INFLATION, INFLATION UNCERTAINTY, AND THE VARIANCE OF MONEY GROWTH: ARE THEY RELATED?

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INFLATION, INFLATION UNCERTAINTY, AND THE VARIANCE OF MONEY GROWTH: ARE THEY RELATED?

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SUMMARY

This thesis complements and extends existing research on inflation uncertainty and its relationship to the level of inflation and money growth uncertainty. Current research suggest that inflation uncertainty is derived from the uncertain relationship between monetary policy and inflation. In effect, uncertainty arises from the unpredictable behavior of the monetary authorities in the accommodation or extrication of inflation within or from the economic system as well as the imprecise impact of money growth on prices. Through the use of a bivariate GARCH approach, a direct measure of money-growth uncertainty is developed.

This measure of money-growth uncertainty is employed as an independent variable in the conditional variance of inflation equation. Money-growth uncertainty is found to have a statistically significant impact on inflation uncertainty across time using four different measures of inflation uncertainty. However, little empirical evidence is found to support the notion of a statistically significant relationship between the level of inflation and money-growth uncertainty.
I. INTRODUCTION

Since Okun's (1971) paper on the relationship between inflation and its variability, several theories have been developed to explain the positive relationship that has been hypothesized to exist between the two. The large majority of the theoretical work, such as, Ball (1992), Devereux (1989), Taylor (1981), and Cukierman and Meltzer (1986), acknowledge the important connection between inflation and/or inflation uncertainty and monetary policy. These papers suggest that inflation uncertainty is derived from the uncertain relationship between monetary policy and inflation. In effect, uncertainty arises from the unpredictable behavior of the monetary authorities in the accommodation or extrication of inflation within or from the economic system as well as the imprecise impact of money growth on prices. Though significant theoretical work exists, with the exception of Holland (1993), there is little empirical work that examines directly the relationship between inflation and/or inflation uncertainty and monetary growth uncertainty. This paper employs a bivariate GARCH approach in an attempt to uncover these important relationships.

The paper proceeds by first reviewing the theoretical work underlying the inflation- and/or inflation uncertainty-money-growth
uncertainty relationship. The empirical approach employed by Holland (1973) is examined in section two. Section three provides a short introduction to the bivariate GARCH formulation. Section four contains models that are used to develop measures of money growth and inflation uncertainty. These models are first estimated parsimoniously using OLS to get initial parameters. The models are then re-estimated in section five employing the bivariate GARCH and the initial values from the OLS estimation. This approach allows equations to be estimated simultaneously and actual measures of money-growth and inflation uncertainty to be developed. Section five also discusses empirical results which reveals a statistically significant relationship between money-growth uncertainty and inflation uncertainty. However, little empirical evidence is found to support the notion of a statistically significant relationship between the level of inflation and money-growth uncertainty.
II. BACKGROUND

Most of the theoretical work in this area focuses on the inflation-inflation uncertainty relationship. However, as mentioned earlier, implicit in most of the literature is the notion that inflation uncertainty is derived from monetary-growth uncertainty. Taylor (1981) develops a contracting model that connects inflation to inflation uncertainty. His model reveals that the degree of accommodation of price shocks by monetary authorities is associated with increased uncertainty about future prices. Cukierman and Meltzer (1986) develop a model which attributes cross-country inflation-inflation uncertainty relationship to the degree of political instability. They argue that policymakers choose the precision of the policy control procedure. Increased political instability leads policymakers to choose less precise control procedures. The imprecision in the control procedure leads to both higher inflation and more inflation uncertainty. Devereux's (1989) model rests on very specific assumptions about the behavior of the monetary authority. Policymakers are assumed to desire higher levels of output than the private sector, and are thereby biased toward higher inflation. Money supply is assumed to be set before real shocks are known. Devereux also assumes that the degree of wage indexation varies inversely with the size of real shocks to the economy.
According to Devereux (1989) when uncertainty about real shocks is high, inflation will be high; uncertainty about real shocks reduces the degree of wage indexation; lower indexation increases the impact of monetary policy and allows monetary authorities to stimulate the economy more effectively. Ball (1992) explicitly connects inflation uncertainty to the dilemma facing monetary authorities. He argues that inflation uncertainty is low when inflation is low and that the Federal Reserve will act to maintain low inflation. However, when inflation is high, the action of the Federal Reserve is unclear: lowering inflation and possibly risking a recession or maintaining current economic conditions. Since, in this case, the course of monetary action is unclear, high inflation implies high inflation uncertainty.

Holland (1993, pp. 39) argues that the "inflation rate and inflation uncertainty are linked by forecasters' uncertainty about the impact of money growth on the price level." His work suggests that the forecast error variance of inflation is a function of the uncertainty about the impact of money on prices, money-growth uncertainty, and the uncertainty of non-monetary shocks. He writes the following equation:

\[
(1) \quad \text{var}(\hat{\epsilon}_t - \epsilon_t) = \text{var}(\epsilon_t) + \text{var}(\epsilon_t) + \text{var}(\epsilon_t)
\]
where \( \bullet t \) is inflation in period \( t \) and \( \bullet' t \) is expected inflation in period \( t \).

The \( A_t \) is a measure of the uncertainty about the impact of money on prices, \( \bullet \), is a measure of money-growth uncertainty and \( \bullet' \) is a measure of the uncertainty of non-monetary shocks. Because Holland does not have a proxy for money growth uncertainty or the uncertainty of non-monetary shocks he assumes that they are both known constants. In effect, Holland examines the impact of the uncertain relationship between money and the price level on the forecast error variance of inflation. The inflation uncertainty measure that he estimates is net of the uncertainty caused by money-growth uncertainty and non-monetary shocks. Holland compares his measure of inflation uncertainty with the measure of inflation uncertainty derived from the Livingston Survey data. The Livingston data allow a direct measure of inflation uncertainty to be observed through the dispersion of forecasts across survey individuals. Holland compares the two measures of inflation uncertainty and finds that they are cointegrated. Given that the two measures are cointegrated, he concludes that his measure of uncertainty is a reasonable estimate of inflation uncertainty.
III. METHODOLOGY

Holland (1994) finds that the uncertainty impact of money on prices does indeed affect the forecast error variance of inflation. However, one of the basic assumptions of the model is that money-growth uncertainty is constant. Recent innovations in econometrics, from Bollerslev (1988), now allow the explicit examination of such assumptions. In what follows, Bollerslev's bivariate GARCH approach is used to examine the relationship between inflation and money growth uncertainty and inflation uncertainty and money-growth uncertainty.

Typically, when attempting to examine issues of inflation uncertainty, researchers have used either forecast dispersion measures from survey data or have employed formal mathematical models to get a proxy for inflation uncertainty. The mathematical model approach falls into either of two categories; linear models where the variance of the prediction error is the estimate of uncertainty and/or univariate ARCH or GARCH models for which conditional heteroskedasticity becomes the estimate of uncertainty.

The development of measures of money-growth uncertainty have proceeded along similar lines, but the problem of finding survey forecasts of money growth uncertainty has been hampered by the lack of a widely
accepted survey forecasts of money growth. Generally, most measures of money growth volatility, such as the one developed by Karamouzis and Lombra (1993), use the Federal Reserve's announced money growth targets as a benchmark. Though these measures of monetary-growth volatility are not estimated for the expressed purpose of finding a proxy for money-growth uncertainty, the squared deviations of money growth from the expressed Federal Reserve target provide a proxy for money growth uncertainty.

In this research, the bivariate GARCH approach is employed to derive estimates of money-growth- and inflation-uncertainty. This approach allows the researcher to develop simultaneously proxies for both money growth and inflation uncertainty and to test the relevant relationships that have been hypothesized to exist between the two. For a N×1 vector y, the Bollerslev (1986) GARCH-M model takes the following general form:

\begin{align}
(1) \quad y_t &= b + G x_t + K h_t + e_t \\
(2) \quad \text{vech}(h_t) &= C + A_t \text{vech}(e_{t-1} e_{t-1}') + B_t \text{vech}(h_{t-1}) \\
\text{et} \mid \Psi_0 &\sim \text{N}(0, h_t) \\
\end{align}

where \text{vech}(\cdot) is the lower column stacking operator of the lower portion of a symmetric matrix, b is an N×1 vector of constants, e_t is an N×1 vector of innovations, C is a 1/2N(N+1)×1 vector, A_t and B_t are 1/2N(N+1)×
$1/2N(N+1)$ matrices, $z$, is a vector of independent variables that explain $y$, and $h$ is the conditional variance generated from past information on $e$, and $h_e$. 
IV. PRELIMINARY OLS ESTIMATION

INFLATION EQUATIONS

Before estimating the GARCH-M model we first estimate OLS models of inflation and money growth. Equations (1) and (2) below are inflation models estimated over the 1960 to 1997 time period using monthly data. The two inflation measures used in this study are calculated by taking first differences of the logarithms of over-all and core CPI price indices. Equation (1) reveals that the model of over-all CPI inflation includes the first, second, seventh and ninth lags of inflation, plus a twelfth order moving average term. The Ljung-Box-Q statistic at \( Q(12) = 5.12 \) and \( Q(24) = 26.59 \), both indicate that the residuals from the model, \( v_t \), are indeed white noise. The Lagrange Multiplier (LM) test suggested by Engle (1982) which tests the null of no ARCH effects is clearly rejected at lags 4, 8, 12, and 18 with \( \ast \) test statistic values of (6.08), (18.15) (19.44) and (21.76) respectively. Equation (2) provides the model for core CPI inflation. The core CPI inflation model includes a first, second, sixth and eighth lag of core inflation as well as a twelfth order moving average term. The Ljung-Box-Q statistic at \( Q(12) = 5.7 \) and at \( Q(24) = 24.88 \) indicate that the residuals are white noise. The LM tests have \( \ast^2 \) test statistic values of (7.6), (12.61), (14.42), and (29.56) at lags 4, 8, 12, and 18. These
results clearly reject the null of no serial correlation of the squared residuals.

\[ (1) \quad \epsilon_i = 0.82 + 0.27\epsilon_{i-1} + 0.22\epsilon_{i-2} + 0.14\epsilon_{i-3} + 0.18\epsilon_{i-4} + 0.21\epsilon_{i-5} + v_i \]
\[ (3.30) \quad (6.23) \quad (5.01) \quad (3.35) \quad (3.96) \quad (4.63) \]

\[ (2) \quad \epsilon_i = 0.42 + 0.36\epsilon_{i-1} + 0.25\epsilon_{i-2} + 0.14\epsilon_{i-3} + 0.14\epsilon_{i-4} - 0.16\epsilon_{i-5} + e_r \]
\[ (2.15) \quad (8.12) \quad (5.64) \quad (3.25) \quad (3.22) \quad (-4.63) \]

**MONEY GROWTH EQUATION**

The OLS model for money growth, in equation (3), is also estimated over the 1960 to 1997 time period using monthly data. There are at least four different measures of money that could be used, \( M_0 \), \( M_1 \), \( M_2 \), and \( L \). Since the mid 1970s, \( M_1 \) has been the most popular measure of money used in research focusing on monetary policy. Over the sample period 1960 to 1997, the log-difference of \( M_1 \) provides the best measure of money growth that captures the Federal Reserve's policy intent. Equation (3) below reveals that the money growth model includes a first, third, seventh and eighth lag of \( M_1 \) money growth and an eleventh- and thirteenth-order moving average terms.

\[ (3) \quad \epsilon_i = 0.85 + 0.62\epsilon_{i-1} + 0.12\epsilon_{i-2} + 0.07\epsilon_{i-3} + 0.20\epsilon_{i-4} - 0.11\epsilon_{i-5} - 0.16\epsilon_{i-6} + z_i \]
\[ (2.62) \quad (15.9) \quad (3.09) \quad (-1.70) \quad (4.36) \]
\[ (-2.40) \quad (-3.19) \]

The Ljung-Box Q statistic at \( Q(12) = 11.05 \) and at \( Q(24) = 28.09 \) both indicate that \( z_i \) is white noise. As with prices, the LM test statistic, which
follows a \( \chi^2 \) distribution, at lags 4, 8, 12, and 18 have values of \((6.5), (6.49), (7.80), \) and \((11.92)\) rejecting the null of no serial correlation of the squared residuals.
V. GARCH-M ESTIMATION AND RESULTS

The bivariate GARCH-M model is first used to examine the Cukierman and Meltzer (1986) argument that a relationship exists between the imprecision in the policy control procedure proxied by money-growth uncertainty and the level of inflation. The following bivariate GARCH-M model is estimated to examine this relationship.

\[
\begin{align*}
    \eta_t &= \eta_{t-1} + \gamma_1 \eta_{t-2} + \gamma_2 \eta_{t-3} - \gamma_3 \eta_{t-4} + \gamma_4 \eta_{t-5} - \gamma_5 \eta_{t-6} - \gamma_6 \eta_{t-7} + \gamma_7 \eta_{t-8} + \epsilon_t \\
    \epsilon_t &= \eta_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \phi_3 \epsilon_{t-3} + \phi_4 \epsilon_{t-4} + \phi_5 \epsilon_{t-5} + \phi_6 \epsilon_{t-6} + \nu_t
\end{align*}
\]

Equation (4) describes the log first-difference of \( M_t \) as a function of first, third, seventh, and eighth lags of log first difference of \( \eta_t \) and eleventh and thirteenth order moving average terms. Estimation of the bivariate GARCH-M model requires a nonlinear process. The initial values for the nonlinear estimation of money growth are taken from the OLS equation (3) above. Equation (5) is a GARCH(1,1) model of the conditional variance of money growth. \( \epsilon_t^2 \) is the variance of unpredictable money.
growth in period $t$. The GARCH formulation implies that the conditional variance of money growth is at time $t$ a function of the squared residuals from equation (4) in $t-1$ and the conditional variance from $t-1$. Included in the conditional variance equation is a dummy variable $D$, which takes on a value of one from 1979:10 to 1983:1 and a value of zero during all other periods. This variable is placed in the model primarily to capture the change in the variance of money growth that occurred during the Federal Reserve policy regime shift of the early 1980s. However, deregulation of the financial services industry, the Savings and Loan bail-out, as well as significant increases in the federal budget deficit all occurred during this same period and may have affected money growth. Equation (5) reveals that the variance, $\sigma_{\epsilon}^2$, is determined by information from past periods only and is a deterministic equation, i.e., there is no error term associated with this equation. The estimated conditional variance of money growth from equation (5) is the time series measure of money-growth uncertainty. Equation (6) describes CPI inflation as a function of the first, second, seventh and ninth lags of CPI inflation, a twelfth-order moving average term and the conditional variance of money growth. The statistical significance of $\ast$, will offer insight as to the relationship between money-growth uncertainty and the level of inflation. Equation (7) is the GARCH(1,1) model of the conditional variance of CPI inflation. $\sigma_{\epsilon}^2$, is the estimated variance of inflation in period $t$. As with the money growth
variance, the conditional variance of inflation in time t is a function of only past information and is a deterministic equation. Equation (8) is a simple constant correlation model of the covariance between z and v.\textsuperscript{1}

Panel A of Table (1) (see appendix A) reports the maximum-likelihood estimates of equations (4a)-(8a) using money growth and the overall CPI inflation measures. The sample is 409 monthly observations from 1963 through 1997.\textsuperscript{2} Equation (4a) provides the basic results for the money growth equation that are similar to the results achieved from the OLS model above. Equation (5a) reveals significant ARCH effects in the conditional variance equation, but no significant GARCH effects, i.e., $*$, is significantly different from zero whereas $^*$, is not. The significance of $D$, suggests that the Federal Reserve's regime shift did indeed impact the variance of money growth over the 1979-1983 period.\textsuperscript{3} Equation (6a) provides similar results to those obtain from the OLS estimation of the inflation equation. However, equation (6a) contains an additional

\textsuperscript{1} Bollerslev (1990) developed the constant correlation model. Several other covariance models were examined, however, the basic results presented in this paper were unaffected.

\textsuperscript{2} The maximum-likelihood equations were estimated over a somewhat shorter period to accommodate the lag structures for both the inflation and money growth equations.

\textsuperscript{3} The dummy variable $D$, was placed initially in equations (3) and (4) and was found to be insignificant. It appears that the change in Federal Reserve's control procedure had a more significant impact on the variance of money growth than on the level of money growth. After estimating the GARCH-M model I examined graphical plots of the conditional variance of money growth. The conditional variance of money growth becomes larger across the 1979 to 1985 period. The change in the Federal Reserve's targeting procedure may account for some of the increase in variance. However, the
independent variable which allows direct examination of the impact of money-growth uncertainty on the level of over-all CPI inflation. $\bullet$, the coefficient associated with the conditional variance of money growth, is not significantly different from zero in the inflation equation. This result indicates that no statistically significant relationship exists between money-growth uncertainty and the level of inflation.$^4$ Equation (7a) reveals that there are no significant GARCH effects for the conditional variance of over-all CPI inflation; however, the t-statistic associated with the lagged squared errors of inflation in equation (7a) is (4.00), indicating that there are significant ARCH effects associated with the errors from the inflation equation. Equation (8a) reveals that the correlation coefficient between $z$ and $v$ is not significant in the conditional covariance equation.

Panel B of Table (1) reports the maximum-likelihood estimates of equations (4b)-(8b) using money growth and the core CPI inflation measures. Here again, the sample is 409 monthly observations from 1963 through 1997. The conditional variance equation for money growth, (5b), reveals only ARCH effects and that the Federal Reserve's regime change variable is again significantly different from zero, as expected. The conditional variance equation (7b) for core CPI reveals that the residuals deregulation of the financial services industry as well as the SkL bailout may have had a significant impact on money growth variance.

$^4$ In an attempt to examine this result for robustness and because money growth uncertainty may affect prices with a significant lag, lags 1 through 24 were examined; however, the same result was obtained.
from the core CPI model are best described by a GARCH(1,1) process, as both the lagged squared residuals and the lagged conditional variance are significantly different from zero. When we examine the impact of money-growth uncertainty on the level of core inflation in equation (6b), we find that no statistically significant relationship exists.

The inflation forecasts from Table (1) are derived from statistical models. However, to further examine the above results, survey inflation forecasts are employed in the derivation of inflation uncertainty measures. The two major inflation survey forecasts that are employed by most research are the Livingston Survey of professional economists and money-managers and the Michigan Survey Research Center (SRC) survey of some 3000 households. The Livingston survey is bi-annual whereas the Michigan survey has been taken monthly since 1978. Both these survey forecasts are less useful for the purpose at hand than are Theis (1984,1990) survey results. Theis provides monthly survey inflation forecasts starting in 1948 and ending in 1990. Theis estimates business price expectations by using data from several periodic surveys of business executives, monthly time series of business expectations for general prices, buying prices, and selling prices. The standard deviation of price expectations are abstracted under the assumption that business expectations are described by a normal distribution where means and standard deviation evolve smoothly over time.
Theis forecasts the expected business selling price and the expected business buying price on a monthly basis over the 1948 to 1990 time period. This study uses these forecasts to calculate forecast errors for business buying prices and business selling prices. The forecast errors are obtained by taking the difference between the over-all CPI inflation and Theirs' business buying and/or business selling price forecasts. The forecast errors are modeled in the GARCH formulation as a simple random walk. The results in Table (2) (see appendix B) are obtained using the GARCH-M framework discussed above. Equations (5c) and (7c) in Panel A of Table (2) (see appendix B) reveal that there are GARCH effects associated with the business selling prices forecast errors variance but no significant ARCH or GARCH effects associated with money growth errors variance over the same time period. Since earlier money growth variance was found to have significant ARCH effects, the Wald test which follows a \( \chi^2 \) distribution, is used to examine the simultaneous the significance of all coefficients in the money growth conditional variance equation. The \( \chi^2 \) test statistic for equation (5c) has a value of \( \chi^2(4) = 408.42 \) clearly suggesting that, in aggregate, the coefficients in

\[ \text{Several other specifications were used in an attempt to model the forecast error from both the buying price and selling price models. The most basic model employed in the GARCH formulation was to make the forecast error a function of a single constant. At the other extreme, the forecast errors were modeled such that the errors from the forecast error equation within the GARCH formulation were white noise as measured by the Ljung-Box \( Q \) test. In all cases, i.e., buying price and selling price forecast errors,} \]
equation (5c) are indeed significantly different from zero. Equation (6c) reveals that money growth uncertainty appears to have no significant statistical impact on the level of the business selling price forecast errors.

Turning to Panel B and the business buying price forecast error, equations (5d) and (7d) reveal that there are ARCH effects present in the variance equation for money growth and GARCH effects associated with the business buying price forecast errors variance. An examination of the equation (6d) reveals that here again, as is the case with core CPI inflation, over-all CPI inflation, and selling price forecast errors, money growth uncertainty has no significant impact on the level of business buying price forecast error.

The results above lend little support to the existence of a statistically significant relationship between the level of inflation and/or the level of the survey forecast error and money-growth uncertainty. The GARCH methodology is, however, sufficiently flexible that it allows the examination of the relationship between inflation uncertainty, as measured by the conditional variance of over-all CPI inflation, and money-growth uncertainty as measured by the conditional variance of the log-difference of $M_t$. Panel A of Table (3) provides the GARCH model results for over-all CPI inflation uncertainty and money-growth uncertainty. The major difference in model specification of Table (3), and

there was no statistically significant relationship between the level of the forecast error
that of Tables (1) and (2) is that equation (6e) does not contain the money growth variance term, $\epsilon^2$, which was found to be non-significant in inflation equations, whereas equation (7e), the inflation conditional variance equation, does contain the money growth variance term. The statistical significance of the coefficient associated with $\epsilon^2$ provides a direct test of the impact of the conditional variance of money growth on the conditional variance of inflation. Equation (7e) reveals that the conditional variance of money growth does indeed have a significant impact on the conditional variance of over-all CPI inflation.\textsuperscript{6} Equation (7e) is similar to the equation that is derived from Holland’s (1993) model of the variance of inflation. However, in this case direct proxies for what Holland calls the uncertain impact of money on prices and money growth uncertainty are provided by the ARCH term and the conditional variance of money growth in equation (7e).

Panel B of Table (3) (see appendix C) provides the results of the GARCH model in the examination of the relationship between the conditional variance of money growth and the conditional variance of core CPI inflation. Here again, the equation of interest is the conditional variance of core CPI inflation, equation (7f), which contains the conditional variance of money growth as an explanatory variable.

and money-growth uncertainty.
Equation (7) reveals that the variance of money growth, the ARCH, nor the GARCH terms have a significant impact on the variance of core inflation individually. However, if we use the Wald test to examine the collective impact of all three variables the test statistic has value of \( \chi^2(3) = 2448.24 \), easily rejecting the null hypothesis. Since the GARCH term has a very large individual t-test value dropping it then recomputing the Wald test will give insight as to the impact of the the ARCH term and the money growth variance term on the variance of core inflation. The recomputing of the Wald test using only the ARCH and money growth variance coefficients gives a test statistic value of (13.99) which is significant at the (0.0009) level.

To examine the above result for robustness and to address the issue of the 1970s being an unusual period because of the oil shocks, the GARCH models using money-growth uncertainty and over-all-CPI inflation uncertainty as well as money-growth uncertainty and core CPI inflation uncertainty are estimated over the 1980 to 1997 time period. Table (4) Panel A reveals that not only is the money growth variance not significant in the price variance equation, there are no significant ARCH or GARCH effect present in the over-all CPI variance across the 1980 to 1997 time period. However, through closer inspection and again by

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\(^6\) Notice that the conditional variance of money growth enters the conditional variance equation for inflation with a one period lag as do the ARCH and GARCH terms. This lag structure may create a problem with multicollinearity.
applying the Wald test, the test results from equation from equation (7g) confirms that while each coefficient individually appears to have no statistically significant impact on the variance of over-all CPI inflation, taken together the Wald test statistic value is $\ast(3) = (17.48)$, clearly rejecting the null hypothesis of no-significant-impact at the (0.0005) significance level. To get a sense of the impact of the money growth variance term on the collective Wald test result, the money growth variance term is dropped and the Wald test is recomputed. The test statistic falls to (2.04) with a significance level of (0.36). This result clearly indicate the money growth variance is important in explaining the variance of over-all CPI inflation.

Turning finally to Panel B, of Table (4) (see appendix D) there is clear evidence of a significant relationship between the conditional variance of core CPI inflation and the conditional variance of money growth. Equation (7h) reveals that the conditional money growth variance term has a positive coefficient and is significant at the (0.03) level over the 1980 to 1997 time period.

The results from the final test of the relationship between inflation uncertainty and money growth uncertainty can be found in Table (5). (see appendix E) Panels (A) and (B) provide the results of the examination of the relationship between Theis' buying and selling price forecast error variances and money-growth uncertainty. Examining equations (6i) and
(6j) reveals that the forecast error models for both selling and buying prices are different from the random walk models of Table (2). Errors from equation (6i) and (6j) are used to develop a forecast error variance for the buying and selling price forecast errors. The criteria for the development of a useful forecast error variance is that the initial error must be white noise. The additional autoregressive and moving average terms that appear in both equations (6i) and (6j) insure that the error terms from each equation are indeed white noise. Moving to the money growth variance equations (5i) and (5j), t-tests of individual coefficients suggest that there are no ARCH or GARCH effects present. The Wald test statistic of collective significance of all coefficient in equation (5i) has a value of \( t^2(4) = 685.13 \). If the coefficient associated with the Federal Reserve regime change is dropped from the test, the test statistic value falls to \( t^2(3) = 455.11 \). In either case, the null hypothesis of no collective significant impact is clearly rejected. The same argument holds true for equation (5j) where the individually coefficients are not significantly different from zero at the (0.05) significance level, but the collective Wald test statistic value is \( t^2(4) = 683.49 \). After dropping the regime-change coefficient, the test statistic value falls to \( t^2(3) = 454.66 \). Again the null hypothesis of no collective significant impact is soundly rejected. Finally, an examination of the forecast error variance equations (7i) and (7j) reveals that there are significant ARCH effects present in the forecast errors for both the selling
and buying price models. The money growth variance term in each equation is marginally significant at the (0.06) and (0.05) significant levels respectively. However, if equations (7i) and (7j) of Table (5) are compared to equations (7c) and (7d) of Table (2) then it is evident that multicollinearity is a problem. When money growth variance terms are added to the variance equations their GARCH terms become insignificant.

To examine the marginal impact of the money growth variance term the Wald test is used to initially examine the collective impact of all coefficients of equation (7i). The Wald test has a test statistic value of $\chi^2(4) = (161.56)$ when coefficient are examined collectively. Dropping the coefficient associated with the money growth variance, the Wald test statistic falls to $\chi^2(3) = (26.18)$. The Wald test statistic for the collective impact of all coefficients of equation (7j) is $\chi^2(4) = (161.61)$. Again, dropping the money growth variance coefficient causes the test statistic to fall to $\chi^2(3) = (28.91)$. Hence, money-growth uncertainty appears to have a statistically significant impact on the forecast error variance for both selling and buying price models.
VI. SUMMARY AND CONCLUSIONS

The result from this research clearly rejects Cukierman and Meltzer (1986) argument that imprecision in the control procedure, i.e., money-growth uncertainty, leads to higher inflation levels. Across both over-all and core measures of CPI inflation, money-growth uncertainty does not have a significant impact on the level of inflation. Further examination of this issue through the use of inflation forecast errors calculated from Theis (1984) and (1996) reveals that money growth uncertainty does not help to explain inflation forecast errors.

This research complements and extends Holland’s (1994) analysis which finds that the uncertain impact of money growth on prices causes inflation uncertainty. Through the use of a bivariate GARCH approach, a direct measure of money-growth uncertainty is developed. This measure of money-growth uncertainty is employed as an independent variable in the conditional variance of inflation equation. Money-growth uncertainty is found to have a statistically significant impact on inflation uncertainty across time using four different measures of inflation uncertainty.
APPENDIX A

Monthly M₄ Money Growth and Over-all CPI Inflation: 1963-1997

A.1 GARCH(1,1)-M - Constant Correlation

(4a) \( \phi = 0.57 + \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} + \psi_5 \psi z_{t-1} - 0.018 z_{t-1} \)

(14.23)* (2.58)* (-0.05) (3.60)* (-2.80)* (-4.66)*

(5a) \( \phi_2 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} - 0.018 z_{t-1} \)

(3.13)* (2.23)* (0.89) (3.51)*

(6a) \( \phi_3 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} - 0.018 z_{t-1} \)

(4.12)* (4.94)* (3.47)* (2.75)* (2.94)* (3.79)* (-0.28)

(7a) \( \phi_4 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} - 0.018 z_{t-1} \)

(5.04)* (4.00)* (-0.23)

(8a) COV = \(*\) \( \) Log-likelihood function = -1234.019 (-1.08


A.2 GARCH(1,1)-M - Constant Correlation

(4b) \( \phi = 0.68 + \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} + \psi_5 \psi z_{t-1} + \psi_6 \psi z_{t-1} - 0.018 z_{t-1} \)

(13.36)* (2.22)* (-1.45) (4.18)* (-2.52)*(3.53)*

(5b) \( \phi_2 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} + \psi_5 \psi z_{t-1} + \psi_6 \psi z_{t-1} - 0.018 z_{t-1} \)

(4.43)* (2.08)* (3.58)*

(6b) \( \phi_3 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} + \psi_5 \psi z_{t-1} + \psi_6 \psi z_{t-1} - 0.018 z_{t-1} \)

(2.98)* (4.85)* (4.46)* (4.45)* (2.68)* (-5.68)* (-0.97)

(7b) \( \phi_4 = \psi z_{t-1} + \psi_1 \psi z_{t-1} + \psi_2 \psi z_{t-1} + \psi_3 \psi z_{t-1} + \psi_4 \psi z_{t-1} + \psi_5 \psi z_{t-1} + \psi_6 \psi z_{t-1} - 0.018 z_{t-1} \)

(6.94)* (3.56)* (23.01)*

(8b) COV = \(*\) \( \) Log-likelihood function = -1068.50 (-0.05

T-statistics are given in parentheses. * = significant at or beyond the 0.05 level.

B.1 GARCH(1,1)-M - Constant Correlation

\[ (4c) \quad \varepsilon_n = 1.05 + \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} - \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} - \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(2.47)^* & \quad (12.98)^* \\
(2.90)^* & \quad (2.03)^*(3.18)^* \\
(-2.21)^* & \quad (-4.14)^*
\end{align*} \]

\[ (5c) \quad \varepsilon_n = \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(1.85) & \quad (0.99) \\
(1.24) & \quad (1.93)
\end{align*} \]

\[ (6c) \quad SPE = 1.13 + \cdots + \varepsilon_n + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(4.14)^* & \quad (6.94)^* \\
(1.11)
\end{align*} \]

\[ (7c) \quad \varepsilon_n = \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(5.04)^* & \quad (5.04)^* \\
(3.77)^* & \quad (-3.23)^*
\end{align*} \]

\[ (8c) \quad COV_n = \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(1.18)
\end{align*} \]


B.2 GARCH(1,1)-M - Constant Correlation

\[ (4d) \quad \varepsilon_n = 1.22 + \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(2.91)^* & \quad (13.29)^* \\
(2.57)^* & \quad (-2.11)^* (3.63)^* (2.45)^* (3.63)^*
\end{align*} \]

\[ (5d) \quad \varepsilon_n = \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(2.52)^* & \quad (1.86) \\
(1.30) & \quad (2.85)^*
\end{align*} \]

\[ (6d) \quad BPE_n = 0.81 + \cdots + BPE_{n-1} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(2.87)^* (8.80)^* \\
(-1.30)
\end{align*} \]

\[ (7d) \quad \varepsilon_n = \cdots + \varepsilon_{n-1} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} + \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(8.91)^* & \quad (4.61)^* \\
(-4.43)^*
\end{align*} \]

\[ (8d) \quad COV_n = \cdots \cdots + \varepsilon_{n-M} + \cdots \cdots + \varepsilon_{n-M} + \cdots \cdots + \varepsilon_{n-M} \]
\[ \begin{align*}
(1.26)
\end{align*} \]

Log-likelihood function = -1036.54

T-statistics are given in parentheses. * = significant at or beyond the 0.05 level.
APPENDIX C

Monthly M1 Money Growth and Over-all CPI Inflation: 1963-1997

C.1 GARCH(1,1) - Constant Correlation

\[ (4e) \quad \rho - 0.53 + \sum_{i=1}^{\infty} \alpha_i - \sum_{i=1}^{\infty} \beta_i - \sum_{i=1}^{\infty} \gamma_i - \sum_{i=1}^{\infty} \delta_i = (2.10)^* (14.88)^* (2.74)^* (1.12) (3.56)^* (2.79)^* (4.45)^* \]

\[ (5e) \quad \phi = \sum_{i=1}^{\infty} \zeta_i^2 + \sum_{i=1}^{\infty} \xi_i + \sum_{i=1}^{\infty} \eta_i = (2.95)^* (2.11)^* (1.41) (3.12)^* \]

\[ (6e) \quad \epsilon = 1.15 + \sum_{i=1}^{\infty} \psi_i + \sum_{i=1}^{\infty} \theta_i + \sum_{i=1}^{\infty} \varphi_i + \sum_{i=1}^{\infty} \psi_i = (3.88)^* (4.86)^* (3.87)^* (2.75)^* (3.04)^* (3.75)^* \]

\[ (7e) \quad \epsilon = \sum_{i=1}^{\infty} \psi_i^2 + \sum_{i=1}^{\infty} \theta_i^2 + \sum_{i=1}^{\infty} \varphi_i^2 = (3.02)^* (3.97)^* (2.90)^* (2.83)^* (2.90)^* \]

\[ (8e) \text{COV} = \sum_{i=1}^{\infty} \zeta_i \quad \text{Log-likelihood function} = -1230.02 \]


C.2 GARCH(1,1) - Constant Correlation

\[ (4f) \quad \rho = 0.56 + \sum_{i=1}^{\infty} \alpha_i + \sum_{i=1}^{\infty} \beta_i + \sum_{i=1}^{\infty} \gamma_i + \sum_{i=1}^{\infty} \delta_i = (2.24)^* (13.94)^* (2.49)^* (0.97) (3.34)^* (-2.72)^* (4.31)^* \]

\[ (5f) \quad \phi = \sum_{i=1}^{\infty} \zeta_i^2 + \sum_{i=1}^{\infty} \xi_i^2 + \sum_{i=1}^{\infty} \eta_i = (3.12)^* (2.23)^* (1.30) \]

\[ (6f) \quad \epsilon = 0.50 + \sum_{i=1}^{\infty} \psi_i^2 + \sum_{i=1}^{\infty} \theta_i^2 + \sum_{i=1}^{\infty} \varphi_i^2 = (3.79)^* (5.17)^* (4.70)^* (4.52)^* (2.28)^* (5.72)^* \]

\[ (7f) \quad \epsilon = \sum_{i=1}^{\infty} \psi_i^2 + \sum_{i=1}^{\infty} \theta_i^2 + \sum_{i=1}^{\infty} \varphi_i^2 = (-0.98) (3.69)^* (20.53)^* (1.40) \]

\[ (8f) \text{COV} = \sum_{i=1}^{\infty} \zeta_i \quad \text{Log-likelihood function} = -1055.61(0.22) \]

T-statistics are given in parentheses. * = significant at or beyond the 0.05 level.

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APPENDIX D

Monthly M₂, Money Growth and Over-all CPI Inflation: 1980-1997

D.1 GARCH(1,1) - Constant Correlation

\[
\begin{align*}
(4g) \quad \epsilon_t &= 0.53 + \cdots \zeta_{t-3} + \cdots \epsilon_{t-1} + \cdots \epsilon_{t-1} - \cdots \epsilon_{t-5} \\
&\quad \quad \quad (2.10)^* (14.88)^* (2.74)^* (1.12)^* (3.50)^* (-2.79)^* \\
(5g) \quad \epsilon_t^2 &= \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots D_t \\
&\quad \quad \quad (2.46)^* (1.52) (-0.21) (2.68)^* \\
(6g) \quad \epsilon_t^2 &= 0.76 + \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 \\
&\quad \quad \quad (2.27)^* (5.80)^* (3.61)^* (3.41)^* (0.70)^* \\
(7g) \quad \epsilon_t^2 &= \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 \\
&\quad \quad \quad (0.63) (1.11) (0.46) (1.28)^* \\
(8g) &\quad \text{COV}_{t} = \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
&\quad \quad \quad (-0.61)^* \\
\text{Log-likelihood function} = -607.39
\end{align*}
\]

Monthly M₂, Money Growth and Core CPI inflation: 1980-1997

D.2 GARCH(1,1) - Constant Correlation

\[
\begin{align*}
(4h) \quad \epsilon_t &= 0.47 + \cdots \epsilon_{t-3} + \cdots \epsilon_{t-1} + \cdots \epsilon_{t-1} - \cdots \epsilon_{t-3} - \cdots \epsilon_{t-1} \\
&\quad \quad \quad (1.64) (10.78)^* (1.32) (2.81) (-2.35)^* (-2.73)^* \\
(5h) \quad \epsilon_t^2 &= \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots D_t \\
&\quad \quad \quad (2.46)^* (1.52) (1.90) (2.92)^* \\
(6h) \quad \epsilon_t^2 &= 0.43 + \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 - \cdots \epsilon_{t-3}^2 \\
&\quad \quad \quad (1.80) (4.28)^* (2.00)^* (2.05)^* (1.83) (-3.46)^* \\
(7h) \quad \epsilon_t^2 &= \cdots \epsilon_t^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 + \cdots \epsilon_{t-1}^2 \\
&\quad \quad \quad (-2.00)^* (1.36) (3.25)^* (2.18)^* \\
(8h) &\quad \text{COV}_{t} = \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
&\quad \quad \quad (0.07)^* \\
\text{Log-likelihood function} = -501.77
\end{align*}
\]

T-statistics are given in parentheses. * = significant at or beyond the 0.05 level.
APPENDIX E


E.1 GARCH(1,1) - Constant Correlation

(4) \( \sigma_p = 1.11 + \cdots x_i \sigma_i + \cdots x_{t-1} \sigma_{t-1} + \cdots x_{t-2} \sigma_{t-2} - \cdots \sigma_{t-H} - \cdots x_{t-H} \sigma_{t-H} \) (2.51)*

(5) \( \sigma_i = \cdots + \cdots z_i \sigma_i + \cdots x_i \sigma_i + \cdots D_i \)

(1.71) (1.17) (1.12) (1.73)

(6) \( \text{SPE}_i = 0.79 + \cdots \text{SPE}_i + \cdots \text{SPE}_{i-1} + \cdots \text{SPE}_{i-2} + \cdots \text{SPE}_{i-H} - \cdots \text{SPE}_{i-H} \) (3.41)* (3.48)* (3.46)* (2.08)* (2.13)* (-2.05)* (1.20)

(7) \( \sigma_i^2 = \cdots + \cdots \sigma_i^2 + \cdots \sigma_i^2 \) + \cdots \sigma_i^2

(2.61)* (3.36)* (-0.65)*

(1.36)

(8) \( \text{COV} = \cdots \cdots \sigma_i \sigma_i \)

Log-likelihood function = -986.63

(-1.31)


E.2. GARCH(1,1) - Constant Correlation

(4) \( \sigma_p = 1.12 + \cdots x_i \sigma_i + \cdots x_{t-1} \sigma_{t-1} + \cdots x_{t-2} \sigma_{t-2} - \cdots \sigma_{t-H} - \cdots x_{t-H} \sigma_{t-H} \) (2.52)*

(5) \( \sigma_i = \cdots + \cdots z_i \sigma_i + \cdots x_i \sigma_i + \cdots D_i \)

(1.74) (1.19) (1.13) (1.77)

(6) \( \text{BPE}_i = -0.41 + \cdots \text{BPE}_i + \cdots \text{BPE}_{i-1} + \cdots \text{BPE}_{i-2} + \cdots \text{BPE}_{i-H} - \cdots \text{BPE}_{i-H} \) (2.32)* (3.32)* (3.35)* (2.35)* (2.03)* (-2.24)* (1.22)

(7) \( \sigma_i^2 = \cdots + \cdots \sigma_i^2 + \cdots \sigma_i^2 \) + \cdots \sigma_i^2

(2.59)* (3.17)* (-0.65)*

(2.00)*

(8) \( \text{COV} = \cdots \cdots \sigma_i \sigma_i \)

Log-likelihood function = -989.04

(-1.30)

T-statistics are given in parentheses. * = significant at or beyond the 0.05 level.

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REFERENCES


