PRODUCT STRATEGIES IN SUPPLY CHAINS

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by

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PRODUCT STRATEGIES IN SUPPLY CHAINS

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Date Approved: June 29, 2015
To my

daughter (Suhana),

wife (Moomal),

and mother (Mohani).
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This doctoral dissertation titled “Product Strategies in Supply Chains,” consists of three essays. In this dissertation, I study firms’ strategic decisions regarding design of products and product lines in different supply chain contexts. I focus on firms’ strategic interactions with supply chain members, including consumers and suppliers, in dynamic environments.

The first essay (Chapter 2) studies how the cost structure of and information asymmetry about an OEM’s in-house option affect her choice of product design quality in a decentralized supply chain where the supplier specifies contract terms. The second essay (Chapter 3) examines the effect of product returns and their potential refurbishing on intertemporal product strategy and profit of a firm facing strategic consumers. We also examine the effect of product returns on the time inconsistency problem faced by the firm. The third essay (Chapter 4) investigates the impact of competition from a third-party remanufacturer on product strategy and profit of an OEM in the presence of strategic consumers. Motivated by general perception among practitioners and the extant literature showing the competition from third-party remanufacturers as undesirable for the OEM, we specifically examine whether competition from a third-party remanufacturer is always undesirable for the OEM.
CHAPTER I

INTRODUCTION

This doctoral dissertation titled “Product Strategies in Supply Chains,” consists of three essays. In this dissertation, I study firms’ strategic decisions regarding design of products and product lines in different supply chain contexts. I focus on firms’ strategic interactions with supply chain members, including consumers and suppliers, in dynamic environments. The first essay of the dissertation studies the effect of supplier power and information structure on an OEM’s product design decisions. The other two essays of the dissertation focus on product returns and remanufacturing in the presence of strategic consumers and competition from third-party remanufacturers. In the following paragraphs, I summarize the research motivations and the main insights of the essays in the dissertation.

Original equipment manufacturers (OEMs) sometimes face the decision of whether to make an essential component of a product in-house or to source it from a supplier. Though sourcing from a supplier with a more favorable cost structure than the OEM could result in higher overall supply-chain profit, the supplier could potentially dictate contract terms and thus leave a lower share of the profit for the OEM. In the first essay (Chapter 2), we investigate implications of the relative cost efficiencies of the supplier and the OEM’s in-house option on the OEM’s choice of product design quality and subsequent contract outcomes. We model the problem as a dynamic game, wherein the OEM chooses product design quality in the first stage (determined by the design quality of a critical component), followed by the supplier offering a contract for supplying the critical component. The supplier has a more favorable cost structure than the OEM’s in-house option for manufacturing the critical component.
Thereafter, the OEM either accepts the supplier’s offer or chooses her in-house option, and sells the product in the consumer market. Contrary to intuition, the supplier’s ability to offer a two-part tariff contract need not always benefit the supplier. In fact, a two-part tariff contract, compared to a price-only contract offered by the supplier, leaves both the OEM and the supplier worse off when the cost competitiveness of the OEM’s in-house option is sufficiently low. We also investigate the impact of information asymmetry regarding the cost structure of the OEM’s in-house option. Counterintuitively, information asymmetry may be desirable not only for the OEM, but also for the supplier.

Consumer product returns are a significant and growing concern in many industries, and firms typically deem returns to be undesirable. Firms may refurbish these returns to recover value, thereby allowing them to extend their product offering over time to new and refurbished products. In the second essay (Chapter 3), we study the impact of returns on the intertemporal product strategy of a firm facing forward-looking or strategic consumers, who anticipate future availability and prices of products, and time their purchases to maximize net utility. Using a two-period model, we find that for sufficiently high return rates, the firm not only offers the refurbished product alone in the second period but also refurbishes all of the first-period returns. Importantly, we show that returns may act as a device for the firm to mitigate the well-known time inconsistency problem. Specifically, when the return rate is sufficiently high, the firm’s incentive to recover value from returns by refurbishing results in a reduction – and eventually elimination – of the incentive to offer the new product in the second period. Thus, a sufficiently high return rate allows the firm to implicitly commit that the new product will be offered exclusively in the first period, and therefore charge a premium for it. As a result, firm profit could increase with the return rate.

In line with the general perception among practitioners, the extant literature
on remanufacturing shows that an OEM’s profit suffers when a third-party remanufacturer competes with the OEM’s remanufacturing operations. Accordingly, the literature recommends ways to deter third-party competition. However, competition for used products (or product cores) can influence the price of new products because strategic (forward-looking) consumers consider the resale value of new products when making their purchase decisions. In the third essay (Chapter 4), we investigate the impact of competition from the third-party remanufacturer on the OEM’s profit in the presence of strategic consumers. Of specific interest is whether competition from the third-party remanufacturer is always undesirable for the OEM when they face strategic consumers. In our model, an OEM offers a new product that depreciates over time. The OEM has an opportunity to acquire and remanufacture depreciated used products and remarket the remanufactured products. A third-party remanufacturer also competes with the OEM for acquisition and remanufacturing of the used products.
2.1. Introduction

Original equipment manufacturers (OEMs) sometimes face the decision of whether to make a critical component of a product in-house or source it from a supplier with superior capability – specifically, lower manufacturing cost for the same design quality (Walker and Weber 1984, 1987). For example, before introducing its flagship smartphone – the Galaxy S5 – Samsung had a choice to either make the processor for the smartphone in-house (Exynos variant) or source the processor from Qualcomm (Snapdragon variant). This was an important decision for Samsung as the processor is a critical component in a smartphone. However, there were trade-offs involved in choosing the source for the processor (Agomuoh 2014). Qualcomm is “the undisputed king of mobile chips, makes some of the most advanced application processors in the industry, and is years ahead of its rivals with 4G LTE technology” (Tibken 2014).

Though sourcing from Qualcomm could potentially generate greater total supply-chain profit when selling to quality-conscious consumers (Eassa 2013), Qualcomm, due to its dominant position, may leave little profit for Samsung. Alternatively, Samsung could make the processor in-house to avoid having to share profits (Eassa 2015), but the total profit generated may be lower. Examples of firms facing such a decision can be found in other product categories as well. In the automotive sector, automobile assemblers often decide whether to source a critical component from a more capable
supplier such as Bosch or make it in-house. Suppliers of the kind discussed above are often in positions to specify contract terms. Other examples include Intel for computer chips, Samsung for mobile displays, and Magna for automotive powertrains. In the context of the supply chain contracting literature, Kostamis and Duenyas (2011), Ozer and Raz (2011), and Ozer and Wei (2006), among others, model supplier-specified contracts.

In addition to the sourcing decision, OEMs typically have choices with regard to product design quality. For instance, Samsung had several options in choosing performance characteristics of the processor for the Galaxy S5: CPU speed (in GHz), CPU instruction set, CPU architecture (32-bit or 64-bit), and semiconductor fabrication technology (expressed in nm). Moreover, product quality may need to be decided before sourcing contracts are signed (Jerath et al. 2015, Shi et al. 2013). This sequence of decisions (product quality followed by contracting) creates a trade-off for an OEM in designing a product. If the OEM designs a high-quality product that only the supplier can manufacture cost-effectively, total supply-chain profit would be higher when selling to quality-conscious consumers, but the supplier can extract a larger share of supply chain profit as the OEM has to rely on the supplier. On the other hand, if the OEM designs a low-quality product that she too can manufacture at a reasonable cost, the OEM can retain a larger share of supply chain profit by forcing the supplier to compete with her in-house option, but total supply-chain profit would be lower. Thus, the cost structures of the OEM and the supplier can influence both the choice of product quality and the dynamics of the sourcing contract.

Finally, firms often possess private information about their own capabilities (such as cost structures), which may help them extract information rent while contracting with other players in the supply chain. Therefore, in such situations, either firms

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1Delphi Automotive for General Motors, Visteon Corporation for Ford Motor Company, and Denso for Toyota Motor Corporation could be considered in-house options.
are averse to sharing such information with other players in the supply chain or the information shared is not credible. If the OEM has private information about the cost structure of her in-house option, she has an incentive to strategically alter her choice of product quality, which, in turn, may affect subsequent contract outcomes.

We aim to answer the following research questions for the setting where the OEM chooses product quality, followed by the supplier deciding contract terms: (1) How do product quality and supply chain profit in the decentralized supply chain differ from those in the vertically integrated supply chain when the supplier offers a price-only contract? How does the cost competitiveness of the OEM’s in-house option impact product quality and supply chain profits in the decentralized supply chain? (2) How does a two-part tariff contract, being more sophisticated than a price-only contract, perform in terms of product quality and supply chain profits? Can a two-part tariff contract coordinate the supply chain? (3) How does asymmetric information regarding the cost structure of the OEM’s in-house option affect product quality and supply chain profits?

We investigate these questions by modeling a three-stage dynamic game between the OEM and the supplier, who has a lower manufacturing cost than the OEM’s in-house option for the same design quality. In the design stage, the OEM decides product quality through her choice of performance characteristics of a critical component used in the product. In the contract stage, the supplier offers a take-it-or-leave-it contract to the OEM. In the selling stage, the OEM either accepts or rejects the supplier’s offer and sets the price of the product to be sold to consumers (if the OEM rejects the supplier’s offer, she makes the component in-house instead of sourcing it from the supplier). We contrast two scenarios: complete information and asymmetric information. In the complete information scenario, we consider two contracts: a price-only contract and a two-part tariff contract. We examine the price-only contract because this contract is a simple and common mechanism governing transactions in
supply chains (Bresnahan and Reiss 1985, Perakis and Roels 2007). We also examine
a two-part tariff contract because this contract has commonly been shown to coordi-
nate supply chains (Cachon and Kok 2010, Cachon and Lariviere 2005). In the
asymmetric information scenario, the OEM has private information about the cost
structure of her in-house option. Our analysis generates the following main insights:

1. Under the price-only contract, we find that the double marginalization problem
may manifest in the form of the OEM choosing lower product quality rather
than lower sales quantity. In particular, if the competitiveness of the OEM’s in-
house option is sufficiently high, the supply chain is coordinated in terms of sales
quantity but not in terms of product quality. However, if the competitiveness of
the OEM’s in-house option is relatively low, the supply chain is coordinated in
terms of product quality but not in terms of sales quantity. Moreover, product
quality, supplier profit, and total supply-chain profit are nonmonotonic in the
competitiveness of the OEM’s in-house option.

2. Contrary to intuition, we find that the supplier’s ability to offer a two-part
tariff contract\(^2\), compared to the price-only contract, may hurt not only the
OEM (as expected) but also the supplier. Specifically, if the competitiveness
of the OEM’s in-house option is sufficiently low, the two-part tariff contract –
compared to the price-only contract – leaves both the OEM and the supplier
worse off. The reason is that the supplier’s ability to offer the two-part tariff
contract induces the OEM to strategically choose low product quality, which,
in turn, lowers total supply-chain profit.

3. Under certain conditions, both the OEM and the supplier – the less-informed
player – earn higher expected profits under asymmetric cost information than

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\(^2\)The ability of a supplier to offer a two-part tariff contract (compared to the price-only contract)
has been typically shown to result in supply chain coordination and the supplier extracting a larger
share of the total supply-chain profit.
under complete information. The reason is that, under asymmetric information, the supplier is unable to perfectly discriminate between a high-cost OEM and a low-cost OEM. The result is that the high-cost OEM may choose higher quality under asymmetric information, leading to higher supply chain profits. This finding is counterintuitive given that the extant literature generally shows that asymmetric information results in: (a) the less-informed player being worse off, and (b) supply chain inefficiency. Thus, we show that asymmetric information may not necessarily be detrimental to the less-informed player in a decentralized supply chain when firms make decisions in a dynamic setting.

2.2. Literature Review

Our paper contributes to four streams of research in the supply chain literature: (i) contracting in decentralized supply chains, (ii) product design quality in supply chains, (iii) supply chain sourcing, and (iv) asymmetric information in supply chains.

In a decentralized supply chain, under simple contracts such as a price-only contract, players often have conflicting interests and thus make decisions that are not supply-chain-optimal (Perakis and Roels 2007, Spengler 1950). Consequently, the problem of double marginalization occurs in which a supplier and a buyer in a decentralized supply chain produce and sell less than the vertically integrated firm (Bresnahan and Reiss 1985). The supply chain contracting literature suggests various mechanisms such as two-part tariffs and buybacks to induce the buyer to order and sell more (Cachon 2003, Tsay et al. 1999). However, this literature has largely considered product design quality as exogenous.

Papers within the contracting literature that consider product design quality as an endogenous decision include Economides (1999), Jerath et al. (2015), Jeuland and Shugan (2008), Shi et al. (2013), and Xu (2009). We study the setting where the downstream player (OEM) decides both product quality and sales quantity taking
into account the contract terms specified by the upstream player (supplier). Our paper is different in that we focus on a supply chain structure where the OEM has an in-house option. Further, we consider asymmetric cost information between the OEM and supplier.

The literature on supply chain sourcing strategies focuses on the manufacturer’s (buyer’s) problem of how to select suppliers, award contracts, and allocate procurement among them. Elmaghraby (2000a) provides an excellent review of earlier work in the field of Operations Management (OM) and Economics on supplier competition and sourcing strategies. Lovejoy (2010) considers the context of a monopolist bringing a new product to market through a multi-tier supply chain with horizontal competition among firms in each tier. Arya et al. (2008) focus on strategic considerations that can influence sourcing decisions and show that, in the presence of rivals, a firm may buy an input for a price even above its in-house cost of production. Novak and Eppinger (2001) study the impact of product complexity on whether to make a component in-house or buy from an external supplier. Our paper is similar in the sense that the buying firm has more than one sourcing options to choose from. However, we endogenize product quality, consider the setting where the supplier specifies contract terms, and also allow for asymmetric information between the supply chain players.

Finally, our work is also related to the literature that examines the effect of asymmetric information in supply chains. Models in the supply chain literature that consider asymmetric information can be classified according to: (a) the parameter for which there is asymmetric information, such as cost or demand, and (b) whether it is a signaling (informed player moving first) or a screening (uninformed player moving first) problem (Chen 2003, Laffont and Martimort 2002). Our work studies a signaling problem in the presence of asymmetric cost information. Papers that consider
sourcing contracts with asymmetric cost information include Corbett (2001), Corbett et al. (2004), Corbett and de Groote (2000), Ha (2001), Iyer et al. (2005), Kaya and Ozer (2009), Kim and Netessine (2013), Kostamis and Duenyas (2011), and Li and Debo (2009). Like our paper, Corbett (2001), Corbett et al. (2004), Ha (2001), and Kostamis and Duenyas (2011) model the downstream player (the OEM in our case) with private cost information. Signaling problems in the OM literature examine equilibria that allow a firm to credibly share her private demand information with another supply chain player and increase overall profit (Cachon and Lariviere 2001, Ha and Tong 2008, Ozer and Wei 2006). In contrast, we study a signaling problem in which the OEM signals her cost structure to the supplier through her choice of product quality.

2.3. Model and Assumptions

2.3.1 Supply Chain

We consider a two-tier supply chain comprising an OEM and a supplier, who are profit-maximizing and risk-neutral. The sequence of decisions is shown in Figure 1. In the first stage (Jerath et al. 2015, Shi et al. 2013) – the design stage – the OEM decides product quality \( q \), defined as a one-dimensional measure of the stream of value that can be derived from the product over its lifetime. For simplicity, we assume that one critical component determines overall product quality. The OEM can either manufacture the component using her in-house option or source it from the supplier through a supply contract (we henceforth use the terms “component” and “product” interchangeably). Let \( o \) and \( s \) denote the OEM and the supplier, respectively. We assume that product quality is observable and contractible. We also assume that the marginal cost of production for a product of quality \( q \) is \( c_i q^2 \), where \( c_i \) is an exogenously given cost parameter for player \( i \in \{ o, s \} \) (Shi et al. 2013). We denote the ratio of the cost parameters by \( k = c_o / c_s \), which reflects the relative cost.
competitiveness of the supplier vis--vis the OEM (or vice versa).

In the second stage – the *contract stage* – the supplier, who is in a position to specify contract terms, offers a take-it-or-leave-it contract to the OEM for supplying the product (Kostamis and Duenyas 2011, Ozer and Raz 2011, Ozer and Wei 2006). We assume that the contracting process is a one-shot interaction. We consider two types of contracts: a price-only contract and a two-part tariff contract. The price-only contract specifies a per-unit wholesale price $w$ charged by the supplier to the OEM. The two-part tariff contract specifies a per unit price $w$ and a lump-sum fee $f$.

To focus on non-trivial cases, we assume that the supplier is more cost competitive than the OEM (i.e., $k > 1$) and that the supplier’s reservation profit is lower than the profit he can earn by contracting with the OEM. For exposition, we set the supplier’s reservation profit to zero.

In the third stage – the *selling stage* – the OEM either accepts or rejects the supplier’s offer and sets the selling price $p$ charged to consumers (or, equivalently, sells $Q$ units of the product).\(^3\) If the OEM accepts the supplier’s offer, she sources the product from the supplier; else the OEM manufactures the product using her in-house option.

We investigate and contrast two scenarios: *complete information* and *asymmetric information*. In the asymmetric information scenario (§2.5), the OEM has private information about the cost structure of her in-house option while the supplier knows it only probabilistically.

### 2.3.2 Consumers

We assume that consumers are heterogeneous in their willingness to pay for quality. We denote this characteristic of consumers by $\theta$. Specifically, a consumer of type $\theta$ is willing to pay at most $\theta q$ for a product of quality $q$. For simplicity, we assume

\[^3\text{Since we assume demand to be deterministic, the price charged by the OEM to consumers has a one-to-one relationship with the sales quantity of the product.}\]
that $\theta$ is uniformly distributed between zero and one. We normalize the market size to one. Furthermore, each consumer buys at most one unit of the product. The net utility that a consumer of type $\theta$ obtains from a product of quality $q$ purchased at price $p$ is $u_\theta = \theta q - p$ (Economides 1999, Jerath et al. 2015, Shi et al. 2013). Being a net-utility-maximizer, a consumer of type $\theta$ buys the product if and only if $\theta q - p \geq 0$. Thus, the sales quantity is $Q = \left(1 - \frac{p}{q}\right)$.

2.3.3 Benchmark: Integrated Supply Chain

We first establish the optimal decisions for the vertically integrated supply chain in which the OEM and the supplier are owned by a single, integrated firm that jointly sets the quality and price of the product to maximize total supply-chain profit. Since $c_s < c_o$, the integrated firm’s optimization problem is

$$\max_{p, q} \left(p - c_s q^2\right) \left(1 - \frac{p}{q}\right). \quad (2.3.1)$$

We denote the solution for the integrated firm by superscript $I$. In Appendix A.2, we summarize our notation for the model parameters (Table 11) and for the equilibrium solutions for each scenario discussed in the paper (Table 3). Proposition 1 summarizes...
the integrated firm’s optimal product quality, sales quantity, and profit, which we use as a benchmark for comparison with the decentralized supply chain. All proofs are included in Appendix A.1.

**Proposition 1.** For the integrated supply chain, the optimal (a) product quality $q^I = \frac{1}{3c_s}$, (b) sales quantity $Q^I = \frac{1}{3}$ and (c) firm profit $\Pi^I = \frac{1}{27c_s}$.

### 2.4. Contracting under Complete Information

In this section, we consider the contracting problem under complete information. At every stage of the game, each player has complete information about previous moves and payoff functions. We consider two types of contracts: the price-only contract and the two-part tariff contract. We consider the price-only contract because this contract is a simple and common mechanism governing transactions in supply chains (Bresnahan and Reiss 1985, Perakis and Roels 2007). We also examine the two-part tariff contract because this contract has commonly been shown to coordinate supply chains (Cachon and Kok 2010, Cachon and Lariviere 2005). We examine how product quality, sales quantity, and supply chain profits in the decentralized supply chain differ from those in the vertically integrated supply chain for both contract types and also compare the performance of the two contracts. For the decentralized supply chain, we denote the profits of the OEM, the supplier, and the total supply chain by $\Pi_o$, $\Pi_s$, and $\Pi_t (= \Pi_o + \Pi_s)$, respectively.

#### 2.4.1 Price-Only Contract

In this section, we consider the case in which the supplier offers a price-only contract characterized by a per-unit wholesale price $w$ charged by the supplier to the OEM. We solve the problem by backward induction, beginning with the selling stage.

**Selling Stage** The OEM’s problem in the selling stage is to decide whether to accept or reject the contract offered by the supplier and set the selling price $p$ (or,
equivalently, sales quantity $Q$) to maximize her profit for given quality $q$ and wholesale price $w$. If the OEM rejects the offer, she manufactures the product using her in-house option. Henceforth, we refer to the profit earned by the OEM when she uses her in-house option, as her reservation profit. Note that the OEM’s reservation profit in our model depends on product quality and is, thus, endogenous. In the selling stage, for given $q$ and $w$, the OEM solves the following optimization problem:

$$\max_{p \in \{q, w\}} \Pi_o(q, w) = \max \begin{cases} \max_{p \in \{q, w\}} (p - w) \left(1 - \frac{p}{q}\right) & \text{if OEM accepts contract,} \\
\max_{p \in \{q, w\}} (p - kcsq^2) \left(1 - \frac{p}{q}\right) & \text{if OEM uses in-house option.}
\end{cases} \tag{2.4.2}$$

The resulting profit for the supplier is

$$\Pi_s(q, w) = \begin{cases} (w - c_sq^2) \left(1 - \frac{p}{q}\right) & \text{if OEM accepts contract,} \\
0 & \text{if OEM uses in-house option.}
\end{cases}$$

The solution of the OEM’s problem (2.4.2) is summarized in Lemma 1.

**Lemma 1.** For given product quality $q$ and wholesale price $w$, the OEM’s best response is to:

(a) enter into the contract with the supplier and set $p^* = \frac{q + w}{2}$ if $w \leq kcsq^2$;
(b) manufacture using her in-house option and set $p^* = \frac{q + kcsq^2}{2}$ if $w > kcsq^2$.

The OEM’s profit at optimal price $p^*$, for given $q$ and $w$, is

$$\Pi_o^*(q, w) = \frac{q}{4} \left(1 - \frac{\min\{w, kcsq^2\}}{q}\right)^2. \tag{2.4.3}$$

**Contract Stage** In the contract stage, the supplier’s problem is to maximize his profit by choosing the wholesale price after taking into account the OEM’s best
response in the selling stage. Since we focus on situations where the supplier’s reservation profit is less than the profit he can earn by contracting with the OEM, the supplier offers the contract with wholesale price not greater than the marginal cost of the OEM’s in-house option, i.e., \( w \leq kc_s q^2 \). Thus, the supplier’s optimization problem in the contract stage is

\[
\max_{w|q} \Pi_s(q) = \max_{w|q} \left( \frac{1}{2} \right) \left( 1 - \frac{w}{q} \right) (w - c_s q^2) \quad \text{s.t.} \quad w \leq kc_s q^2.
\]

(2.4.4)

**Lemma 2.** For given product quality \( q \), the supplier’s best response is to choose wholesale price \( w^* \) such that:

(a) \( w^* = kc_s q^2 \) if \( q < \frac{1}{(2k-1)c_s} \);

(b) \( w^* = \frac{q + c_s q^2}{2} \) if \( q \geq \frac{1}{(2k-1)c_s} \).

Lemma 2 states that if the product quality chosen by the OEM is above a threshold, the supplier sets wholesale price \( w^* = \frac{q + c_s q^2}{2} \). On the other hand, if the product quality chosen by the OEM is below this threshold, the supplier sets the wholesale price such that the OEM is indifferent between accepting and rejecting the contract (i.e., \( w^* = kc_s q^2 \)). While in the latter case, the OEM earns only her reservation profit, in the former case she earns profit greater than or equal to her reservation profit.

The total supply-chain profit at the supplier’s optimal wholesale price \( w^* \) for a given quality chosen by the OEM is

\[
\Pi^*_t(q) = \begin{cases} 
\frac{q}{4} \left[ (1 - c_s q)^2 - ((k - 1) c_s q)^2 \right] & \text{if } q < \frac{1}{(2k-1)c_s}, \\
\frac{3q}{16} (1 - c_s q)^2 & \text{if } q \geq \frac{1}{(2k-1)c_s}.
\end{cases}
\]

(2.4.5)

If product quality were exogenous, the integrated firm’s profit would be \( \Pi^I(q) = \frac{q}{4} (1 - c_s q)^2 \). Clearly \( \Pi^*_t(q) < \Pi^I(q) \), implying that if product quality were exogenous, the price-only contract would not coordinate the supply chain. This conclusion
is consistent with the classic result in the supply chain literature that price-only contracts do not coordinate supply chains.

**Design Stage** In the design stage, the OEM sets product quality taking into account the supplier’s best response in the contract stage. Using superscript $P$ to denote the optimal solution under the price-only contract, the OEM’s optimization problem in the design stage is

$$
\Pi^P_o = \max \begin{cases} 
    \max_q \frac{q}{4} (1 - kc_s q)^2 & \text{s.t. } q < \frac{1}{(2k - 1)c_s}, \\
    \max_q \frac{q}{16} (1 - cs q)^2 & \text{s.t. } q \geq \frac{1}{(2k - 1)c_s}.
\end{cases}
$$

(2.4.6)

**Lemma 3.** Under the price-only contract:

(a) $q^P = \frac{1}{3kc_s}$, $\Pi^P_o = \frac{1}{27kc_s}$, and $\Pi^P_s = \frac{k-1}{27kc_s}$ for $k < 4$;

(b) $q^P = \frac{1}{3cs}$, $\Pi^P_o = \frac{1}{108cs}$, and $\Pi^P_s = \frac{1}{54cs}$ for $k \geq 4$;

(c) $\frac{\partial \Pi^P_o}{\partial k} \leq 0$ for all $k$;

(d) $\Pi^P_s \big|_{k<4} < \Pi^P_s \big|_{k\geq4}$, $\frac{\partial \Pi^P_s}{\partial k} \big|_{\{k<2\}} > 0$ and $\frac{\partial \Pi^P_s}{\partial k} \big|_{\{2\leq k<4\}} \leq 0$.

Lemma 3 shows how the competitiveness of the OEM’s in-house option relative to the supplier affects product quality and profits when the supplier offers the price-only contract. Regardless of the OEM’s competitiveness, under complete information, the OEM and the supplier enter into the contract and manufacturing is performed by the more cost-efficient player – the supplier. The OEM’s profit can be expressed as $\Pi^P_o = q^P M^P_o Q^P$, where $M^P_o$ is OEM’s margin per unit quality per unit quantity (see Table 4 in Appendix A.2). Similarly, the supplier’s profit is $\Pi^P_s = q^P M^P_s Q^P$, where $M^P_s$ is supplier’s margin per unit quality per unit quantity.

Since the OEM chooses product quality before she enters into the contract with the supplier, she faces a trade-off: should she choose lower quality $\left(\frac{1}{3kc_s}\right)$ that maximizes her reservation profit, or should she choose higher quality $\left(\frac{1}{3cs}\right)$ that maximizes total
supply-chain profit. If the OEM chooses the lower quality, she can retain a higher margin and sell a larger quantity of the product \((M_o^P = Q^P = \frac{1}{3})\) since the supplier is forced to compete with the OEM’s in-house option and, thus, to offer a competitive wholesale price. On the other hand, if the OEM chooses the higher quality, the supplier would set a higher wholesale price, which, in turn, would result in a lower margin for the OEM and a lower sales quantity \((M_o^P = Q^P = \frac{1}{6})\). As a result, the optimal product quality (see Figure 2) is non-monotonic in the OEM’s competitiveness. In particular, when the OEM’s competitiveness is sufficiently low \((k \geq 4)\), the OEM chooses higher quality than when it is sufficiently high \((k < 4)\).

Furthermore, the supplier’s profit is significantly lower for \(k < 4\) than for \(k \geq 4\). This is because when the OEM’s competitiveness is sufficiently high \((k < 4)\), the OEM, in order to retain a higher margin, chooses lower product quality, which shrinks total supply-chain profit. Further, the supplier’s profit is non-monotonic in \(k\) for \(k < 4\), first increasing and then decreasing in \(k\), and peaking at \(k = 2\). This is because the competitiveness of the OEM’s in-house option influences the supplier’s profit through two countervailing forces: as the OEM’s competitiveness decreases (i.e., \(k\) increases), the supplier is able to retain a higher proportion of the total supply-chain profit; however, the OEM chooses decreasing product quality to maximize her profit.
(endogenous) reservation profit, which, in turn, reduces total supply-chain profit. Effectively, for $2 < k < 4$, the profits of both the OEM and the supplier decrease with $k$.

Proposition 2 contrasts the performance of the decentralized supply chain under the price-only contract with that of the integrated firm (§2.3.3).

**Proposition 2.** *Comparison of the performance of the decentralized supply chain under the price-only contract with that of the integrated firm:*

(a) $q^P < q^I$ and $Q^P = Q^I$ for $k < 4$;
(b) $q^P = q^I$ and $Q^P < Q^I$ for $k \geq 4$;
(c) $\Pi^P_t < \Pi^I$ for all $k$.

As expected, we find that the price-only contract does not coordinate the supply chain (i.e., $\Pi^P_t < \Pi^I$). Given the two decisions (product quality and sales quantity) made in our supply chain context, there are multiple degrees of coordination or lack thereof. For our context, where product quality is an endogenous decision and the OEM has an in-house option, the classic double marginalization problem may manifest in the form of the OEM choosing lower product quality rather than lower sales quantity.

The OEM lowers either product quality or sales quantity depending on the competitiveness of her in-house option. When her competitiveness is sufficiently low (i.e., $k \geq 4$), the OEM’s choice of product quality is the same as that of the integrated firm ($q^P = q^I$) but the sales quantity is lower ($Q^P < Q^I$). The reason for the OEM not lowering the quality is that her in-house option is so inferior that she relies on the supplier for manufacturing the product and, hence, chooses product quality that maximizes total supply-chain profit. In contrast, when her competitiveness is sufficiently high (i.e., $k < 4$), the OEM’s choice of product quality is lower than that of the integrated firm ($q^P < q^I$) but the sales quantity is the same ($Q^P = Q^I$). The OEM chooses the lower quality because it maximizes her reservation profit.
2.4.2 Two-part Tariff Contract

In this section, we consider the scenario in which the supplier offers a two-part tariff contract \( \{w, f\} \), where \( w \) is the per-unit price and \( f \) is the lump-sum fee. We examine whether the two-part tariff contract, being more sophisticated than the price-only contract, can improve the performance of the supply chain – including whether the two-part tariff contract can coordinate the supply chain, as has been shown in the supply chain literature (Cachon and Lariviere 2005). The sequence of decisions is the same as that shown in Figure 1. We solve the problem by backward induction, beginning with the selling stage.

**Selling Stage** The OEM’s problem in the selling stage is to set the selling price (or equivalently, sales quantity) to maximize her profit for given quality \( q \) and contract parameters \( \{w, f\} \). Thus, in the selling stage, the OEM solves

\[
\max_{p\{q,w,f\}} \Pi_o(q, w, f) = \max \begin{cases} 
\max_{p\{q,w,f\}} (p - w) \left( 1 - \frac{p}{q} \right) - f & \text{if OEM accepts contract,} \\
\max_{p/q} (p - kc_s q^2) \left( 1 - \frac{p}{q} \right) & \text{if OEM uses in-house option.}
\end{cases} 
\tag{2.4.7}
\]

The resulting profit for the supplier is

\[
\Pi_s(q, w, f) = \begin{cases} 
(w - c_s q^2) \left( 1 - \frac{p}{q} \right) + f & \text{if OEM accepts contract,} \\
0 & \text{if OEM uses in-house option.}
\end{cases} 
\tag{2.4.8}
\]

The optimal solution of (2.4.7) is the same as that expressed in Lemma 1 in §2.4.1 because the best response \( p^* \) of the OEM in the selling stage does not depend on the
lump-sum fee $f$. At the optimal price $p^*$, the OEM’s profit for given $q$, $w$, and $f$ is

$$
\Pi^*_o(q, w, f) = \max \begin{cases} 
\frac{q}{4} \left( 1 - \frac{w}{q} \right)^2 - f, \\
\frac{q}{4} (1 - kc_s q^2)^2
\end{cases}
\text{using in-house option}
\text{accepting contract}
$$

**Contract Stage** The supplier’s optimization problem in the contract stage is

$$
\max_{\{w,f\}} \Pi_s(q) = \max_{\{w,f\}} \left( \frac{1}{2} \left( 1 - \frac{w}{q} \right) (w - c_s q^2) + f \right) \\
s.t. \quad \frac{q}{4} \left( 1 - \frac{w}{q} \right)^2 - f \geq \frac{q}{4} (1 - kc_s q^2)^2,
$$

where the constraint is that the OEM must not be worse off if she accepts the contract.

**Lemma 4.** *For given product quality $q$, the supplier’s best response is to offer a two-part tariff contract $\{w^*, f^*\}$ such that $w^* = c_s q^2$ and $f^* = \frac{q}{4} (1 - c_s q^2)^2 - \frac{q}{4} (1 - kc_s q^2)^2$. *

Regardless of the OEM’s choice of product quality in the design stage, under the two-part tariff contract, the supplier sets the per-unit price equal to his own marginal cost and the lump-sum fee such that the OEM is only left with her reservation profit (i.e., the profit from using her in-house option). Note that, for given $q$, $\Pi^*_s(q) = \frac{q}{4} (1 - c_s q^2)^2 = \Pi^I(q)$, implying that if product quality were exogenous, the two-part tariff contract would coordinate the supply chain. This conclusion is consistent with the classic result in the supply chain literature that two-part tariff contracts can coordinate supply chains (Cachon and Lariviere 2005). In a two-part tariff contract, a supplier achieves coordination through marginal-cost pricing, and the lump-sum fee helps allocate profits between the OEM and the supplier. However, we show that this classic result may not hold when the contract is offered (by the supplier, in our case) subsequent to product quality being chosen by the offeree (the OEM, in our case).
Design Stage  We use superscript $2P$ to denote the optimal solution under the two-part tariff contract. In the design stage, the OEM sets product quality by solving the following optimization problem:

$$
\Pi_o^{2P} = \max_q \frac{q}{4} (1 - kc_s q)^2.
$$

(2.4.10)

Lemma 5. Under the two-part tariff contract:

$$q^{2P} = \frac{1}{3kc_s}, \quad Q^{2P} = \frac{1}{3} + \frac{k-1}{6k}, \quad \Pi_o^{2P} = \frac{1}{27kc_s}, \quad \text{and} \quad \Pi_s^{2P} = \frac{(k-1)(5k-1)}{108k c_s}.$$

Proposition 3 compares the performance of the two-part tariff contract, the price-only contract, and the integrated supply chain.

Proposition 3. Comparisons among the performance of the price-only contract, the two-part tariff contract, and the integrated supply chain:

(a) $q^{2P} = q^P < q^I$ for $k < 4$ and $q^{2P} < q^P = q^I$ for $k \geq 4$;

(b) $Q^{2P} > Q^P = Q^I$ for $k < 4$ and $Q^{2P} > Q^P > Q^I$ for $k \geq 4$;

(c) $\Pi_o^{2P} = \Pi_o^P$ for $k < 4$ and $\Pi_o^{2P} < \Pi_o^P$ for $k \geq 4$;

(d) $\Pi_s^{2P} > \Pi_s^P$ for $k < 4$ and $\Pi_s^{2P} < \Pi_s^P$ for $k \geq 4$;

(e) $\Pi^I > \Pi_t^{2P} > \Pi_t^P$ for $k < 4$ and $\Pi^I > \Pi_t^P > \Pi_t^{2P}$ for $k \geq 4$.

Clearly, the two-part tariff contract fails to achieve \textit{product quality coordination}, i.e., the product quality chosen by the OEM is always less than that chosen by the integrated firm. As a result, the total supply-chain profit under the two-part tariff contract is always less than the profit of the integrated supply chain. However, the sales quantity under the two-part tariff contract exceeds that of the integrated firm; the two contract parameters available to the supplier enable him to induce the OEM to choose a higher quantity in order to compensate for the lower product quality.

Under the two-part tariff contract, we have $w^* = c_s q^*^2$ and $Q^* = 1 - \frac{c_s}{q^*} = \frac{(q^* - w^*)}{2q^*}$ (see Lemma 4 and Lemma 1(a)), which yields the following relationship: $2Q^* + c_s q^* = 1$. Thus, the optimal sales quantity and product quality are partial substitutes. If the quality chosen by the OEM is lower, the supplier offers the contract such that it induces the OEM to order a higher quantity.
As expected, the two-part tariff contract helps the supplier earn higher profit (leading to higher total supply-chain profit) than the price-only contract for \( k < 4 \). The reason is that when \( k < 4 \), the two-part tariff contract induces the OEM to choose a higher sales quantity but the same product quality as in the price-only contract, thereby resulting in a higher total supply-chain profit. Compared to the price-only contract, the two-part tariff contract provides the supplier an additional degree of freedom in retaining all supply chain profits other than the reservation profit of the OEM.

A key result in the supply chain coordination literature is that price-only contracts lead to non-supply-chain-optimal decisions in the supply chain (i.e., double marginalization), and more sophisticated contracts – such as a two-part tariff contract – can be employed by the supplier to improve supply chain performance (Cachon and Kok 2010). Further, the more sophisticated contracts are typically known as working to the advantage of the offeror (the supplier, in our case) and to the likely disadvantage of the offeree (the OEM, in our case).

However, in the presence of the OEM’s in-house option and endogenous product quality, the two-part tariff contract may not always improve total supply-chain profit,
nor the supplier’s profit, compared to the price-only contract. In fact, the two-part tariff contract, compared to the price-only contract, leaves not only the OEM but also the supplier worse off if the competitiveness of the OEM is sufficiently low ($k \geq 4$). The reason for this counterintuitive result is as follows. Under the two-part tariff contract, the supplier sets the price equal to his own marginal cost, and the lump-sum fee such that the OEM is only left with her reservation profit. The OEM takes into account the supplier’s incentive to leave – through the lump-sum fee – only her reservation profit and therefore chooses product quality that maximizes this (endogenous) reservation profit. The end result when $k \geq 4$ is that the OEM chooses lower product quality (as compared to the price-only contract), which shrinks total supply-chain profit, effectively hurting both the OEM and the supplier. Thus, in the presence of the OEM’s in-house option and endogenous product quality, the supplier’s ability to offer a two-part tariff contract could prove to be detrimental for the supplier himself.

2.5. Contracting under Asymmetric Information

In this section, we examine the impact of asymmetric information about the cost structure of the OEM’s in-house option on product quality and supply chain profits. The supplier’s cost parameter $c_s$ is common knowledge but the OEM’s cost parameter $c_o (= k_j c_s)$ is private information to the OEM. The supplier, however, has a probabilistic prior belief about the value of $k_j$. For analytical tractability, we assume that $k_j$ assumes one of two values: $k_l$ (low) or $k_h$ (high), where $k_h > k_l$. We restrict our attention to the price-only contract with $k_j < 4$, so as to focus on scenarios where the OEM’s in-house option is reasonably competitive. We denote a variable $x$ in the asymmetric information scenario by $\tilde{x}$, where $x \in \{q, w, p, Q, \Pi_o, \Pi_s, \Pi_t\}$.

---

\(^5\)Cachon and Kok (2010), in a different supply chain context, find that a two-part tariff contract can leave competing manufacturers (suppliers) worse off and the retailer better off than a price-only contract. We show that both supply chain tiers may be worse off under a two-part tariff contract compared to a price-only contract.
The supplier believes that the cost parameter of the OEM is either \(khc_s\) with probability \(\alpha\) or \(klc_s\) with probability \((1-\alpha)\). We refer to the OEM with cost parameter \(khc_s\) as the high-cost OEM and the OEM with cost parameter \(klc_s\) as the low-cost OEM.

We model the game of asymmetric information by introducing a prior move by nature that determines the OEM’s type. In the transformed game, nature moves first and decides the OEM’s type. In the design stage, the OEM sets product quality, which may signal her cost structure. The supplier observes the OEM’s choice of product quality and updates his beliefs about the OEM’s cost structure. In the contract stage, the supplier offers a price-only contract. In the selling stage, the OEM either accepts the contract or manufactures the product using her in-house option and sets the selling price of the product.

Let \(\widetilde{\Pi}_j^o[q, w]\) and \(\widetilde{\Pi}_j^s[q, w]\) denote the profits of the OEM and the supplier, respectively, when the supplier faces OEM type \(j \in \{h, l\}\), the OEM chooses quality \(\tilde{q}\), the supplier offers the contract with wholesale price \(\tilde{w}\), and the OEM optimally sets the selling price \(\tilde{p}\). Also, let \(\widetilde{\Pi}_o[q, w], \widetilde{\Pi}_s[q, w]\) and \(\widetilde{\Pi}_t[q, w]\) denote the expected profits of the OEM, the supplier, and the (total) supply chain. Let \(Pr(a|b)\) denote the conditional probability of event \(a\) occurring given that event \(b\) has already occurred. We find the equilibria of this sequential game by using the solution concept of Perfect Bayesian Equilibrium (PBE). A PBE is a set of strategies and beliefs such that, at any stage of the game, the chosen strategies are optimal given the beliefs, and the beliefs, in turn, are consistent with the optimal strategies.

**Selling Stage** In the selling stage, for given \(\tilde{q}\) and \(\tilde{w}\), the OEM’s optimal selling price is the same as that expressed in Lemma 1 in §2.4.1. Note that in the selling stage, the OEM makes her price decision with complete information. Hence, the solution for this stage has to only be subgame perfect. Similar to (2.4.3) in §2.4.1,
the optimal profit of OEM type $j$ for given $\tilde{q}$ and $\tilde{w}$ is

$$
\tilde{\Pi}_o[\tilde{q}, \tilde{w}] = \frac{\tilde{q}}{4} \left( 1 - \frac{\min \{ \tilde{w}, k_j c_s \tilde{q}^2 \} }{\tilde{q}} \right)^2.
$$

(2.5.11)

**Contract Stage**  If the supplier had complete information about the cost structure of the OEM’s in-house option and given that we restrict our analysis to $k_j < 4$, it would be optimal for the supplier to offer a contract with $\tilde{w} = k_l c_s \tilde{q}^2$ to the low-cost OEM and with $\tilde{w} = k_h c_s \tilde{q}^2$ to the high-cost OEM (note that Lemma 2(a) in §2.4.1 applies under complete information when $k_j < 4$). However, the supplier might not know which type of OEM he is facing. The supplier observes the OEM’s choice of product quality and updates his beliefs about the OEM’s cost structure in accordance with Bayes’ rule. Let $Pr (k_h | \tilde{q})$ be the supplier’s posterior (updated) belief that the OEM is high-cost and $Pr (k_l | \tilde{q})$ be his posterior belief that the OEM is low-cost, given that the OEM has chosen quality $\tilde{q}$ in the design stage. For given quality and the supplier’s posterior beliefs, there are two possibilities. First, that the supplier sets a low wholesale price $\tilde{w} \leq k_l c_s \tilde{q}^2$ and both the high- and low-cost OEMs contract with the supplier. Second, that the supplier sets a high wholesale price $k_l c_s \tilde{q}^2 < \tilde{w} \leq k_h c_s \tilde{q}^2$ and only the high-cost OEM contracts with the supplier. Neither OEM type contracts with the supplier if $\tilde{w} > k_h c_s \tilde{q}^2$. Thus, the supplier’s optimization problem in the contract stage (also see (2.4.4) in §2.4.1), given his posterior beliefs, is

$$
\max_{\tilde{w} | \tilde{q}} \tilde{\Pi}_s[\tilde{q}, \tilde{w}] = \max \begin{cases} 
\max_{\tilde{w} | \tilde{q}} \left( 1 - \frac{\tilde{w}}{\tilde{q}} \right) \left( \frac{\tilde{w} - c_s \tilde{q}^2}{2} \right) & \text{s.t. } \tilde{w} \leq k_l c_s \tilde{q}^2, \\
\max_{\tilde{w} | \tilde{q}} Pr (k_l | \tilde{q}) \left( 1 - \frac{\tilde{w}}{\tilde{q}} \right) \left( \frac{\tilde{w} - c_s \tilde{q}^2}{2} \right) & \text{s.t. } k_l c_s \tilde{q}^2 < \tilde{w} \leq k_h c_s \tilde{q}^2.
\end{cases}
$$

(2.5.12)

**Lemma 6.** Let $\tilde{w}_h (\tilde{q}) = \min \left\{ k_h c_s \tilde{q}^2, \frac{\tilde{q}+c_s \tilde{q}^2}{2} \right\}$ and $\tilde{w}_l (\tilde{q}) = \min \left\{ k_l c_s \tilde{q}^2, \frac{\tilde{q}+c_s \tilde{q}^2}{2} \right\}$. In the contract stage, the supplier’s optimal strategy for given $\tilde{q}$ is to offer the price-only contract at either $\tilde{w}_h (\tilde{q})$ or $\tilde{w}_l (\tilde{q})$ or a mix of both.
Note that \( \bar{w}_h \) and \( \bar{w}_l \) are functions of the quality \( (\bar{q}) \) chosen by the OEM. For brevity, we henceforth use \( \bar{w}_h \) and \( \bar{w}_l \) to imply \( \bar{w}_h (\bar{q}) \) and \( \bar{w}_l (\bar{q}) \), respectively. Note that \( \bar{w}_l \leq \bar{w}_h \). The optimal strategy of the supplier depends on the OEM’s choice of product quality and the resulting posterior belief about the OEM’s type.

**Design Stage**

Taking into account the supplier’s best response \( \bar{w}^* \subseteq \{\bar{w}_h, \bar{w}_l\} \) in the contract stage, the high- and low-cost OEMs’ respective optimization problems in the design stage are

\[
\max_{\bar{q}} \bar{\Pi}_o^h [\bar{q}, \bar{w}^*] = \max_{\bar{q}} \left( Pr (\bar{w}_h | \bar{q}) \bar{\Pi}_o^h [\bar{q}, \bar{w}_h] + Pr (\bar{w}_l | \bar{q}) \bar{\Pi}_o^l [\bar{q}, \bar{w}_l] \right) \tag{2.5.13}
\]

and

\[
\max_{\bar{q}} \bar{\Pi}_o^l [\bar{q}, \bar{w}^*] = \max_{\bar{q}} \left( Pr (\bar{w}_h | \bar{q}) \bar{\Pi}_o^h [\bar{q}, \bar{w}_h] + Pr (\bar{w}_l | \bar{q}) \bar{\Pi}_o^l [\bar{q}, \bar{w}_l] \right), \tag{2.5.14}
\]

where \( Pr (\bar{w}_h | \bar{q}) \) and \( Pr (\bar{w}_l | \bar{q}) \) are the conditional probabilities of the supplier offering wholesale prices \( \bar{w}_h \) and \( \bar{w}_l \), respectively, given that the OEM has chosen quality \( \bar{q} \) in the design stage.

Let \( \tilde{q}_h = \frac{1}{3 k_h c_s} \) and \( \tilde{q}_l = \frac{1}{3 k_l c_s} \). Since \( k_h > k_l \), we refer to \( \tilde{q}_h \) as low quality and \( \tilde{q}_l \) as high quality. Further, let \( \bar{w}_{hl} = k_h c_s \tilde{q}_l^2 \), \( \bar{w}_{hh} = k_h c_s \tilde{q}_h^2 \), and \( \bar{w}_{hl} = \min \left\{ k_h c_s \tilde{q}_l^2, \frac{\tilde{q}_l + c_s \tilde{q}_l^2}{2} \right\} \).

Substituting for \( \tilde{q}_h \) and \( \tilde{q}_l \), we get: \( \bar{w}_{hl} = \frac{1}{9 k_l c_s} \); \( \bar{w}_{hh} = k_h c_s \tilde{q}_h^2 = k_h \left( \frac{1}{9 k_l c_s} \right) \) when \( k_h < \frac{3 k_l + 1}{2} \), and \( \bar{w}_{hl} = \frac{\tilde{q}_l + c_s \tilde{q}_l^2}{2} = \left( \frac{3 k_l + 1}{2 k_l} \right) \left( \frac{1}{9 k_l c_s} \right) \) when \( k_l \geq \frac{3 k_l + 1}{2} \). Thus, we have \( \bar{w}_{hl} > \bar{w}_{ll} \). We therefore refer to \( \bar{w}_{hl} \) as high wholesale price and \( \bar{w}_{ll} \) as low wholesale price when the OEM chooses high quality \( \tilde{q}_l \). Lemma 7 refines the strategy spaces of the OEM and the supplier.

**Lemma 7.** (a) In the design stage, the OEM’s optimal strategy space is

\[ \{ \{\tilde{q}_l, \tilde{q}_h\}|k_h, \tilde{q}_l|k_l \}. \]

(b) In the contract stage, \( Pr (\bar{w}_{hh} | \tilde{q}_h) = 1. \)
Lemma 7(a) states that, in the design stage, the optimal strategy of the high-cost OEM is to choose either low quality ($\tilde{q}_h$) or high quality ($\tilde{q}_l$) or a mix of both, whereas the optimal strategy of the low-cost OEM is to always choose high quality. The high-cost OEM has an incentive to choose high quality over low quality because she earns a higher profit if the supplier happens to charge low wholesale price ($\tilde{w}_{ll}$) on observing high quality, i.e., $\tilde{\Pi}_h\big[\tilde{q}_l, \tilde{w}_{ll}\big] > \tilde{\Pi}_h\big[\tilde{q}_h, \tilde{w}_{hh}\big]$. However, she also faces the risk of earning a lower profit if the supplier happens to charge high wholesale price ($\tilde{w}_{hl}$) on observing high quality, i.e., $\tilde{\Pi}_h\big[\tilde{q}_l, \tilde{w}_{hl}\big] < \tilde{\Pi}_h\big[\tilde{q}_h, \tilde{w}_{hh}\big]$.

Lemma 7(b) states that, in the contract stage, offering wholesale price $\tilde{w}_{hh}$ is a strictly dominant strategy for the supplier given that the OEM has chosen low quality ($\tilde{q}_h$) in the design stage. This is so because the supplier would know with certainty that he is facing a high-cost OEM.

The supplier faces a tradeoff on observing high quality ($\tilde{q}_l$). If the supplier offers the contract with high wholesale price and the OEM turns out to be high-cost, the supplier gets the contract and earns $\tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{hl}\big]$; however, if the OEM turns out to be low-cost, the supplier fails to get the contract. On the other hand, if the supplier offers the contract with low wholesale price, the supplier gets the contract regardless of OEM type, and earns $\tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{ll}\big]$ if the OEM turns out to be high-cost and $\tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{ll}\big]$ if the OEM turns out to be low-cost. Since $\tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{hl}\big] > \tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{ll}\big] = \tilde{\Pi}_s\big[\tilde{q}_l, \tilde{w}_{ll}\big]$ (see Table 6 in Appendix A.2), the supplier faces a trade-off on observing high quality ($\tilde{q}_l$): either offer high wholesale price but with the risk of not getting the contract, or offer low wholesale price but with the certainty of getting the contract.

2.5.1 Equilibria

We denote the probability of the high-cost OEM choosing low quality ($\tilde{q}_h$) by $\beta$ and high quality ($\tilde{q}_l$) by $(1 - \beta)$, i.e., $Pr(\tilde{q}_h|k_h) = \beta$ and $Pr(\tilde{q}_l|k_h) = (1 - \beta)$. Also, given that the OEM has chosen high quality ($\tilde{q}_l$), we denote the probability of the supplier
offering high wholesale price ($\tilde{w}_{hl}$) by $\gamma$ and low wholesale price ($\tilde{w}_{ll}$) by $(1-\gamma)$, i.e., $Pr(\tilde{w}_{hl}|\tilde{q}_l) = \gamma$ and $Pr(\tilde{w}_{ll}|\tilde{q}_l) = (1-\gamma)$.

After eliminating implausible equilibria, the extensive form of the quality-signaling game is shown in Figure 4. Table 6 (see Appendix A.2) summarizes the profits of the OEM and the supplier for all plausible equilibria of the game. Note that if the supplier offers high wholesale price ($\tilde{w}_{hl}$) on observing high quality ($\tilde{q}_l$) and the OEM turns out to be low-cost, contracting does not occur and the supplier gets only his reservation profit.
Similarly, the high-cost OEM’s problem is to determine the probability distribution over her set of actions $\tilde{q}_h$ and $\tilde{q}_l$, taking into account the supplier’s best response in the contract stage. Thus, the high-cost OEM solves the following:

$$\max_{\gamma, \beta} \begin{cases} \gamma \left[ \alpha (1 - \beta) \tilde{\Pi}^h_s [\tilde{q}_l, \tilde{w}_{hl}] + (1 - \alpha) \tilde{\Pi}^l_s [\tilde{q}_l, \tilde{w}_{hl}] \right] & \text{if } \tilde{w}^* = \tilde{w}_{hl}, \\ (1 - \gamma) \left[ \alpha (1 - \beta) \tilde{\Pi}^h_s [\tilde{q}_l, \tilde{w}_{ul}] + (1 - \alpha) \tilde{\Pi}^l_s [\tilde{q}_l, \tilde{w}_{ul}] \right] & \text{if } \tilde{w}^* = \tilde{w}_{ul}. \end{cases}$$

(2.5.15)

We find the PBEs for the game by solving equations (2.5.15) and (2.5.16) simultaneously. As shown in Proposition 4, we find two PBEs: a pooling equilibrium and a semiseparating equilibrium.

**Proposition 4.** Define $X = \frac{2k_l(k_l-1)}{k_h-1}(3k_l-k_h)$ and $Y = \frac{8k_l(k_l-1)}{(3k_l-1)^2}$. Also, let

$$\zeta_p = \{ \{ \alpha < X \} \text{ and } \{ k_h < \frac{3k_l+1}{2} \} \} \cup \{ \{ \alpha < Y \} \text{ and } \{ k_h \geq \frac{3k_l+1}{2} \} \},$$

$$\zeta_{m_1} = \{ \{ \alpha \geq X \} \text{ and } \{ k_h < \frac{3k_l+1}{2} \} \}, \text{ and } \zeta_{m_2} = \{ \{ \alpha \geq Y \} \text{ and } \{ k_h \geq \frac{3k_l+1}{2} \} \}.$$

Under asymmetric information about the cost structure of the OEM’s in-house option, following are the equilibria and optimal strategy profiles for the OEM and the supplier:

(a) For $\zeta_p$, a pooling equilibrium occurs in which both the low-cost and high-cost OEMs’ strategy is to choose high quality ($\tilde{q}_l$) and the supplier’s strategy is to offer low wholesale price ($\tilde{w}_{ul}$) if he observes high quality ($\tilde{q}_l$) and to offer wholesale price $\tilde{w}_{hl}$ if he observes low quality ($\tilde{q}_h$);

Recall that choosing low quality ($\tilde{q}_h$) is the truth-telling strategy for the high-cost OEM. Similarly, choosing high quality ($\tilde{q}_l$) is the truth-telling strategy for the low-cost OEM. In contrast to the high-cost OEM, the low-cost OEM does not have an incentive to deviate from her truth-telling strategy.
(b) For $\zeta_m \cup \zeta_{m_2}$, a semiseparating equilibrium occurs. The low-cost OEM’s strategy is to choose high quality ($\bar{q}_l$). The high-cost OEM’s strategy is to choose low quality ($\bar{q}_h$) with probability $\beta^*$ and high quality ($\bar{q}_l$) with probability $(1 - \beta^*)$, where $\beta^* = \frac{\alpha - X}{\alpha (1 - X)}$ for $\zeta_m$ and $\beta^* = \frac{\alpha - Y}{\alpha (1 - Y)}$ for $\zeta_{m_2}$. The supplier’s strategy is to offer wholesale price $\bar{w}_{hh}$ if he observes low quality ($\bar{q}_h$), and to offer high wholesale price ($\bar{w}_{hl}$) with probability $\gamma^*$ and low wholesale price ($\bar{w}_{ll}$) with probability $(1 - \gamma^*)$ if he observes high quality ($\bar{q}_l$), where $\gamma^* = \frac{4k_l^2}{k_h (6k_l - k_h)}$ for $\zeta_m$ and $\gamma^* = \frac{16k_l^7 (k_h - k_l)}{k_h (k_l + 1)(7k_l - 1)}$ for $\zeta_{m_2}$.

Figure 5: Parameter Settings where the Pooling or the Semiseparating Equilibrium Occurs

Figure 5 illustrates parameter settings where the pooling or the semiseparating equilibrium occurs. Which of the equilibria occurs can be explained by whether the supplier, on observing high quality, has an incentive to set high wholesale price. If the supplier sets high – rather than low – wholesale price, he gets a higher margin but risks either losing the contract with the low-cost OEM or inducing the high-cost OEM to choose a lower sales quantity. When the difference between the costs of the high-cost OEM and the low-cost OEM (i.e., $k_h - k_l$) is relatively low, the increase in the supplier’s margin (from setting high wholesale price) is too low to compensate for the risk of either losing the contract or facing a decreased sales quantity, effectively resulting in the pooling equilibrium. However, when difference between $k_h$ and $k_l$
is above a threshold value, the increase in the supplier’s margin (from setting high wholesale price) outweighs the aforementioned risk, resulting in the semiseparating equilibrium. Moreover, the higher the probability (\(\alpha\)) of the OEM being high-cost, the lower are the odds of the supplier losing the contract when he sets high wholesale price. Therefore, the semiseparating equilibrium occurs above a threshold value of \(\alpha\) (for given \(k_h\) and \(k_l\)).

Proposition 5 outlines the impact of asymmetric information on product quality and profits in the decentralized supply chain, as compared to the scenario of complete information. As in Corbett et al. (2004) and Kaya and Ozer (2009), to enable the comparison, we denote the expected values of product quality and supply chain profits under complete information as follows:

\[
q^{PE} = \alpha q^P(k_h) + (1 - \alpha) q^P(k_l),
\]

\[
\Pi^{PE}_o = \alpha \Pi^o(k_h) + (1 - \alpha) \Pi^o(k_l),
\]

\[
\Pi^{PE}_s = \alpha \Pi^s(k_h) + (1 - \alpha) \Pi^s(k_l),
\]

\[
\Pi^{PE}_t = \Pi^{PE}_o + \Pi^{PE}_s.
\]

For the asymmetric information scenario, we label the expected equilibrium values of variables by superscript \(A\).

**Proposition 5.** Asymmetric information about the OEM’s cost structure (under the price-only contract), as compared to the scenario of complete information, results in:

(a) higher expected product quality \(\tilde{q}^A > q^{PE}\);

(b) higher expected OEM profit \(\tilde{\Pi}^A_o > \Pi^{PE}_o\) under the pooling equilibrium; \(\tilde{\Pi}^A_o = \Pi^{PE}_o\) otherwise;

(c) higher expected supplier profit \(\tilde{\Pi}^A_s > \Pi^{PE}_s\) if \((k_h - 1)(k_l - 1) > 1\); \(\tilde{\Pi}^A_s \leq \Pi^{PE}_s\) otherwise;

(d) higher expected total supply-chain profit \(\tilde{\Pi}^A_t > \Pi^{PE}_t\) under the pooling equilibrium or if \((k_h - 1)(k_l - 1) > 1\); \(\tilde{\Pi}^A_t \leq \Pi^{PE}_t\) otherwise;

(e) the possibility that contracting might not occur between the low-cost OEM and the

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*We use the term “expected” because the players’ moves are probabilistic in the semiseparating equilibrium.*
In the complete information scenario (with \( k < 4 \)), the OEM does not choose a higher quality than her reservation-profit-maximizing quality because the supplier, knowing the cost of the OEM’s in-house option, would then charge a higher wholesale price, which would leave the OEM worse off. However, in the asymmetric information scenario, the high-cost OEM chooses a higher quality than her reservation-profit-maximizing quality – with certainty in the pooling equilibrium and with a positive probability \( (1 - \beta^*) \) in the semiseparating equilibrium. Thus, expected product quality is higher under asymmetric information than under complete information (Proposition 5(a)).

While it is expected that the OEM may benefit from asymmetric information about her cost structure (Proposition 5(b)), under certain conditions, asymmetric information results in an increase in expected total supply-chain profit (Proposition 5(d); see Figures 6 and 7). This finding is in contrast to the supply chain literature that typically shows that asymmetric information leads to a loss of supply-chain efficiency (Chen 2003, Corbett 2001, Corbett et al. 2004, Ha 2001, Kaya and Ozer 2009). A nuanced exception is Kostamis and Duenyas (2011), who show that a supply

---

**Figure 6**: Regions of Expected Profit Increase under Asymmetric Information as compared to Complete Information \((\alpha = 0.5)\)
chain comprising an OEM and a supplier might be better off with two dimensions of asymmetric information rather than just one when the OEM possesses private information about the demand forecast and/or her production cost.

Even more strikingly, we find that asymmetric information can lead to strictly higher expected profit even for the supplier when \((k_h - 1)(k_l - 1) > 1\) (Proposition 5(c) and Figures 6 and 7). This finding is in contrast to the supply chain literature that typically shows that asymmetric information results in the less-informed player being worse off because the more-informed player is able to extract information rent.

\[ k_l = 2, \alpha = 0.5 \]

\[ k_l = 2, \alpha = 0.5 \]

Figure 7: Increase in Expected Profits under Asymmetric Information as compared to Complete Information

In fact, under the pooling equilibrium (i.e., for \(\zeta_p\)) and if \((k_h - 1)(k_l - 1) > 1\), both the high-cost OEM and the supplier are strictly better off under asymmetric information as compared to complete information. The reason for this counterintuitive result is as follows: so as not to risk losing the contract, the supplier may offer low wholesale price \((\tilde{w}_{hl})\) on observing high quality \((\tilde{q}_l)\). Anticipating this, even the high-cost OEM may choose high quality. In the pooling equilibrium, despite the high-cost OEM choosing high quality, the low wholesale price \((\tilde{w}_{hl})\) chosen by the supplier keeps the sales quantity the same as that under complete information (i.e., the sales quantity when the high-cost OEM chooses low quality \((\tilde{q}_h)\) and the supplier responds
with wholesale price \( w_{hh} \)).\(^8\) Thus, high quality coupled with the preservation of sales quantity by the low wholesale price \((\tilde{w}_{ll})\), leads to an increase in total supply-chain profit (as compared to complete information).\(^9\) Moreover, the high-cost OEM earns a higher profit since both high quality and low wholesale price lead to a higher margin for her. Finally, the supplier earns a higher profit because he gets a higher margin when the high-cost OEM chooses high quality \((\tilde{q}_l)\) and the supplier responds with low wholesale price \((\tilde{w}_{ll})\), as compared to if the high-cost OEM chose low quality \((\tilde{q}_h)\) and the supplier responded with wholesale price \(w_{hh}\).

In the complete information scenario, contracting always occurs between the OEM and the supplier. In contrast, in the asymmetric information scenario, contracting may not always occur (Proposition 5(e)). Specifically, if the supplier offers high wholesale price \((\tilde{w}_{hl})\) and the OEM turns out to be a low-cost OEM, contracting does not occur and the supplier gets only his reservation profit.

### 2.6. Conclusion

In this paper, we investigate implications of the relative cost efficiencies of a supplier and an OEM’s in-house option on the OEM’s choice of product design quality and subsequent contract outcomes in a supply chain where the supplier is in a position to specify contract terms. We model the problem as a dynamic game, wherein the OEM chooses product design quality (determined by the design quality of a critical component), followed by the supplier offering a contract for supplying the critical component. Thereafter, the OEM either accepts the supplier’s offer or chooses her in-house option, and sells the product in the consumer market.

We contrast two scenarios: \textit{complete information} and \textit{asymmetric information}. In

\(^8\)If the high-cost OEM were to choose high quality \((\tilde{q}_l)\) under complete information, the supplier would set high wholesale price \((\tilde{w}_{hl})\) and, consequently, the sales quantity would be lower than if the supplier set low wholesale price \((\tilde{w}_{ll})\).

\(^9\)In the pooling equilibrium, since the sales quantity and sourcing decision are the same as under complete information, the change in total supply-chain profit from asymmetric information (relative to complete information) depends only on the change in product quality.
the complete information scenario, we consider two contracts: a price-only contract and a two-part tariff contract. Contrary to intuition, we show that the supplier’s ability to offer a two-part tariff contract, compared to the price-only contract, may hurt not only the OEM (as expected) but also the supplier. Specifically, if the competitiveness of the OEM’s in-house option is sufficiently low, the two-part tariff contract – compared to the price-only contract – leaves both the OEM and the supplier worse off.

In the asymmetric information scenario, we examine the impact of OEM’s private information about the cost structure of her in-house option on product design quality and supply chain profits. We show that asymmetric information, under certain conditions, is beneficial not only for the OEM, but also for the supplier – the less-informed player. This finding is counterintuitive given that the extant literature generally shows that asymmetric information results in: (a) the less-informed player being worse off, and (b) supply chain inefficiency.

The insights obtained from our analysis are, of course, to be considered in the context of our model setup and assumptions. Relaxing some of these assumptions will afford deeper insights into the effects of endogenous product quality and the dynamic nature of contracting games. For instance, it would be interesting to study product design quality in a supply chain setting where the OEM is in the position to specify contract terms and the supplier has private information about his cost structure. Further, the production cost may depend not only on product design quality but also on investments in process improvement. Finally, the contracts considered could extend beyond the price-only and two-part tariff contracts analyzed in our work.
CHAPTER III

THE VALUE OF PRODUCT RETURNS: INTERTEMPORAL PRODUCT MANAGEMENT WITH STRATEGIC CONSUMERS

3.1. Introduction

Consumer product returns are an inevitable part of the exchange process between firms and consumers. Consumers return products for reasons such as defects, performance not meeting expectations, or remorse (Ferguson and Toktay 2006, Guide et al. 2006). In 2013, the value of consumer returns in the US alone exceeded $267 billion, or 8.6% of gross sales (TRE 2013). In many industries, such as electronics and computers, consumer returns are substantial and have been growing steadily over the past few years (Tibben-Lembke 2004). According to a 2011 Consumer Electronics Association study (CEA 2011), 27% of the consumers returned a newly purchased CE device. The most common reason for return was functional defects. Interestingly, the most popular exchange consumers made was for the same model and brand. Firms often see such returns as a costly component of doing business (Petersen and Kumar 2009, Stock et al. 2006). At the same time, returns provide an opportunity for firms to extend their product lines in the future by adding refurbished products.

In this paper, we study how a firm balances the trade-offs in devising a strategy to manage and perhaps even take advantage of product returns. Several factors influence the trade-offs. First, though refurbished products may cannibalize demand for new products, they may help the firm capture the low-end segment of the market that would not purchase a new product (Atasu et al. 2010, Guide and Li 2010, Ovchinnikov 2011). Second, refurbished products usually appear in the market several months
after new products are introduced (Guide et al. 2006). This gives rise to potential strategic behavior by consumers, who are increasingly becoming more informed and sophisticated (Li et al. 2014, Su 2007). Strategic consumers make purchase decisions based not only on what is offered today, but also on what is expected to be offered in the future (Besanko and Winston 1990). Firms have an incentive to drop prices in the future and consumers, in anticipation, reduce their willingness to pay for the product today (this effect is also referred to as the “time inconsistency problem”). The result is a reduction in the firm’s total profit (Bulow 1982, Coase 1972).

To the best of our knowledge, the impact of consumers’ strategic behavior has not received significant attention in prior research on product returns. On the one hand, product returns may further exacerbate the time inconsistency problem for the firm because returns provide an option for the firm to extend the product line in the future by offering the refurbished product in addition to the new product. On the other hand, product returns may mitigate the time inconsistency problem since refurbished products act as substitutes for new products in the future. Thus, it is unclear: (a) how product returns affect a firm’s intertemporal product line and refurbishing decisions in the presence of strategic consumers, and (b) how product returns affect a firm’s time inconsistency problem. The following three questions summarize the focus of our research:

1. How does strategic consumer behavior influence a firm’s product line and refurbishing decisions?

2. How do the firm’s product strategy and profit change with the product return rate and the perceived quality of the refurbished product?

3. How is the firm’s profit influenced by its (in)ability to credibly commit to its future decisions?

To answer these questions, we develop a two-period game-theoretic model where the
firm cannot credibly commit to its future decisions, allowing us to capture strategic consumer behavior in an intertemporal setting. In the first period, the firm offers a new product to consumers. Consumers, who are utility-maximizing, strategic, and heterogeneous in their valuations of the product, make their purchase decisions taking into account not only the net utility from purchasing in the first period but also anticipated (future) utility from purchasing in the second period. A fraction of the new units sold in the first period end up as consumer returns due to functional or cosmetic defects. The firm replaces returned units with functioning new units (consumers in our model have the incentive to seek replacements for defective units). In the second period, based on the number of new products sold in the first period and the number of consumer returns, the firm decides: (1) the quantity of returned units to refurbish, and (2) the quantity of new products to produce.

Our analysis shows that, for low return rates, the firm offers only the new product in the second period to avoid cannibalization by the refurbished product. Conversely, for sufficiently high return rates, the firm offers only the refurbished product in the second period. More importantly, we find that in the presence of strategic consumers, the impact of product returns on the firm’s profit is muted, and sometimes even positive. The reason is that a higher return rate helps the firm implicitly commit to not offer the new product in the future. This induces more consumers to buy early, thereby benefiting the firm. A novel contribution of our work is the recognition of product returns as a possible commitment device. The literature shows that allowing product returns can help reduce a consumer’s purchase risk (specifically, uncertainty about product valuation), and therefore a moderate product return rate may be optimal for the firm (Davis et al. 1998, Ketzenberg and Zuidwijk 2009, Shulman et al. 2011). However, consumers in our model do not face the risk of owning a product that does not meet their utility expectation. Therefore, our analysis provides an alternative explanation for the increase in firm profit with an increase in the return
The rest of the paper is organized as follows. In §3.2, we position our research in the context of the relevant literature. We discuss key assumptions of our model in §3.3 and derive the firm’s optimal product strategy in §3.4. In §3.5, we contrast the firm’s optimal product strategy and profit with those in the scenario where the firm can credibly commit to its future decisions. In §3.6, we discuss extensions of the model. We conclude with managerial insights in §3.7.

3.2. Literature Review

We consider the product strategy of a firm that caters to strategic consumers and where product returns from an earlier period can be refurbished for sale in a later period. Thus, our work is related to two streams of research: (1) closed-loop supply chains (CLSCs), where the management of product returns has received significant attention; and (2) the durable goods literature, which examines the challenges of selling durable products over multiple periods.

The refurbishing of product returns presents an important and growing challenge for firm operations. The internal competition between new and refurbished products is an essential concern discussed in the CLSC literature (Guide and Li 2010, Vorasayan and Ryan 2006). The complexity in managing product returns lies in the fact that the new and the refurbished products are both substitutes and complements of each other (Atasu et al. 2008). The complementarity between new and refurbished products arises in the sense that the number of cores available for refurbishing is limited by the number of new products sold in earlier periods (Ferguson and Toktay 2006, Ferrer and Swaminathan 2006). Indeed, if refurbishing is sufficiently attractive, the firm might have an incentive to deliberately underprice new products to generate more returns (Debo et al. 2005).
Papers in the CLSC stream have captured various relationships between production and remanufacturing decisions made over multiple periods. For example, in a multi-period setting, a firm’s strategy (pricing and quantity) for new and refurbished products may depend on potential competition from other firms offering new or refurbished products (Atasu et al. 2008) and on competition for the recovery of end-of-life products (Majumder and Groenevelt 2001). However, the impact of consumers’ strategic behavior has not received significant attention in this stream. We believe this is an important issue for two reasons: (i) consumers are increasingly aware that returns from the sales of new products today may induce the firm to offer refurbished products in the future (MacRumors 2012), and (ii) current purchase decisions of strategic consumers are influenced by their anticipation of the firm’s product strategy in the future (Su 2007). We show that strategic consumer behavior significantly influences a firm’s strategy for managing product returns and its decision regarding which products to offer. Specifically, we show that a firm can use returns as an implicit indication that the production of new products will be curbed in the future, which, in turn, increases a consumer’s willingness to purchase the new product earlier.

The issue of strategic consumer behavior – and its implications on a firm’s intertemporal product strategies – has been of longstanding interest in the durable goods literature. While a durable goods manufacturer might announce that she will not continue production in the future, she has the incentive to produce additional units, reduce prices, and attract new consumers when that future arrives (Coase 1972). This inconsistent behavior of the firm over time is known as “time inconsistency,” with the profit of the firm being negatively affected if consumers are strategic or forward-looking (Bulow 1982, Stokey 1981). In our model, product returns provide the firm an opportunity to extend the product line in the future with the refurbished
product that can be offered in addition to the new product. Arguably, this opportunity of extending the product line could further exacerbate the time inconsistency problem for the firm. However, in the future, refurbished products also act as a substitute for new products, which may help mitigate the time inconsistency problem. Thus, the impact of product returns on a firm’s time inconsistency problem is unclear and worth studying.

Several commitment devices have been proposed to counter the time inconsistency problem. These include: leasing as opposed to selling (Bulow 1982, Desai and Purohit 1998), planned obsolescence (Bulow 1986, Waldman 1993), or choice of product architecture (Ramachandran and Krishnan 2008). We find that as returns increase beyond a threshold, the firm would prefer to offer a greater quantity of refurbished products in the future; this enables the firm to implicitly commit to limit, and eventually eliminate, the offering of new products in the future. In summary, we propose that product returns can be a novel and practical way of addressing the time inconsistency problem under appropriate circumstances.

3.3. The Model

3.3.1 Model Assumptions and Settings

We consider a two-period model to characterize the dynamics between production and consumption decisions made at different points of time. In the first period, the firm offers a new product at price $p_1$ and consumers, who are utility-maximizing and strategic (or forward looking), decide whether to buy the product in the first period or wait until the second period. The firm’s returns policy allows consumers to exchange a defective product for another new unit at no cost; a fraction $\alpha'$ of all new products sold are returned due to functional or cosmetic defects (because consumers are utility-maximizing, they have the incentive to seek replacements for defective units; we relax this assumption in §3.6 to allow for some consumers to seek refunds
instead of replacements). Effectively, the firm receives a fraction \( \alpha = \alpha' / (1 - \alpha') \) of net sales of the new product as returns. For analytical exposition, we refer to \( \alpha \) as the return rate; however, note that \( \alpha \) is convex in \( \alpha' \) (for example, when \( \alpha' = 20\% \), \( \alpha = 25\% \)).

The firm can refurbish these returned units and sell them in the second period. However, the firm may choose to refurbish only a fraction of the returns. The prices of the new and the refurbished products in the second period are \( p_2 \) and \( p_r \), respectively. Let \( q_1 \), \( q_2 \), and \( q_r \) be the net sales (we henceforth use “sales” to imply “net sales”, i.e., quantity produced, less returns) of the new product in the first period, the new product in the second period, and the refurbished product in the second period, respectively. The sequence of decisions in our model is shown in Figure 8. We assume that, at the beginning of the first period, the firm cannot credibly commit to its future decisions. As we later show in §3.5, if the firm were able to do so, it will optimally commit to not offer the new product in the second period.

\[\text{Figure 8: Timeline of Decisions}\]

We assume that the return rate for the new product remains the same in both the periods. Further, we assume that refurbished products do not have defects, and
are therefore not returned. This is consistent with the fact that refurbished products are typically individually tested while new products are typically tested by random sampling (Atasu et al. 2008). Due to technological obsolescence, product returns in the second period are salvaged by the firm at a marginal profit of zero (without loss of generality). In §3.6, we show that relaxing these assumptions does not significantly change our main qualitative insights.

**Products and Costs:** We assume that the firm does not incur any fixed costs but incurs constant marginal costs for producing the new product and refurbishing a returned product. We denote the marginal costs of production for the new and the refurbished products by $c_n$ and $c_r$ respectively. The marginal production cost for the new product is the same in both the periods. The marginal cost of refurbishing a returned product is lower than the marginal cost of manufacturing a new product (i.e. $0 \leq c_r < c_n$).

The product is perfectly durable, that is, the product delivers the same level of service in each period, regardless of its age. We define product quality $v$ as a one-dimensional measure of the value the product delivers over its lifetime. Empirical research on refurbished products shows that consumers value a refurbished product less than its new counterpart (Guide and Li 2010, Subramanian and Subramanyam 2012). Consumers perceive the qualities of the new and the refurbished products to be $v_n$ and $v_r$, respectively, where $v_n > v_r$. We assume $v_n > c_n$ and $v_r > c_r$, which allows us to focus on non-trivial situations where the firm is active in at least one of the two periods.

**Consumers:** Consumers are heterogeneous in their valuations of product quality. We denote this characteristic of consumers by $\theta$, where a consumer of type $\theta$ is willing to pay $v\theta$ for a product of quality $v$. Thus, a consumer of type $\theta$ obtains net utility $v\theta - p$ from a product of quality $v$ offered at price $p$. For simplicity, we assume that $\theta$ is uniformly distributed between zero and one. We normalize the total market size
over the two-period planning horizon to one. Each consumer demands at most one unit of the product (new or refurbished), and consumers do not own any product at the beginning of the first period. Both the firm and consumers discount future costs, revenues, and utilities at the rate of $\rho$ per period, where $0 < \rho < 1$. Because of the durability of the product, a consumer who buys a new product in the first period exits the market.

3.3.2 Dynamics of the Game

**Sequence of Decisions:** The sequence of decisions is shown in Figure 8. First, the firm sets the price of the new product ($p_1$) at the beginning of the first period. After observing the first-period price of the new product, consumers anticipate the (future) prices of the new and the refurbished products in the second period to be $p'_2$ and $p'_r$, respectively. Based on these anticipated prices (and availability) of the new and the refurbished products, consumers decide whether to buy the new product in the first period or wait until the second period. Subsequently, in the second period, the firm decides which products to offer (new, refurbished, or both), and at what prices. Consumers who did not purchase the new product in the first period consider the product(s) offered in the second period in making their purchase decisions.

The four options for the consumers are: (1) buy a new product in the first period, (2) wait and buy a new product in the second period, (r) wait and buy a refurbished product in the second period, and (0) buy none of the products. In the first period, strategic consumers — in addition to having the foresight to consider future purchasing options — also correctly anticipate the optimal pricing reactions of the firm in the second period, based on consumers’ purchase decisions in the first period. To obtain first-period consumption decisions and second-period product offerings (and prices) that are consistent with each other, we focus on subgame perfect Nash equilibrium outcomes of this game, wherein consumers’ anticipated future prices $p'_2$ and $p'_r$, are
identical to the optimal prices $p_2^*$ and $p_r^*$ chosen by the firm. Given prices $p_1$, $p_2$, and $p_r$, the discounted net utilities of consumer type $\theta$ from the different purchasing options are given by:

$$
\begin{align*}
  u_1^\theta &= v_n\theta - p_1 \\
  u_2^\theta &= \rho(v_n\theta - p_2) \\
  u_r^\theta &= \rho(v_r\theta - p_r).
\end{align*}
$$

(3.3.17)

Demands: Let $\theta_{ij}$ represent a marginal consumer who is indifferent between the two actions $i$ and $j$, where $i$ and $j$ each represent one of the four consumer options introduced earlier ($i,j \in \{1,2,r,0\}$). For any given set of prices $p_1$, $p_2$, and $p_r$, the marginal consumers — obtained from equating respective utilities in (3.3.17) — are given in Table 1.

<table>
<thead>
<tr>
<th>$\theta_{ij}$</th>
<th>$j = 2$</th>
<th>$j = r$</th>
<th>$j = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$</td>
<td>$(p_1 - \rho p_2)/(v_n(1-\rho))$</td>
<td>$(p_1 - \rho p_r)/(v_n - \rho v_r)$</td>
<td>$p_1/v_n$</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>-</td>
<td>$(p_2 - p_r)/(v_n - v_r)$</td>
<td>$p_2/v_n$</td>
</tr>
<tr>
<td>$i = r$</td>
<td>-</td>
<td>-</td>
<td>$p_r/v_r$</td>
</tr>
</tbody>
</table>

If the firm offers both the new and the refurbished products in the second period, the demands for the different products are: $q_1 = 1 - \theta_{12}$, $q_2 = \theta_{12} - \theta_{2r}$, and $q_r = \theta_{2r} - \theta_{r0}$. If the firm does not offer the new product in the second period, the demands for the products are: $q_1 = 1 - \theta_{1r}$, and $q_r = \theta_{1r} - \theta_{r0}$. If the firm does not offer the refurbished product in the second period, the demands are $q_1 = 1 - \theta_{12}$ and $q_2 = \theta_{12} - \theta_{20}$. Finally, if the firm does not offer either the new or the refurbished product in the second period, the demand in the first period is given by $q_1 = 1 - \theta_{10}$. Note that the sales quantity of the refurbished product in the second period is constrained by the number of new products returned in the first period, i.e., $q_r \leq \alpha q_1$.

Profit Maximization: We solve the firm’s problem by backward induction, starting with the second period and ending with the first period, which yields a subgame perfect Nash equilibrium of prices and consumption decisions. In the second period,
consumers who did not buy in the first period make their purchase decisions after observing prices $p_2$ and $p_r$ for the new and the refurbished products, respectively. Note that $q_1$ consumers have already exited the market by purchasing the new product in the first period. Let $\Pi_2(p_1)$ denote the second-period profit of the firm for a given first-period price of the new product $p_1$ (or, equivalently, sales quantity $q_1$). We use asterisks to denote optimal solutions/values. The firm’s optimization problem in the second period is given by:

$$
\Pi^*_2(p_1) = \max_{\{p_2, p_r\}|p_1} \left[ q_2(\tilde{p})(p_2 - (1 + \alpha)c_n) + q_r(\tilde{p})(p_r - c_r) \right]
$$

s.t.

$$q_2(\tilde{p}) \geq 0,$$

$$\alpha q_1 \geq q_r(\tilde{p}) \geq 0,$$

where $\tilde{p}$ refers to the vector of prices $\{p_2, p_r\}$ in the second period, and $q_2(\tilde{p})$ and $q_r(\tilde{p})$ are the demands for the new and the refurbished products in the second period as functions of prices, for a given $p_1$ (or, equivalently, $q_1$). Since refurbished products are derived from the returns generated by new product sales in the first period, we impose the constraint $\alpha q_1 \geq q_r$. Note that the effective marginal cost of producing the new product is $(1 + \alpha)c_n$ since the firm has to produce $(1 + \alpha)$ units of the new product for each unit eventually sold. We denote the optimal second-period prices for the new and the refurbished products by $p^*_2(p_1)$ and $p^*_r(p_1)$, respectively.

In the first period, consumers make their purchase decisions after observing price $p_1$ and anticipating second-period prices $p^*_2(p_1)$ and $p^*_r(p_1)$. In a subgame perfect Nash equilibrium, consumers’ anticipated prices and the firm’s optimal prices are identical, that is, $p^*_2(p_1) = p^*_2(p_1)$ and $p^*_r(p_1) = p^*_r(p_1)$. The firm’s objective in the first period is to maximize its total profit over the two-period planning horizon by setting price $p_1$ for the new product, taking into account optimal second-period prices $p^*_2(p_1)$ and $p^*_r(p_1)$. Let $\Pi$ denote the total (two-period) discounted profit of the firm from selling the new product in the first period, and the new product and/or the
refurbished product in the second period. The firm’s problem at the beginning of the first period is given by:

\[
\Pi^* = \max_{p_1} [\Pi_1 (p_1) + \rho \Pi_2^* (p_1)] \\
= \max_{p_1} [q_1 (p_1) (p_1 - (1 + \alpha) c_n) + \rho \Pi_2^* (p_1)]
\]

subject to:

\[ q_1 (p_1) \geq 0, q_2^* (p_1) \geq 0, \]
\[ \alpha q_1 (p_1) \geq q_r^* (p_1) \geq 0. \]

where \( q_2^* (p_1) \) and \( q_r^* (p_1) \) are the resulting sales quantities from the second-period prices \( p_2^* (p_1) \) and \( p_r^* (p_1) \) that optimize the firm’s second-period profit in Problem 3.3.18 above.

3.4. Analysis
3.4.1 Second Period Optimization

The state of the market at the start of the second period is defined by the number of new products sold in the first period \( q_1 \), which, in turn, determines the number of units returned in the first period \( \alpha q_1 \). In the second period, given \( q_1 \) and \( \alpha \), the firm decides whether to produce any more new products and whether to refurbish the returned units from the first period. The firm chooses one of the following product strategies in the second period: offer both the new and the refurbished products (Product Line); offer only the refurbished product (Refurbished Only); offer only the new product (New Only); offer none of the products (None). We characterize the optimal second-period product strategies in Proposition 6.

**Proposition 6. Second-Period Product Strategy.**

There exist \( \bar{\alpha}, q_1(\alpha) \) and \( q_1(\alpha) \) (with \( q_1 < \bar{q}_1 \)), such that in the second period it is optimal to offer:

i) both the new and the refurbished products if \( \alpha > \bar{\alpha} \) and \( q_1 < q_1(\alpha) \) (Product Line);

ii) only the refurbished product if \( \alpha > \bar{\alpha} \) and \( q_1 \leq q_1 < \bar{q}_1 \) (Refurbished Only);
iii) only the new product if $\alpha \leq \bar{\alpha}$ and $q_1 < \bar{q}_1$ (New Only); and,
iv) none of the products if $q_1 \geq \bar{q}_1$ (None).

Proof. All Proofs are in the Appendix.

Proposition 6 demarcates the conditions under which the firm should pursue a specific strategy in the second period. If the firm has saturated the market by selling a substantial number of new products in the first period ($q_1 \geq \bar{q}_1$), then the remaining consumers’ willingness to pay in the second period is so low that it is not cost-effective for the firm to sell any more products. As a result, even if the firm has a substantial number of units returned in the first period, it is optimal to not offer any product in the second period (i.e., Proposition 6(iv)). Thus, the firm offers a product in the second period only if $q_1 < \bar{q}_1$. Moreover, the decision to offer the refurbished product in the second period depends on the relative efficiencies of producing the new and the refurbished products. When $\alpha \leq \bar{\alpha}$, the cost per unit perceived quality of the new product ($c_n(1 + \alpha)/v_n$) is lower than that of the refurbished product ($c_r/v_r$). Therefore, if $\alpha \leq \bar{\alpha}$ and $q_1 < \bar{q}_1$, the firm offers only the new product (i.e., Proposition 6(iii)).

If $\alpha > \bar{\alpha}$, then in the second period, offering the refurbished product becomes attractive for the firm. However, the sales quantity in the first period determines whether the firm should continue to offer the new product as well. If $q_1 \geq \underline{q}_1$, the willingness to pay of the remaining consumers is relatively low and the firm offers only the refurbished product in the second period (i.e., Proposition 6(ii)). However, if $q_1 < \underline{q}_1$, despite the superior cost-efficiency of the refurbished product as compared to the new product, the firm offers the new product as well because not only is the market not as saturated from sales in the first period, but also the willingness to pay of the remaining consumers is relatively high. Therefore, the firm offers both the new and the refurbished products (i.e., Proposition 6(i)). These product strategies are illustrated in Figure 9.
3.4.2 Complete Two-Period Solution

In the previous section, we characterized the firm’s second-period product strategies given the first-period sales quantity ($q_1$). In this section, we analyze the complete two-period game between the firm and the consumer. Consumers make their purchase decisions in the first period, taking into account not only the price of the new product in the first period, but also the anticipated product offering and prices in the second period. Such strategic behavior by consumers influences the firm’s product strategy and pricing decisions over both periods. Proposition 7 presents the firm’s optimal product strategy over both periods.


The firm always offers the new product in the first period.

Further, there exist $v_r^N(\alpha)$, $v_r^R(\alpha)$ such that the firm’s optimal strategy in the second period is to offer:

i) only the new product if $v_r \leq v_r^N$ (New Only);

ii) both the new and the refurbished products if $v_r^N < v_r < v_r^R$ (Product Line); and,
iii) only the refurbished product if $v_r \geq v_r^R$ (Refurbished Only).

We discuss the optimal product strategy derived in Proposition 7 in conjunction with Figure 10, which numerically illustrates the relationship between the thresholds $v_r^N$ and $v_r^R$ and the return rate $\alpha$. Since the new product is always offered in the first period, Figure 10 focuses on the product strategy in the second period.

![Figure 10: Optimal Second Period Product Strategy](image)

When the perceived quality of the refurbished product is low ($v_r \leq v_r^N$), producing the new product in the second period is more profitable than refurbishing product returns. Additionally, when the return rate $\alpha$ is low, the limited supply of returns from the first period further limits the opportunity to offer the refurbished product in the second period. Moreover, when $\alpha$ is low, the effective marginal cost of producing the new product $c_n (1 + \alpha)$ is also low. Therefore, when the perceived quality of the refurbished product and the return rate are low, the firm offers only the new product in both periods. In contrast, when both $v_r$ and $\alpha$ are sufficiently high, consumers perceive the refurbished product to be closer in quality to the new product ($v_r \geq v_r^R$) and the effective cost of producing new products $c_n (1 + \alpha)$ is high. Under
these conditions, refurbishing is more profitable than producing the new product, and
there is an ample supply of returns for refurbishing. Therefore, the firm offers only
the refurbished product in this region.

In the intermediate range, \( v_r^N < v_r < v_r^R \), offering a product line in the second
period is optimal. In this range, refurbished products are perceived to be of sufficiently
high quality that it is worthwhile for the firm to produce them; at the same time, the
new product can be sold without severe cannibalization by the refurbished product.
As a result, offering the vertically differentiated product line (i.e., both the new and
the refurbished products) is optimal. It is also worth noting from Figure 10 that
offering the product line is optimal when \( v_r \) is large but \( \alpha \) is low; this is so because
although refurbished products are highly profitable to offer in this situation, low
returns constrain their production. Therefore, the new product is included as part of
the product line in the second period to capitalize on the market opportunity.

We also study the firm’s optimal usage of returns for refurbishing. While Proposition
7 shows that the firm refurbishes returns only if \( v_r^N < v_r < v_r^R \), an important question
pertaining to the firm’s operations strategy is: how many of the returns should be
refurbished? Figure 11 answers this question. Naturally, the firm should offer the re-
furbished product if its perceived quality is sufficiently large (\( v_r > v_r^N \) in Proposition
7(ii)), refurbishing some of the returns as \( v_r \) exceeds this threshold, and refurbishing
all of the returns if \( v_r \) is sufficiently higher.

It is reasonable to expect that it would be optimal to refurbish only a fraction
of the returns if the return rate \( \alpha \) is large. Intriguingly, we find that as \( \alpha \) exceeds a
certain threshold, the firm should refurbish all of the returns. The reason is that a
high return rate adds to the effective cost \( c_n (1 + \alpha) \) of producing the new product;
the firm therefore limits the quantity of the new product in the first period, thus
curtailing the supply of returns for refurbishing. Furthermore, a high \( \alpha \) also increases
the cost of offering new products in the second period, making it more attractive to
Figure 11: Optimal Returns Management
\( (c_n = 0.5, c_r = 0.35, v_n = 1.0, \rho = 0.5) \)

offer as many refurbished units as possible in the second period before the new product is considered. Thus, the firm optimally refurbishes more of the returns even as the the return rate increases. In summary, the attractiveness of offering the refurbished product increases with the return rate, as is evident from the firm’s optimal product strategy. For sufficiently high return rates coupled with a sufficiently high perceived value of the refurbished product, the firm not only offers the refurbished product alone in the second period to avoid cannibalization from the new product, but also refurbishes all of the returns.

Overall, the firm’s optimal strategy is to always offer the new product in the first period and at least one version of the product — new or refurbished — in the second period. As the product does not improve over time, delaying its launch to the second period is naturally suboptimal. Furthermore, unless the firm sets a suboptimally low price for the new product in the first period, it is impossible for the firm to induce the entire segment of prospective consumers to buy in the first period itself. Therefore, the firm offers at least one product (new or refurbished, or both) in the second period. While this helps simplify the strategy space for the firm, it also implies that the firm
must deal with the time inconsistency problem, which we discuss in §3.5.

3.5. Return Rate and the Time Inconsistency Problem

In this section, we analyze the impact of product return rate $\alpha$ on the firm’s total profit. Since firms not only incur the cost of refurbishing product returns but also have to charge lower prices for refurbished products as compared to new products, returns are deemed undesirable and firms strive to reduce them (CEA 2011). However, we show that in the presence of strategic consumers, the opportunity to offer a refurbished product as a substitute for the new product may offset the negative effects of product returns. In fact, under certain conditions, an increase in product returns might even lead to an increase in firm profit.

The primary impact of product returns is an effective increase in the unit production cost, $c_n(1 + \alpha)$. The secondary effect, as we show in the following analysis, is that product returns in the first period decrease — and eventually eliminate — the firm’s incentive to produce the new product in the second period. This enables the firm to implicitly commit to not producing the new product in the second period. In §3.5.1, we derive the firm’s optimal strategy in the scenario where it can credibly commit to its future production and pricing decisions, and contrast it to the firm’s strategy derived in §3.4, where it cannot make such a commitment. Subsequently, in §3.5.2, we analyze the impact of the return rate on overall firm profit under both these scenarios. Our analysis shows that returns have the potential to solve the well-known time inconsistency problem for durable products.

3.5.1 Commitment and Product Strategy

Base-Case: When the Firm can Credibly Commit. We first consider the scenario in which the firm can credibly commit to its future production and pricing decisions at the beginning of the first period (we refer to this as the “commitment
The commitment scenario helps us identify drivers of firm’s profit in no-commitment scenario. We use the superscript $C$ to denote optimal solutions/values for the commitment scenario. The firm’s optimization problem is given by:

$$
\Pi^C = \max_{p_1, p_2, p_r} \left[ q_1(\vec{p}) (p_1 - (1 + \alpha) c_n) + \rho q_2(\vec{p}) (p_2 - (1 + \alpha) c_n) + \rho q_r(\vec{p}) (p_r - c_r) \right]
$$

s.t. 
$$
q_1(\vec{p}) \geq 0, \ q_2(\vec{p}) \geq 0, \ \alpha q_1(\vec{p}) \geq q_r(\vec{p}) \geq 0 \tag{3.5.20}
$$

where $\vec{p}$ is the vector of prices $\{p_1, p_2, p_r\}$ for the products in the first and second periods, and $q_1(\vec{p}), q_2(\vec{p}),$ and $q_r(\vec{p})$ are the corresponding demands for these products.

**Proposition 8. Complete Two-Period Product Strategy: Commitment Scenario.**

*If the firm can credibly commit to its future decisions at the beginning of the first period, it will not produce the new product in the second period, i.e., $q_2^C = 0$. Further, there exist $\bar{\alpha}, \bar{v}_r(\alpha)$, such that the firm’s optimal strategy in the second period is to:*

i) *not refurbish, i.e., $q_r^C = 0$, if $\alpha \leq \bar{\alpha}$;*

ii) *refurbish some of the returns, i.e., $\alpha q_1^C > q_r^C > 0$, if $\alpha > \bar{\alpha}$ and $v_r < \bar{v}_r$; and,*

iii) *refurbish all of the returns, i.e., $\alpha q_1^C = q_r^C > 0$, if $\alpha > \bar{\alpha}$ and $v_r \geq \bar{v}_r$.*

If the firm is able to commit to its future product strategy at the beginning of the first period, the firm optimally commits to not offer the new product in the second period. As prior work has shown, such a commitment allows the firm to steer sales of new products to the first period and, therefore, maximize profit (Bulow 1982). Further, when the return rate is sufficiently low (i.e., $\alpha \leq \bar{\alpha}$ in Proposition 8(i)), the firm also commits to not offer the refurbished product in the second period. Although this results in waste of returns, it induces more consumers to purchase the new product in the first period rather than wait for the (cheaper) refurbished product in
the second period. At the other extreme, if the return rate is significantly high and if the perceived quality of the refurbished product is sufficiently high (Proposition 8(iii)), it is optimal for the firm to refurbish all of the returns. These second-period strategies are depicted in Figure 12 below.

![Figure 12: Optimal Second Period Product Strategy: Commitment Scenario](image)

Effect of Commitment on Product Strategy: Comparing the results in Propositions 7 and 8 yields an important insight regarding the impact of commitment on the firm’s product strategy, especially in the second period. If the firm has the ability to credibly commit to its future actions, it will optimally commit to not offer the new product in the second period (i.e., $q^C_2 = 0$). However, if the firm cannot make such a commitment, the firm will indeed (optimally) offer the new product in the second period when $v_r < v^R_r$ (as shown in Proposition 7(ii) and illustrated in Figure 10 in §3.4.2). Furthermore, when $\alpha < \bar{\alpha}$, the firm would like to make a stronger commitment that no product — neither new nor refurbished — will be offered in the second period. However, in the absence of the ability to credibly make such a commitment, the firm always offers some combination of products (new, refurbished, or both) in
the second period.

What makes such a commitment desirable for the firm? Consumers with relatively high valuations of product quality buy the new product in the first period. Therefore, if the firm wants to sell the new product in the second period too, it has to lower the price to induce the remaining consumers — who have lower valuations of product quality — to buy the product. If the firm cannot credibly commit to its future decisions, strategic consumers take this intertemporal price difference into account and, thus, are less willing to purchase the product in the first period. However, if the firm can credibly commit to not offer the new product in the second period, it can earn a higher profit by inducing more consumers purchase the new product in the first period. It is worth noting that the commitment scenario in Bulow (1982) turns out to be a special case of the commitment scenario in our paper.

3.5.2 Returns as a Commitment Device

Managers typically perceive returns as a costly component of doing business (CEA 2011). However, we show in this section that product returns can enable a firm to implicitly commit to its future decisions, even if the firm cannot explicitly make such a commitment. We begin by comparing the firm’s profits when it can credibly commit and when it cannot.

Proposition 9. Returns as a Commitment Device.

i) The firm’s optimal profit $\Pi^*$ in the no-commitment scenario is never greater than the firm’s optimal profit $\Pi^C$ in the commitment scenario.

ii) However, there exists $\alpha_c$ such that the profits in the two scenarios are identical for $\alpha \geq \alpha_c$.

Proposition 9 shows that the time inconsistency problem exists for sufficiently low return rates, but the problem is completely eliminated for sufficiently high return rates. If the firm were able to credibly commit to its second-period strategy in the
first period, the firm would optimally not produce the new product in the second period. In contrast, if the firm is unable to credibly commit to its future decisions, it may be compelled to offer the new product in the second period as well, resulting in a lower profit. However, when the return rate is sufficiently high \((\alpha \geq \alpha_c)\), the firm’s profits in the commitment and no-commitment scenarios become identical. This is so because when the supply of returns from first-period sales is abundant, the firm optimally offers only the refurbished product in the second period even if it cannot make a credible commitment that it will not offer the new product in the second period (Proposition 6(ii)). Therefore, a high return rate allows the firm to implicitly commit to not offer the new product in the second period. Thus, a sufficiently high return rate eliminates the time inconsistency problem. This is illustrated in Figure 13, wherein the profits in the commitment and no-commitment scenarios are identical for \(\alpha \gtrsim 0.21\).

![Figure 13: Optimal Profits in the Commitment and No-Commitment Scenarios](https://example.com/figure13)

Additionally, product returns can have a counterintuitive effect on the firm’s overall profit: in a certain range of return rates, the firm’s overall (two-period) profit can increase with the return rate \(\alpha\) (for \(0.13 \lesssim \alpha \lesssim 0.18\) in Figure 13). In contrast, when
the firm can credibly commit to its future decisions, its profit *always* monotonically decreases in $\alpha$. This finding is of theoretical as well as practical importance. We explore this phenomenon further, and outline the conditions under which firm’s profit increases with $\alpha$ in Proposition 10.

**Proposition 10. Return Rates and Profits in the Commitment and No-Commitment Scenarios.**

1) **Commitment Scenario.** When the firm can credibly commit to its future decisions, its profit monotonically decreases in the return rate $\alpha$.

2) **No-Commitment Scenario.** When the firm cannot credibly commit to its future decisions, there exists $\hat{v}_r(\alpha)$ such that the firm’s profit can be non-monotonic in $\alpha$ for $v_r > \hat{v}_r$.

Practitioners logically view returns in a negative light because returns directly result in increased costs — the effective cost of producing the new product, the cost of refurbishing returns, and the loss of revenue due to the lower perceived value of the refurbished product. However, this negative view of returns is unequivocally valid only in the scenario where the firm can make a credible commitment about its future actions (Proposition 10(i)). When the firm cannot make such a commitment, the role of returns in mitigating the time inconsistency problem has an indirect (positive) impact on the firm’s profit. A higher return rate could increase firm profit by mitigating — and even eliminating — the time inconsistency problem faced by the firm; a sufficiently high return rate allows the firm to implicitly commit that the new product will be offered exclusively in the first period, and therefore be able to charge a premium for it. Further, under the conditions identified in Proposition 10(ii) and illustrated in Figure 13, as the return rate increases, the positive effect of returns in mitigating the time inconsistency problem may increasingly dominate the negative impact of returns on the production cost; this results in an increase in firm profit within a certain range $[\alpha_l, \alpha_h]$ of the return rate.
Perceived Quality of the Refurbished Product and the Value of Returns

Product returns can be an effective solution to the time inconsistency problem because the firm can use returns to implicitly commit that new products will effectively be crowded out of production in the second period. However, the value of product returns depends on the cost of refurbishing and the perceived value of the refurbished product. To understand the impacts of these two factors, we numerically consider the range of return rates $[\alpha_l, \alpha_h]$ over which firm profit increases with the return rate $\alpha$ (see Figure 14).

First, the range of return rates over which the firm’s profit increases (with respect to $\alpha$) is wider for larger values of $v_r$, the perceived value of the refurbished product. The implication is that if a firm can improve consumers’ perception of the refurbished product (e.g., through marketing efforts), not only does refurbishing become more attractive, but also a higher rate of consumer returns may become more desirable. It is also worth noting that when $v_r$ is high, the $[\alpha_l, \alpha_h]$ range can be quite significant (between 9% and 18% in Figure 14). In such a situation, the firm would, counter-intuitively, rather have a return rate of 18% than a much lower return rate of 9%.

Second, the $[\alpha_l, \alpha_h]$ range also widens with a decrease in the cost of refurbishing $c_r$.

Our work complements prior research on product returns in the sense that we provide an alternative explanation for why a moderate amount of returns may be preferable to lower returns. The extant literature shows that though generous return policies increase returns, they also reduce a consumer’s purchase risk (specifically, uncertainty about product valuation), thereby implying that a moderate amount of returns is optimal (Davis et al. 1998, Ketzenberg and Zuidwijk 2009). In this paper, we assume that the quality of the product is common knowledge and consumer valuations are deterministic, although a particular new unit may be defective and therefore exchanged for a functioning new unit. In other words, consumers in our model do not face the risk of owning a product that does not meet their utility
expectations. Thus, our analysis provides an alternative explanation for the possible increase in a firm’s profit with the return rate.

3.6. Robustness

We examine the robustness of the results to our assumptions regarding product return rates, the salvage value of returns, and consumers always exchanging a defective new unit for another new unit.

First, we relax the assumption that the return rate of the new product is the same in both the periods. It is plausible that the firm can improve (reduce) product return rates over time either from learning-by-doing as the firm becomes more experienced in producing the new product, or from improving quality control processes or using improved production technology. Therefore, we allow a lower return rate for the new product in the second period as compared to the first period. Let $\alpha_2$ be the return rate for the new product in the second period such that $\alpha_2 = k_2 \alpha$, where $0 < k_2 \leq 1$. A lower $k_2$ implies that producing the new product in the second period becomes more
attractive.\(^1\) We find that our results on the firm’s product strategy (identified in §3.4) continue to hold and that product returns continue to play a role as a commitment device. As \(k_2\) decreases, the return rate above which the firm’s profit can increase (see Proposition 10(ii)), becomes larger because the effective cost of producing the new product in the second period decreases.

Second, we relax the assumption that refurbished products are non-defective. Recall that we justified this assumption given the cumulative diagnostics and individual testing performed on refurbished products (Atasu et al. 2008, Guide et al. 2006). We allow the return rate \(\alpha_r\) for the refurbished product in the second period to be a fraction of the return rate \(\alpha\) for the new product. Thus, \(\alpha_r = k_r\alpha\) where \(k_r \geq 0\).

Again, we find that our results on the firm’s product strategy and the role of product returns as a commitment device continue to hold. In particular, we observe that as \(k_r\) increases, the return rate above which the firm’s profit can increase, becomes larger. This is so because a larger \(k_r\) means that the effective cost of refurbishing is larger, which impairs the ability of returns to mitigate the time inconsistency problem.

Third, we allow for a positive (exogenous) salvage value \(0 \leq s \leq c_n\) for product returns that accompany sales of the new product in the first and second periods.\(^2\) For returns resulting from first-period sales, the firm now has the option to refurbish them (at a margin of \(p_r - c_r\)) or salvage them for a fixed value of \(s\) per unit. Further, second period returns can now be salvaged for a positive value \(s\), whereas they did not yield any value in our main model. We would expect a positive salvage value for product returns to unequivocally improve firm profit. However, this is not always true and profit can decrease with exogenous \(s\). To understand this, recall that the reason returns serve as a commitment device is that returns make it attractive for

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\(^1\) We assume that \(k_2\) is sufficiently high such that the firm does not trivially postpone all production to the more efficient second period.

\(^2\) The salvage value \(s\) is bounded above by \(c_n\); otherwise, the firm can trivially make infinite profit by producing infinite units of the new product and salvaging all of them. Further, in this extension and as in our main model, refurbished products are not defective and, therefore, are not returned.
the firm to produce the refurbished product instead of the new product in the second period. However, when $s$ is higher, the firm’s incentive to refurbish is lower; in other words, the firm has a greater incentive to produce the new product in the second period. Therefore, as $s$ increases, the firm’s ability to use returns as a commitment device diminishes. In particular, we observe that as $s$ increases, the return rate above which the firm’s profit can increase, becomes larger.

*Fourth*, we relax the three aforementioned assumptions simultaneously. The resulting setting is that the firm produces $q_1 (1 + \alpha)$ units of the new product in the first period, of which $q_1$ units are sold and $\alpha q_1$ units are returned. In the second period, the firm refurbishes $q_r (1 + \alpha_r)$ units that were returned in the first period. $q_r$ units of the refurbished product are sold and $\alpha_r q_r$ units are again returned in the second period. In the second period, the firm also produces $q_2 (1 + \alpha_2)$ units of the new product, of which $q_2$ units are sold and $\alpha_2 q_2$ units are returned. Thus, the total quantity of returns not refurbished is $(\alpha q_1 - q_r + \alpha_2 q_2)$, for which the firm obtains a salvage value $s$ per unit. Our qualitative results again continue to hold.

*Finally*, we relax the assumption that each returned new unit is exchanged for another new unit, by considering two types of returns: returns that result in an exchange, and returns that result in a refund. Let $\alpha_e \leq \alpha$ be the proportion of consumers who exchange a defective new unit with another new unit. Thus, $\alpha - \alpha_e$ is the proportion of consumers who return defective new units for a refund. Yet again, we find that our results on the firm’s product strategy (identified in §3.4) continue to hold and that product returns continue to play a role as a commitment device. Further, we observe that when $\alpha_e < \alpha$, firm profit is lower than when $\alpha_e = \alpha$ (i.e., our main analysis). This is so because consumers who receive refunds do not contribute to firm profit in the first period.
3.7. **Conclusion**

Consumer returns — products returned due to functional or cosmetic defects — are a significant and growing concern in product categories such as computers and electronics. These returns represent a costly component of doing business, and firms often refurbish the returns to recover value. While refurbishing might allow the firm to extend its product line in a cost-efficient way in the future, it could simultaneously give rise to strategic behavior by consumers. This issue has largely been overlooked in the deep literature on closed-loop supply chains. Although refurbished products are perceived to be of lower quality as compared to new products, offering them in the future can make waiting — for better prices and wider choices — an attractive option for strategic consumers.

In this paper, we developed a model to capture the effect of strategic behavior by consumers on the intertemporal product strategy of a firm facing consumer returns. Specifically, we develop a two-period game-theoretic model that captures the temporal separation observed in practice between when a new product is launched by a firm and when its refurbished version is offered. We characterize the firm’s intertemporal product strategy when it cannot credibly commit to its future actions, and contrast it with the strategy in the scenario where the firm can credibly commit. If the firm can credibly commit to a future strategy, it will simply announce that the new product will not be offered in the future, as a way to encourage consumers to buy the new product earlier at a higher price. In reality, such commitments are seldom credible (Coase 1972) and, as a result, the firm ends up with a lower total profit.

Our model-based analysis shows that a high return rate can restore this commitment capability when refurbishing is attractive: when refurbishing is economical to the firm and when consumers’ perceived value of the refurbished product is high, the firm responds to a high return rate by simultaneously increasing the number of refurbished units and choking the production of new units in the second period. This
provides an implicit, and to our knowledge, a novel way to commit that the new product will be scarce (or, in the extreme, not available at all) if consumers choose to wait. A high return rate allows the firm to implicitly commit that the new product will be offered exclusively in the first period, and therefore be able to charge a premium for it. As a result, we find the counterintuitive result that the firm’s profit may increase with the return rate under certain conditions.

We find that our qualitative findings are robust to model assumptions such as return rate differences across products (new and refurbished) and periods. Future research can further extend our work along several dimensions. First, we assume that the return rates and the perceived values of the products are independent; however, consumers may update their opinions of product quality based on the return rates. Second, while we focus on a market full of strategic consumers to deduce the role of returns as a commitment device, a more general formulation of the problem could allow for a mix of strategic and non-strategic consumers. Third, because of our research focus, we assume the product return rate to be common knowledge between the firm and consumers. While consumers may be able to infer return rates from product discussion forums (MacRumors 2012), annual reports (for publicly traded firms; typically 10-Ks), and from industry studies (such as those cited in this paper), the firm may have asymmetrically better information on the return rate. Fourth, according to the 2011 Consumer Electronics Association study (CEA 2011), when returning a CE device, the most popular exchange consumers make is for the same model and same brand (38%), followed by a different model but the same brand (13%). 17% return a product for a different brand and 27% request some form of monetary compensation such as store credit or reimbursement. Although our analysis, including the extensions, treats the majority (65%) of these instances, the consideration of multiple products offered by the same firm or competition with another firm’s products will extend the coverage of the return instances in practice. Finally, the effect of return
rates on the incentive to create better products and processes remains unexplored
and is a valuable direction to consider in future work.
4.1. Introduction

According to a Gartner research report (Gartner 2015), consumers in mature markets upgrade their smartphones every 18 to 20 months. Consequently, as the report forecasts, the worldwide market for refurbished phones that are sold to end users is set to grow to 120 million units by 2017, with an equivalent wholesale revenue of around $14 billion. This is up from 56 million units in 2014, with an equivalent wholesale revenue of $7 billion. While only seven percent of smartphones end up in official recycling programs, 64 percent get a second lease of life with 41 percent being traded in or sold privately.

The report also reckons that the growing number of privately sold phones will stir up competition in the take-back market and drive refurbishers to engage in more aggressive marketing campaigns and new incentives. Therefore, original equipment manufacturers (OEMs) often try to deter the entry of independent third-party remanufacturers by various means. Such efforts begin at the design phase of new products. OEMs design their products such that acquisition and remanufacturing of used products by third-party remanufacturers becomes expensive and difficult. For example, the MacBook Pro with Retina Display 13” consists of proprietary pentalobe screws (making opening the device unnecessarily difficult), the battery assembly is entirely, and very solidly, glued into the case (complicating replacement), the screws and cable holding the trackpad are buried under battery (making it impossible to replace the trackpad without first removing the battery), the Retina display is a fused unit
(requiring the entire assembly to be replaced) with no protective glass (making it susceptible to break), the proprietary SSD isn’t a standard drive, and the RAM is soldered to the motherboard (making it much harder to extract and replace) (iFixit 2013). Similarly, OEMs selling smartphones and tablets too solder/glue components such as battery and memory card to logic board, fuse the display with the front glass, and use proprietary screws, making disassembly and repair difficult. Moreover, OEMs such as Apple do not share repair manuals with consumers and third-party remanufacturers. Finally, OEMs try to be proactive in acquiring the used products from consumers.

Moreover, consumers are increasingly becoming more informed and sophisticated (Li et al. 2014, Su 2007). When consumers buy products, they consider not only the products (among new, used and remanufactured products) and their selling prices but also future resale value of these products (Reardon 2015).

In this paper we show that an OEM who remanufactures used products can be better off with competition in remanufacturing from an independent third-party remanufacturer. The reason is that competition in acquisition of the used products for remanufacturing increases the resale value of the new products. As a result, the OEM can charge a higher price for the new products, thereby earning a higher overall profit.

4.2. Literature Review

Our paper spans two streams of research: (a) secondhand (used-product) markets, and (b) remanufacturing of used products.

The durable goods literature studies the behavior of a firm selling products that depreciate over time. A firm selling new products has an incentive to induce consumers to replace their used products with the new products. In the presence of a well-functioning secondhand market, consumers holding used products can sell their used
products to other consumers. Products are traded from high-valuation consumers to low-valuation consumers in a competitive secondary market, allowing consumers to update to their preferred quality. This stream of literature mainly investigates whether and when a firm benefits from the secondhand market, and whether and when a firm has incentives to eliminate the secondhand market, that is, to behave in a fashion such that there are no old products available to serve as potential substitutes for the new products. The effect of a secondhand market on the demand for the new products can be decomposed into two components: a positive resale value effect due to the option value of selling new units as they become old; and a negative substitution effect due to the (imperfect) substitutability of new and used products (Hendel and Lizzieri 1999, Waldman 1996a, Waldman 1996b, Waldman 1997). The literature is equivocal in answering whether a firm benefits from the secondhand market. While Levinthal and Purohit (1989) and Waldman (1996a) show that the presence of a secondhand market can cause a reduction in the profitability of a monopolist firm, Hendel and Lizzieri (1999) show, to the contrary, that a firm benefits from a smoothly functioning secondhand market. The literature also identifies various ways firms try to eliminate the secondhand markets such as leasing (Bulow 1982, Waldman 1997), planned obsolescence (Bulow 1986, Levinthal and Purohit 1989, Waldman 1993, Waldman 1996a, Waldman 1996b), restricting a consumer's ability to maintain the good (Hendel and Lizzieri 1999) and trade-ins (Fudenberg and Tirole 1998). However, this stream of research ignores the remanufacturing of used products and the effect of competition in remanufacturing on resale value of products and resulting impact on firm profit.

Existing research in the remanufacturing literature has largely investigated whether and when the OEMs should remanufacture the used products (also called “cores”), and whether and when the OEMs allow third-party players to remanufacturer the
used products (Atasu et al. 2008, Debo et al. 2005, Ferguson and Toktay 2006, Majumder and Groenevelt 2001). To make sound decisions, the OEMs must take into account the following: first, the presence of remanufactured products may cannibalize the demand of an OEM's new products since remanufactured products may act as low-end substitutes for the new product. Second, if the OEM chooses not to remanufacture, third-party players may collect and remanufacture the used products, creating a competition for the OEM's new products.

Literature in remanufacturing models price of used products either zero (Atasu et al. 2008, Ferguson and Toktay 2006, Majumder and Groenevelt 2001), or as a function of quantity of used products available (Debo et al. 2005) or both quantity and quality of used products (Oraiopoulos et al. 2012). Majumder and Groenevelt (2001) consider core allocation mechanism between the OEM and local remanufacturers as exogenously given and do not consider the competition for used items. However, the literature in remanufacturing implicitly assumes that the competition in remanufacturing does not influence the resale (residual) value of the new product. In our model, the price of used products depends not only on quantity and quality of used products but also on competitive environment in remanufacturing.

The literature in remanufacturing concludes that the entry of a third-party remanufacturer is detrimental for the OEM and that it is profitable for the OEM to remanufacture or collect cores to preempt third parties (Atasu et al. 2008, Debo et al. 2005, Ferguson and Toktay 2006). Majumder and Groenevelt (2001) show that OEM’s profits are higher in monopoly than in competition from a local remanufacturer. Therefore, OEM has the incentive to restrict competition from local remanufacturer by making cost of remanufacturing high and is willing to forego some of the benefits of remanufacturing in order to restrict the local remanufacturer. Ferrer and Swaminathan (2006) show that a low cost of remanufacturing causes higher participation by the OEM in the secondary market. Ferguson and Toktay (2006) find that
as the third party remanufacturer becomes more competitive and the cannibalization threat increases, the OEM increases her efforts to deter the entry of independent remanufacturers through collection of cores even if remanufacturing is not profitable for the OEM. Debo et al. (2005) show that when independent firms remanufacture the cores, the OEM incorporates lower remanufacturability (defined by the number of cores available for remanufacturing) to reduce the number of cores independent remanufacturers can collect, effectively deterring the competition. They show that keeping all else equal, a manufacturer is better off without competition in the market for remanufactured products. Atasu et al. (2008) show that under competition (either from another firm offering the new product or from a local remanufacturer) remanufacturing can become an effective marketing strategy, allowing the OEM to defend its market share via price discrimination. They show that remanufacturing is more beneficial under competition than in a monopoly setting; the tougher the competition, the more profitable is remanufacturing. This is so because remanufactured products help the OEM compete for the low-valuation consumer, who would otherwise be lost to competitors.

Oraiopoulos et al. (2012) consider resale value as an endogenous decision (dependent on quantity and quality of used products and competition from the OEM’s new products), they also show that as the number of third-party remanufacturers increases, the OEM profit increases. However, they show this result in a situation where the OEM does not participate in remanufacturing and charges a relicensing fee – an additional lever through which the OEM extracts profits from remanufacturing – to the third-party remanufacturer.

In contrast, we show that an OEM, who also remanufactures used products, can be better off with encouraging competition in remanufacturing from third-party (independent) remanufacturers. Competition can affect the OEM’s profit in two ways: the profit from remanufacturing decreases; the profit from the new product may increase
due to an increase in resale value of the new products.

4.3. Model

Consider a discrete time, two-period world inhabited by an OEM, a third-party remanufacturer (3PR) and consumers.

**Consumers:** Market consists of two consumer segments. We henceforth refer to these segments by “high segment” and “low segment” respectively. These segments differ in their preferences for product quality. We denote this characteristics of the high and low segments by \( \theta_h \) and \( \theta_l \), respectively, where \( \theta_h > \theta_l \). In particular, the high and low segments get per period utility \( \theta_hq \) and \( \theta_lq \), respectively, from a product of quality \( q \). Moreover, we denote the size of the high and low segments by \( n_h \) and \( n_l \), respectively, and assume that the size of each segment remains constant over time.

In each period, a consumer uses either zero or one unit of the products (out of new, used and remanufactured products). Consumers are strategic in the sense that they make purchase decisions in order to maximize their intertemporal net utility.

In the first period, the OEM offers a new product of quality \( q \) at price \( p_1 \) and consumers, taking into account second period options, decide whether to buy the new product. The new product can be used for two periods. However, the product depreciates over time and the units of the new product sold in the first period become “used products” in the second period. We denote the quality of the used product by \( \delta q \), where \( \delta \) can be interpreted as durability of the new product. If a consumer of type \( \theta \) keeps the used product, he derives utility \( \theta \delta q \) from using it in the second period.

However, in the second period, a consumer holding a used product can buy a new product and sell his used product to either the OEM or the 3PR, who, in turn, can remanufacture the acquired used products and sell the remanufactured products to consumers. For exposition, we assume that consumers cannot trade the used products among themselves and that the OEM and the 3PR need to remanufacture
the used products in order to resell them to consumers. We model the second period interactions among the OEM, the 3PR and consumers by a two stage game.

In the first stage, the OEM offers the new product at price $p_2$ to consumers including those who hold the used products, and consumers, taking into account expected prices of the used and the remanufactured products, decide whether to buy the new product. The cost and quality of the new product remains the same in both the periods. We denote the marginal cost of producing the new product by $c_n$.

In the second stage, the OEM and the 3PR acquire the used products from consumers and offer remanufactured products of quality $\delta_r q$ and $\tilde{\delta}_r q$, respectively. In line with empirical findings, we assume that (a) the remanufactured products are of lower quality than the new product but of higher quality than the used product, and (b) the remanufactured products offered by the 3PR are not of higher quality than those offered by the OEM. In particular, we assume $1 > \delta_r \geq \tilde{\delta}_r > \delta \geq 0$. The cost of remanufacturing a used product for the OEM and the 3PR are $c_r$ and $\tilde{c}_r$, respectively. The OEM and the 3PR compete to acquire the used products by setting prices $p_u$ and $\tilde{p}_u$, respectively. A consumer sells his used product to the player (between the OEM or the 3PR) who pays a higher price. We assume that if both the OEM and the 3PR set the same price, the consumer sells the product to the OEM. This assumption is reasonable because OEMs, in practice, have greater access to used products due to their established relationships with consumers. Let the quantities of the used products acquired by the OEM and the 3PR be $Q_u$ and $\tilde{Q}_u$, respectively. The OEM and the 3PR offer the remanufactured products at prices $p_r$ and $\tilde{p}_r$, respectively. Let the quantities of remanufactured products offered by the OEM and the 3PR be $Q_r$ and $\tilde{Q}_r$, respectively.

The OEM and the 3PR discount their profits and consumers their net utilities by a common discount factor $\rho$. We denote total profits of the OEM by $\Pi$, and the first and second period profits of the OEM by $\Pi_1$ and $\Pi_2$, respectively. Finally, we denote
profits of the OEM and the 3PR from acquisition and remanufacturing of the used products by $\Pi_r$ and $\tilde{\Pi}_r$, respectively.

To focus on the remanufacturing of the used products, we consider parameter settings in which the OEM has an incentive to offer the new product to the consumers holding the used products, and the OEM and the 3PR have incentives to acquire and remanufacture the used products. In particular, we restrict our attention to parameter settings in which it is optimal for the OEM to offer the new product to the high segment in each period but not optimal to offer the new product to the low segment in either of the periods. Moreover, we assume $\theta_h (1 - \delta) q + \theta_l \delta_r q > c_n + c_r$ to ensure that, in the second period, the OEM has an incentive to offer the new product to the high segment and the remanufactured product to the low segment. Finally, we also assume $\theta_l \delta_r q > c_r$ and $\theta_l \tilde{\delta}_r q > \tilde{c}_r$ to ensure that the OEM and the 3PR have incentives to acquire and remanufacture the used products, and sell the remanufactured products to the low segment in the second period.

4.4. Analysis

We first solve for acquisition and remanufacturing of the used products in the absence of competition from the 3PR (monopoly) and presence of competition from the 3PR (competition). Subsequently, we analyze the offering of the new products by the OEM in each period.

4.4.1 Second Period: Acquisition and Remanufacturing

4.4.1.1 Monopoly

In the second stage, the OEM (in the absence of competition from the 3PR) sets price and quantity of the used products to be acquired, and price and quantity of the
remanufactured products to be sold in the consumer market. Thus, the OEM solves

\[ \max_{p_r, Q_r, Q_r} \Pi_r = \max_{p_r, Q_u, Q_r} Q_r \left( p_r - c_r \right) - Q_u p_u \]

\[ \text{s.t.} \quad Q_r \leq Q_u \leq n_h, \]

where the constraint implies that the OEM cannot remanufacture more than the number of used products acquired by her and that the OEM cannot acquire more than the number of used products available in the market. Proposition 11 outlines the optimal solution of the OEM’s problem (4.4.21). We denote the optimal solution by superscript \( m \).

**Proposition 11.** In the monopoly, the following is the optimal solution:

(a) acquisition price of used products \( p_u^m = 0 \);

(b) acquired quantity of used products \( Q_u^m = \min \{ n_h, n_l \} \);

(c) price of remanufactured products \( p_r^m = \theta_l \delta_r q \);

(d) sales quantity of remanufactured products \( Q_r^m = \min \{ n_h, n_l \} \);

(e) the OEM profit: \( \Pi_r^m = \min \{ n_h, n_l \} (\theta_l \delta_r q - c_r) \).

Proposition 11 highlights two main points. First, the number of used products the OEM acquires is just equal to the number of remanufactured products she offers. In other words, the OEM does not have any incentive to acquire the used products more than she requires for the remanufacturing. Second, when the OEM is a monopoly, the price at which the OEM buys the used products is zero.

### 4.4.1.2 Competition

In the second stage of the second period, the OEM and the 3PR set prices and quantities of the used products to be acquired, and prices and quantities of the remanufactured products to be sold in the consumer market. Thus, the OEM and the
3PR solve

\[
\max_{p_u, \tilde{Q}_u, \tilde{p}_r, \tilde{Q}_r} \Pi_r = \max_{p_u, Q_u, p_r, Q_r} Q_r (p_r - c_r) - Q_u p_u
\]
\[
\text{s.t.} \quad Q_r \leq Q_u
\]
\[
Q_u + \tilde{Q}_u \leq n_h \quad (4.4.22)
\]

and

\[
\max_{\tilde{p}_u, \tilde{Q}_u, \tilde{p}_r, \tilde{Q}_r} \tilde{\Pi}_r = \max_{\tilde{p}_u, Q_u, \tilde{p}_r, \tilde{Q}_r} \tilde{Q}_r (\tilde{p}_r - \tilde{c}_r) - \tilde{Q}_u \tilde{p}_u
\]
\[
\text{s.t.} \quad \tilde{Q}_r \leq \tilde{Q}_u
\]
\[
Q_u + \tilde{Q}_u \leq n_h \quad (4.4.23)
\]

respectively. The constraints in (4.4.22) imply that the OEM cannot remanufacture more than the number of used products acquired by her and that the number of used products acquired by her cannot exceed the number of used products available in the market minus the number of used products acquired by the 3PR. Similarly, the constraints in (4.4.23) imply that the 3PR cannot remanufacture more than the number of used products acquired by him (the 3PR) and that the number of used products acquired him cannot exceed the number of used products available in the market minus the number of used products acquired by the OEM.

Proposition 12 outlines the equilibrium solution of the OEM’s problem (4.4.22) and the 3PR’s problems (4.4.23). We denote the optimal solution by superscript \( c \).

**Proposition 12.** In the competition, the following is the equilibrium solution:

(a) acquisition price of used products \( p_u^c = \min \{ \min \{ n_h, n_l \} \left( \frac{\min \{ n_h, n_l \} (\theta_t \delta_r q - \tilde{c}_r)}{n_h} \right), \min \{ n_h, n_l \} (\theta_t \delta_r q - c_r) \} \);

(b) quantity of used products acquired by the OEM: \( Q_u^c = n_h \) if \( \theta_t \delta_r q - c_r \geq \theta_t \tilde{\delta}_r q - \tilde{c}_r \); else \( Q_u^c = 0 \);

(c) quantity of used products acquired by the 3PR: \( \tilde{Q}_u^c = 0 \) if \( \theta_t \delta_r q - c_r \geq \theta_t \tilde{\delta}_r q - \tilde{c}_r \); else \( \tilde{Q}_u^c = n_h \);

(d) quantity of remanufactured products sold by the OEM: \( Q_r^c = \min \{ n_h, n_l \} \) if \( \theta_t \delta_r q - c_r \geq \theta_t \tilde{\delta}_r q - \tilde{c}_r \); else \( Q_r^c = 0 \);
(e) quantity of remanufactured products sold by the 3PR: \( \tilde{Q}_c^r = 0 \) if \( \theta_l \tilde{\delta}_r q - c_r \geq \theta_l \delta_r q - \tilde{c}_r \); else \( \tilde{Q}_c^r = \min \{ n_h, n_l \} \);

(f) price of remanufactured products offered by the OEM: \( p_c^r = \theta_l \delta_r q \);

(g) price of remanufactured products offered by the 3PR: \( \tilde{p}_c^r = \theta_l \tilde{\delta}_r q \);

(h) the OEM profit: \( \Pi_c^r = \min \{ n_h, n_l \} \left[ \theta_l \delta_r q - c_r - \left( \theta_l \tilde{\delta}_r q - \tilde{c}_r \right) \right] \) if \( \theta_l \delta_r q - c_r > \theta_l \delta_r q - \tilde{c}_r \); else \( \Pi_c^r = 0 \);

(i) the 3PR profit: \( \tilde{\Pi}_c^r = \min \{ n_h, n_l \} \left[ \theta_l \tilde{\delta}_r q - \tilde{c}_r - \left( \theta_l \delta_r q - c_r \right) \right] \) if \( \theta_l \delta_r q - c_r < \theta_l \tilde{\delta}_r q - \tilde{c}_r \); else \( \tilde{\Pi}_c^r = 0 \).

Note that when the OEM is a monopoly, the OEM sets the price of the used product \( p_u^m = 0 \). In contrast, when the OEM faces competition from the 3PR, the price of the used product is \( p_u^c > 0 \). Thus, competition for acquisition of the used products raises the price of the used product (\( p_u^c > p_u^m \)). Moreover, in the presence of competition, each player has the incentive to deter the competition in remanufacturing by acquiring all the used products even if the player does not remanufacture all the used products. Finally, competition in acquisition of the used products reduces the profit of the OEM from remanufacturing, that is, \( \Pi_c^r < \Pi_m^r \). In particular, when \( \theta_l \delta_r q - c_r > \theta_l \tilde{\delta}_r q - \tilde{c}_r \), the OEM’s profit from remanufacturing is positive but decreases as the 3PR becomes more competitive. Similarly, when \( \theta_l \delta_r q - c_r < \theta_l \tilde{\delta}_r q - \tilde{c}_r \), the 3PR’s profit from remanufacturing is positive but decreases as the OEM becomes more competitive. When \( \theta_l \delta_r q - c_r = \theta_l \tilde{\delta}_r q - \tilde{c}_r \), competition between the OEM and the 3PR intensifies, and none of the player makes profit from remanufacturing.

4.4.2 Second Period: New Product

4.4.2.1 Monopoly

In the first stage of the second period, the OEM sets price of the new product and sells it to the high segment, which already owns the used product. In the beginning

\(^1\)We define competitiveness of a player by difference between the perceived quality of the remanufactured product offered by the player minus her cost of remanufacturing.
of the second period, a high segment consumer has three options. First, he can keep the used product and receive a second-period net utility $\theta h \delta q$ from using it. Second, he can buy a new product at price $p_2$ and sell the used product at a price $p_u^m$ to the OEM, thereby receiving a second-period net utility $\theta h q - p_2 + p_u^m$. Third, he can buy a remanufactured product offered by the OEM at price $p_r^m$ and sell the used product at a price $p_u^m$, thereby receiving a second-period net utility either $\theta h \delta r q - p_r^m + p_u^m$.

To induce a high-segment consumer to buy the new product again in the second period, the OEM must set price of the new product in the second period such that the consumer is not worse off buying the new product in the second period; that is, $\theta h q - p_2 + p_u^m \geq \max \{\theta h \delta q, \theta h \delta r q - p_r^m + p_u^m\}$. Thus, the OEM solves

$$\max_{p_2} \Pi_2 = \max_{p_2} Q_2 (p_2 - c_n) + \Pi_r^m$$
$$\text{s.t.} \quad \theta h q - p_2 + p_u^m \geq \max \{\theta h \delta q, \theta h \delta r q - p_r^m + p_u^m\}$$  \hspace{1cm} (4.4.24)

Note that in the monopoly $p_u^m = 0$ and $p_r^m = \theta l \delta r q$ (Proposition 11). Proposition 13 outlines the equilibrium solution of the OEM’s problem (4.4.24).

**Proposition 13.** In the monopoly, the following is the equilibrium solution for the new product offered in the second period:

(a) price of the new product: $p_2^m = \theta h q - \max \{\theta h \delta q, (\theta h - \theta l) \delta r q\}$;

(b) the OEM profit: $\Pi_2^m = n_h (\theta h q - \max \{\theta h \delta q, (\theta h - \theta l) \delta r q\} - c_n) + \min \{n_h, n_l\} (\theta l \delta r q - c_r)$.

4.4.2.2 **Competition**

In the first stage of the second period, the OEM sets price of the new product and sells it to the high segment, which already owns the used product. In the beginning of the second period, a high segment consumer has three options. First, he can keep the used product and receive a second-period net utility $\theta h \delta q$ from using it. Second, he can buy a new product at price $p_2$ and sell the used product at a price $p_u^c$ (to
Thus, the OEM solves
\[ \max_{p} \quad \Pi_2 = \max_{p} Q_2 (p_2 - c_n) + \Pi^c_2 \]
subject to
\[ \theta_h q - p_2 + p_u^c \geq \max \{ \theta_h \delta q, \theta_h \delta r q - p_r^c + p_u^c \} \text{ if } \theta_l \delta r q - c_r \geq \theta_l \delta r q - \bar{c}_r \text{ and } \theta_h q - p_2 + p_u^c \geq \max \{ \theta_h \delta q, \theta_h \delta r q - \bar{p}_r^c + p_u^c \} \text{ if } \theta_l \delta r q - c_r < \theta_l \delta r q - \bar{c}_r, \]
where \( p_r^c = \theta_l \delta r q \) and \( \bar{p}_r^c = \theta_l \delta r q \). Thus, the OEM solves

\[ \theta_h q - p_2 + p_u^c \geq \max \{ \theta_h \delta q, \theta_h \delta r q - p_r^c + p_u^c \} \text{ if } \theta_l \delta r q - c_r \geq \theta_l \delta r q - \bar{c}_r \text{ and } \theta_h q - p_2 + p_u^c \geq \max \{ \theta_h \delta q, \theta_h \delta r q - \bar{p}_r^c + p_u^c \} \text{ if } \theta_l \delta r q - c_r < \theta_l \delta r q - \bar{c}_r. \]

(4.4.25)

Proposition 14 outlines the equilibrium solution of the OEM's problem (4.4.25).

**Proposition 14.** In the competition, the following is the equilibrium price of the new product offered in the second period:

(a) \( p^c_2 = \theta_h q - \max \{ \theta_h \delta q - p^c_u, (\theta_h - \theta_l) \delta q \} \) if \( \theta_l \delta r q - c_r \geq \theta_l \delta r q - \bar{c}_r; \)

(b) \( p^c_2 = \theta_h q - \max \{ \theta_h \delta q - p^c_u, (\theta_h - \theta_l) \bar{\delta} r q \} \) if \( \theta_l \delta r q - c_r < \theta_l \bar{\delta} r q - \bar{c}_r. \)

Proposition 15 outlines the optimal profit of the OEM in the second period under competition.

**Proposition 15.** In the competition, the following is the OEM profit in the second period,

(a) \( \Pi^c_2 = n_h (\theta_h q - \theta_h \delta q - c_n) + \min \{ n_h, n_l \} (\theta_l \delta r q - c_r) \) if \( \min \{ n_h, n_l \} (\theta_l \delta r q - c_r) \leq \)

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\( \theta_h \delta q - (\theta_h - \theta_l) \delta_r q \) and \( \theta_l \delta_r q - c_r \geq \theta_l \delta_r q - \bar{c}_r \) or \( \min_{n_h} \left\{ \frac{\theta_l \delta_r q - c_r}{n_t} \right\} \leq \theta_h \delta q - (\theta_h - \theta_l) \delta_r q \) and \( \theta_l \delta_r q - c_r < \theta_l \delta_r q - \bar{c}_r \);

(b) \( \Pi^c_2 = n_h (\theta_h q - (\theta_h - \theta_l) \delta_r q - c_n) + \min \left\{ n_h, n_l \right\} \left( \theta_l \delta_r q - c_r - \left( \theta_l \delta_r q - \bar{c}_r \right) \right) \) if \( \min_{n_h} \left\{ \frac{\theta_l \delta_r q - c_r}{n_h} \right\} > \theta_h \delta q - (\theta_h - \theta_l) \delta_r q \) and \( \theta_l \delta_r q - c_r \geq \theta_l \delta_r q - \bar{c}_r \);

(c) \( \Pi^c_2 = n_h \left( \theta_h q - (\theta_h - \theta_l) \bar{\delta}_r q - c_n \right) \) if \( \min_{n_h} \left\{ \frac{\theta_l \delta_r q - c_r}{n_h} \right\} > \theta_h \delta q - (\theta_h - \theta_l) \delta_r q \) and \( \theta_l \delta_r q - c_r < \theta_l \delta_r q - \bar{c}_r \).

4.4.3 First Period

In the first period, the OEM sets price of the new product and sells it to the high segment. If a high-segment consumer buys the new product in the first period, the consumer again buys the new product in the second period at price \( p^*_2 \) and sells the used product at price \( p^*_u \); as a result, the consumer gets a net utility \((1 + \rho) \theta_h q - p_1 - \rho (p^*_2 - p^*_u) \), where \( p^*_2 = p^*_2 \) and \( p^*_u = p^*_u \) in the monopoly and \( p^*_2 = p^*_2 \) and \( p^*_u = p^*_u \) in the competition. On the other hand, if the consumer does not buy the new product in the first period, he buys the new product at price \( p^*_2 \); as a result, the consumer gets a net utility \( \rho (\theta_h q - p^*_2) \).

To induce the consumer to buy in the first period, the OEM must set price of the new product in the first period such that the consumer is not worse off buying the new product in the first period; that is, \((1 + \rho) \theta_h q - p_1 - \rho (p^*_2 - p^*_u) \geq \rho (\theta_h q - p^*_2) \), which yields \( \theta_h q - p_1 + \rho p^*_u \geq 0 \). Thus, the maximum price the OEM can charge for the new product in the first period is consumer’s utility from using the new product in the first period plus the present value of the used product price (i.e. resale price of the new product offered in the first period). Thus, the OEM solves

\[
\max_{p_1} \Pi_t = \max_{p_1} Q_1 (p_1 - c_n) + \rho \Pi^*_2
\]

s.t.

\[
\theta_h q - p_1 + \rho p^*_u \geq 0. \tag{4.4.26}
\]

Proposition 16 outlines the equilibrium solution of the OEM’s problem (4.4.26).
Proposition 16. The following is the equilibrium solution in the first period:

(a) the optimal price of the new product: \( p^*_1 = \theta_h q + \rho p^*_u \);
(b) the optimal total profit of the OEM: \( \Pi^*_t = n_h (\theta_h q + \rho p^*_u - c_n) + \rho \Pi^*_2 \),

where \( p^*_u = p^*_u \) and \( \Pi^*_2 = \Pi^*_2 \) in the monopoly, and \( p^*_c \) and \( \Pi^*_c \) in the competition.

Since in the monopoly \( p^*_u = p^*_u \), the optimal new product price in the first period is \( p^*_u = \theta_h q \) and the OEM profit \( \Pi^*_m = n_h (\theta_h q - c_n) + \rho |n_h (\theta_h q - c_n) - \theta_h q| + \rho n_h (\theta_h q - c_n) + \min \{n_h, n_l\} (\theta_l \delta_r q - c_r) \).

Similarly, in the competition \( p^*_u = p^*_u \) and the optimal new product price in the first period is \( p^*_u = \theta_h q + \rho p^*_u \) and the OEM profit \( \Pi^*_c = n_h (\theta_h q + \rho p^*_u - c_n) + \rho [n_h (p^*_2 - c_n) + \min \{n_h, n_l\} (\theta_l \delta_r q - c_r)] - \rho n_h p^*_u \), where \( p^*_2 \) is the optimal price of the new product in the second period (refer Proposition 14) and \( p^*_u \) is optimal price of the used product (refer Proposition 12).

Proposition 17 outlines the effect of competition on overall profit of the OEM.

Proposition 17. Effect of competition from the 3PR on the OEM profit:

(a) \( \Pi^*_t > \Pi^*_m \) if \( \theta_l \delta_r q - c_r \geq \theta_l \delta_r q - \tilde{c}_r \) and \( \theta_h q > (\theta_h - \theta_l) \delta_r q \) or if \( \theta_l \delta_r q - c_r < \theta_l \delta_r q - \tilde{c}_r \) and \( \theta_h q > (\theta_h - \theta_l) \delta_r q \);
(b) \( \Pi^*_t = \Pi^*_m \).

The Proposition 17 states that the OEM is strictly better off (i.e. \( \Pi^*_c > \Pi^*_m \)) with competition from the 3PR if either \( \theta_h q > (\theta_h - \theta_l) \delta_r q \) and the OEM succeeds in acquiring and remanufacturing the used products or \( \theta_h q > (\theta_h - \theta_l) \delta_r q \) and the 3PR succeeds in acquiring and remanufacturing the used products. This implies that as long as keeping the used product gives a higher utility than buying the remanufactured product, the OEM is better off with competition; in that case, the OEM needs to price the new product in the second period keeping in mind only the used product, not the remanufactured product.
The following example illustrates the situation in which competition from a third-party remanufacturer benefits the OEM.

**Example 1.** Let \( \theta_h = 0.8, \theta_l = 0.5, n_h = 1, n_l = 1, q = 10, \delta = 0.4, \delta_r = 0.7, \tilde{\delta}_r = 0.6, c_n = 5.0, c_r = 2, \tilde{c}_r = 2 \) and \( \rho = 0.5 \).

**Monopoly:** The OEM optimally sets price of the used product at \( p_u^m = 0 \) and price of the remanufactured product at \( p_r^m = 3.5 \), resulting in \( \Pi_r^m = 1.5 \). In the second period, a high segment consumer has the following options: keep his used product, thereby getting a net utility 3.2; buy the new product at price \( p_2^m \) and sell the used product at price \( p_u^m \), thereby getting a net utility \( 8.0 - p_2^m + p_u^m = 8.0 - p_2^m \); buy the remanufactured product at price \( p_r^m \) and sell the used product at price \( p_u^m \), thereby getting a net utility \( 5.6 - p_r^m + p_u^m = 2.1 \). To incentivize the high segment consumer to buy the new product again in the second period, the OEM optimally sets \( p_2^m = 4.8 \) and, thus, earns a second-period profit \( \Pi_2^m = 1.3 \). In the first period, a high segment consumer’s willingness to pay for the new product is sum of the utility he gets from using it in the first period and the discounted used product price he gets in the second period. Thus, the OEM optimally sets \( p_1^m = 8.0 \) and, thus, earns a total profit \( \Pi_1^m = 3.65 \).

**Competition:** The OEM and the 3PR would earn \( 1.0 - p_u \) and \( 1.5 - \tilde{p}_u \) respectively if they acquire the used products at price \( p_u \) and \( \tilde{p}_u \) respectively. In the equilibrium, the OEM succeeds in acquiring and remanufacturing the used products. The OEM optimally sets price of the used product at \( p_u^c = 1.0 \) and price of the remanufactured product at \( p_r^c = 3.5 \), resulting in \( \Pi_r^c = 0.5 \) and \( \tilde{\Pi}_r^c = 0 \). In the second period, a high segment consumer has the following options: keep his used product, thereby getting a net utility 3.2; buy the new product at price \( p_2^c \) and sell the used product at price \( p_u^c \), thereby getting a net utility \( 8.0 - p_2^c + p_u^c = 9.0 - p_2^c \); buy the remanufactured product at price \( p_r^c \) and sell the used product at price \( p_u^c \), thereby getting a net utility \( 5.6 - p_r^c + p_u^c = 3.1 \). To incentivize the high segment consumer to buy the new product...
again in the second period, the OEM optimally sets $p_2 = 5.8$ and, thus, earns a second-period profit $\Pi_2 = 1.3$. In the first period, a high segment consumer’s willingness to pay for the new product is sum of the utility he get from using it in the first period and the discounted value of used product price he get in the second period. Thus, the OEM optimally sets $p_1 = 8.0 + \rho p_u = 8.5$ and, thus, earns a total profit $\Pi_1 = 4.15$.

A higher price of the used product enables the OEM charge a higher price for the new product in the first period. Moreover, under certain conditions, a higher price of the used product also enables the OEM charge a higher price for the new product in the second period. However, the reasons are different. In the first period, consumers are willing to pay a higher price for the new product because they expect a higher resale price of the product in the second period. In the second period, consumers are sometime willing to pay a higher price for the new products because in the second period the higher price of the used products makes selling the used products more attractive than keeping them.

If the OEM does not allow the entry of the third-party remanufacturer, the OEM cannot credibly commit to future resale value of the new product (i.e. $p_u$). If strategic consumers anticipate lower resale value (or no resale value, to be specific) of the new product, they lower their willingness to pay for the new product in the first period and sometimes in the second period as well.

However, by allowing the entry of the third-party remanufacturer, the OEM subjects herself to a competitive pressure from the third-party remanufacturer for acquisition of the used products. The competition from the 3PR plays a crucial role: it increases price of the used product and reassures consumers about the future resale value of the new product (when the new product becomes a used product). This assurance of a higher resale value of the new products increases consumers’ willingness to pay for the new products. In fact, the competition may have a positive spillover effect on the prices of the new product in both the periods, increasing the profits from
the new product sales. Though competition from the 3PR decreases the OEM’s prof-
its from the remanufacturing, the benefits for the OEM in the form of a higher price
of the new product can more than offset the losses in the remanufacturing. Thus,
overall, the competition from the 3PR may benefit the OEM.

The OEMs often consider third-party remanufacturers as a threat and try to deter
the entry of the third-party remanufacturers by various means such as designing their
products in such a way that it makes remanufacturing difficult and expensive for the
third-party remanufacturers. On the contrary, keeping the cost of remanufacturing
for the third-party remanufacturers low can actually benefit the OEM if it intensifies
the competition in acquisition of the used products. In conclusion, an OEM who
remanufactures used products can be better off with competition in remanufacturing
from an independent third-party remanufacturer.
APPENDIX A

PROOFS AND TABLES OF CHAPTER II

A.1. Proofs

A.1.1 Proposition 1

The integrated firm solves (4.4.21). First-order conditions (FOCs) with respect to
\( p \) and \( q \) yield \( p = \frac{q + c_s q^2}{2} \) and \( 2c_s q^2 (q - p) = p (p - c_s q^2) \), respectively. Solving
these FOCs simultaneously, we get two stationary points: \( \{ q^* = \frac{1}{3c_s}, p^* = \frac{2}{9c_s} \} \) and \( \{ q^* = \frac{1}{c_s}, p^* = \frac{1}{c_s} \} \). The stationary point \( \{ q^* = \frac{1}{3c_s}, p^* = \frac{2}{9c_s} \} \) yields zero sales quan-
tity and we therefore ignore it. The stationary point \( \{ q^* = \frac{1}{c_s}, p^* = \frac{1}{c_s} \} \) is the
unique optimal solution since the Hessian matrix \( H \) is negative definite at this point.
Thus, the optimal solution is \( \{ q^I = \frac{1}{3c_s}, p^I = \frac{2}{9c_s} \} \), and the optimal sales quantity
and profit are \( Q^I = \left( 1 - \frac{p^I}{q^I} \right) = \frac{1}{3} \) and \( \Pi^I = \frac{1}{27 c_s} \), respectively. ■

A.1.2 Lemma 1

The OEM solves (2.4.2) at the selling stage. If the OEM accepts the contract offered
by the supplier, the OEM solves \( \max_{p \mid \{w, q\}} (p - w) \left( 1 - \frac{p}{q} \right) \). The FOC for this problem
with respect to \( p \) yields \( p^* = \frac{q + w}{2} \). Since the second order condition (SOC) for
the problem is \( \frac{\partial^2 \Pi_o}{\partial p^2} = -\frac{2}{q} \leq 0 \), the unique optimal solution is \( p^* = \frac{q + w}{2} \) and the
corresponding optimal profits of the OEM and the supplier are \( \Pi_o^* = \frac{q}{4} \left( 1 - \frac{w}{q} \right)^2 \) and
\( \Pi_s^* = \frac{1}{2} \left( 1 - \frac{w}{q} \right) \left( w - c_s q^2 \right) \), respectively.

If the OEM rejects the contract and manufactures the product using her in-house
option, the OEM solves \( \max_{p \mid q} (p - k c_s q^2) \left( 1 - \frac{p}{q} \right) \). The unique optimal solution is
\( p^* = \frac{q + k c_s q^2}{2} \) and the corresponding optimal profits of the OEM and the supplier are
\( \Pi_o^* = \frac{q}{4} \left( 1 - k c_s q \right)^2 \) and \( \Pi_s^* = 0 \), respectively.
The OEM accepts the contract if \( \frac{q}{4} \left(1 - \frac{w}{q}\right)^2 \geq \frac{q}{4} \left(1 - kc_s q\right)^2 \), i.e., if \( w \leq kc_s q^2 \).

\[
\Pi_2 = \frac{q}{4} (1 - kc_s q)^2 \quad \text{and} \quad \Pi^*_s = \frac{(k-1)c_s q^2}{2} \quad (1-c_s q), \quad \text{respectively. When} \quad q \geq \frac{1}{(2k-1)c_s}, \quad \text{the optimal wholesale price is} \quad w^* = \frac{q + c_s q^2}{2}, \quad \text{and the resulting profits of the OEM and the supplier are} \quad \Pi^*_o = \frac{q}{16} (1 - c_s q)^2 \quad \text{and} \quad \Pi^*_s = \frac{q}{8} (1 - c_s q)^2, \quad \text{respectively.}
\]

\section{A.1.3 Lemma 2}

In the contract stage, the supplier solves (2.4.4). When \( q < \frac{1}{(2k-1)c_s} \), the optimal wholesale price is \( w^* = kc_s q^2 \), and the resulting profits of the OEM and the supplier are \( \Pi^*_o = \frac{q}{4} (1 - kc_s q)^2 \) and \( \Pi^*_s = \frac{(k-1)c_s q^2}{2} (1-c_s q) \), respectively. When \( q \geq \frac{1}{(2k-1)c_s} \), the optimal wholesale price is \( w^* = \frac{q + c_s q^2}{2} \), and the resulting profits of the OEM and the supplier are \( \Pi^*_o = \frac{q}{16} (1 - c_s q)^2 \) and \( \Pi^*_s = \frac{q}{8} (1 - c_s q)^2 \), respectively.

\section{A.1.4 Lemma 3}

In the design stage, the OEM solves (2.4.6). Let the case corresponding to the constraint \( q < \frac{1}{(2k-1)c_s} \) be denoted by “Case 1” and the case corresponding to the constraint \( q \geq \frac{1}{(2k-1)c_s} \) be denoted by “Case 2”.

Case 1 yields the optimal solution \( q^1 = \frac{1}{3kc_s} \), and the corresponding optimal profit of the OEM is \( \Pi^1_o = \frac{1}{27kc_s} \). Note that \( q^1 < \frac{1}{(2k-1)c_s} \) since \( k > 1 \). Case 2 yields the optimal solution \( q^{2a} = \frac{1}{(2k-1)c_s} \) when \( k < 2 \) and \( q^{2b} = \frac{1}{3c_s} \) when \( k \geq 2 \). The corresponding optimal profit of the OEM is \( \Pi^2_o = \frac{(k-1)^2}{4(2k-1)c_s} \) when \( k < 2 \) and \( \Pi^2_o = \frac{1}{108c_s} \) when \( k \geq 2 \).

To determine which of these quality choices (among \( q^1 \), \( q^{2a} \), and \( q^{2b} \)) is optimal for the OEM, we need to compare the OEM’s profits corresponding to quality choices \( q^1 \) and \( q^{2a} \) for \( k < 2 \), and quality choices \( q^1 \) and \( q^{2b} \) for \( k \geq 2 \).

For \( k < 2 \), we have: \( \Pi^1_o - \Pi^{2a}_o = \frac{1}{27kc_s} - \frac{(k-1)^2}{4(2k-1)c_s} \). At \( k = 1 \), \( \Pi^1_o - \Pi^{2a}_o = \frac{1}{27kc_s} > 0 \). At \( k = 2 \), \( \Pi^1_o - \Pi^{2a}_o = \frac{1}{54c_s} - \frac{1}{108c_s} > 0 \). Furthermore, \( \frac{\partial(\Pi^1_o - \Pi^{2a}_o)}{\partial k} = -\frac{1}{27k^2c_s} - \frac{2(k-1)(2-k)}{4(2k-1)c_s} < 0 \). Thus, for \( k < 2 \), the optimal quality is \( q^P = q^1 = \frac{1}{3kc_s} \). The corresponding optimal profits of the OEM and the supplier are \( \Pi^o_o = \frac{1}{27kc_s} \) and \( \Pi^o_s = \frac{k-1}{27k^2c_s} \), respectively.

For \( k \geq 2 \), we have: \( \Pi^1_o - \Pi^{2b}_o = \frac{1}{27kc_s} - \frac{1}{108c_s} \). For \( 2 \leq k \leq 4 \), since \( \Pi^1_o > \Pi^{2b}_o \), the optimal quality is \( q^P = q^1 = \frac{1}{3kc_s} \) and, the corresponding optimal profits of the...
OEM and the supplier are $\Pi_o^P = \frac{1}{27 k c_s}$ and $\Pi_s^P = \frac{k-1}{27 k^2 c_s}$, respectively. For $k \geq 4$, since $\Pi_o^{2b} \geq \Pi_o^1$, the optimal quality is $q^P = q^{2b} = \frac{1}{3c_s}$ and, the corresponding optimal profits of the OEM and the supplier are $\Pi_o^P = \frac{1}{108c_s}$ and $\Pi_s^P = \frac{1}{54c_s}$, respectively. The overall solution of (2.4.6) is summarized in Table 4 in Appendix A.2.

Since $\Pi_o^P = \frac{1}{27 k c_s}$ when $k < 4$, $\frac{\partial \Pi_o^P}{\partial k} |_{k<4} = -\frac{1}{27 k^2 c_s} |_{k<4} < 0$; $\frac{\partial \Pi_o^P}{\partial k} |_{k\geq4} = 0$.

Since $\Pi_s^P |_{k<4} = \frac{k-1}{27 k^2 c_s}$, we have: $\frac{\partial \Pi_s^P}{\partial k} |_{k<4} = -\frac{k-2}{27 k^3 c_s}$ and $\frac{\partial^2 \Pi_s^P}{\partial k^2} |_{k<4} = \frac{2(k-3)}{27 k^4 c_s}$. It follows that: $\frac{\partial \Pi_s^P}{\partial k} |_{k<2} > 0$, $\frac{\partial \Pi_s^P}{\partial k} |_{k=2} = 0$ and $\frac{\partial \Pi_s^P}{\partial k} |_{\{2<k<4\}} < 0$. Moreover, in the region $k < 4$, the supplier’s profit attains its maximum value $\Pi_s^P = \frac{1}{108c_s}$ at $k = 2$. Finally, since $\Pi_s^P |_{k\geq4} = \frac{1}{54c_s}$, it follows that $\Pi_s^P |_{k<4} < \Pi_s^P |_{k\geq4}$.

A.1.5 Proposition 2

We know that the optimal quality and sales quantity for the integrated firm are $q^I = \frac{1}{3c_s}$ and $Q^I = \frac{1}{3}$, respectively (Proposition 1). Also, under the price-only contract, we have $q^P = \frac{1}{3c_s}$ when $k < 4$ and $q^P = \frac{1}{3c_s}$ when $k \geq 4$ (Lemma 3). Thus, $q^P < q^I$ for $k < 4$ and $q^P = q^I$ for $k \geq 4$. Moreover, $Q^P = \frac{1}{3}$ when $k < 4$ and $Q^P = \frac{1}{6}$ when $k \geq 4$ (Lemma 3). Thus, $Q^P = Q^I$ for $k < 4$ and $Q^P < Q^I$ for $k \geq 4$.

We also know that $\Pi^I = \frac{1}{27 c_s}$ (Proposition 1). From Lemma 3, we get $\Pi^P = \Pi^P_o + \Pi^P_s = \frac{2k-1}{27 k c_s}$ for $k < 4$ and $\frac{\partial \Pi^P}{\partial k} |_{k<4} = -\frac{2(k-1)}{27 k^2 c_s} |_{k<4} < 0$. Thus, $\Pi^P \leq \Pi^I$ when $k < 4$. Moreover, when $k \geq 4$, we have $\Pi^P = \Pi^P_o + \Pi^P_s = \frac{1}{36c_s} < \Pi^I$.

A.1.6 Lemma 4

In the selling stage, the OEM solves (2.4.7). If the OEM accepts the contract, the unique optimal selling price is $p^* = \frac{9+w}{2}$. Substituting $p^*$ in (2.4.7) and (2.4.8), respectively, yields $\Pi_o^* = \frac{2}{3} \left(1 - \frac{w}{q}\right)^2 - f$ and $\Pi_s^* = \frac{1}{2} \left(1 - \frac{w}{q}\right) \left(w - c_s q^2\right) + f$. If the OEM manufactures using her in-house option, the unique optimal selling price is $p^* = \frac{q+k c_s q}{2}$. Substituting $p^*$ in (2.4.7) and (2.4.8), respectively, yields $\Pi_o^* = \frac{2}{3} \left(1 - k c_s q\right)^2$ and $\Pi_s^* = 0$. In the contract stage, the supplier solves (2.4.9), yielding $w^* = c_s q^2$ and $f^* = \frac{2}{3} \left(1 - c_s q\right)^2 - \frac{2}{3} \left(1 - k c_s q\right)^2$. The OEM’s profit is $\Pi_o^* = \frac{2}{3} \left(1 - k c_s q\right)^2$ and the
supplier’s profit is \( \Pi_s^* = f^* \).

\[ \text{A.1.7 Lemma 5} \]

In the design stage, the OEM solves (2.4.10), yielding optimal values \( q^{2P} = \frac{1}{3} \), \( Q^{2P} = \frac{1}{3} + \frac{k-1}{6k} \), \( \Pi_o^{2P} = \frac{1}{27kcs} \) and \( \Pi_s^{2P} = f^{2P} = \frac{(k-1)(5k-1)}{108k^2cs} \).

\[ \text{A.1.8 Proposition 3} \]

Parts (a) to (d) of the Proposition follow algebraically from comparisons among the optimal solutions for the integrated firm (Proposition 1), the price-only contract (Lemma 3), and the two-part tariff contract (Lemma 5).

From Lemma 5, we have \( \Pi_t^{2P} = \Pi_o^{2P} + \Pi_s^{2P} = \frac{(3k-1)^2}{108k^3cs} \). For \( k < 4 \), we have \( \Pi_t^{2P} - \Pi_t^{P} = \frac{(k-1)^2}{108k^3cs} > 0 \). For \( k \geq 4 \), we have \( \Pi_t^{2P} - \Pi_t^{P} = \frac{(3k-1)^2-3k^3}{108k^4cs} \). Since \( \Pi_t^{2P} - \Pi_t^{P} |_{k=4} < 0 \) and \( \frac{\partial (\Pi_t^{2P} - \Pi_t^{P})}{\partial k} |_{k=4} < 0 \), we have \( \Pi_t^{2P} - \Pi_t^{P} < 0 \) for \( k \geq 4 \). Finally, since \( \Pi_t^{P} < \Pi^I \) (Proposition 2(c)) and \( \Pi_t^{2P} - \Pi^I = -\frac{(4k-1)(k-1)^2}{108k^3cs} < 0 \), part (e) follows.

\[ \text{A.1.9 Lemma 6} \]

In the selling stage, for given \( \tilde{q} \) and \( \tilde{w} \), the OEM’s best response \( \tilde{p}^* \) is the same as that expressed in Lemma 1. In particular, if \( \tilde{w} \leq k_jc_s\tilde{q}^2 \), OEM type \( j \) enters into the contract with the supplier and sets selling price \( \tilde{p}^* = \frac{\tilde{q}+\tilde{w}}{2} \). If \( \tilde{w} > k_jc_s\tilde{q}^2 \), OEM type \( j \) manufactures using her in-house option and sets \( \tilde{p}^* = \frac{\tilde{q}+k_jc_s\tilde{q}^2}{2} \). The optimal profit of OEM type \( j \) for given \( \tilde{q} \) and \( \tilde{w} \) is expressed in (2.5.11).

If \( \tilde{w} > k_hc_s\tilde{q}^2 \), the OEM – regardless of her cost structure – rejects the contract. Therefore, \( \tilde{w} > k_hc_s\tilde{q}^2 \) cannot be optimal for the supplier. In the range \( k_lc_s\tilde{q}^2 < \tilde{w} \leq k_hc_s\tilde{q}^2 \), the supplier can contract only with the high-cost OEM and, from Lemma 2(a), it follows that it is optimal for the supplier to set \( \tilde{w}^* = \tilde{w}_h(\tilde{q}) = \min \left\{ k_hc_s\tilde{q}^2, \frac{\tilde{q}+c_s\tilde{q}^2}{2} \right\} \).

In the range \( \tilde{w} \leq k_lc_s\tilde{q}^2 \), the supplier can contract with both the high- and the
low-cost OEM. From Lemma 2(a) and the concavity of the supplier’s profit with respect to $\tilde{w}$ (see (2.4.4) in §2.4.1), it follows that it is optimal for the supplier to set $\tilde{w}^* = \tilde{w}_l(\tilde{q}) = \min \left\{ k_l c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}$.

Thus, the supplier’s optimal strategy is to choose $\tilde{w}^*$ from the space $\{\tilde{w}_h(\tilde{q}), \tilde{w}_l(\tilde{q})\}$. The optimal mix within this space is discussed in Proposition 4. ■

A.1.10 Lemma 7

Since $\tilde{w}^* \subseteq \{\tilde{w}_h, \tilde{w}_l\}$, where $\tilde{w}^* = \tilde{w}_h(\tilde{q}) = \min \left\{ k_h c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}$ and $\tilde{w}_l(\tilde{q}) = \min \left\{ k_l c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}$, the following relationship holds: $Pr(\tilde{w}_l|\tilde{q}) = 1 - Pr(\tilde{w}_h|\tilde{q})$. In the design stage, the high-cost OEM solves (2.5.13), which can be rewritten as (see (2.5.11))

$$\max \tilde{q} Pr(\tilde{w}_h|\tilde{q}) \left( \frac{\tilde{q}}{4} \right) \left( 1 - \frac{\min \left\{ k_h c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}}{\tilde{q}} \right)^2 + \max \tilde{q} Pr(\tilde{w}_l|\tilde{q}) \left( \frac{\tilde{q}}{4} \right) \left( 1 - \frac{\min \left\{ k_l c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}}{\tilde{q}} \right)^2.$$

Similarly, the low-cost OEM solves (2.5.14), which, using the relationship $k_h > k_l$, can be rewritten as

$$\max \tilde{q} \left( \frac{\tilde{q}}{4} \right) \left[ 1 - \frac{\min \left\{ k_l c_s \tilde{q}^2, \frac{\tilde{q} + c_s \tilde{q}^2}{2} \right\}}{\tilde{q}} \right]^2 = \max \left\{ \begin{array}{l} \max \tilde{q} \frac{\tilde{q}}{4} (1 - k_l c_s \tilde{q})^2 \quad \text{s.t. } \tilde{q} < \frac{1}{(2k_l - 1)c_s}, \\ \max \tilde{q} \frac{\tilde{q}}{16} (1 - c_s \tilde{q})^2 \quad \text{s.t. } \tilde{q} \geq \frac{1}{(2k_l - 1)c_s}. \end{array} \right.$$
is to charge \( \bar{\omega}^* (\tilde{q}_y) = \bar{\omega}_{hy} = \min \left\{ k_h c_s \tilde{q}_y^2, \frac{\tilde{q}_y + c_s \tilde{q}_y^2}{2} \right\}, \) i.e., \( Pr(\bar{\omega}_{hy} | \tilde{q}_y) = 1. \) Thus, the high-cost OEM’s problem is \( \max_{\tilde{q}_y} \tilde{\Pi}^h_o [\tilde{q}_y, \bar{\omega}_{hy}] = \max_{\tilde{q}_y} (\tilde{q}_y) \left[ 1 - \frac{\min \left\{ k_h c_s \tilde{q}_y^2, \frac{\tilde{q}_y + c_s \tilde{q}_y^2}{2} \right\}}{\tilde{q}_y} \right]^2 \), which yields the optimal choice of product quality for the high-cost OEM as \( \tilde{q}_y^* = \tilde{q}_h = \frac{1}{3k_h c_s} \neq \tilde{q}_l, \) since \( k_h < 4. \)

However, the high-cost OEM also has an incentive to choose \( \tilde{q}_l \) instead of \( \tilde{q}_h \) since she earns a higher profit by choosing \( \tilde{q}_l \) if the supplier happens to choose \( \bar{\omega}_l = k_l c_s \tilde{q}_l^2. \) Consequently, if the supplier observes \( \tilde{q}_l, \) he is unable to perfectly identify the OEM’s type. Thus, the high-cost OEM’s optimal strategy space is \( \{\tilde{q}_l, \tilde{q}_h\} \) and the supplier’s optimal strategy is to offer \( \bar{\omega}_{hh} = k_h c_s \tilde{q}_h^2 \) if the OEM chooses \( \tilde{q}_h, \) i.e., \( Pr(\bar{\omega}_{hh} | \tilde{q}_h) = 1. \)

\[ \Box \]

A.1.11 Proposition 4

After excluding the dominated strategies of the OEM and the supplier, the remaining possible strategies and equilibria are as shown in Figure 4. To determine whether an equilibrium is plausible, we check whether either the OEM or the supplier has an incentive to unilaterally deviate from her or his strategy. The supplier and the high-cost OEM solve (2.5.15) and (2.5.16), respectively.

Using the expressions in Table 6, we have the following: \( \tilde{\Pi}^h_o [\tilde{q}_l, \bar{\omega}_l] = \tilde{\Pi}^l_o [\tilde{q}_l, \bar{\omega}_l] = \tilde{\Pi}^l_o [\tilde{q}_l, \bar{\omega}_ll] > \tilde{\Pi}^h_o [\tilde{q}_h, \bar{\omega}_hh] > \tilde{\Pi}^h_o [\tilde{q}_l, \bar{\omega}_l]. \) Also, \( \tilde{\Pi}^h_s [\tilde{q}_h, \bar{\omega}_{hh}] > \tilde{\Pi}^l_s [\tilde{q}_l, \bar{\omega}_l] = 0 \) and \( \tilde{\Pi}^h_s [\tilde{q}_l, \bar{\omega}_l] = \tilde{\Pi}^l_s [\tilde{q}_l, \bar{\omega}_l] > 0. \) Moreover, \( \tilde{\Pi}^h_s [\tilde{q}_h, \bar{\omega}_{hh}] > \tilde{\Pi}^h_s [\tilde{q}_l, \bar{\omega}_l] \) iff \((k_h - 1)(k_l - 1) \leq 1. \)

Separating Equilibrium \( \{\{\tilde{q}_h | k_h, \tilde{q}_l | k_l\}, \{\bar{\omega}_{hh} | \tilde{q}_h, \bar{\omega}_{hh} | \tilde{q}_l\}\}: \) This equilibrium is implausible since the supplier has an incentive to deviate from the strategy of choosing high wholesale price (\( \bar{\omega}_{hh} \)) to choosing low wholesale price (\( \bar{\omega}_l \)) if he observes high quality (\( \tilde{q}_l \)), i.e., \( \tilde{\Pi}^l_s [\tilde{q}_l, \bar{\omega}_l] > \tilde{\Pi}^l_s [\tilde{q}_l, \bar{\omega}_{hh}] = 0. \)
Separating Equilibrium \{\{\tilde{q}_h|k_h, \tilde{q}_l|k_l\} \cup \{\tilde{w}_{hh}|\tilde{q}_h, \tilde{w}_{hl}|\tilde{q}_l\}\}: This equilibrium is implausible since the high-cost OEM has an incentive to deviate from choosing low quality (\(\tilde{q}_h\)) to choosing high quality (\(\tilde{q}_l\)), i.e., \(\tilde{\Pi}_o^h[\tilde{q}_l, \tilde{w}_{ll}] > \tilde{\Pi}_o^h[\tilde{q}_h, \tilde{w}_{hh}]\).

Pooling Equilibrium \{\{\tilde{q}_l|k_h, \tilde{q}_l|k_l\} \cup \{\tilde{w}_{hh}|\tilde{q}_h, \tilde{w}_{hl}|\tilde{q}_l\}\}: This equilibrium is implausible since the high-cost OEM has an incentive to deviate from choosing high quality (\(\tilde{q}_l\)) to choosing low quality (\(\tilde{q}_h\)), i.e., \(\tilde{\Pi}_o^h[\tilde{q}_h, \tilde{w}_{hh}] > \tilde{\Pi}_o^h[\tilde{q}_l, \tilde{w}_{hl}]\).

Pooling Equilibrium \{\{\tilde{q}_l|k_h, \tilde{q}_l|k_l\} \cup \{\tilde{w}_{hh}|\tilde{q}_h, \tilde{w}_{hl}|\tilde{q}_l\}\}: Since \(\tilde{\Pi}_o^h[\tilde{q}_l, \tilde{w}_{ll}] > \tilde{\Pi}_o^h[\tilde{q}_h, \tilde{w}_{hh}]\), the high-cost OEM does not have an incentive to deviate from choosing high quality (\(\tilde{q}_l\)). Moreover, the supplier would not have an incentive to deviate from choosing low wholesale price (\(\tilde{w}_{ll}\)) to choosing high wholesale price (\(\tilde{w}_{hl}\)) on observing high quality (\(\tilde{q}_l\)) if

\[
\alpha \tilde{\Pi}_s^h[\tilde{q}_l, \tilde{w}_{ll}] + (1 - \alpha) \tilde{\Pi}_s^l[\tilde{q}_l, \tilde{w}_{ll}] > \alpha \tilde{\Pi}_s^h[\tilde{q}_l, \tilde{w}_{hl}] + (1 - \alpha) \tilde{\Pi}_s^l[\tilde{q}_l, \tilde{w}_{hl}].
\] (A.1.27)

Denote \(X = \frac{2k_l(k_l-1)}{(k_h-1)(3k_l-k_h)}\) and \(Y = \frac{8k_l(k_l-1)}{(3k_l-1)^2}\). When \(k_h < \frac{3k_l+1}{2}\), (A.1.27) holds if \(\alpha < X\); and when \(k_h \geq \frac{3k_l+1}{2}\), (A.1.27) holds if \(\alpha < Y\).

Thus, the pooling equilibrium \{\{\tilde{q}_l|k_h, \tilde{q}_l|k_l\} \cup \{\tilde{w}_{hh}|\tilde{q}_h, \tilde{w}_{hl}|\tilde{q}_l\}\} occurs for \(\zeta_p\), where

\[
\zeta_p = \{\alpha < X\} \cup \{k_h < \frac{3k_l+1}{2}\} \cup \{\alpha < Y\} \cup \{k_h \geq \frac{3k_l+1}{2}\}.
\]

Semiseparating Equilibrium \{\{\tilde{q}_h, \tilde{q}_l|k_h, \tilde{q}_l|k_l\} \cup \{\tilde{w}_{hh}|\tilde{q}_h, \tilde{w}_{hl}, \tilde{w}_{ll}|\tilde{q}_l\}\}: In this equilibrium, the high-cost OEM and the supplier choose mixed strategies. The high-cost OEM’s strategy is to randomize between choosing low quality (\(\tilde{q}_h\)) and high quality (\(\tilde{q}_l\)). Similarly, on observing high quality (\(\tilde{q}_l\)), the supplier’s optimal strategy is to randomize between offering high wholesale price (\(\tilde{w}_{hl}\)) and low wholesale price (\(\tilde{w}_{ll}\)).
The high-cost OEM’s optimal strategy is to choose low quality ($\tilde{q}_h$) with probability $\beta^*$ and high quality ($\tilde{q}_t$) with probability $(1 - \beta^*)$ such that the supplier, on observing high quality ($\tilde{q}_t$), is indifferent between offering high wholesale price ($\tilde{w}_{hl}$) and low wholesale price ($\tilde{w}_{ll}$), that is (see (2.5.15)),

$$\alpha (1 - \beta^*) \Pi^h_s[\tilde{q}_t, \tilde{w}_{hl}] + (1 - \alpha) \Pi^l_s[\tilde{q}_t, \tilde{w}_{hl}] = \alpha (1 - \beta^*) \Pi^h_s[\tilde{q}_t, \tilde{w}_{ll}] + (1 - \alpha) \Pi^l_s[\tilde{q}_t, \tilde{w}_{ll}] .$$

(A.1.28) yields $\beta^* = \frac{\alpha - X}{\alpha(1 - X)}$ when $k_h < \frac{3k_l + 1}{2}$ and $\beta^* = \frac{\alpha - Y}{\alpha(1 - Y)}$ when $k_h \geq \frac{3k_l + 1}{2}$, where $X = \frac{2k_l(k_l - 1)}{(k_h - 1)(3k_l - k_h)}$ and $Y = \frac{8k_l(k_l - 1)(3k_l - k_h)}{(3k_l - 1)^2}$. When $k_h < \frac{3k_l + 1}{2}$, we have $0 \leq \beta^* \leq 1$ for $\alpha \geq X$, and when $k_h \geq \frac{3k_l + 1}{2}$, we have $0 \leq \beta^* \leq 1$ for $\alpha \geq Y$. Thus, for $\zeta_{m1} \cup \zeta_{m2}$, where $\zeta_{m1} = \{ \{ \alpha \geq X \} \text{ and } \{ k_h < \frac{3k_l + 1}{2} \} \}$, and $\zeta_{m2} = \{ \{ \alpha \geq Y \} \text{ and } \{ k_h \geq \frac{3k_l + 1}{2} \} \}$, the high-cost OEM chooses $\tilde{q}_h$ with probability $\beta^*$ and $\tilde{q}_t$ with probability $(1 - \beta^*)$.

The supplier’s optimal strategy is to offer high wholesale price ($\tilde{w}_{hl}$) with probability $\gamma^*$ and low wholesale price ($\tilde{w}_{ll}$) with probability $(1 - \gamma^*)$ such that the high-cost OEM is indifferent between choosing low quality ($\tilde{q}_h$) and high quality ($\tilde{q}_t$), that is (see (2.5.16)),

$$\Pi^h_o[\tilde{q}_h, \tilde{w}_{hh}] = \gamma^* \Pi^h_o[\tilde{q}_t, \tilde{w}_{hl}] + (1 - \gamma^*) \Pi^h_o[\tilde{q}_t, \tilde{w}_{ll}] .$$

(A.1.29) yields $\gamma^* = \frac{4k_l^2}{k_h(5k_l - k_h)}$ when $k_h < \frac{3k_l + 1}{2}$ and $\gamma^* = \frac{16k_l^2(k_h - k_l)}{k_h(k_h + 1)(7k_l - 1)}$ when $k_h \geq \frac{3k_l + 1}{2}$. Since we restrict our attention to $k_j < 4$, $\gamma^*$ always satisfies $0 \leq \gamma^* \leq 1$.

Thus, the semiseparating equilibrium $\{ \{ \tilde{q}_h, \tilde{q}_t \} \mid k_h, \tilde{q}_t \} \cup \{ \tilde{w}_{hh}, \tilde{q}_h, \tilde{w}_{hl}, \tilde{w}_{ll} \} \{ \tilde{q}_t \}$ occurs for $\zeta_{m1} \cup \zeta_{m2}$.

\section*{A.1.12 Proposition 5}

Using the expressions in Table 4 in Appendix A.2 for the scenario of complete information, we have $q^{P^e} = \alpha \left( \frac{1}{3k_h c_s} \right) + (1 - \alpha) \left( \frac{1}{3k_l c_s} \right)$, $\Pi^{P^e}_o = \alpha \left( \frac{1}{27k_h c_s} \right) + (1 - \alpha) \left( \frac{1}{27k_l c_s} \right)$, $\Pi^{P^e}_s = \alpha \left( \frac{k_h - 1}{27k_h c_s} \right) + (1 - \alpha) \left( \frac{k_l - 1}{27k_l c_s} \right)$ and $\Pi^{P^e}_t = \Pi^{P^e}_o + \Pi^{P^e}_s = \alpha \left( \frac{2k_h - 1}{27k_h c_s} \right) + (1 - \alpha) \left( \frac{2k_l - 1}{27k_l c_s} \right)$.
Within the asymmetric information scenario, we consider both the pooling equilibrium (i.e., for $\zeta_p$) as well as the semiseparating equilibrium (i.e., for $\zeta_{m1} \cup \zeta_{m2}$).

**Product Quality:** In the pooling equilibrium, $q^A = \frac{1}{3k_c s}$ and $q^A - q^{PE} = \frac{0}{3k_c s}$. In the semiseparating equilibrium, $q^A = (1 - \beta^*) \left( \frac{1}{3k_c s} \right) + (1 - \alpha^*) \left( \frac{1}{3k_c s} \right)$ and $q^A - q^{PE} = (1 - \beta^*) \left( \frac{1}{3k_c s} - \frac{1}{3k_c s} \right)$. Thus, $q^A > q^{PE}$.

**OEM Profit:** The expected profit of the OEM is

$$\Pi^A_o = \alpha \Pi^h_o [q^*, \tilde{w}^*] + (1 - \alpha) \Pi^l_o [q^*, \tilde{w}^*]$$

$$= \alpha \left[ \beta^* \Pi^h_o [q_h, \tilde{w}_{hh}] + (1 - \beta^*) \left( \gamma^* \Pi^h_o [q_l, \tilde{w}_{hl}] + (1 - \gamma^*) \Pi^h_o [q_l, \tilde{w}_{ul}] \right) \right]$$

$$+ (1 - \alpha) \left[ \gamma^* \Pi^l_o [q_l, \tilde{w}_{hl}] + (1 - \gamma^*) \Pi^l_o [q_l, \tilde{w}_{ul}] \right] .$$

(A.1.30)

In the pooling equilibrium (i.e., $\beta^* = 0$ and $\gamma^* = 0$), $\Pi^h_o [q^*, \tilde{w}^*] = \Pi^l_o [q^*, \tilde{w}^*] = \frac{1}{27k_c s}$ and, thus, $\Pi^A_o - \Pi^{PE}_o = \alpha \left( \frac{1}{27k_c s} - \frac{1}{27k_c s} \right) > 0$. In the semiseparating equilibrium, from (A.1.29) and (A.1.30) we get $\Pi^h_o [q^*, \tilde{w}^*] = \frac{1}{27k_c s}$ and $\Pi^l_o [q^*, \tilde{w}^*] = \frac{1}{27k_c s}$, yielding $\Pi^A_o - \Pi^{PE}_o = 0$.

**Supplier Profit:** The expected profit of the supplier is

$$\Pi^A_s = \alpha \left[ \beta^* \Pi^h_s [q_h, \tilde{w}_{hh}] + (1 - \beta^*) \left( \gamma^* \Pi^h_s [q_l, \tilde{w}_{hl}] + (1 - \gamma^*) \Pi^h_s [q_l, \tilde{w}_{ul}] \right) \right]$$

$$+ (1 - \alpha) \left[ \gamma^* \Pi^l_s [q_l, \tilde{w}_{hl}] + (1 - \gamma^*) \Pi^l_s [q_l, \tilde{w}_{ul}] \right] .$$

(A.1.31)

In the pooling equilibrium (i.e., $\beta^* = 0$ and $\gamma^* = 0$), (A.1.31) yields $\Pi^A_s = \frac{k_l - 1}{27k_c s}$ and $\Pi^A_s - \Pi^{PE}_s = \alpha \left( \frac{k_l - 1}{27k_c s} - \frac{k_h - 1}{27k_c s} \right)$, which is $> 0$ if $(k_h - 1)(k_l - 1) > 1$. In the semiseparating equilibrium, (A.1.28) and (A.1.31) yield $\Pi^A_s = \alpha \beta^* \left( \frac{k_h - 1}{27k_c s} \right) + (1 - \alpha^*) \left( \frac{k_l - 1}{27k_c s} \right)$ and $\Pi^A_s - \Pi^{PE}_s = \alpha (1 - \beta^*) \left( \frac{k_h - 1}{27k_c s} - \frac{k_h - 1}{27k_c s} \right)$, which again is $> 0$ if $(k_h - 1)(k_l - 1) > 1$. Thus, irrespective of the type of equilibrium, $\Pi^A_s > \Pi^{PE}_s$ if
\((k_h - 1)(k_l - 1) > 1.\)

**Total Supply-Chain Profit:** The expected total supply-chain profit is \(\tilde{\Pi}^A_t = \tilde{\Pi}^A_o + \tilde{\Pi}^A_s\). In the pooling equilibrium, we have

\[
\tilde{\Pi}^A_t - \Pi_{PE}^t = \alpha \left( \frac{1}{27k_1c_s} - \frac{1}{27k_hc_s} \right) + \alpha \left( \frac{k_l - 1}{27k_l^2c_s} - \frac{k_h - 1}{27k_h^2c_s} \right) > 0.
\]

In the semiseparating equilibrium, we have \(\tilde{\Pi}^A_t - \Pi_{PE}^t = \alpha \left( 1 - \beta^* \right) \left( \frac{k_l - 1}{27k_l^2c_s} - \frac{k_h - 1}{27k_h^2c_s} \right)\), which is > 0 if \((k_h - 1)(k_l - 1) > 1.\) Thus, \(\tilde{\Pi}^A_t > \Pi_{PE}^t\) either under the pooling equilibrium (i.e., for \(\zeta_p\)), or if \((k_h - 1)(k_l - 1) > 1.\)

**Contract Outcome:** In the semiseparating equilibrium, if the supplier offers high wholesale price \((\tilde{w}_{hl})\) on observing high quality \((\tilde{q}_l)\) and the OEM happens to be low-cost, contracting does not occur. \(\blacksquare\)
## A.2. Tables

Table 2: Notation (1)

<table>
<thead>
<tr>
<th>Description</th>
<th>Complete Information</th>
<th>Asymmetric Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Integrated Firm</td>
<td>Price-Only Contract</td>
</tr>
<tr>
<td>Cost Parameter: Supplier</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_s$</td>
</tr>
<tr>
<td>Cost Parameter: OEM</td>
<td></td>
<td>$kc_s$</td>
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<tr>
<td>Probability: OEM Type</td>
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Table 3: Notation (2)

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<tr>
<th>Description</th>
<th>Integrated Firm</th>
<th>Price-Only Contract</th>
<th>Two-Part Tariff Contract</th>
<th>Asymmetric Information (Expected Values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>$q^I$</td>
<td>$q^P$</td>
<td>$q^{2P}$</td>
<td>$\tilde{q}^A$</td>
</tr>
<tr>
<td>Quantity</td>
<td>$Q^I$</td>
<td>$Q^P$</td>
<td>$Q^{2P}$</td>
<td>$Q^A$</td>
</tr>
<tr>
<td>Contract Parameters</td>
<td>–</td>
<td>$w^P$</td>
<td>$w^{2P}$, $f^{2P}$</td>
<td>$\tilde{w}^A$</td>
</tr>
<tr>
<td>Selling Price</td>
<td>$p^I$</td>
<td>$p^P$</td>
<td>$p^{2P}$</td>
<td>$\tilde{p}^A$</td>
</tr>
<tr>
<td>OEM Profit</td>
<td>–</td>
<td>$\Pi_o^P$</td>
<td>$\Pi_o^{2P}$</td>
<td>$\Pi_o^A$</td>
</tr>
<tr>
<td>Supplier Profit</td>
<td>–</td>
<td>$\Pi_s^P$</td>
<td>$\Pi_s^{2P}$</td>
<td>$\Pi_s^A$</td>
</tr>
<tr>
<td>Total Supply-Chain Profit</td>
<td>$\Pi^I$</td>
<td>$\Pi_t^P$</td>
<td>$\Pi_t^{2P}$</td>
<td>$\Pi_t^A$</td>
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Table 4: Complete Information: Price-only Contract

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>$q^P$</th>
<th>$w^P$</th>
<th>$p^P$</th>
<th>$Q^P$</th>
<th>$M_o^P$</th>
<th>$M_s^P$</th>
<th>$\Pi_o^P$</th>
<th>$\Pi_s^P$</th>
<th>$\Pi_t^P$</th>
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</thead>
<tbody>
<tr>
<td>$k &lt; 4$</td>
<td></td>
<td>$\frac{1}{3kC_s}$</td>
<td>$\frac{1}{9kC_s}$</td>
<td>$\frac{2}{9kC_s}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{k-1}{3k}$</td>
<td>$\frac{1}{27kC_s}$</td>
<td>$\frac{k-1}{27k^2C_s}$</td>
<td>$\frac{2k-1}{27k^2C_s}$</td>
</tr>
<tr>
<td>$k \geq 4$</td>
<td></td>
<td>$\frac{1}{3C_s}$</td>
<td>$\frac{2}{9C_s}$</td>
<td>$\frac{5}{18C_s}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{108C_s}$</td>
<td>$\frac{1}{54C_s}$</td>
<td>$\frac{1}{36C_s}$</td>
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Table 5: Complete Information: Two-Part Tariff Contract

<table>
<thead>
<tr>
<th>Variable</th>
<th>$q^{2P}$</th>
<th>$w^{2P}$</th>
<th>$f^{2P}$</th>
<th>$p^{2P}$</th>
<th>$Q^{2P}$</th>
<th>$\Pi^P_o$</th>
<th>$\Pi^P_s$</th>
<th>$\Pi^P_t$</th>
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</thead>
<tbody>
<tr>
<td>$\frac{1}{3kC_s}$</td>
<td>$\frac{1}{9k^2C_s}$</td>
<td>$\frac{(k - 1)(5k - 1)}{108k^3C_s}$</td>
<td>$\frac{3k + 1}{18k^2C_s}$</td>
<td>$\frac{1}{3} + \frac{k - 1}{6k}$</td>
<td>$\frac{1}{27kC_s}$</td>
<td>$\frac{(k - 1)(5k - 1)}{108k^3C_s}$</td>
<td>$\frac{(3k - 1)^2}{108k^3C_s}$</td>
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Table 6: Asymmetric Information: Strategy Sets and Profits

<table>
<thead>
<tr>
<th>OEM Type</th>
<th>Quality, ( \tilde{q}^* )</th>
<th>Wholesale Price, ( \tilde{w}^* )</th>
<th>Profit of OEM, ( \tilde{\Pi}_o^l[\tilde{q}^<em>, \tilde{w}^</em>] )</th>
<th>Profit of Supplier, ( \tilde{\Pi}_s^l[\tilde{q}^<em>, \tilde{w}^</em>] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-Cost</td>
<td>( \tilde{q}_h = \frac{1}{3k_h c_s} )</td>
<td>( \tilde{w}_{hh} = k_h c_s \tilde{q}_h^2 )</td>
<td>( \tilde{\Pi}_o^h[\tilde{q}<em>h, \tilde{w}</em>{hh}] = \frac{1}{27k_h c_s} )</td>
<td>( \tilde{\Pi}_s^h[\tilde{q}<em>h, \tilde{w}</em>{hh}] = \frac{k_h - 1}{27k_h^2 c_s} )</td>
</tr>
<tr>
<td></td>
<td>( \tilde{q}_l = \frac{1}{3k_l c_s} )</td>
<td>( \tilde{w}_{hl} = \frac{k_h c_s \tilde{q}_l^2}{2} )</td>
<td>( \tilde{\Pi}_o^h[\tilde{q}<em>l, \tilde{w}</em>{hl}] = \frac{1}{12k_l c_s} \left( 1 - \frac{k_h}{3k_l} \right)^2 )</td>
<td>( \tilde{\Pi}_s^h[\tilde{q}<em>l, \tilde{w}</em>{hl}] = \left( \frac{k_h - 1}{18k_l^2 c_s} \right) \left( 1 - \frac{k_h}{3k_l} \right) )</td>
</tr>
<tr>
<td>Low-Cost</td>
<td>( \tilde{q}_l = \frac{1}{3k_l c_s} )</td>
<td>( \tilde{w}_{ll} = \frac{k_l c_s \tilde{q}_l^2}{2} )</td>
<td>( \tilde{\Pi}_o^l[\tilde{q}<em>l, \tilde{w}</em>{ll}] = \frac{1}{27k_l c_s} )</td>
<td>( \tilde{\Pi}_s^l[\tilde{q}<em>l, \tilde{w}</em>{ll}] = \frac{k_l - 1}{27k_l^2 c_s} )</td>
</tr>
</tbody>
</table>
Table 7: Asymmetric Information: Perfect Bayesian Equilibria and Optimal Strategy Profiles

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Equilibrium Strategy</th>
<th>Equilibrium Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooling</td>
<td>$\tilde{q}_l$</td>
<td>$\tilde{w}_{hh}$ if $\tilde{q}<em>h$; $\tilde{w}</em>{ll}$ if $\tilde{q}_l$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{q}_l$</td>
<td>${\alpha &lt; X } \land { k_h &lt; \frac{3k_l+1}{2} }$ \lor ${\alpha &lt; Y } \land { k_h \geq \frac{3k_l+1}{2} }$</td>
</tr>
<tr>
<td>Semiseparating</td>
<td>$\tilde{q}_h$ w/Pr $\beta^<em>$ &amp; $\tilde{q}_l$ w/Pr $(1 - \beta^</em>)$</td>
<td>$\tilde{w}<em>{hh}$ if $\tilde{q}<em>h$; $\tilde{w}</em>{hl}$ w/Pr $\gamma^*$ &amp; $\tilde{w}</em>{ll}$ w/Pr $(1 - \gamma^*)$ if $\tilde{q}_l$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{q}_l$</td>
<td>${\alpha \geq X } \land { k_h &lt; \frac{3k_l+1}{2} }$ \lor ${\alpha \geq Y } \land { k_h \geq \frac{3k_l+1}{2} }$</td>
</tr>
</tbody>
</table>
APPENDIX B

PROOFS AND TABLES OF CHAPTER III

B.1. Notation: Strategy Space

In the Appendix, we use the following notation to simplify our discussion. The possible product strategies of the firm are given by $WXYZ$, where $W \in \{N, \emptyset\}$, $X \in \{N, \emptyset\}$, and $Y \in \{R_S, R_A, \emptyset\}$ denote whether the firm offers the new product in the first period, the new product in the second period, and the refurbished product in the second period, respectively. The subscript for $R$ denotes whether Some ($S$) or All ($A$) of the first-period returns are refurbished.

Table 8: Firm’s Product Strategy Space

<table>
<thead>
<tr>
<th>Product Strategy Notation</th>
<th>1st Period</th>
<th>2nd Period</th>
<th>Refurbish</th>
</tr>
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<tbody>
<tr>
<td>New Product</td>
<td>New Product</td>
<td>Refurbished</td>
<td></td>
</tr>
<tr>
<td>$NNR_S$</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>$NNR_A$</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>$N\emptyset R_S$</td>
<td>√</td>
<td>×</td>
<td>√</td>
</tr>
<tr>
<td>$N\emptyset R_A$</td>
<td>√</td>
<td>×</td>
<td>√</td>
</tr>
<tr>
<td>$NN\emptyset$</td>
<td>√</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>$\emptyset N\emptyset$</td>
<td>×</td>
<td>√</td>
<td>×</td>
</tr>
<tr>
<td>$N\emptyset\emptyset$</td>
<td>√</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

B.1.1 Proof of Proposition 6

For a given sales quantity of the new product in the first period $q_1$, sales quantities in the second period are given by:

\[ q_2(p) = 1 - q_1 - \frac{p_2 - p_r}{v_n - v_r} \]
\[ q_r(p) = \frac{p_2 - p_r}{v_n - v_r} - \frac{p_r}{v_r} \]
We substitute $q_2(\bar{p})$ and $q_r(\bar{p})$ in the firm’s Problem (3.3.18) and solve for $p_2$ and $p_r$; note that $\Pi_2$ is jointly concave in $p_2$ and $p_r$. The second period optimal prices and sales quantities are summarized in Table 9 in Appendix B.2 (as functions of $q_1$).

For simplicity, let $\mu_n = v_n - (1 + \alpha) c_n$ and $\mu_r = v_r - c_r$. Define $\bar{\alpha} = (c_r v_n / c_n v_r) - 1$, $\bar{q}_1(\alpha) = \max \{(\mu_n - \mu_r) / (v_n - v_r), \mu_n / (v_n + 2\alpha v_r)\}$, and $\bar{q}_1(\alpha) = \max \{\mu_n / v_n, \mu_r / v_r\}$. We now derive conditions under which the different product strategies are optimal in the second period.

i) **Product Line** $(q_1^* > 0, q_r^* > 0)$: The first-order conditions (FOCs) of the firm’s second-period Problem (3.3.18) yield $q_2(q_1) = (1/2) ((\mu_n - \mu_r) / (v_n - v_r) - q_1)$ and $q_r(q_1) = (1 + \alpha)c_n v_r - c_r v_n) / 2v_r (v_n - v_r)$, if $\alpha q_1 > q_r$. The FOCs yield $q_2(q_1) = (v_n (1-q_1) - 2\alpha q_1 v_r - (1 + \alpha) c_n) / 2v_n$ and $q_r(q_1) = \alpha q_1$, if $\alpha q_1 = q_r$. Thus, if $\alpha > \bar{\alpha}$ and $q_1 < q_1^*$, we have $q_2^* > 0$ and $q_r^* > 0$, i.e., the firm offers the product line in the second period.

ii) **Refurbished Only** $(q_2^* = 0, q_r^* > 0)$: The FOCs yield $q_r(q_1) = (v_r (1-q_1) - c_r) / 2v_r$ if $\alpha q_1 > q_r$ and $q_2 = 0$, and yield $q_r(q_1) = \alpha q_1$ if $\alpha q_1 = q_r$ and $q_2 = 0$. Thus, if $\alpha > \bar{\alpha}$ and $q_1 \leq q_1 < q_1^*$, we have $q_2^* = 0$ and $q_r^* > 0$, i.e., the firm offers only the refurbished product in the second period.

iii) **New Only** $(q_2^* > 0, q_r^* = 0)$: The FOCs yield $q_2(q_1) = (v_n (1-q_1) - (1 + \alpha) c_n) / 2v_n$ if $0 < q_1 < \bar{q}_1$ and $q_r = 0$. The FOCs yield $q_2 = \mu_n / 2v_n$ if $q_1 = 0$ (implying $q_r = 0$). Thus, if $\alpha \leq \bar{\alpha}$ and $q_1 < \bar{q}_1$, we have $q_2^* > 0$ and $q_r^* = 0$, i.e., the firm offers only the new product in the second period.

iv) **None** $(q_2^* = 0, q_r^* = 0)$: The FOCs yield $q_2(q_1) < 0$ and $q_r(q_1) < 0$ if $q_1 \geq \bar{q}_1$. Thus, if $q_1 \geq \bar{q}_1$, we have $q_2^* = 0$ and $q_r^* = 0$, i.e., the firm offers none of the products in the second period.

### B.1.2 Proof of Proposition 7

We first present a result that narrows the set of strategies that could be optimal for the firm.
Theorem 2. The following product strategies are not optimal for the firm:

i) offering the new product in the first period and no product in the second period (i.e., Strategy $N\emptyset\emptyset$, where $q_1^* > 0$, and $q_2^* = q_r^* = 0$)

ii) offering no product in the first period and only the new product in the second period (i.e., Strategy $\emptyset N\emptyset$, where $q_1^* = q_r^* = 0$, and $q_2^* > 0$).

Proof. We first show that Strategy $\emptyset N\emptyset$ cannot be optimal because it is strictly dominated by Strategy $NN\emptyset$. Then we show that Strategy $N\emptyset\emptyset$ is suboptimal because it results in nonpositive firm profit.

For Strategy $NN\emptyset$, in which $q_r = 0$, the optimal solutions are $q_1^* = 2(1-\rho)\mu_n/v_n(4-3\rho)$, $q_2^* = \mu_n(2-\rho)/2v_n(4-3\rho)$, and $\Pi^*_{NN\emptyset} = (\mu_n(2-\rho)^2/4v_n(4-3\rho)$. For Strategy $\emptyset N\emptyset$, in which $q_1 = q_r = 0$, we have $q_2^* = \mu_n/2v_n$ and $\Pi^*_{\emptyset N\emptyset} = \rho \mu_n^2/4v_n < \Pi^*_{NN\emptyset}$. Therefore, Strategy $\emptyset N\emptyset$ is strictly dominated by Strategy $NN\emptyset$ and cannot be optimal.

Next, suppose Strategy $N\emptyset\emptyset$ is optimal, where $q_2 = q_r = 0$. In the first period, the marginal consumer who is indifferent between purchasing and not purchasing is located at $\theta_m = 1 - q_1^*$. By definition, in Strategy $N\emptyset\emptyset$, this marginal consumer obtains zero utility from buying the new product, i.e., $p_1^* = v_n\theta_m = v_n(1 - q_1^*)$. In the second period, the firm will have no incentive to sell the new product to any consumer with $\theta \leq \theta_m$ if $v_n\theta_m \leq (1 + \alpha)c_n$. Combining these two conditions, we obtain $p_1^* \leq c_n (1 + \alpha)$, implying that the firm makes nonpositive profit. Therefore, Strategy $N\emptyset\emptyset$ is suboptimal.

Theorem 2 shows that the firm should always offer the new product in the first period and the new and/or the refurbished product in the second period. From Proposition 6, this implies that $q_1 < \tilde{q}_1$. The remaining feasible product strategies are listed in Table 10 (note that Table 9 provides the corresponding optimal second-period quantities and prices for the strategies listed in Table 10). Conditions for the different second-period product strategies listed in the statement of Proposition 7 are derived below:
i) New Only: Let \( v_N^r(\alpha) = \frac{v_n c_r}{(1 + \alpha) c_n} \). Observe that \( v_r \leq v_N^r(\alpha) \implies \alpha \leq \bar{\alpha} = \frac{(c_r v_n / c_n v_r)}{c_r} - 1 \). Thus, from Proposition 6 (iii), when \( \alpha \leq \bar{\alpha} \) and \( q_1 < q_1^* \), the firm offers only the new product in the second period.

ii) Product Line: The firm offers both the new and the refurbished products in the second period, refurbishing some of the returns (i.e., \( 0 < q_1^* < \alpha q_1^* \), and \( q_2^* > 0 \)) if \( c_r > \bar{c}_r \) and \( v_r < v_{r RS}^r(\alpha) \), where

\[
\bar{c}_r = \frac{v_r (1 + \alpha) c_n}{v_n} - \frac{4 \alpha (1 - \rho) v_r (v_n - v_r) \mu_n}{v_n^2 (4 - 3 \rho)}
\]

\[
v_{r RS}^r(\alpha) = v_n - \frac{v_n (4 - 3 \rho) ((1 + \alpha) c_n - c_r)}{v_n (4 - 3 \rho) - 2 (1 - \rho) \mu_n}
\]

The firm offers both the new and the refurbished products in the second period, refurbishing all of the returns (i.e., \( 0 < q_1^* = \alpha q_1^* \), and \( q_2^* > 0 \)) if \( c_r \leq \bar{c}_r \) and \( v_r < v_{r RS}^r(\alpha) \), where

\[
\frac{2 v_n (v_n + 2 \alpha v_r) ((1 - \rho) \mu_n + \rho \alpha (\frac{v_r (1 + \alpha) c_n}{v_n} - c_r))}{\mu_n (v_n^2 (4 - 3 \rho) + 4 \rho \alpha^2 v_r (v_n - v_r))} < 1 \quad (B.1.32)
\]

Roots of (B.1.32) at equality are \( X + \sqrt{Y} \) and \( X - \sqrt{Y} \), where

\[
X = \frac{-2 \alpha^3 c_n \rho + \alpha^2 (2 \rho (v_n + c_r) - 3 \rho c_n) - \alpha (\rho c_n + 2 (1 - \rho) \mu_n)}{4 \rho \alpha^2}
\]

and

\[
Y = \frac{\alpha^2 (4 \rho v_n ((2 - \rho) \mu_n + 2 \rho c_r) + (2 (1 - \rho) \mu_n - 2 \rho c_r (v_n + c_r) + \rho (1 + 2 \alpha) c_e)^2)}{(4 \rho \alpha^2)^2},
\]

where \( c_e = (1 + \alpha) c_n \). Since \( X^2 - Y = -v_n ((2 - \rho) \mu_n + 2 \rho c_r) / 4 \rho \alpha^2 < 0 \), the only nonnegative root is \( v_{r RA}^r(\alpha) = X + \sqrt{Y} \), and (B.1.32) holds when \( v_r < v_{r RA}^r(\alpha) \).

Let \( v_{r}^r(\alpha) = \max \{v_{r RA}^r(\alpha), v_{r RS}^r(\alpha)\} \). It is straightforward to show that \( v_{r RA}^r \geq v_{r RS}^r \iff c_r \leq \bar{c}_r \). Therefore, when \( v_{r RA}^r < v_{r RS}^r \) (implying \( c_r > \bar{c}_r \)), the firm offers the
product line, refurbishing some of the returns if \( v_r < \bar{v}_r = v_{rR} \). On the other hand, when \( v_{rA} \geq v_{rS} \) (implying \( c_r \leq \bar{c}_r \)), the firm offers the product line, refurbishing all of the returns if \( v_r < \bar{v}_r = v_{rA} \). From part (i) above, only the new product is offered if \( v_r \leq \bar{v}_r \). Thus, the firm offers the product line in the second period if \( \bar{v}_r < v_r < v_{rR} \).

iii) Refurbished Only: When \( v_r > v_{rR} \), we have \( q^*_2 = 0 \) and \( q^*_r > 0 \). Thus, the firm offers only the refurbished product in the second period. Note that some or all of the returns may be refurbished under this strategy.

B.1.3 Proof of Proposition 8

New Product in the Second Period. We first show that if the firm can commit to its future decisions, it is never optimal for the firm to offer the new product in the second period, that is, \( q^C_2 = 0 \). In the second period, there are three possibilities:

Case (i) \( q_r = 0 \); Case (ii) \( \alpha q_1 > q_r > 0 \); Case (iii) \( \alpha q_1 = q_r > 0 \). We show that in each of these cases, \( q^C_2 = 0 \).

Case (i): When \( q_r = 0 \), the FOCs of the firm’s Problem (3.5.20) yield \( q_1(q_2) = \mu_n/2v_n - \rho q_2 \) with corresponding profit \( \Pi(q_2) = (\mu_n^2 - 4 \rho (1 - \rho) q_2^2 v_n^2)/4v_n \). Since \( \Pi \) decreases in \( q_2 \), we have the optimal \( q^C_2 = 0 \).

Case (ii): When \( \alpha q_1 > q_r > 0 \), the FOCs yield \( q_2 = (v_n c_r - v_r (1 + \alpha) c_n)/2v_n (v_n - v_r) \) and \( q_r = ((1 + \alpha) c_n v_r - c_r v_n) / 2v_n (v_n - v_r) \). Observe that \( q_r > 0 \Rightarrow q_2 < 0 \). Therefore, the optimal \( q^C_2 = 0 \).

Case (iii): When \( \alpha q_1 = q_r > 0 \), solving Problem (3.5.20) yields the following:

\[
q_2 = \begin{cases} 
\frac{\alpha(\rho(\alpha v_r (c_r + v_n - v_r) + v_n (c_r - v_r)) - v_n v_r + (1 + \alpha) c_n v_r (1 - \rho (2 + \alpha)))}{2v_n^2 - 2 \rho (v_n^2 - \alpha^2 v_r (v_n - v_r))} & \text{if } c_r < \bar{c}_r \\
\frac{-\alpha v_r (v_n - (1 + \alpha) c_n)}{2v_n^2} & \text{if } c_r \geq \bar{c}_r
\end{cases}
\]
where \( \tilde{c}_r = v_n c_n (1+\alpha - \alpha (v_n - v_r)) v_n^2 / v_n^2 \). Observe that: (i) \( \partial q_2 / \partial c_r > 0 \) for \( c_r < \tilde{c}_r \); (ii) \( q_2 < 0 \) for \( c_r \geq \tilde{c}_r \) because \( v_n > (1+\alpha) c_n \); and (iii) \( \lim_{c_r \to \tilde{c}_r^-} q_2 = \lim_{c_r \to \tilde{c}_r^+} q_2 \). Therefore, \( q_2 < 0 \forall c_r \), and the optimal \( q_{rC} = 0 \).

### Refurbished Product in the Second Period.
Since the firm optimally does not offer the new product in the second period in the commitment scenario, we substitute \( q_2 = 0 \) in the FOCs of the firm’s Problem (3.5.20), which yields:

\[
q_1 = \frac{\mu_n - \rho v_r}{2(v_n - \rho v_r)},
\]

\[
q_r = \frac{(1+\alpha)c_n c_r - c_r v_n}{2v_r (v_n - \rho v_r)}.
\]

\( i \): If \( \alpha \leq \bar{\alpha} = (c_r v_n / c_n v_r) - 1 \), then \( q_r \leq 0 \). Therefore, the optimal \( q_{rC} = 0 \), i.e., the firm does not offer the refurbished product in the second period.

\( ii \): If \( \alpha > \bar{\alpha} \), we have \( \alpha q_1 > q_r > 0 \) if \( \alpha v_r^2 - v_r A - c_r v_n < 0 \), where \( A = \alpha (v_n + \rho c_r) - (1+\alpha)^2 c_n \). In other words, if \( v_n < \tilde{v}_r \), where \( \tilde{v}_r = (A + \sqrt{A^2 + 4\alpha \rho c_r v_n}) / 2\alpha \rho \), the optimal \( q_{rC} < \alpha q_{1C} \), i.e., the firm refurbishes some of the returns in the second period.

\( iii \): If \( \alpha > \bar{\alpha} \) and \( v_r \geq \tilde{v}_r \), we have \( q_r > \alpha q_1 \). Therefore, the optimal \( q_{rC} = \alpha q_{1C} \), i.e., the firm refurbishes all of the returns in the second period.

\[ \square \]

### B.1.4 Proof of Proposition 9

#### Profit Comparison.
To show \( \Pi^* \leq \Pi^C \), we follow the logic in Bulow (1986, p. 733).

Note that in the commitment scenario, the firm sets prices for both the periods at the beginning of the first period. Therefore, in the commitment scenario, the firm’s Problem (3.5.20) is solved as a one-shot game between the firm and consumers.

In contrast, in the no-commitment scenario, the firm sets prices for the second period at the beginning of the second period; this problem is given in (3.3.2). Thus, the solution for the complete two-period Problem (3.3.19) in the no-commitment scenario must be a subgame perfect Nash equilibrium.

Let \( S \) be the set of feasible solutions \( \{p_1, p_2, p_r\} \) in the no-commitment scenario and \( S^C \) be the set of feasible solutions in the commitment scenario. Clearly, \( S \subseteq S^C \).
Therefore, $\Pi^* \leq \Pi^C$.

**Threshold Return Rate $\alpha_c$.** In the commitment scenario (where the optimal $q^C_2 = 0$), there are three possible strategies: *Strategy N∅∅* ($q^C_r = 0$), *Strategy N∅RS* ($\alpha q^C_1 > q^C_r > 0$), and *Strategy N∅RA* ($\alpha q^C_1 = q^C_r > 0$). The (tight) upper bounds on the return rate $\alpha$ for Strategies N∅∅, N∅RS, and N∅RA to be feasible, are $(v_n-c_n)/c_n$, $(v_n-c_n/c_n$ and $(v_n-c_n)/(c_n-r(v_n-c_r))$, respectively. Note that $(v_n-c_n)/c_n < (v_n-c_n)/(c_n-r(v_n-c_r))$.

Thus, if $\alpha \geq (v_n-c_n)/c_n$, only *Strategy N∅RA* is feasible, that is, $q^C_2 = 0$ and $\alpha q^C_1 = q^C_r > 0$.

In the no-commitment scenario, there are five possible product strategies (see Table 10). The (tight) upper bounds on the return rate $\alpha$ for these strategies are as follows: $(v_n-c_n)/c_n$ for *Strategy NNRs* and *Strategy NN∅*, ($(v_n-c_n)-(v_n-c_r))/c_n$ for *Strategy NNRs*, ($(v_n-c_n)-(v_n-c_r))/c_n$ for *Strategy N∅RS*, and $(v_n-c_n)/(c_n-r(v_n-c_r))$ for *Strategy N∅RA*. Note that ($(v_n-c_n)-(v_n-c_r))/c_n < (v_n-c_n)/(c_n-r(v_n-c_r)) < (v_n-c_n)/c_n < (v_n-c_n)/(c_n-r(v_n-c_r))$. Thus, if $\alpha \geq (v_n-c_n)/c_n$, only *Strategy N∅RA* is feasible, that is, $q^C_2 = 0$ and $\alpha q^C_1 = q^C_r > 0$.

Thus, if $\alpha \geq (v_n-c_n)/c_n = \alpha_c$, only *Strategy N∅RA* is feasible in both the commitment and the no-commitment scenarios, and the profits are identical in the two scenarios, i.e., $\Pi^* = \Pi^C$. This is because, when $\alpha \geq \alpha_c$, the new product is optimally not offered in the second period in the no-commitment scenario as well.

**B.1.5 Proof of Proposition 10**

**Commitment Scenario.** We consider the following exhaustive cases:

*Case (i)* If $q_r = 0$, then $\Pi^C = (v_n-(1+\alpha)c_n)^2/4v_n$, and $\partial \Pi^C/\partial \alpha = -c_n q^C_1 < 0$.

*Case (ii)* If $\alpha q_1 = q_r > 0$, we have:

$$
\frac{\partial \Pi^C}{\partial \alpha} = -q^C_1 \left[ \frac{c_n (v_n - \rho v_r) + \rho (v_n + \rho v_r \alpha) + v_r \alpha (v_n - \rho v_r))}{v_n + \rho v_r \alpha (\alpha + 2)} \right] < 0
$$
Case (iii) If $\alpha q_1 > q_r > 0$, we have:

$$\frac{\partial \Pi^C}{\partial \alpha} = -c_n q_1 C < 0$$

**No-Commitment Scenario.** To prove the possible non-monotonicity of firm profit $\Pi^*$ with respect to $\alpha$, we restrict our attention to situations in which the firm refurbishes all of the returns. That is, $\alpha q_1 = q_r > 0$, making it more likely for the optimal profit $\Pi^*$ in the no-commitment scenario to approach the optimal profit $\Pi^C$ in the commitment scenario (using Proposition 9).

When $\alpha q_1 = q_r > 0$ in the no-commitment scenario, the firm follows one of the two strategies: *Strategy NNR$_A$* (i.e. $\alpha q_1^* = q_r^* > 0$, $q_2^* > 0$) or *Strategy NNR$_A$* (i.e. $\alpha q_1^* = q_r^* > 0$, $q_2^* = 0$). For these situations, we show that: (i) $\Pi^*$ is decreasing in $\alpha$ at $\alpha = 0^+$, and (ii) there exist $\hat{v}_r$, $\alpha_s$ such that $\Pi^*$ increases in $\alpha$ at $\alpha_s$ for $v_r > \hat{v}_r$.

(i) When $\alpha \to 0^+$, *Strategy NNR$_A$* is always optimal and we have:

$$\Pi^* = \frac{(2 - \rho)^2 (v_n - c_n)^2}{4v_n (4 - 3\rho)}$$

$$\left. \frac{\partial \Pi^*}{\partial \alpha} \right|_{\alpha \to 0^+} = -\frac{(v_n - c_n) (c_n (4 (1 - \rho) (v_n - \rho v_r) + \rho^2 v_n) + 4c_r v_n \rho (1 - \rho))}{2v_n^2 (4 - 3\rho)} < 0$$

(ii) We know from Proposition 9 that when $\alpha \geq \alpha_c$, the firm’s profit in the no-commitment scenario equals the profit in the commitment scenario. This is because, when $\alpha \geq \alpha_c$, the new product is optimally not offered in the second period in the no-commitment scenario as well. Therefore, we focus on the value of $\alpha$ beyond which the firm can implicitly commit that $q_2^* = 0$ and, thus, where $\Pi^*$ is liable to increase in $\alpha$ (approaching the optimal profit $\Pi^C$ in the commitment scenario). Accordingly, let $\alpha_s = \inf \{ \alpha : q_2^* (\alpha) = 0 \}$ and $\mu_n^s = v_n - c_n (1 + \alpha_s)$. At $\alpha_s$, we have

$$\left. \frac{\partial \Pi^*}{\partial \alpha} \right|_{\alpha = \alpha_s} = \frac{\mu_n^s (v_n (2 - \rho) (\mu_n^s - \alpha_s c_n) - 2\alpha_s v_r (1 - \rho) \mu_n^s - 2\alpha_s v_r ((1 - \rho) (v_n - c_n) + \alpha_s c_n))}{2\alpha_s (v_n + 2\alpha_s v_r)^2}.$$
The right hand side is > 0 when

\[ v_r > \hat{v}_r = \frac{v_n (2 - \rho) (\mu_n^* - \alpha_s c_n)}{2\alpha_s (2 (1 - \rho) \mu_n^* + (2 - \rho) \alpha_s c_n)} \]

\[ \Box \]

### B.2. Optimal Quantities and Prices

Table 9: Optimal Sales Quantities and Prices in the Second Period for a given \( q_1 \)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>( q_2^* (q_1) ) and ( q_r^* (q_1) )</th>
<th>( p_2^* (q_1) ) and ( p_r^* (q_1) )</th>
</tr>
</thead>
</table>
| **NNR** | \[
q_2^* = \frac{1}{2} \left( 1 - q_1 - \frac{(1 + \alpha) c_n - c_r}{v_n - v_r} \right) \\
q_r^* = \frac{(1 + \alpha) c_n v_r - c_r v_n}{2 v_r (v_n - v_r)}
\] | \[
 p_2^* = \frac{v_n (1 - q_1) + (1 + \alpha) c_n}{2} \\
p_r^* = \frac{v_r (1 - q_1) + c_r}{2}
\] |
| **NRA** | \[
q_2^* = \frac{v_n (1 - q_1) - 2\alpha q_1 v_r - (1 + \alpha) c_n}{2 v_n}
q_r^* = \alpha q_1
\] | \[
 p_2^* = \frac{v_n (1 - q_1) + (1 + \alpha) c_n}{2} \\
p_r^* = \frac{v_r [v_n (1 - q_1) + (1 + \alpha) c_n - 2\alpha q_1 (v_n - v_r)]}{2 v_n}
\] |
| **N∅** | \[
q_2^* = 0 \\
q_r^* = \frac{v_r (1 - q_1) - c_r}{2 v_r}
\] | \[
 p_r^* = \frac{v_r (1 - q_1) + c_r}{2}
\] |
| **N∅A** | \[
q_2^* = 0 \\
q_r^* = \alpha q_1
\] | \[
 p_r^* = v_r (1 - q_1 - \alpha q_1)
\] |
| **N∅N∅** | \[
q_2^* = \frac{v_n (1 - q_1) - (1 + \alpha) c_n}{2 v_n}
q_r^* = 0
\] | \[
 p_2^* = \frac{v_n (1 - q_1) + (1 + \alpha) c_n}{2} \\
p_r^* = \frac{v_n + (1 + \alpha) c_n}{2}
\] |
| **∅N∅** | \[
q_2^* = \frac{v_n - (1 + \alpha) c_n}{2 v_n}
q_r^* = 0
\] | \[
 p_2^* = \frac{v_n + (1 + \alpha) c_n}{2}
\] |
| **N∅∅** | \[
q_2^* = 0 \\
q_r^* = 0
\] | \[
 p_r^* = 0
\] |
Table 10: Optimal Sales Quantity and Price of the New Product in the First Period

<table>
<thead>
<tr>
<th>Strategy</th>
<th>( q_1^* )</th>
<th>( p_1^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( NN\overline{R}S )</td>
<td>( \frac{2(1 - \rho)\mu_n}{v_n (4 - 3\rho)} )</td>
<td>( (1 - q_1^<em>)v_n - \rho ((1 - q_1^</em>)v_n - (1 + \alpha) c_n) )</td>
</tr>
<tr>
<td>( NN\overline{R}A )</td>
<td>( \frac{v_n}{v_n (4 - 3\rho)} ) \ \ \ \ [2 - \frac{v_n (1 + \alpha) c_n - v_n c_r}{v_n (4 - 3\rho) + 4\rho^2 v_r (v_n - v_r)}] )</td>
<td>( (1 - q_1^<em>)v_n - \rho ((1 - q_1^</em>)v_n - (1 + \alpha) c_n) )</td>
</tr>
<tr>
<td>( N\overline{R}S )</td>
<td>( \max \left{ \frac{2(1 - \rho)\mu_n - \rho\mu_r}{4v_n - 3\rho v_r}, \frac{\mu_n - \mu_r}{v_n - v_r} \right} )</td>
<td>( (1 - q_1^<em>)v_n - \rho ((1 - q_1^</em>)v_r - c_r) )</td>
</tr>
<tr>
<td>( N\overline{R}A )</td>
<td>( \max \left{ \frac{\mu_n + \rho\alpha\mu_r}{2(v_n + \rho\alpha v_r (2 + \alpha))}, \frac{\mu_n}{v_n + 2\alpha v_r} \right} )</td>
<td>( (1 - q_1^<em>)v_n - \rho \alpha q_1^</em> v_r )</td>
</tr>
<tr>
<td>( NN\emptyset )</td>
<td>( \frac{2(1 - \rho)\mu_n}{v_n (4 - 3\rho)} )</td>
<td>( (1 - q_1^<em>)v_n - \rho ((1 - q_1^</em>)v_n - (1 + \alpha) c_n) )</td>
</tr>
</tbody>
</table>
C.1. Proofs

C.1.1 Proof of Proposition 11

Since, the OEM is a monopoly both in the acquisition of the used products and in the selling of the remanufactured products, she extracts whole consumer surplus from both the high-segment consumers, who sell the used products, and the low-segment consumers, who buy the remanufactured products. Thus, the OEM optimally sets $p_u^m = 0$ and $p_r^m = \theta_l \delta_r q$.

Since $\theta_l \delta_r q - c_r > p_u^m$, the OEM has an incentive to sell as many remanufactured products as she can. The number of remanufactured products the OEM can sell is constrained by either the quantity of the used products (i.e. supply constraint) available or the size of the low-segment market (i.e. demand constraint) or both. Thus, $Q_r^m = \min \{n_h, n_l\}$. Moreover, the OEM has no incentive to acquire more than the number of used products she is going to remanufacture, that is, $Q_u^m = Q_r^m$. Substituting the optimal solution in the OEM’s profit function $\Pi_r = Q_r (p_r - c_r) - Q_u p_u$ yields optimal profit as shown in Proposition 11(e).

C.1.2 Proof of Proposition 12

Suppose $n_h \leq n_l$. Since $n_h \leq n_l$, the OEM and the 3PR have sufficient number of low-segment consumers available to sell their remanufactured products and, thus, can extract whole surplus from the low segment. Therefore, the OEM and the 3PR optimally set $p_c^r = \theta_l \delta_r q$ and $\tilde{p}_r^c = \theta_l \tilde{\delta}_r q$, respectively, for their remanufactured products. As long as $\theta_l \delta_r q - c_r > p_u$ and $\theta_l \tilde{\delta}_r q - \tilde{c}_r > \tilde{p}_u$, the OEM and the 3PR have incentives to sell as many remanufactured products as they can. Since $n_h \leq n_l$, the
number of remanufactured products a player can sell is constrained by the quantity of the used products (supply constraint) she acquires. Thus, the OEM and the 3PR optimally set $Q^c = Q_u$ and $\tilde{Q}^c = \tilde{Q}_u$.

Since $n_h \leq n_l$, the supply of the used products is constrained and each player competes to acquire as many used products as she can by setting acquisition price of the used products. Consumers sell their used products to a player who pays a higher price. When both the players set the same price, we assume that consumers sell their used products to the OEM. This is a reasonable assumption since OEMs usually have a greater access to used products. Thus,

$$Q_u = \begin{cases} n_h & \text{if } p_u \geq \tilde{p}_u \\ 0 & \text{if } p_u < \tilde{p}_u \end{cases} \quad \text{(C.1.33)}$$

and

$$\tilde{Q}_u = \begin{cases} 0 & \text{if } p_u \geq \tilde{p}_u \\ n_h & \text{if } p_u < \tilde{p}_u \end{cases} \quad \text{(C.1.34)}$$

Each player has an incentive to set lowest price but high enough to acquire all the used products as long as the player makes nonnegative profit. Thus, the OEM’s objective is to minimize $p_u$ such that $p_u \geq \tilde{p}_u$ and $p_u \leq \theta_l \delta_r q - c_r$. Similarly, the 3PR’s objective is to minimize $\tilde{p}_u$ such that $\tilde{p}_u > p_u$ and $\tilde{p}_u \leq \theta_l \tilde{\delta}_r q - \tilde{c}_r$.

We can easily show that when $\theta_l \delta_r q - c_r \geq \theta_l \tilde{\delta}_r q - \tilde{c}_r$, the equilibrium price of the used product is $p^e_u = \theta_l \tilde{\delta}_r q - \tilde{c}_r$, and the corresponding number of used products acquired and remanufactured by the OEM and the 3PR are $Q^e_u = Q^e_r = n_h$ and $\tilde{Q}^e_u = \tilde{Q}^e_r = 0$ respectively. Similarly, when $\theta_l \delta_r q - c_r < \theta_l \tilde{\delta}_r q - \tilde{c}_r$, the equilibrium price of the used product is $p^e_u = \theta_l \delta_r q - c_r$, and the corresponding number of used products acquired and remanufactured by the OEM and the 3PR are $Q^e_u = Q^e_r = 0$ and $\tilde{Q}^e_u = \tilde{Q}^e_r = n_h$ respectively.
Suppose $n_h > n_l$. If the total number of remanufactured products offered by the OEM and the 3PR combined is more than the number of low-segment consumers available, the equilibrium price of the remanufactured products are $p^r_c = 0$ and $\tilde{p}^r_c = 0$, respectively, and each player makes negative profit. Therefore, in the equilibrium, the total number of the remanufactured products by the OEM and the 3PR combined is $Q^r_c + \tilde{Q}^r_c = n_l$ and, thus, the OEM and the 3PR set $p^r_c = \theta_l\delta_rq$ and $\tilde{p}^r_c = \theta_l\tilde{\delta}_rq$, respectively. As long as $\theta_l\delta_rq - c_r > p_u$ and $\theta_l\tilde{\delta}_rq - \tilde{c}_r > \tilde{p}_u$, the OEM and the 3PR have incentives to sell as many remanufactured products as they can. Since $n_h > n_l$, the number of remanufactured products a player can sell is either constrained by the number of used products (supply constraint) she acquires or the remaining market size ($n_l - \tilde{Q}_r$ for the OEM and $n_l - Q_r$ for the 3PR). Thus, the OEM’s and the 3PR’s best responses are to remanufacture $Q_r = \min \left\{ Q_u, n_l - \tilde{Q}_r \right\}$ and $\tilde{Q}_r = \min \left\{ \tilde{Q}_u, n_l - Q_r \right\}$ respectively. Note that the optimal solution may have multiple Nash equilibria if $Q_u > 0$, $\tilde{Q}_u > 0$ & $Q_u + \tilde{Q}_u > n_l$.

Since consumers sell their used products to a player who pays a higher price (along with the assumption that consumers sell their used products to the OEM when $p_u = \tilde{p}_u$), either $Q_u = 0$ & $\tilde{Q}_u = 0$, and, thus, we get unique Nash equilibria. Thus, expressions (C.1.33) and (C.1.34) also apply when $n_h > n_l$. Each player has an incentive to set lowest price but high enough to acquire all the used products as long as the player makes nonnegative profit. Thus, the OEM’s objective is to minimize $p_u$ such that $p_u \geq \tilde{p}_u$ and $n_hp_u \leq n_l (\theta_l\delta_rq - c_r)$. Similarly, the 3PR’s objective is to minimize $\tilde{p}_u$ such that $\tilde{p}_u > p_u$ and $n_h\tilde{p}_u \leq n_l (\theta_l\tilde{\delta}_rq - \tilde{c}_r)$.

We can easily show that when $\theta_l\delta_rq - c_r \geq \theta_l\tilde{\delta}_rq - \tilde{c}_r$, the equilibrium price of the used product is $p^c_u = \frac{\mu}{n_h} \left( \theta_l\tilde{\delta}_rq - \tilde{c}_r \right)$; the corresponding number of used products acquired and remanufactured by the OEM are $Q^c_u = n_h$ and $Q^c_r = n_l$ respectively, and by the 3PR are $\tilde{Q}^c_u = 0$ and $\tilde{Q}^c_r = 0$ respectively. Similarly, when $\theta_l\delta_rq - c_r < \theta_l\tilde{\delta}_rq - \tilde{c}_r$, the equilibrium price of the used product is $p^c_u = \frac{\mu}{n_h} (\theta_l\delta_rq - c_r)$; the corresponding
number of used products acquired and remanufactured by the OEM are $Q_u^c = 0$ and $Q_r^c = 0$ respectively, and by the 3PR are $\tilde{Q}_u^c = n_h$ and $\tilde{Q}_r^c = n_l$ respectively.

Thus, $p_u^c = \min \left\{ \min_{n_h, n_l} \left( \theta_l \delta_r q - \tilde{c}_r \right), \min_{n_h, n_l} \left( \theta_l \delta_r q - c_r \right) \right\}$. Substituting the optimal solution in the OEM’s profit function $\Pi_r = Q_r (p_r - c_r) - Q_u p_u$ yields optimal profit as shown in Proposition 12(h). Similarly, substituting the optimal solution in the 3PR’s profit function $\tilde{\Pi}_r = \tilde{Q}_r (\tilde{p}_r - \tilde{c}_r) - \tilde{Q}_u \tilde{p}_u$ yields optimal profit as shown in Proposition 12(i).

Table 12 summarizes outcome of the acquisition and remanufacturing of the used products under competition.

C.1.3 Proof of Proposition 13

If the high segment consumer keeps the used product, he receives a second-period net utility $\theta h \delta q$ from using it. If the consumer buys the new product at price $p_2$ and sells the used product at price $p_u^m$, he receives a second-period net utility $\theta h q - p_2 + p_u^m$. If the consumer sells the used product at price $p_u^m$ and buys the OEM’s remanufactured product at price $p_r^m$, he receives a second-period net utility $\theta h \delta_r q - p_r^m + p_u^m$. To induce the consumer to buy the new product again in the second period, the OEM must set the price of the new product in the second period such that the consumer is not worse off buying the new product in the second period; that is, $\theta h q - p_2 + p_u^m \geq \max \{ \theta h \delta q, \theta h \delta_r q - p_r^m + p_u^m \}$. Thus, the OEM solves (4.4.24).

We know that in the monopoly $p_u^m = 0$ and $p_r^m = \theta l \delta_r q$ (Proposition 11). Moreover, the OEM would like to set price of the new product as high as possible given the constraint. Thus, the OEM optimally sets $p_2^m = \theta h q - \theta h \delta q$ if $\theta h \delta q \geq (\theta h - \theta l) \delta_r q$ and $p_2^m = \theta h q - (\theta h - \theta l) \delta_r q$ if $\theta h \delta q < (\theta h - \theta l) \delta_r q$. Thus, $p_2^m = \theta h q - \max \{ \theta h \delta q, (\theta h - \theta l) \delta_r q \}$. Moreover, optimal sales quantity is $Q_2^m = n_h$. Thus, the optimal profit of the OEM is $\Pi_2^m = n_h (\theta h q - \max \{ \theta h \delta q, (\theta h - \theta l) \delta_r q \} - c_n) + \min \{ n_h, n_l \} (\theta h \delta_r q - c_r)$, as shown in Proposition 13(b).
C.1.4 Proof of Proposition 14

If the high segment consumer keeps the used product, he receives a second-period net utility \( \theta_h \delta q \) from using it. If the consumer buys the new product at price \( p_2 \) and sells the used product at price \( p^c_u \), he receives a second-period net utility \( \theta_h q - p_2 + p^c_u \). If the consumer sells the used product at price \( p^c_u \) and buys the OEM’s remanufactured product at price \( p^c_r \), he receives a second-period net utility \( \theta_h \delta r - p^c_r + p^c_u \). If the consumer sells the used product at price \( p^c_u \) and buys the 3PR’s remanufactured product at price \( \tilde{p}^c_r \), he receives a second-period net utility \( \theta_h \delta r - \tilde{p}^c_r + p^c_u \). To induce the consumer to buy the new product again in the second period, the OEM must set the price of the new product in the second period such that the consumer is not worse off buying the new product in the second period; that is, \( \theta_h q - p_2 + p^c_u \geq \max \{ \theta_h \delta q, \theta_h \delta r - p^c_r + p^c_u \} \) if \( \theta_l \delta r - c_r \geq \theta_l \tilde{\delta} r - \tilde{c}_r \) and \( \theta_h q - p_2 + p^c_u \geq \max \{ \theta_h \delta q, \theta_h \tilde{\delta} r - \tilde{p}^c_r + p^c_u \} \) if \( \theta_l \delta r - c_r < \theta_l \tilde{\delta} r - \tilde{c}_r \), where \( p^c_r = \theta_l \delta r q \) and \( \tilde{p}^c_r = \theta_l \tilde{\delta} r q \). Thus, the OEM solves (4.4.25).

Note that the OEM would like to set price of the new product as high as possible given the constraints. Thus, the OEM optimally sets \( p^c_2 = \theta_h q - \max \{ \theta_h \delta q - p^c_u, (\theta_h - \theta_l) \tilde{\delta} r q \} \) if \( \theta_l \delta r - c_r \geq \theta_l \tilde{\delta} r - \tilde{c}_r \), and \( p^c_2 = \theta_h q - \max \{ \theta_h \delta q - p^c_u, (\theta_h - \theta_l) \tilde{\delta} r q \} \) if \( \theta_l \delta r - c_r < \theta_l \tilde{\delta} r - \tilde{c}_r \), where \( p^c_u \) is the optimal price of the used product in the competition (refer Proposition 12).

C.1.5 Proof of Proposition 15

Note that optimal sales quantity is \( Q^c_h = n_h \). Therefore, the OEM profit is \( \Pi^c_h = n_h (p^c_2 - c_n) + \min \{ n_h, n_l \} (\theta_l \delta r - c_r) - n_h p^c_u \), where \( p^c_u \) is the optimal price of the used product in the competition (refer Proposition 12) and \( p^c_2 \) is the optimal price of the new product of the second period in the competition (refer Proposition 14).

Note that if \( \theta_l \delta r - c_r \geq \theta_l \tilde{\delta} r - \tilde{c}_r \), then \( p^c_2 = \theta_h q - \max \{ \theta_h \delta q - p^c_u, (\theta_h - \theta_l) \delta r q \} \), where \( p^c_u = \min_{n_h, n_l} \left( \theta_l \tilde{\delta} r q - \tilde{c}_r \right) (14(a)) \).
If $\theta_1 \delta_r q - c_r \geq \theta_2 \tilde{\delta}_r q - \tilde{c}_r$ and $\theta_h \delta q - \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \tilde{\delta}_r q - \tilde{c}_r) \geq (\theta_h - \theta_l) \delta r q$, then $p^*_2 = \theta_h q - (\theta_h - \theta_l) \delta r q$ and $\Pi^*_2 = n_h (\theta_h q - \theta_h \delta q - c_n) + \min \{n_h, n_t\} (\theta_l \delta_r q - c_r)$.

If $\theta_1 \delta_r q - c_r \geq \theta_2 \tilde{\delta}_r q - \tilde{c}_r$ and $\theta_h \delta q - \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \tilde{\delta}_r q - \tilde{c}_r) < (\theta_h - \theta_l) \delta r q$, then $p^*_2 = \theta_h q - (\theta_h - \theta_l) \delta r q$ and $\Pi^*_2 = n_h (\theta_h q - (\theta_h - \theta_l) \delta r q - c_n) + \min \{n_h, n_t\} (\theta_l \delta_r q - c_r - \theta_l \tilde{\delta}_r q - \tilde{c}_r)$.

Note that if $\theta_1 \delta_r q - c_r < \theta_2 \tilde{\delta}_r q - \tilde{c}_r$, then $p^*_2 = \theta_h q - \max \left\{ \theta_l \delta q - p^*_u, (\theta_h - \theta_l) \tilde{\delta}_r q \right\}$, where $p^*_u = \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \delta_r q - c_r)$ (14(b)).

If $\theta_1 \delta_r q - c_r < \theta_2 \tilde{\delta}_r q - \tilde{c}_r$ and $\theta_h \delta q - \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \tilde{\delta}_r q - c_r) \geq (\theta_h - \theta_l) \tilde{\delta}_r q$, then $p^*_2 = \theta_h q - \left( \theta_h \delta q - \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \delta_r q - c_r) \right)$ and $\Pi^*_2 = n_h (\theta_h q - \theta_h \delta q - c_n) + \min \{n_h, n_t\} (\theta_l \delta_r q - c_r)$.

If $\theta_1 \delta_r q - c_r < \theta_2 \tilde{\delta}_r q - \tilde{c}_r$ and $\theta_h \delta q - \frac{\min \{n_h, n_t\}}{n_h} (\theta_l \delta_r q - c_r) < (\theta_h - \theta_l) \tilde{\delta}_r q$, then $p^*_2 = \theta_h q - (\theta_h - \theta_l) \tilde{\delta}_r q$ and $\Pi^*_2 = n_h \left( \theta_h q - (\theta_h - \theta_l) \tilde{\delta}_r q - c_n \right)$.

**C.1.6 Proof of Proposition 16**

If a high-segment consumer buys the new product in the first period, the consumer again buys the new product in the second period at price $p^*_2$ and sells the used product at price $p^*_u$; as a result, the consumer gets a net utility $(1 + \rho) \theta_h q - p_1 - \rho (p^*_u - p^*_2)$. If the consumer does not buy the new product in the first period, he has the following four options in the second period: (i) buy the new product at price $p^*_2$ and thereby get a net utility $\rho (\theta_h q - p^*_2)$; (ii) buy the remanufactured product offered by the OEM (if $\theta_1 \delta_r q - c_r \geq \theta_2 \tilde{\delta}_r q - \tilde{c}_r$) at price $p^*_r$ and thereby get a net utility $\rho (\theta_h \delta q - p^*_r)$; (iii) buy the remanufactured product offered by the 3PR (if $\theta_1 \delta_r q - c_r < \theta_2 \tilde{\delta}_r q - \tilde{c}_r$) at price $\tilde{p}^*_r$ and thereby get a net utility $\rho \left( \theta_h \tilde{\delta}_r q - \tilde{p}^*_r \right)$; (iv) buy none and thereby get zero utility.

From the second period analysis, we already know that at optimality $\theta_h q - p^*_2 + p^*_u = \max \left\{ \theta_h \delta q, \theta_h \delta q - p^*_u + p^*_2 \right\}$ if $\theta_1 \delta_r q - c_r \geq \theta_2 \tilde{\delta}_r q - \tilde{c}_r$ and $\theta_h q - p^*_2 + p^*_u =$
max \( \{ \theta_h \delta q, \theta_h \delta_r q - \tilde{p}_r^* + p_u^* \} \) if \( \theta_i \delta_r q - c_r < \theta_i \delta_r q - \tilde{c}_r \); that is, \( \theta_h q - p_2^* \geq \theta_h \delta_r q - p_r^* \) if \( \theta_i \delta_r q - c_r \geq \theta_i \delta_r q - \tilde{c}_r \) and \( \theta_h q - p_2^* \geq \delta_r q - \tilde{p}_r^* \) if \( \theta_i \delta_r q - c_r < \theta_i \delta_r q - \tilde{c}_r \).

To induce the consumer to buy the new product in the first period, the OEM must set price of the new product in the first period such that the consumer is not worse off buying the new product in the first period; that is, \( (1 + \rho) \theta_h q - p_1 - \rho p_u^* \leq p_2^* \) or \( \theta_h q - p_1 + \rho p_u^* \leq 0 \). Thus, the OEM solves (4.4.26).

Since, the OEM would like to set price of the new product as high as possible given the constraint \( \theta_h q - p_1 + \rho p_u^* \geq 0 \), the OEM optimally sets \( p_1^* = \theta_h q + \rho p_u^* \).

Moreover, optimal sales quantity is \( Q_1^* = n_h \). Substituting the optimal solution in \( \Pi_1 = Q_1 (p_1 - c_n) + \rho \Pi_2^* \) yields optimal profit of the OEM as shown in Proposition 16(b).

C.1.7 Proof of Proposition 17

In the monopoly, the optimal first period price of the new product is \( p_1^m = \theta_h q \) and the OEM profit

\[
\Pi_1^m = n_h (\theta_h q - c_n) + \rho [n_h (\theta_h q - \Delta^m - c_n) + \min \{ n_h, n_l \} (\theta_i \delta_r q - c_r)],
\]

where

\[
\Delta^m = \max \{ \theta_h \delta q, (\theta_h - \theta_i) \delta_r q \}.
\]

Moreover, let \( K = n_h (\theta_h q - c_n) + \rho (n_h (\theta_h q - c_n) + \min \{ n_h, n_l \} (\theta_i \delta_r q - c_r)). \) Then

\[
\Pi_1^m = K - \rho n_h \Delta^m.
\]

Similarly, in the competition, the optimal first period price of the new product is \( p_1^c = \theta_h q + \rho p_u^c \) and the OEM profit

\[
\Pi_1^c = n_h (\theta_h q + \rho p_u^c - c_n) + \rho [n_h (p_2^c - c_n) + \min \{ n_h, n_l \} (\theta_i \delta_r q - c_r)] - \rho n_h p_u^c.
\]

Let \( \Delta^c = p_2^c - \theta_h q \), where \( p_2^c \) is optimal second period price of the new product in the
competition (refer Proposition 14). Thus,

\[ \Delta^c = \begin{cases} 
\max \left\{ \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \delta r, q - \bar{c}_r \right), \left( \theta_h - \theta_l \right) \delta r, q \right\} & \text{if } \theta_l \delta r, q - c_r \geq \theta_l \bar{r}, q - \bar{c}_r, \\
\max \left\{ \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \delta r, q - c_r \right), \left( \theta_h - \theta_l \right) \bar{r}, q \right\} & \text{if } \theta_l \delta r, q - c_r < \theta_l \bar{r}, q - \bar{c}_r.
\end{cases} \]

Then \( \Pi^c_t = K - \rho n_h \Delta^c \).

Thus, \( \Pi^c_t = \Pi^m_t + \rho n_h \left( \Delta^m - \Delta^c \right) \). Therefore \( \Pi^c_t \geq \Pi^m_t \) if and only if \( \Delta^m \geq \Delta^c \).

Assuming \( \delta r \geq \bar{\delta}_r \), we establish relationship between \( \Pi^c_t \) and \( \Pi^m_t \) for each parameter setting as follows:

If \( \theta_l \delta r, q - c_r \geq \theta_l \bar{r}, q - \bar{c}_r \) and \( \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \geq \left( \theta_h - \theta_l \right) \delta r, q \), then \( \Delta^c = \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \rho n_h \left( \theta_h \delta q - \left( \theta_h - \theta_l \right) \delta r, q \right) \).

If \( \theta_l \delta r, q - c_r \geq \theta_l \bar{r}, q - \bar{c}_r \) and \( \theta_h \delta q > \left( \theta_h - \theta_l \right) \delta r, q > \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \), then \( \Delta^c = \left( \theta_h - \theta_l \right) \delta r, q \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \rho n_h \left( \theta_h \delta q - \left( \theta_h - \theta_l \right) \delta r, q \right) \).

If \( \theta_l \delta r, q - c_r \geq \theta_l \bar{r}, q - \bar{c}_r \) and \( \left( \theta_h - \theta_l \right) \delta r, q \geq \theta_h \delta q \), then \( \Delta^c = \left( \theta_h - \theta_l \right) \delta r, q \) and \( \Delta^m = \left( \theta_h - \theta_l \right) \delta r, q \). Thus, \( \Pi^c_t = \Pi^m_t \).

If \( \theta_l \delta r, q - c_r < \theta_l \bar{r}, q - \bar{c}_r \), \( \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \geq \left( \theta_h - \theta_l \right) \bar{r}, q \), and \( \theta_h \delta q > \left( \theta_h - \theta_l \right) \bar{r}, q \), then \( \Delta^c = \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \rho n_h \left[ \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \delta r, q - c_r \right) \right] > \Pi^m_t \).

If \( \theta_l \delta r, q - c_r < \theta_l \bar{r}, q - \bar{c}_r \), \( \theta_h \delta q \geq \left( \theta_h - \theta_l \right) \bar{r}, q \), and \( \theta_h \delta q > \left( \theta_h - \theta_l \right) \bar{r}, q \), then \( \Delta^c = \left( \theta_h - \theta_l \right) \bar{r}, q \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \theta_h \delta q - \left( \theta_h - \theta_l \right) \bar{r}, q > \Pi^m_t \).

If \( \theta_l \delta r, q - c_r < \theta_l \bar{r}, q - \bar{c}_r \), and \( \left( \theta_h - \theta_l \right) \delta r, q \geq \theta_h \delta q > \left( \theta_h - \theta_l \right) \bar{r}, q > \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \), then \( \Delta^c = \left( \theta_h - \theta_l \right) \bar{r}, q \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \theta_h \delta q - \left( \theta_h - \theta_l \right) \bar{r}, q > \Pi^m_t \).

If \( \theta_l \delta r, q - c_r < \theta_l \bar{r}, q - \bar{c}_r \), and \( \left( \theta_h - \theta_l \right) \delta r, q \geq \theta_h \delta q > \theta_h \delta q - \min \left\{ \frac{\{n_h, m\}}{n_h} \right\} \left( \theta_l \bar{r}, q - \bar{c}_r \right) \), then \( \Delta^c = \left( \theta_h - \theta_l \right) \delta r, q \) and \( \Delta^m = \theta_h \delta q \). Thus, \( \Pi^c_t = \Pi^m_t + \theta_h \delta q - \left( \theta_h - \theta_l \right) \delta r, q > \Pi^m_t \).
\[
\min_{n_h} \left\{ n_h \right\} (\theta_i \delta_r q - c_r), \text{ then } \Delta^c = (\theta_h - \theta_l) \tilde{\delta}_r q \text{ and } \Delta^m = (\theta_h - \theta_l) \delta_r q. \text{ Thus, } \Pi^c_t = \Pi^m_t + \rho n_h \left[ (\theta_h - \theta_l) \delta_r q - (\theta_h - \theta_l) \tilde{\delta}_r q \right] > \Pi^m_t.
\]

If \( \theta_i \delta_r q - c_r < \theta_i \tilde{\delta}_r q - \tilde{c}_r \), \((\theta_h - \theta_l) \delta_r q \geq (\theta_h - \theta_l) \tilde{\delta}_r q \geq \theta_h \delta q \), then \( \Delta^c = (\theta_h - \theta_l) \tilde{\delta}_r q \) and \( \Delta^m = (\theta_h - \theta_l) \delta_r q \). Thus, \( \Pi^c_t = \Pi^m_t + \rho n_h \left[ (\theta_h - \theta_l) \delta_r q - (\theta_h - \theta_l) \tilde{\delta}_r q \right] \geq \Pi^m_t \).

In conclusion, \( \Pi^c_t > \Pi^m_t \) if \( \theta_i \delta_r q - c_r \geq \theta_i \tilde{\delta}_r q - \tilde{c}_r \) and \( \theta_h \delta q > (\theta_h - \theta_l) \delta_r q \) or if \( \theta_i \delta_r q - c_r < \theta_i \tilde{\delta}_r q - \tilde{c}_r \), \( \theta_h \delta q > (\theta_h - \theta_l) \tilde{\delta}_r q \); else \( \Pi^c_t = \Pi^m_t \).
### C.2. Tables

Table 11: Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>Quality of new product</td>
</tr>
<tr>
<td>( c_n )</td>
<td>Marginal cost of producing new product</td>
</tr>
<tr>
<td>( p_1 ) (( p_2 ))</td>
<td>Price of new product in first (second) period</td>
</tr>
<tr>
<td>( Q_1 ) (( Q_2 ))</td>
<td>Sales quantity of new product in first (second) period</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Discount factor for second period</td>
</tr>
<tr>
<td>( \delta q )</td>
<td>Quality of used product</td>
</tr>
<tr>
<td>( p_u ) (( \tilde{p}_u ))</td>
<td>Price of used product set by OEM (3PR)</td>
</tr>
<tr>
<td>( Q_u ) (( \tilde{Q}_u ))</td>
<td>Quantity of used product acquired by OEM (3PR)</td>
</tr>
<tr>
<td>( \delta_r q ) (( \tilde{\delta}_r q ))</td>
<td>Quality of remanufactured product offered by OEM (3PR)</td>
</tr>
<tr>
<td>( c_r ) (( \tilde{c}_r ))</td>
<td>Cost of remanufacturing for OEM (3PR)</td>
</tr>
<tr>
<td>( p_r ) (( \tilde{p}_r ))</td>
<td>Price of remanufactured product set by OEM (3PR)</td>
</tr>
<tr>
<td>( Q_r ) (( \tilde{Q}_r ))</td>
<td>Sales quantity of remanufactured product offered by OEM (3PR)</td>
</tr>
<tr>
<td>( h ) (( l ))</td>
<td>High-end (low-end) consumer segment</td>
</tr>
<tr>
<td>( \theta_h ) (( \theta_l ))</td>
<td>Willingness to pay of high-end (low-end) consumer segment</td>
</tr>
<tr>
<td>( n_h ) (( n_l ))</td>
<td>Number of consumers in high-end (low-end) consumer segment</td>
</tr>
<tr>
<td>( \Pi_t )</td>
<td>Total profit of OEM</td>
</tr>
<tr>
<td>( \Pi_1 ) (( \Pi_2 ))</td>
<td>Profit of OEM in first (second) period</td>
</tr>
<tr>
<td>( \Pi_r ) (( \tilde{\Pi}_r ))</td>
<td>Profit of OEM (3PR) from acquisition and remanufacturing of used product</td>
</tr>
<tr>
<td>( m ) (( c ))</td>
<td>Superscript to represent an optimal solution in the monopoly (competition)</td>
</tr>
</tbody>
</table>
Table 12: Competition: Acquisition and Remanufacturing of Used Products

<table>
<thead>
<tr>
<th>$Q^c_u$</th>
<th>$Q^c_r$</th>
<th>$p^c$</th>
<th>$\tilde{Q}^c_u$</th>
<th>$\tilde{Q}^c_r$</th>
<th>$\tilde{p}^c$</th>
<th>$\tilde{p}^c_u$</th>
<th>Constraint</th>
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</thead>
<tbody>
<tr>
<td>$n_h \leq n_l$</td>
<td>$n_h$</td>
<td>$n_h$</td>
<td>$\theta_l \delta_r q$</td>
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<td>0</td>
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<td>$\theta_l \delta_r q - c_r \geq \theta_l \delta_r q - \tilde{c}_r$</td>
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<tr>
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<td>$n_h$</td>
<td>$n_l$</td>
<td>$\theta_l \delta_r q$</td>
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<td>$\frac{\theta_l}{\tilde{p}_l} (\theta_l \delta_r q - \tilde{c}_r)$</td>
<td>$\theta_l \delta_r q - c_r &lt; \theta_l \delta_r q - \tilde{c}_r$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q^c_u$</th>
<th>$Q^c_r$</th>
<th>$p^c$</th>
<th>$\tilde{Q}^c_u$</th>
<th>$\tilde{Q}^c_r$</th>
<th>$\tilde{p}^c$</th>
<th>$\tilde{p}^c_u$</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
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<td>$n_h \leq n_l$</td>
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<td>0</td>
<td>$\theta_l \delta_r q$</td>
<td>$\theta_l \delta_r q$</td>
<td>$\theta_l \delta_r q - c_r$</td>
<td>$\theta_l \delta_r q - c_r &lt; \theta_l \delta_r q - \tilde{c}_r$</td>
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<tr>
<td>$n_h &gt; n_l$</td>
<td>0</td>
<td>0</td>
<td>$\theta_l \delta_r q$</td>
<td>$\theta_l \delta_r q$</td>
<td>$\frac{\theta_l}{\tilde{p}_l} (\theta_l \delta_r q - c_r)$</td>
<td>$\theta_l \delta_r q - c_r &lt; \theta_l \delta_r q - \tilde{c}_r$</td>
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Table 13: Solution for New Product of the Second Period in Competition

<table>
<thead>
<tr>
<th>Optimal Price</th>
<th>Constraints</th>
</tr>
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<tbody>
<tr>
<td>$p^c_2 = \theta h q - (\theta h \delta q - p^c_u)$</td>
<td>$\theta h \delta q - p^c_u \geq (\theta h - \theta l) \delta_r q$</td>
</tr>
<tr>
<td>$p^c_2 = \theta h q - (\theta h \delta q - p^c_u)$</td>
<td>$\theta h \delta q - p^c_u \geq (\theta h - \theta l) \delta_r q$</td>
</tr>
<tr>
<td>$p^c_2 = \theta h q - (\theta h - \theta l) \delta_r q$</td>
<td>$\theta h \delta q - p^c_u &lt; (\theta h - \theta l) \delta_r q$</td>
</tr>
<tr>
<td>$p^c_2 = \theta h q - (\theta h - \theta l) \delta_r q$</td>
<td>$\theta h \delta q - p^c_u &lt; (\theta h - \theta l) \delta_r q$</td>
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</tbody>
</table>
Bibliography


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Reardon, M. (2015). A hot market for preowned gadgets is a boon to savvy consumers.


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