TWO-STAGE COMBINATORIAL OPTIMIZATION FRAMEWORK FOR AIR TRAFFIC FLOW MANAGEMENT UNDER CONSTRAINED CAPACITY

A Thesis
Presented to
The Academic Faculty

by

Bosung Kim

In Partial Fulfillment of the Requirements for the Degree
Doctor of Philosophy in the
School of Aerospace Engineering

Georgia Institute of Technology
May 2015

Copyright © 2015 by Bosung Kim
TWO-STAGE COMBINATORIAL OPTIMIZATION FRAMEWORK FOR AIR TRAFFIC FLOW MANAGEMENT UNDER CONSTRAINED CAPACITY

Approved by:

Professor John-Paul Clarke,
Committee Chair
School of Aerospace Engineering
Georgia Institute of Technology

Professor Magnus Egerstedt
School of Electrical and Computer Engineering
Georgia Institute of Technology

Professor Eric Feron
School of Aerospace Engineering
Georgia Institute of Technology

Professor Alan Erera
School of Industrial and Systems Engineering
Georgia Institute of Technology

Professor Evangelos Theodorou
School of Aerospace Engineering
Georgia Institute of Technology

Date Approved: January 7 2015
To my loving and supporting family.
ACKNOWLEDGEMENTS

It is difficult to find perfect words to thank my advisor, family, and friends for supporting me. They have encouraged my growth both as a scholar and as a person with their academic and life wisdom.

First and foremost, I would like to thank my advisor, Dr. John-Paul Clarke, for his professional guidance and belief in me that always gave me confidence to continue my research. His practical and theoretical knowledge of air transportation, and valuable suggestions always set me on a right path. Without his mentorship I would not have acquired the knowledge, nor the skills, to complete my Ph.D. studies.

I would also like to express my deepest gratitude to Jim Brooks, who has been a great inspiration, and friend from the moment I joined the research lab. I would also like to thank the members of my committee, Dr. Feron, Dr. Theodorou, Dr. Erera, and Dr. Egerstedt, for their valuable insights and suggestions.

Next, I want to acknowledge all my friends in the Air Transportation Laboratory for their constant encouragement and want to say good luck with your future research and careers. I would also like to say thank all my friends at Georgia Tech and in Korea.

Lastly, I want to thank my family who devoted their lives to inspire me.
# TABLE OF CONTENTS

DEDICATION .................................................. iii

ACKNOWLEDGEMENTS ...................................... iv

LIST OF TABLES ............................................. vii

LIST OF FIGURES ........................................... ix

SUMMARY ..................................................... xi

## I  INTRODUCTION ........................................ 1

1.1 Motivation ............................................. 1

1.2 Problem Statement ..................................... 2

1.2.1 Terminal Airspace Under Constrained Capacity .......... 2

1.2.2 En Route Airspace Under Constrained Capacity .......... 8

1.3 Thesis Contribution and Outline ......................... 11

## II  LITERATURE REVIEW ............................... 15

2.1 Terminal Airspace Flow Management .................. 15

2.2 En Route Airspace Flow Management .................. 18

## III  AIR TRAFFIC FLOW MANAGEMENT MODEL FOR TERMINAL AIRSPACE ........... 21

3.1 Limited Resources .................................... 21

3.1.1 Runways ........................................... 21

3.1.2 TRACON boundary fixes ............................ 23

3.2 Approach .............................................. 27

3.3 Mathematical Formulations ........................... 29

3.3.1 Fix Assignments Model(1st Stage Model) .............. 29

3.3.2 Runway Assignments Model(2nd Stage Model) ......... 36

3.4 Cost Functions ......................................... 41

3.4.1 Cost Functions with Environmental Impacts .......... 41

3.5 Results ................................................. 45
3.5.1 Fix Assignments Model (1st Stage Model) ............... 45
3.5.2 Runway Assignments Model (2nd Stage Model) ........ 53
3.6 Review ................................................. 67

IV AIR TRAFFIC FLOW MANAGEMENT MODEL FOR EN ROUTE AIRSPACE ..................................................... 70

4.1 Limited Resources and CTOP ................................. 70
4.2 Two-Stage Combinatorial Model ............................. 74
  4.2.1 Optimal Slot Allocation Model (1st Stage Model) ..... 76
  4.2.2 Optimal Slot Assignments Model (2nd Stage Model) . 89
4.3 Computational Study and Analysis ............................ 94
  4.3.1 Simulation Parameter ................................... 94
  4.3.2 Comparison of Modified Branch and Bound algorithm to Heuristic Algorithm ....................................... 100
  4.3.3 Numerical Examples and Results ........................ 101

V CONCLUSION AND FUTURE RESEARCH ......................... 106

5.1 Conclusion for Terminal Airspace Model ................. 106
5.2 Conclusion for En Route Model ............................. 107
5.3 Applications to Real-World Problems ....................... 108
5.4 Suggested Future Directions .................................. 110

Appendices ......................................................... 113

APPENDIX A — EN ROUTE OPTIMIZATION IMPLEMENTATION ................................................................. 113

REFERENCES .................................................. 118
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Example Trajectory Option Set for a Single Flight</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>Minimum separation requirements in distance</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>Approach speed by weight class of aircraft</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>Sample of ICAO’s databank</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>Representative schedule at ATL</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>The average delays in seconds of baseline and optimization for fuel minimization</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>The average delays in seconds of baseline and optimization for emission minimization</td>
<td>52</td>
</tr>
<tr>
<td>8</td>
<td>Representative schedule at DTW</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>Minimum separation requirements in seconds</td>
<td>55</td>
</tr>
<tr>
<td>10</td>
<td>The number of arrivals assigned to each runway and the average delays in seconds for Baseline, RA/FCFS, and RA/SEQ</td>
<td>57</td>
</tr>
<tr>
<td>11</td>
<td>The number of departures assigned to each runway and the average delays in seconds for the baseline, RA/FCFS, and RA/SEQ</td>
<td>62</td>
</tr>
<tr>
<td>12</td>
<td>Emissions changes for different operations</td>
<td>65</td>
</tr>
<tr>
<td>13</td>
<td>Average run time in seconds to simulate a given schedule of arrivals</td>
<td>65</td>
</tr>
<tr>
<td>14</td>
<td>Sample arrival schedule of ATL</td>
<td>68</td>
</tr>
<tr>
<td>15</td>
<td>Results of the decomposed and combined optimization</td>
<td>69</td>
</tr>
<tr>
<td>16</td>
<td>Each Flight’s Earliest Entry Time at Each FCA for Example 1</td>
<td>79</td>
</tr>
<tr>
<td>17</td>
<td>Each Flight’s Earliest Entry Time at Each FCA for Example 2</td>
<td>80</td>
</tr>
<tr>
<td>18</td>
<td>Each Flight’s Earliest Entry Time at Each FCA for Example 3</td>
<td>81</td>
</tr>
<tr>
<td>19</td>
<td>Simulation Environment for Parameter Analysis</td>
<td>96</td>
</tr>
<tr>
<td>20</td>
<td>Given Capacity of Each FCA for Parameter Analysis</td>
<td>96</td>
</tr>
<tr>
<td>21</td>
<td>Runtime in Seconds of Heuristic, IP, and Modified Branch and Bound Method to Solve a Given Schedule</td>
<td>100</td>
</tr>
<tr>
<td>22</td>
<td>Simulation Environment</td>
<td>102</td>
</tr>
<tr>
<td>23</td>
<td>Given Capacity of Each FCA</td>
<td>102</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Case Study for Different Flight Schedule Density by Varying the Number</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>of Competitor’s Flights</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computational Resource Environment</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>Computation Times for Each Case</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>Cost Comparison for Each Case</td>
<td>104</td>
<td></td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Examples of the unbalanced traffic stream</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Arrival flows heading to ATL</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Unevenly distributed traffic delay driven by severe weather</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>CTOP Framework [53]</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Conventional paths in TRACON of DTW for south flow operations</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>South flow operations in DTW TRACON</td>
<td>23</td>
</tr>
<tr>
<td>7</td>
<td>Atlanta STAR procedure and radar track of traffic</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>Sample arrival-departure capacity trade-off curve</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>Overall Procedure of Terminal Airspace Optimizations</td>
<td>27</td>
</tr>
<tr>
<td>10</td>
<td>FLCON 8 STAR chart for arrivals coming from the northeast of ATL</td>
<td>32</td>
</tr>
<tr>
<td>11</td>
<td>Mission profile (not to scale)</td>
<td>42</td>
</tr>
<tr>
<td>12</td>
<td>ATL arrival-departure capacity trade-off curve(from FAA’s Airport Capacity Benchmark Report 2004)</td>
<td>46</td>
</tr>
<tr>
<td>13</td>
<td>Assigned arrival fix for minimizing fuel</td>
<td>49</td>
</tr>
<tr>
<td>14</td>
<td>Normalized total fuel for each time window</td>
<td>50</td>
</tr>
<tr>
<td>15</td>
<td>Assigned arrival fix for minimizing emission</td>
<td>51</td>
</tr>
<tr>
<td>16</td>
<td>Normalized total emission for each time window</td>
<td>53</td>
</tr>
<tr>
<td>17</td>
<td>Runway layout of DTW</td>
<td>54</td>
</tr>
<tr>
<td>18</td>
<td>The difference between the number of arrivals assigned to each runway</td>
<td>58</td>
</tr>
<tr>
<td>19</td>
<td>Normalized total emissions for arrivals (note: emissions are normalized by the maximum emissions of all the time windows.)</td>
<td>59</td>
</tr>
<tr>
<td>20</td>
<td>Percentage of emissions for TRACON transit, surface operations, and queuing delays(note: emissions are normalized by the baseline value of each time window.)</td>
<td>61</td>
</tr>
<tr>
<td>21</td>
<td>The difference between the number of departures assigned to each runway</td>
<td>63</td>
</tr>
<tr>
<td>22</td>
<td>Normalized total emissions for departures (note: emissions are normalized by the maximum emissions of all the time windows.)</td>
<td>64</td>
</tr>
</tbody>
</table>
23 Percentage of emissions for TRACON transit, surface operations, and queuing delays (note: emissions are normalized by the baseline value of each time window.) ............................................. 65
24 Optimization architecture of airport surface operations in the presence of uncertainty ............................................. 66
25 Overview of CTOP Assignment Algorithm ............................. 71
26 Multiple Flow Constrained Areas in a CTOP ............................ 74
27 Optimization Procedure .................................................... 75
28 Tree Generation with Grouping by a FCA entry .......................... 76
29 Tree Traversal of Example 1 ................................................. 79
30 Tree Traversal of Example 2 ................................................. 81
31 Tree Traversal of Example 3 ................................................. 82
32 Branch and Bound Algorithm Example ................................... 85
33 Modified Branch and Bound Algorithm Example ...................... 88
34 Arrival Delay Calculation using SOS2 Variables ....................... 94
35 FCA Definition in the Given Simulation .................................. 95
36 Moving Window Strategy ..................................................... 96
37 Computational Time for Minimum Sum of All Assigned Slots’ Times Solution with Different Window Size and Moving Step Size .............. 97
38 Minimum Sum of All Assigned Slots’ Times Solution with Different Window Size and Moving Step Size ............................................. 98
39 Minimum Arrival Delay Solution with Different Window Size and Moving Step Size ............................................. 99
40 Comparison on Solutions with Different Flight Schedules ................ 105
41 Overall Flow Chart for En Route Models ................................. 113
42 Sample Tree Traversal in Implementation .................................. 114
43 Handling Available Slots for Different Capacity(a) ....................... 115
44 Handling Available Slots for Different Capacity(b) ....................... 116
45 Handling Available Slots for Different Capacity(c) ....................... 116
46 Handling Available Slots for Different Capacity(d) ....................... 117
SUMMARY

Air traffic flow management is a critical component of air transport operations because at some point in time, often very frequently, one or more of the critical resources in the air transportation network has significantly reduced capacity, resulting in congestion and delays for airlines, individuals and other entities who use the network. Typically, these “bottlenecks” are noticed at a given airport or terminal area, but they also occur in en route airspace. The two-stage combinatorial optimization framework for air traffic flow management under constrained capacity that is presented in this thesis, represents an important step toward the full consideration of the combinatorial nature of air traffic flow management decision that is often ignored or dealt with via priority-based schemes. It also illustrates the similarities between two traffic flow management problems that heretofore were considered to be quite distinct.

The runway systems at major airports are highly constrained resources. From the perspective of arrivals, unnecessary delays and emissions may occur during peak periods when one or more runways at an airport are in great demand while other runways at the same airport are operating under their capacity. The primary cause of this imbalance in runway utilization is that the traffic flow into and out of the terminal areas is asymmetric (as a result of airline scheduling practices), and arrivals are typically assigned to the runway nearest the fix through which they enter the terminal areas. From the perspective of departures, delays and emissions occur because arrivals take precedence over departures with regard to the utilization of runways (despite the absence of binding safety constraints), and because arrival trajectories often include level segments that ensure “procedural separation” from arriving traffic.
while planes are not allowed to climb unrestricted along the most direct path to their destination. Similar to the runway systems, the terminal radar approach control facilities (TRACON) boundary fixes are also constrained resources of the terminal airspace. Because some arrival traffic from different airports merges at an arrival fix, a queue for the terminal areas generally starts to form at the arrival fix, which are caused by delays due to heavy arriving traffic streams. The arrivals must then absorb these delays by path stretching and adjusting their speed, resulting in unplanned fuel consumption; however, these delays are often not distributed evenly. As a result, some arrival fixes experience severe delays while, similar to the runway systems, the other arrival fixes might experience no delays at all. The goal of this thesis is to develop a combined optimization approach for terminal airspace flow management that assigns a TRACON boundary fix and a runway to each flight while minimizing the required fuel burn and emissions. The approach lessens the severity of terminal capacity shortage caused by and imbalance of traffic demand by shunting flights from current positions to alternate runways. This is done by considering every possible path combination. To attempt to solve the congestion of the terminal airspace at both runways and arrival fixes, this research focuses on two sequential optimizations. The fix assignments are dealt with by considering, simultaneously, the capacity constraints of fixes and runways as well as the fuel consumption and emissions of each flight. The research also develops runway assignments with runway scheduling such that the total emissions produced in the terminal area and on the airport surface are minimized.

The two-stage sequential framework is also extended to en route airspace. When en route airspace loses its capacity for any reason, e.g., severe weather condition, air traffic controllers and flight operators plan flight schedules together based on the given capacity limit, thereby maximizing en route throughput and minimizing flight operators’ costs. However, the current methods have limitations due to the lack of
consideration to the combinatorial nature of air traffic flow management decision. One of the initial attempts to overcome these limitations is the Collaborative Trajectory Options Program (CTOP), which will be initiated soon by the Federal Aviation Administration (FAA). The developed two-stage combinatorial optimization framework fits this CTOP perfectly from the flight operator’s perspective. The first stage is used to find an optimal slot allocation for flights under satisfying the ration by schedule (RBS) algorithm of the FAA. To solve the formulated first stage problem efficiently, two different solution methodologies, a heuristic algorithm and a modified branch and bound algorithm, are presented. Then, flights are assigned to the resulting optimized slots in the second stage so as to minimize the flight operator’s costs.
CHAPTER I

INTRODUCTION

1.1 Motivation

Air Traffic Flow Management (ATFM) is the service that ensures the efficient utilization of airspace resources. For efficient operations, ATFM manages traffic flows and allocates these to scarce capacity resources while maximizing throughput and minimizing the costs; however, these are very difficult objectives to achieve in real world operations simultaneously. Frequently one of the critical resources in the air transportation network has significantly reduced capacity, resulting in congestions and delays for flight operators and other entities who use the network. Prior to this research, many optimization models in the field of ATFM are developed. One such area is an optimization of the terminal airspace operations. Because bottlenecks are typically noticed at given airports or terminal areas, efficient operations of the terminal airspace can play a key role in the ATFM. The most critical and insufficient resources of terminal airspace operations are runways and metering fixes. Due to the limited number of runways and metering fixes of the terminal airspace, we can observe easily that arriving flights form a queue at these spots resulting in congestions of the terminal airspace. To absorb these delays and ensure required spaces between consecutive arrivals, the air traffic controller should make flights change their predetermined routes or speeds, and sometimes arrivals need to hold in the air making a predetermined holding pattern in severe case while burning unnecessary fuel. In addition to a waste of fuel, these changes of trajectories and speeds cause not only inconvenience but also difficulties in terms of terminal airspace operations to both flight operators and air traffic controllers. In this thesis, an optimization model for not only assigning
a fix and a runway to each flight but also scheduling aircraft operations at the fix and on the runway is suggested.

Similar to the terminal airspace problem, these bottlenecks also occur in en route airspace due to severe weather or failed navaids. However, en route airspace problems are more difficult to resolve in terms of a problem scope. That is, once congestions in the en route airspace is expected, we need to consider every Origin and Destination (O-D) pair of affected flights unlike the terminal airspace problems resulting in much bigger problem domain. To address en route issues, the FAA recently developed CTOP under Collaborative Decision Making (CDM) philosophy. CDM is a joint government-industry initiative for improvement of ATFM performance through increased information exchange among aviation community stakeholders. To archive this objective, CDM is comprised of representatives from government, general aviation, air carriers, private industry and academia to create technological and procedural solutions to the ATFM challenges faced by the National Airspace System (NAS). In the CTOP based on this CDM philosophy, system operator, the FAA, and system users or flight operators, airlines, make decisions in such a way to minimize users’ inconvenience while maximizing system throughput. An two-stage optimization model from flight operator side for this CTOP framework is developed and presented in this thesis. The proposed two-stage combinatorial optimization framework for ATFM under constrained capacity presents an important step towards the full consideration of the combinatorial nature of ATFM decisions that is often ignored.

1.2 Problem Statement

1.2.1 Terminal Airspace Under Constrained Capacity

Flights destined to and departing from congested airports often experience severe delays, particularly during peak traffic times. These delays occur mainly at two locations: the runways and arrival fixes. Since runway systems are a limited resource
Figure 1: Examples of the unbalanced traffic stream
Figure 2: Arrival flows heading to ATL
and require a long time, huge cost, and actual ground to build, many airports are facing a shortage of runway capacity as the volume of traffic grows. Additionally, runways are major bottlenecks for arrivals because of the joining of several arriving traffic streams. At congested airports, such delays are often attributed to insufficient runway capacity. At some airports, however, the management of airfield resources, not the shortage of airfield resources, is the primary contributor to delays. Specifically, while some parts of the existing runway system may have excess traffic, others are operating at less than capacity. This sub-optimal utilization of vital runway resources may be the result of an imbalance in traffic flow into and out of a terminal area. Such is the situation illustrated in Fig. 1(a), at the Hartsfield-Jackson Atlanta International Airport (ATL), where the heaviest traffic flow into the terminal area is from the northeast, and the next most voluminous flow is from the northwest. During peak traffic periods, when air traffic controllers, hereafter referred to as “controllers”, are conducting triple independent landing operations, they dedicate specific runways for aircraft entering through the northeast and northwest corners of the terminal area and direct aircraft entering through the southeast and southwest corners to the remaining landing runways. Nevertheless, these dedicated landing runways may still require re-routing to shunt aircraft from the heavy traffic streams to the under-served landing runways as shown in Fig. 2. Similarly, at the Detroit Metropolitan Wayne County Airport (DTW), illustrated in Fig. 1(b), the majority of the traffic enters the terminal area from the west during peak periods; and at many other hub airports, airline scheduling practices result in geographically-biased traffic flow, sub-optimal utilization of runway resources, and the production of excess gaseous emissions.

Similar to runway systems, arrival fixes are one of the bottleneck spots for arrivals because arriving flights from different airports merge at the arrival fixes to form a few pre-defined traffic streams known as Standard Terminal Arrival Routes (STARs). In some airports, however, a lack of resources and bottlenecks are not the main reasons
Figure 3: Unevenly distributed traffic delay driven by severe weather
for unbearable delays. Instead, the reason may be the unbalanced demands of traffic flows may lead to severe delays at certain arrival fixes whereas other fixes in the same terminal area simultaneously do not suffer from a lack of capacity and, as a result, experience no such delays. As mentioned, the air traffic demand from four arrival fixes/posts of ATL is very unbalanced. The northeast arrival fix experiences much more traffic volume than the other three arrival posts, and this imbalance of traffic volume can lead to huge delays on the northeast operation of ATL under peak volume periods. Sometimes, this challenge can deteriorate unbearably due to severe weather conditions. If the northeast arrival fix at ATL, which is the busiest one, is blocked or loses a part of its capacity due to a bad ceiling and visibility, the detours that are assigned without thoughtful careful consideration of traffic demand and capacity can result in severe delays at other arrival fixes. As illustrated in Fig. 3(a), the northeast fix of Charlotte Douglas International Airport (CLT) experiences severe delays. As a result, aircraft, sometimes as many as ten or twelve, keep their position at holding areas and form a queue to enter the terminal area. Furthermore, these delays are propagated to the northeast airfield systems, resulting in the developing/forming of a second queue. The other arrival fixes at CLT, on the other hand, simultaneously operate at less than their capacities, idling resources. Similar to CLT, Philadelphia International Airport (PHL) experiences unbalanced traffic demands as presented in Fig. 3(b). More flights head to the northwest post and encounter much longer queues of arrivals than other posts. Note that, in Fig. 3, both airports were under a severe weather condition caused by a winter storm and a Ground Delay Program (GDP) was activated at both airports by the FAA. Even though a GDP was initiated to address expected congestions, imbalance of traffic demands among arrivals from each corner post causes severe delays to the specific terminal airspace resources while the others didn’t experience those congestions.
1.2.2 En Route Airspace Under Constrained Capacity

The en route airspace is one of the critical resources in the air transportation network. Similar to bottlenecks observed at the terminal areas, very frequently congestions and delays are induced by imbalance between the given traffic demand and resource capacity. The FAA currently alleviates these en route delays and congestions due to imbalance in one of three ways: (a) by re-routing flights prior to their departure; (b) by re-routing aircraft that are already en route; or (c) by regulating the rate at which aircraft enter constrained areas via Ground Delay Programs (GDPs), Airspace Flow Programs (AFPs), Ground Stops (GSs), or Miles-In-Trail (MIT) restrictions. There are several shortcomings with the aforementioned approaches to alleviating en route delays and congestions. First, GDPs have less impact on en route delays and congestions. A major portion of the flights that are actually passing through the constrained airspace are not included in the programs because GDPs are developed for capacity decreases on terminal airspace and they are not going to the airports selected for the GDPs. According to the FAA, implementing a GDP at the ten airports that contribute the most to traffic congestions may only impact less than a third of the traffic [50]. Second, assigning delays and re-routing flights are determined independently resulting in losing the opportunity to maximize throughput by assigning delays and rerouting flights together. For example, AFPs lessen congestions by assigning delays to affected flights (ground delays prior to departure or airborne holding after departure) or by assigning a trajectory that will take flights out of the AFPs. However, AFPs can’t make a decision considering both options simultaneously. Third, GSs and MIT are not procedures that require much coordination between the system operator (FAA) and the system users (flight operators). Due to these centralized NAS resource management, it is difficult for the system operator to reflect the flight operators’ preferences into the traffic flow management system.

To overcome these shortcomings, the FAA has introduced CTOPs as one of the
Figure 4: CTOP Framework [53]

practices of CDM. CDM is a NAS operation paradigm where decisions of ATFM are based on a shared view of the NAS and the consideration of the consequences on the system itself and its stakeholders resulting from these decisions. There are two main principles in CDM. First, more shared information will lead to better decisions. Second, tools and procedures need to be in place to enable system providers, the FAA, and system users, the flight operators, to exchange information and changes of conditions in easier way. By exchanging information and preferences, CDM wants stakeholders to learn from each other and build a common knowledge, resulting in decisions that are the most valuable to the NAS as well as themselves. Since CTOPs are designed under this CDM paradigm, the underlying philosophy of CTOPs is to give flight operators of the NAS as much input as possible into delays and re-route decisions by allowing them to submit their trajectory preferences, assigning each flight to its most preferred possible trajectory given the constraints, and then allowing flight operators of the NAS to swap route trajectories/delays among their own assigned slots.
Table 1: Example Trajectory Option Set for a Single Flight

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Alt</th>
<th>Speed</th>
<th>Relative Trajectory Cost (RTC)</th>
<th>Trajectory Valid Start Time (TVST)</th>
<th>Trajectory Valid End Time (TVET)</th>
<th>Required Minimum Notification Time (RMNT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORD..ELX.. ..JHW..RKA.. ..LGA</td>
<td>350</td>
<td>480</td>
<td>0</td>
<td>13:00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ORD..ELX.. ..JHW..RKA.. ..LGA</td>
<td>370</td>
<td>480</td>
<td>10</td>
<td>13:00</td>
<td>15:00</td>
<td>-</td>
</tr>
<tr>
<td>ORD..TVC.. ..RKA..IGN.. ..LGA</td>
<td>350</td>
<td>480</td>
<td>20</td>
<td>13:00</td>
<td>16:00</td>
<td>45</td>
</tr>
<tr>
<td>ORD..TVC.. ..RKA..IGN.. ..LGA</td>
<td>370</td>
<td>480</td>
<td>25</td>
<td>13:00</td>
<td>15:50</td>
<td>45</td>
</tr>
<tr>
<td>ORD..ASP.. ..YYZ..ROC.. ..RKA..IGN.. ..LGA..</td>
<td>350</td>
<td>480</td>
<td>40</td>
<td>13:00</td>
<td>16:00</td>
<td>-</td>
</tr>
<tr>
<td>ORD..ASP.. ..YYZ..ROC.. ..RKA..IGN.. ..LGA..</td>
<td>370</td>
<td>480</td>
<td>45</td>
<td>13:00</td>
<td>15:00</td>
<td>-</td>
</tr>
</tbody>
</table>

in Flow Constrained Areas (FCAs).

As shown in Fig. 4, Air Traffic Control System Command Center (ATCSCC) of the FAA will identify constraints in the NAS and issue a CTOP for the en route airspace which has constrained capacity (defined by a start time, a stop time, affected areas, and associated capacities). Flight operators of the NAS will then submit their trajectory options and indicate their preferences via Trajectory Option Set (TOS) messages that can be sent at any time and as their preferences change. This flexibility will allow flight operators of the NAS to lessen the impact of the programs by responding tactically with flight substitutions and additional options based on
individual company business rules and whether a re-route or delay better suits their needs. An example trajectory option set for a single flight is shown in Table 1. As can be seen, each option in the set will include a trajectory, an acceptable delay for that trajectory in form of RTC in minutes, constraints of trajectory availability in terms of start and end times, and the time prior to departure that flight operators must be notified of the trajectory assigned. Using these TOS messages, flight operators of the NAS can inform their trajectory preferences as well as trajectory constraints to the FAA. Then, the FAA will assigned each flight to its most preferred trajectory based on submitted TOS messages on a flight by flight basis under the assignments rule of CTOPs described in Chapter 4.

1.3 Thesis Contribution and Outline

The main contributions of this thesis are to manage the given scarce resources efficiently involving the terminal airspace and the en route airspace operations. For the terminal airspace, the contributions of this thesis are as follows:

1. This research provides a combined optimization solution for maximizing terminal airspace efficiency and making greener terminal airspace operations by formulating and solving a two-stage combinatorial optimization model that minimizes total consumed fuel and generated emissions due to the terminal airspace operations under the given capacity and safety constraints.

2. This research develops an optimization algorithm addressing not only the costs of current operations directly related with decision but also the costs of the consequential operations, as opposed to most other studies where only runways and TRACON boundary fixes are considered.

3. This research creates potentially tractable and implementable optimization approaches under the current Traffic Management Advisor (TMA) framework of
the National Aeronautics and Space Administration (NASA) by decomposing a problem into a two-stage model in accordance with time sequence of decisions. In addition, it also creates implementable emissions computation framework using aircraft performance data provided by Base of Aircraft Data (BADA), emissions database developed by the International Civil Aviation Organization (ICAO), and aircraft registration database published by the FAA.

These contributions with a thoughtful implementation in real application of the terminal airspace management lead to a more efficient and greener solution method for the terminal airspace operations from the cruise phase to the gate of the airport terminal.

In addition to above implications of the terminal airspace optimizations, this research effort provides three significant contributions to the field of the en route airspace optimizations:

1. This research suggests a novel combinatorial optimization model from flight operator side within the CTOP framework proposed and developed by the FAA. By decomposing a problems into two-stage sequential models, it captures the combinatorial nature of the CTOPs decision making process under the given assignment rules and competition.

2. This research provides an efficient solution methodology to account competition with other flight operators in the first stage model. To obtain an optimal slot allocation solution in reasonable runtime, a heuristic approach and a modified branch and bound approach are developed and analyzed.

3. This research aims at minimizing flight operators’ costs associated with all operations from departure to arrival, thereby maximizing their revenue, by solving a large scale Mixed Integer Programming (MIP) problem.
Using these findings and developments together, we are able to implement an optimal response system of CTOPs corresponding to the required actions of the flight operator, i.e., submitting TOS messages for each flight, in close to real time for moderate schedule density.

This thesis is divided into two major sections. After presenting previous works on the ATFM problems for the terminal and en route airspace along with a review of some current program for addressing capacity issues at the terminal and en route airspace in the United States in Chapter 2, the first major section begins with Chapter 3 describing the developed optimization models for TRACON boundary fix assignments and runway assignments. The fix assignments model assigns an TRACON boundary fix to each flight while minimizing the total consumed fuel and generated emissions by accounting for not only the capacity of arrival and departure TRACON boundary fixes but also the capacity of runways for the given terminal areas and airport configuration. With regard to the runway assignments model, the proposed model assigns a runway to aircraft and optimizes the sequence of terminal airspace operations on the same runway simultaneously while maximizing both runway utilization and throughput, thereby minimizing the environmental impact of aircraft operations in the terminal area and on the airport surface, by addressing the required safety and operational constraints.

Following the terminal airspace optimization, the second major section, beginning with Chapter 4, proposes an optimization model addressing traffic flow management problem with regard to en route trajectory selection after investigation of CTOPs. The analysis of CTOP process motivates the need for optimizations from flight operator side. To minimize the impact of programs, a two-stage sequential optimization model is proposed with consideration of combinatorial nature in decision making process. After identifying and obtaining an optimal set of slots in the given FCAs in the first stage model, the second stage model finds an optimal assignment solution by
addressing every cost factor resulting from assignment decisions including costs associated with departure delays, arrival delays, and en route operations in form of a MIP problem. Finally, the findings and conclusions of this thesis are provided in Chapter 5, where the suggested possible research directions for future are also presented.
2.1 *Terminal Airspace Flow Management*

The terminal area optimizations have been well studied in the literature, and a number of optimization models have been developed. Starting from Dear’s [31, 32] scheduling and sequencing model, more than 60 papers have been published on the topic of arrival scheduling and sequencing. For example, Brinton [20] attempted to solve the scheduling problem of arrivals by introducing an implicit enumeration algorithm to deal with the non-linearity of the problem constraints instead of using a linear programming technique, while Carr et al. [26, 27] optimized the sequence and schedule of arrivals by taking individual airline priorities into account among incoming flights and compared their optimal solutions to first-come, first-served (FCFS) policy solutions of a fast-time simulation approach. Beasley et al. [16, 17] presented a MIP model that has served as a base formulation of the runway scheduling problem. In addition, they provided computational results showing that their formulation works well. In the literature, these computational results of an exact algorithm have been used as a reference to compare the performance of heuristic methods. Anagnostakis et al. [9] suggested a method for solving the take-off/landing problem in which both sequence determination and time assignment problems are solved separately. To account for the effects of uncertainties, Solveling et al. [61] formulated a two-stage stochastic model for runway operation planning, in which the first stage was used to find an optimal sequence of arrivals and departures based on only the aircraft weight class. Then, each flight was assigned to the resulting optimized sequence in the second stage. These runway-centric solutions were based on observations that
the main throughput bottleneck at an airport was the runway system [40]. However, as Erzberger [35] and Gilbo [36] discovered in their research, bottlenecks may occur elsewhere in the terminal area, resulting in severe delays not only on runways but also at the arrival fixes located at the boundary of the TRACON. Moreover, the researchers considered runway systems as a resource that is independent from the entire airspace. Thus, the options for improving operational efficiency were limited. To overcome this issue, Bianco et al. [19] developed a multi-resource scheduling and sequencing problem, including arrival fixes and runways; Saraf and Slater [58] then investigated dynamic hierarchical scheduling with a wide view of terminal airspace; and Gilbo [36] suggested an optimization scheme for an arrival and departure fixes optimization that minimized cumulative queuing delays for all traffic flow. However, he did not capture the possible benefits of assigning fixes to flights. Kim et al. [46,47] researched an optimal runway assignment algorithm that minimizes environmental impacts by considering the sequencing and assignment problem on arrivals simultaneously, and they expanded their model to include departure operations. In addition to runway assignment problems, Kim and Clarke [45] also proposed the TRACON boundary fix assignment model to balance traffic demand with the terminal airspace system capacity by considering the terminal airspace operations from the cruise phase to the runways. To handle multiple runways, Boeing developed the multiple runway planner, which was analyzed by Berge et al. [18].

As mentioned, the majority of traditional research has focused on a runway system and TRACON airspace system. Because congestions develop from the runways to the terminal area, these segments need to be considered for any airfield airspace resource optimization. However, severe congestions could result from an imbalance of traffic demand, as shown in Fig. 3. To alleviate this undesirable situation, we need to consider the interaction between the runway system and the TRACON airspace system. Within this new approach to consider the terminal airspace operations, the suggested
optimization model distributes the delays of arrival fixes by assigning fixes to flights and absorbing the delays in the cruise phase, where flights fly most efficiently, then assigns runways to flights and simultaneously optimizes the sequence of operations on the same runways precisely. The proposed model could cause controllers to assign a longer ground routes than scheduled, but this decision to assign a detour will be issued before the aircraft cross the top of descent (TOD) point. Therefore, this additional travel distance will occur in cruise phase rather than in the descent and low altitude operations, which require higher fuel flow rate to travel. Moreover, The suggested concept/method is consistent with the current Traffic Management Advisor (TMA) operation developed by NASA [38,63]. Thus, it is much easier to implement this approach into current technology.

Another interesting area of the research is how to model the objective functions of the optimization. Traditionally researchers have focused on minimizing fuel consumption and “wheels-on” delay by maximizing runway throughput. However, these metrics (i.e., fuel and wheels-on delays) fail to capture the impact of decisions pertaining to airport surface operations and their resulting emissions when considering the terminal airspace operations. In fact, because the highest production of $HC$ and $CO$ occurs during ground taxi operations, the joint optimization of runway assignments and schedules minimize operations in the terminal airspace and on the airport surface. The review of the literature revealed few studies that propose optimization models for the joint assignment and scheduling of runways. However, many researchers have attempted to quantify and analyze the pollutants associated with aviation stakeholders and ground taxi operations. The Environmental Protection Agency (EPA) estimated types of emissions generated by U.S. airports, while EUROCONTROL analyzed environmental benefits for the free-route airspace project and the relationship between environmental impacts and delays for both ground and airborne operations [6,23,42]. For surface movement, Miller and Clarke [52] investigated the rapid rise in aviation
emissions from ground operations and the reduction in emissions from improved sur-
face operations. Solveling et al. [60] attempted to formulate an objective function
considering environmental impacts. However, it is limited to certain specific aircraft
types such as boeing 747, 757, and 777, whereas there exist a variety of aircraft in
real airports.

2.2 En Route Airspace Flow Management

Similar to the traffic flow management of the terminal airspace, a number of re-
searchers have examined the traffic flow management of the en route airspace and
developed various optimization models to balance demands with the given system ca-
pacity. Odoni has given a systematic description of the ground holding problem while
Terrab has studied the ground holding strategies [54,64]. Implementing this concept,
the FAA monitored airports and issued a GDP to flight operators of the NAS when it
finds a severe capacity demand imbalance. However, the system was a very centralized
paradigm on the system operator, the FAA, resulting in inefficient uses of the valuable
NAS resources. To overcome these shortcomings, GDPs have been scheduled and run
using the CDM framework [10,13,70]. As mentioned previously, the CDM embodies
a new philosophy to improve the efficiency of the NAS resources management by a
closer collaboration among the system operator (the FAA), flight operators (the air
carriers), and academia. This framework is based on mutual exchange of data and
more flexible and efficient collaborative procedures. Under this philosophy, Chang et
al. provide a description of the GDP enhancements resulting from the CDM initia-
tive at a level of detail that is relevant to the quantitative modeler, while Kotnyek
and Richetta evaluate and show validity of the given static stochastic ground holding
problem under the CDM framework [28,48]. Moreover, Ball et al. presented and
showed that their stochastic integer program formulation can be solved using an LP
and still provide integer solutions due to the unimodal structure of constraints matrix [12]. They claimed that the proposed model is valuable to the enhanced GDPs under CDM framework due to this fast runtime in spite of consideration of stochastic nature. Regarding the efficient way to allocate the constrained capacity to flight operators of the NAS, many researchers have investigated allocation algorithms including Ration by Schedule (RBS) and Ration by Distance (RBD) rule [11,14,51,67]. Under active GDPs based on CDM framework, flight operators should make a decision with respect to rescheduling their flights under GDPs and Carlson presented an IP model and a real-time solution algorithm that assists an airline in making these rescheduling decisions at its hub airport, the location with the largest number of operations and therefore the greatest opportunity for improvement [24]. To investigate and improve the operations in terms of fairness in the CDM framework, Vossen and Ball researched trading slots scheme of GDPs among flight operators [66,68,69].

Unlike aforementioned studies solving capacity demand imbalance from the system operator perspective, Abdelghany et al. developed a genetic algorithm and a heuristic algorithm for GDPs and AFPs, respectively, to minimize the schedule disruptions and operational costs from the flight operators perspective [7, 8]. In addition, Vlachou and Lovell proposed and analyzed two allocation schemes for air carriers to express some preference structure for their flights that are specifically affected by APFs [65]. For CTOPs specifically, Kim and Hansen have proposed a modeling framework and compared four resource assignment schemes featuring different user preference inputs and allocation mechanisms [44]. However, they focused on the utilization of the NAS resources from the system operator perspective. As a result, it is limited to evaluations and examination of the problem from the biased perspective. To the best of our knowledge, there is no attempt to model CTOPs and optimize the given problem from the flight operator perspective. With respect to the benefits of CTOPs,
Kamine et al. evaluated and estimated improvement resulting from assigning route-out trajectory which does not enter the given FCAs using the AFP framework while Ye et al. analyzed several strategies by varying MIT restrictions for the airspace and rerouting in collaboration with the air operations center [43,72].
CHAPTER III

AIR TRAFFIC FLOW MANAGEMENT MODEL FOR TERMINAL AIRSPACE

3.1 Limited Resources

3.1.1 Runways

Arguably, the two most important functions of controllers who manage terminal airspace and airport surface operations are the assignment of flights (arrivals and departures) to runways and the scheduling of landings and take-offs with the enforcement of safety separation rules. The goal in a runway assignment is to spread traffic volume across available runways, thereby maximizing their utilization. When assigning flights to runways, controllers have to account for many factors, including their origins and destinations (i.e., arrival and departure fixes and the gates to which they are assigned), the traffic volume on the various runways, and differences in aircraft performance. The scheduling of arrivals and departures results in a sequence of take-offs and landings with associated separations between successive runway operations. The minimum separation is generally 5 NM for all aircraft in the en-route airspace and 3 NM within the terminal airspace. However, for arrivals, the minimum separation between the final approach fix (FAF) and the runway depends on combinations of the weight class of the leading and trailing aircraft, ranging from 2.5 NM to 6 NM. Furthermore, as approach and landing speeds are specific to the aircraft type and weight, the sequence of operations on a runway affects runway throughput.

The conventional approach to a runway assignment is to assign each flight to a runway based on the distance from its arrival fix to the runway as illustrated in
Fig. 5. For example, DTW has five arrival fixes: SPICA for flow from the northeast, MIZAR for flow from the southwest, GEMNI and WEEDA for flow from the southeast, and POLAR for flow from the northwest. While this strategy is useful for evenly-distributed traffic flow, it results in unbalanced runway utilization at DTW when heavy traffic flows from the west, as is the case during certain peak periods.

For a more efficient utilization of runway resources of DTW where typically the outer runways, 21L and 22R, are dedicated to arrivals and inner runways, 21R and 22L, are dedicated to departures, each arrival fix must have distinct paths to at least two runways, and similarly, each runway must have distinct paths to at least two departure fixes. Achieving this flexibility requires that several paths be added to the previously described conventional paths. Figure 6(a) depicts the current operating standards to the south within the Detroit TRACON, located in the Cleveland Air Route Traffic Control Center (ARTCC). The new paths for this flow within this TRACON, illustrated in Fig. 6, operate in the same manner as those that follow the
The south flow operation procedure of DTW TRACON

Figure 6: South flow operations in DTW TRACON

standard procedures for DTW [3]. Compared to the conventional track map, the new track map is more complex. Each fix is connected to two runways via distinct paths that provide an opportunity for controllers to assign arrivals and departures to runways based on traffic scenarios. Despite the several common points in Fig. 6(b), the vertical components of the associated trajectories are separated by at least 1,000 ft thereby ensuring that the required vertical separation is satisfied and that each path can be considered independently. Moreover, because these paths pass over DTW, at altitudes ranging from 7,000 to 8,000 feet, we can model the runway assignment problem for arrivals separately from the runway assignment problem for departures. While this change in the operating paradigm could result in additional workload for controllers upstream from the terminal airspace, we can envision that the overall workload will be the same as or less than the workload associated with tactically balancing runway utilization, which will be reduced and in some cases eliminated.

3.1.2 TRACON boundary fixes

Similar to the runway systems, TRACON boundary fixes are important and limited resources of the terminal areas because they are also bottlenecks for arrival traffic
Figure 7: Atlanta STAR procedure and radar track of traffic streams. As illustrated in Fig. 7, ATL has four main STARs as arrival traffic routes, with each dot representing the position information of arriving flights extracted from the Enhanced Traffic Management System (ETMS) for September 30, 2005. Flights from different origins enter the initial STAR fix and then follow the pre-defined STAR route so that they merge at the TRACON boundary arrival fixes and form fewer streams. Unlike arrival traffic, delays are propagated in the totally opposite direction. Delays on the surface, which affect runway operations, are propagated toward arrival fixes and routes defined in STAR. This unfavorable situation is exacerbated when severely bad weather occurs or an imbalance of traffic volume over the arrival fixes.
Figure 8: Sample arrival-departure capacity trade-off curve

exists. As depicted in Fig. 3(a), in early January, 2013, the northeast arrival fix of CLT experienced severe delays resulting in an arrival queue at the holding area, and these delays were propagated through the associated STAR routes. As a result, a group of flights formed a second holding queue while the other fixes operated below their capacity. This undesirable impacts of traffic imbalance cause additional fuel consumption resulting from fuel-inefficient operations, such as an increase of thrust to maintain the given altitude and speed at holding areas, to flight operators as well as additional workload resulting from management of a set of flights in holding areas, such a dense traffic, to air traffic controllers.

To resolve these issues, we need to optimize the terminal area utilization by balancing the traffic volume over each arrival fix on the boundary of the TRACON areas. To achieve effective fix assignments, air traffic controllers must consider many factors, including the capacity of the airfield resources and the traffic demand of the terminal airspace. Fig. 8 describes a sample of an airport arrival-departure runway capacity trade-off curve. Because airport surface operates interactively with the runways and taxiways, both the arrival and departure operations affect each other, as
shown in Fig. 8. On the other hand, because the fix capacity is determined by two parameters, the required crossing speed at TRACON boundary fixes and distance buffer of a pair of consecutive flights forced by minimum separation requirements or Miles-In-Trail (MIT) restrictions, weather conditions could result in each fix having a different capacity. However, if only some arrival fixes suffer huge traffic volume, we can alleviate these congestions by diverting some of the arriving flights heading to busy arrival fixes to other arrival fixes that have available capacity to accommodate these diverted flights. Obviously, every pair of origin and destination has preferred usually the shortest paths, so shunting flights leads to longer en route paths, requiring additional fuel burn. However, if we change the route of some flights before they start descent operations, those flights provide a buffer time which absorbs the delays at busy arrival fixes while we can minimize the additional fuel burn. Shunting flights prior to descent operations is also beneficial in that it avoids anticipated delays at the initially assigned arrival fixes with huge traffic volume. In addition, the suggested approach is more effective when severe weather prohibits the use of a terminal area or terminal areas since all flights planning to travel through that area should be shunted. In this case, the alternative route should be chosen by the suggested approach. With respect to the performance index of this demand balancing approach, we need to take into account all consequential costs associated with our decisions made through fix assignments. To capture these, the performance index consists of three parts: the inside costs of TRACON transition, the outside costs of TRACON transition, and the queuing delay costs at fixes on the TRACON boundary. The outside costs of TRACON transition take into account the estimated cost of fuel and emissions from the current position to the assigned arrival fix; these costs incurred by a few flights could increase as a result of the diverting route scheme of the approach. In spite of this disadvantage, the main benefit comes from much lower queuing delay and inside costs of TRACON transition because this approach prevents huge delays for flights
Figure 9: Overall Procedure of Terminal Airspace Optimizations in arrival traffic streams by absorbing terminal area delays in the cruise phase.

3.2 Approach

As discussed, runway systems and TRACON boundary fix systems are places where multiple traffic streams merge together resulting in bottlenecks. To utilize these limited resources efficiently, an optimization of runway and TRACON boundary fix assignments is required. However, it is difficult to assign TRACON boundary fixes and runways to each flight due to the scale of problems. In addition to difficulties of computational complexity, assigning runways to flights in more than a two hours look-ahead window requires schedule changes inevitably due to uncertainty such as winds, trajectory, and speed. To address terminal airspace optimization problems with aforementioned considerations, a two-stage sequential optimization framework is proposed as shown in Fig. 9.

The proposed two-stage planning approach consists of two optimization models: the fix assignment model (the first stage) that optimizes terminal airspace operations from the cruise phase and the landing phase under the given capacity constraint, aircraft performance information, and schedule of flights; the runway assignment model (the second stage) that optimizes terminal airspace operations from the TRACON boundary fixes to the gates of the airport terminals under the given solution from the
fix assignment model, safety constraint, aircraft performance information, and schedule of flights. This solution methodology is appropriate for the terminal airspace operation problems, since the first planning stage finds the fix assignment solution based on more robust information (arrival-departure capacity trade-off curve in the resolution of 15 minutes) than the precise schedule of flights, which is the information needed in the second planning stage. However, this approach could lead to a hidden pitfall and reduced savings if we do not consider the interaction, at each stage, between each terminal airspace operations such that the holding operations at TRACON boundary fix affects to the descent operations. To avoid this limitation of the decomposed approach, the first stage model should assign TRACON boundary fixes to flights by accounting capacity of the fixes as well as capacity of the runways. For example, the number of TRACON boundary fixes, in general, is bigger than the number of runways resulting in much higher capacity. As a result, more flights need to hold at TRACON boundary fixes if we do not account the capacity of runway systems when optimizing the operations from the TOD to the TRACON boundary fixes. Similar to the first stage model, the second stage model also must address consequence of runway assignments. That is, runway assignment solutions determine not only the operations between TRACON boundary fixes and runways but also the operations between runways and the gates of airport terminal. Thus, the second stage model should take account of taxi operations on airport surface. This approach provides an easily implementable optimization model under the current TMA framework by decomposing a problem into a two-stage model in accordance with time sequence of decisions. Thus, controllers focus on the fix assignment problems only for flights which are before the TOD while we can assign runways and schedule runway operations optimally at same time for flights which are after the TOD using the fix assignment results. Using this approach we can resolve terminal airspace congestions by absorbing the expected terminal airspace delays at the cruise phase. Even though unresolved
congestions, unavoidably, are remained or newly caused, we can account these issues at the second stage optimization. Therefore, it will also provide distributed workload to controllers by reducing, in some cases eliminating, conflict-detection and resolution efforts in the terminal airspace.

3.3 Mathematical Formulations

3.3.1 Fix Assignments Model (1st Stage Model)

\( A \) the set of arrivals
\( D \) the set of departures
\( N \) the set of time windows
\( \Delta \) interval of time window
\( F^A \) the set of arrival fixes
\( F^D \) the set of departure fixes
\( F^AI \) the set of initial STAR fixes
\( T^O_i(l) \) estimated time to arrival fix via initial STAR fix \( l \) for flight \( i \in A \)
\( T^l_i(k) \) estimated time between fix \( k \) to the closest runway flight for \( i \in A \cup D \)
\( C^A_i(l) \) estimate of fuel consumed to arrival fix via initial STAR fix \( l \) for flight \( i \in A \)
\( C^I_i(k) \) estimate of fuel consumed to the closest runway via fix \( k \) for flight \( i \in A \)
\( \phi(r^A_n) \) airport arrival-departure capacity trade-off curve at time window \( n \)
\( r^A_{n,\max} \) maximum arrival capacity at time window \( n \)
\( c_{kn} \) fix capacity at time window \( n \) for \( k \in F^A \cup F^D \)
\( f^AO_i \) scheduled initial STAR fix for flight \( i \in A \)
\( g^AO_{ik} \) scheduled arrival fix for flight \( i \in A \)
\( g^DO_{ik} \) scheduled departure fix for flight \( i \in D \)
\( d^R_i \) scheduled departing time for flight \( i \in D \)
\( ff^\text{hold}_i \) fuel flow rate at holding area for flight \( i \in A \)
\( ff^\text{idle}_i \) fuel flow rate on the ground for flight \( i \in D \)
The objective of the proposed fix assignment model is to optimize terminal area utilization while satisfying the capacity constraints of the fixes and runways. The optimization model for fix assignments is formulated by an IP technique. To simplify the model, the following assumptions are made:

1. All aircraft must follow pre-determined transition paths during transition.

2. No aircraft is allowed to change its path once it is assigned to a fix or starts descending or departing operations.

3. All arriving flights are assigned to the closest runway.

### 3.3.1.1 Decision Variables

To formulate the fix assignment problem, the following decision variables are defined:

\[ f_{il}^A = \begin{cases} 1 & \text{if flight } i \text{ is assigned to initial STAR fix } l \\ 0 & \text{otherwise} \end{cases} \]

\[ g_{ikn}^A = \begin{cases} 1 & \text{if flight } i \text{ will transit arrival fix } k \text{ during time window } n \\ 0 & \text{otherwise} \end{cases} \]

\[ g_{ikn}^D = \begin{cases} 1 & \text{if flight } i \text{ will transit departure fix } k \text{ during time window } n \\ 0 & \text{otherwise} \end{cases} \]

\[ s_{in}^A = \begin{cases} 1 & \text{if aircraft } i \text{ is assigned to land during time window } n \\ 0 & \text{otherwise} \end{cases} \]

\[ s_{in}^D = \begin{cases} 1 & \text{if aircraft } i \text{ is assigned to take-off during time window } n \\ 0 & \text{otherwise} \end{cases} \]

\[ t_i^R = \text{landing/take-off time for flight } i \in A \cup D \]

\[ t_i^F = \text{actual fix crossing time for flight } i \in A \cup D \]

\[ e_i^F = \text{new estimated fix crossing time for flight } i \in A \cup D \]
As previously mentioned, each STAR has multiple initial entry fixes and one common TRACON boundary fix. For example, ATL’s FLCON 8 STAR procedure for the northeast traffic stream has three entry fixes: SOT, MOL, and SPA. These three arrival streams merge at ODF and head to one common arrival fix, DIRTY, which is located on the TRACON boundary, as illustrated in Fig. 10. To determine whether flight $i$ is assigned to initial STAR fix $l$, the decision variable $f_{il}^A$ is defined. In addition to this, we also define the assignment variable $g_{ikn}^A$ to indicate that flight $i$ is assigned to arrival fix $k$ during time window $n$. These two variables have a close relationship because each initial STAR fix has an associated TRACON boundary fix. Therefore, once an initial STAR fix is determined, we know which TRACON boundary fix will be assigned. An additional parameter, the time window, is considered in order to account for TRACON boundary fix capacity constraints. Similar to TRACON boundary fix assignment variables, $g_{ikn}^D$ variables for departures fix assignments are defined. Likewise, $s_{in}^A$ and $s_{in}^D$ indicate whether flight $i$ occupies time slot $n$ for landings/take-offs. In addition to this, $e_i^F$ is defined as the new estimated time of arrival (ETA) at a TRACON boundary arrival/departure fix. Since the given scheduled ETA is not necessarily the same as the ETA for the new assigned TRACON boundary arrival/departure fix, we need this variable to compute holding time at the TRACON boundary arrival/departure fixes.

### 3.3.1.2 Objective Functions

Maximization of terminal airspace throughput and minimization of terminal airspace delays are the most common and simple objectives in the literature. In the proposed
Figure 10: FLCON 8 STAR chart for arrivals coming from the northeast of ATL model, minimization of total consumed fuel and emissions is suggested as the performance index of the model. Because this model alleviates unevenly distributed delays of TRACON boundary fixes by shunting flights to alternative one, we should consider the additional fuel consumption resulting from these detours.

\[
\begin{align*}
\text{min} & \left\{ \sum_{i \in A \cup D} \text{delay} (i) + \sum_{i \in A} \text{inside} (i) + \sum_{i \in A} \text{outside} (i) \right\} \\
\text{delay} (i) & = \begin{cases} 
(t_i^F - e_i^F) f_i^{hold} & \forall i \in A \\
(t_i^R - d_i^R) f_i^{idle} & \forall i \in A 
\end{cases} \\
\text{inside} (i) & = \sum_{n \in N} \sum_{k \in F} C_i^I (k) g_{ikn} \\
\text{outside} (i) & = \sum_{l \in F} C_i^O (l) f_{il}
\end{align*}
\]

As shown in Eqs. (1), the performance index (or objective function) of the model consists of three parts. The first part is the costs of holding delays at TRACON boundary arrival fixes and ground delays on airport surface for departing operations.
To compute holding delays at TRACON boundary arrival fixes, we need to calculate ETAs at newly assigned TRACON boundary arrival fixes by the equation given below. The second part of objective function is the costs associated with the transition from TRACON boundary fixes to runway thresholds for the descending operations that occurs inside the TRACON. The last term is the costs for the transitions from current positions to TRACON boundary arrival fixes that take place outside of the TRACON. Regarding the costs of departing operations, only the costs of departing delays is considered. If we consider the costs associated with ascending operations, the majority of the costs will come from departing operations since the fuel flow rate for take-off and climb-out procedures during departing operations is much higher than it is for arriving operations. To avoid this undesirable result, the transition cost from runways to TRACON boundary departure fixes for departing flights is excluded. Similarly, the costs after flights cross TRACON boundary departure fixes are not considered. More detailed methods of calculating fuel and emissions costs are presented in next subsection.

\[
ETA = \text{planned arrival fix crossing time} \\
- \text{transit time from current positions to arrival fixes via scheduled initial STAR fix} \\
- \text{transit time from current positions to arrival fixes via assigned initial STAR fix}
\]

3.3.1.3 Constraints

To meet the given operational restriction and capacity, the sets of constraints are formulated. These sets of constraints can be categorized according to their purpose: assignment, capacity, and timing

- Assignment constraints
The first set of constraints, Eqs. (5), depicts that only one initial STAR fix can be assigned to each flight. Similarly, Eqs. (6) represents that only one time slot of TRACON boundary arrival fix can be occupied by each flight. In addition to this, the assigned initial STAR fix should match the corresponding TRACON boundary arrival fix. The set of Eqs. (7) is the assignment constraints for departing flights.

The next two sets of constraints, Eqs. (8) and (9), determine the time window of landing/take-off for arriving and departing flights, respectively.

- Capacity constraints
Each TRACON boundary arrival/departure fix has its own capacity, and the number of assigned flights during the given time window must be less than or equal to its capacity. This requirement is enforced by Eqs. (10) and (11). For the runway capacity constraints, we need to determine how many arrivals and departures are allowed during the given time window. As mentioned in Section 3.1.2, the runway arrival/departure capacity depends on the airport capacity trade-off curve. For this capacity curve to be implemented in the model, the defined capacity decision variables, $r^A_n$ and $r^D_n$, are subject to the airport capacity trade-off curve by Eqs. (12) and (13). Note that Eqs. (13) consists of several inequality equations that use standard linearization technique to capture nonlinearity of airport capacity trade-off curve. The next two sets of constraints, Eqs. (14) and (15), account for how many flights are allowed to land and take off during the given time windows. Finally, the last two constraints require that actual landing/take-off be less than or equal to the runway capacity determined by the previous capacity curve constraints.
• Timing constraints

\[
\begin{align*}
\Delta \sum_{n \in N} (n-1)s_{in} & \leq t_i^R \leq \Delta \sum_{n \in N} ns_{in} \quad \forall i \in A \cup D \quad (18) \\
e_i^F & = \Delta \sum_{n \in N} \sum_{k \in F^A} (n-1)g_{ikn} - \sum_{l \in F^{AI}} T_i^O (l) f_{il}^{AO} + \sum_{l \in F^{AI}} T_i^O (l) f_{il}^{A} \quad \forall i \in A \quad (19) \\
t_i^F & = \Delta \sum_{n \in N} \sum_{k \in F^A} (n-1)g_{ikn} \quad \forall i \in A \quad (20) \\
t_i^R & = t_i^F - \sum_{n \in N} \sum_{k \in F^D} T_i^T (k) g_{ikn} \quad \forall i \in D \quad (21) \\
t_i^R & = \Delta \sum_{n \in N} \sum_{k \in F^D} (n-1)g_{ikn} \quad \forall i \in D \quad (22)
\end{align*}
\]

Equation (18) imposes landing/take-off time assignments based on \( t_i^R \) using the discretized time window. The next three constraints, Eqs. (19), (20), and (21), define ETAs, actual TRACON boundary arrival fix crossing times, and actual runway times for arriving flights, respectively. Similarly, Eqs. (23) and (24) define the actual TRACON boundary departure fix crossing times and actual take-off times for departing flights, respectively.

### 3.3.2 Runway Assignments Model (2nd Stage Model)

- \( A \) the set of arrivals
- \( R_{Arr} \) the set of runways for arrivals
- \( F_{Arr} \) the set of arrival fixes
- \( E_i^H \) estimates of emission index at holding area for flight \( i \in A \)
- \( E_i^S \) estimates of emission index on surface for flight \( i \in A \)
- \( F_i^H \) estimates of fuel flow rate at holding area for flight \( i \in A \)
- \( F_i^S \) estimates of fuel flow rate on surface for flight \( i \in A \)
- \( C_i^T \) estimates of emission during terminal transit for flight \( i \in A \)
$T^S_i(k)$ taxi time between runway and gate for flight $i \in A$, runway $k \in R_{Arr}$

$T^T_i(l, k)$ transit time for flight $i \in A$, runway $k \in R_{Arr}$, fix $l \in F_{Arr}$

$S^F_i$ scheduled time at arrival fix for flight $i \in A$

After solving the fix assignment problem, we need to address more precise schedule in terms of time resolution as flights approach to the TRACON. To achieve this goal, the runway assignment problem is formulated as a MIP model. To simplify the model, the following assumptions are made:

1. Aircraft must follow pre-determined transition paths and procedures within the TRACON.

2. Aircraft are not allowed to change their paths once they begin to fly the assigned path.

3. Within TRACON area, the trailing aircraft are not allowed to overtake the leading aircraft.

4. The altitude at crossing point in the transit path satisfies the vertical separation requirements.

3.3.2.1 Definitions of decision variables

To facilitate the formulation, the following decision variables are defined:

$t^R_i = \text{landing time for flight } i \in A$

$t^F_i = \text{arrival fix crossing time for flight } i \in A$

$p_{ij} = \begin{cases} 
1 & \text{if aircraft } i \text{ arrives at runway before aircraft } j \quad i, j \in A \\
0 & \text{otherwise}
\end{cases}$

$q_{ij} = \begin{cases} 
1 & \text{if aircraft } i \text{ crosses at arrival fix before aircraft } j \quad i, j \in A \\
0 & \text{otherwise}
\end{cases}$
\[ r_{ik} = \begin{cases} 
1 & \text{if aircraft } i \text{ is assigned to runway } k \quad i \in A, k \in R_{Arr} \\
0 & \text{otherwise} 
\end{cases} \]

where \( t^R_i \) and \( t^F_i \) are continuous variables and \( p_{ij}, q_{ij}, \) and \( r_{ik} \) are binary variables. For comparison purposes, three different scenarios in Section 3.5.2 are studied by excluding some of the variables in the decision set. For instance, the FCFS policy is enforced by setting the variables \( p_{ij} \) and \( q_{ij} \) based on a given schedule of arrivals and the nearest runway policy (i.e., in the baseline simulation) by fixing the variables \( r_{ik} \).

### 3.3.2.2 Constraints

Aircraft \( i \) can arrive in front of aircraft \( j \) or behind aircraft \( j \) at runways and fixes. If aircraft \( i \) arrives before aircraft \( j \), \( p_{ij} = 1 \), aircraft \( j \) automatically arrives after aircraft \( i \), \( p_{ji} = 0 \), and vice versa. In addition, each aircraft must be assigned only one runway. Eq. (25) represents these constraints mathematically.

\[
\begin{align*}
& p_{ij} + p_{ji} = 1, \quad \forall i, j \in A, \ i \neq j, \ p_{ij} \in \{0, 1\} \\
& q_{ij} + q_{ji} = 1, \quad \forall i, j \in A, \ i \neq j, \ q_{ij} \in \{0, 1\} \\
& \sum_{k \in R_{Arr}} r_{ik} = 1, \quad \forall i \in A, \ r_{ik} \in \{0, 1\}
\end{align*}
\]

Each aircraft must be separated from its preceding aircraft by at least the minimum required spacing. The most important constraints for fix and runway assignments are the minimum spacing requirements presented in Eq. (26) and Eq. (27). If both flights \( i \) and \( j \) are assigned to the same runway \( k \) and flight \( i \) arrives before flight \( j \), \( r_{ik}, r_{jk}, \) and \( p_{ij} \) are set equal to 1, which activates the second inequality. Otherwise, the first inequality is activated, accomplished by introducing a big dummy number \( M \) and imposing the separation rule regardless of the runway assignment and the flight sequence. \( S^R_{ij} \) and \( S^R_{ji} \) represent the minimum time spacing for aircraft pairs \( i - j \) and
on the same runway, respectively.

\[
\begin{align*}
    t^R_i - t^R_j & \geq S_{ji}^R - M (2 + p_{ij} - r_{ik} - r_{jk}), \quad \forall i, j \in A, \forall k \in R_{Arr}, \ i \neq j \\
    -t^R_i + t^R_j & \geq S_{ij}^R - M (3 - p_{ij} - r_{ik} - r_{jk}), \quad \forall i, j \in A, \forall k \in R_{Arr}, \ i \neq j
\end{align*}
\]  

(26)

Similar to runway spacing, minimum spacing for TRACON boundary arrival fixes is written as

\[
\begin{align*}
    f_{il} f_{jl} (t^F_i - t^F_j) & \geq f_{il} f_{jl} (S_{ji}^F - M q_{ij}), \quad \forall i, j \in A, \forall l \in F_{Arr}, \ i \neq j \\
    f_{il} f_{jl} (-t^F_i + t^F_j) & \geq f_{il} f_{jl} (S_{ij}^F - M (1-q_{ij})), \quad \forall i, j \in A, \forall l \in F_{Arr}, \ i \neq j
\end{align*}
\]  

(27)

where \( f_{il} \) is a known variable of an assigned fix and \( S_{ij}^F \) and \( S_{ji}^F \) are the minimum allowable interval between aircraft pairs \( i-j \) and \( j-i \) at the same arrival fix. Because the separation constraints are expressed in time, each required separation distance must be converted to a corresponding interval. The time constraints for arrival fixes are straightforward and determined by the minimum separation distance at the fix entry points divided by the aircraft speed. Calculating the required time of separation on the runway is more complicated. The runway threshold separation requirements in the distance between each arrival pair are listed in Table 2. The approach/landing speeds for each weight class of aircraft are summarized in Table 3. Given the speed of the leading aircraft, \( v_i \), the speed of the trailing aircraft, \( v_j \), the distance from the FAF to the runway, \( l \), and the minimum separation distance, \( d_{ij} \), the time separation is calculated by the following equations.

\[
S_{ij}^R = \begin{cases} 
    \frac{d_{ij} + l}{v_j} - \frac{l}{v_i} & \text{if } v_i \geq v_j \\
    \frac{d_{ij}}{v_j} & \text{if } v_i < v_j
\end{cases}
\]  

(28)

For arriving flights, the landing time is determined by adding a pre-calculated transition time between the given TRACON boundary arrival fix and the assigned runway threshold to the given TRACON boundary arrival fix crossing time. \( T^T_i (l, k) \) is a
Table 2: Minimum separation requirements in distance

<table>
<thead>
<tr>
<th>(NM)</th>
<th>Heavy</th>
<th>B757</th>
<th>Large</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>B757</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Large</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>Small</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 3: Approach speed by weight class of aircraft

<table>
<thead>
<tr>
<th>(knot)</th>
<th>Heavy</th>
<th>B757</th>
<th>Large</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach Speed</td>
<td>150</td>
<td>130</td>
<td>130</td>
<td>90</td>
</tr>
</tbody>
</table>

pre-calculated table of transition time for the TRACON boundary fix $l$ and runway $k$ pair.

$$t_i^R - t_i^F = \sum_{k \in R_{Arr}} \sum_{l \in F_{Arr}} T_i^{T}(l, k) f_{dl} r_{ik}, \forall i \in A$$  \hspace{1cm} (29)

For practical purpose flight information is updated at a given cycle and an initial delay constraint must be imposed so that the optimization model will not assign more flights to the congested runway. The following constraint requires that no flight be scheduled on runway $k$ before initial delay $d_k$.

$$t_i^R \geq \sum_{k \in R_{Arr}} d_k r_{ik}, \forall i \in A$$  \hspace{1cm} (30)

3.3.2.3 Objective functions

Given an airport configuration and flight schedule, the objective function of the MIP model, which minimizes the total emissions for arrivals in the terminal airspace and on the airport surface, is presented in Eq. (31). Total emissions consist of three types: those produced during TRACON transition from the TRACON boundary fix to the runway, those produced during surface operations from the runway to the gate, and those resulting from airborne queuing delays at the TRACON boundary arrival fix. $C_i^{T}$ represents the estimate of emissions during TRACON transition.
The emissions for airborne queuing and surface operations are calculated by emission index multiplying fuel flow rate and time in that mode. $E^H_i$ and $F^H_i$ in Eq. (31) denote the emission index and the fuel flow rate during the airborne holding of flight $i$, respectively, while $E^S_i$ and $F^S_i$ represent the corresponding values on the surface. In addition, $S^F_i$ is the estimated time of arrival (ETA) at the TRACON boundary arrival fix, which is used to calculate the queuing delays at the TRACON boundary fix. Similar to $T^T(l,k)$, $T^S_i(k)$ is the unimpeded taxi time for flight $i$ from runway $k$ to its gate. The formulation for departures is created by replacing the set of arrivals with the set of departures and parameters associated with arrival operations with those associated with departure operations. The method for calculating emissions is explained in detail in the next section.

$$\min \left\{ \sum_{i \in A} C^T_i + \sum_{i \in A} E^H_i F^H_i (t^F_i - S^F_i) + \sum_{i \in A} \sum_{k \in R_{Arr}} E^S_i F^S T^S_i (k) r_{ik} \right\}$$

(31)

3.4 Cost Functions

3.4.1 Cost Functions with Environmental Impacts

Proponents of the Next Generation Air Transportation System (NextGen) list reduced environmental impact, in particular, the reduction of gaseous emissions, as a critical motivator for the development of the proposed “new and improved” air transportation system. Their rationale is as follows. The production of greenhouse gases is directly related to the amount of fuel burned. Approximately 3.5% of total greenhouse gas emissions can be attributed to the aviation industry. In the worst case scenario in which the aviation industry does not take positive action to reduce aircraft/engine emissions, the total could reach 15% by the year 2050 [55]. Engine emissions include nitrogen oxides NO$_x$, sulfur dioxides SO$_2$, and carbon oxides CO$_x$. NO$_x$ is the primary cause of smog, contributing to the formation of condensation trails, cirrus clouds, and acid rain. The impact of NO$_x$ is estimated to be two to four
times as great as that of $CO_2$, and its impact at high altitudes (8-13km) is even greater than at low altitudes [56]. In addition, engine emissions substantially impact local air quality. The amount of engine emissions produced depends on the engine power setting, the duration of operations at that power setting, and the altitude. Based on the International Civil Aviation Organization (ICAO) engine emissions databank, $NO_x$ production per unit time (the $NO_x$ index) is greatest during the take-off phase while the corresponding $HC$ and $CO$ indices are greatest during ground taxi operations [5]. Thus, a reduction in delays during any phase of a flight will reduce the quantity of emissions produced by an aircraft.

Emissions are dependent on engine power settings and resulting fuel burn. Only three segments of the generic aircraft mission profile shown in Fig. 11—descent, approach, and taxi-in— are relevant to arrival operations while taxi-out, take-off, and initial climb-out are relevant to the analysis of departure emissions. Since each flight mode can have various engine power settings and vertical profiles, the associated emissions are calculated separately by the following equations.

$$E = t_j \times f_j \times \sum_k EI_{jk}$$  (32)
Table 4: Sample of ICAO’s databank

<table>
<thead>
<tr>
<th>Engine Identification: AE3007A1/1</th>
<th>Take-Off</th>
<th>Climb-Out</th>
<th>Approach</th>
<th>Idle/Taxi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emission Index for $HC$(g/kg)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>3.88</td>
</tr>
<tr>
<td>Emission Index for $CO$(g/kg)</td>
<td>0.74</td>
<td>0.55</td>
<td>6.8</td>
<td>40.07</td>
</tr>
<tr>
<td>Emission Index for $NO_x$(g/kg)</td>
<td>16.10</td>
<td>14.01</td>
<td>7.12</td>
<td>4.17</td>
</tr>
<tr>
<td>Fuel Flow Rate(kg/sec)</td>
<td>0.3805</td>
<td>0.3163</td>
<td>0.1125</td>
<td>0.0459</td>
</tr>
</tbody>
</table>

$$E_j:$$ emissions during mode $j$, (g)

$$t_j:$$ time in mode $j$, (sec)

$$ff_j:$$ mode-specific fuel flow rate in mode $j$, (kg/sec)

$$EI_{jk}^k:$$ emission index for pollutant $k$ in mode $j$, (g/kg)

Accurate fuel flow rates and emission indices are crucial to obtaining accurate emission estimates. This section describes how to use the publicly available fuel flow rates and emission indices found in the ICAO Aircraft Engine Emissions Databank [2, 5] to calculate emissions. Typical fuel flow rates and emission indices from the ICAO databank are shown in Table 4. Even though the ICAO databank provides sufficient data for computing emissions, it does not cover the entire range of engine power settings that are used during terminal airspace and airport surface operations. Thus, the BADA is used to compute the fuel flow rate for descent and the emission indices obtained for each flight condition (e.g., altitude and speed) by Boeing Method II (BM II) to compute the amount of generated emissions [15].

BADA has been developed as an aircraft performance model by EUROCONTROL. According to EUROCONTROL, the database includes data for over one hundred aircraft types and uses information from both flight and operating manuals. Thus, with BADA, the fuel flow rate for the descent mode can be extracted for a specific flight condition by computing the relationship between the mode and engine thrust. Even though BADA and ICAO data cover the entire range of engine
operating conditions, recent studies have shown that BADA fuel flow estimates differ from manufacturer data at low altitudes. By comparing BADA estimates with airline fuel consumption data obtained from Flight Data Recorder (FDR) system, Senzig et al. [59] showed significant differences between estimated and actual fuel consumption below 10,000 feet above field elevation (AFE). Robinson and Kamgar-pour [57] have also reported significant inconsistencies between BADA predictions and the published manufacturer values for fuel burn from the aircraft operating manuals of several Boeing and Airbus aircraft. To correct these inconsistencies in fuel flow estimates computed using BADA, we implemented Robinson’s calibrating methodology. Although Senzig’s approach could be implemented using data published by the U.S. Department of Transportation’s Volpe Center (Volpe), the lack of accessible data for a variety of Boeing aircraft was a major obstacle to implementation [33].

Airborne holding and surface operation emission indices and fuel flows, \( E_i^H, F_i^H, E_i^S \) and \( F_i^S \) in the objective function are obtained by using BADA, BM II, and Robinson’s methodology. Because operating conditions vary over time during descent, approach, take-off, and climb-out, emissions can be computed using the following formula:

\[
C_i^T = \sum_{k \in R_{Ar}} \sum_{l \in F_{Ar}} \sum_{t_{lk}=0}^{T^T(t_{lk})} f_{ilr_{ik}} \sum t_{lk} = 0 \quad E_i^T(t_{lk}) F_i^T(t_{lk}) time_{step}, \quad \forall i \in A
\]  

(33)

Given the assumptions defined in the previous section, \( E_i^T(t_{lk}) \) and \( F_i^T(t_{lk}) \) can be estimated along the transition path.

Similarly, after obtaining the fuel flow rate, \( C_i^I(k) \) and \( C_i^O(l) \) of fix assignments model are calculated by Eqs. (34) and (35). To minimize generated pollutants, we substitute Eqs. (36) and (37) for Eqs. (34) and (35) while delay costs in Eqs. (2) are obtained by multiplying delays by \( E_i^{hold} \) and \( E_i^{idle} \) because the holding area and
### Table 5: Representative schedule at ATL

<table>
<thead>
<tr>
<th>Day</th>
<th>September 30th, 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runway configuration</td>
<td>8L, 9R</td>
</tr>
<tr>
<td>The number of arrival flights</td>
<td>869</td>
</tr>
<tr>
<td>Schedule hour</td>
<td>07:00~22:00</td>
</tr>
</tbody>
</table>

Surface operations can be assumed as stationary operations.

\[
C_I^I (k) = \sum_{t=0}^{T_I^I(k)} f f_i^T(t) \times time_{step} \tag{34}
\]

\[
C_O^I (l) = \sum_{t=0}^{T_O^I(l)} f f_i^T(t) \times time_{step} \tag{35}
\]

\[
C_I^O (k) = \sum_{t=0}^{T_I^O(k)} E_i(t) \times f f_i(t) \times time_{step} \tag{36}
\]

\[
C_O^O (l) = \sum_{t=0}^{T_O^O(l)} E_i(t) \times f f_i(t) \times time_{step} \tag{37}
\]

### 3.5 Results

#### 3.5.1 Fix Assignments Model (1st Stage Model)

The proposed first stage model for ATL is implemented using the fifteen-hour flight schedule on September 30th, 2005 from Aviation System Performance Metrics (ASPM) as a representative schedule, presented in table 5. Note that the 5th runway, which is available now, was under construction on the given representative day. Because the publicly available airport capacity trade-off curve information is published based on the airport configuration of 2004 by FAA [1], we need to consider the schedule with that configuration, which is 4 parallel runways. As depicted in Fig. 12, FAA’s report provides capacity curves for instrument flight rules (IFR) and visual flight rules (VFR). For the historical data of the given day from ETMS, VFR is applied during all periods of the schedule.

To generate a baseline schedule, the flight data of ATL are extracted from the
ETMS database for each 30-minute window. Because an estimate of the TRACON boundary fix crossing times is not available in the ETMS database, the simulation estimated it based on the current positions of flights and the given STAR routes. Specifically, after considering every possible nominal path between a current position and the initial entry fix points of each STAR, each flight is assigned to the nearest initial entry fix of each STAR as well as to the associated TRACON boundary arrival fix. Here, a 30-minute period is chosen as the time duration for each window for the following reasons. Assigning a different TRACON boundary arrival fix near airports will cause significant extra fuel consumption. Therefore, we need to make a decision in an earlier phase, such as the cruise phase, rather than during the descent phase, before the flight crosses the TOD points. To ensure efficient assignments, the threshold distance, 200 nautical miles (NM), is defined and the flights inside the threshold area are excluded in the assignment problem, while we need to include those flights in timing problems for capacity consideration. In addition, because the cruise speed of most commercial aircraft does not exceed 500 knots, generally, in true airspeed (TAS), this 30-minute time window can cover our areas of interest, which are up to 500 NM from the destinations.

Figure 12: ATL arrival-departure capacity trade-off curve (from FAA’s Airport Capacity Benchmark Report 2004)
To evaluate the performance of the proposed model, we investigate the following two simulations:

1. Baseline simulation

2. Fix assignment (FA)

The baseline simulation produces the conventional fix assignments in order to compare them with the proposed optimization results. For the given fix assignments and capacity, the baseline simulation determines the timing decision variables to satisfy the capacity constraints at each arrival fix and each runway. The second simulation is an optimization algorithm model of fix assignments that considers every possible path combination from a current position to the initial entry fix for the STARs and assigns a TRACON boundary fix as well as a time slot to each flight. The proposed model automatically attempts to find a minimum fuel or emission cost solution even if this requires longer transition paths for some flight. In the following discussion, the fix assignment simulation is referred to as FA.

As previously mentioned, the proposed model takes into account a 30-minute rolling planning window. In this time configuration, some flights in one current time window could already be considered in the optimization of a previous time window if the flight has a slower cruise speed. To prevent duplicated assignments, those flights are considered only for capacity constraints and timing constraints in the next time window without reassigning the fix and the associated time slot. For the TRACON boundary arrival fix of ATL in the optimization, four arrival fixes are considered: DIRTY for the flow from the northeast, ERLIN for the flow from the northwest, HONIE for the flow from the southwest, and CANUK for the flow from the southeast.

3.5.1.1 Minimizing fuel cost

As key elements of this part, the consumed fuel costs of the entire operations from the current position of each flight to the destination are considered in the mathematical
Table 6: The average delays in seconds of baseline and optimization for fuel minimization

<table>
<thead>
<tr>
<th>Hour</th>
<th># of flight</th>
<th>Delay Baseline</th>
<th>FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>07:00~07:30</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>07:30~08:00</td>
<td>4</td>
<td>55.15</td>
<td>86.79</td>
</tr>
<tr>
<td>08:00~08:30</td>
<td>8</td>
<td>62.26</td>
<td>62.26</td>
</tr>
<tr>
<td>08:30~09:00</td>
<td>14</td>
<td>71.46</td>
<td>68.20</td>
</tr>
<tr>
<td>09:00~09:30</td>
<td>3</td>
<td>84.73</td>
<td>80.55</td>
</tr>
<tr>
<td>09:30~10:00</td>
<td>9</td>
<td>111.60</td>
<td>106.93</td>
</tr>
<tr>
<td>10:00~10:30</td>
<td>16</td>
<td>74.65</td>
<td>59.88</td>
</tr>
<tr>
<td>10:30~11:00</td>
<td>34</td>
<td>109.12</td>
<td>79.39</td>
</tr>
<tr>
<td>11:00~11:30</td>
<td>37</td>
<td>86.58</td>
<td>84.65</td>
</tr>
<tr>
<td>11:30~12:00</td>
<td>46</td>
<td>281.29</td>
<td>343.29</td>
</tr>
<tr>
<td>12:00~12:30</td>
<td>29</td>
<td>331.98</td>
<td>284.59</td>
</tr>
<tr>
<td>12:30~13:00</td>
<td>28</td>
<td>94.31</td>
<td>75.23</td>
</tr>
<tr>
<td>13:00~13:30</td>
<td>32</td>
<td>101.23</td>
<td>67.77</td>
</tr>
<tr>
<td>13:30~14:00</td>
<td>27</td>
<td>90.79</td>
<td>81.48</td>
</tr>
<tr>
<td>14:00~14:30</td>
<td>26</td>
<td>100.69</td>
<td>71.06</td>
</tr>
<tr>
<td>14:30~15:00</td>
<td>34</td>
<td>69.63</td>
<td>65.41</td>
</tr>
<tr>
<td>15:00~15:30</td>
<td>35</td>
<td>106.76</td>
<td>92.23</td>
</tr>
<tr>
<td>15:30~16:00</td>
<td>44</td>
<td>179.95</td>
<td>105.27</td>
</tr>
<tr>
<td>16:00~16:30</td>
<td>33</td>
<td>116.93</td>
<td>102.96</td>
</tr>
<tr>
<td>16:30~17:00</td>
<td>36</td>
<td>135.77</td>
<td>118.83</td>
</tr>
<tr>
<td>17:00~17:30</td>
<td>40</td>
<td>152.61</td>
<td>79.18</td>
</tr>
<tr>
<td>17:30~18:00</td>
<td>37</td>
<td>90.18</td>
<td>100.06</td>
</tr>
<tr>
<td>18:00~18:30</td>
<td>29</td>
<td>117.01</td>
<td>89.48</td>
</tr>
<tr>
<td>18:30~19:00</td>
<td>42</td>
<td>76.00</td>
<td>65.17</td>
</tr>
<tr>
<td>19:00~19:30</td>
<td>53</td>
<td>284.38</td>
<td>208.75</td>
</tr>
<tr>
<td>19:30~20:00</td>
<td>37</td>
<td>328.09</td>
<td>422.95</td>
</tr>
<tr>
<td>20:00~20:30</td>
<td>38</td>
<td>284.97</td>
<td>264.55</td>
</tr>
<tr>
<td>20:30~21:00</td>
<td>34</td>
<td>138.94</td>
<td>116.96</td>
</tr>
<tr>
<td>21:00~21:30</td>
<td>19</td>
<td>83.03</td>
<td>82.93</td>
</tr>
<tr>
<td>21:30~22:00</td>
<td>45</td>
<td>129.09</td>
<td>88.56</td>
</tr>
</tbody>
</table>

Total 869 256.13 227.13
formulation. As noted in previous section, three operational costs are taken into account as the performance index: the outer transition costs from the current position to the TRACON boundary fixes, the queuing delay costs at the TRACON boundary fix, and the TRACON transition costs from the TRACON boundary fixes to the runways.

Table 6 and Fig. 13 present the assignment results and corresponding delays. Clearly, the TRACON boundary fixes at north of ATL, DIRTY and ERLIN, are much busier than the TRACON boundary fixes at south of ATL, HONIE and CANUK. More specifically, the northeast post boundary fix, DIRTY, is busy during all of the simulation time periods, and the traffic stream from the northwest post boundary fix, HONIE, reaches its peak between 19:00 and 20:00 due to the traffic volumes from the western U.S. states. Interestingly, the proposed model assigns more flights to the fixes located at the western TRACON boundary of ATL for the following reason. The model attempts to find the optimal solution that minimizes the total fuel consumed during the terminal operations even though the ground path for some arrivals is longer than the baseline ground path. Because the east operational configuration was active for the given schedule, the traffic streams coming from the east should be assigned the level-off segment which is parallel with the runways. This segment requires much

![Figure 13: Assigned arrival fix for minimizing fuel](image)

49
more fuel due to the level-off operation at the low altitude. Thus, the model tries to assign more flights to the western TRACON boundary arrival fixes, which lengthens their cruise paths, but eliminates any level-off path if the longer cruise path requires less fuel than a closer TRACON boundary arrival fix would require. The total average queuing delays at the TRACON boundary fixes in the model declined to 88.7% of the baseline simulation.

Fig. 14 represents the total consumed fuel costs for each time window, and these values are normalized by the maximum fuel costs among all the time windows. This shows that the proposed model results in efficient fix assignments requiring less fuel than baseline simulation assignments in all the time windows. Because VFR is active for all time periods, few benefits from the reduction of delays at TRACON boundary arrival fixes are found. As a result, the fuel savings associated with the decreased

![Figure 14: Normalized total fuel for each time window](image)
delays are also limited for the given traffic demand and capacity conditions. Overall fuel consumption declined to 95.76% of the baseline.

3.5.1.2 Minimizing emission cost

To evaluate the model with respect to emission minimization, the objective function with emission costs is used as explained in previous section. Table 7 and Fig. 15 reflect the result of the emission minimizing optimization. Similar to minimization of fuel consumption, the optimization results show that the model reduces delays and corresponding emissions. In terms of assignment results, the proposed model assigns more flights to the TRACON boundary fixes located at the western part of ATL for the same reasons that the model for minimum fuel simulation assigned flights to the west. In general, $NO_x$ and $CO_2$ are proportional to fuel consumption, whereas $HC$ and $CO$ are inversely proportional to fuel consumption. Therefore, $HC$ and $CO$ are likely caused primarily by the taxi operation on surface. Because this model does not account for surface operation costs, most emission costs are presented as resulting from $NO_x$ and $CO_2$. As a result, the emission costs of the operations in the given scope could be considered as weighted fuel costs, and this is a reason why the assignment result and the associated delays are very similar with results of the simulation for minimum fuel. Fig.16 also reveals results that are similar to those of

![Figure 15: Assigned arrival fix for minimizing emission](image-url)
Table 7: The average delays in seconds of baseline and optimization for emission minimization

<table>
<thead>
<tr>
<th>Hour</th>
<th># of flight</th>
<th>Delay Baseline</th>
<th>Delay FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>07:00~07:30</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>07:30~08:00</td>
<td>4</td>
<td>55.15</td>
<td>55.15</td>
</tr>
<tr>
<td>08:00~08:30</td>
<td>8</td>
<td>62.26</td>
<td>47.63</td>
</tr>
<tr>
<td>08:30~09:00</td>
<td>14</td>
<td>71.46</td>
<td>76.66</td>
</tr>
<tr>
<td>09:00~09:30</td>
<td>3</td>
<td>84.73</td>
<td>80.55</td>
</tr>
<tr>
<td>09:30~10:00</td>
<td>9</td>
<td>111.60</td>
<td>127.54</td>
</tr>
<tr>
<td>10:00~10:30</td>
<td>16</td>
<td>74.65</td>
<td>55.49</td>
</tr>
<tr>
<td>10:30~11:00</td>
<td>34</td>
<td>109.12</td>
<td>77.48</td>
</tr>
<tr>
<td>11:00~11:30</td>
<td>37</td>
<td>86.58</td>
<td>94.17</td>
</tr>
<tr>
<td>11:30~12:00</td>
<td>46</td>
<td>278.03</td>
<td>345.08</td>
</tr>
<tr>
<td>12:00~12:30</td>
<td>29</td>
<td>331.98</td>
<td>285.57</td>
</tr>
<tr>
<td>12:30~13:00</td>
<td>28</td>
<td>94.31</td>
<td>81.31</td>
</tr>
<tr>
<td>13:00~13:30</td>
<td>32</td>
<td>91.86</td>
<td>78.83</td>
</tr>
<tr>
<td>13:30~14:00</td>
<td>27</td>
<td>90.79</td>
<td>82.76</td>
</tr>
<tr>
<td>14:00~14:30</td>
<td>26</td>
<td>100.69</td>
<td>65.29</td>
</tr>
<tr>
<td>14:30~15:00</td>
<td>34</td>
<td>69.63</td>
<td>65.13</td>
</tr>
<tr>
<td>15:00~15:30</td>
<td>35</td>
<td>106.76</td>
<td>92.89</td>
</tr>
<tr>
<td>15:30~16:00</td>
<td>44</td>
<td>179.95</td>
<td>104.70</td>
</tr>
<tr>
<td>16:00~16:30</td>
<td>33</td>
<td>116.93</td>
<td>113.99</td>
</tr>
<tr>
<td>16:30~17:00</td>
<td>36</td>
<td>135.77</td>
<td>131.94</td>
</tr>
<tr>
<td>17:00~17:30</td>
<td>40</td>
<td>156.36</td>
<td>100.00</td>
</tr>
<tr>
<td>17:30~18:00</td>
<td>37</td>
<td>90.18</td>
<td>104.51</td>
</tr>
<tr>
<td>18:00~18:30</td>
<td>29</td>
<td>117.01</td>
<td>88.59</td>
</tr>
<tr>
<td>18:30~19:00</td>
<td>42</td>
<td>79.57</td>
<td>64.74</td>
</tr>
<tr>
<td>19:00~19:30</td>
<td>53</td>
<td>284.38</td>
<td>244.08</td>
</tr>
<tr>
<td>19:30~20:00</td>
<td>37</td>
<td>340.25</td>
<td>400.56</td>
</tr>
<tr>
<td>20:00~20:30</td>
<td>38</td>
<td>288.92</td>
<td>251.57</td>
</tr>
<tr>
<td>20:30~21:00</td>
<td>34</td>
<td>138.94</td>
<td>121.42</td>
</tr>
<tr>
<td>21:00~21:30</td>
<td>19</td>
<td>83.03</td>
<td>86.72</td>
</tr>
<tr>
<td>21:30~22:00</td>
<td>45</td>
<td>129.09</td>
<td>85.73</td>
</tr>
<tr>
<td>Total</td>
<td>869</td>
<td>256.13</td>
<td>233.31</td>
</tr>
</tbody>
</table>
Figure 16: Normalized total emission for each time window

the simulation of the minimization of fuel consumption. Overall fuel consumption and delays declined to 96.75% and 91.09% of the baseline simulation.

3.5.2 Runway Assignments Model (2nd Stage Model)

As flights approach to the assigned TRACON boundary fix from the fix assignment model, the runway assignment model takes the place of optimizations. To demonstrate the application of the model, it is implemented at DTW, assigning flights to runways and optimizing arrival and departure operations. The runway layout of DTW is visualized in Fig. 17. For the numerical study, representative input data are used from the eight-hour flight schedule on January 10, 2007, from the Aviation System Performance Metrics (ASPM) database, summarized in Table 8. Runway configuration 21L,22R——21R,22L, in which runways 21L and 22R are dedicated to arrivals and 21R and 22L to departures, was in operation on that day. This configuration is
one of the most often used runway configurations in DTW operations. Because the assigned TRACON boundary arrival/departure fix and the time that flights cross the boundary fix are not available in the ASPM database, the simulation estimated nominal TRACON boundary arrival fix assignments for arrivals based on their airport of origin. Specifically, after creating a nominal path between the origin airport and DTW and assuming a great circle route, we mapped the heading to the corresponding arrival fix. The scheduled landing time is calculated by subtracting the unimpeded taxi-in time from the scheduled gate-in time while the scheduled time at the TRACON boundary arrival fix is estimated by subtracting the transition time between the TRACON boundary arrival fix and the nearest runway pair from the scheduled landing time. Information for departures was generated in a similar manner.

Figure 17: Runway layout of DTW
Table 8: Representative schedule at DTW

<table>
<thead>
<tr>
<th>Day</th>
<th>January 10, 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runway configuration</td>
<td>21L,22R, 21R,22L</td>
</tr>
<tr>
<td>The number of arrivals</td>
<td>331</td>
</tr>
<tr>
<td>The number of departures</td>
<td>310</td>
</tr>
<tr>
<td>Schedule hour</td>
<td>13:00~21:00</td>
</tr>
</tbody>
</table>

Table 9: Minimum separation requirements in seconds

<table>
<thead>
<tr>
<th></th>
<th>21L</th>
<th>22R</th>
</tr>
</thead>
<tbody>
<tr>
<td>(seconds)</td>
<td>Heavy</td>
<td>B757</td>
</tr>
<tr>
<td>Heavy</td>
<td>96</td>
<td>141.8</td>
</tr>
<tr>
<td>B757</td>
<td>96</td>
<td>110.8</td>
</tr>
<tr>
<td>Large</td>
<td>60</td>
<td>69.3</td>
</tr>
<tr>
<td>Small</td>
<td>60</td>
<td>69.3</td>
</tr>
</tbody>
</table>

As noted in Section 3.3.2, the most important requirement that every pair of consecutive aircraft must satisfy is that of minimum separation. The time separations in Table 9 are derived using the information in Table 2 and Table 3. Because the FAFs for runways are located at varying distances from DTW, the derived time separations also vary slightly. The aircraft data and the N-number field of ASPM data are used to determine the specified engine model from the FAA N-number database [4]. Then the emission index and the fuel flow rate are obtained from the ICAO Aircraft Engine Emissions Databank and BADA with correction by BM II and Robinson’s calibrating methodology, discussed in Section 3.4.1.

To evaluate the performance of the proposed model, we conducted the following three simulations.

1. Baseline simulation (the nearest runway to the arrival fixes except WEEDA to runways with FCFS policy)

2. Runway assignment with FCFS (RA/FCFS)

3. Runway assignment with sequencing (RA/SEQ)
The baseline simulation provides an estimate of the fuel consumed and the emissions produced in a conventional runway assignment. With a given runway configuration, each flight, except flights from the WEEDA arrival fix, is assigned to the nearest runway according to the FCFS rule based on wheels-on time. Since two paths from the WEEDA arrival fix to the runways are from the standard terminal arrival route (STAR) published by the FAA, flights from this fix are assigned to the runway with less traffic load.

To evaluate the effect of sequencing at arrival fixes and runways, two simulations with different sequencing algorithms are conducted. In the second simulation, flights are assigned to runways by minimizing total emissions, but without optimizations of the sequence at the TRACON boundary arrival fixes and runways, so the original arrival sequence retains. Thus, the only difference between the results of the optimization model and those of the baseline simulation is that the former attempted to find the minimum overall emissions solution by taking longer or shorter TRACON transition and taxi paths into account if they were able to help reduce overall emissions and congestion. Third simulation employed the fully functional model, which accounts for runway assignments and optimal sequencing to achieve greater reductions in emissions and delays. In the following discussion of this section, the full simulation is noted as RA/SEQ and the second simulation as RA/FCFS.

Because of practical implementation constraints, the optimization model uses a 30-minute rolling planning window and passes the delays of each runway and TRACON boundary fix to the next iteration in the form of additional constraints using Eq. (30). In addition, the impact of optimization is assessed by simulating arrival and departure operations, separately. According to the FAA, landings and take-offs may be conducted independently for a given runway configuration, so this optimization framework does not violate any operational constraints.
Table 10: The number of arrivals assigned to each runway and the average delays in seconds for Baseline, RA/FCFS, and RA/SEQ

<table>
<thead>
<tr>
<th>Hour</th>
<th># of flight</th>
<th>Baseline</th>
<th>RA/FCFS</th>
<th>RA/SEQ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>21L 22R</td>
<td>21L 22R</td>
<td>21L 22R</td>
</tr>
<tr>
<td>13:00~13:30</td>
<td>6</td>
<td>4 2   0.00</td>
<td>5 1   11.51</td>
<td>5 1   11.51</td>
</tr>
<tr>
<td>13:30~14:00</td>
<td>15</td>
<td>10 5  24.42</td>
<td>11 4 36.50</td>
<td>11 4 37.24</td>
</tr>
<tr>
<td>14:00~14:30</td>
<td>36</td>
<td>21 15 82.01</td>
<td>19 17 53.10</td>
<td>20 16 51.69</td>
</tr>
<tr>
<td>14:30~15:00</td>
<td>14</td>
<td>9 5  87.07</td>
<td>8 6  66.97</td>
<td>9 5  87.54</td>
</tr>
<tr>
<td>15:00~15:30</td>
<td>3</td>
<td>1 2   0.00</td>
<td>2 1  30.28</td>
<td>2 1  30.28</td>
</tr>
<tr>
<td>15:30~16:00</td>
<td>31</td>
<td>8 23 124.51</td>
<td>14 17 75.63</td>
<td>15 16 70.72</td>
</tr>
<tr>
<td>16:00~16:30</td>
<td>32</td>
<td>14 18 204.34</td>
<td>15 17 126.45</td>
<td>17 15 130.53</td>
</tr>
<tr>
<td>16:30~17:00</td>
<td>12</td>
<td>7 5  26.21</td>
<td>7 5  41.49</td>
<td>6 6  47.08</td>
</tr>
<tr>
<td>17:00~17:30</td>
<td>7</td>
<td>7 0   6.53</td>
<td>6 1  13.23</td>
<td>6 1  13.23</td>
</tr>
<tr>
<td>17:30~18:00</td>
<td>37</td>
<td>20 17 108.17</td>
<td>21 16 82.01</td>
<td>21 16 100.47</td>
</tr>
<tr>
<td>18:00~18:30</td>
<td>40</td>
<td>21 19 369.55</td>
<td>18 22 301.53</td>
<td>19 21 342.78</td>
</tr>
<tr>
<td>18:30~19:00</td>
<td>6</td>
<td>5 1  88.33</td>
<td>3 3  48.89</td>
<td>5 1  130.40</td>
</tr>
<tr>
<td>19:00~19:30</td>
<td>10</td>
<td>2 8   5.49</td>
<td>4 6  39.43</td>
<td>4 6  39.43</td>
</tr>
<tr>
<td>19:30~20:00</td>
<td>37</td>
<td>10 27 304.64</td>
<td>18 19 98.48</td>
<td>19 18 85.95</td>
</tr>
<tr>
<td>20:00~20:30</td>
<td>40</td>
<td>13 27 661.36</td>
<td>20 20 298.00</td>
<td>19 21 200.53</td>
</tr>
<tr>
<td>20:30~21:00</td>
<td>5</td>
<td>3 2  387.45</td>
<td>2 3  43.63</td>
<td>3 2  90.20</td>
</tr>
<tr>
<td>Total</td>
<td>331</td>
<td>155 176 224.56</td>
<td>172 159 125.79</td>
<td>179 152 129.12</td>
</tr>
</tbody>
</table>
**Figure 18:** The difference between the number of arrivals assigned to each runway

### 3.5.2.1 Emissions for arrivals

Similar to the first stage model, the objective function accounts for the total emissions produced by aircraft from the boundary of TRACON to the assigned gates. Thus, the emissions in the objective function consist of three parts from the aircraft operation modes: TRACON transit, airborne queuing delays, and taxi. $NO_x$ and $CO_2$ are proportional to fuel consumption, in general, while $HC$ and $CO$ are somewhat inversely proportional to fuel consumption, as listed in table 4. Because of the incomplete combustion, the majority of $HC$ and $CO$ emitted is from the taxi mode even though the fuel consumption is low in the taxi mode.

The results of this model for arrivals are presented in Table 10 and Fig. 18. The figure visualizes the difference of traffic volume between runways 21L and 22R for the
three simulations. An extremely unbalanced runway assignment occurred in the baseline simulation during 15:30∼16:00 and 19:30∼20:30, when most flights in the heavier traffic streams from the west were assigned to runway 22R based on the nearest runway assignment rule in the baseline simulation. Thus, runway 22R was overloaded, resulting severe congestion, whereas runway 21L was under-served. However, this biased runway assignment is resolved in the RA/FCFS and RA/SEQ with optimized runway assignment. The average delays in Table 10 is calculated by subtracting ETA at the runway from the wheel-on time of optimization result and then dividing the sum of those by the number of flights in the given time interval. In most time intervals, the average delays in RA/FCFS and RA/SEQ were less than those in baseline. Particularly in the time intervals with high traffic volume such as 15:30∼16:00 and 19:30∼20:30, the average delays decreased significantly when compared with the
baseline simulation. In certain time intervals with low traffic volume, however, the average delays of RA/FCFS and RA/SEQ were slightly higher than those of baseline. The total average delays of RA/FCFS and RA/SEQ declined to 56.0% and 57.5% of the baseline. Another finding is that the average delays of RA/SEQ were higher than those of RA/FCFS in certain time intervals. The reasons for this increase are two-fold: First, the proposed model attempts to find the optimal solution—one that minimizes total emissions even though for some arrivals their TRACON transit distance is longer than the baseline; thus, the wheels-on delay can increase. However, total emissions decrease as a result of reduced taxi time and queuing delays at the airspace boundary fix. Second, the initial conditions of each time interval differ because the delays in previous time interval propagate to current time interval.

As presented in Fig. 19, the emissions of RA/FCFS and RA/SEQ were significantly less than those of the baseline. The RA/SEQ produced the least amount of pollutants in all but two iterations, 16:30~17:00 and 18:30~19:00. During these two time intervals, the emission level of the RA/SEQ was slightly higher than that of the RA/FCFS because of the delays propagated from the previous time interval. The fully functional model produced the least amount of emissions during the entire simulation period. Overall emissions declined to 75.79% and 70.12% of the baseline for RA/FCFS and RA/SEQ, respectively.

To further investigate the effects of the sequencing algorithm, the results of time windows with representative of the balanced and unbalanced traffic volume are compared, as shown in Fig. 20. It is important to note that most savings in emissions result from the runway assignment for the unbalanced traffic stream case represented in Fig. 20 (b) and (c), while noting that effective sequencing can affect significantly to reduction of emissions for the balanced traffic case described in Fig. 20 (a). Among the three types of emissions, those from airborne queuing delays exhibit significant differences while those from surface operations and TRACON transit vary only slightly.
Specifically, the emissions from airborne queuing delays decreased significantly as a result of the balanced runway utilization. In addition, the emissions from surface operations decreased because of shorter taxi distance and less congestion on airport surface.

### 3.5.2.2 Emissions for departures

To evaluate the proposed model with regard to departures, it is implemented using the departure schedule and generated inputs using the same technique used for arrivals. The results are presented in Table 11 and Fig. 21. As was the case for arrivals, an imbalance in the traffic demand occurs, resulting in inefficient runway assignments in the baseline simulation. The proposed models address this issue, with the RA/SEQ providing the greatest reduction in emissions.

As summarized in Table 12, the benefits in terms of emissions reduction for arriving traffic is much greater than for departing traffic. After all, with much higher engine power settings, the fuel flow rate is greater during take-off and initial climb. Therefore, despite the significant delay decrease, the total emissions reduction is smaller than it is for arrivals. Although from the perspective of controllers, the baseline simulation is easier to implement in actual operations, significant delays and emissions result. Thus, the proposed model could be the basis for a decision support tool that
Table 11: The number of departures assigned to each runway and the average delays in seconds for the baseline, RA/FCFS, and RA/SEQ

<table>
<thead>
<tr>
<th>Hour</th>
<th># of flight</th>
<th>Baseline</th>
<th>RA/FCFS</th>
<th>RA/SEQ</th>
<th>Baseline</th>
<th>RA/FCFS</th>
<th>RA/SEQ</th>
<th>Baseline</th>
<th>RA/FCFS</th>
<th>RA/SEQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:00～13:30</td>
<td>8</td>
<td>2 6 4.28</td>
<td>2 6 4.28</td>
<td>2 6 4.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13:30～14:00</td>
<td>35</td>
<td>22 13 91.73</td>
<td>18 17 63.23</td>
<td>18 17 61.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:00～14:30</td>
<td>36</td>
<td>29 7 397.61</td>
<td>18 18 69.48</td>
<td>18 18 59.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:30～15:00</td>
<td>9</td>
<td>2 7 63.91</td>
<td>4 5 32.33</td>
<td>4 5 32.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15:00～15:30</td>
<td>23</td>
<td>11 12 28.70</td>
<td>11 12 28.70</td>
<td>11 12 28.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15:30～16:00</td>
<td>33</td>
<td>9 24 195.88</td>
<td>15 18 40.79</td>
<td>16 17 50.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:00～16:30</td>
<td>8</td>
<td>2 6 12.75</td>
<td>3 5 5.75</td>
<td>3 5 5.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:30～17:00</td>
<td>4</td>
<td>1 3 0.00</td>
<td>1 3 0.00</td>
<td>1 3 0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17:00～17:30</td>
<td>41</td>
<td>20 21 106.73</td>
<td>21 20 94.34</td>
<td>20 21 82.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17:30～18:00</td>
<td>27</td>
<td>14 13 24.99</td>
<td>13 14 24.99</td>
<td>14 13 23.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18:00～18:30</td>
<td>3</td>
<td>2 1 0.00</td>
<td>2 1 0.00</td>
<td>2 1 0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18:30～19:00</td>
<td>7</td>
<td>3 4 0.00</td>
<td>3 4 0.00</td>
<td>3 4 0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19:00～19:30</td>
<td>35</td>
<td>8 27 273.05</td>
<td>16 19 60.33</td>
<td>16 19 60.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19:30～20:00</td>
<td>34</td>
<td>9 25 534.97</td>
<td>14 20 30.51</td>
<td>14 20 33.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20:00～20:30</td>
<td>3</td>
<td>1 2 0.00</td>
<td>1 2 0.00</td>
<td>1 2 0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20:30～21:00</td>
<td>4</td>
<td>2 2 12.00</td>
<td>2 2 12.00</td>
<td>2 2 12.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>310</strong></td>
<td><strong>137 173 187.76</strong></td>
<td><strong>144 166 47.84</strong></td>
<td><strong>145 165 46.12</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 21: The difference between the number of departures assigned to each runway controllers could use to meet emissions and increased capacity goals of both NextGen and SESAR.

3.5.2.3 Discussion

When the RA/FCFS is used for arrivals, the average computation time for each window is 18.3 seconds; and when the RA/SEQ is used, it is 175.8 seconds. When the RA/FCFS is used for departures, the average runtimes for each time window is 4.7 seconds; and when the RA/SEQ is used, it is 207.1 seconds. These findings indicate that flight sequencing requires additional computation times because of the quadratic increase in the number of decision variables, $p_{ij}$ and $q_{ij}$; however, the number of decision variables for runway assignment $r_{ik}$ increases linearly. To examine computational complexity in terms of problem size, simulations with various time intervals are studied. The computation times for these simulations are presented in Table 13.
As shown previously, the results of the arrival operation show that the suggested optimization model improves runway utilization and minimizes the impact on the local environment, and the results of the departure operation show that it has less impact on the environment. The unimpeded taxi time is used as the basis for the surface fuel burn and emissions estimates in the optimization model. The literature has shown that because taxi operations, whether from pushback to take-off or from touchdown to the gate, exhibit great uncertainties, using the unimpeded taxi time may overestimate or underestimate the benefits [22, 25, 41]. To assess the influence of these surface movement uncertainties on the performance of our model, it is included as a component of a comprehensive model for the optimization of airport surface traffic in the presence of uncertainty (as part of a NASA contract). The flowchart shown in Fig. 24 describes how the simulation data were propagated and

Figure 22: Normalized total emissions for departures (note: emissions are normalized by the maximum emissions of all the time windows.)
Figure 23: Percentage of emissions for TRACON transit, surface operations, and queuing delays (note: emissions are normalized by the baseline value of each time window.)

Table 12: Emissions changes for different operations

| Compared with baseline | Arrivals | | | | |
|------------------------|----------|----------|----------|----------|
|                        |          | TRACON   | Surface  | Delay     | Total     |
| RA/FCFS                | 103.80%  | 97.81%   | 30.48%   | 75.79%   |
| RA/SEQ                 | 102.55%  | 96.43%   | 17.13%   | 70.12%   |

| Compared with baseline | Departures | | | | |
|------------------------|------------|----------|----------|----------|
|                        |            | TRACON   | Surface  | Delay     | Total     |
| RA/FCFS                | 100.17%    | 99.95%   | 25.42%   | 97.5%    |
| RA/SEQ                 | 100.14%    | 99.89%   | 20.64%   | 97.36%   |

updated within the system and which information was required to and generated at each optimization module. The core component of the simulation environment is the high-fidelity fast-time airport surface simulation, Airspace Concept Evaluation System (ACES) Terminal Model Enhancement (TME), developed by the Saab Sensis Corporation. ACES-TME is an enhancement of the ACES simulation platform developed by NASA. As shown in Fig. 24, runway assignment model outputs an optimal

Table 13: Average run time in seconds to simulate a given schedule of arrivals

<table>
<thead>
<tr>
<th>Simulation window size</th>
<th>RA/FCFS</th>
<th>RA/SEQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-minute</td>
<td>0.3</td>
<td>7.2</td>
</tr>
<tr>
<td>15-minute</td>
<td>2.8</td>
<td>13.7</td>
</tr>
<tr>
<td>20-minute</td>
<td>7.3</td>
<td>49.6</td>
</tr>
<tr>
<td>30-minute</td>
<td>18.3</td>
<td>175.8</td>
</tr>
</tbody>
</table>
solution for the given schedule generated by ACES-TME.

To investigate the effects of the runway assignment model, the runway assignment model is excluded from the whole simulation framework for a sub-set of the runs and then compared the results to those of the runs with the full optimization. A comparison of the results reveals that the proposed model has a positive effect on taxi operations for both arrivals and departures [37]. Specifically, average taxi out time and delay increase by approximately 1.5% and 12.3%, respectively, when the runway assignment model is excluded. For arrivals, average taxi in time and delay increase by approximately 10.9% and 6.3% respectively, without the optimization result of runway assignment model. These results indicate that the runway assignment model provides benefits in terms of reducing taxi time and delay for arrivals and departures as well as associated environmental benefits.
3.6 Review

In this chapter, the two-stage sequential framework is implemented to address the ATFM problem for the terminal airspace. Because of complexity of the problem, this decomposed approach provides great savings on not only solution time but also the ability to account the capacity and schedule changes of the later part resulting from weather changes, speed changes and path stretching of flights. Moreover, TRACON boundary fix assignments are highly coupled with runway assignments and runway sequence as presented in Eq. (27) and Eq. (29). To formulate this in forms of linear programming problems, we should introduce more variables to account products of these decision variables, as shown in Eq. (38) and Eq. (39), resulting in the huge increase of binary decision variables as well as solution time.

\[
\begin{align*}
\begin{cases}
    x_1 x_2 = y \\
    x_1, x_2, y \in \{0, 1\}
\end{cases}
\end{align*} \quad \iff \quad 
\begin{align*}
\begin{cases}
    y \leq x_1 \\
    y \leq x_2 \\
    y \geq x_1 + x_2 - 1
\end{cases}
\end{align*}
\tag{38}
\]

\[
\begin{align*}
\begin{cases}
    x_1 x_2 = y \\
    x_1 \in \{0, 1\} \\
    0 \leq x_2 \leq u \\
    0 \leq y \leq u
\end{cases}
\end{align*} \quad \iff \quad 
\begin{align*}
\begin{cases}
    y \leq ux_1 \\
    y \leq x_2 \\
    y \geq x_2 - u (1 - x_1)
\end{cases}
\end{align*}
\tag{39}
\]

To investigate how the decomposed model works compared with the combined optimization, we can take a simple case of nine flights with ETAs that is densely packed as given in table 14. For ease of comparison, all aircraft are assumed same weight class. In addition, total flight times from current positions to runways are used as an unified cost function for each stage to avoid disagreement of optimization
Table 14: Sample arrival schedule of ATL

<table>
<thead>
<tr>
<th>Flight index</th>
<th>Class</th>
<th>fix</th>
<th>ETA at fix</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1</td>
<td>Large</td>
<td>DIRTY</td>
<td>100</td>
</tr>
<tr>
<td>f2</td>
<td>Large</td>
<td>ERLIN</td>
<td>140</td>
</tr>
<tr>
<td>f3</td>
<td>Large</td>
<td>DIRTY</td>
<td>150</td>
</tr>
<tr>
<td>f4</td>
<td>Large</td>
<td>HONIE</td>
<td>160</td>
</tr>
<tr>
<td>f5</td>
<td>Large</td>
<td>CANUK</td>
<td>180</td>
</tr>
<tr>
<td>f6</td>
<td>Large</td>
<td>ERLIN</td>
<td>190</td>
</tr>
<tr>
<td>f7</td>
<td>Large</td>
<td>HONIE</td>
<td>200</td>
</tr>
<tr>
<td>f8</td>
<td>Large</td>
<td>DIRTY</td>
<td>210</td>
</tr>
<tr>
<td>f9</td>
<td>Large</td>
<td>DIRTY</td>
<td>250</td>
</tr>
</tbody>
</table>

objectives. Solving the decomposed problem are as follows:

- Based on the given flight schedule, the first stage model calculates the extra flight time for each flight when taking an alternative TRACON boundary fix.
- Using the computed travel time, the first stage model finds TRACON boundary fix assignments minimizing the unified cost function.
- The second stage model computes new ETA for each flight based on the assigned TRACON boundary fix. In here, we assume that there is no uncertainty during transition from current positions to assigned TRACON boundary fixes.
- Based on this newly calculated ETA, the second stage model optimize runway assignments and runway schedule minimizing the total travel time between TRACON boundary fixes and runways as well as airborne queuing delays at TRACON boundary fixes.

For the combined optimization, the technique of Eq. (38) and Eq. (39) are utilized in order to express nonlinear terms of Eq. (29) and Eq. (27) as linear decision variables. Even though it is a small terminal airspace problem compared to the numerical example addressed in previous section with respect to the number of flights, two approaches provide the same solutions as presented in table 15. This result does not
Table 15: Results of the decomposed and combined optimization

<table>
<thead>
<tr>
<th>Flight index</th>
<th>assigned fix</th>
<th>assigned runway</th>
<th>fix time</th>
<th>runway time</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1</td>
<td>DIRTY</td>
<td>100</td>
<td>8L</td>
<td>600</td>
</tr>
<tr>
<td>f2</td>
<td>ERLIN</td>
<td>140</td>
<td>8L</td>
<td>380</td>
</tr>
<tr>
<td>f3</td>
<td>DIRTY</td>
<td>170</td>
<td>8L</td>
<td>670</td>
</tr>
<tr>
<td>f4</td>
<td>HONIE</td>
<td>160</td>
<td>9R</td>
<td>400</td>
</tr>
<tr>
<td>f5</td>
<td>CANUK</td>
<td>180</td>
<td>9R</td>
<td>680</td>
</tr>
<tr>
<td>f6</td>
<td>ERLIN</td>
<td>210</td>
<td>8L</td>
<td>450</td>
</tr>
<tr>
<td>f7</td>
<td>HONIE</td>
<td>230</td>
<td>9R</td>
<td>470</td>
</tr>
<tr>
<td>f8</td>
<td>DIRTY</td>
<td>240</td>
<td>8L</td>
<td>740</td>
</tr>
<tr>
<td>f9</td>
<td>DIRTY</td>
<td>310</td>
<td>8L</td>
<td>810</td>
</tr>
</tbody>
</table>

guarantee that the decomposed problem-solving framework provides an optimal solution. However, we would expect at least sub-optimal solution with the significantly reduced solution time.
CHAPTER IV

AIR TRAFFIC FLOW MANAGEMENT MODEL FOR EN ROUTE AIRSPACE

4.1 Limited Resources and CTOP

Major concerns in the field of the ATFM are how to balance demand with the given system capacity. As discussed in the section of the terminal airspace optimization, a part of flights should be delayed or shunted to absorb expected terminal airspace congestions. However, in real world, it is extremely difficult to issue these delays and detours to a part of flights due to fairness issue. As a result, the FCFS policy is used in the actual field because it is the fairest rule when considering sequencing problems, in general. In spite of this fairness issue, we know the FCFS policy is not the best one in terms of maximizing utilization. One way to resolve this drawback is to build a global collaborative community with a wide variety of stakeholders and create a new platform or framework under a global agreement with those stakeholders. The result of the efforts of this practice forms a CDM community which is a joint government/industry initiative aimed at improving the ATFM through increased information exchange among aviation community stakeholders such as government, general aviation, airlines, private industry and academia. Within this CDM concept, they work together to create technological and procedural solutions to the ATFM challenges faced by the NAS.

To minimize the impacts of en route capacity shortages, the FAA has developed the CTOP based on discussion with the CDM community. Under this CTOP framework, let’s take a look this problem from the perspective of flight operators. To minimize the impacts of CTOPs and maximize revenue we, as a flight operator, need to determine
Figure 25: Overview of CTOP Assignment Algorithm

a trajectory and the required delays while recognizing the competition with other flight operators. Estimating the costs associated with the available trajectory sets and determining the best TOS, however, is difficult due to the combinatorial nature of the decision process.

As described in Fig. 25, the FAA addresses the CTOPs on a flight by flight basis in order of an Initial Arrival Time (IAT). Specifically, the FAA determines; the IATs at all FCAs based on the trajectories in the TOS submitted by flight operators, sorts flights based on their earliest IAT, determines the best trajectory among the TOS submitted based on the required ground delays (calculated by the FAA), and the relative trajectory cost (RTC), which is submitted by flight operators, and then assigns flights to their best trajectory option minimizing the adjusted cost which is the sum of the required ground delays and the RTC. Unlike the RBS algorithm used in GDPs, a CTOP RBS algorithm has a slightly different rule that can lead
to significantly different assignment results because of the feature reflecting flight operators’ inputs in the form of the RTC during the slot assignment decision process.

For the sake of simplicity, let’s assume that there is only one FCA and that the given flight can transit through this FCA via two different trajectory options. Let’s also assume that trajectory option 1 requires a 30 minutes delay on the ground and that there are no required ground delays for trajectory option 2. Based on a typical RBS algorithm, the second trajectory option is a better option than trajectory 1 based on the delay. However, if trajectory option 1 produces a much lower operating cost such as transit time and fuel burn to the destination, which are enough to compensate for the 30 minutes delay, the air carrier would want the trajectory option 1 rather than the option 2. In this situation, the flight operators can advise the FAA that they prefer the trajectory option 1 by submitting a larger RTC value for the trajectory option 2 (i.e. that a RTC value is bigger than 30 minutes). As shown in this example, the RTC value submitted by the flight operators plays a key role in the assignment results by injecting their business preference to the FAA.

From the perspective of flight operators, there are three fundamental questions to be addressed in the CTOP process.

• What is the best set of slots within a CTOP that a NAS user will get under constrained capacity and competition with other NAS users?
• How can a NAS user obtain its best set of slots in a CTOP with the FAA’s CTOP RBS assignment algorithm?
• Which slot should be assigned to each flight to minimize the impacts of en route capacity and maximize revenue?

In order to answer these questions, we need to consider the following observations. First, a CTOP will only be initiated when the demand for a given airspace exceeds capacity such as that which occurs during severe weather conditions. Second, there is uncertainty with respect to the strategy employed by other flight operators. After
the FAA identifies constrained areas as well as their associated capacity and issues a CTOP, the available slots for the FCAs will be assigned to each flight based on the TOSs that have been submitted by flight operators. Let’s assume that other flight operators do nothing and submit only one most preferable trajectory for each flight. Let’s assume further that we decide to include a “NOSLOT” trajectory option as one of $F_A$ flights’ TOS candidates, which is to fly out of the FCAs. Under these assumptions, other flight operators will obtain one slot for every flight; however, for a severe delay case $F_A$ flights are more likely to get a “NOSLOT” trajectory because these trajectories do not require any ground delays since they are not traversing any of the constrained areas. Moreover, if other flight operators submit their TOSs based, in some way, on a “greedy” strategy, $F_A$ flights are going to be assigned to more delayed slots or “NOSLOT” trajectories. To avoid these unnecessary slot losses during the CTOP assignment procedure, we first need to obtain as many as possible of the best available slots (less delay). Then, we can manipulate actual flight assignments to the obtained slots by swapping flights or submitting the “NOSLOT” trajectory option.

As mentioned, solving a CTOP problem from the perspective of a flight operator is difficult because of the combinatorial nature and existence of competition. Moreover, a solution can be problematic due to the potential size of the problem. The rationale for the CTOPs initiated by the FAA is to maximize the throughput of en route airspace over “multiple FCAs” by issuing both delays and re-routes as illustrated in Fig. 26. It is possible that multiple FCAs will exist and to deal with this issue, we decompose the problem into two sequential sub-problems as shown in Fig. 27. The first stage model finds the best slot allocation that minimizes $F_A$’s cost under the given capacity constraints, the existence of competition with other flight operators, and a set of trajectory candidates. The second stage model finds the best slot assignments based on the results of the first stage solution. Since, in the scope of the proposed CTOPs, the FAA allows internal slot changes by swapping flights or submitting “NOSLOT”
trajectory options after the initial slot allocation, the first stage model plays the role of initial slot allocations in a CTOP, and the second stage model acts as a swapping algorithm. This decomposed approach provides benefits in terms of not only reduced computation time but also a more effective model capturing the CTOP RBS algorithm.

4.2 Two-Stage Combinatorial Model

\( F_{AS} \) the set of our flights with only one FCA entry

\( F_{AM} \) the set of our flights with multiple FCA entries

\( F_A \) the set of our flights, \( F_A = F_{AM} \cup F_{AS} \)

\( F_O \) the set of other flight operators’ flights

\( F \) the set of flights, \( F = F_{AM} \cup F_{AS} \cup F_O \)
Figure 27: Optimization Procedure

- $C$ the set of FCAs
- $I$ the time index set
- $R_f$ the set of routes for flight $f$
- $\Delta$ time interval for time slots
- $e_{f,c}^{\text{FCA}}$ the earliest nominal entry time at the FCA $c$ for flight $f \in F_{AM}$
- $e_f^{\text{FCA}}$ the earliest nominal entry time at the FCA for flight $f \in F_{AS} \cup F_{O}$
- $t_{f,r,c}^{\text{FCA}}$ nominal entry time at the FCA $c$ when the flight $f$ takes route $r$
- $t_{f,r}^{\text{dest}}$ nominal arrival time at destination when the flight $f$ takes route $r$
- $s_f^{\text{arr}}$ the scheduled arrival time for the flight $f$
- $IAT_f$ the earliest initial arrival time for flight $f$
- $l_{c,i}$ the capacity for slot $i$ of the FCA $c$
- $o_{c,i}$ the occupied capacity for slot $i$ of the FCA $c$ by exemption
- $h_{f,c}$ FCA flag, 1 if flight $f$ has a route passing through the FCA $c$, 0 otherwise
- $z_{f,r}^{\text{en-route}}$ en route cost of the route $r$ for the flight $f$
- $z_f^{\text{dep-delay}}$ departure delay cost for the flight $f$
- $z_f^{\text{arr-delay}}$ arrival delay cost for the flight $f$
- $M$ big dummy number
4.2.1 Optimal Slot Allocation Model (1st Stage Model)

4.2.1.1 Heuristic Approach

In the first stage model, we need to solve the following problem.

Given:
- A set of aircraft within a planning horizon
- The definition and capacity of FCAs determined by the FAA
- Schedule of flights
- A set of possible trajectories

Find an optimal FCA slot allocation that:
- minimizes total costs

By considering:
- competition with other NAS users
- possible trajectory combinations

As noted previously, the most important factor of this stage is consideration of competition with other flight operators. Unfortunately, other flight operators’ strategy and therefore their trajectory candidates are unknown, so to prevent unnecessary slot losses resulting in significant delays, a “greedy” algorithm is needed for the optimal slot allocation.
This decision process can be represented in form of a tree, however, a full branching at every trajectory for each flight would be extremely difficult to solve due to the huge number of combinations. To reduce the computational complexity, when multiple trajectories enter a single FCA, only the trajectory with the earliest feasible arrival at the FCA is considered. This is a valid approach because the FAA assigns a single available slot to each flight regardless of the number of submitted trajectories. Even if the earliest feasible arrival at the FCA produces a higher operating cost, this approach is still valid because we can change an actual slot assignment for each flight in the later stage. Let’s assume that we have only two flights in a CTOP and that flight 1 and 2 both have a total of n trajectory candidates as illustrated in Fig. 28. Let’s also assume that both flights have k FCA entries, where k is the number of FCAs in the CTOP. While the number of possible trajectory combinations is $n \times n = n^2$, the proposed approach results in $k^2$ combinations. Because the typical number of trajectory candidates is large, e.g. 30 or higher number, and k is a relatively small number, e.g. 2 or 3, this approach promises significant savings in terms of the number of possible combinations.

To reduce the computational complexity for a “greedy” slot acquisition, the following basic rules for the selection of trajectory candidate are utilized:

- To obtain a better assignment order in a CTOP, the TOSs will contain for each flight, one trajectory associated with the earliest IAT among the given trajectory candidate database.
- To maximize the number of assigned slots, the TOSs will not contain a “NOSLOT” trajectory option.

With the above basic rules, the following heuristic tree search algorithm is developed which complies with the FAA’s CTOP RBS policy.

This heuristic algorithm is motivated by the fact that, in general, the best slot would be earliest arrival time at the FCAs with regard to reducing delay; however,
**Heuristic Algorithm**

Tree traversal

Sort flights based on the earliest IAT regardless of the FCAs

\[ f = 0 \]

while \((f \neq \text{end of a planning horizon})\) do

  if all trajectories of a flight \(f\) enter the same FCA then
    Do a slot allocation based on greedy strategy (the earliest feasible slot)
  else
    Branch further by investigating the earliest feasible slot for each route and choosing the earliest arrival for each FCA as a candidate for slot allocations
  end if

  \[ f = f + 1 \]

end while

Choose the best trajectory combination which has the minimum cost.

There are instances where the earliest arrival would not be the best when multiple FCAs exist. To replicate the CTOP RBS algorithm and to find an optimal solution, our heuristic algorithm uses the flight schedule as the basis for the planning horizon, and traverses the tree generated by the previous reduced tree approach shown in Fig. 28. Each planning horizon starts from a flight which has multiple FCA entries, and ends when the flight has only one FCA entry. Obviously, a long planning horizon requires more time to solve, but the trade-off is that it produces a higher quality solution. After sorting flights by the earliest IAT as the FAA’s CTOP assignment order, the algorithm branches at a node (a flight) which has multiple FCA entries. This branching approach will capture the combinatorial nature of the problem to minimize the total delay costs. During tree traversal, if a current flight belongs to other flight operators or belongs to \(F_A\) with only one FCA entry for complete trajectory database, the algorithm executes a slot allocation based on a “greedy” strategy (the earliest slot allocation). Otherwise, it will find the earliest feasible slot for the entire trajectory database and select the earliest for each FCA as candidates for slot allocation. After calculating the total required costs for every combination, it chooses the allocation combination that minimizes the delay costs.
Table 16: Each Flight’s Earliest Entry Time at Each FCA for Example 1

<table>
<thead>
<tr>
<th>Flight No</th>
<th>Owned by</th>
<th>IAT</th>
<th>The earliest entry time at FCA1</th>
<th>The earliest entry time at FCA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>competitors</td>
<td>7:30</td>
<td>7:30</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>$F_A$</td>
<td>7:35</td>
<td>7:35</td>
<td>7:35</td>
</tr>
<tr>
<td>3</td>
<td>$F_A$</td>
<td>7:40</td>
<td>7:40</td>
<td>7:40</td>
</tr>
<tr>
<td>4</td>
<td>competitors</td>
<td>7:45</td>
<td>N/A</td>
<td>7:45</td>
</tr>
<tr>
<td>5</td>
<td>$F_A$</td>
<td>7:50</td>
<td>7:50</td>
<td>7:50</td>
</tr>
<tr>
<td>6</td>
<td>$F_A$</td>
<td>7:55</td>
<td>7:55</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Figure 29: Tree Traversal of Example 1

Example  Let’s assume that two FCAs are defined and that there are 6 flights in a CTOP. Flights 1 and 4 are competitor’s flights while flights 2, 3, 5, and 6 are $F_A$ flights. Let’s further assume that there are a total of 10 FCA slots as follows:

- FCA 1 - 8:00, 8:10, 8:20, 8:30, 8:40
- FCA 2 - 8:07, 8:18, 8:27, 8:36, 8:45

In the following examples, the sum of assigned slots’ time is defined as a performance index. In addition, only flights 2, 3, and 5 have multiple FCA entries which results in a tree branching at these nodes.
Example 1

Let’s consider that the earliest IAT for all flights is earlier than the given available slots of FCAs as noted in the table 16. Because all IATs are earlier than the available slots, every slot of both FCAs is feasible. After full enumerations for a planning horizon, the proposed heuristic algorithm finds the minimum cost solution as shown in Fig. 29 which represents the minimum delay cost associated with the trajectories analyzed. An interesting trajectory selection is the decision regarding flight 3. Although the slot time of FCA 1 (8:10) is earlier than one of FCA 2 (8:18) and both slots are available for flight 3 at node 3, taking the later slot of FCA 2 is more beneficial when considering whole trees. This result is influenced by flight 4. Since flight 4 requires a slot for FCA 2 and flight 5 can take any slots from both the FCAs, the proposed approach finds an optimal solution by taking the later slot of FCA 2 for flight 3 and giving an earlier slot of FCA 1 to flight 5.

Example 2

This example is identical to example 1 except for the earliest entry time given for flights 2 and 5. While the CTOP assignment order remains the same, these changes impact the feasibility and produce significant changes to the minimum cost solution. Unlike the trajectory decision for flight 3 in example 1, taking an earlier slot at FCA 2 (8:07) is more beneficial for $F_A$ in this case study. This decision is driven by the

<table>
<thead>
<tr>
<th>Flight No</th>
<th>Owned by</th>
<th>IAT</th>
<th>The earliest entry time at FCA1</th>
<th>The earliest entry time at FCA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>competitors</td>
<td>7:30</td>
<td>7:30</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>$F_A$</td>
<td>7:35</td>
<td>7:35</td>
<td>8:15</td>
</tr>
<tr>
<td>3</td>
<td>$F_A$</td>
<td>7:40</td>
<td>7:40</td>
<td>7:40</td>
</tr>
<tr>
<td>4</td>
<td>competitors</td>
<td>7:45</td>
<td>N/A</td>
<td>7:45</td>
</tr>
<tr>
<td>5</td>
<td>$F_A$</td>
<td>7:50</td>
<td>8:25</td>
<td>7:50</td>
</tr>
<tr>
<td>6</td>
<td>$F_A$</td>
<td>7:55</td>
<td>7:55</td>
<td>N/A</td>
</tr>
</tbody>
</table>
fact that the competitor can take only the slots at FCA 2 and that an earlier slot at FCA 1 is infeasible for flight 5. Due to this the proposed approach results in taking the earliest slot at each branching node (flight 2, flight 3, and flight 5).

Example 3

This example includes additional changes on the earliest entry time and, as expected, a different minimum cost solution is obtained. Unlike the previous two examples, flight 3 will enter a later slot at FCA 2 (8:25) resulting in big delay difference (17

Table 18: Each Flight’s Earliest Entry Time at Each FCA for Example 3

<table>
<thead>
<tr>
<th>Flight No</th>
<th>Owned by</th>
<th>IAT</th>
<th>The earliest entry time at FCA1</th>
<th>The earliest entry time at FCA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>competitors</td>
<td>7:30</td>
<td>7:30</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>$F_A$</td>
<td>7:35</td>
<td>7:35</td>
<td>8:15</td>
</tr>
<tr>
<td>3</td>
<td>$F_A$</td>
<td>7:45</td>
<td>7:40</td>
<td>8:25</td>
</tr>
<tr>
<td>4</td>
<td>competitors</td>
<td>7:45</td>
<td>N/A</td>
<td>7:45</td>
</tr>
<tr>
<td>5</td>
<td>$F_A$</td>
<td>8:15</td>
<td>8:15</td>
<td>8:40</td>
</tr>
<tr>
<td>6</td>
<td>$F_A$</td>
<td>8:26</td>
<td>8:26</td>
<td>N/A</td>
</tr>
</tbody>
</table>
minutes) between taking a slot at FCA 1 and FCA 2. Thus, the developed heuristic algorithm assigns the earlier slot at FCA 1 (8:10) to flight 3 and assigns the later slot at FCA 2 (8:18) to flight 2 instead of manipulating the slot allocation of flight 3. As shown through these three examples, different constraints can influence the optimal solution significantly.

4.2.1.2 Modified Branch and Bound Approach

In previous section, the heuristic model for the first stage problem is presented. As discussed, this approach provides significant savings in terms of computational complexity. By grouping trajectories based on FCA entry, the problem domain reduces significantly. In spite of this advantage, the limitation exists resulting in an approximation algorithm. The most commonly used exact algorithm to solve combinatorial problems and NP-hard discrete optimization problems is the branch and bound method. This algorithm was first proposed by Land and Doig [49], and has become
popular [29]. For implementing the branch and bound algorithm, the following mathematical first stage model is formulated in IP technique:

**Decision Variables**

\[ x_{f,c} = \text{assigned entry time at FCA } c \text{ for flight } f \quad \forall f \in F, c \in C \]

\[ y_{f,c,i} = \begin{cases} 
1 & \text{if flight } f \text{ is assigned to slot index } i \text{ of FCA } c \\
0 & \text{otherwise} 
\end{cases} \quad \forall f \in F, c \in C, i \in I \]

\( x_{f,c} \) is an integer variable for assigned entry time at the given FCA while \( y_{f,c,i} \) is a binary variable that represents which specific slot of FCA \( c \) is assigned to flight \( f \). Thus, if flights will not pass through a certain FCA then associated decision variables, \( x_{f,c} \) and \( y_{f,c,i} \) will have zero accordingly.

**Objective**

\[
\min \left\{ \sum_{f \in F} \sum_{c \in C} a_f x_{f,c} \right\}
\] (40)

In the first stage model, we need to consider only the cost associated with our flights under the given competition with other flight operators. Therefore, the proposed objective function needs to consider only costs associated with flights in \( F_A \) and this is done by setting \( a_f = 1 \) if flight \( f \in F_A \) or 0 otherwise.

**Constraints**

\[
0 = \sum_{i=1}^{e_{f,c} - \Delta} h_{f,c} y_{f,c,i} \quad \forall f \in F, c \in C
\] (41)

\[
1 = \sum_{c \in C} \sum_{i \in I} y_{f,c,i} \quad \forall f \in F_{AM}
\] (42)

\[
h_{f,c} = \sum_{i \in I} y_{f,c,i} \quad \forall f \in F_{AS} \cup F_O, c \in C
\] (43)

\[
l_{c,i} \geq \sum_{f \in F} y_{f,c,i} \quad \forall i \in I, c \in C
\] (44)
As described in Eq.(41), (42), and (43), each flight cannot be assigned to the slot earlier than its earliest nominal arrival time at a FCA. Moreover, each flight is not able to be assigned to multiple slots. In addition, if a flight has only one FCA entry for entire trajectory candidates, this flight is assigned to a slot of the associated FCA accordingly. With respect to capacity constraints, the number of flights assigned to each slot for each FCA has to be less than capacity of it as described in Eq.(44). Eq.(45) and (46) are used to link two groups of decision variables $x_{f,c}$ and $y_{f,c,i}$. In order to capture the FAA’s CTOP RBS rule, Eq.(47) is introduced. By adding big dummy number $M$, this equation is active only when both flights pass through same FCA and both flights are included in the set of RBSMap, whereas the other case results in a trivial constraint. RBSMap is generated when all the following rules are satisfied:

1. The earliest FCA entry time of $f_1$ is earlier than $f_2$.

2. If both flights are in the set of $F_{AS} \cup F_O$, then both flights have to fly via same FCA.

3. For the given FCA $c$, $e_{f_1,c}^{FCA} \leq e_{f_2,c}^{FCA}$

The branch and bound algorithm consists of a series of enumerations of solving LP problems by adding an extra constraint at each node resulting in separation of feasible search domain. That is, the set of candidate solutions at each branching node is calculated by forming a rooted tree. Then, the algorithm fathoms every branch of this tree, which represents subsets of the solution set. By enumerating this process
until the end of tree, the branch and bound algorithm searches the complete domain of decision variables for a given problem for an optimal solution. However, entire tree enumeration is generally impossible due to the exponential growth of solution tree resulting from combinatorial nature. To reduce computational complexity, the branch and bound algorithm adopts the following three pruning criteria:

1. infeasibility

2. integer solution

3. boundness

Fig. 32 shows an example of the branch and bound algorithm and these three pruning criteria. Pruning by infeasibility is presented at node 4 and 5. After introducing an additional constraint, $x_2 \geq 1$ and $x_3 \geq 1$, the problem becomes infeasible.
At these nodes, we don’t have to evaluate deeper branches because there is no way to make the problem feasible by introducing additional constraints. Thus, if we encounter infeasibility then we could stop the enumeration of that node. Pruning by integer solution is the case when a solution of the relaxed LP program is feasible in the original solution domain. As shown in node 3 and node 7, if the algorithms found an integer solution at any node we can consider that the tree branch associated with the node is fathomed. Due to the strategy of the branch and bound algorithm, we can find greater than or equal to the costs associated with the current solution by exploring deeper if it exists for minimization problems. Therefore, if we meet an integer solution at any node we can stop the search process for that node and explore the other branch to find better solutions. A last pruning criterion is boundness. As discussed in pruning by integer solution, we can update our upper bound costs when the algorithm finds any feasible integer solution at any node. After obtaining the first feasible integer solution at node 3, we can set the associated cost, $z = 14.6$, as the current upper bound. Then, we can update this upper bound value at note 7 when we found a better solution resulting in a smaller cost, $z = 13.0$. Using this upper bound, we can stop our further exploration at node 8. In the branch and bound algorithm, we solve a relaxed LP problem at each node and based on LP theory the solution from a relaxed LP problem can be considered as a lower bound for minimization. Thus, before enumerating the candidate solutions of a branch further, we need to check whether a current lower bound is bigger than an upper bound, and the branch is discarded if it cannot produce a better solution than the best feasible solution found so far through exploring the solution tree by the branch and bound algorithm.

**Modified Branch and Bound Algorithm**

Extensive researches of the branch and bound algorithm have been studied in literature [21, 30, 34, 39]. By an increase of computing power, in addition to three
**Modified Branch and Bound Algorithm**

<table>
<thead>
<tr>
<th>Solve a relaxed LP problem and set the resulting cost as a lower bound.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>while</strong> (there exists no more tree branch to explore) <strong>do</strong></td>
</tr>
<tr>
<td>- Solve the given IP problem based on the greedy algorithm on a flight by flight basis in order of an IAT and set the resulting cost as an upper bound of the IP.</td>
</tr>
<tr>
<td>- Add an extra constraint to the original problem and solve a relaxed LP problem.</td>
</tr>
<tr>
<td>- <strong>if</strong> the solution from a relaxed LP problem is infeasible <strong>then</strong></td>
</tr>
<tr>
<td>- Stop to search.</td>
</tr>
<tr>
<td>- <strong>else</strong></td>
</tr>
<tr>
<td>- Set a current objective value as a lower bound.</td>
</tr>
<tr>
<td>- <strong>if</strong> the lower bound is greater than the upper bound so far <strong>then</strong></td>
</tr>
<tr>
<td>- Stop to search.</td>
</tr>
<tr>
<td>- <strong>else</strong></td>
</tr>
<tr>
<td>- <strong>if</strong> a current solution is an integer feasible solution <strong>then</strong></td>
</tr>
<tr>
<td>- Set a current objective value as an upper bound.</td>
</tr>
<tr>
<td>- Stop to search</td>
</tr>
<tr>
<td>- <strong>else</strong></td>
</tr>
<tr>
<td>- Branch further at the current node to next tree level.</td>
</tr>
<tr>
<td>- <strong>end if</strong></td>
</tr>
<tr>
<td>- <strong>end if</strong></td>
</tr>
<tr>
<td><strong>end if</strong></td>
</tr>
<tr>
<td><strong>if</strong> there is no node in same tree level <strong>then</strong></td>
</tr>
<tr>
<td>- Delete the last two added extra constraints.</td>
</tr>
<tr>
<td>- Go back to next node in previous tree level.</td>
</tr>
<tr>
<td><strong>else</strong></td>
</tr>
<tr>
<td>- Delete the last one added extra constraint.</td>
</tr>
<tr>
<td>- Go to next node in same tree level</td>
</tr>
<tr>
<td><strong>end if</strong></td>
</tr>
<tr>
<td><strong>end while</strong></td>
</tr>
</tbody>
</table>

Choose the upper bound as an optimal solution.
pruning criteria, the branch and bound algorithm is used in NP-hard discrete optimization problems as an exact algorithm. In spite of great developments in hardware, a combinatorial optimization problem is still considered as a very difficult problem, specifically, for big size problems. Because the proposed CTOP is designed to address and resolve multiple FCAs simultaneously, it is obvious that the search domain of the first stage IP model is extensive resulting in computational difficulties. To reduce this inevitable computational complexity, a modified branch and bound algorithm is proposed. Overall algorithm is exactly same except the feature pruning by boundness. As shown in Fig. 32, pruning by boundness of the typical branch and bound algorithm is executed only after the algorithm finds at least an upper bound which is an integer feasible solution. As a result, the branch and bound algorithm explores without pruning by boundness until node 3 which the algorithm obtains an integer feasible solution. However, the proposed branch and bound algorithm starts with the

**Figure 33:** Modified Branch and Bound Algorithm Example
upper bound found from the heuristic greedy algorithm. Therefore, the search space of feasible domain decreases significantly by performing boundness criteria starting from the initial branch exploration. In here, the heuristic greedy algorithm means the proposed heuristic algorithm in the heuristic approach of the first stage model excluding branch strategy at the multiple FCA entries node. For example, when the algorithm reaches the node which has multiple FCA entries it assigns the FCA which has a smaller slot time. Since it requires just a bit of time to calculate an upper bound using this approach, there is no impact on initialization by adding this feature at the starting node. However, savings are quite solid. For the case of example in Fig. 29 an upper bound is computed by assigning flight $f_2$, $f_3$, $f_5$, and $f_6$ to a 8:07 slot of FCA 2, a 8:10 slot of FCA 1, a 8:20 slot of FCA 1, and a 8:30 slot of FCA 1, respectively, resulting in total 33:07 instead of the optimal solution, 32:55. This value is fourth-smallest solution out of total 8 combinations. For the case of example in Fig. 30 the greedy upper bound will generate the optimal solution resulting in huge reduction on search spaces. Similar to these two examples, an upper bound found from the heuristic greedy algorithm at example 3 depicted in Fig. 31 is fourth-smallest solution out of 8 combinations. Because the suggested heuristic greedy algorithm provides a very tight upper bound, we can reduce computational complexity significantly. As shown in Fig. 33, if an upper bound at node 0 is 13.5 then we don’t have to evaluate entire left side tree on the contrary to Fig. 32. Therefore, the modified branch and bound algorithm requires only 5 relaxed LP problems to solve instead of 9 relaxed LP problems of the original branch and bound algorithm.

4.2.2 Optimal Slot Assignments Model (2nd Stage Model)

In the second stage model, the following problem is to be solved:
Given:
- Slot allocation solutions from the 1st stage model
• Full set of trajectory options
• Schedule of flights

Find optimal slot assignments that:
• Satisfies the given slot capacity constraints
• Minimizes the total costs

By:
• Swapping the slot assignments
• Submitting “NOSLOT” trajectory options to fly out of FCAs

After obtaining the best slot allocation from the first stage model, we need to find the optimal slot assignments in this planning stage that minimize total costs of out all flights. To achieve this goal, mainly two methods are employed here, swapping the slot internally among $F_A$ flights and submitting “NOSLOT” trajectory options.

The second stage model is formulated as a MIP problem. In this planning stage, we need to consider only allocated slots from the first stage model as capacity constraints without considering competition of other flight operators. As a result, it is much easier to solve than the first stage model in terms of the computational complexity. The challenge of the second stage model is the modeling of the cost functions. To capture all the possible costs of this decision process, the cost function consists of three parts: the costs associated with en route operations, the costs resulting from departure delays, and the costs caused by arrival delays. With regard to arrival cost modeling, it can be represented by missed connection costs and these missed connection costs generally can be modeled as a non-linear function using a combination of linear and step functions as shown in Fig. 34. To capture this non linearity, Special Ordered Sets of type 2 (SOS2) variables are employed as described in the following section.
4.2.2.1 Assumption

To simplify the model, the followings are assumed.

- The set of trajectories for each flight is given.
- The definition and capacity of the FCA is given.
- The en route costs, departure delay costs, and arrival delay costs are given.

4.2.2.2 Decision Variables

To facilitate the formulation, the following decision variables are defined. $v_{f,r}$ and $y_{f,c,i}$ are binary variables that represent which trajectory is chosen as the best trajectory for flight $f$ and which slot of the FCA $c$ is assigned to flight $f$, respectively. The estimates of a departure delay for flight $f$ are $d_{f}^{dep}$. Unlike a departure delay, there is no decision variable associated with an arrival delay. To estimate an arrival delay, SOS2 variables, $w_{f,k}$, are introduced.

$$v_{f,r} = \begin{cases} 1 & \text{if route } r \text{ is chosen as best route for flight } f \\ 0 & \text{otherwise} \end{cases} \quad \forall f \in F_A, r \in R_f$$

$$y_{f,c,i} = \begin{cases} 1 & \text{if flight } f \text{ is assigned to slot index } i \text{ of FCA } c \\ 0 & \text{otherwise} \end{cases} \quad \forall f \in F_A, r \in R_f, i \in I$$

$$d_{f}^{dep} = \text{estimates of departure delay for flight } f, \forall f \in F_A$$

$$w_{f,k} = \text{SOS2 variable for estimates of arrival delay for flight } f, \forall f \in F_A, k \in \{0, 1, 2\}$$

4.2.2.3 Cost Function

Given the en route costs, departure delay costs, and arrival delay costs, the objective function of the second stage model, which minimizes the total costs of flights caused through the entire operations from origin airports to destination airports, is presented in the following Eq. (48). The total costs of each individual flight consists of three types: those required during en route operations from origins to destinations such as fuel costs, those resulting from departure delays at origins such as passenger costs, and
those resulting from arrival delays at destinations such as missed connection costs.

The en route costs are determined by multiplying $v_{f,r}$ and an en route cost index for each trajectory option. Since $v_{f,r}$ is a binary decision variable and the sum of $v_{f,r}$ is 1, only the en route costs associated with the selected trajectory is accounted for accordingly. Regarding the departure and arrival delay costs, they are computed by a simple multiplication between the delays and the cost index per unit time.

\[
\text{Total cost} = \min \{ z_{\text{en-route}} + z_{\text{dep-delay}} + z_{\text{arr-delay}} \} \quad (48)
\]

\[
z_{\text{en-route}} = \alpha \sum_{f \in F_A} \sum_{r \in R_f} z_{f,r}^{\text{en-route}} v_{f,r} \quad (49)
\]

\[
z_{\text{dep-delay}} = \beta \sum_{f \in F_A} z_{f}^{\text{dep-delay}} d_{f}^{\text{dep}} \quad (50)
\]

\[
z_{\text{arr-delay}} = (1 - \alpha - \beta) \sum_{f \in F_A} z_{f}^{\text{arr-delay}} d_{f}^{\text{arr}} \quad (51)
\]

### 4.2.2.4 Constraints

The Eq. (52) determines the earliest nominal FCA entry time which is used to determine the feasibility of FCA slot assignments. To find valid feasibility constraints, only the trajectories which enter the given FCA are considered. Based on these constraints, a flight cannot be assigned to a slot earlier than its earliest nominal FCA entry time by setting $y_{f,c,i} = 0$ for the slots earlier than $e_{f,c}^{FCA}$ as described in Eq. (53).

\[
e_{f,c}^{FCA} = \min_{h_{f,r,c}=1} \{ h_{f,r,c} e_{f,r,c}^{FCA} \} \quad \forall f \in F_A, c \in C \quad (52)
\]

\[
\sum_{i=1}^{e_{f,c}^{FCA}} y_{f,c,i} = 0 \quad \forall f \in F_A, c \in C \quad (53)
\]

To avoid duplicate slot assignments, only one $v_{f,r}$ can have a value of 1, therefore, limiting the results to only one optimum trajectory for each flight as described in Eq. (54). Similar to the trajectory selection decision variables, a flight cannot be assigned to multiple slots. Moreover, if the best trajectory does not enter any FCAs then all $y_{f,c,i}$ are set to zero. To satisfy these two statements simultaneously, the
inequality Eq. (55) is employed instead the usual equality equation for assignment constraints. For assignment consistency between trajectory selections and slot assignments, if the selected best trajectory enters one of the FCAs, the flight should be assigned to the slot of the FCA accordingly. Otherwise, all $y_{f,c,i}$ is set to zero and this is achieved by Eq. (56).

$$
\sum_{r \in R_f} v_{f,r} = 1 \quad \forall f \in F_A
$$

(54)

$$
\sum_{c \in C} \sum_{i \in I} y_{f,c,i} \leq 1 \quad \forall f \in F_A
$$

(55)

$$
\sum_{r \in R_f} h_{f,r,c} v_{f,r} = \sum_{i \in I} y_{f,c,i} \quad \forall f \in F_A, c \in C
$$

(56)

As noted, the first stage model will find the best slot set for $F_A$. Therefore, in the second stage model flights should be assigned to occupied slots only, or they need to fly out of FCAs by submitting a “NOSLOT” trajectory option. In the latter case, the sum of $y_{f,c,i}$ could be less than what $F_A$ obtained from the first stage model. To enforce these two possible cases, the following inequality is introduced:

$$
\sum_{f \in F_A} y_{f,c,i} \leq l_{c,i} \quad \forall i \in I, c \in C
$$

(57)

If the chosen best trajectory does not enter a FCA, then departure delays at the associated origins should be zero. Otherwise, the departure delays could have any value. To satisfy both cases and to determine departure delays, the Eq. (58) are introduced into the second model as constraints with big dummy number M.

$$
d_{f}^{\text{dep}} \leq M \sum_{r \in R_f} \sum_{c \in C} h_{f,r,c} v_{f,r} \quad \forall f \in F_A
$$

(58)

$$
\sum_{c \in C} \sum_{i \in I} \Delta (i) y_{f,c,i} \leq d_{f}^{\text{dep}} + \sum_{r \in R_f} \sum_{c \in C} h_{f,r,c}^{\text{FCA}} v_{f,r} \leq \sum_{c \in C} \sum_{i \in I} \Delta (i + 1) y_{f,c,i} \quad \forall f \in F_A
$$

(59)

Unlike the departure delays, arrival delays at destinations are not defined in the set of decision variables. In order to estimate the arrival delays for flights, SOS2 variables
are introduced as illustrated in Fig. 34. The left hand side of Eq. (60) represents actual arrival time of flight \( f \) at destinations. If the actual arrival time is less than the scheduled arrival time, then \( d_{f}^{arr} = 0 \) by setting \( w_{f,2} = 0 \). Otherwise, \( d_{f}^{arr} \) will be determined accordingly by setting \( w_{f,0} = 0 \).

\[
\begin{align*}
    d_{f}^{dep} + \sum_{r \in R_f} t_{f,r}^{dest} w_{f,r} & = 0 \cdot w_{f,0} + s_{f}^{arr} w_{f,1} + (M + s_{f}^{arr}) w_{f,2} & \forall f \in F_A & (60) \\
    d_{f}^{arr} & = 0 \cdot w_{f,0} + 0 \cdot w_{f,1} + M w_{f,2} & \forall f \in F_A & (61) \\
    \sum_{k \in \{0,1,2\}} w_{f,k} & = 1 & \forall f \in F_A & (62) \\
    w_{f,k} & \leq 1 & \forall f \in F_A, k \in \{0,1,2\} & (63)
\end{align*}
\]

**4.3 Computational Study and Analysis**

**4.3.1 Simulation Parameter**

The proposed optimization model can be implemented for a CTOP with multiple FCAs. To demonstrate the model, it is implemented over two FCAs as shown in Fig 35, FCAA05 (sector boundaries of ZMP, ZAU, ZOB, and ZID) and FCACDF (sector boundaries of ZID, ZDC, and ZTL). The given slots of these FCAs were allocated to
each flight operation in the first planning stage and then assigned the allocated slots to $F_A$ flights to minimize the total costs in the second planning stage.

To investigate both the impact of the size of the planning horizon and a moving window strategy in terms of solution quality and computational complexity, the representative input data from a 7 hour flight schedule are extracted from CTOP TMI messages on 17 July 2013, and they are summarized in the table 19.

In addition, the capacity of the given FCAs defined by the FAA are extracted from CTOP FCA messages to create a realistic numerical example as summarized in table 20. With regard to a nominal capacity, we assumed that each FCA has 30 slots per 15 minute interval.
Table 19: Simulation Environment for Parameter Analysis

<table>
<thead>
<tr>
<th>FCAs</th>
<th>FCAA05, FCACDF</th>
</tr>
</thead>
<tbody>
<tr>
<td># of $F_A$ flights</td>
<td>106</td>
</tr>
<tr>
<td># of exempted flights</td>
<td>3</td>
</tr>
<tr>
<td># of other NAS users’ flights</td>
<td>266</td>
</tr>
<tr>
<td>CTOP schedule</td>
<td>07/17/2013 16:00 ~ 23:00</td>
</tr>
</tbody>
</table>

Table 20: Given Capacity of Each FCA for Parameter Analysis

<table>
<thead>
<tr>
<th>FCAA05</th>
<th>FCACDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time bin</td>
<td># of slots</td>
</tr>
<tr>
<td>00 min ~ 15 min</td>
<td>4</td>
</tr>
<tr>
<td>15 min ~ 30 min</td>
<td>3</td>
</tr>
<tr>
<td>30 min ~ 45 min</td>
<td>4</td>
</tr>
<tr>
<td>45 min ~ 60 min</td>
<td>3</td>
</tr>
<tr>
<td>Nominal condition</td>
<td>30 slots / 15 min</td>
</tr>
</tbody>
</table>

As mentioned previously, the size of the planning horizon could have a significant effect on the solution. A longer planning horizon will lead to a better solution than a shorter planning horizon, but it will require significantly greater computation time. For practical purposes, the proposed model optimizes a subset of the schedule as an input, and then it moves a part of windows as shown in the Fig. 36. In other words, after solving each subset of the given schedule the model freezes the solution of the earlier parts of the subset, and then considers the later parts again when solving the subsequent windows.

Figure 36: Moving Window Strategy
next subset of schedule. This approach is motivated by the conjecture that decisions made for earlier parts of the schedule have less influence on the decisions of the later part of the schedule if each subset is sufficiently long.

The simulation results of the first stage model minimizing the sum of an assigned slot time is presented in Fig. 37. The x axis, “chunk size”, represents the size of each optimization window. Since the first stage model considers the flights which enter multiple FCAs as the branching nodes, the number of branching flights in each optimization window is critical with regard to computational time. Thus, a chunk size represents how many branch nodes (how many flights having trajectories which enter more than two FCAs) are included in the subset of the schedule, and it is chosen as the parameter for the analysis purposes with regard to the impact of the planning
Figure 38: Minimum Sum of All Assigned Slots’ Times Solution with Different Window Size and Moving Step Size

horizon. As a result, each window might have the different number of flights in the planning horizon. A moving step size is chosen as the parameter in the same manner, and it represents how many branch nodes are included in the freezing part of the subset after obtaining the solution. Thus, a smaller moving step size results in more overlaps between two consecutive subsets of schedule. The relationship between the computational time and our parameters (a chunk size and a moving step size) is shown in Fig. 37. As expected, an increase of chunk size results in an almost exponential growth of computational time. In addition, a smaller moving step size requires a longer computational time.

To assess the quality of solutions with a different chunk size and moving step size, the simulation with different cost functions of the first stage model are executed as
shown in Fig. 38 and Fig. 39. Regardless of the cost functions, a bigger chunk size results in a better solution leveling-off around a chunk size of 14, and a smaller moving step converges earlier than a larger (around a chunk size of 13). This result is quite obvious because it is a highly congested situation. For the 7 hours schedule, there are a total of 375 flights but only 175 available slots for the two FCAs. This highly congested situation decreases the chance that $F_A$ can occupy better slots than other flight operators as explained in previous examples. Moreover, this situation makes it more difficult to find a better slot allocation by changing combinations between earlier flights of the schedule and later ones. Based on the results of these simulations, we can support the earlier conjecture that the decisions made for earlier parts of the schedule would not affect much to later parts if each subset of the schedule is large.
<table>
<thead>
<tr>
<th># of flights</th>
<th>chunk size</th>
<th>Heuristic</th>
<th>IP</th>
<th>Modified Branch and Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>6</td>
<td>0.002</td>
<td>6.44</td>
<td>0.41</td>
</tr>
<tr>
<td>70</td>
<td>8</td>
<td>0.011</td>
<td>39.87</td>
<td>1.62</td>
</tr>
<tr>
<td>90</td>
<td>11</td>
<td>0.029</td>
<td>252.45</td>
<td>5.59</td>
</tr>
<tr>
<td>110</td>
<td>20</td>
<td>10.08</td>
<td>1002.54</td>
<td>41.17</td>
</tr>
<tr>
<td>130</td>
<td>27</td>
<td>1537.46</td>
<td>* &gt; 1 hour</td>
<td>75.07</td>
</tr>
<tr>
<td>150</td>
<td>32</td>
<td>* &gt; 10 hours</td>
<td>*</td>
<td>595.86</td>
</tr>
<tr>
<td>170</td>
<td>35</td>
<td>*</td>
<td>*</td>
<td>1949.36</td>
</tr>
</tbody>
</table>

4.3.2 Comparison of Modified Branch and Bound algorithm to Heuristic Algorithm

To compare three approaches: the heuristic algorithm, an IP with the typical branch and bound algorithm, and an IP with the modified branch and bound algorithm, a part of given schedule used in section 4.3.1 is extracted and optimized. Unlike the heuristic algorithm, the IP model regardless solution methods considers other flight operators’ flight as decision variables resulting in much difficulties in computational point of view. As a result, the total number of flights in the given schedule needs to be considered when interpreting the result. Table 21 shows how many flights, how many multiple FCA entries flight is in the given CTOP and the runtime associated with each algorithm. As discussed in previous section, the runtime of the heuristic algorithm has exponential growth based on a chunk size while the IP model doesn’t have that steep increase rate. As expected, introduction of an upper bound at node 0 results in huge reduction on runtime comparing with the typical branch and bound algorithm. Because this approach provides an ability to prune branches in very early stages by comparing a lower bound solution found from a relaxed LP problem with the upper bound found from a heuristic greedy strategy, the savings could be significant.
In other words, the gap between the upper bound solutions and the optimal solution is not big.

To demonstrate the gains achieved by the modified branch and bound method, we can compare the runtime against the other approaches. As shown in previous section, the heuristic algorithm provides reasonable solution times as well as solution convergence under a relatively big chunk size. Unlike the heuristic algorithm, the modified branch and bound algorithm requires much more times to solve the problem when considering relatively small number of flights. Within this range of the number of flights, the proposed heuristic algorithm is dominant over the modified branch and bound algorithms. This difference in runtime for small problems is due to the nature of the branch and bound algorithm. Since a relaxed LP problem is needed to solve at each node, the branch and bound algorithm requires building more problems while the heuristic approach doesn’t. In addition, the heuristic approach doesn’t consider single FCA entry flights including ours and other flight operators’ as decision variables resulting in less independent variables. However, the branch and bound algorithm requires computing the optimal slot allocation for every flight in the given schedule by solving a LP model. The overhead required to consider all flights is the driving factor behind the longer runtime for the smaller windows. In spite of this disadvantage, the proposed branch and bound algorithm is dominant over the heuristic one for larger problems. Because of pruning by an upper bound from very earlier phase of branch trees, the combination of solutions increases not too super fast like the heuristic approach. Thus, it requires much small amounts of times to solve the given schedule of flights.

4.3.3 Numerical Examples and Results

To evaluate the proposed model with a different schedule, in this section, an 11 hour flight schedule on 27 December 2013 from a CTOP TMI message is used, and inputs
Table 22: Simulation Environment

<table>
<thead>
<tr>
<th>FCAs</th>
<th>FCAA05, FCACDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTOP schedule</td>
<td>12/27/2013 16:00 ~ 12/28/2013 03:00</td>
</tr>
</tbody>
</table>

Table 23: Given Capacity of Each FCA

<table>
<thead>
<tr>
<th>FCAA05</th>
<th>FCACDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time bin</td>
<td># of slots</td>
</tr>
<tr>
<td>00 min ~ 15 min</td>
<td>7</td>
</tr>
<tr>
<td>15 min ~ 30 min</td>
<td>7</td>
</tr>
<tr>
<td>30 min ~ 45 min</td>
<td>7</td>
</tr>
<tr>
<td>45 min ~ 60 min</td>
<td>7</td>
</tr>
<tr>
<td>Nominal condition</td>
<td>30 slots / 15 min</td>
</tr>
</tbody>
</table>

are generated based on the flight schedule and definition of the FCAs for eight cases. In the selected schedule, there are total 89 $F_A$ flights with 7 exemptions. In addition, other flight operators’ flights are generated. Using the number of $F_A$ flights in each 1 hour period, other flight operators’ flights are generated randomly as noted in the table 24. For example, for case 1; for the 8 $F_A$ flights between 16:00 and 17:00, 0 (= $8 \times 0$) flights of other flight operators are generated for same period. Other flight operators’ flights for cases 2, 3, 4, 5, 6, 7, and 8 are 8 (= $8 \times 1$), 16 (= $8 \times 2$), 24 (= $8 \times 3$), 32 (= $8 \times 4$), 40 (= $8 \times 5$), 48 (= $8 \times 6$), and 56 (= $8 \times 7$) respectively in the same manner.

The simulations were executed in the environment as summarized in Table 25. In these simulations, a chunk size of 20 and a moving step size of 6 are chosen for the parameters. As summarized in table 26, there is no significant difference in the computational time for the first and second stage model between all the cases. Since the proposed heuristic approach branches only at the flights which have multiple FCA entries, the number of other flight operators’ flights has minimal impact on the first stage computational time. In addition, the second stage model accounts for $F_A$ flights only and the slots that are already occupied through the first stage model,
**Table 24:** Case Study for Different Flight Schedule Density by Varying the Number of Competitor’s Flights

<table>
<thead>
<tr>
<th>Case</th>
<th># of $F_A$ flights</th>
<th># of exempted flights</th>
<th># of competitor’s flights</th>
<th>Total flights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>89</td>
<td>7</td>
<td>0 (= 89 x 0)</td>
<td>96</td>
</tr>
<tr>
<td>Case 2</td>
<td>89</td>
<td>7</td>
<td>89 (= 89 x 1)</td>
<td>185</td>
</tr>
<tr>
<td>Case 3</td>
<td>89</td>
<td>7</td>
<td>178 (= 89 x 2)</td>
<td>274</td>
</tr>
<tr>
<td>Case 4</td>
<td>89</td>
<td>7</td>
<td>267 (= 89 x 3)</td>
<td>363</td>
</tr>
<tr>
<td>Case 5</td>
<td>89</td>
<td>7</td>
<td>356 (= 89 x 4)</td>
<td>452</td>
</tr>
<tr>
<td>Case 6</td>
<td>89</td>
<td>7</td>
<td>445 (= 89 x 5)</td>
<td>541</td>
</tr>
<tr>
<td>Case 7</td>
<td>89</td>
<td>7</td>
<td>534 (= 89 x 6)</td>
<td>630</td>
</tr>
<tr>
<td>Case 8</td>
<td>89</td>
<td>7</td>
<td>623 (= 89 x 7)</td>
<td>719</td>
</tr>
</tbody>
</table>

**Table 25:** Computational Resource Environment

<table>
<thead>
<tr>
<th></th>
<th>Ubuntu 12.04 LTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating system</td>
<td>Ubuntu 12.04 LTS</td>
</tr>
<tr>
<td>Computing power</td>
<td>Intel i7-3630QM @ 2.4GHz x 8, 6GB Memory</td>
</tr>
<tr>
<td>Solver</td>
<td>IBM ILOG CPLEX Optimization Studio V12.5.1</td>
</tr>
<tr>
<td>Window size of 1st stage</td>
<td>20</td>
</tr>
<tr>
<td>Moving step size of 1st stage</td>
<td>6</td>
</tr>
<tr>
<td>Cost function for 1st stage</td>
<td>Minimizing arrival delays</td>
</tr>
<tr>
<td>Cost function for 2nd stage</td>
<td>Minimizing arrival delays</td>
</tr>
</tbody>
</table>

thus, there is no impact on computational difficulty resulting from the number of other flight operators’ flights. For the sake of simplicity and comparison, the sum of arrival delays is used as a cost function of the first and second stage model.

To investigate impacts on solution by varying the level of competition, i.e. schedule density, the shortest en route time trajectory of each flight is chosen as a baseline simulation. In other words, it represents the set of the shortest path for each flight resulting in no arrival delay solution if there exists no capacity constraint. Thus, the baseline simulation provides an estimate of the arrival delay costs produced in a current trajectory submission practice. Unlike the computational time, total arrival delay costs have big impacts with the level of competition. Specifically, they increase rapidly by introducing more other flight operators’ flights. Compared to the baseline
Table 26: Computation Times for Each Case

<table>
<thead>
<tr>
<th>Case</th>
<th>1st stage computational time</th>
<th>2nd stage computational time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>169.72</td>
<td>0.577</td>
</tr>
<tr>
<td>2</td>
<td>193.75</td>
<td>0.615</td>
</tr>
<tr>
<td>3</td>
<td>203.33</td>
<td>0.585</td>
</tr>
<tr>
<td>4</td>
<td>215.97</td>
<td>0.688</td>
</tr>
<tr>
<td>5</td>
<td>208.83</td>
<td>0.767</td>
</tr>
<tr>
<td>6</td>
<td>215.60</td>
<td>0.652</td>
</tr>
<tr>
<td>7</td>
<td>227.36</td>
<td>0.798</td>
</tr>
<tr>
<td>8</td>
<td>237.09</td>
<td>0.952</td>
</tr>
</tbody>
</table>

Table 27: Cost Comparison for Each Case

<table>
<thead>
<tr>
<th>Case</th>
<th>baseline solution</th>
<th>1st stage solution</th>
<th>2nd stage solution</th>
<th>savings compared with baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.200</td>
<td>1.967</td>
<td>1.283</td>
<td>59.9%</td>
</tr>
<tr>
<td>4</td>
<td>5.983</td>
<td>3.700</td>
<td>2.517</td>
<td>57.9%</td>
</tr>
<tr>
<td>5</td>
<td>17.70</td>
<td>11.02</td>
<td>7.233</td>
<td>59.1%</td>
</tr>
<tr>
<td>6</td>
<td>44.10</td>
<td>32.80</td>
<td>21.13</td>
<td>61.1%</td>
</tr>
<tr>
<td>7</td>
<td>79.02</td>
<td>63.80</td>
<td>28.25</td>
<td>64.2%</td>
</tr>
<tr>
<td>8</td>
<td>120.3</td>
<td>103.1</td>
<td>45.27</td>
<td>62.4%</td>
</tr>
</tbody>
</table>

simulation, however, the two-stage sequential model achieves significant savings in arrival delay costs as summarized in Fig. 40 and table 27. The proposed model results in much less arrival delays in all but two cases, case 1 and case 2 due to very minimal competition. However, the results indicate that the more intense competition exists, the more savings are achieved. Total arrival delays decline to less than 40% of the baseline depending on schedule density. As visualized in Fig. 40, the first stage is much more beneficial less congested schedule while the second stage is much more efficient for highly congested schedule. The reason is that it is much harder to find better slot under too much competition with other flight operators, but as more delays are required we can take routing-out advantage in the second stage.

Because the trajectory database used in this simulation contains “NOSLOT” trajectory option for one fourth of $F_A$ flights, the rest of those who have no “NOSLOT”
trajectory have to wait until the slots in the FCAs are available resulting in exponential growth of arrival delay costs. If the trajectory database provides more trajectory containing “NOSLOT” options, the second model will assign more flights to those trajectories, leading the more reduction on arrival delay costs if possible.

**Figure 40:** Comparison on Solutions with Different Flight Schedules
CHAPTER V

CONCLUSION AND FUTURE RESEARCH

In this thesis, two areas of the ATFM were researched. Firstly, the modeling and optimization of the terminal airspace was performed to maximize throughput of resources in addition to minimization of fuel and emission costs. To achieve these goals, two sequential optimization models are formulated using MIP technique. A second area of the ATFM is the en route airspace. Specifically, two stage sequential optimization models for CTOPs are developed from the perspective of flight operators. To solve the first stage model, two solution approaches, the heuristic algorithm and the modified branch and bound algorithm, are also developed while the second stage model is formulated in an IP problem.

5.1 Conclusion for Terminal Airspace Model

To address congestions in the terminal airspace, two optimization models are presented. The first stage model, optimization of fix assignments, accounts unevenly distributed traffic demand and resulting queuing delays from the cruise phase to the runways of destination airports. The fix assignment model minimizes the total consumed fuel as well as environmental impact during entire terminal operations by assigning the TRACON boundary fixes to the given flights. To address the fuel consumption and environmental impact, TRACON transit costs, outside of TRACON transit costs, and queuing delay costs are used as the objective function of the proposed model by implementing BADA and ICAO database. The fix assignment model is formulated as an IP problem, and the presented simulation results of a 15 hours schedule at ATL show the validation and benefits of the proposed model. For fuel minimizing case, fuel consumption and associated delays decrease by 4.3% and 12.3%
comparing with the baseline, respectively while emission reduction decreases slightly, about 3.3%. This study will be necessary and more effective when the terminal airspace loses its capacity by severe weather. After obtaining the optimal TRACON boundary fixes assignment, a next model, optimization of runway assignments, addresses congestions of inside of the TRACON area. The proposed runway assignment model jointly optimized the terminal airspace operations, from TRACON boundary fixes to runways, and airport surface operations, from runways to gates of airport terminals, by reducing emissions and improving runway utilization. To address the environmental impact, the total emissions produced from the boundary of the TRACON to airport gates is used as the objective function in a model that optimizes runway assignments and the sequence of take-offs and landings on runways. The integrated simulations of an eight-hour flight schedule at DTW indicated that arrival emissions could decrease by 29.9% when compared with the baseline simulation. For departures, the overall emissions decreased slightly, about 2.6%. Due to the lack of actual fuel burn and emissions, the reduction of emissions is considered as the best case scenario. Because of the complexity of the problem, the full functional model was hard to implement in a real-time simulation, but it found an optimal solution within a reasonable time. Results also demonstrated that the proposed model could optimally balance runway utilization, resulting in an increase in runway throughput and a reduction in delay. Therefore, the proposed optimization for planning terminal operations could improve airport efficiency and the local environments.

5.2 Conclusion for En Route Model

Two sequential models that find an optimal slot allocation in the first stage model and an optimal slot assignment in the second stage model for the constrained capacity of multiple FCAs are presented. To address the CTOP RBS algorithm and competition with other flight operators, the first stage model minimizes the given
cost functions using a developed heuristic tree search algorithm. In addition to the heuristic algorithm, a more efficient solution methodology by modifying the branch and bound algorithm is also presented. After finding optimal slot allocation from the first stage model, the second stage model solves the MIP problem and assigns our flights to the acquired slots from the solution of the first stage model by swapping flights internally and by submitting “NOSLOT” trajectory options. The presented simulation results of a 7 hour flight schedule with two FCAs, validates the heuristic tree search algorithm for the first planning stage. Moreover, the comparison between the heuristic algorithm and the modified branch and bound algorithm also shows that reduction of search space by finding a tight upper bound, close to the optimal solution, has significant impacts on savings of computational complexity. The integrated simulations of an 11 hour flight schedule indicated that arrival delay costs could decrease more than 60% depending on flight schedule density when compared with the baseline simulation. It also demonstrated the proposed model could find the solution in a reasonable time with a realistic computational environment.

### 5.3 Applications to Real-World Problems

As presented in this thesis, two ATFM problems are addressed using a same approach, the two-stage sequential framework. For the terminal airspace, the model solves TRACON boundary fix assignments as the first stage problem, then accounts runway assignments and runway scheduling as the second stage problem. Similarly, the en route problem is addressed using same manner, finding optimal slot allocations in the first stage and finding optimal slot assignments in the second stage. For both areas, the suggested problem-solving framework provides significant benefits in terms of runtime as well as ability to capture real-world situation. Due to complexity, it is almost impossible to solve them within reasonable time. Moreover, non-linearity of the real-world problems results in difficulty to build a model representing real-world
problems. For example, TRACON boundary fix assignments are highly coupled with runway assignments and runway sequence as presented in Eq. (27). Similarly, en route problems have the same issue due to the gap between FAA’s slot allocation procedure and flight operators’ slot assignments. To minimize the impacts of CTOPs, flight operators need to know which set of slots of FCAs will be allocated. However, it is significantly difficult to capture this gap by formulating a combined optimization model. Even if we are able to model a combined optimization overcoming aforementioned issues, we need to consider the uncertainty of real-world applications such as weather changes resulting in capacity loss of terminal airspace, actual aircraft arrival time variation resulting from speed changes and path stretching, and other flight operators’ action during FAA’s CTOP assignment procedure. Due to these unknown factors and variations from the given schedule, recourse actions are inevitable to minimize the impacts of them. The proposed two-stage sequential framework provides the way to handle these obstacles by decomposing the problem into two stages; the first stage addresses the given problem macroscopically under limited information such as other flight operators’ actions and flight arrival schedule while the second stage solves the problem with the detailed information after clearing uncertainty such as the updated flight arrival schedule and FAA’s CTOP slots allocations. Therefore, any problem having this decomposable structure can be solved using the proposed problem-solving framework.

Applications to real-world problems range from areas such as cloud computing, airport gate assignments, to logistics. For cloud computing, to maximize utilization of physical servers and minimize electric power consumption optimization of virtual machine assignments and scheduling is needed. However, it is extremely difficult to solve the given problem due to complexity resulting from problem size, dynamic fluctuations of virtual machine demands, fragmentation of hardware resources across multiple physical hosts, and load balancing for multiple resources simultaneously. One
way to address this problem is to apply the proposed framework. That is, after solving
the virtual machine partitioning problem for sets of physical hardware macroscopically
using less detailed information from couple hour look-ahead demand estimation, more
precise assignments and scheduling of virtual machine for each physical server is
accounted when detailed demand information is available. Using this approach we
can find the least number of physical servers from the first stage solution resulting in
savings of power consumption while making a healthier cloud system by distributing
virtual machines as evenly as possible in the second stage. Similar to applications of
cloud computing, we are able to utilize this framework to airport gate assignments
as well as logistics. When the limited information or inaccurate information are
available, macroscopic assignments (gate assignments for sets of gates and vehicle
assignments for sets of areas) are performed. Then, more precise gate assignments and
vehicle routing problems are addressed when more accurate input data are realized
in the later stage.

5.4 Suggested Future Directions

Both the terminal airspace model and the en route airspace model have possible fu-
ture works. To begin with, the terminal airspace model relied on the assumption that
all flight operators such as airlines will follow the system operator (the FAA)’s recom-
mendation, change of the trajectory, to meet capacity limits of the terminal airspace
resources, TRACON boundary fixes and runways, and to minimize the operational
costs including fuel consumptions and emission impacts, excluding consideration of
fairness. Because the proposed model for the terminal airspace minimizes the objec-
tive costs consisting of consumed fuel and emitted pollutants, it is clearly evident that
the algorithm puts old and big aircraft above the others in terms of priority during
optimizations. In general, this type of limitations is inevitable when optimizing the
airspace operations due to the nature of trade-off. To minimize fuel consumption over
the entire operations of the given schedule, we need to provide priority to the aircraft who consumes more fuel to fly resulting in more emissions, such as outdated aircraft and huge size of aircraft. Therefore, we need to study the fairness among flight operators when considering the optimization of the terminal airspace. This could be done by adding constraints limiting the number of flights needed to change their trajectory based on the ratio of traffic volume by each flight operator. Fairness is one of active topics in the field of scheduling and assignments. [62, 71] In addition to this possibility, we can consider stochastic nature of the terminal airspace problem in future. There exist a lot of uncertainties causing variation from the deterministic world to stochastic one. For example, weather causes the change of capacity of the terminal airspace by affecting the regulations over airport from VFR to IFR, or vice-versa. To account this uncertainty, the stochastic fix assignment model could be formulated in standard two-stage MIP technique by addressing capacity uncertainty in the second stage model in form of expectation of the costs.

Regarding en route airspace model, there are several areas of possible researches that would advance the optimization work presented here. Algorithms to reduce the computational complexity would allow dense schedules to be solved within less runtime. The en route airspace model is developed based on the assumptions that the ETA information of flights of other flight operators are available and they submit only one trajectory, the most preferred one such as the shortest path like current approaches. However, these assumptions will not hold all the time due to consideration of competition with other entities who use the given air transportation network. In addition, modeling or expecting competitors’ actions is extremely difficult in the form of typical stochastic approaches. Thus, applying a sort of game theory would also be potential research topics to capture real completion of flight operators in CTOPs. Because CTOPs are recently developed programs and they are still in the test phase, huge varieties of application and extension could be possible. For example, the FAA
is trying to use CTOPs to ration flights for terminal airspace congestions by issuing multiple FCAs enclosing TRACON boundary in the application of Optimization of Airspace and Procedures in the Metroplex (OAPM). The OAPM is being developed to provide solutions on a regional scale, rather than focusing on a single airport or set of procedures. As a part of the OAPM features, the Continuous Descent Approach (CDA) of metroplex area is an essential function and rationing demand of flights at the boundary of the terminal airspace is required to guarantee the CDA operations and to maximize benefits from the CDA operations. To absorb the expected delays resulting from the terminal airspace operations, CTOPs could be a perfect solution in the OAPM framework.
EN ROUTE OPTIMIZATION IMPLEMENTATION

To support the en route airspace optimization models, more detailed information is presented here in terms of implementation.

A.1 CTOP Data Handling and Input Generation

The proposed models are incorporated into three programs: an input generation program, an optimal slot allocation program, and an optimal slot assignment program. As shown in Fig.41, the input generation program receives the messages from a CTOP and the trajectory database as inputs, and then generates input files for the optimal slot allocation module. The optimal slot allocation module (1st stage model), determines the best slot allocation using the developed heuristic tree search algorithm. Finally, the allocated slots from the optimal slot allocation module are
delivered to the optimal slot assignment program (2nd stage model), and the optimal slot assignments are computed.

Regarding the FAA’s CTOP message handling, FCA_RESYNCH_REPLY.*.xml contains information defining the FCA such as ID, name, and position information. The extracted latitude and longitude information which defines the FCA is used to determine if the aircraft route intercepts the FCA and subsequently the entry time. CTOP_TMI_RESYNCH_REPLY.xml contains information regarding the capacity of the FCA and flight information such as the aircraft ID (ACID), the origin, the destination, the earliest runway time of departure (ERTD) and the earliest FCA entry time. The capacity information of each FCA is extracted for generating FCAList.txt which is one of the input files of the 1st stage model.

A.2 Tree Traversal

A pre-order depth-first traversal is used in implementation of en route airspace optimization model. This type of tree traversal is conducted recursively at each node,
starting with the root node, as shown in Fig.42. The pre-order traversal process is described in the following systematic way.

1. Visit the root.

2. Traverse the left sub-tree.

3. Traverse the right sub-tree.

**A.3 FCA Capacity Handling**

After extracting capacity information of each FCA from `FCAList.txt`, the model generates available slot lists for each FCA. The model then extracts flight information from `CTOPFlightList.txt`. If there are exemption flights in the list, the model assigns those flights to the most preferable slots before solving the problem. The current program assumes that capacity bin size is 15 minutes, which is a default value of the FAA’s CTOP message, and how to locate the available slots based on each capacity described in Fig.43, 44, 45, and 46.

![Figure 43: Handling Available Slots for Different Capacity(a)](image-url)
5 flights / 15 min: slot at 1.5, 4.5, 7.5, 10.5, and 13.5 min

\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\end{array}

6 flights / 15 min: slot at 1.25, 3.75, 6.25, 8.75, 11.25, and 13.75 min

\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\end{array}

7 flights / 15 min: slot at 1.071, 3.214, 5.357, 7.5, 9.643, 11.786, and 13.929 min

\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\end{array}

8 flights / 15 min: slot at 0.9375, 2.8125, 4.6875, 6.5625, 8.4375, 10.3125, 12.1875, and 14.0625 min

\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\end{array}

\textbf{Figure 44:} Handling Available Slots for Different Capacity(b)

9 flights / 15 min: slot at 0.833, 2.5, 4.167, 5.833, 7.5, 9.167, 10.833, 12.5, and 14.167 min

\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\end{array}

10 flights / 15 min: slot at 0.75, 2.25, 3.75, 5.25, 6.75, 8.25, 9.75, 11.25, 12.75, and 14.25 min

\begin{array}{cccccccc}
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
\end{array}

11 flights / 15 min: slot at 0.682, 2.045, 3.409, 4.773, 6.136, 7.5, 8.864, 10.227, 11.591, 12.955, and 14.318 min

\begin{array}{cccccccc}
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
\end{array}

12 flights / 15 min: slot at 0.625, 1.875, 3.125, 4.375, 5.625, 6.875, 8.125, 9.375, 10.625, 11.875, 13.125, and 14.375 min

\begin{array}{cccccccc}
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\
\end{array}

\textbf{Figure 45:} Handling Available Slots for Different Capacity(c)
Figure 46: Handling Available Slots for Different Capacity (d)

13 flights / 15 min: slot at 0.577, 1.731, 2.885, 4.038, 5.192, 6.346, 7.5, 8.654, 9.808, 10.962, 12.115, 13.269, and 14.423 min

```
1 1 1 0 1 1 1 1 1 1 1 1
```

14 flights / 15 min: slot at 0.536, 1.607, 2.679, 3.75, 4.821, 5.893, 6.964, 8.036, 9.107, 10.179, 11.25, 12.321, 13.393, and 14.464 min

```
1 1 1 1 1 1 1 1 1 0 1 1 1 1 1
```

15 flights / 15 min: slot at 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5, 10.5, 11.5, 12.5, 13.5, and 14.5 min

```
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```
REFERENCES


