\xi}{\partial \pi} \right) \theta \, dt + \frac{\partial \Psi}{\partial x}(x(T))\eta(T). \quad (22)$$

In order to unravel boundary conditions remember that $\eta(0) = 0$ since that variation starts at x_0 which is fixed. Next, consider $\nu(T)$. The boundary condition dictates the terminal value of the costate; this gives us $\xi(T)$, a function of $x(T)$. We can apply that to the variation in $\xi(T)$ as well to solve for $\nu(T)$. We have

$$\tilde{\xi}(T) = \xi(T) + \epsilon\nu(T)$$
$$\Rightarrow \nu(T) = \frac{1}{\epsilon} \left[\tilde{\xi}(T) - \xi(T) \right]$$
$$= \frac{1}{\epsilon} \left[\frac{\partial \psi}{\partial x}(x(T) + \epsilon\eta(T)) - \frac{\partial \psi}{\partial x}(x(T)) \right]$$
$$= \frac{1}{\epsilon} \left[\frac{\partial \psi}{\partial x}(x(T)) + \epsilon \frac{\partial^2 \psi}{\partial x^2}(x(T))\eta(T) - \frac{\partial \psi}{\partial x}(x(T)) \right]$$
$$= \frac{\partial^2 \psi}{\partial x^2}(x(T))\eta(T). \quad (23)$$

If we also set the dynamics of our new costates as

$$\dot{\lambda}_x = -\frac{\partial L}{\partial x} - \lambda_x \frac{\partial f_x}{\partial x} - \lambda_\xi \frac{\partial f_\xi}{\partial x} \quad (24)$$
$$\dot{\lambda}_\xi = -\lambda_x \frac{\partial f_x}{\partial \xi} - \lambda_\xi \frac{\partial f_\xi}{\partial \xi}, \quad (25)$$

then our derivative has reduced to

$$\delta\hat{J}_\pi(\pi; \theta) =$$
$$= -\lambda_x(T)\eta(T) + \lambda_\xi(0)\nu(0) - \lambda_\xi(T) \frac{\partial^2 \psi}{\partial x^2}(x(T))\eta(T)$$
$$+ \int_0^T \left(\lambda_x \frac{\partial f_x}{\partial \pi} + \lambda_\xi \frac{\partial f_\xi}{\partial \pi} \right) \theta \, dt + \frac{\partial \Psi}{\partial x}(x(T))\eta(T).$$

Setting

$$\lambda_x(T) = \frac{\partial \Psi}{\partial x}(x(T)) - \lambda_\xi(T) \frac{\partial^2 \psi}{\partial x^2}(x(T)) \quad (26)$$
$$\lambda_\xi(0) = 0, \quad (27)$$

we are left with

$$\delta\hat{J}_\pi(\pi; \theta) = \int_0^T \left(\lambda_x \frac{\partial f_x}{\partial \pi} + \lambda_\xi \frac{\partial f_\xi}{\partial \pi} \right) \theta \, dt$$

which should equal zero for all values of θ to achieve optimality. To keep track of this in the compact way presented in the theorem, let

$$\mu = \int_t^T \left(\lambda_x \frac{\partial f_x}{\partial \pi} + \lambda_\xi \frac{\partial f_\xi}{\partial \pi} \right) dt. \quad (28)$$

Differentiating μ with respect to time gives the expression in Eqs. 17 and 16, which concludes the proof. ■

4. APPLICATION TO HUMAN MOVEMENT

Now consider the case where the template cost function, J_u is given by Eq. 5. We think of this as an appropriate abstraction that captures the natural variation found in the way humans execute a given movement because it derives from an established kinesthetic theory of movement style. We then use a second cost function J_π to figure out an optimal setting of these stylistic "knobs" for some excerpt of real movement. Thus, we will extract values for the weighting parameters that will do a good job of recreating the style of the motion excerpt between any start and end pose.

We constrain the weight matrices in J_u to be of the form

$$\begin{aligned} Q &= qI_l \\ R &= rI_m \\ P &= pI_n \\ S &= sI_l, \end{aligned}$$

where I_n is an $n \times n$ identity matrix. Thus, we can write our optimization parameter as $\pi = [q, r, p, s]^T$.

We choose $L(x, \rho)$ and $\Psi(x(T), \rho(T))$ in Eq. 14 as

$$L(x, \rho) = \frac{1}{2}||y - \rho||^2 \tag{29}$$

$$\Psi(x(T), \rho(T)) = \frac{1}{2}||y - \rho||^2 \bigg|_T . \tag{30}$$

Our state, input, output, and references are, respectively: $x = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, ..., \theta_l, \dot{\theta}_l]'$[1], $u = [u_{\theta_1}, u_{\theta_2}, ..., u_{\theta_l}]'$, $y = [\theta_1, \theta_2, ..., \theta_l]'$, $\sigma = [\sigma_1, \sigma_2, ..., \sigma_l]'$, and $\rho = [\rho_1, \rho_2, ..., \rho_l]'$ where σ is chosen to be a linear interpolation between the endpoints of ρ (this is the nominal move) and ρ is real data we wish to describe with the output of our original system, y. The system dynamics matrices A, B, and C are the canonical matrices for a completely controllable double integrator drift-less system. The ρ we employ is expected to be motion from a human dancer recorded on a motion capture system. This corresponds to the angles over time of joint segments defined by an .asf/.amc file pair: x, y, and z rotations for each bone represented in the skeleton extracted from the raw data.

Note that now our notion of system dynamics is a little unusual because the original state x ends up with three co-states: ξ the co-state which enforces the output to follow the trajectory specified by Q, R, P, and S, and then another two co-states, one each for x and ξ, that enforce π to be the optimal set of weights to match the empirical signal ρ.

Per the optimality conditions in Thm. 3.1, the following derivatives are necessary:

$$\frac{\partial \psi}{\partial x}(x(T)) = C'SCx(T) - C'S\sigma(T) \tag{31}$$

$$\frac{\partial^2 \psi}{\partial x^2}(x(T)) = C'SC \tag{32}$$

$$\tag{33}$$

$$\frac{\partial L}{\partial x} = x'C'C - \rho'C \tag{34}$$

$$\frac{\partial \Psi}{\partial x}(x(T)) = x(T)'C'C - \rho(T)'C \tag{35}$$

$$\frac{\partial f_x}{\partial x} = A - B(R + B'PB)^{-1}B'PA \tag{36}$$

$$\frac{\partial f_x}{\partial \xi} = -B(R + B'PB)^{-1}B' \tag{37}$$

$$\begin{aligned} \frac{\partial f_\xi}{\partial x} = {} & A'PB(R + B'PB)^{-1}B'PA \\ & -C'QC - A'PA \end{aligned} \tag{38}$$

$$\frac{\partial f_\xi}{\partial \xi} = A'PB(R + B'PB)^{-1}B' - A' \tag{39}$$

$$\frac{\partial f_x}{\partial \pi} = \left[\begin{array}{cccc} \frac{\partial f_x}{\partial \pi_1} & \frac{\partial f_x}{\partial \pi_2} & \frac{\partial f_x}{\partial \pi_3} & \frac{\partial f_x}{\partial \pi_4} \end{array} \right]$$

$$\frac{\partial f_x}{\partial \pi_1} = 0_{nx1} \tag{40}$$

$$\frac{\partial f_x}{\partial \pi_2} = BH^{-2}B'(\pi_3 Ax + \xi) \tag{41}$$

$$\frac{\partial f_x}{\partial \pi_3} = -BH^{-1}B'Ax + BB'BH^{-1}B'(\pi_3 Ax + \xi) \tag{42}$$

$$\frac{\partial f_x}{\partial \pi_4} = 0_{nx1} \tag{43}$$

$$\frac{\partial f_\xi}{\partial \pi} = \left[\begin{array}{cccc} \frac{\partial f_\xi}{\partial \pi_1} & \frac{\partial f_\xi}{\partial \pi_2} & \frac{\partial f_\xi}{\partial \pi_3} & \frac{\partial f_\xi}{\partial \pi_4} \end{array} \right]$$

$$\frac{\partial f_\xi}{\partial \pi_1} = C'(\sigma - Cx) \tag{44}$$

$$\frac{\partial f_\xi}{\partial \pi_2} = -\pi_3 A'BH^{-2}B'(\pi_3 Ax + \xi) \tag{45}$$

$$\begin{aligned} \frac{\partial f_\xi}{\partial \pi_3} = {} & (2\pi_2 A'BH^{-1}B'A - A'A)x \\ & +A'BH^{-1}B'(\pi_3 Ax \\ & +\xi)\pi_3 A'BB'BH^{-2}B'(\pi_3 Ax + \xi) \end{aligned} \tag{46}$$

$$\frac{\partial f_\xi}{\partial \pi_4} = 0_{nx1} \tag{47}$$

where $H = R + B'PB$.

Now, to assemble the Hamiltonian dynamics (x with all three of the costates), let $w_z = \lambda_x'$ and $w_\xi = \lambda_\xi'$. We

[1]In practice, it may be most effective to perform this analysis on each joint angle individually as different joints may move with different qualities, a notion Laban described as simultaneity versus succession.

then have:

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \\ \dot{w_x} \\ \dot{w_\xi} \end{bmatrix} = \begin{bmatrix} M & & 0 & 0 \\ & & 0 & 0 \\ -C'C & 0 & -M' & \\ 0 & 0 & & \end{bmatrix} \begin{bmatrix} x \\ \xi \\ w_x \\ w_\xi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C'Q & 0 \\ 0 & C' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_\theta \\ \rho \end{bmatrix} \tag{48}$$

where the entries for M describe the dynamics of $[x, \xi]'$ under Eqs. 5 and 6 and are given by

$$M_{11} = A - B(R + B'PB)^{-1}B'PA \tag{49}$$

$$M_{12} = -B(R + B'PB)^{-1}B' \tag{50}$$

$$M_{21} = A'PB(R + B'PB)^{-1}B'PA - C'QC - A'PA \tag{51}$$

$$M_{22} = A'PB(R + B'PB)^{-1}B' - A'. \tag{52}$$

Then, setting $z = [x, \xi, w_x, w_\xi]'$ and $\zeta = [\sigma, \rho]'$ gives dynamics of the following form:

$$\dot{z} = \mathcal{M}z + \mathcal{N}\zeta. \tag{53}$$

Thus,

$$z(T) = e^{\mathcal{M}T}z_0 + \int_0^T e^{\mathcal{M}(T-t)}\mathcal{N}\zeta(t)dt = \Phi z_0 + \eta \tag{54}$$

and let

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} \text{ and } \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}.$$

Rearranging and plugging in the following boundary conditions (along with x_0)

$$\xi(T) = C'SCx(T) - C'S\sigma(T)$$

$$w_x(T) = C'SCw_\xi(T) - C'Cx(T) + C'\rho(T)$$

$$w_{\xi 0} = 0$$

we can see that

$$\begin{bmatrix} x(T) \\ \xi(0) \\ w_x(0) \\ w_\xi(T) \end{bmatrix} = \mathcal{A}^{-1}\left(\begin{bmatrix} \Phi_{11} \\ \Phi_{21} \\ \Phi_{31} \\ \Phi_{41} \end{bmatrix} x_0 + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix} + \mathcal{B} \begin{bmatrix} \sigma(T) \\ \rho(T) \end{bmatrix} \right)$$

where

$$\mathcal{A} = \begin{bmatrix} I_n & -\Phi_{12} & -\Phi_{13} & 0 \\ C'SC & -\Phi_{22} & -\Phi_{23} & 0 \\ C'C & -\Phi_{32} & -\Phi_{33} & -C'SC \\ 0 & -\Phi_{42} & -\Phi_{43} & I_n \end{bmatrix} \tag{55}$$

$$\mathcal{B} = \begin{bmatrix} 0 & 0 \\ C'S & 0 \\ 0 & C' \\ 0 & 0 \end{bmatrix} \tag{56}$$

and where the invertibility of $\mathcal{A}$ is guaranteed by the complete controllability of the system.

Thus, we have $\dot{z}$ and z_0 where y recreates ρ optimally according to the structure of J_π as best as the structure of J_u will allow.

We now implement this setup in nine dimensions on the motion of the right leg of a human mover. The motion of the leg is captured via a Vicon Motion Capture system, smoothed with 3 passes of a sliding average of width 2, and is represented as a vector $\rho \in \mathbb{R}^9$. This results in a 18×18 system. The state $x \in \mathbb{R}^9$ is initialized to match ρ at $t = 0$ (i.e., $x_0 \approx [\rho(0); \rho'(0)]$) and normalized to start at zero and last for one unit of time (i.e., $\rho(0) = 0$ and $T = 1$). Performing gradient descent with Armijo step size on π (using $\alpha = \beta = 0.5$), we get an output y that mimics ρ as shown in Fig. 3.

Zooming in on a single joint angle, namely, motion of the left femur in the z-direction, the following values where computed for π: $q = 1.53$, $r = 0.018$, $p = 0.0001$, and $s \triangleq 1$). The convergence of the machinery derived here is shown in Fig. 4. These values for the optimization parameter π prescribe a relative location for the trajectory ρ in Laban's dynamosphere (Fig. 1). For example, the leg motion in the this motion is then considered *direct*, *light*, and *sudden*. The ability of the method to recreate the empircal is shown, joint-by-joint, in Fig. 3, where the top right corner corresponds to the femur in the z-direction.

The simulated motion was animated alongside the computed reference trajectory and the original real data in order to demonstrate how well the approximations of linear dynamics and quadratic costs are able to capture the essence of the original movement. The animation shows that the "Simulated Data" endows the simple linear interpolation of the reference with a more life-like quality. A snapshot is shown in Fig. 5[2].

[2]The video may be viewed at `http://people.virginia.edu/~ael8a/`.

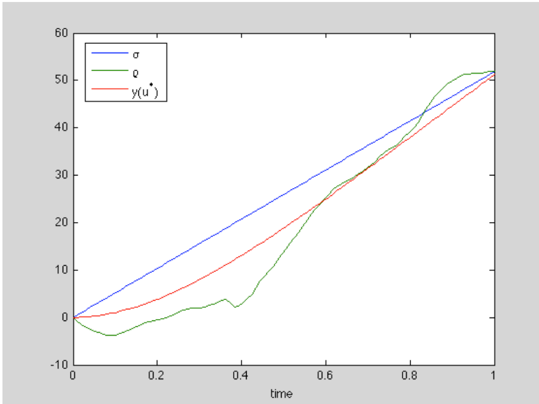

Figure 3: This figure shows the result of the simulation: an output y (in red) which tracks data of a subject's femur (in solid blue) as closely as possible using the mapping between Laban's motion factors and a cost function.

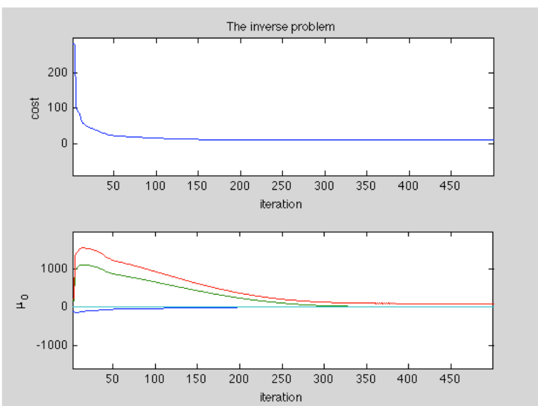

Figure 4: The bottom figure shows that the solution satisfies the optimality condition in Eq. 16, and the upper figure shows the corresponding converging cost function for one of the nine dimenions of motion simulated here.

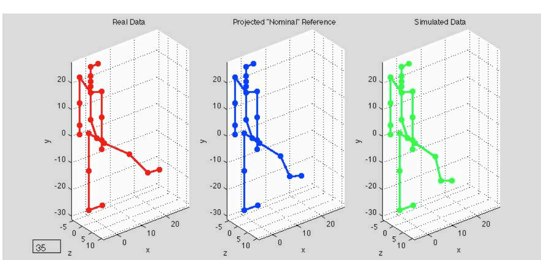

Figure 5: This figure shows, left to right, a snapshot of the animation of: the original movement, the nominal reference trajectory, and our recreation via the method presented here.

5. TOWARD A METHOD FOR STYLE-BASED SEGMENTATION

In this paper we asked the question: "Given a movement, can we extract its quality?" In answer, we posited a nested pair of cost functions: one that sets up a definition of quality as it applies to movement and the other that describes a metric for success. These cost functions set up useful abstractions for the goal of extracting a very specific kinesthetic concept of motion quality. Solving for optimality conditions and gathering three costates for our original state, we derived an algebraic solution to the posed question and implemented it on real human motion capture data.

The next obvious question is: "Given a series of movements, of a known quality, can we extract the distinct movements contained therein?" This amounts to an optimal timing control problem that endows this work with an extra knob for matching – along an entire unsegmented, trajectory. And provides a natural way to define what is meant by the motion segmentation problem, which is inherently ill-posed. Instead of constraining the problem via statistical metrics or arbitrary mathematical constructs, our idea is to have a human mover define a motion, seeding the problem with one known "movement" or even "moveme" that our machinery can then interpret stylistically in order to segment a longer series of movements from this mover. Such a segmentation would provide a formulation for motion primitives that are kinesthetically defined, thus endowing cyberphysical systems with a vocabulary of movement that resonates with the human experience of movement.

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