PROBABILISTIC FATIGUE CRACK LIFE PREDICTION
IN A DIRECTIONALLY-SOLIDIFIED NICKEL SUPERALLOY

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PROBABILISTIC FATIGUE CRACK LIFE PREDICTION
IN A DIRECTIONALLY-SOLIDIFIED NICKEL SUPERALLOY

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To my parents, Mary Jane and Shelby, for their infinite patience, constant support, and undying belief in what I have yet to achieve.

“Wherever Thou wilt, Lord, but not to India.”
— Thomas the Apostle

“That’s what you think.”
— Jesus
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## List of Symbols or Abbreviations

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<th>Description</th>
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<tr>
<td>APB</td>
<td>Antiphase boundary.</td>
</tr>
<tr>
<td>CCG</td>
<td>Creep crack growth.</td>
</tr>
<tr>
<td>DS</td>
<td>Directionally solidified.</td>
</tr>
<tr>
<td>e</td>
<td>Efficiency, thermodynamic.</td>
</tr>
<tr>
<td>EIFS</td>
<td>Equivalent initial flaw size.</td>
</tr>
<tr>
<td>EPFM</td>
<td>Elastic-plastic fracture mechanics.</td>
</tr>
<tr>
<td>ESE(T)</td>
<td>Eccentrically-loaded single edge (tension) specimen.</td>
</tr>
<tr>
<td>ET</td>
<td>Elevated temperature.</td>
</tr>
<tr>
<td>FCGR</td>
<td>Fatigue crack growth rate.</td>
</tr>
<tr>
<td>FCP</td>
<td>Fatigue crack propagation.</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element model.</td>
</tr>
<tr>
<td>HCF</td>
<td>High cycle fatigue.</td>
</tr>
<tr>
<td>IGT</td>
<td>Industrial gas turbine.</td>
</tr>
<tr>
<td>LCF</td>
<td>Low cycle fatigue.</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear elastic fracture mechanics.</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo.</td>
</tr>
<tr>
<td>M(T)</td>
<td>Middle-crack (tension) specimen.</td>
</tr>
<tr>
<td>PF</td>
<td>Probability of failure.</td>
</tr>
<tr>
<td>POD</td>
<td>Probability of detection.</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions per minute.</td>
</tr>
<tr>
<td>RT</td>
<td>Room temperature.</td>
</tr>
<tr>
<td>SC</td>
<td>Single crystal.</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress Intensity Factor.</td>
</tr>
<tr>
<td>$\sigma_y'$</td>
<td>Cyclic yield strength.</td>
</tr>
</tbody>
</table>
T

TTf

TMF

TTCI

ΔK

ΔKeff

ΔKth

ΔKth

C

C

C

C

C

C

C

K

K

K

K

m

m/cyc

mA

R

r

γ

γ

μ

σ

D

<001>

[001]

{001}

Temperature.

Turbine inlet temperature.

Thermo-mechanical fatigue.

Time to crack initiation.

Stress intensity range.

Effective stress intensity range.

Threshold stress intensity range.

Crack growth rate, mm or in per cycle.

Crack growth law (Paris) coefficient.

Walker exponent.

Creep crack growth rate coefficient.

Stress intensity factor.

Critical stress intensity factor.

Crack opening stress intensity.

Crack growth law (Paris) exponent.

meters per cycle (crack growth rate).

Creep crack growth rate exponent.

Load ratio.

Cyclic plastic zone size.

Gamma phase matrix of superalloy.

Strengthening precipitate phase.

A TCP phase (microstructure).

A TCP phase (microstructure).

Grain diameter.

Family of primary crystallographic directions.

A primary crystallographic direction.

Family of crystallographic planes.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>TCP</td>
<td>Topologically close-packed.</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Statistical skewness of distribution.</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>Statistical kurtosis (peakedness) of distribution.</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Statistical mean.</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Statistical standard deviation.</td>
</tr>
<tr>
<td>(\xi(x))</td>
<td>A spatially varying stochastic process.</td>
</tr>
<tr>
<td>(-3\sigma)</td>
<td>“Minus three sigma” value of a statistical parameter.</td>
</tr>
<tr>
<td>(CV)</td>
<td>Coefficient of variation.</td>
</tr>
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SUMMARY

Accurate life prediction of fracture-critical components is vital to maintaining both personnel safety and economic viability for many engineering structures and systems. As the number of such components becomes large, quantifying the longevity of a system becomes difficult due to the natural variations in properties of the materials that make up those components. Therefore, in order to represent adequately the risk engendered by a system, the analyst must utilize the tools provided by probability and statistics to determine the chances or likelihood of a failure. With the input of a thorough statistical characterization of the variability in the relevant material properties (and operational demands), a probabilistic model can produce a clear assessment of risk as its output. The research documented herein has focused on producing both the input and output for a probabilistic life prediction model of directionally-solidified nickel-base superalloy gas turbine rotor blades based on subcritical crack growth as the primary damage mechanism.

To characterize the variation in fatigue crack growth rate (FCGR), fatigue experiments and post-test metallography examinations were performed on thin panels of a General Electric proprietary nickel superalloy produced by directional solidification. The majority of the tests were performed at 760°C (1400°F) to match engine conditions provided by General Electric, with a few tests performed at room temperature for reference. Stable FCGR was measured during all of the tests, and the variation therein was characterized from specimen to specimen as well as from casting to casting (as each casting produced nine specimens), in order to examine blade to blade as well as within-blade property changes. A computer code was written in FORTRAN to perform a basic Monte Carlo analysis of fatigue crack size and life distributions, using a distribution of initial flaw sizes produced in a parallel research program. The code enables an engineer to assess the influence of various factors on
the risk of blade failure after a certain operational life and establish appropriate inspection protocols to maintain engine safety.
CHAPTER 1

INTRODUCTION

Probabilistic analysis is emerging as the most acceptable and quantifiable language in life prediction of critical engineering hardware. Here, "critical" is an admittedly broad term which may be founded upon a machine's potential to affect human safety and life — as is the case with aircraft structures or nuclear pressure vessels, for example — or upon a large capital investment and extended service expectation by the possessor of such equipment — such as the multi-million dollar turbine engines employed by power generation utilities. (This is, of course, not to imply that safety is not at risk when capital equipment undergoes a catastrophic failure!) It is precisely with these latter applications, industrial gas turbines (IGTs), that the following document is concerned.

The application of probabilistic methods to component- and system-level life prediction enables the engineering analyst to speak in the language of the business and financial analysts for the ultimate goal of product life management. Before the advent of probabilistic life prediction, the designers of machiner tended to couch design limits in terms of conservative estimates, maximum allowable, and fairly arbitrary safety factors. Some single number, typically an average from a handful of tests, was accepted as the nominal value of any given engineering property, and a single value would be calculated as the design or limit load that would, in all likelihood, avoid an unforeseen failure. Similarly, for time- or cycle-dependent phenomena, a deterministic calculation produced a single-point determination of the time required to fail a component or system.

Minimizing the risk of failure in this type of design methodology generally results in
overdesign, producing a heavier, more expensive, sometimes less efficient machine than would otherwise have been designed. During the course of service, often modifications are made to a machine to improve its performance or meet design goals missed, and this normally comes at the cost of reducing the safety margin. However, without some kind of quantifiable assessment of whether or not failure will occur, it is almost impossible to say exactly what the effect of this reduced margin is, other than to say quite vaguely that the design is “closer” to failure than before. By using probabilistic design or life prediction, an engineering organization can balance the cost of a proposed “upstream” design change with the “downstream” cost to the likelihood of failure. Similarly, in the case of fielded equipment, rigorously quantifying the ultimate impact of several candidate life-enhancement endeavors is the only way to ensure that the limited funding available is judiciously spent for the maximum return in safety and economics.

Unfortunately, in order for a technically defensible probabilistic life prediction to be performed, it is first necessary to undertake a laboratory testing effort that can be so extensive (and expensive) as to call the entire program into question. The inputs for any probabilistic mechanical model are the forms and parameters of the statistical distribution functions for any variables that are considered random. Determining these distribution functions for any given condition (and there are often several different conditions under study) requires a significant number of mechanical experiments, as the “accuracy” of, or confidence in, the model is in large part a function of the sheer quantity of data points generated. Using a probabilistic life prediction model to show, for example, that new inspection equipment will reduce the risk of premature failure from 0.2% to 0.15% cannot justify the capital investment if the confidence level of the model, due to insufficient experimental data, is only 50%! 

2
In the case of gas turbine engines, the problem of data availability is compounded by the constant evolution of the alloys utilized in the often life-limiting hot section. While the cooler compressor section of the engine is subjected to high loads and vibration-inducing aerodynamic conditions that have spawned an entire government-wide initiative on high cycle fatigue (HCF) — for example, Reference [1] — the combination of rotational loads and exposure to temperatures approaching their melting points subjects the disks and blades downstream from the combustor to some of the most severe operating conditions of any engineering application. Engine startup-shutdown cycles, which combine the rotational acceleration with a rapid increase in temperature, impose a severe low cycle fatigue (LCF) loading on the disks and blades, while subsequent operation at high centrifugal loads and temperatures leads to creep deformation and damage, primarily in the blade airfoil.

Because the performance and economy of these costly machines is tied directly to the ability of the turbine components to survive ever-higher thermomechanical demands, nickel-base (and some cobalt-base) superalloys for blade and disk applications have been the subject of constant research and development. Casting houses and turbine manufacturers alike have identified alloying elements and casting techniques that will improve high-temperature strength, oxidation resistance, and creep strength. This has led to generations of turbines being fielded with a wide array of proprietary alloys, each with a unique chemistry, microstructure, and set of mechanical properties. As a result, there simply have not been sufficient research programs dedicated to the meticulous characterization of every commercialized superalloy that is required for a probabilistic life prediction model. Since the purpose of building such detailed life prediction tools is to reap the economic gains that can be realized with performance improvements down to tenths of a percent while maintaining failure rates down to hundredths of a percent, it would not pass muster to base the
model upon generalizations about entire "classes" of alloy, as is often done with steel, in order to reduce the amount of testing required. The particular alloy under study herein, General Electric's GTD-111, is widely used in industrial gas turbines, and yet the extant literature on its life-determining characteristics is discouragingly scarce [2]. This research program was intended in part to help fill that data void and to identify critical areas for further focused experimentation.

In addition to basic lack of data, the form of GTD-111 under study possesses a microstructure that significantly enhances its resilience to turbine operating conditions, but simultaneously poses unique challenges to fatigue crack life prediction. In turbine rotor blade applications, prolonged radial loading at elevated temperature leads to creep becoming a primary failure mechanism. In conventionally-cast, equiaxed material with a random grain boundary distribution, grain boundaries oriented transverse to the radial load serve as major damage loci through creep cavitation and creep crack growth. Elimination of the transverse grain boundaries results in significant improvement of creep damage resistance over equiaxed alloys [3]. In aerospace and some industrial applications, the grain boundary problem is often eliminated through the use of single crystal (SC) castings, which eliminate all grain boundaries from the cast component. However, very large component sizes, such as IGT blades (also called "buckets") can lead to manufacturability problems in SC castings, driving up the cost of the part. In many cases, the direct ancestor of SC casting technology, directional solidification (DS), can provide an economic alternative while retaining excellent creep strength. As a rough comparison, an equiaxed IGT cooled turbine blade costs on the order of $10,000; its SC equivalent over three times as much, with the DS version in between [4]. Even though the DS blades present significant cost savings over SC blades, the potential value in life extension of these components is still quite notable.
With the aim of demonstrating a preliminary life prediction model for probabilistic fatigue crack growth and laying the groundwork for a more detailed mechanical basis for such a framework, this document comprises 9 chapters. Having presented the motivation and context for a probabilistic life prediction model for gas turbine blades in Chapter 1, it lays out a more detailed examination of the current literature in Chapter 2. Some details of the metallurgy and microstructure of nickel-base superalloys will be covered to provide a mechanistic context for the fatigue crack growth modeling problem. The evolution of life prediction methodologies will be covered, focusing primarily on fatigue life as represented by fracture mechanics-based crack propagation models. Finally, the transition from deterministic methodologies (and their rudimentary association with statistical principles) to a probabilistic formulation of the fatigue life problem will be illustrated through a review of the significant steps in the development of such models.

Chapter 3 presents the design of, and rationale for, the specimens used for fatigue crack propagation (FCP) testing; two non-standard specimen configurations were utilized for reasons that will be explained. Chapter 4 details the experimental methods of this research program, including fatigue crack growth rate testing conditions and measurement techniques. Some additional, specialized analytical techniques required for this project are discussed in Chapter 5 to clarify the manner in which data was obtained and processed. The results of FCP testing are presented in Chapter 6, pointing out the effects of load ratio, temperature, and thickness. As the information required for a probabilistic model includes the statistical characterization of property distributions, the analysis of the experimental results will be discussed in a statistical context.

The results of the mechanical study are used in the construction of a probabilistic fatigue crack growth model as discussed in Chapter 7. Some relevant information about the nature
of turbine engine design and operation will also be presented as it pertains to the particular IGTs in question and the subsequent variables required for modeling fatigue cracking in an entire IGT fleet. Having described the model, several simulations of crack growth in sets of engines are presented, and the influence of different operational and material variables are discussed. The most significant conclusions reached with this research are summarized in Chapter 8, and recommendations for further work in this area are given in Chapter 9.
CHAPTER 2

BACKGROUND

Prior to beginning discussion of the current effort to model fatigue crack life in industrial gas turbines, some background material will be presented in order to establish a frame of reference and define some commonly used terms. The conditions inside a gas turbine engine present one of the most challenging environments in engineering design, so the basics of turbine layout and operation will be covered. Relevant operational and mechanical nuances of an IGT will be presented, since some of the terms in the life prediction model are unique to this arena. Next, approaches to establishing safe life limits for critical turbine hardware will be discussed. In light of the multitude of available lifing methods, a discussion of the unique features of the present allow follows, as these features are a key motivation for developing the proposed methodology. Finally, a number of previous efforts in probabilistic fatigue are explored to provide guidance in the analysis and interpretation of the fatigue crack growth data that will be obtained.

2.1 GAS TURBINE ENGINES

2.1.1 Gas Turbine Fundamentals

While the mechanical details involved in the actual assembly and function of a gas turbine engine are numerous and complex, conceptually a turbine is quite simple. The heart of a turbine engine is essentially a single shaft that couples a rotating compressor at the front with a drive turbine (the specific section of the machine that gives the turbine engine its name) at the back. Both the compressor and the turbine are comprised of one or more rotors,
which are rotating disks with a large number of wing-like airfoils or "blades" attached to the circumference. In the compressor section, the blades are doing work upon the air that is drawn in, increasing the pressure, temperature, and density of the air and forcing it aft into the heart of the engine; in the turbine, the air does work upon the blades, pushing against the airfoils and spinning the rotors like windmills. Between the compressor and the turbine, the combustor combines fuel with the compressed air to generate the hot, high pressure combustion gas which expands through the turbine and drives the shaft, which in turn powers the compressor and closes the energy loop. A simple schematic of a turbine is shown in Figure 2.1.

![Figure 2.1: Schematic of a single-stage turbine and relative stage temperatures](image)

Figure 2.1: Schematic of a single-stage turbine and relative stage temperatures
The front end of a turbine engine, the compressor or "cold" section, is primarily an aerodynamic challenge, as the ingested air is being forced "uphill" against a steep pressure gradient. While gas temperatures increase a few hundred degrees in the cold section, as shown in Figure 2.1, the temperatures are still low enough to enable the use of lightweight titanium alloys, such as Ti-6-4 and Ti-6-2-4-2, for both the blades and the disks. The "hot" section or the actual turbine itself, however, experiences extremely high temperatures of the combustion gas, and requires the use of more heat-resistant alloys. Nickel and cobalt alloys are the most common for high temperature applications, for reasons that will be explored later.

Turbine engine performance and efficiency both improve with increased maximum (post-combustion) temperature. While the actual efficiency of a turbine engine is limited by mechanical and thermodynamic losses, the general trend in efficiency increase with temperature can be illustrated by the Carnot equation,

\[ e = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{min}}} \]  \hspace{1cm} (2.1)

where \( e \) is the ideal efficiency of a thermodynamic cycle, and \( T_{\text{max}} \) and \( T_{\text{min}} \) are the highest and lowest temperatures experienced by the working fluid. Also, the work output of a given cycle is generally proportional to the \( (T_{\text{max}} - T_{\text{min}}) \) term in the numerator. Thus, performance and efficiency increases can be achieved by increasing the "firing temperature" of the gas that exits the combustion section. Because of this, as turbines became more advanced, even the specialized superalloys developed for such conditions were unable to withstand operation at the combustion gas temperatures and high centrifugal stresses without some means of cooling. To reduce the metal temperatures and prevent premature failure, sophisticated "secondary flow" schemes have been developed in which compressed, lower-temperature air is bled out of an intermediate compressor stage and channeled down.
around the main shaft or out near the external casing into the combustor and turbine sections, where it is used to cool critical components. The airfoils that make up the rotor blades and nozzle vanes are also often cooled internally in the first, hottest stages of the turbine section. For blades, some of the turbine cooling air is contained by seal plates pressed up against the turbine disk and forced into small passages in the blade-disk attachment slot where it enters cooling passages inside the blade through holes in the bottom or shank of the blade. This cooling air reduces the temperature of the metal airfoils from the inside and is then ejected out through holes in the airfoil into the main engine airflow. The use of cooling air enables the hot section components to operate in environments that would otherwise not be survivable by the base metal. However, the extreme temperature gradient created across the thin airfoil section separating cooling air from combustion gas can cause severe thermal stresses associated with the engine startup cycle.

Saving the turbine blades from elevated temperature failures comes at the cost of overall engine efficiency and performance, unfortunately; any air bledd out of the compressor for blade cooling is air that is not being used to perform work [5]. In order to extend the temperature capability of turbine blades without suffering performance setbacks from excessive cooling air penalties, the blades can be coated with a thermal barrier coating (TBC), such as yttria-stabilized zirconia (YSZ), to protect them from direct exposure to the heat of the combustion gases. In addition to simple thermal protection, coatings are also required to prevent the blades from suffering environmental attack. These environmental barrier coatings (EBCs), such as the popular NiCoCrAlY blend, protect against high temperature oxidation and hot corrosion [6]. However, these fairly brittle coatings bring new failure mechanisms into the life analysis, and oftentimes cracking in the turbine blades originates in the protective coating. Indeed, as coatings enable the turbine blades to survive
ever higher firing temperatures, the useful life of the coating becomes the controlling factor that dictates the inspection and repair interval for the engine; furthermore, the nature of the coating and its failure mechanism increases the uncertainty in blade life, making life prediction more difficult [4].

2.1.2 Industrial Gas Turbines

While gas turbine engines are most commonly associated with aerospace propulsion, they have in the last few decades become significantly more widespread in the power generation industry. Gas turbines, or combustion turbines as they are more commonly called in this arena, used for power generation are very similar to their aerospace counterparts, except they are much larger in size and operate at a lower rotational speed. A General Electric MS9001F model IGT is shown in Figure 2.2; the technician in the foreground gives an indication of scale. IGTs also differ from their aerospace counterparts by their typical modes of operation. While an aircraft turbofan engine may receive a thorough off-wing inspection approximately every 5,000 hours with the opportunity to repair and replace blades, an IGT bucket will see several times that operating interval before its condition is adequately assessed [7]. The operation of an IGT can fall into one of two general categories: base mode, and cyclic mode. A base mode unit is used for sustained, primary power generation; it operates for hundreds of hours at a time without interruption. A cyclic unit, on the other hand, is normally used to supplement another primary energy source — typically older coal- or oil-fired plants — during times of “peak” power demand, and is frequently started and stopped as the extra power is required. The two modes differ substantially in nature, but approximately the same operating hours take place between depot maintenance: for the particular IGTs under study, base units operate an average of 500 hours for every startup,
and are brought in for maintenance at 24,000 hours; the cyclic units only run an average 30 hours per startup, but at a maintenance interval of 900 cycles (startups), accumulate only 12.5% more operating hours, at 27,000. It should be noted that this length of operation without maintenance is on the same order as the life limits for some fracture-critical aircraft engine components, so the rotors in IGTs must be designed with longevity in mind.

![General Electric MS9001F turbine assembly](image)

Figure 2.2: General Electric MS9001F turbine assembly

Efficiency, not surprisingly, is one of the primary reasons IGTs are growing in popularity for power generation plants. Traditional steam-driven generator power plants achieve overall thermodynamic efficiencies of only approximately 25% for most coal-fired plants and 33% for nuclear plants. IGTs, on the other hand, are capable of simple cycle efficiencies on their own of nearly 40%; when coupled with a smaller steam turbine to make use of the waste exhaust heat, they are capable of closer to 60% overall efficiency [8]. Energy efficiency controls the return on investment in an IGT, and as described by Eq. (2.1), the efficiency of an IGT increases with maximum temperature, or the turbine inlet temperature.
(TIT). It has been estimated that an increase of 50°C (90°F) in TIT results in approximately 2-4% higher efficiency and an 8-13% increase in power generation. The current level of IGT technology has achieved firing temperatures approaching 1425°C (2600°F). The drive to increase IGT performance and cost effectiveness through higher firing temperatures comes, however, at the expense of the components and materials used in the combustor and turbine sections.

2.2 LIFE MANAGEMENT

In light of the harsh operating conditions within gas turbines as described above, it is not uncommon for cracks to form at highly stressed locations, such as the turbine blade cooling holes subjected to centrifugal and thermal loads. If a fatigue crack in a turbine blade were to grow critical and lead to fracture and blade release, the kinetic energy unleashed would cause severe damage to the engine and possibly endanger lives. In a typical IGT, a blade failure could cause up to $3 million in damage, according to one case study [9]. Therefore, life models are needed to determine when these fracture-critical components should be inspected and repaired to guarantee safety and minimize economic losses.

2.2.1 Traditional Fatigue Approach

Traditional lifing of fracture-critical turbine engine components has been based what is known as the "safe life" design approach. This method generally considers crack initiation by LCF as the failure criterion, and is therefore also commonly known as the time to crack initiation (TTCI) approach, or safe crack initiation life [10]. Under the safe life approach, the life of a critical component is established by its predicted LCF life to initiate some "significant" or measurable crack, commonly on the order of 0.4 mm [11, 12]. Typically, strain-life LCF data are applied to high strain locations that have been identified
on a fracture critical component through analysis in order to predict the fatigue life of the component as controlled by LCF failure at the critical location.

However, even if the life prediction input parameters, such as strain range and temperature, are constant or well characterized, the fatigue life data is rarely if ever so. Significant scatter exists in the experimental fatigue data, and basing the fatigue life prediction on some single, nominal curve can lead to dangerous risks or undue conservatism. Repetitive fatigue testing at several conditions is required to quantify the risk of failure in a given analysis; an excellent discussion of a number of experimental procedures and statistical methods is available in Reference [13]. Once a number of specimens (roughly 15 or more) have been tested at each of several different load or strain levels, statistical distributions (the lognormal distribution is widely employed for fatigue lives) are fitted to the data for each condition, as shown in Figure 2.3. Rather than a single curve through the center of the data, individual curves are fitted to specific probabilities of failure at each condition, creating isolines of probability. In order to ensure an adequately conservative design, the "minus three sigma" (-3σ) curve, or three standard deviations below the mean life, is used as a life limit for the part, and even then a safety factor (typically 3) is applied to reduce the life even further. This implies that, safety factor aside, only one in 750 parts (the failure rate at the -3σ limit) would fail, or conversely, that almost 99.9% of the parts would be retired without knowing anything of their fatigue condition, possibly disposing of significant remaining viable life. In addition, this basic fatigue approach cannot predict the life of a part with a pre-existing (and possibly undetected) crack-like flaw. While the statistical methods may seem thorough, there is still a great deal of information not revealed with this method.
Figure 2.3: Statistical distributions applied to fatigue data [13]

2.2.2 Fracture Mechanics & Damage Tolerance

Utilizing a fracture mechanics based methodology surpasses fatigue design in its ability to predict the useful life of a fracture critical component, particularly where inspection data and crack detection are available. Using fracture mechanics to describe fatigue crack growth enables the designer to predict the rate at which an existing crack will grow under service loads, and the time required for the crack to reach instability or final fracture. Once the total life to a critical crack size is determined, some portion thereof is determined to be the useful life, much like the safety factor applied to traditional fatigue life as above. However, unlike the traditional fatigue approach, the damage tolerant fatigue life is renewable at each inspection. The components are inspected for cracking at regular intervals, and any component not displaying cracking beyond a certain limit can be returned to service with a quantifiable remaining life. The acceptable interval between inspection intervals is
based on crack propagation analysis of the largest possible undetected flaw, such that no undetected flaw will grow beyond the rejection criterion before the next inspection. Thus, any component with no fatigue crack indications can be reasonably assumed to perform through the next service interval and, at worst, return to inspection with a subcritical but rejectable crack. Other components which actually display fatigue cracking at inspection may be returned to service for another interval if the detected crack will not propagate beyond some limit prior to the next inspection. The damage tolerance approach, therefore, allows for a better understanding of fatigue damage in a component, and enables better decisions about fitness for duty, which is particularly important in life extension of machines at the end of their design life.

2.2.3 Probabilistic Life Models

As discussed above, fatigue data, like any other physical property, display scatter from test to test. For example, in FCGR data, a factor of two between minimum and maximum FCGR has often been employed as a rule of thumb, but it can be even greater as constraint conditions change, or in materials with anisotropic or heterogeneous microstructures (such as DS superalloys, as will be shown later). The existence of significant scatter in fatigue data for all manner of materials has driven efforts throughout industry and academia to model fatigue as a non-deterministic process. When attempting to predict the life of a hypothetically cracked structural member for a large number of fielded components, a designer must account for the fact that each member can possess different fatigue initiation and crack propagation resistances. This component property variability can arise from such factors as heat treatment non-uniformity, tool wear during machining, and environmental and aging effects in service. Probabilistic modeling of fatigue allows the designer to asses
the risk of the weakest structural members of a population failing after a certain measure of usage due to this component variability, without knowing specifically which component will fail.

Several software packages, whether public/academic, commercial, or company proprietary, have been developed to treat probabilistic fracture mechanics; due to their numbers, only a few will be discussed here. One of the earlier programs, PROF (PRobability Of Failure) [14] was developed for the US Air Force and models probabilistic distributions of initial and post-repair flaw sizes, probability of detection distributions, and load spectra. A significant result of published simulations is that, by attaching cost figures to the processes of inspection, repair, and failure in the model, the user is able to optimize inspection and repair schedules to minimize the overall life cycle cost.

One of the most prominent codes that has been validated by most aircraft engine manufacturers in the last several years is DARWIN (Design Assessment of Reliability With INspection) [15]. Developed by Southwest Research Labs (SwRI) in response to hard alpha defects in titanium rotors, DARWIN imports models from many commercially available finite element modeling (FEM) packages and performs a zone-based Monte Carlo (MC) simulation of crack growth from random initial flaws. DARWIN has been designed specifically for use with 2-D models of axisymmetric rotors, although application to 3-D components such as blades is planned. The primary focus of the MC simulation is the size, orientation, and location of the initial defect in the rotor. The fracture mechanics crack growth module of the program is deterministic; as described above for models in $a-N$ space, a distribution is applied to the deterministic life prediction taken as the median behavior.
Another package available for FEM use is a new addition to the popular ANSYS software called PDS (Probabilistic Design System) [16]. ANSYS is commonly used by engineering companies to perform component stress, thermal, and fatigue analysis of 2- and 3-D models. The ANSYS-PDS package can perform MC simulation as well as Response Surface approximations, which are less computationally intensive. While this software is versatile and thorough, it also requires the use of parallel computing systems for a model of any complexity, occupying dozens of workstations for hours or even days to perform MC simulations of a large, refined FEM mesh.

Finally, several more general-purpose probabilistic analysis packages exist which can be used in conjunction with various types of engineering computation models, including NESSUS, NESTEM, ProFES, and QRAS. NESSUS was developed by NASA and SwRI and has subsequently been enhanced into NESTEM [17]. Both of these packages generate probability of failure for component models fed into the program by sampling random variables such as load and temperature, and they also generate parameter sensitivity information. ProFES [18], developed at Los Alamos National Laboratory, also utilizes FEM files from other programs, such as ANSYS, and applies probabilistic distributions to selected variables in the FEM data file. It has the capability to perform MC simulations, but as with ANSYS-PDS this would be computationally intensive, so Response Surface, first-order (FORM) and second-order reliability method (SORM) are included. QRAS [17], a NASA code, integrates the results from codes such as the above and, based on user input on system hierarchy, calculates overall system reliability for complex systems with multiple failure modes. While the ability to couple such codes with complex finite element models makes them very powerful tools, the fact that they may also be used with simple numerical codes, such as a one-dimensional FORTRAN simulation of crack growth, promises
extremely fast trade and sensitivity studies at early design stages.

In contrast to the scarcity of published data on GTD-111 that will be presented later, the manuscripts produced over the last 25 years on probabilistic fatigue life prediction are legion, and the subject continues to grow in popularity (see, for example, Reference [19]). This is certainly appropriate, inasmuch as well-deserved attention is being brought to bear on the growing problems in the life extension of aging aircraft fleets and improved management of power generation systems. However, it has also produced so many different “camps” of fatigue probabilists, each convinced of the veracity and superiority of their own methodology, that no single approach receives the collective attention of the field, and it leaves one to wonder just how a broadly accepted, robust “industry standard” can ever be adopted by the governing agencies (e.g., FAA). That said, it would seem that the development of yet another methodology would be redundant at best. However, there are unconventional characteristics to the alloy under study (particularly in its given airfoil application) that are not directly addressed by currently available software. While some commercial package will likely be employed in the future to some adaptation of the current approach, it is still useful and informative to explore and develop a methodology based on the system at hand. Below, the structure and properties of nickel-base superalloys (and their directionally solidified variants) will be presented, and their implications for fatigue modeling will be discussed.

### 2.3 DEVELOPMENT AND CHARACTERISTICS OF NI-BASE SUPERALLOYS

Nickel-base superalloys dominate materials selection for turbine component design due to their high temperature strength, fatigue and fracture resistance, creep strength, and corrosion and oxidation resistance. Other materials being studied for turbine applications —
simple intermetallic alloys, such as nickel or titanium aluminides, and monolithic or reinforced industrial ceramics, such as silicon nitride and silicon carbide — have the allure of increasing an engine’s power-to-weight ratio and, in the case of ceramics, increasing the operating temperature or reducing cooling air. However, their low fracture toughness and extremely poor tolerance of stress concentrations and impact damage currently indicate that nickel-base superalloys will be the material of choice in turbine design for the foreseeable future.

2.3.1 Composition and Microstructure

The history of Ni-base superalloys began in 1941 with the development of Nimonic 80, a fairly simple Ni-Cr alloy that contained small amounts of Ti and Al which formed the Ni3(Ti,Al) precipitates, known as γ′, which form the backbone of the superalloys’ excellent high temperature properties. Since then, a wide range of other alloying elements have been added to alloy compositions in order to enhance creep and oxidation properties, for example. While the strength and utility of these alloys in the extreme environment of the turbine cannot be questioned, the relative complexity of their composition receives some good-natured ribbing from metallurgists, calling them an “alphabet soup” for containing a dozen or more alloying elements. The compositions of a number of Ni-base superalloys are shown in Table 2.1.

The microstructure of Ni-base superalloys basically consists of an austenitic FCC matrix, the γ phase; an ordered L12 precipitate, γ′; and various carbides (M23C6, MC, and M6C, where M represents a metal element) and borides distributed throughout. The primary matrix constituents are Ni and Cr in solid solution, with Co, W, Mo, Al, and Ta contributing to further solid solution strengthening. The M23C6 carbide is generally a grain
Table 2.1: Common Blading Alloy Compositions [20]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Composition by Weight Percent</th>
<th>Ni</th>
<th>Cr</th>
<th>Co</th>
<th>Mo</th>
<th>W</th>
<th>Ta</th>
<th>Nb</th>
<th>Ti</th>
<th>C</th>
<th>B</th>
<th>Zr</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Selected Equiaxed Alloys</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rene 80</td>
<td>Bal.</td>
<td>14.0</td>
<td>9.5</td>
<td>4.0</td>
<td>4.0</td>
<td></td>
<td></td>
<td>3.0</td>
<td>5.0</td>
<td>0.17</td>
<td>0.015</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>IN 738LC*</td>
<td>Bal.</td>
<td>16.0</td>
<td>8.5</td>
<td>1.7</td>
<td>2.6</td>
<td>1.7</td>
<td>0.9</td>
<td>3.4</td>
<td>3.4</td>
<td>0.11</td>
<td>0.010</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>IN 939*</td>
<td>Bal.</td>
<td>22.5</td>
<td>19.0</td>
<td></td>
<td>2.0</td>
<td>1.4</td>
<td>1.0</td>
<td>1.9</td>
<td>3.7</td>
<td>0.15</td>
<td>0.009</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>GTD-111*</td>
<td>Bal.</td>
<td>14.0</td>
<td>9.5</td>
<td>1.5</td>
<td>3.8</td>
<td>2.8</td>
<td></td>
<td>3.0</td>
<td>4.9</td>
<td>0.10</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Selected Directionally Solidified Alloys</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW 1426</td>
<td>Bal.</td>
<td>6.5</td>
<td>10.0</td>
<td>1.7</td>
<td>6.5</td>
<td>4.0</td>
<td></td>
<td>6.0</td>
<td></td>
<td>0.10</td>
<td>0.015</td>
<td>0.10</td>
<td>1.5 Hf, 3.0 Re</td>
</tr>
<tr>
<td>DS GTD-111*</td>
<td>Bal.</td>
<td>14.0</td>
<td>9.5</td>
<td>1.5</td>
<td>3.8</td>
<td>2.8</td>
<td></td>
<td>3.0</td>
<td>4.9</td>
<td>0.10</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Selected Single Crystal Alloys</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PW 1480</td>
<td>Bal.</td>
<td>10.0</td>
<td>5.0</td>
<td></td>
<td>4.0</td>
<td>12.0</td>
<td></td>
<td>5.0</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMSX-4</td>
<td>Bal.</td>
<td>6.5</td>
<td>9.0</td>
<td>0.6</td>
<td>6.0</td>
<td>6.5</td>
<td></td>
<td>5.6</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td>3.0 Re, 0.1 Hf</td>
</tr>
<tr>
<td>SC 16*</td>
<td>Bal.</td>
<td>16.0</td>
<td></td>
<td>3.0</td>
<td></td>
<td>3.5</td>
<td></td>
<td>3.5</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Primarily Industrial Gas Turbine Alloys
boundary strengthener, hindering creep cavitation and grain boundary sliding, although it has been known to be able to develop a deleterious cellular structure, which results in a lower creep rupture strength. In addition, during the evolution of superalloy composition, topologically close-packed (TCP) phases, such as σ, μ, and laves phases, began to develop as plate-like structures along {111}-type planes as a result of poor compositional quality. While new composition control procedures have largely eliminated their presence, they can still develop under some conditions and adversely affect creep strength and ductility. These TCP phases also promote crack propagation due to their brittle nature [21]. A trade off exists in that while an increase in Cr improves hot-corrosion resistance, at a certain solubility limit it promotes the formation of the σ phase.

The γ’ precipitate is largely what gives Ni-base superalloys their superior high temperature strength properties. The history of these superalloys has seen a general increase in the volume fraction of the γ’ precipitate, as well as a slight size increase and a progression toward a more cubic morphology. The cubic form of γ’ found in modern superalloys is indicative of an increased lattice parameter mismatch with the γ matrix; the cubes form with ⟨001⟩-type facets. Recent study of GTD-111 has shown a generally cubic morphology of γ’ [21]. In addition, smaller, secondary gamma prime precipitates have arisen in the microstructure. The γ’ is a long-range ordered precipitate and therefore contributes to anti-phase boundary (APB) strengthening through dislocation interaction. It is this order-based strengthening mechanism which brings about superalloys’ unique strength-versus-temperature behavior as described below.
2.3.2 Directional Solidification and Single Crystals

Superalloys have been developed specifically for high temperature applications, and the primary damage mechanism that arises under such conditions is creep. By serving as damage loci for creep cavitation and enhanced deformation through sliding, grain boundaries oriented non-normally to the principal stress axis are a superalloy component’s greatest weakness. To eliminate this problem for cast components, VerStyder developed the process of directional solidification in 1966 [22]. By drawing a casting slowly out of a furnace while extracting heat axially through a chill plate at the bottom of the mold, as shown in Figure 2.4, it is possible to control solidification such that it occurs in a stable, one-dimensional (from a macroscopic perspective) fashion. This produces a columnar grain structure in which all grain boundaries are nominally oriented parallel to the direction of solidification, with the intention that this will coincide with the direction of primary loading. While the elimination of transverse grain boundaries greatly improves creep properties of DS blades, it is still necessary to strengthen the longitudinal grain boundaries with distributed carbides to hinder grain boundary sliding. Formation of these carbides requires the presence of B, Zr, and C, just as with polycrystalline castings, as can be seen in Table 2.1. The main compositional change introduced with DS alloys is the addition of Hf to enhance intermediate temperature ductility [23]. Introduction of directional solidification techniques improved operating temperatures by approximately 50°C while maintaining the same stress rupture capability. Thermal fatigue resistance is also improved by roughly an order of magnitude over polycrystalline castings [24].

DS alloys form with a primary <001> axis aligned with the solidification direction and the other two <001>-type axes oriented randomly from grain to grain. Solidification dendrites are evident within the individual grains, as shown in Figure 2.5, with primary
Figure 2.4: Schematic of directional solidification method, showing optional single crystal selector for SC castings [23]

(PDA) and secondary (SDA) dendrite arms (and in some cases tertiary), separated by an interdendritic eutectic. The spacing between PDA and SDA are both functions of local cooling rate.

An enhancement to the DS casting technique uses a corkscrew-shaped flue at the bottom of the casting mold, known as a single crystal selector, to promote the persistence of only one grain or crystal as the solidification front moves into the body of the casting. The entire component casting is then a single crystal,* and the complete elimination of grain boundaries negates the need for the grain boundary strengthening elements Hf, B, Zr, and C. Removal of these alloying elements increases the melting temperature of superalloys by

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*Technically, both DS and SC castings are produced through directional solidification, as the mold is extracted from a furnace to apply a controlled cooling gradient, and the microstructural difference was originally highlighted by the terms "columnar crystal" (CC) versus "single crystal" (SC). Current reference to a DS material, then, actually refers to a CC grain structure, and SC is reserved for castings which use the single crystal selector.

24
80 to 100°C [23]. The addition of approximately 3% Re to the SC alloys enables a higher concentration of Al and Ti and thus a higher volume fraction of γ'. Furthermore, the Re stabilizes the γ' and impedes coarsening, one of the main microstructural degradations in service. Use of SC blades enables a 20 - 50°C increase in operating temperature over DS alloys [23]. However, SC casting techniques are quite challenging from a quality control perspective, and a number of defects can arise which render a component unusable. Casting challenges are exacerbated by the very large component sizes required in IGT applications. While cooled SC turbine blades are more and more common in the first stage rotor of many aerospace propulsion engines where smaller components reduce the risk of casting defects, the application of such technology to industrial turbines can be cost prohibitive. In such circumstances, the use of DS blades provides a sufficient improvement over polycrystalline properties in a more economical manner.
2.3.3 Mechanical Properties

The γ matrix of Ni-base superalloys has a low stacking fault energy which promotes planar slip and inhomogeneous deformation until cross-slip is thermally activated. As mentioned above, γ' also contributes greatly to the strength of superalloys. A lattice parameter mismatch of approximately 0.2 - 1% strengthens the matrix through the induced lattice strains which hinder dislocation motion. The precipitate strengthens the system through the formation of both superlattice and APB faults during shear as a result of its long range order [23]. At high volume fractions and optimum morphology of γ', the dislocations do not climb past or bow between precipitates, but rather cut through them, as shown in Figure 2.6. The creation of the APB in γ' causes pairing of dislocations as they move through the precipitate in shear. This leads to an increase in strength of γ' with temperature as thermally-assisted cross-slip can move one of the pair of dislocations to a cubic [001] plane, effectively pinning the pair. This increase in precipitate strength helps counteract the general reduction of lattice strength with temperature, and can actually cause an increase in strength with increasing temperature over a certain range. Eventually a high enough temperature is reached that primary cube-slip is activated. At this point, strength
begins to decrease rapidly, as shown in a schematic of yield strength versus temperature representative of alloys like the one currently under study, shown in Figure 2.7.

![Graph showing yield strength versus temperature](image)

**Figure 2.7:** Yield strength versus temperature trend for a nickel-base superalloy

Compared to some conventionally cast, equiaxed alloys with a random grain orientation distribution, DS alloys have a very heterogeneous microstructure and anisotropic (or more accurately, orthotropic) properties. Most measures of mechanical properties are considered in two categories, longitudinal and transverse. In general, DS alloys for blade applications are tailored to have maximum strength in the longitudinal direction, which will coincide with the applied load in service, although this can change with temperature as different damage mechanisms become dominant. For the particular alloy under study, yield and ultimate tensile strengths at room temperature (RT) are both superior in the longitudinal direction (by roughly 8% and 46%, respectively). However, in the critical temperature range (particularly 760°C) for maximum strength (the "hump" in Figure 2.7), transverse yield strength actually exceeds that in longitudinal by 11% [21]. At 850°C, longitudinal yield strength is once again higher than transverse, but the ultimate tensile strength becomes
superior in the longitudinal direction by about 10%. The elastic modulus of DS and SC superalloys is extremely low compared to equiaxed alloys, falling generally, in the longitudinal (solidification or [100]) direction, at just over half the stiffness of conventionally polycrystals. This low modulus actually contributes to the alloys’ superior thermal fatigue resistance by reducing the magnitude of stresses associated with a given thermal strain.

2.3.4 Fatigue Crack Growth in Ni-Base Superalloys

Modeling FCP in a DS superalloy such as this presents unique challenges due to both the nature of the material and the complexity of the operating conditions. A survey of the results of some previous studies on the advanced alloys employed in turbine hot sections will highlight some trends — and sometimes the lack thereof — in FCP behavior in superalloys. Compared with the multitude of published studies on fatigue crack propagation in equiaxed and SC superalloys, there have been very few such investigations in their DS counterparts. Many studies of crack growth in DS materials have been confined to pure metals or simple alloys. For the DS superalloy under study (and many other DS blading alloys), the size of the cyclic plastic zone, \( r_c \), is very small compared to the average grain diameter. The zone size is determined by

\[
    r_c = \frac{1}{\pi} \left( \frac{\Delta K}{2\sigma_y^c} \right)^2
\]

where \( \Delta K \) is the stress intensity range and \( \sigma_y^c \) is the cyclic yield strength, used in lieu of the monotonic yield strength in the case of materials that display cyclic hardening or softening [26]. Crack propagation is generally a function of the damage mechanics that take place inside the cyclic plastic zone [27, 28, 26], and it is proposed that for large portions of crack growth in a DS alloy outside of the influence of the widely-scattered grain boundaries, the mechanics of FCP are very similar between DS and SC materials. While SC turbine
blades contain no grain boundaries, typical DS blades contain only tens of grain boundaries, as compared to the thousands of grain boundaries found in conventional polycrystalline castings. With this in mind, and since the general chemistry, processing, and microstructure of DS and SC superalloys are essentially the same, some insight into the damage of DS superalloys can be gleaned from examining SC research results.

2.3.4.1 Effect of Precipitates

As mentioned above, these Ni-base superalloys are precipitation hardened with the strengthening precipitate $\gamma'$, which largely controls their high temperature strength through its influence on dislocation movement. The size, spacing, and morphological distributions of $\gamma'$ can be influenced through chemistry adjustments and heat treatment of the cast components, so understanding its influence on FCP is an important link in predicting a material's fatigue capability. In a study of FCP in polycrystalline Waspaloy, in which two different average sizes of $\gamma'$ were produced through heat treatment, Antolovich and Jayaraman showed that an increase in precipitate size from 10 nm to approximately 100 nm resulted in FCGR increased by up to a factor of five [29]. Furthermore, it was shown that a finer $\gamma'$ distribution increased FCP resistance [27]. Since $\gamma'$ pins the slip planes on which it resides, a more uniform distribution would enhance FCP resistance; in fact, preliminary observations by Boyd-Lee indicate that slip bands tend to occur through lattice planes pinned by a below-average $\gamma'$ density [27]. However, Sengupta et al., in a study of FCP in the SC alloy CMSX-4 at 650°C, showed the opposite effect; in this case, an increase in $\gamma'$ size decreased near-threshold FCGR and increased $\Delta K_{th}$ [30]. Although the current understanding is unclear, nevertheless the role of $\gamma'$ should be kept in mind when modeling the fatigue strength of high temperature components, considering the coarsening of the precipitate.
that has been observed after high temperature aging as mentioned above.

2.3.4.2 Effect of Grain Structure

It is generally accepted from a number of studies that the effect of grain size on FCP for a given alloy composition is that an increase in average grain size reduces the FCGR in the material, as summarized by Bowman [29]. There appears to be both an intrinsic and extrinsic mechanism for this phenomenon [29, 31]. Intrinsically, larger grain sizes promote heterogeneous deformation at the crack tip, promoting single slip, and as a result, slip reversibility is increased; it has been argued by Antolovich et al that damage accumulation ahead of the crack tip proceeds more slowly under these circumstances as less dislocation interaction eases dislocation motion, and LCF life in the damage zone is increased [29]. Externally, this heterogeneous deformation mechanism in turn produces larger crystallographic facets on the fracture surface, resulting in a significantly more tortuous crack path for coarse-grained material [31]. This can retard crack growth and increase threshold through a reduced crack driving force (crack deflection) and increased roughness-induced closure. There is still some debate as to whether the crack path tortuosity is a primary cause of the reduced FCGR, or merely a secondary product of the inherent crack path selection mechanism [27] which fundamentally drives the instantaneous FCGR. However, experimental results which show that the difference between coarse and fine grain material crack growth rates diminishes at a higher load ratio suggest that closure is indeed a source of the difference [29]. It is interesting to note that Patel et al found that the coarse-grained specimens displayed considerably more scatter in FCGR than fine-grained specimens of the same alloy [31], a finding which would suggest interesting implications in probabilistic modeling of a large diameter columnar-grained casting.
While these effects are fairly consistent in long fatigue crack growth, the interaction of small cracks with grain boundaries is characteristically abnormal. Small cracks growing by a Stage I, crystallographic single-slip process following persistent slip bands, which are generally blocked by high angle grain boundaries. Thus, while a large spacing between grain boundaries can promote longer fatigue life through the crack deflection described above, more closely spaced grain boundaries may hinder the propagation of small cracks. This has in fact been observed in Waspaloy and Astroloy FCP specimens, in which smaller grain size improved small crack growth resistance even though it was detrimental to long crack growth resistance below 400°C [27]. According to some dislocation models, FCGR in small cracks depends largely on the distance to the approached grain boundary, accelerating as grain boundaries trap dislocations, although it is disputed whether superalloys follow this trend [27]. However, in studies of nickel-base bi-crystal specimens, Wang et al showed that FCGR was highest at an “optimum” distance from the grain boundary, reaching a minimum as the crack tip reached the boundary and accelerating again once across [32]. This was attributed to the grain boundary-induced redistribution of stresses ahead of the crack tip.

Another aspect of grain size and orientation that must be considered in crack growth through DS materials is that as the crack tip moves from one grain into the next, the fatigue damage process zone moves from one orientation of slip systems to another. Since SC components have an orthotropic system of properties that extend throughout the material, a number of studies have been performed on the effect of crystallographic orientation on various damage mechanisms such as creep, fatigue, and crack propagation. Many studies considered crack growth variations as the loading axis (and normal axis to the nominal
crack plane) was varied between [001], [011], and [111] type directions. Of primary interest to studies of DS material is testing conducted with the load axis coinciding with [001], since the components are specifically designed with this intent. While all of the columnar grains have nominally parallel [001] solidification axes, the other two ([010] and [100]) axes will vary in orientation from grain to grain, so the direction of fatigue crack propagation can vary crystallographically between [010] and [011]. In CMSX-2 single crystals oriented as above at 700°C, Antolovich et al. [33] observed different FCGR responses to an air environment (compared to vacuum) between FCP in the [010] versus the [011] crack growth direction; however, they drew no conclusions as to the cause citing scatter in the data. Defresne et al. [34] and Ai et al. [35] both observed significant increase in FCGR in the [010] direction over the [011] direction, at 650°C and 950°C, respectively, in CMSX-2. In work on fatigue in different orientations of SC superalloy, Anton [36] suggested an inverse relationship between FCGR and elastic modulus; the anisotropy of elastic modulus between [011] and [010], with the former being the stiffer direction, could contribute to the faster FCP in [010].

2.3.4.3 Effect of Operating Conditions

It is a common observation that an increase in load ratio, $R$, results in an increased FCGR for a given $\Delta K$, for example in references [37, 38, 39]. It should be observed here that the more pronounced the shift in FCGR with $R$, the more significant the effect of closure. Specifically, an increase in $R$ for a fixed $\Delta K$ increases both $K_{max}$ and $K_{min}$, and if closure effects set some $K_{c}$ from which $\Delta K_{eff}$ is defined, then as $K_{max}$ approaches $K_{c}$ from below, $\Delta K_{eff}$ also increases. Furthermore, if apparent threshold is influenced by closure effects, then as an increase in $R$ increases $\Delta K_{eff}$ as just described, then what appeared to
be $\Delta K_{th}$ at a lower load ratio can begin to display Region II FCP, and the apparent $\Delta K_{th}$ effectively decreases, as has been observed by Doker [40] and Shyam et al. [39].

An increase in operating temperature changes the nature of FCP in superalloys because of the change in dominant damage mechanism as described above. At room temperature, heterogeneous slip leads to FCP along crystallographic planes (of [111]-type) as in Stage I crack growth; in fact, such behavior can be observed in some superalloys up to 760°C at higher cyclic frequencies [41]. Koss and Chan explain this extended crystallographic cracking as a result of a build-up of normal stress on the slip planes due to an inability of coplanar slip to relax the stresses ahead of the crack tip [42]. This fracture mechanism promotes a tortuous Stage I type crack path and effective Mode I shielding. As temperature increases, secondary slip systems are activated and deformation becomes more homogeneous, promoting more typical Mode I loading, Stage II crack propagation. At intermediate temperatures (650°C), Henderson et al. [43] observed that crack propagation was more likely to occur along the $\gamma\cdot\gamma'$ interface (for (001) load oriented specimens) or in the matrix (for (011) and (111) oriented specimens), "meandering" around the particles and slowing crack growth. At 850°C, the crack was able to cut through the precipitates, reduce tortuosity, and grow more rapidly for the (011) and (111) specimens. However, for the (001) specimens, the increase in temperature from 650°C to 850°C actually led to a slight decrease in FCGR at a frequency of 10 Hz for cracking that remained largely within the matrix, rather than the acceleration produced by a change of damage mechanisms at other orientations (there was no $\gamma'$ cutting at either temperature for this orientation). Thus, it should be understood that the effect of temperature on FCP is tied to the active damage mechanism at a given temperature. Furthermore, different thermal effects can produce conflicting results in FCGR, depending on the interaction between temperature, frequency,
environment and alloy. For instance, Doker et al. [40] and Antolovich and Lerch [44] report an increase in FCGR as temperature increases from an intermediate elevated temperature (in this case 700°C) to even higher temperatures (900°C in the case of Doker). However, Lupinc and Onofrio [45], in FCP tests on <001>-loaded CMSX-2 single crystals, found that while FCGR increased very slightly from 650°C to 750°C, further increase to 950°C did increase FCGR at high ΔK but decreased FCGR at low ΔK. Further investigation of FCP at 950°C in vacuum and air showed that the air environment actually decreased FCGR, strongly implicating an oxide-induced retardation, since their tests were performed at R=0.05 and therefore quite susceptible to closure effects at lower ΔK. For the DS GTD-111 under current study, Yoon [46] found a fairly consistent increase in FCGR from 650°C to 760°C and again to 870°C in LT-loaded specimens tested at 0.5 Hz. While it is not clear what the effect of an increase from RT to ET is on FCGR (although Chan [42] suggests no change between RT and 982°C), there has generally been identified some intermediate temperature range — such as 650°C, 700°C, and 760°C in various experiments — corresponding to the range of effective ET γ’ strengthening that is evident in Figure 2.7, above which FCP resistance drops off appreciably in the absence of extrinsic toughening effects.

The frequency of loading and time held at load have ambiguous effects on FCGR, as oxidation and creep can contribute to increased damage at the crack tip, but at the same time, can lead to oxide induced crack closure and crack tip blunting which may counteract the deleterious effects. For example, the work of Lupinc and Onofrio [45] showed that at 950°C in air, a 5 second hold produced an across-the-board retardation of FCGR as compared to testing at 4 Hz, suggesting the hold time enhanced fracture surface oxidation effects; in vacuum, however, the addition of hold time produce little effect at low ΔK but a progressively greater acceleration as ΔK increased, suggesting the creep damage was dependent
not only on temperature but on the strain ranges ahead of the crack tip. For (001) oriented specimens 650°C, Henderson et al. showed little change in FCGR between 1 Hz and 10 Hz, but that the 1 Hz testing showed a slight near-threshold improvement, possibly due to oxide induced closure as above [43]. At 850°C, the slower loading frequency (1 Hz) showed a noticeably decreased FCGR. In the non-(001) orientations, however, the slower frequency produced an increase in FCGR for lower ∆K at 650°C, but an increase in apparent threshold (and thus decreased near-threshold FCGR) at 850°C. In equiaxed 718 superalloy tested in air, Lynch et al. [47] report little effect of frequency or hold time at 500°C, but similar effects as above at 700°C: lower frequencies or hold times increased threshold and lower FCGR at low ∆K, with accelerated FCGR at high ∆K for low frequencies (but no further acceleration with the addition of hold time). The influences of individual environmental or operating variables, it seems, cannot be described alone with clear trends, but must be considered as a whole based on both intrinsic and extrinsic factors.

2.3.5 GTD-111 Studies

The General Electric proprietary alloy GTD-111 was developed in the mid-1970’s to supersede the common blading alloy, IN738 LC, yet as recently as 1998 Daleo et al. [48] commented on the lack of available data on GTD-111 and the situation has not appreciably improved in the intervening few years. An extensive literature survey produced under two dozen original published studies on the alloy, fewer than half of them discussing the directionally solidified version of the alloy developed in the mid-1980’s and none of those considering fatigue crack propagation. Embley and Russel [49] did consider fatigue damage in the realm of TMF testing and, as was mentioned earlier, issued the warning that isothermal LCF testing was insufficient to characterize the behavior of the material under
thermo-mechanical cycling. In experiments on GTD-111 and IN738 LC, they identified the cyclic $\sigma_{\text{max}}$ and $T_{\text{max}}$ as the critical parameters in predicting TMF damage. Viswanathan et al. [4] have included LCF data for equiaxed GTD-111 in their crack initiation life prediction code REMLIFE, and have stated their intention to extend the code to the DS alloy.

Daleo and Wilson [48] have performed extensive work on microstructural evolution in service-exposed GTD-111 in order to understand its degradation in time, but more pressingly to determine whether extant data on the more common René 80 aeroengine alloy would suffice for maintenance purposes. They identified GTD-111 as compositionally very similar to René 80, with alterations to the proportions of C, Ta, and Mo to improve hot corrosion properties. (Gaudenzi et al. [50] only describe the hot corrosion resistance of GTD-111 as being equivalent to IN738.) Daleo and Wilson describe the dissolution of secondary and coarsening of primary $\gamma$' precipitates with service exposure, and point out that such degradation tends to lower creep strength and possibly fatigue resistance. Coarsening of $\gamma$' was marginal at 816°C but more pronounced at 899°C, as was the development of $\gamma$'/M23C6 carbide films along grain boundaries. Sajjadi et al. [51] also studied the composition and microstructure of GTD-111, identifying a key difference between it and IN738 LC as the increase in volume fraction of $\gamma$' from about 45% to over 60%. While the strength of GTD-111 is subsequently higher, its ductility is lower, which is also due in large part to its higher refractory metal content. By 1994, Hale [52] presented work on the development of repair processes for equiaxed GTD-111 blades, once again citing scarcity of available data as a motivating factor. While the condition of the base metal is not to be ignored, it is primarily the sustainability of the protective coating that determines the life of the blades in this extreme environment, and a large part of research related to GTD-111 has been directed thereto. Cheruva et al. [53] present a computer algorithm, COATLIFE, for
predicting the cyclic oxidation life of blade coatings. Metz et al. [54] investigated a number of different coating systems and their oxidation resistance, while Chan et al. [55, 56] have performed extensive work on coating degradation and life.

In 1989, Peterson [57] discussed the development of the directional solidification process for GTD-111, also presenting the results of some mechanical tests and machining trials. Comparing the information in a recent related patent [58] with compositional analysis of equiaxed GTD-111 by Sajjadi et al. [51], the notable differences between the two forms of the alloy include the addition of .04 wt.% Zr, increasing B to 0.12 wt.% from 0.01 wt.%, and the removal of the 0.23 wt.% Fe. Already Swaminathan et al. [59] have published work on the refurbishment of DS GTD-111 first stage blades, as well as equiaxed GTD-111 and IN738 counterparts, showing that significant improvement to microstructure, tensile and creep properties are possible in both forms of GTD-111 but not in IN738. Currently work is underway through a power generation consortium, the Electric Power Research Institute (EPRI), to develop a laser-welding repair procedure for DS GTD-111 blades [60]. Still, research efforts remained focused on the coatings applied to the blades to protect them from the increased firing temperatures; Boestman et al. [61] highlight the fact that the introduction of DS GTD-111 and the advanced GT-29 Plus coating enabled the increase of firing temperatures from 927°C to 965°C compared to IN738 coated with a simple Pt-Al system. Daleo and Boone [6] investigated the failure mechanisms of coating systems used on DS GTD-111, making the significant observation that fatigue cracks in turbine blades often initiate in the coating systems, which are generally more brittle than the base alloy. Daleo et al. [62] and Cheruvu et al. [63] go on to discuss the utility of coating/base alloy interdiffusion as a highly accurate measure of service temperature exposure.

While fatigue data is almost nonexistent, creep properties have been studied in both
forms of the alloy. Schilke et al. [64] have reported that GTD-111 exceeds IN738 by 20°C in creep rupture and has superior LCF resistance. Woodford [65] identified the unique differentiation between creep strengths in different orientations of DS GTD-111: the longitudinal direction, which sees constant centrifugal load in service, performed better at lower stress and higher temperature, whereas the transverse and diagonal orientations were superior at higher stress and lower temperature. He also developed a methodology for accelerated creep testing for quick evaluation of the effects of microstructural evolution in service; he concluded that the creep strength of GTD-111 is not significantly affected by long term exposure to elevated temperature, but that it may become embrittled. Duleo et al. [66] present additional accelerated creep testing methods for equiaxed GTD-111 buckets removed from service.

In conclusion, although the effects of creep and coating degradation cannot be ignored, a significant need exists for quantification of fatigue and crack propagation behavior is the fairly new but increasingly employed DS GTD-111 turbine bucket alloy.

2.3.6 Implications of DS Microstructure

The microstructure of DS alloys such as the GTD-111 under study presents unique characteristics that must be considered in the modeling of FCG. Firstly, the extremely large diameter of the columnar grains (ranging from roughly 3 to 10 mm), which is two to three orders of magnitude larger than most conventional equiaxed alloys, would imply that visually detectable cracks of considerable size may display the anomalous behavior of small cracks. According to Suresh, the transition from the more microstructure-sensitive behavior of small cracks to more microstructure-insensitive long crack behavior takes place when the size of the cyclic plastic zone \( r_c \) becomes comparable to some characteristic
microstructural dimension, typically taken as grain size. Fatigue cracks which might otherwise be considered physically "long" have been observed to display the crystallographic Stage I crack growth mechanism when the crack tip plastic zone was still smaller than the grain diameter [26]. Under loading conditions in which crystallographic slip dominates, a grain-to-grain variation in crack growth resistance due to local slip system orientation could produce long-range differences in observed crack growth. This effect would be particularly significant in thin members such as airfoils, which could consist of only a single grain through the thickness. In addition to generally propagating at rates much higher than predicted by the Paris law for long cracks, short cracks tend to display much more scatter in fatigue crack growth rate data. “Oscillatory” crack growth rate behavior has been commonly observed in short cracks as they interact with local microstructure features; the acceleration and deceleration of crack growth can occur over a length that is several times the spacing of the relevant microstructural barriers [67].

The dendritic microstructure in Figure 2.5 is indicative of inhomogeneous composition and mechanical properties in DS superalloys like GTE-111. It has been shown that the γ’ phase coarsens from the core of the PDA out toward the interdendritic regions [21]. The interdendritic regions have also been shown to contain micropores [68]. During solidification, elements such as Ta, Mo, and W tend to microsegregate; due to the low diffusivities of these elements, this compositional inhomogeneity cannot practically be removed through heat treatment. Hence, deleterious precipitates like σ and laves phases as well as brittle carbides become more common in interdendritic regions. Furthermore, segregation of these alloying elements to the periphery of the dendrites causes a change in the alloy structure parameters, due to differences in lattice parameter of the different solutioned alloying
elements [68]. Since the lattice parameter mismatch is a significant source of the strengthening characteristics of the γ' precipitate, it can be expected that a gradient of structure parameter will give rise to a microstructural strength variation as well. Altogether this represents a periodic microstructure of dendrite cores and interdendritic regions with varying mechanical properties, within a long-range variation in slip systems between very large grains. These features will influence the approach to modeling material variability as it affects life prediction.

2.4 FATIGUE CRACK LIFE PREDICTION

As discussed in Section 2.2, modern damage tolerance approaches to life prediction focus on the stable propagation of subcritical cracks. In this section, some of the basic mechanics and their mathematical representations will be presented as they will be employed later in the modeling effort.

Characterization and prediction of FCP is important in the life prediction of many turbine components for two reasons. First, a significant portion of the total fatigue life of a component may be spent with a subcritical, stably-propagating fatigue crack; while many life prediction schemes consider “failure” to consist of the existence of even a small fatigue crack, there are often cases in which a cracked component can be returned to service without significantly reducing safety. Secondly, the incorporation of the crack propagation life into the overall maintenance and life management protocol enables the use of the often large amount of field-generated data on cracks detected during inspection for use in adaptive life prediction schemes. The monitoring of subcritical cracks in service provides a wealth of information about the conditions being experienced by a fleet of machine; initial
predictions of what should be observed at inspection and maintenance intervals are compared with actual measurements, and the errors and trends are used to correct and refine the original model. Without consideration of this stable, easily measured damage mechanism as an important part of a component’s serviceable life, the “failure event” of the component becomes an obscure transition from un-failed to failed, allowing little or no insight into the underlying process.

Fatigue cracks are initiated after a certain amount of damage accumulation due to HCF or LCF. Stress concentrations produce localized damage under fatigue loading, and after the material’s capacity for reversed plastic deformation has been exhausted, the damage manifests itself as a microscopic fatigue crack. Often the fatigue damage occurs at a surface detail, and the cyclic plastic deformation along slip bands produce intrusions and extrusions which, as they grow larger, result in a stress discontinuity that begins to behave like a crack [13]; this process can also occur in the interior of a component at grain boundaries where slip system misorientations result in local deformation incompatibility and stress concentrations. In the case of HCF, the load cycles are typically being applied at such a high frequency that once a crack initiates, it propagates so rapidly to failure that the overall fatigue life is generally considered to be initiation life, and the proportion of propagation life is negligible. Propagation is, however, typically a significant portion of LCF life, often because the number of cycles required to initiate the crack is so small due to the high stresses and inelastic strains undergone by the material. In many cases, especially where stress concentrations due to design details or machining flaws are concerned, the local damage of LCF is strain controlled, and once a crack initiates, the local deformation is more easily accommodated by the more compliant crack, which then propagates in a fairly stable manner.
2.4.1 Crack Growth Models

Fatigue crack growth in the absence of significant nonlinear deformation (such as large-scale plasticity or time-dependent creep deformation) is most often characterized by a power law relationship based on the stress intensity range, $\Delta K$. The stress intensity factor (SIF), $K$, is calculated for a given cracked component based on its geometry, constraints, loading conditions, and crack size, and enables fracture mechanics data generated in a laboratory specimen to be applied to any component of arbitrary geometry and loading. When plotted against the stress intensity range (which is typically done on log-log axes), FCP behavior tends to fall into three regions along a sigmoidal curve shown in Figure 2.8 [69]. Although the origins, and indeed the physical veracity, of the FCP threshold are a matter of significant debate, for engineering design purposes it is generally accepted that there is a threshold stress intensity range, $\Delta K_{th}$, below which a crack will not propagate. At low stress intensity ranges in Region I, as $\Delta K$ approaches $\Delta K_{th}$, the FCGR decelerates precipitously. At high stress intensity ranges in Region III, as $K_{max}$ approaches the material’s fracture toughness $K_C$, the FCGR increases rapidly, and general design practice dictates that Region III be avoided altogether. In Region II, stable crack growth generally follows a linear relationship on a log-log plot, and it is around this region that many crack growth models are built.

A number of different models are available to characterize crack growth rate, primarily in Region II, and for some models, in two or even all three regions. The original and most typical model, developed by Paris and Erdogan [70] and now known as the Paris law, describes the linear region of the log-log plot of Figure 2.8 as the power law

$$\frac{da}{dN} = C \Delta K^m$$

(2.3)
Figure 2.8: Schematic of a $da/dN - \Delta K$ plot showing three regions: near-threshold (I), linear or Paris (II), and critical (III) [69]

Modifications have been made to this basic equation to account for Regions I and III, with considerable interest in the former as a significant portion of propagation life may be spent as a small crack growing at these very low rates.

Different load ratios have been shown to result in different crack growth rates for a given $\Delta K$, largely due closure effects as first observed by Elber [71]. As a result of the plastic deformation that takes place at the tip of a crack under load (see below), there is often a “plastic wake” of material along the surfaces of the crack that retains some residual deformation. As the crack is unloaded, this material on the crack faces can come into contact before reaching the minimum load of the cycle, thus reducing the effective total load range experienced by the crack tip. Such closure can also be brought about by oxide formation on newly formed fracture surfaces in a deleterious environment, lateral displacement of fracture surface asperities under multiaxial loading, and other mechanisms. Elber proposed that only the portion of the load cycle above a certain stress intensity $K_{op}$ at which point the crack was “open” and the faces no longer in contact, contributed to fatigue damage and crack propagation. Various mechanistic and empirical models have been proposed.
to account for this effect in the crack growth rate equation. One such model, which will be employed later, was developed by Walker [72], and can be represented as

$$\frac{da}{dN} = C \left( \frac{\Delta K}{(1 - R)^{1/2}} \right)^m$$

(2.4)

It should be noted that models such as the Walker equation which account for closure or load ratio are empirically based, and as the operative deformation mechanisms which give rise to crack wake closure can vary from material to material, the parameters for any given model must be determined experimentally for each new material employed. Furthermore, the operating environment, including temperature, frequency, and corrosive elements, can also change the relevant mechanisms and thus require a new determination of the empirical crack growth law parameters for the particular condition under study.

2.4.2 Nonlinear Fracture Mechanics

The stress intensity factor $K$ used in the aforementioned crack growth models is built upon the assumptions of linear elastic fracture mechanics (LEFM), which begin to break down as elevated temperatures promote nonlinear deformation, and even more so as cyclic frequency decreases or time held under load increases. Conditions in turbine hot sections often push the limits of LEFM due to the high operating temperatures and prolonged run times, and the LEFM parameter $K$ may not always be appropriate [73]. While a rigorous examination of nonlinear fracture mechanics is beyond the scope of this text, such risks and limitations should be kept in mind when discussing a LEFM treatment of turbine life predictions; further elaboration of elastic-plastic fracture mechanics (EPFM) and time-dependent crack growth can be found in Saxena [74], with further development of such techniques for elevated temperature crack propagation by Orange [73].

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Some basic concepts in nonlinear fracture mechanics should be introduced for the purpose of later discussion of crack life modeling in the turbine environment. As mentioned above, the time-dependent plastic deformation associated with creep is a prevalent damage mechanism in the turbine hot section. At elevated temperature, the extremely high stresses in the crack tip region can, under a load sustained for even a short duration, result in sufficient localized creep damage to change the stress and strain fields ahead of the crack so as to render LEFM invalid. Creep deformation at the crack tip can actually lead to crack blunting and retard crack the crack growth otherwise expected by LEFM analysis [46]. If the load is held for prolonged periods, severe creep damage at the crack tip can cause creep cavitation and rupture of the creep damage zone, resulting in crack propagation through creep crack growth. Based on early work by Rice and others in EPFM, which developed the strain energy-based parameter called the J-integral for correlating crack growth data under elastic-plastic conditions, the related parameters \( C^* \) and \( C(t) \) have been proposed for correlating creep crack growth data [74]. Time dependent crack growth under creep loading is often estimated with an equation of similar form to the Paris law, e.g.,

\[
\frac{da}{dt} = A (C^*)^n
\]  

(2.5)

Krishna et al. [75] showed that, for their test data on creep crack growth in DS superalloy at 760°C, \( \Delta K \) proved quite satisfactory in correlating creep crack growth rate with a simple power law as above. Work by Orange describes other parameters based on strain energy or strain-modified LEFM parameters for reducing crack growth data under more complicated thermomechanical cycling [73]. Under some circumstances of combined cyclic and sustained/creep loading, it has been shown [73, 3] that a linear superposition of the two crack
growth rates,

\[
\left( \frac{da}{dN} \right)_{\text{tot}} = \left( \frac{da}{dN} \right)_{\text{LEFM}} + \left( \frac{da}{dt} \right)_{\text{creep}} \times \tau_{\text{cy}}
\]  

(2.6)

where \( \tau_{\text{cy}} \) is the length of sustained load during the cycle, can acceptably predict total crack growth rate, although some correction factors are required as the strain range grows large [73].

2.4.3 Equivalent Initial Flaw Size

Fatigue cracking can be broken into two phases of initiation and propagation: prediction of the second phase has been discussed above. Variation in time to fatigue crack initiation can be controlled by a number of different factors, and there are as many ways to model the early stages of fatigue cracking. Indeed, a major aspect of modeling fatigue crack-controlled component life is determining the initial flaw size from which the crack is then propagated in the simulation. If crack initiation is largely a result of the early brittle fracture of hard precipitates or inclusions, as is often the case in aluminum alloys, a true initial crack size can be modeled based on microstructural characterization of the material. Significant work in this area includes that of Laz and Hillberry [76] and DeBartolo and Hillberry [77]. In some cases, cracks initiate from machining marks or casting defects and can similarly be modeled through study of the fabrication processes. Where LCF or thermo-mechanical fatigue (TMF) at a stress concentration is the root cause of cracking, the time to an arbitrarily-sized initial crack may be the random variable that precedes the crack propagation simulation.

However, establishing initial fatigue conditions based on knowledge of the controlling damage mechanisms is not always possible. Where component service experience yields fatigue crack data in the absence of prior predictions or expectations, the observed crack
size distributions and knowledge of the service history of each component can be utilized to regress a population of initial cracks that are assumed to exist at the outset of component service. Such an approach was put forth by Johnson et al. to determine an estimated distribution of times to crack detection as well as a hypothetical population of initial flaws at time zero, called the equivalent initial flaw size (EIFS) distribution [78]. Whether regressed from field inspection data or inferred from measurements of constituent particles, the distribution of EIFS serves as another random input variable to a fracture mechanics based probabilistic fatigue analysis.

2.5 ANALYSIS OF RANDOM FCP

2.5.1 Crack Size/Life Distributions vs. Growth Rate Modeling

Probabilistic crack growth models can be separated into two basic categories. The first category considers the dispersion in crack growth test data in $a - N$ space, without any subsequent data manipulation. Repetitive FCP tests are performed at identical conditions (temperature, load ratio, etc.) in order to generate a distribution of $a - N$ curves. For statistical analysis, all of the crack size measurements are made at the same exact cycle counts, in order to develop distributions of crack size at each given lifetime, or alternatively the number of cycles are recorded for each specimen to reach fixed increments of crack length, in order to develop distributions of lifetimes at each given crack size. The distributions so generated are presented schematically in Figure 2.9. (For both of these approaches, a common initial crack size is required to define "zero cycles" in order to normalize the results.) Knowing the form and relative dispersion of the distribution in, for example, life required to reach a critical crack size, the designer can perform a deterministic crack growth analysis based on the median crack growth properties and then superimpose the appropriate
normalized distribution on the median result [15]. One of the most significant applications of this approach was performed by Virkler [79], a large set of $a - N$ data produced by 68 identical tests of 2024-T3 aluminum alloy. The Virkler data was a watershed in the field of probabilistic crack growth modeling and is still used as a guidepost for many model validation efforts. Virkler took measurements at fixed crack sizes at relatively fine increments and recorded the number of cycles $N$ to reach that crack length; subsequent statistical analysis of the cycles to reach certain crack lengths, $N(a)$, indicated that the 3-parameter lognormal distribution best fit the data. An even larger set of data was generated several years later by Ghonem and Dore [80], running three sets of 60 identical FCP tests, each set at a different $R$-ratio and stress level, in 7075-T6 aluminum; they used the alternate method of measuring crack length at fixed cycle counts, which generated distributions of crack size at a given time.

![Diagram with graphs showing crack size distributions and life distributions with crack size][1]

**Figure 2.9:** Schematic variation of (a) crack size distributions with number of cycles and (b) life distributions with crack size [81]

These approaches, however, are not easily adapted for varying operating conditions other than those at which the tests are performed; in order to account for such effects as load
interaction, representative spectrum loading tests would also have to be performed to generate new distributions. Therefore, models in the second category use reduced test data in $\frac{da}{dN} - \Delta K$ space to “fill in the gaps” of the crack growth curve between the periodic observations described above. Building a probabilistic FCP model typically centers on developing statistical distributions for the coefficients and exponents of traditional deterministic crack growth laws for subsequent numerical simulation. In building such a model from replicate tests, a frequently employed procedure involves fitting the Paris law (Eq. (2.3)) to the $\frac{da}{dN} - \Delta K$ data (or, alternatively, fitting an integrated form of the Eq. (2.3) to the raw $a - N$ data) from each experiment. Repeating this procedure for a large number of experiments produces distributions of the coefficient $C$ and exponent $m$. This approach was applied to the Vichler data by Ostergaard [82], who found a pronounced negative correlation between $m$ and $\log C$ which was subsequently incorporated into his simulation. Such a relationship between the Paris law parameters has been observed elsewhere [83], and today $m$ and $\log C$ are generally modeled as a joint bivariate distribution.

According to Elling [84], another form of models in this category which allow for more complex representation of the data often take the basic form of

$$\frac{da}{dN} = f(\Delta K)\xi(x)$$

(2.7)

in which $f()$ represents an arbitrary crack growth law (such as the Paris or Walker forms), and $\xi$ is a stochastic process superimposed upon the deterministic law. This stochastic process is determined through special statistical analysis of the data and may be a function of position or crack length, time, or a combination of space and time. In some cases — for example, Ishikawa et al. [85] — the model is ultimately presented in a form that appears to fall under the first $a - N$ space category by formulating a probabilistic FCPG law, normalizing the space and stress variables, and integrating it to yield a solution for crack size
as a function of time. Distributions of the crack population are formed, ultimately, through the transformation by this integrated crack growth law of the input distributions of crack growth resistance and applied load.

In describing the previous, Paris law-based approach, Ellyin places in this category the particular case in which the exponent \( m \) is held constant and only a distribution of \( C \) is determined \(^{[84]}\); however, looking closely, one can see this actually would fall under this stochastic process approach. Holding \( m \) fixed and making \( C \) random is identical to Eq. (2.7) in which the \( f'(\Delta K) \) is the Paris law with a median value of \( C \) multiplied by the random function \( \xi \) which has the median 1.0 and a distribution of the form determined by analysis of \( C \). Examples of this stochastic crack growth rate approach can be found in recent experiments on Ti-6-4 \(^{[86, 87]}\) and lamellar gamma titanium aluminide \(^{[88]}\) by Soboyejo et al.. Constant \( \Delta K \) experiments have been performed in order to characterize the variability in crack growth rate, which subsequently yielded a distribution of a normalized crack growth coefficient \( C \) for use in simulation.

2.5.2 Inter-specimen and Intra-specimen Scatter

When building a probabilistic crack growth model based on the crack growth rate law or crack propagation resistance as described above, one must make a fundamental decision as to the nature or origin of the scatter in crack propagation. As described by Ichikawa \(^{[83]}\) and others, analysis of random crack growth data and subsequent modeling thereof can be broken down into two approaches: the inter-specimen variability approach and the intra-specimen variability approach. This binary consideration of variability in crack growth does not parallel the two categories above of \( a - N \) space versus \( \Delta a - \Delta K \) space approaches, but there is a notable (though inexact) relationship with the two subcategories of the crack
growth rate approaches. When distributions of crack growth law parameters are generated by curve-fitting separate crack growth laws to each set of data from multiple specimens as described above (e.g., Ostergaard’s approach [82]), the implicit assumption is that scatter in FCGR is a specimen-to-specimen phenomenon. The fitted FCP law represents the average behavior for the entire specimen, and any sampling of the parameters represents inter-specimen variation. (As an exception to the rule, Shen et al. [86] generated a multiple sets of Paris law parameters from a single specimen by curve-fitting small subsets of the data over narrow bands of \( \Delta K \).) Inter-specimen variability is typically the result of batch-to-batch variations in material processing, non-uniformities in thermal profile during heat treatment, and other such factors.

Intra-specimen variability approaches, on the other hand, consider scatter in crack growth data to arise from local variations in space of the material’s resistance to crack growth [83]. Therefore, models of the form of Eq. (2.7) in which the stochastic function \( \xi \) is taken to be a function of position or crack length (and even, one can argue, time) within a given component or specimen, represent the intra-specimen variability approach. The works of Shen [86] and Soboyejo [87], which study variability in a microscopic scale, fall under this approach. A number of cumulative damage based models (for example, Ellyin [81]), which simulate FCP as the sequential LCF failures of discrete ligaments ahead of the crack tip (typically the size of the fatigue damage process zone) with varying fatigue strengths, also represent intra-specimen variability. Intra-specimen variability results from material inhomogeneity and the distribution of physical features which promote or hinder crack propagation, such as grain boundaries, clusters of precipitates, microsegregation of constituents, etc.
Choosing either of these approaches in a testing simulation program is as much an empirical decision as a philosophical one. Assuming that for large engineering structures, microstructural variations will “come out in the wash” (i.e., that over large distances, the expectation value of the stochastic process $\xi$ is unity) can lead to the decision that inter-specimen variability is the only important factor. Yoon and Yang [89] have actually shown in simulations that the effect of a microscopic stochastic process diminished at long fatigue lives, though it increased variance in crack population at shorter lives. Some have argued, on the other hand, that inter-specimen variability may be modeled as a result of intra-specimen variability [83], and one may be fundamentally inclined to believe that representation of the microstructural damage process is important. In the end, the decision should be guided by the data at hand: Dolinski [90] points out that the noticeable crossing and intermingling of crack growth curves in the Vinkler as well as the Ghonem and Dore data sets is indicative of a “stochastic inhomogeneity” in the material broached by the crack front, producing intra-specimen scatter. But at the same time, consideration of the specimen-to-specimen trends, as represented by the average behavior for each individual specimen or component, is also essential to accurate simulation. Vinkler showed [79] that sampling random crack growth rates from the entire set of data without consideration of within-specimen trends led to a marked underestimation of variability in crack growth life as well. He pointed out that some specimens displayed on the whole slower or faster than average crack growth for most of the crack growth test, and that ignoring such a trend resulted in the simulation of a “meandering” crack growth rate which kept each simulated crack growth curve unnaturally close to the mean behavior.
CHAPTER 3

MATERIAL

As described in Chapter 1, directional solidification of turbine hot-section superalloy components provides improved creep strength more easily and economically than single-crystal casting methods. This is especially true when large casting sizes, such as are found in IGTs, lead to difficulties in obtaining quality single crystal castings. Therefore, to study the particular IGT applications under consideration, all research was performed on DS GTD-111, a General Electric proprietary alloy. The composition of DS GTD-111 was given in Table 2.1. In order to produce specimens of similar microstructure to the turbine bucket application, slabs of GTD-111 were cast by General Electric’s normal supplier, PCC Airfoils, using the directional solidification method in a rectangular mold approximately 32×197 mm in cross-section and 254 mm in the direction of solidification. Following casting, the slabs were heat-treated by the casting house according to the same General Electric specifications for the turbine buckets. While the specific steps are proprietary, it consists of a high temperature solution anneal in vacuum, fan argon cooling to room temperature, and then an intermediate temperature aging treatment followed by fan air cooling. Heat treatment results were verified against the material specification by testing to a hardness level of 42 HRc [21].

Several slabs of GTD-111 were directionally cast to produce specimens for a number of different researchers at Georgia Tech. In all four slabs used to produce the specimens for this research, the FCG panels were drawn from one side of the casting, in order to accommodate a number of other different specimens and specimen types for other researchers. A
typical casting cut-up plan is shown in Figure 3.1. All of the FCG specimens were oriented such that the loading axis was parallel to the solidification direction, and the fatigue crack would grow transverse to the columnar grain structure. Small samples were drawn from a number of the castings and examined by another researcher, both metallographically and under a scanning electron microscope, to compare microstructural features of the castings with those of a blade drawn from service, including secondary dendrite arm spacing and \( \gamma' \) size and distribution. \[91\] The slab castings were found to appear nominally identical in microstructure to the fielded blade.

### 3.1 SPECIMEN DESIGN

Specimen design for this project was driven by the early stage requirement that fatigue crack growth tests be performed at negative R-ratios (tension-compression load cycles). Many of the ASTM standard specimen designs are pin- or roller-loaded and therefore not suitable for compressive loading. With this in mind, a modified niddle-crack tension [M(T)] specimen design was originally selected for testing. The specimens were to be flat panels loaded by a rigid hydraulic wedge grip, in order to enable compressive loading.
Two specimen profile designs and two specimen thicknesses were selected to be machined from one slab for the first batch of eight specimens, in order to test each configuration twice for a specimen configuration down-select; the profiles were a straight-sided and a dogbone configuration. Both specimens were 177.8 mm long and had a maximum width of 50.8 mm, with the dogbone specimen tapering down to a 38.1 mm wide gage section. A 3.81 mm wide central notch with a root radius of approximately 0.125 mm was produced in all specimens by electro-discharge machining (EDM) for fatigue crack initiation.

The dogbone configuration was a candidate specimen due to early concerns about the specimen failing by an unexpected fatigue crack initiation at the edge of the wedge grip-specimen interface in the straight-sided specimen, as had happened in previous experience with intermetallics. (Elevated temperature testing with water-cooled grips can exacerbate this problem: since many alloys possess higher fracture toughness and crack propagation resistance at elevated temperatures as compared to room temperature, the applied load required for slow, stable crack growth in the hot gage section may be more than sufficient to initiate and rapidly propagate a crack at a surface damage location caused by the teeth on the wedge grips in the cold section of the specimen.) However, a wider gage section was preferred from the standpoint of providing as much material as possible per specimen for fatigue crack propagation and therefore generating more data per test.

The two thicknesses selected were 2.54 mm (0.10 in) and 4.76 mm (0.188 in). As the primary focus for this research was in-service cracking of airfoil sections, an important specimen criterion was that the thickness be kept to a minimum in order to match the stress state in the airfoil application. The airfoil wall thickness in the Frame 7 first stage blades varies from approximately 1.5 mm to 2.5 mm over the region in which cracking is being investigated, as seen in Figure 3.2; therefore, in order to best represent the early
stage of crack growth, the thinner specimen configuration was desired. However, since the

![Diagram of cooling air passage](image)

**Figure 3.2:** Cutout section of blade trailing edge, showing wall thickness around cooling passage

test matrix was to include compressive loading down to $R = -1$, buckling of such a thin
specimen was a real concern, hence the thicker candidate specimen. (Note that while the
Euler buckling coefficient, which depends on cross-sectional modulus, varies with the cube
of specimen thickness, it only varies linearly with specimen width. Since the load required
to reach a given stress — and stress intensity factor — also varies only linearly with width,
the specimen width does not influence the susceptibility to buckling.)

Upon validation of the wider (straight-sided), thinner specimen as a successful design,
additional specimens were machined from three new slabs, producing 27 straight-sided
specimens of 50.8 mm width and 2.54 mm thickness, shown in Figure 3.3. Of these, 21
were tested with mixed success prior to a specimen redesign due to testing difficulties with
the $M(T)$ specimen design. In order for FCGR data to be considered valid, ASTM Stan-
dard B647 [92] on FCG testing sets limits on the eccentricity of the overall crack with
respect to the specimen centerline. Due to the fairly inhomogeneous nature of the material,
the cycles to crack initiation and rate of propagation at either side of the central starter
notch were never quite the same. Since the average grain diameter in the DS material was:
very close to the width of the notch, it was very likely in each specimen that either crack (left or right) would initiate in a different grain, with a different local slip system orientation. Additionally, the distance from either notch root to the nearest grain boundary could be different from side to side, further influencing the local damage mechanics. Finally, any initial asymmetry or local microstructural differences at the two crack tips could result in an unequal response to load shedding at the beginning of the test, specifically through relatively different amounts of crack growth retardation, which often exacerbated the imbalanced condition. This resulted in a number of specimens growing highly asymmetric cracks by the time crack initiation and load-shedding was complete, in some cases with one crack front completely arrested. Therefore, the requirement of negative R-ratio testing was set aside, and a single edge-crack configuration was considered. Also, in order to reuse the remaining M(T) specimens as completely as possible, modified extended compact
tension [EC(T)] specimens were produced by essentially slicing the M(T) specimens in half by wire-EDM and extending the starter notch, as shown in Figure 3.4. EC(T) specimens had been used previously in this lab with good success [93], and a published stress intensity solution was available. Fourteen such pin-loaded specimens were produced from 7 M(T) specimens, almost equally distributed between the three slab castings.

The modified EC(T) specimen design used is somewhat non-standard as compared to the ASTM standard eccentrically-loaded single edge tension, or ESE(T), specimen described in E647. The EC(T) specimen itself is the predecessor to the ESE(T) configuration, and a detailed stress analysis was available from Piascik and Newman [94]. While the EC(T) configuration allows for a variable loading pin location relative to the cracked edge, the ESE(T) standard design in E647 specifies the loading pin distance from the edge and

Figure 3.4: Modified EC(T) design configuration (dimensions in inches)
provides a single applicable stress intensity factor solution. Thus, the EC(T) design was chosen over the ASTM specimen in order to allow for a flexible pin placement, so that an optimum range of $\Delta K$ values could be achieved during the FCG test. (With the ESE(T) configuration, the applied $\Delta K$ during a constant-load test would have increased from near-threshold to near-critical over a shorter distance than was possible with the EC(T) specimen, using a less eccentric loading pin location.)

Deviation from the even the standard EC(T) configuration was necessary in order to accommodate the induction coils already being used to heat the gage section. While the EC(T) stress intensity solution allows for a variable load-line location, it still specifies a particular specimen height (or more accurately, a particular longitudinal distance from crack plane to loading pin) as a proportion of the specimen width, as shown in Figure 3.5. Since the M(T) specimens already had central notches EDM’d in them, the most logical path to producing edge-notched specimens was to bisect them longitudinally; however, given the resultant 25 mm width of the new specimen, the specimen length prescribed by the NASA report would have been prohibitively short, possibly causing the grips to interfere with the induction coils, or at the very least suffering peripheral heating which could cause fatigue failure of the clevis. Therefore the modified EC(T) specimens were left at the same overall length as the original M(T) specimens, and the SIF solution was verified with an in-house finite element model, as will be discussed in Chapter 5. The loading pin and hole diameters were also increased in order to reduce the stresses in the pins and minimize the possibility of fatigue failure at the grip. Finally, the initial notch in the specimen was increased over the NASA specification in order optimize the amount of crack extension possible during the test. A fatigue crack initiated from a shorter notch as recommended would require a higher load to ensure propagation; however that load
Figure 3.5: Standard NASA EC(T) specimen configuration [94]

would then cause the applied $K$ to approach its critical value over a much shorter distance, due to the nature of the SIF solution geometry factor. As mentioned before, pin location and initial notch size were varied in order to provide the maximum number of crack size measurements possible for a given test. Therefore, the new notches machined by wire-EDM were nominally 7.62 mm deep, with two exceptions. The M(T) specimen identified as 9-9 had already been pre-cracked to roughly 7mm from the centerline prior to EDM bisection; therefore, no new EDM notch was machined in the resultant EC(T) specimens. Thus the EC(T) specimens 9-9a and 9-9b were fabricated with a nominal 1.8 mm notch.
CHAPTER 4

EXPERIMENTAL METHODS

4.1 TEST MATRIX

In order to produce a probabilistic fatigue crack life prediction model that can account for not only material variability but also different operating conditions (such as temperature, load ratio, and hold time), a test matrix is required that is not only broad in its experimental variables, but deep as well to satisfy statistical requirements. The initial test matrix proposed included four load ratios and four temperatures, as shown in Table 4.1. Ideally, a sufficient number of specimens for thorough statistical analysis would be tested at each condition; the required number of tests is determined largely by the desired confidence level, but also by the dispersion in the data actually measured. However, a tiered test matrix was conceived that would balance statistical analysis with trend analysis based on a more moderate number of specimens, due to financial constraints. The aim of the tiered

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>Load Ratio (R)</th>
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<tbody>
<tr>
<td></td>
<td>-1  0.05  0.5  0.75</td>
</tr>
<tr>
<td>25</td>
<td>5   5   10  5</td>
</tr>
<tr>
<td>650</td>
<td>5   5   5   5</td>
</tr>
<tr>
<td>760</td>
<td>10  5   20  5</td>
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<tr>
<td>815</td>
<td>5   5   5   5</td>
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Table 4.1: Original proposed test matrix

matrix was to establish a primary test condition — in this case, \( R = 0.5 \) and \( T = 760°C \) — at which the form and parameters of the material property distributions (i.e., fatigue crack
growth law coefficient and exponent) would be characterized with reasonable confidence. Next, two secondary test conditions — $R = 0.5$, $T = 25^\circ C$ and $R = -1$, $T = 760^\circ C$ — would be studied with a moderate amount of specimens to establish with some certitude whether the form of the fatigue crack growth rate distribution had fundamentally changed, or merely translated and increased or decreased in variance. The number of repetitive tests at those conditions would also help accurately quantify the effects of temperature and load ratio on the deterministic crack growth rate equation at the heart of the model. (Moving from the primary test condition to the two secondary test conditions in the test matrix essentially represents determining the partial derivative of the fatigue crack growth rate with respect to the two “orthogonal” axes, $\frac{\partial}{\partial R}$ and $\frac{\partial}{\partial T}$.) Finally, the tertiary test matrix conditions would be studied with a small number of specimens simply to ensure any nonlinearity in the effects of $R$ and $T$ on the fatigue crack growth rate were captured in the model.

Due to time and equipment constraints, it was agreed that, with the exception of a few initial specimens being tested at room temperature, only the $T=760^\circ C$ testing would be carried out at Georgia Tech, with the remainder of the matrix being conducted by a subcontractor at GE’s discretion. Furthermore, as a limited amount of GTD-111 material was cast for a number of different research projects, in the end a total of only 35 specimens were fabricated for this project, including the initial eight specimens used for configuration down-select, as described in Chapter 3. It should be recalled that four of those specimens were of a thicker gage section than the other 31, and a few of those tests were conducted at room temperature.

Also as mentioned in Chapter 3, the specimen configuration was changed in the later stages of the project due to concerns about crack growth asymmetry and data validity,
which will be explained further in Chapter 6. While the experimental conditions, measurement techniques, and data analysis methods were essentially the same between the original and modified specimen configurations and theoretically should not adversely influence the statistical modeling, it is nonetheless an additional, unplanned variable. However, as the project neared completion and it became apparent that no further specimens were going to be produced, the decision to convert the remaining specimens seemed justified in light of concerns that as much of the remaining data generation be valid as possible. Therefore, the final test matrix as performed is as shown in Table 4.2. In this matrix, “Thk” refers to

<table>
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<th>Temp. (°C)</th>
<th>Load Ratio (R)</th>
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<tbody>
<tr>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>25°C</td>
<td>1M</td>
</tr>
<tr>
<td>760°C</td>
<td>5M</td>
</tr>
</tbody>
</table>

the (non-selected) 4.76 mm thick specimens, “M” refers to 2.54 mm thick M(T) specimens, and “E” refers to 2.54 mm thick EC(T) specimens.

4.2 EQUIPMENT

4.2.1 Fatigue Loading

Testing was performed, except where noted, according to ASTM E647. Fatigue loads were applied to the specimens with a closed-loop servo-hydraulic 20 kip-capacity load frame built by SATEC and controlled by an MTS TestStar II system. For the original M(T) configuration, specimens were held by water-cooled hydraulic wedge grips; the wedge inserts were 38.1 mm wide with diamond-point teeth and gripped approximately 50 mm (longitudinally) of each end of the specimens with a nominal pressure of 10 MPa. In order
to avoid any lateral instability or buckling, the load frame cross-head was kept as low as possible to keep the load train accordingly short. A picture of a specimen being heated by the induction coils can be seen in Figure 4.1, with the Questar microscopes seen in the foreground.

![Figure 4.1: Hydraulic grips, induction heater, and Questar microscopes used in FCG testing](image)

When the decision was made to change to the EC(T) configuration, a programmatic goal became to avoid a full re-fixturing of the load frame grip system to a dedicated clevis system, since it was uncertain whether other test programs would need testing time on the hydraulic grips, as had happened in the past. Therefore, simple clevis adapters were designed and fabricated for temporary conversion of the hydraulic wedge grips to accommodate the pin-loaded specimens. The design drawing for the clevis adapters is shown in Figure 4.2. As discussed in Chapter 3, the EC(T) specimen configuration used in this
Figure 4.2: Clevis adapters used with hydraulic wedge grips for modified EC(T) specimen testing (dimensions in inches)

program was modified from the ASTM standard, and the loading pins used were 9.4mm (0.375 inches) in diameter. Both the clevis adapters and the pins were machined from 316 stainless steel.

4.2.2 Thermal Environment

In order to simulate the high temperature conditions experienced by the first stage turbine blades, most of the FCG testing was performed at 760°C (with the exception of a few initial room temperature tests). The use of hydraulically clamped specimens for the purposes of compressive load ratios restricted the selection of heating methods that could be used. The hydraulic wedge grips are not designed to operate in such elevated temperature environments (and in any event are too large to fit inside most furnaces used for such tests). However, the limited material available and subsequent small size of the specimens also did not allow for the placement of any available furnace between the hydraulic grips. After a few initial mini-furnace designs and susceptor heaters were considered, the
decision was made to heat the specimen directly with an induction coil. While a number of heat-treatment companies suggest guidelines for induction coil design, tailoring induction heating to a new component is still a trial-and-error "art" more than a rigorous "science." Several induction coils were turned and tested before a successful design was found. The configuration used, as shown in Figure 4.1, was fabricated from $\frac{1}{2}$-inch copper tubing turned in a rounded rectangle approximately 64 mm $\times$ 10 mm, with two turns each above and below the crack plane. Spacing between the two pairs of turns was approximately 15 mm vertically to allow for oblique crack lighting, while spacing within the pairs was approximately 2-3 mm between turns. This induction coil was used for both M(T) and E(T) specimen configurations.

Induction heat has been used successfully many times in smooth-bar LCF and tension testing, but there was some concern about using induction heat on a cracked specimen, due to the possibility of creating a locally super-heated zone at the crack tip. Once a successful induction coil had been fabricated, a specimen that had undergone extensive fatigue crack growth propagation at room temperature was instrumented with a number of thermocouples, including one welded directly over the crack tip on the back face, to study the thermal profile in the gage section. In addition, a thermal imaging camera was used to film and photograph the cracked specimen at elevated temperature to look for any anomalies in the temperature distribution; an image of the thermal profile is shown in Figure 4.3. In the thermal image, each color tone represents a 20°C temperature band, so the single-color region shown in the specimen gage section nominally represents a 750-770°C temperature distribution, which is within acceptable limits. Neither the thermocouples nor the thermal imaging method showed any noteworthy problems with the thermal profile.
Prior to the commencement of this research program, two Questar long-distance microscopes on precision screw-driven 3-axis stages had been procured for optical measurement of surface crack length. The Questar microscopes were controlled by a single desktop computer using software that indicated the position of the microscope stage to a precision of 3 μm (or 0.0001 in). Each microscope’s field of view was captured by a grayscale CCD video camera, which in turn fed the digital image to a video capture card in the computer and/or to a separate black and white CRT monitor. A cross-hair superimposed on the screen was used to align the microscope with the crack tip.

Since the M(T) specimen configuration being used had two independently-growing crack fronts, both Questar microscopes were used to follow crack propagation, each microscope individually following the left or right crack tip propagation. Additionally, as
each crack tip grew at a similar but still independent rate, there was some time delay between measurements of either crack tip, due to the time needed to acquire in the cross-hairs the crack tip location. It was originally hoped that the use of both microscopes would allow more controlled monitoring of the two crack fronts, without the need to stop the test to traverse a single microscope from one end of the crack to the other. However, the erratic behavior of the two crack fronts made synchronized measurements essentially impossible. Therefore, the distance to either crack tip was measured in an alternating sequence throughout the test for the majority of the specimens, as will be explained in Chapter 5.

Furthermore, the nature of the crack growth behavior prevented the measurement of data in the "optimum" form for statistical analysis purposes. As discussed in Chapter 2, it has been the goal of previous studies in probabilistic fatigue crack propagation to make all crack size measurements at constant cycle counts or constant size values from one test to the next. By acquiring data in this manner, distributions of the crack size at a given lifetime or of the lifetime to a given crack size could be determined from the $a-N$ data independent of any PCG rate calculations. Unfortunately, as mentioned, the difficulty in measurement of both crack fronts at a given cycle count made the first option impossible. Additionally, the second type of analysis is premised upon an identical evolution of stress intensity factor over the crack size increments; however, even if one crack front could be measured precisely at fixed lengths, the tendency of the other crack front to behave quite differently between specimens (thus altering the local stress intensity conditions) would render such analysis invalid.
CHAPTER 5

ANALYTICAL METHODS

In addition to the experimental methods already described and the guidelines provided by ASTM E647, additional analytical techniques were employed in order to process the raw crack size measurement data. As far as these procedures are non-standard, they are described in detail here.

5.1 ASYMMETRIC CRACK MEASUREMENTS

Initiation and propagation of a fatigue crack in any material is a random process, and the random effects can appear even greater in a highly heterogeneous material, inasmuch as the apparent dispersion from test to test (and notch to notch) can be greater due to local microstructural differences. The use of the M(T) specimen configuration requires that cracks initiate at approximately the same time at either end of the central starter notch in order to adhere to the crack symmetry requirements laid out in ASTM E647. However, since crack propagation is an accelerating process — with the stress intensity factor, and therefore the FCGR, increasing with crack size — any significant delay between the initiation times of the two cracks can produce an irrecoverable asymmetry. It is a common example problem in introductory physics courses to describe the relative positions of two accelerating objects in a “race.” In any case where two items of interest are accelerating (in this case, crack tips), the item in the lead not only has the advantage of position, but also the advantage of speed; having started first, the lead item has always been accelerating longer, and therefore moving faster. While scatter in the observed FCGR did contribute to
marginal increases and decreases in the growth at either crack tip, it was most often not
eight to reverse or even come close to eliminating any early asymmetry in cases where
there was significant lag between crack initiation at either notch.

In order to describe the material crack growth characteristics as accurately as possible
for specimens that, for the most part, violated the ASTM symmetry requirements, a
non-standard stress intensity factor solution was required. The solution for an asymmetric
central crack in a plate of finite width subject to uniform tension was digitized from The
Stress Analysis of Cracks Handbook, and is shown in Figure 5.1 [95]. Points were selected
from all of the graphs in the figure using cross-hairs and coded into a VisualBasic for
Applications (VBA) macro in Microsoft (MS) Excel for ease of use in data reduction. For
an asymmetric crack with a “major” and “minor” crack half, two different sets of curves
exist for determination of the geometry factor $F(g)$ at either crack tip in the stress intensity
equation

$$K_I = \sigma \sqrt{\pi a F(g)}$$

(5.1)

so that the SIF at the major crack tip, $K_{I, MA}$ is higher than the SIF at the minor crack tip,
$K_{I, MB}$, according to the geometry of the cracked specimen. In this case, $F(g)$ is a function of
the overall half-crack length, the specimen width, and the eccentricity of the center of the
crack relative to the specimen centerline.

The raw crack size measurement data that was fed into the asymmetric crack data re-
duction consisted of the elevated temperature optical measurements of the two crack tips
using the Questar microscopes in an alternating series. That is, measurements of crack tip
position were made for one crack tip at a time while the test ran continuously (to avoid
creep damage effects), so that while one crack tip was measured, the precise location of
the opposite crack tip was unknown. This is illustrated through an example of typical test
Figure 5.1: Stress intensity factor solution used for M(T) specimens [95]
data shown in Table 5.1. An estimate of the opposite tip’s position was calculated through

<table>
<thead>
<tr>
<th>N (cyc)</th>
<th>a_left (mm)</th>
<th>a_right (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21,000</td>
<td>2.602</td>
<td>3.074</td>
</tr>
<tr>
<td>22,850</td>
<td>2.820</td>
<td>3.332</td>
</tr>
<tr>
<td>24,900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26,650</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Example of typical alternating data acquisition.

simple linear interpolation (against cycle count) of its prior and posterior measurements. For example, in Table 5.1, for the measurement at 24,900 cycles of the left crack at 2.820 mm, the corresponding value for the right crack tip for use in SIF calculation is given by

$$\bar{a}_{right}^{i+1} = a_{right}^{i+1} + \left( \frac{N_{i+1} - N_{i+1}}{N_{i+1} - N_{i}} \right) (a_{right}^{i+1} - a_{right}^{i+1})$$

$$= 3.074 + \frac{24,900 - 22,850}{26,650 - 22,850} (3.332 - 3.074)$$

$$= 3.213$$

Here, the hat over \(\bar{a}\), indicates that it is an interpolated value, not a measured value. Since crack growth rate was to be calculated and reported individually for each of the two crack fronts, the lack of simultaneous measurements was not a significant shortfall, because for a given geometry factor \(F(g)\), the precise location of the opposite crack tip had a second order effect due to the fine spacing of measurements. Even an error as large as half the entire crack growth increment for that measurement produces a negligible effect on the opposite side.

Prior to determination of the stress intensity factor, the optical crack sizes were corrected for two factors: thermal expansion and parallax error. Each specimen was measured

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with a dial caliper for thickness and overall width. Then, at elevated temperature, a full traverse of the specimen width was performed with each microscope and an optical reading of gage section width was recorded. The difference between the two width measurements included the effects of both thermal expansion between room temperature and 760°C and an apparent reduction in optical width due to the angle between the specimen face normal and the focal axis of the microscope. (Due to space constraints and the size of the microscope tripods and stages, it was not possible to position them directly in front of the specimen.) A scale factor for each microscope was determined as

$$f_{optic} = \frac{2W_{caliper}}{2W_{optical}} \quad (5.3)$$

The reduced data, then, is based on the corrected crack size

$$a_{core} = a_{optical} \cdot f_{optic} \quad (5.4)$$

which translates ET off-axis optical measurements into RT mechanical measurements. This was performed for both microscopes (and thus crack tips) and for every test. It should be noted that ASTM E647 contains no specific instructions as to how to reduce elevated temperature fatigue crack growth data in order to account for thermal expansion of a metallic specimen. Physical measurements of specimen dimensions are invariably made at room temperature, even though optical measurement is based on the ET crack size. While this inconsistency is minor, as specimen expansion at 760°C is less than 1%, it is a simple numerical step to eliminate the inconsistency, and rather than scaling up the room temperature specimen dimensions to account for expansion, the optical crack size measurements were scaled down to room temperature, very slightly decreasing the apparent crack growth rate.
5.2 FINE ELEMENT MODELING OF EC(T) SPECIMEN

As mentioned in Chapter 3, problems encountered with the M(T) specimen configuration (arrest and asymmetry) prompted conversion of the remaining specimens to the single edge crack EC(T) configuration. Although the actual specimens used were longer (as a proportion of the transverse dimension) than the prescribed EC(T) configuration, it was initially assumed that, due to small deflections there would be minimal impact of the increased length on the SIF. During the course of testing, however, it became apparent that the new EC(T) data was not collapsing onto the old M(T) data, using the published EC(T) SIF solution. Therefore, a finite element model (FEM) of the actual specimen design was constructed in order to correct the SIF.

Using the freely distributed CASCA and FRANC2D software packages from Cornell University, a two-dimensional FEM of the modified EC(T) specimen was created; the model accounted for the increased length, the extended starter notch, and the pin-loading conditions. The specimen was broken down into several sub-regions with user-specified boundary nodes until the Automesh function in CASCA produced satisfactory results; a screen image of the FEM mesh is shown in Figure 5.2. Although the DS superalloy is orthotropic rather than isotropic, the FEM material properties were specified in FRANC2D to be isotropic, in order to have the same basis as the isotropic, LEFM-assumptions inherent in the other SIF solution used for the M(T) specimen. The elastic modulus and Poisson’s ratio were input as the nominal, longitudinally-loaded values for DS GTD-111, 103 GPa and 0.394 respectively. Magnified views of the attachment and notch/gage regions are shown in Figure 5.3. The pin-loading condition was approximated by applying a parabolic pressure distribution along the outer (farthest from the crack plane) 180° of the hole. Since crack was to be modeled as a planar crack normal to the applied load (and
Figure 5.2: Undeformed finite element mesh of EC(T) specimen in FRANC2D

Figure 5.3: Close up views of finite element mesh in (a) attachment region and (b) notched gage section
therefore bisecting the specimen symmetrically), the remaining boundary conditions were
specified as follows: fixed $X$ and $Y$ position of the corner node at the intersection of the
crack plane (symmetry plane) and the specimen edge opposite the notch, to preclude arbi-
trary specimen displacement; and fixed $Y$ position of the adjacent corner node, on the
interior crack plane, in order to preclude arbitrary rotation (as the deformation should be
symmetric). The nominal experimental load of 1780 $N$ was applied to the uncracked spec-
imen to verify the adequacy of the boundary conditions and the relatively symmetry and
uniformity of stress distribution prior to proceeding.

A crack was defined along the element edges on the specimen's plane of symmetry
using FRANC2D, starting with a small (0.3 mm) crack near the notch root as shown in
Figure 5.4. The SIF was calculated in FRANC2D using the J-Integral method, and this
procedure was repeated for several crack growth increments across the useable ligament of
the specimen. Although the crack growth increment for each step was manually determined
by selecting a target node, the anticipated crack growth direction indicated by the maximum
stress plane was consistently normal to the loading axis.

The FRANC2D calculations for the SIF are plotted as a function of $a/c$ and compared to
the published EC(T) solution in Figure 5.6. It can be seen that deviation from the published
solution is at a maximum near the extended starter notch and decreases monotonically to
near zero. Therefore, the correction to the SIF for data analysis purposes was applied as
an effective stress concentration superimposed on the published SIF solution as a function
of distance from the notch. This was effected by fitting a fourth-order equation to the set
of FRANC2D-calculated SIF values divided by the corresponding values obtained with the
standard solution, as shown in Figure 5.7. It should be clear from the figures that the
change in SIF between the published solution and the tailored FEM simulations is only

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Figure 5.4: Finite element mesh of initial crack from notch; crack plane and singularity elements have been emphasized

Figure 5.5: Stress contours ($S_{yy}$ in units of MPa) for initial 0.5mm crack from notch
Figure 5.6: Comparison of FRANC2D and standard EC(T) SIF solutions

Figure 5.7: Stress concentration factor versus distance from notch as determined by FRANC2D over the standard SIF solution
marginal, and in the end it did not resolve the discrepancy between the EC(T) and M(T) specimen results. This issue will be explored further in Chapter 6.
CHAPTER 6

EXPERIMENTAL RESULTS

As described in Chapter 4, fatigue crack growth rate tests were performed on two specimen configurations from three slabs. The two specimen configurations, M(T) and EC(T), produced unexpectedly different FCGR results, so they are presented separately, and possible sources of the difference will be explored later. Furthermore, some early observations from the early testing of prototype specimens will be presented first even though those tests are not included in the ultimate statistical analysis and modeling. Some statistical data characterizing the FCG responses will be presented for the two specimen types in anticipation of the modeling efforts described in Chapter 7. Finally, trends in the data and qualitative observations of crack growth behavior during testing will be discussed herein.

6.1 PROTOTYPE SPECIMENS

As described in Chapter 3, prior to selection of the 2.5 mm thick, straight-sided specimens, a set of prototype specimens with two thicknesses and two gage widths (one dogbone configuration) were fabricated and tested. These tests were run primarily as feasibility studies, and also made use of only one Questar microscope so that the microscope had to traverse back and forth across the specimen, resulting in coarser measurements. Therefore, these test results do not stand up to the rigor of the final analysis and are not included in the statistical characterization based on the final three castings – a situation further reinforced by the fact that much of the prototype specimen data comes from a different specimen thickness, and the majority of it from a different configuration. However, there are still
some qualitative observations to be made, even though the sample size of this data set is quite small.

6.1.1 Room Temperature FCP

Although the focus of this program was on elevated temperature testing at 760°C (1400°F), a few of the first specimens were initially tested at room temperature in order to validate the experimental setup and tune the system settings to the new material and specimen design. Due to the rough nature of this initial data and the limited number of specimens, as well as the program’s intent to characterize elevated temperature behavior only, no rigorous characterization of the room temperature data has been performed. However, it is presented in Figure 6.1 for qualitative examination, and for illustration of the differences between ET and RT crack growth rate. One feature to note about the room temperature data is the minimum FCGR achieved in some of the tests, generally beginning at or below $2 \times 10^{-8} \text{ m/cyc}$, and in one case even below $1 \times 10^{-8} \text{ m/cyc}$. As will be shown below, these values were rarely achieved in elevated temperature testing due to crack closure effects.

More importantly, though, the RT data displays a markedly higher FCGR than the ET data at equivalent values of $\Delta K$ for both $R = 0.05$ and $R = -1$. This could be explained by at least two possible phenomena. First, it is often witnessed in metals that while strength may decrease with temperature, toughness often increases to some extent with temperature. Thus, the higher FCGR at RT could be attributed to lower toughness ahead of the crack tip — especially considering that strain rate effects may even be manifesting at a loading frequency of 10 Hz, reducing the alloy’s ability to blunt the crack through plastic deformation. On the other hand, what may actually be revealed by the data is not so
much an increase in FCGR as a decrease in effective $\Delta K$ as a result of the closure effects already mentioned at ET. Oxides forming on the fracture surface at ET can cause closure and a reduced crack driving force, as will be discussed below. Without the aid of more elaborate thermomechanical cycle-based testing, however, it is not possible now to point conclusively at one phenomenon over the other.

6.1.2 Thickness Effect

Examination of Figure 6.1 does not suggest any thickness effect on FCGR at room temperature. The 2.5 mm specimen is well-centered in the 4.8 mm specimen data at R = 0.05, and although data is limited at R = -1, those two data sets overlap quite well also. However, three of the prototype specimens were tested at the same load ratio (R = -1) at elevated temperature, and the same cannot be said for that data. As seen in Figure 6.2, the thicker specimen displayed an increased FCGR for a given $\Delta K$ as compared to the thinner specimens.

This may be explained in light of the same increased constraint at the crack tip that causes plane-strain fracture toughness to be lower than plane-stress toughness; however, that would not account for this effects emergence only at elevated temperature. Nonetheless, it is a phenomenon to be considered in future testing.

6.2 M(T) SPECIMEN RESULTS

Apart from the special SIF and the alternating visual measurement method described in Chapter 5, the test data was acquired according to ASTM E-647. Standard E-647 presents multiple candidate data reduction techniques for converting raw $a - N$ measurements to $R_{\infty}-\Delta K$ data, including algorithms which fit groups of $a - N$ points to quadratic equations and derive the slope at the median point. In this research, the basic secant-slope method was
Figure 6.1: FCGR results, room temperature, M(T) specimen

Figure 6.2: Prototype specimen data at 760°C, R = -1, displaying thickness effect
chosen, for practical as well as philosophical reasons. The secant slope method calculates the FCGR as the slope of a simple line between two points in a $-N$ space, i.e.,

$$
\left( \frac{da}{dN} \right)_\text{sec} = \frac{a_{i+1} - a_i}{N_{i+1} - N_i}
$$

(6.1)

The SIF range, $\Delta K$, coupled with this particular value is calculated based on the average, $\bar{a}$, of the two crack sizes.

As a practical matter, the secant-slope method was preferred given the limited number of data points available per test (itself a function of specimen size and other experimental conditions), since the polynomial-fitting methods effectively truncate the range of data points, sacrificing a handful of the first and last $a-N$ points of the set. In terms of the philosophy of the modeling effort, the curve-fitting methods were rejected due to their inherent data smoothing, inasmuch as this program sought to investigate scatter. As will be discussed later, during early conceptualization of the crack growth model, the more erratic point-to-point variability in FCGR was planned to be an explicit variable in the simulation, so this type of data was desired. Information about such variability is lost in the common smoothing techniques employed in published data [27].

FCGR data for all of the final M(T) configuration specimens tested at 760°C are presented first in Figure 6.3, where significant influence of R-ratio can be seen. It should be noted that in this plot, the data for $R = -1$ are plotted against the full range of $\Delta K$ — that is, using the actual “negative” SIF in compression as $K_{min}$ rather than setting it equal to zero for negative $R$ values according to the ASTM standard. This has the effect of doubling the apparent $\Delta K$ as compared to the ASTM definition. For illustrative purposes, the data in Figure 6.3 are re-plotted against $K_{max}$ in Figure 6.4. For a given value of $\Delta K$, the values of $K_{max}$ for any $R = -1$ and 0.05 tests are almost the same, putting the two data sets on nearly equal footing in terms of SIF; it can be seen then in Figure 6.4 that the two data
Figure 6.3: FCGR results, all R-ratios, $T = 760^\circ$C, M(T) specimen
Figure 6.4: FCGR results of Figure 6.3 presented versus $K_{\text{max}}$. 
sets essentially collapse upon each other. This indicates that, at these conditions (760°C, no hold time), the compressive loads of the fully reversed cycle have negligible effect of FCGR beyond the damage already induced by the tensile loading. Furthermore, the relative positions of the \( R = 0.05, 0.5, \) and \( 0.75 \) data sets further demonstrate the role that closure may play in FCP. The first two aforementioned sets are not greatly separated, but there is a noticeable shift then to the third, \( R = 0.75, \) set. This indicates that there is not as large a difference in crack driving force between \( R = 0.5 \) and 0.05 as between \( R = 0.5 \) and 0.75, suggesting that crack tip closure probably occurs at some load just below 50% of the maximum under these conditions.

The test data for each \( R \)-ratio are presented separately in Figures 6.5–6.8 for clarification of inter-specimen variability at a given test condition. It is clear from the legends for each figure that different numbers of specimens were tested at each condition, and not all slabs were tested at each condition; this was an undesired consequence of the experimental changes mid-program due to the difficulties encountered with the specimen configuration. Furthermore, the \( R = 0.5 \) condition received more attention during the first stage of testing, but customer interest became focused on \( R = 0.05 \) by the time the new specimen design was implemented.

All of the factors mentioned so far that limited specimen numbers at each condition hinder rigorous statistical characterization; however, using a suitable equation to collapse the data for all conditions onto one model allows for a sufficiently robust assessment of material behavior for subsequent modeling. As mentioned in Section 2.4, the Walker equation is one such model that accounts for load ratio effects. Examination of Eq. (2.4) readily suggests that all of the data can be plotted against an effective SIF range,

\[
\Delta K_{\text{eff}} = \frac{\Delta K}{(1-R)^{1/2}}
\]  

(6.2)
Figure 6.5: FCGR results, $R = -1$, $T = 760^\circ C$, M(T) specimen

Figure 6.6: FCGR results, $R = 0.05$, $T = 760^\circ C$, M(T) specimen
Figure 6.7: FCGR results, R = 0.5, T = 760°C, M(T) specimen

Figure 6.8: FCGR results, R = 0.75, T = 760°C, M(T) specimen
given proper selection of a system-specific Walker exponent, \( c_w \), and then a single crack growth law can be applied to all sets of \( R \) as

\[
\frac{da}{dN} = f(\Delta K_{\text{eff}})
\]  

(6.3)

In order to accommodate both positive and negative load ratios, it is actually necessary to choose two Walker exponents, \( c_w^+ \) and \( c_w^- \). For this data set, the positive exponent \( c_w^+ \) was optimized first by iterating on its value to achieve the highest possible correlation coefficient, \( R^2 \), for the Walker equation applied to the positive load ratio data. Next, the \( R = -1 \) data was added to the set and \( c_w^- \) iterated, again until the highest \( R^2 \) was achieved. The optimized Walker coefficients are presented in Table 6.1 and the “Walkerized” FCGR data are shown in Figure 6.9. It should be noted that a few of the specimen data sets, which were presented in Figures 6.3 - 6.8 but displayed some erratic behavior due to crack arrest at either front, were censored from the data set used to calculate the Walker parameters as well as in the statistical analysis below.

### Table 6.1: Optimized Walker exponents for M(T) specimen data.

<table>
<thead>
<tr>
<th>( R )</th>
<th>( c_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0 )</td>
<td>0.709</td>
</tr>
<tr>
<td>( &lt; 0 )</td>
<td>1.005</td>
</tr>
</tbody>
</table>

6.2.1 Statistical Analysis

Once the data has been adjusted to bring together sets from different load ratios, it is clear that a significant amount of dispersion remains, inasmuch as each load ratio data set displayed a fair degree of dispersion internally which was then uniformly translated by the Walker transformation. Having this single set of Walkerized data, the crack growth rate
Figure 6.9: "Walkerized" FCGR results, M(T) specimen

characteristics of each specimen were then modeled with a simple Paris equation (Eq. (2.3)) based on $\Delta K_{eff}$.

$$\frac{da}{dN} = C\Delta K_{eff}^{m}$$  \hspace{1cm} (6.4)

By doing so, it is assumed that all of the acquired data falls into Region II crack growth (see Figure 2.8) and not into Region I; this assumption has been made for the purposes of this analysis simply by visual assessment of the plotted data, and further research into near-threshold behavior may be required before this assumption can be fully verified.

Rather than attempting a statistical analysis of each of four fairly small sets of data (for each load ratio) further subdivided as three castings, an analysis is performed on the collapsed set of all 24 crack growth data arrays by use of the Walker equation. Fitting the Paris law to the set of Walkerized data produces 24 equations, represented by 24 sets of Paris coefficients and exponents. The variation in these exponents is thus represented as
variability in the sets of these coefficients and exponents. As will be explained in Chapter 7, this variation can be considered as a composite of a variability between castings (Slabs 6, 7, and 9) and a variability within each casting (e.g., Specimen 6-1, 6-6, and 6-8 for R = 0.5, etc.). Therefore, summary statistics are presented in Table 6.2 for the entire set of specimens regardless of casting, as well as basic statistics that characterize the intra-casting behavior of the specimens from each slab. For the mean, μ, value of log C, each linear value of C is also presented below the logarithm in parentheses. (Note that this is the antilog of the mean of log C, not the mean of C; it has been presented as such in order for the Paris law parameters to obey the semilog correlation.)

As already mentioned, this program struggled to balance the conflicting goals of adequate statistical analysis and broad characterization of several test conditions with a limited number of specimens. As a result, there are simply not enough data points to confidently claim any particular distribution function (e.g., normal, Weibull, lognormal, etc.) over another to describe the relevant parameters (log C and m). Therefore, only basic descriptive statistics are presented here, and as will be discussed later, a normal distribution for m and lognormal for C are assumed for subsequent modeling purposes. These distributions are fairly safe assumption as born out by the literature, at the very least for the exponent m [82, 96, 97, 98]. However, it is wise at this point to echo the admonition of a contemporary engineering probabilist, that it is often a blunder of engineers to assume normal behavior where it is not justified [99]. Thus, the values of skewness, γ, and kurtosis, κ are presented for the set of all specimens (higher order statistical moments become very inaccurate for sample sizes as small as single slab sets) to give the reader some indication of the concentration of data and the presence of any notable “tails.” The relatively minor skewness and the kurtosis approaching 3 lend some confidence to the assumption of normality for the
distribution.

It has also been shown that, for a set of crack growth rate curves for a given system, there is a correlation between the Paris law parameters $m$ and $C$, often found to follow a power law relationship (or a linear relationship between $m$ and $\log C$) [82, 96]. This relationship is essential to accurate modeling of crack growth behavior, and therefore has been determined for the set of M(T) specimen data to follow

$$\log C = -1.540m - 6.823$$  \hspace{1cm} (6.5)

Use of this relationship will be discussed in Chapter 7. The relationship between $m$ and $\log C$ is shown in Figure 6.10.

\[\text{Figure 6.10: Linear relationship between Paris parameters } m \text{ and } \log C \text{ for M(T) specimens}\]
Table 6.2: Statistics for Walkerized M(T) specimen data

<table>
<thead>
<tr>
<th></th>
<th>Total Set</th>
<th>Slab 6</th>
<th>Slab 7</th>
<th>Slab 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>log(C)</td>
<td>μ</td>
<td>log(C)</td>
<td>μ</td>
</tr>
<tr>
<td></td>
<td>τ0</td>
<td></td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>-13.76 (1.74×10^{-14})</td>
<td>-14.27 (6.03×10^{-15})</td>
<td>4.805 (1.05×10^{-14})</td>
<td>-13.89 (1.05×10^{-14})</td>
</tr>
<tr>
<td>σ</td>
<td>1.211</td>
<td>0.783</td>
<td>0.598</td>
<td>0.391</td>
</tr>
<tr>
<td>CV</td>
<td>0.088</td>
<td>0.174</td>
<td>0.042</td>
<td>0.081</td>
</tr>
<tr>
<td>γ</td>
<td>0.15</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>κ</td>
<td>2.06</td>
<td>2.13</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
6.3 EC(T) SPECIMEN FCG RESULTS

Based on customer input, all of the remaining specimens that were converted to EC(T) configuration were tested at \( R = 0.05 \). It was anticipated that the results from this set of tests would simply supplement the existing results at this load ratio from the M(T) specimens, and a thorough statistical analysis of FCGR at one experimental condition could be performed on this composite set. Given that the finite element model was in very close agreement with the published EC(T) solution, as described in Chapter 5, it was expected that the applicability of the SIF parameter between configurations would be maintained. Indeed, the ability of \( \Delta K \) to correlate results between laboratory tests and field applications of an entirely different configuration is one of the cornerstones of the utility of LEFM!

However, as can be seen from the EC(T) data presented in Figure 6.11, the EC(T) data is closer to the \( R = 0.5 \) M(T) data than the \( R = 0.05 \) data. The precise cause of this discrepancy is unknown, although it may be related to the lower constraint or the change in T-stress in the EC(T) specimen as compared with the M(T) configuration.* The EC(T) specimen is subjected to more of a bending stress than is the M(T), and this can lead to a reduced effect of crack closure (and therefore, at load ratios affected by closure in the M(T) specimen, an accelerated crack growth rate as is seen in the EC(T) specimen). If this is indeed the case, then the fact that crack closure is likely to occur in the vicinity of \( R = 0.5 \) (as described above) for the M(T) specimen may have some relationship to the fact that the EC(T), \( R = 0.05 \) data appears to be more in line with the M(T), \( R = 0.5 \) data. This would result from the fact that, in the relative absence of closure for the EC(T) at \( R = 0.05 \), the effective driving force would be very similar to the M(T) at \( R = 0.5 \).

Since all of the EC(T) specimens were tested at \( R = 0.05 \), there is of course no need to

*Personal communication with Dr. Jim Newman, University of Alabama.
Figure 6.11: FCGR results, $R = 0.05$, $T = 760^\circ$C, EC(T) specimen
collapse the data through the Walker equation. In fact, it is not even likely that the Walker coefficient determined for the M(T) specimen above can be justifiably applied to the EC(T) specimen data in order to present it against $\Delta K_{eff}$. Since the Walker coefficient is system specific based on the system's response to load ratio changes (which is, in turn, a result of dominant deformation mechanism, fracture surface roughness, etc.), then if the difference between M(T) and EC(T) data sets is indeed a result of different closure effects, it stands to reason that it would respond differently to load ratio changes and thus require a unique $c_w$. Lacking any data for other load ratios for this specimen, such a parameter could not be determined.

6.3.1 Statistical Analysis

Summary statistics for the EC(T) specimens are presented in Table 6.3. As for the M(T) specimens, statistical parameters are reported for the entire set of specimens (irrespective of casting) as well as for each individual casting; again, higher order statistical moments are only reported for the full set of data due to limited sample size. Compared to the M(T) data set, the EC(T) data is more skewed, with a negative tail instead of the slight positive one calculated above. The "peakedness" of the data is closer to 3 than it was for M(T), which is the theoretical value for a normal distribution.

A similar correlation between the Paris law parameters $m$ and $\log C$ to that found for the M(T) specimens exists for the EC(T) specimens as well. Since the EC(T) was not congruent with the M(T) data, the relationship describing the correlation is not surprisingly (even if not quite understandably) different as well, and was determined to be

$$\log C = -1.264m - 6.859$$

(6.6)

The correlated parameters are plotted in Figure 6.12.
<table>
<thead>
<tr>
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<th>Total Set</th>
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<th>Slab 7</th>
<th>Slab 9</th>
</tr>
</thead>
<tbody>
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<td>( \log(C) )</td>
<td>-10.68</td>
<td>-10.67</td>
<td>-10.59</td>
<td>-10.74</td>
</tr>
<tr>
<td>( m )</td>
<td>3.02</td>
<td>2.99</td>
<td>3.00</td>
<td>3.07</td>
</tr>
<tr>
<td>( \mu )</td>
<td>(2.11 \times 10^{-11})</td>
<td>(2.14 \times 10^{-11})</td>
<td>(2.56 \times 10^{-11})</td>
<td>(1.81 \times 10^{-11})</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.588</td>
<td>0.576</td>
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<td>0.496</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.460</td>
<td>0.420</td>
<td>0.713</td>
<td>0.429</td>
</tr>
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<td>CV</td>
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<td>0.054</td>
<td>0.087</td>
<td>0.046</td>
</tr>
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<td>( \gamma )</td>
<td>2.47</td>
<td>2.24</td>
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<tr>
<td>( \kappa )</td>
<td>-0.57</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 6.12: Linear relationship between Paris parameters $m$ and $logC$ for EC$_{(T)}$ specimens

6.4 QUALITATIVE OBSERVATIONS ON FCP

6.4.1 Transients and Crack Arrest

Crack growth arrest was a fairly notable occurrence during elevated temperature testing of DS GTD-111. As can be seen by comparing Figures 6.1 and 6.9, the range of FCGR values measured during RT testing was, in general, lower by a factor of 2-4 times than the crack growth rates measured at ET. This was due to the material's tendency to arrest at one or both of the crack tips quite readily when trying to shed load in order to achieve lower initial crack growth rates. First, in order to grow a pre-crack at the central notch, it was generally necessary to initiate a crack either at RT or at a higher maximum load at ET than was used during subsequent ET testing; however, making changes to the system in order to transition to the ultimate test conditions often retarded or arrested crack growth if great care
was not taken. If the precrack was initiated (or was propagating) at ET and load-shedding was required to achieve the desired load amplitude, load decrements were applied quite conservatively compared to the E647 recommendations of no more than 10% load steps, generally less than 5%. However, even this caution could not guarantee continued propagation, and as there were two crack fronts per specimen, maintaining crack growth at both of them was only half as likely; this led to the frequently asymmetric crack growth conditions that plagued this research, as it often took a number of adjustments to load cycling in order to achieve non-arresting load-shedding for both crack fronts. Further aggravating the situation was the fact that, once a crack had arrested during even the slightest load shedding, it became difficult to re-activate crack growth, even at loads up to roughly 10% higher than were being applied prior to the arrest! This kind of behavior was previously observed in ET testing of a superalloy by Padula et al. [31], and was attributed to some sort of “oxide induced coaming.”

This “coaxing” phenomenon was also observed during thermal transients at constant load amplitudes. In an effort to avoid problems with crack arrest due to load shedding, some of the specimens were crack initiated at room temperature prior to further pre-cracking at elevated temperature. However, in a few instances, it was observed that upon raising the temperature from RT to ET, the crack (usually on both fronts) would spontaneously arrest. Additionally, during two of the prototype specimen tests at positive load ratios, ET testing was halted and returned to nominally zero load and then to RT. Upon returning to ET at zero load and then to the previous maximum load amplitude, the crack appeared to have arrested, and a load increase of greater than 10% was required to restart crack propagation. Although the precise physical mechanism behind it is unclear, his phenomenon deserves further study as it may have significant implications in turbine engine life prediction in
which thermal cycles are as important as mechanical cycles.

6.4.2 Crack Path Tortuosity

As described in Section 2.3.4, crack path tortuosity has been investigated for its role in crack growth rate, whether as a cause (through mechanical closure) or a by-product (by way of crack path selection mechanism) of an alteration in crack growth rate under certain conditions. While no analytical work has been performed yet on the tested specimens to quantify the degree of tortuosity, some remarks will be made about the observed behavior. The fatigue crack path was notably more tortuous at room temperature and at high values of ΔK (Region III) than at the nominal ET, Region II test conditions. While only a few of the prototype specimens were tested at RT, a number of the final specimens were pre-cracked at RT before data was acquired at ET. An example of the former is shown in Figure 6.13, in which a markedly tortuous crack path is followed and a significant bifurcation is seen as well. In Figure 6.14, on the other hand, an example of the latter transition from ET to RT crack propagation is shown; at the ET condition, the crack path takes a noticeably less tortuous path with far less frequent crack deflections. Recall from Section 2.3.4, the deformation mechanism at RT is heterogeneous based on planar slip, and a crystallographic crack path is therefore favored. At ET the deformation becomes more homogeneous as cross-slip is activated in the damage zone, and crack growth proceeds in a direction generally normal to the maximum tensile stress.

Initially it was speculated that a better understanding of the FCGR variability might be gained by consideration of the “true” crack growth rate, accounting for the deflected length (or actual surface area created) of the tortuous crack path rather than simply using the projected length (on the load-normal plane). This approach could be taken further by adjusting
Figure 6.13: Room temperature crack path, prototype specimen (R = 0.05)

Figure 6.14: Crack path at RT and ET (R = 0.05)
the SIF based on projected crack length and adjusting it for the mixed-mode loading effect of crack deflection, as described by Suresh [26]. However, this was not pursued in part because of the lesser extent of tortuosity at the primary ET test conditions, and also because examination of the fracture surface of specimens tested to failure revealed that the surface intercept of the fracture surface did not always accurately describe the through-thickness crack path. Crack path facets typically observed on the surface during ET testing were on the order of 0.25 mm, or an order of magnitude less than the specimen thickness, and noticeable crack deflections up or down were not necessarily reflected across the entire crack front. Larger scale deflections, in which the crack deviated from the normal plane for a significant portion of its length (and the angled crack path was reflected through the entire thickness), might cause some concern as in some cases they violate the crack normality requirements of E647. However, work by Chan et al. [41] and Dibione et al. [100] has shown that, for single crystal nickel superalloys, continued use of the projected crack length for deflections up to 30° is still acceptable, so further analysis of the role of deflection was not pursued at this time.

6.4.3 Role of Grain Boundaries

A number of specimens (particularly the bulk of the EC(T) configuration) were polished and metallographically etched prior to testing in order to identify the location of grain boundaries to determine what role, if any, they played in FCP. This move was initially inspired by an early metallographic examination of one of the first M(T) specimens tested, which revealed a noticeable change in crack path across a grain boundary, as seen in Figure 6.15. It seemed reasonable that, as the relative orientation of two crystallographic axes was free to rotate between grains,
Figure 6.15: Change in crack path across grain boundary

Figure 6.16: Illustration of two possible through-thickness grain structures with the same surface intercepts
Unfortunately, examination of the surface intercepts of grain boundaries proved inconclusive at this time. Although the specimen thickness was nominally half of the average grain diameter typically observed in this material, the grain intercepts observed on the front and back surfaces of each specimen were far from perfectly aligned, which might suggest a clean, through-thickness grain boundary. However, even this “ideal” situation would not guarantee that FCGR measurements took place largely in single grains with periodic interruptions by single grain boundaries largely normal to the crack growth direction, since grain triple points may be hidden below the surface and what is actually observed at the surface is crack growth through a bicrystal. Figure 6.16 presents two simple examples of very different grain structures that otherwise look identical at the surface in terms of grain boundaries. The former situation described above, in which FCGR data is essentially the sum of distinct regions of single crystal crack growth, is shown in the first schematic, whereas what may be a much more complicated situation is shown in the second. Indeed, even in Figure 6.15 it can be seen that the boundary between the two grains at the surface is not perfectly discrete, but rather a transition as there is overlap or intermingling between a few of the outer dendrites of either grain. A thorough examination of the role of grain boundaries and different grain orientations, it seems, will require much more extensive (and likely destructive) metallographic work, as well as possible FEM work to characterize the stress and strain fields ahead of a crack moving through a triple point or a bicrystal, for example.
CHAPTER 7

PROBABILISTIC CRACK GROWTH MODELING

In addition to supplementing the limited available material property data on this particular alloy through fatigue crack growth testing, the second major deliverable of this project was to formulate a fracture mechanics-based life prediction methodology for the first stage turbine buckets. This chapter describes the development of a first-pass probabilistic model that utilizes the scattered data presented in Chapter 6.

7.1 MODELLING APPROACH

As described in Section 2.2, there are a number of different approaches to life management, both deterministic and probabilistic. Particularly in the relatively new arena of probabilistic life prediction, it seems as though each major research or manufacturing organization has its own model development philosophy, and its own, internally developed life model is favored to the exclusion of the lessons learned through the efforts of other groups. Furthermore, the newly developed models have not been in existence long enough, or been implemented widely enough, to establish with any clarity the superiority of one approach over another. Where powerful and massively-parallel computing resources are available, a full Monte Carlo simulation of a 3-D thermal-mechanical finite element model of a fracture-critical component may be performed. On a workstation without the benefit of parallel processing, a limited number of FEM simulations coupled with Response Surface method might be favored. In cases where the damage state can be reduced to a one-dimensional problem, a direct analytical or simple numerical solution may be fairly
straightforward.

When putting forth a new probabilistic life prediction methodology, it is all but impossible to declare whether the answers it provides are “right;” paradoxically, true verification of the model having achieved its objective — enabling the maximum economic utilization of a component without suffering a number of unacceptable failures — could only come by operating the population of components to failure and observing the predicted life distribution. However, where the fundamental physics of the problem can be reasonably verified through a limited number of laboratory or field tests, a well-built probabilistic methodology can, at the very least, enable the user to simulate the relative influences of the input distributions on the evolution of risk with time. The effects of a proposed inspection schedule or component retirement protocol on a fleet of machines operating under various conditions can be examined numerically prior to implementation.

Due to the time and resource limitations inherent in a project of this scope, a fairly simple predictive model was developed to demonstrate the concepts and capabilities of the framework, rather than pursue the cutting edge of probabilistics at the expense of the underlying material study of this particular alloy. Since the research was sponsored with a particular damage mode and location in mind — namely, fatigue crack propagation from the first trailing edge cooling hole — damage was represented one-dimensionally by a single crack size $a$. A number of input variables that control crack growth, such as initial flaw size, grain size, FCP resistance, and certain operational parameters, are randomly generated based on experimental measurements and used in the subsequent simulation of the crack growth throughout the turbine blade’s life. Repetition of this simulation over a full set of blades for a number of engines establishes a crack life data set that can be used
for statistical analysis, generating distributions of crack size or lifetimes. A simple, FORTRAN based program was selected over more complicated software options with the intent that (1) the response functions (primarily, the evolution of SIF with crack size for a given blade geometry) could be easily reprogrammed, and (2) in future versions of the model the probabilistic segments of the code could be supplanted with more powerful, external probabilistic packages (such as NESSUS), which would interface with the deterministic core to perform advanced analysis techniques (such as Response Surface, FORM/SORM, etc.).

Description of the various elements of the simulation program developed follows below.

7.2 SELECTING AND MODELING RANDOM VARIABLES

In any real engineering design, nearly every parameter can be considered as a random variable, for such is the nature of all things man-made or even naturally occurring. Slight variations will exist in every detail: the local airfoil wall thickness may vary between castings due to casting core placement error; the local temperature at the fatigue crack may be affected by cooling air deprivation brought on by dust or other blockage; the vibratory stresses themselves are almost a study in chaos theory. Part of the challenge of this project is identifying which parameters might have the greatest impact on component life distribution by virtue of their own variability. Those parameters modeled as random variables in the current version of the model have been selected to highlight the results of the research and are by no means an exhaustive set. Such a life prediction model can be made arbitrarily complex, making an ever-greater number of parameters non-deterministic; however, as the number of random variables increases, the number of simulations required to ensure an accurate sampling of the myriad permutations increases, so parameter randomization must be performed judiciously. All of the parameters included in this model can, of course, be
varied at the programmer or user level from simulation to simulation in order to perform trade studies; the parameters that are varied from iteration to iteration are described here.

7.2.1 Initial Conditions

A subtask was established in the project for another researcher to use field inspection data of in-service crack sizes in turbine buckets at the targeted location, combined with records of each turbine's service history, to determine the EIFS distribution. As described by Wilson [101], a fracture mechanics-based crack growth equation was developed to include the three most relevant service variables — engine starts, total engine operating hours, and full-load trips — and, after numerical optimization of the various influence coefficients, was used to “grow” the observed service cracks back to the start of engine service to establish the EIFS distribution. Since the crack regression algorithm accounts for varying operating conditions from one user to the next in normalizing the field data, the resultant initial flaw sizes should be independent of the engine customer or proposed usage for any given simulation. However, two different engine frame types operating under the same mode, with identical hours, starts, and trips, may display different fatigue crack indications in service, due to the different buckets used in each engine. This is especially true since different manufacturing methods are used to fabricate the cooling holes, such as shaped tube electrolytic machining (STEM-drilling) and in-situ casting. As mentioned above, the initial fatigue quality of a component can depend on surface conditions brought about by manufacturing processes, so these variables need to be included in the model. Therefore, field inspection data is segregated according to engine frame type (or bucket design), and a different EIFS distribution is regressed for each different frame.

In the Monte Carlo simulation scheme developed here, the initial flaw or crack size for
each simulation is generated by random selection from the EIFS distribution, as described by statistical parameters for each frame by the inspection data regression. As with all of the other random variables in this program, a value for the non-deterministic parameter is calculated by generating a random value for a standard normal distribution (with mean of 0 and standard deviation of 1) using a subroutine based on the system’s built-in (uniform) random number generator; this normalized value is then multiplied by the standard deviation of the relevant parameter and added to the mean value. The engine frame type is a user-input character variable, and remains constant for each engine simulated while a new initial flaw realization is produced for each blade. The ability to change engine frame types from batch to batch of simulations enables the user to determine any possible necessity for different inspection schedules for the engine frames due to differences in initial bucket quality. Should microstructural studies of the material yield specific distribution information about crack initiating constituent particles, the distribution parameters for the simulated EIFS can be updated with minimal effort.

7.2.2 Fatigue Crack Growth Resistance

Given an initial flaw size in a component subjected to cyclic loading, the ensuing fatigue response will be a function of the flaw size, the load history, and the material’s local resistance to fatigue crack propagation. As described in Section 2.2.3, there are a number of methods for modeling this response. The resistance of a given material or component to fatigue crack propagation can basically be represented by the parameters of the governing crack growth rate equation. In this case, the fairly simple Walker equation is used to represent crack growth rates in the linear regime while accounting for the effect of load ratio, as described by Eq. (2.4). Therefore, variability in crack propagation resistance can
be simulated by random selection of the parameters $C$ and $m$, the Paris coefficient and exponent. It was shown in Chapter 6 that the measured FCGR data behaved according to the widely observed correlation between the Paris law parameters. Retaining this correlation is essential to accurate simulation of fatigue behavior, for independent sampling of $C$ and $m$ could produce wildly inaccurate results. As described by Annis, it is a common but grave error to sample from the marginals of a bivariate distribution, assuming the two marginal distributions to be statistically independent [99]. Figure 7.1 shows how the resultant joint distribution of two variables is notably different if they are independent or correlated. In

![Correlated, Bivariate Normal](image)

**Figure 7.1:** Effect of correlation on the production of a joint probability distribution from two marginal distributions; solid line represents correlated distribution, dashed line uncorrelated [99]

this case, an improperly selected $C - m$ pair giving, for example, both a high intercept, $C_6$, and a high slope, $m$ — a statistically unlikely event, according to the negative correlation — could produce a crack growth rate curve orders of magnitude higher than realistically
possible. Thus, it is not sufficient simply to know the means and standard deviations of the Paris coefficient and exponent, but the relationship between the two is required as well.

To represent this relationship in this first version of the model, a simple formula based on a linear regression of the experimental data was utilized after the method put forth by Ostergaard and Hillberry [82]. In this approach, the correlation between the parameters is represented according to the experimentally determined (regressed) equation, such as Eq. (6.5). Since the correlation between the two is not perfect, the residuals from this linear regression are then themselves statistically analyzed as the parameter $F$, defined by

$$\log F = -\left[\log C - \log \bar{C}\right]$$  \hfill (7.1)

where $\log \bar{C}$ is the regressed value of $\log C$, and a distribution is defined for the subsequent simulation of $F$. For both the M(T) and EC(T) specimen data, the residuals (in $\log C$) about the correlation equation can be represented by a normal distribution with a mean of zero and a standard deviation of 0.026. Thus in the algorithm, for a given realization of local fatigue crack growth properties, a random value of $m$ is generated from its marginal distribution. The corresponding coefficient $C$ is then generated by applying a randomly simulated deviate $F$ to the result of the linear relationship of Eq. (6.5).

Furthermore, it should be noted that a correlated bivariate distribution can be modeled more directly using the Cholesky decomposition method, which relies on the covariance matrix for the random variables in question [102]. This approach has been used recently, for example, to simulate the shape distributions of hard precipitates in aluminum alloy, in which the major and minor axes of the ellipsoid particles were characterized by a strongly correlated bivariate distribution [77]. Again, as with other alternative, more rigorous statistical techniques that could be used in a framework such as this, the pursuit of optimum statistical methods had to defer to the establishment of baseline data and methodologies at
this stage, and further development of the model is planned.

7.2.3 Microstructural Characteristics

The procedure described above merely lays out the mathematics of simulating fatigue properties based upon the statistics of the measured data; to simulate random material, some algorithm must be established for laying out the variation of the relevant properties in space. How this is accomplished depends not only on a priori understanding of the underlying mechanistic principles that govern the crack growth response, but also on a posteriori analysis of the experimental data – both what the data says and how it was taken. Indeed, proper consideration of the modeling approach beforehand should guide the experimentalist to the appropriate methods and goals.

As mentioned in Section 2.2.3, a number of recent research efforts have been directed at measuring fatigue crack propagation at very fine crack growth increments in order to characterize the variability in crack growth rate over the course of the experiment. However, as can be readily shown by some numerical exercises on the experimental data, the statistical results of such a treatment of FCGR hinge upon the selection of the crack growth measurement distance or cyclic interval, as recently shown by Bahr [103]. A process may appear fairly smooth when considered on a large scale, only to appear more like a series of fits and starts when examined very closely. Two of the experiments described herein had crack size measurements performed at noticeably different average crack growth increments, approximately 0.24 mm for Specimen 9-7 and 0.6 mm for Specimen 6-8. Examining the relative variability of the two tests, as measured by the calculated value of $\frac{dK}{da}$ normalized by the regressed value for that $\Delta K$ and shown in Figure 7.2, it is readily observed that the finer measurement spacing produced greater apparent variability. (Consideration of data scat-
Figure 7.2: Effect of measurement spacing on the relative apparent scatter in FCGR: 0.24 mm measurements (a) and 0.6 mm measurements (b)
ter at this level would lend itself to a stochastic process approach as described earlier by Eq. (2.7), using the spatial distribution of the FCGR residuals to characterize the random process $\xi(x)$. Continual subdivision of the measurements would lead to increased apparent scatter, and furthermore, selection of a particular resolution seems to be entirely arbitrary.

While the scale of the dendritic-interdendritic microstructure and property variation discussed in Section 2.3.6 might suggest the dendrite core as an appropriate spatial unit, two difficulties emerge in that approach. First, the spacing of the measurements made in this program (driven by resolution considerations) was on the order of the primary dendrite spacing, approximately 250 $\mu$m; however, in order to adequately characterize a periodic phenomenon, measurements must be made at a frequency greater than the Nyquist frequency, or twice the highest component frequency of the relevant phenomenon (104). That is, measurement spacing of better than 150 $\mu$m would be required, which in turn would necessitate a more accurate measurement system. The second problem with use of the dendritic scale is that even if an alternative measurement system were used, the thickness of the specimen — being several times the dendrite spacing — could lead to microstructural averaging and cloud the apparent signal anyway. For example, if a potential drop (PD) measurement system were reporting a smooth, monotonically increasing signal, it could indicate the steady progression of an even crack front through the specimen regardless of the dendritic structure before it. On the other hand, it could be the result of a jagged crack front in which some segments are moving quickly through weak eutectic channels while others progress more slowly through tougher dendrite cores, the entire ensemble moving steadily forward in alternating turns. Any fine fluctuations in the PD signal would likely be lost in system noise, and regardless, without observation of the internal crack front, it would not be possible to separate out the influences of the individual crack front segments as they

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passed through local microstructure. Therefore, the dendritic microstructural scale, while scientifically appealing, suffers a seeming impossibility of measurement, at least given the specimens and equipment at hand.

Therefore, the scale of variability for this model was chosen at least based on a measurable and physically justifiable characteristic: grain diameter. Current work supported by the US Air Force on modeling the variability of FCP in titanium alloys based on the grain size as a governing length scale for variability has shown positive results.* It was pointed out in Chapter 6 that efforts to clearly identify grain boundaries in the FCP specimens, in order possibly to correlate observed variations in FCGR with different grains, were unsuccessful, so no clear length scale of variability has been observed. However, since the thickness of the specimens is smaller than the typical grain diameter by roughly a factor of two, and the distance over which crack growth occurred in a typical tests is on the order of a grain diameter, then at this stage of modeling exercises the assumption has been implemented that the inter-specimen variability within a given casting represents the grain-to-grain variability in the casting. Inasmuch as several specimens cut from a portion of a single slab casting display significant differences in FCGR, their variations to at least describe a spatial distribution of properties through the thickness of a single casting, and for now the grain boundary appears to be an acceptable choice of boundaries between those properties.

Therefore, in modeling the spatial variability for the current simulation, an array of distances to grain boundaries from the trailing edge is generated for each simulated blade according to a simple normal distribution of grain diameters measured metallographically. Since the placement of the blade trailing edge relative to the first grain boundary can happen

*Personal communication with Dr. Jeff Calceterra, Air Force Research Lab.
anywhere in the first grain with equal probability, the distance to the first grain boundary is equal to the first generated grain diameter multiplied by a uniformly distributed fraction between 0 and 1; the subsequent distance to each boundary thereafter is simply the diameter of the intervening grain.

So, at the outset of the process a single blade, i, is simulated by generating its average, "global" properties: mean FCGR law slope, \( \mu_{i}^{\text{FCGR}} \); global standard deviation in FCGR law slope, \( \sigma_{i}^{\text{FCGR}} \) (i.e., the standard deviation between the blade's grains); and the average grain diameter for the blade, \( D_{i}^{g} \). The blade to blade variability is modeled based on the variability between (the average values of) the slabs, as a whole, from which the experimental specimens were produced. For example, the global FCGR law slope for each blade \( \mu_{i}^{\text{FCGR}} \), is determined according to the statistics of the slab to slab averages of the experimental data — e.g., the values 4.805, 4.582, and 3.77 for \( \mu_{i}^{\text{FCGR}} \) in Table 6.2. Furthermore, the standard deviation within the blade has a statistical description of its own, because the standard deviation between experimental specimens within a given slab varies from slab to slab; therefore, the standard deviation in FCGR slope between simulated grains within a blade is modeled as varying from blade to blade. While difficult to describe, this intercasting and intergranular property simulation algorithm is conceptually fairly simple, and thus is shown schematically in Figure 7.3, with the relationships between interspecimen/interslab and interblade/intergranular statistics described explicitly therein.

Unfortunately, this is not a complete nor entirely accurate representation of the physical phenomenon, as it implicitly assumes that each segment of crack growth occurs in a single grain or crystal. However, it is not only possible but likely that certain periods of crack growth will occur in bi-crystals (through the thickness) of varying character and across triple points, as was shown in Figure 6.16 (the second, more complicated schematic). The
Figure 7.3: Flowchart illustrating intergranular variation simulation.
role of such regions in FCP require further study, but the basic methodology of subdividing the simulated component into different property regions based on relevant physical measurements is still logically sound. Future versions of such a model could incorporate simulations of spatially distributed triple points and non-normal grain boundaries (i.e., FCP through a bi-crystal) with associated FCGR characteristics without changing the fundamental algorithm.

7.2.4 Mechanics of Crack Propagation

Two subroutines accomplish the growth of the simulated crack over the life of the blade; they are shown as the first two crack increment process blocks in the flowchart of Figure 7.4. The first subroutine represents the basic LEFM crack propagation of the engine startup cycle. As a result of the temperature effects on FCGR mentioned in Chapter 6, it has coded to accept the temperature during load-up as an input so that in the future it can be easily adapted to behave differently for cold, warm, or hot starts, although current the temperature of the entire simulation is held fixed at 760°C to match the experimental data. Formulae for SIF are to be entered at the programmer level based on FEM analysis of cracked blades, and are functions of crack size and engine speed; different engines (with different blade designs) are likely to have different SIF formulae.

Since IGT’s tend to operate continuously for long periods of time, creep damage can occur ahead of the crack tip and lead to creep crack growth (CCG). Therefore, a second subroutine was created to simulate further growth of the crack during operation after the initial LEFM-based increment. It is anticipated that results of concurrent research into CCG in DS GTD-111 will be used in the future to provide a mechanistic basis for this subroutine. However, in the meantime, a simple SIF-based approach to CCG has been adapted from
Wilson's research in regressing the EIFS distribution [101]. The FCGR equation used to generate the EIFS distributions that will feed into this model included a term for the effects of hold time based on the SIF, essentially just changing the units for \( C \) in Eq. (2.3) to \( \text{m/hr} \) instead of \( \text{m/cyc} \), and using \( K_{\text{max}} \) as the driving force instead of \( \Delta K \). Therefore, to be consistent, the CCG subroutine simply uses the optimized coefficients from that research to estimate a crack growth increment during hold time from \( K_{\text{max}} \).

7.2.5 Operational Variables

For the purposes of this investigation, the development of the model benefitted from the fact that the operational loads under consideration are fairly predictable and repeatable. The variability from engine to engine can largely be reduced to two variables — mode and hours per start — and there is only one random event of real concern, when an engine undergoes full-load trips. In some cases, the engine start events can be further divided into "hot," "cold," and "warm" starts of varying degree. However, at this stage, lacking any mechanical test data to describe the load interaction effects that may result from such events, the current version of the model does not simulate trips or different start types, and characterizes only the difference in modes and operating times. Whether the engine is a cyclic or base unit, the variable storing the hours between engine startups is a constant, average value, which represents the assumption that slight variability between the lengths of each operational interval are not a primary driver of variability (i.e., that regardless of variations in operational hours, the influence of running time is dominated by its mean value or expectation).

The difference in operational mode only determines, for the purposes of this simulation,
whether inspection intervals are based on number of startup cycles (for cyclic mode) or operating hours (for base mode). Thus, inspection of the engine’s blades becomes the primary operational variable simulated in this model, and inspection intervals can be varied from batch to batch of simulations in order to determine the influence of inspection frequency on the risk of failure of a blade. Furthermore, since inspection is a human process with a certain amount of error involved, detection of a crack at each inspection is modeled based on the popular probability of detection (POD) concept. The POD curve is essentially a cumulative distribution function (CDF) versus crack size, representing the probability that a crack of a given size will be detected on inspection, and generally varies with the process used and operator skill. The POD curve can be altered quite simply at the programmer level and is currently modeled as a lognormal distribution of crack size. To simulate inspection, a crack length \( a_{\text{detect}} \) is randomly generated from the POD curve to represent the maximum crack size that might be missed when inspecting the current blade; crack detection thus occurs if

\[
a(N_{\text{insp}}) > a_{\text{detect}}
\]  

(7.2)

Undetected cracks remain in the simulation to continue growing until the next inspection interval. Detected cracks may or may not be returned to service, depending on whether it exceeds a predefined variable \( a_{\text{repair}} \), as shown in Figure 7.4. This value can be changed within the program to model the effect of allowing turbine blades to remain in service with a certain subcritical crack size; if it can be shown that a detected crack will not grow too close to a critical value before the next overhaul, it may be acceptable to forego repair during the current inspection. Here it should be pointed out that, depending on the definitions of \( a_{\text{repair}} \) and the POD curve, it may be entirely possible that a crack large enough to warrant repair or replacement of the blade could go undetected, and this could lead to a rare
in-service failure event. It is precisely this type of risk that this program was developed to predict. Finally, repair or replacement of a blade with a detected crack is simulated by resetting the crack size to a new initial flaw size, determined by either the EIFS distribution for the given engine type (as it was at the outset), or for a repaired blade, a new EIFS specific to a repaired blade. At this stage, however, it is unknown exactly what the characteristics of such an EIFS distribution would be, and it is quite likely that a new stress intensity factor formula would have to be defined for a repaired blade (for example, in the case of a blade with a small crack repaired by grinding a small filleted region from the trailing edge). Therefore, the current simulation simply replaces a retired blade with a new blade, generating a new flaw size from the engine’s EIFS distribution.

7.2.6 Overall Crack Growth Simulation Algorithm

A flowchart describing the crack growth simulation algorithm, including the LEFM and creep-based crack growth, blade inspection, and checks for blade failure is shown in Figure 7.4. The failure event is defined by $K_{\text{max}} > K_c$, where $K_{\text{max}}$ is determined by the crack size and the relevant stress intensity function for the blade design, and $K_c$ is the fracture toughness, modeled as a normally distributed random variable. If a blade failure does occur during simulation, a counter is incremented for use later if the user wishes to determine $P_f$ based on the number of blades simulated, but the simulations of the remaining blades in the engine continue, so that crack size and life distributions can still be determined for the pre-failure condition. The final value of the crack size in the failed blade is multiplied by 10 prior to storage, however, so that if it is inadvertently used in post-processing, it will stand out visually in any crack growth curves.

The top-level organization of the entire code is shown in Figure 7.5. The number of
Figure 7.4: Flowchart describing cycle-by-cycle crack growth modeling
blade simulations performed is dictated by two user-controlled variables: the number of blades per engine, $N_{\text{blades}}$, and the number of engines simulated, $N_{\text{sim}}$. During the simulation, a raw, unformatted data file is created in the .\temp directory which contains the engine number, blade number, time, cycles, and crack size for every blade simulated. For MC estimation of $P_f$, it will be necessary to run thousands of engine simulations, in which case the amount of raw data generated for crack growth curves would be quite cumbersome, and the primary variable of interest would just be the number of failures, $n_{\text{fail}}$, divided by the number of simulations. Otherwise, to produce data suitable for analyzing crack size, $a(N)$ or $d_t$, or time to crack size, $t(a)$ or $N(a)$, distributions, a post-processing subroutine is included for generating the desired output from the raw data files by specifying a time $t$ (or cycles $N$), or a target crack size $a$. The requested data is then stored in a formatted data file in the working directory for importation into a spreadsheet or statistical analysis program.

7.3 CRACK PROPAGATION MODELING RESULTS

A wide range of variables can be manipulated statistically in the MC simulation algorithm developed, producing countless combinations of different material and operational conditions. A few basic variations have been simulated for comparison here in order to demonstrate some of the potential results of the model.

7.3.1 Operating Mode

One of the most significant variables between engines is whether it operates as a base unit or a cyclic unit, as discussed in Chapter 2. While cyclic and base units may accumulate a similar number of operating hours between inspection intervals — 900 cycles at roughly 30 hours each for cyclic units, coming to 27,000, and specified intervals of 24,000 hours for base units, accumulating only 50 cycles on average — the cyclic unit is subjected to many


**Figure 7.5:** Top-level flowchart of life prediction algorithm.
more load cycles and more LCF damage. Creep crack growth may become a significant cause of damage for a base unit. Two simulations of an engine, one in cyclic and one in base operation, are shown in Figure 7.6; the initial conditions (including the EIFS distribution for the 7FA frame) and operating loads are the same for both engines. Damage accumulation appears less severe in the base unit in Figure 7.6(a) simply because, when plotted against operating hours, the base unit simply has not accumulated many operating cycles. Plotted against cycles, the two engines appear quite similar; in this case, damage from CCG does not appear very significant, the FCGR for the base engine seeming only slightly elevated compared to the cyclic unit.

Examining the base loaded engine more closely in Figure 7.7, it appears fairly unremarkable. The evolution of scatter in the crack size data does not change markedly from its random initial conditions set by the EIFS distribution, largely due to the minimal accumulation of cycles (and subsequently, the fact that the cracks likely have not passed a single grain boundary) over several thousand hours. Recall that at this stage of modeling, CCG was modeled quite simply as a $K$-correlated crack growth rate using average values of the regression coefficients, $C_n$ and $m_b$, determined by Wilson [101] for the CCG contribution to total crack growth,

$$\frac{da}{dt} = C_nK^{m_b}$$  \hspace{1cm} (7.3)

Note also that since the low cyclic frequency does make CCG a significant contributor to the crack growth in Figure 7.7, the individual $a-N$ curves appear very similar to each other. This is a result of the fact that, at this stage of the model, CCG is a non-stochastic phenomenon.

On the other hand, the cycle-driven damage of the cyclic unit, shown in Figure 7.8, reveals a good degree of the variability in crack growth as a result of the stochastic FCP law.
Figure 7.6: Simulations of cyclic and base units, as a function of (a) operating hours, (b) engine starts
Figure 7.7: Partial sample of individual $a-N$ curves for base loaded engine

employed in the model. Note the intermingling of $a-N$ curves, driven in part by smaller initial flaws growing in less FCP-resistant grains, and in part by changes in FCP properties as initially similar curves pass into different grains. Such intermingling has been cited as a fair indicator of realistic modeling of the phenomena in stochastic crack growth [83].

7.3.2 Influence of Initial Flaw Size

The EIFS distributions generated for each frame by Wilson [101] used a deterministic crack growth equation to grow measured cracks "backwards," thus producing the equivalent initial cracks that serve as initial conditions for this model. Of course, in service, the material from blade to blade (and within each blade) will be non-homogeneous, so use of this crack growth model will apply the effects of random crack growth properties in addition to the random initial flaw size. To study the effects of EIFS distribution characteristics,
four simulations of cyclic engines were run, incrementally changing the initial conditions from the EIFS of a 7FA frame ($\mu = 0.202\text{mm}, \sigma = 0.097\text{mm}$) to that of a 7FA+ frame ($\mu = 0.04\text{mm}, \sigma = 0.009\text{mm}$). As these represent, respectively, a higher mean flaw size with a higher coefficient of variation (CV = 0.48) and a lower mean flaw size with a lower CV (0.23), the intermediate "steps" simulated a higher mean flaw size with a lower CV, and conversely a lower mean flaw size with a higher CV. In this manner, the influence of the two can be examined separately. The results are calculated according to the crack size distribution at 2500 cycles (shortly before the third inspection).

The effects on mean, standard deviation, and CV in crack size distribution are presented in Table 7.1 against the same characteristics of the EIFS distribution. The resultant distributions were determined from 20 engine simulations for each condition, or 1840 simulated blades. The resulting distributions are also presented graphically in Figure 7.9. It is inter-
Table 7.1: Statistical influences of EIFS parameters

<table>
<thead>
<tr>
<th></th>
<th>CV = 0.23</th>
<th>CV = 0.48</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{a}_{i} = 0.04 \text{ mm} )</td>
<td>( \mu = 0.526 \text{ mm} )</td>
<td>( \mu = 0.525 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.059 \text{ mm} )</td>
<td>( \sigma = 0.07 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( CV = 0.11 )</td>
<td>( CV = 0.13 )</td>
</tr>
<tr>
<td>( \bar{a}_{i} = 0.202 \text{ mm} )</td>
<td>( \mu = 0.871 \text{ mm} )</td>
<td>( \mu = 0.865 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.143 \text{ mm} )</td>
<td>( \sigma = 0.214 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( CV = 0.16 )</td>
<td>( CV = 0.25 )</td>
</tr>
</tbody>
</table>

Figure 7.9: Resultant crack size distributions from variations in EIFS distribution
esting to note that, while an increase in initial flaw CV resulted in an expected increase in 
\( a(N) \) CV — and this is visually evident in moderately small probability levels in Figure 7.9 — it is actually the lower CV distribution for both cases that produces the largest “rogue” crack at the right tails. The increase in CV is also noticeably more significant for the higher mean crack size case, but the physical (or modeling) mechanism behind this is not certain.

7.3.3 Role of Grain Structure

The role of the grain-based approach to variability was examined through three simulations that represented fundamentally different microstructure characteristics. The first simulation was of a standard cyclic engine using the 7FA EIFS distribution. The second simulation had the intergranular variation in FCGR eliminated, ensuring \( \sigma_n^* \) of Figure 7.3 was zero setting \( \sigma_n^* \) and \( \sigma_o^* \) equal to zero in the code preamble. The overall level of variability of the experimental data was maintained, however, by setting \( \sigma_{jn} = 0.46 \), the variability of the entire three-slab set of data in Table 6.3. Thus, the interspecimen variability of the data (irrespective of the originating casting) became the entire blade-to-blade variability of the simulation. Finally, effect of the coarse grain structure on the material inhomogeneity was examined by producing a much finer grain structure (\( \mu_D = 0.5 \text{mm}, \sigma_D = 0 \)) with the same intergranular variation in FCP resistance as the standard simulation. It was hypothesized that as the crack grew much larger than the scale of variation, the dispersion in crack size would reduce due to “microstructural averaging” [79].

The results are presented as crack size distributions at 12,500 cycles in Figure 7.10. These distributions were calculated from five engine simulations, or 460 blades. One notable feature is the significant “dip” in the center of the baseline distribution; while it may
Figure 7.10: Crack size distributions at 12,500 cycles for three different microstructures

be an artifact of simulation from insufficient sampling, it also appears in the second simulation, suggesting it may be something inherent in the structure of the data or simulation. Comparing the two cases, in which scatter is in large part intergranular (baseline) or entirely blade-to-blade (second case), it is clear, interestingly, that the effect of the intergranular variation in FCGR is actually to reduce the spread in crack size data over long crack lives. This might be expected, since the likelihood of several grains in a row having consistently high or low FCP resistance is very small; the change in properties through the microstructure actually tends to "correct" itself back towards the mean behavior. Finally, this corrective effect is most obvious in the final simulation, in which the same level of variation on a much finer scale (i.e., sampled an order of magnitude more times over the life of the crack) essentially reduces the "sampling error" of properties at the crack tip. After covering the span of many more grains, the resultant distribution is highly concentrated
and very smooth.

7.4 SUMMARY

The current model covers random effects in many aspects of turbine blade life prediction, from microstructural to operational variables. While fairly simple in concept and structure, it is flexible and robust, able to simulate any number of changes that may be under the designer’s control to assess the impact on damage evolution and risk. The challenge now lies, of course, in more thorough experimental study of all of the inputs into the model in order to ensure the accuracy of the output.
CHAPTER 8
CONCLUSIONS

The purpose of this research project was two-fold: to investigate the FCP properties of DS GTD-111 superalloy in order to provide data where previously there was a notable void; and to propose an approach to probabilistic life prediction of a specific material degradation issue in fielded components. Extensive mechanical testing was performed and a number of notable observations have been made about the behavior of the system:

• A heterogeneous material such as the coarse-grained, orthotropic DS superalloy can display significant variability in fatigue crack growth rate from specimen to specimen. Since the specimens used were cut in stacks from large castings, the inter-specimen scatter from a single casting essentially represents a spatial variation in FCP resistance.

• DS GTD-111 displays an accelerated FCGR at RT as compared to ET. This could be explained by either or both of two phenomena: increased crack growth increment as a result of decreased fracture toughness, or increased sensitivity to strain rate, at lower temperature; or a reduction of effective ΔK at elevated temperature as a result of fracture surface oxidation effects, causing oxide-induced closure at the crack tip. Without more specific thermomechanical testing, it is not clear at this time precisely which mechanism is controlling. However, the fact that fatigue damage at RT tends to occur by single slip along preferential crystallographic planes and not by cleavage normal to the maximum stress point toward the latter mechanism being more likely.
The high sensitivity to thermal excursions and tendency to arrest after a thermal cycle support this suggestion.

- Closure plays a significant role in FCP at ET, as evidenced by the marked shift in crack growth curves as R-ratio changes. The effect of closure, when combined with a change in constraint conditions at the crack tip, may actually change the crack growth properties between center crack and edge crack specimens, as suggested from current experience between the M(T) and EC(T) configurations.

- The specimen free surface intercept of the fracture surface can display a change in crack path across the specimen, particularly at a grain boundary. While through-thickness morphology may not follow the same trend to such an extent as to require a more detailed analysis of deflected stress intensity factors, it is nonetheless indicative of a change in the deformation behavior at the crack tip between grains.

The experimental data and a study of the literature guided the development of a simple but useful probabilistic crack growth model based on random initial flaw sizes and FCP resistance. A number of simulations of fatigue cracks in IGTs revealed the following:

- Based on the results of two concurrent research projects in probabilistic fracture mechanics approaches to life prediction, including the effects of engine hold time at temperature, it appears that creep crack growth is not a significant driver of fatigue cracking. This observation is based on forward simulations using the influence coefficients for time-dependent cracking derived from regression of the field data.

- The effect of variance in initial flaw size on the scatter in crack size distributions is proportionately greater for a larger average initial flaw. When the mean initial
flaw size was nearly an order of magnitude smaller, the effect of increased COV was marginal compared to the effect on the larger flaw.

- A spatially varying inhomogeneity in FCP resistance actually serves to reduce the variance in the distribution of large cracks. Re-sampling from the stochastic material properties over a number of grains tends to naturally correct, making the continuation of markedly faster or slower crack growth statistically unlikely and driving the system behavior toward the mean. By the same token, a finer grain structure increases this sampling rate and can result in an even narrower crack size distribution.
CHAPTER 9

RECOMMENDED FUTURE WORK

This project has represented a first step for this research group into the realm of probabilistic fracture mechanics, and was conceived as the first phase of an extended study into the life prediction and risk management of IGT turbine blades. Many characteristics of the alloy studied have been explored and discussed, and a general methodology for using scattered experimental data in a probabilistic life prediction model has been presented. It is to be expected that each discovery can suggest a new area of investigation, and with that in mind a number of recommendations for future research are proposed herein.

9.1 MATERIAL CHARACTERIZATION

9.1.1 Mechanical Testing

All of the FCG testing performed for this project was conducted under isothermal conditions, primarily at the elevated temperature most relevant to the case-specific cracking observed in the engines. A fairly extensive test matrix was proposed for the overall research effort (including work to be done outside of Georgia Tech), but even the tests planned to study hold time effects on crack growth rate are still isothermal. In reality, the loading at the tip of a crack in a turbine blade during a startup-run-shutdown-cool cycle is very complex, and involves thermomechanical stresses brought on by the combined load and temperature excursions. In addition, some unusual arresting behavior related to thermal excursions were discussed in Chapter 6, supporting the notion that the thermal cycle of an engine startup and shutdown is as important as the centrifugal load. Detailed TMF testing
that simulates the entire strain-temperature profile of engine operation should be performed for the different types of engine cycle, so that the effects of randomly distributed startups and trips can be accurately modeled.

The most surprising and disappointing result of the experimental work was the large discrepancy between the M(T) and EC(T) specimen data, despite significant efforts to compensate through FEM analysis of the modified specimen and verification of the load frame's measurements. In discussions with Dr. Jim Newman, who was part of extensive work on the specimen configuration at NASA, it was mentioned that the significant difference in crack tip constraint between the two specimens is still a concern. In light of the relatively rough fracture surface, oxidation, and closure effects observed in this material, it is likely that any change in loading conditions may have adversely affected the results. What this suggests about the applicability of the stress intensity factor between specimens at most conditions is unclear and cause for concern if verified, but an entire study on cross-specimen effects in FCP behavior would be justifiable and widely useful.

9.1.2 Metallography/Fractography

Detailed microstructural or fractographic characterization of the material was not performed. Indeed, the extant literature on the applicability of fracture surface analysis to prediction of FCGR presents mixed opinions. However, in order to further justify a microstructure-based approach to modeling random crack growth, more quantitative relationships between grain structure, crack path selection, and observed crack growth rate should be determined if possible. In the presence of accurate FCGR measurements, it should be possible with destructive metallography — sectioning a number of planes through the specimen — to characterize not only the grain structure through the specimen (and behind the observed surface
crack path), but the 3-D morphology of the fracture surface as it changes across the microstructure. Material investigation into the oxide-induced thermal “coaxing” phenomenon could also be assisted by metallographic and scanning electron microscopy studies of specimens that are subject to thermal cycles, both in cracked and uncracked configurations.

9.2 COMPUTER MODELING

As mentioned in Chapter 7, there almost seems to be no limit to the number of variables that can be used to model a complex component’s response to even more complex environmental and service inputs; nor are there any a priori clear criteria for which of those variables should be deterministic and which random variables. Continued development of the fatigue crack simulation program is planned, including (most pressing) the addition of other, non-normal distribution functions. Other proposed enhancements to the model are as follows.

9.2.1 Microstructure Evolution

The effects of time in the current model are handled only through per-cycle creep crack growth, and this only encompasses the damage occurring in the local crack tip region. However, as the total number of operating hours increases for the turbine bucket, a cumulative effect of time at temperature causes a microstructural degradation throughout the material, even remote from the crack tip region. As stated in Chapter 2, prolonged exposure to the operating temperatures and stresses of the first stage bucket can cause coarsening of the \( \gamma' \) precipitates (one indication of creep damage). It was also discussed how coarsening of the \( \gamma' \) can, in turn, cause a decrease in resistance to both creep and fatigue crack growth [23, 27]. The parameters which govern crack growth (the only damage function) in this model, however, are selected at time zero for each simulated blade, and do not evolve with
time, thus implicitly assuming no general time-dependent damage to the material outside of

 crack propagation. A future release of this model should incorporate the ability to predict

 the microstructural evolution undergone by the turbine bucket due to total exposure time

 and represent the subsequent effects on the crack propagation laws.

 9.2.2 Mechanistic Enhancement

 A number of simplifying assumptions have been made at this stage, and more rigorous
 mechanical simulation of the crack propagation itself will be implemented in the future.
 The current creep crack growth algorithm is a very rudimentary, LEFM-based equation.
 Significant work has been done in this department on characterizing CCG in this mate-
 rial, and a new subroutine should be developed to model this source of damage in a more
 accurate and physically justifiable manner.

 The synergistic effects of creep and fatigue cracking also need to be included to ac-
 curately model thermo-mechanical events. In some cases, hold times have actually been
 shown to retard crack growth at ET [46]; in such a case, it would be rather ad hoc to in-
 clude a negative creep crack growth increment to account for this. Rather, the crack size
 variable should be rendered multidimensional in order to carry with it information about
 current crack tip conditions brought about by creep conditions, such as crack tip blunting
 for far-field creep strains for example.

 Similarly, a multidimensional crack size variable could also include the effects of load
 history on LEFM-based damage. The most significant variable missing from the current
 model is the trip or overload event, and in order to accurately represent such occurrences
 with closure or plasticity-based models, some parameter describing, for example, the end
 of the overload yield zone will be necessary.

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REFERENCES


