Project #: E-25-M29  Cost share #: E-25-322  Rev #: 18
Center #: 10/24-6-R7239-0A0  Center shr #: 10/22-1-F7239-0A0  OCA file #:
Contract#: N00014-91-J-1888  Mod #: P00009  Work type : RES
Prime #:  Document : GRANT
Subprojects ? : Y  Contract entity: GTRC
Main project #:

Project unit: MECH ENGR  Unit code: 02.010.126  CFDA: 12.AAA
Project director(s): BERTHELOT Y H  MECH ENGR  (404)894-7482
JARZYNSKI J  MECH ENGR  (404)-

Sponsor/division names: NAVY  Sponsor/division codes: 103  / OFC OF NAVAL RESEARCH
Sponsor amount  / 025
New this change Total to date
Contract value (0.10) 723,944.90
Funded 88,595.90 723,944.90
Cost sharing amount 34,453.00

Does subcontracting plan apply ?: N

Title: LASER IMAGING OF THE VIBRATIONS OF SUBMERGED ELASTIC SHELLS

PROJECT ADMINISTRATION DATA

OCA contact: Jacquelyn L. Bendall 894-4820

Sponsor technical contact  Sponsor issuing office
PHILLIP B. ABRAHAM  GAIL BOGER
(703)696-2586

CODE 1132SM  CODE 1513:GDB
OFFICE OF NAVAL RESEARCH  OFFICE OF NAVAL RESEARCH
800 NORTH QUINCY STREET  800 NORTH QUINCY STREET
ARLINGTON, VA 22217-5000  ARLINGTON, VA 22217-5000

Security class (U,C,S,T,S) : U  ONR resident rep. is ACO (Y/N): Y
Defense priority rating : N/A  ONR supplemental sheet
Equipment title vests with: Sponsor  GIT X

Administrative comments - MODIFICATION NO. P00009 ADDS FUNDING IN THE AMOUNT OF $88,595.90. FUNDS WILL BE BUDGETED IN SUBPROJECT E-25-X75.
NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 01/03/97

Project No. E-25-M29
Project Director BERTHELOT Y H
School/Lab MECH ENGR
Center No. 10/24-6-R7239-0A0
Sponsor NAVY/OFC OF NAVAL RESEARCH
Contract/Grant No. N00014-91-J-1888
Contract Entity GTRC

Prime Contract No. __________________________
Title LASER IMAGING OF THE VIBRATIONS OF SUBMERGED ELASTIC SHELLS
Effective Completion Date 961231 (Performance) 961231 (Reports)

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Comments:_________________________________________________________________

Subproject Under Main Project No. __________
Continues Project No. __________

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NOTE: Final Patent Questionnaire sent to PDPI.
LETTER REPORT TO ONR
Under Contract N00014-91-J-1888

Period of 07/01/91 to 10/31/91

General Information:

- Project title: Laser Imaging of the Vibrations of Submerged Elastic Shells.
- Scientific Officer: Dr. Phillip B. Abraham, Code 1132SM
- Principal Investigator: Dr. Yves H. Berthelot, Associate Professor (404) 894-7482
- Co-P. Investigator: Dr. Jacek Jarzynski, Professor (404) 894-7479
- Graduate Student: Hyun-Gwon Kil, Ph. D. Candidate
- Institution: Georgia Institute of Technology School of Mechanical Engineering
- Starting Date: 07/01/91

(a) Description of Research Goals:

The main thrust of the project is to develop optical techniques in structural acoustics. This contract is a follow-up on a previous contract (N00014-88-K-0431) during which a fiber optic laser interferometer was developed for measurements of structural vibrations in air. The two specific objectives under the present contract are:

- To develop a prototype fiber optic laser probe for underwater measurements of in-plane motion of the vibrating structure. Specific improvements of the probe over the one previously developed for measurements in air will be its smaller size, increased ruggedness, capability to make underwater measurements, and improved automation of the scan around the (more complex, e.g. ribs) structure.

- To develop a prototype laser probe for simultaneous measurements in air of out-of-plane surface motion at several location. Such a probe could be used as a non-invasive sensor for active vibration control. The probe could become a very practical tool in industrial noise control (e.g., machinery noise control, or fuselage vibration control in aircrafts, etc...).
(b) Significant Results in the Past Year:

- The laser probe designed under contract N00014-88-K-0431 for measurements in air has been improved (smaller size, increase ruggedness) and adapted for underwater measurements of in-plane vibrations.

(c) Plans for Next Year’s Research:

- Make measurements with the new probe in a water tank with a cylindrical shell excited by a shaker.

- Compare with numerical predictions

- Select another graduate student (possibly supported under the DoD/ASERT program) to work on the laser measurement technique in air, with an emphasis on simultaneous measurements at several points of out-of-plane structural vibration.
LETTER REPORT TO ONR  
Under Contract N00014-91-J-1888

Period of 10/01/92 to 09/30/93

General Information:
- Project title: Laser Imaging of Shell Vibrations
- Scientific Officer: Dr. Phillip B. Abraham, Code 332SM
- Principal Investigator: Dr. Yves H. Berthelot, Associate Professor (404) 894-7482
- Co-I. Investigator: Dr. Jacek Jarzynski, Professor (404) 894-7479
- Graduate Student: Hyun-Gwon Kil, Ph. D. Candidate
  Ming Yang, Ph. D. Candidate
  F. Guillot, M.S. Candidate
- Institution: Georgia Institute of Technology
  School of Mechanical Engineering
- Starting Date: 10/01/91

(a) Description of Research Goals:

This contract is a follow-up on a previous contract (N00014-88-K-0431) during which a fiber optic laser interferometer was developed for measurements of structural vibrations in air. The two specific objectives under the present contract are:

- To automate the scanning and focusing of the system so that extensive data can be gathered on shells.

- To develop a prototype laser probe for simultaneous measurements in air of out-of plane and in-plane motion at 4 points on the vibrating surface of the shell. Such a probe could be used as a non-invasive sensor for active vibration control in structural acoustics.
(b) Significant Results in the Past Year:

- A complete set of measurements has been taken on a freely suspended cylindrical shell with the fully automated laser system (measurements of in-plane surface motion). The data has been analyzed with an extended Prony method and, for the first time, longitudinal and shear waves as well as evanescent waves have been identified by direct non-contact measurements. Also, mode conversion at the free end of the shell has been observed.

- A new probe has been assembled to measure simultaneously in-plane and out-of-plane surface displacements on the shell. Preliminary tests on a piezoelectric cylinder are in good agreement with theoretical predictions of surface motion. Several critical issues have been identified: focusing on a few bright speckles, simultaneous alignment of the laser beams for in-plane and out-of-plane detection, and miniaturization of the probe heads.

- A 4 channel fast A/D converter has been tested on simulated data for direct digital signal detection (instead of the phase-locked loops being currently used) by means of a digital Hilbert transform (DHT).

- A joint research program with the Naval Surface Warfare Center (Ed Balizer) has been established to determine the physical mechanism of the giant electrostriction observed in some polymers. Samples with known structural differences will be synthesized at NSWC and the electrostrictive response of the samples will be measured at Georgia Tech using laser Doppler interferometry.

(c) Plans for Next Year's Research:

- Finish the measurements and analysis of evanescent waves on the cylindrical shell. In particular, study the structural field near the shaker and near the boundaries, and possibly near a rib. Study mode conversion.

- Continue the development of the vector laser probe so that it can measure simultaneously surface motion at several points (3 points, possibly 4). Optimize the system: Study the effects of the angle of incidence on the surface (number of bright speckles).

- Test the all-digital Hilbert transform demodulation technique on some experimental data.

- Establish the material properties of the polymers of interest with the NSWC collaboration.
OFFICE OF NAVAL RESEARCH
PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS REPORT
01 October 1992 through 30 September 1993

R&T Number: 432 p 006

Contract/Grant Title: N0001491J1888

Scientific Officer: Abraham

Principal Investigator: Berthelot

Mailing Address: School of Mechanical Engineering, Georgia Tech., Atlanta, GA 30332

Phone Number: (404) 894-7482

FAX Number: (404) 894-7790

E-Mail Address: yves.berthelot@me.gatech.edu

a. Number of Papers Submitted to Referred Journal but not yet published: 0

b. Number of Papers Published in Referred Journals: (List Attached): 1

c. Number of Books or Chapters Submitted but not yet Published: 0

d. Number of Books or Chapters Published (List Attached): 0

e. Number of Printed Technical Reports & Non-Referred Papers (List Attached): 1

f. Number of Patents Filed: 0

g. Number of Patents Granted (List Attached): 0

h. Number of Invited Presentations at Workshops or Professional Society Meeting (List Attached): 0

i. Number of Presentations at Workshops or Professional Society Meetings (List Attached): 2

j. Honors/Awards/Prizes for Contract/Grant Employees: (List Attached, may include Society Awards/Offices, Promotions, Faculty Awards/Offices, etc.) 1

(*) Numbers reflect activity pertinent to contract N0001491J1888 only.
k. Providing the following information will assist with statistical purposes.

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1. Degrees Granted (List Attached): 1

* Underrepresented or minority groups include Blacks, Hispanics, and Native Americans. Asians are not considered an underrepresented or minority group in science and engineering.

** Supported at least 25% this year on contract/grant.
TECHNOLOGY TRANSFER

Technology transfer is an important measure of the relevance of scientific endeavors. ONR scientific officers need to be aware of any such transfer, and they will use it to the benefit of their programs. Please describe any recent (approximately last three years) direct or indirect interactions you had with Navy, other DoD, or industrial scientists and engineers; describe only those interactions that resulted in their use of methodology, data, software, or other developments produced or directly derived from your ONR grant/contract. Also describe similar technology transfer, if any, that resulted without any such interactions.

1) Naval Research Laboratory
   Development of a 3D Laser Doppler System for Structural Acoustics. [B. Houston/J. Vignola]

2) Naval Surface Weapons Center
   Joint investigation of the material properties of some polymers by laser interferometry [with Ed. Balizer].

Enclosure (2)
LIST OF PUBLICATIONS/REPORTS/PATENTS/GRADUATES

1. Papers Published in Referred Journals:


2. Books (and sections thereof) Published:

None

3. Technical Report, Non-Refereed Papers:


4. Presentations:


5. Patents Granted:

Enclosure (3)

6. Degrees Granted (name, date, degree):

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<td>Y. Berthelot</td>
<td>Georgia Tech</td>
<td>Woodruff Faculty Fellow</td>
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Enclosure (4)
OTHER SPONSORED RESEARCH
(Include title, sponsors's name, dollar amount and start and end dates for the award)


Enclosure (5)
FUNDING BALANCE

Please indicate the remaining ONR grant/contract resources you have in your institution as of 30 SEP 93. This information is important to the Scientific Officer's planning process and to ONR tracking of expenditure rates.

$124,021 including the AASERT award for 1993-1996.

Enclosure (6)
Dear Phil:

Enclosed with this letter is the viewgraph that you requested when we spoke over the telephone last week. It should convey the main features of the proposed 3 dimensional laser probe.

A laser beam is split into 5 beams and frequency shifted 5 times by 5 Acousto-optic (AO) cells. The frequency shift is usually of the order of 40 MHz. The 5 beams are focused to a small spot on the vibrating surface. The surface is rough and light is scattered in all directions producing an interference pattern (due to the phase changes caused by the roughness). Such a pattern is called a speckle pattern. A large core multimode fiber collects some of the scattered light. At the multimode fiber, beating occurs at the sum and difference optical frequencies of each of the 5 beams. The bandwidth of the photodiode is limited so that only frequencies in the hundreds of MHz or below are picked up. If we select the 5 frequencies to be 40,000, 40,100, 40,500, 40,700, and 40,150 MHz, the beat frequencies of interest are 100, 150, 200, 350, 400, 550, 600, and 700 kHz. Of those beat frequencies, it turns out that only the first three are of interest. From geometrical considerations, it can be seen that the beat between beam 1 and 2 (see figure) occurs at 100 kHz and serves as the carrier of the phase signal corresponding to the in-plane motion in the y direction. Similarly, the beat between beam 3 and 4 (see figure) occurs at 200 kHz and serves as the carrier for the phase signal corresponding to the in-plane motion in the x direction. The beat between beam 1 and 5 occurs at 150 kHz. The geometry is more involved and it can be shown that the 150 kHz component serves as a carrier for a combination of phase changes due to in-plane motion in the y direction (which is known) and of phase changes due to out-of-plane motion. Proper signal processing techniques exist to demodulate such signals and extract information about the 3D vector vibration of the surface.
Let me also discuss some of the main issues in my view regarding such a device. First, is the question of novelty. Although 3D laser probes are commercially available, they are designed to measure turbulence in fluids and not structural waves. The two problems are quite different and require different type of measurement systems.

Second, is the question of compactness, ruggedness, and practicality. Ideally, one would want to use a miniature diode diode laser. However, to get a good signal, one needs roughly 1 milliwatt of optical power at the output of each of the five beams illuminating the surface. Losses occur also at the AO (Acousto-optic Bragg cells) and elsewhere in the system so that the required laser power should be at least of the order of 50 milliwatts. In addition, the coherence length should be greater than 10 centimeters or so (i.e., greater than the path difference between any two light paths to the photodiode). The point is that the technology for such a diode laser is not there at present or it is really pushing the limit. Driving 5 AO-cells is not a trivial matter for compactness. Each cell requires an extremely stable quartz oscillator. All crystal oscillators require a power supply. It becomes evident that it is not a trivial matter to put the entire system of diode laser+ 5 AO cells + 5 crystal oscillators, + power supplies, 5 lenses, + detection lens + photodiode, all on a single unit close to the vibrating surface. The idea I am trying to convey is that it is technologically feasible but certainly challenging. Trying to detect one of the in-plane component and the vertical component requires “only” 3 AO cells and it is easier, albeit not trivial.

Thirdly is the idea of remote control for focusing since the probe will presumably be located at some inaccessible points. A major problem in the design of the laser probe is the focusing of the laser beams. Alignment and focusing are critical. A major thrust of our research effort at Georgia Tech has been to alleviate that problem by designing a probe “pre-aligned” in 2 directions so that good alignment can be achieved with only a single axis automated system. The system works at present when measuring the in-plane motion only (our design with a single AO cell). An important task before us is to see how the system will perform when trying to measure both in-plane and out-of-plane motion simultaneously. It may turn out that a single axis alignment mechanism is insufficient. This would seriously complicate the design of a workable probe capable of measuring 1 in-plane and 1 out-of-plane component simultaneously. It would also make it virtually impossible to design a simple and rugged probe with 5 AO cells capable of imaging 2 in-plane and 1 out-of-plane component. It is necessary to proceed slowly and understand the limitations of the system before designing the 5 AO cell laser probe.
I hope that this letter will help you in presenting the issues facing the design of a laser Doppler probe for structural acoustics. It is an exciting area of research which has, I believe, great potential. There is no doubt that the technology for optics, electro-optics, and acousto-optics is growing very rapidly especially in the field of miniaturization and high power semiconductor lasers and these progress will continue to have a direct impact on this avenue of research.

Sincerely yours,

Yves H. Berthelot
OPTICAL BENCH ARRANGEMENT

Power Supply

Crystal Oscillator 40.0 MHz

AO cell

Crystal Oscillator 40.1 MHz

AO cell

Crystal Oscillator 40.3 MHz

AO cell

Fiber Coupler

Fiber Coupler

Fiber Coupler

Single made fibers

Tree coupler

Tree coupler

Tree coupler

4 probe heads
NON-INTRUSIVE LASER MONITORING OF VIBRATIONS OF ELASTIC SHELLS

OBJECTIVES:

- Measure simultaneously both in-plane/out-of-plane surface wave motion.
- Measure (CW or transients) simultaneously at several locations on the surface with an array of optical probes.
- Measure evanescent waves and mode conversion near a rib.

Overall objective: Combine laser sensor with active control device to reduce sound radiation from vibrating structures.

ACCOMPLISHMENTS:

- Laser probe is operational for in-plane measurements (>1nm). The probe was validated with measurement and wave-vector data analysis for a cylindrical shell (detection of shear/flexural waves).
- Automation of the laser probe focusing and scanning.
- A probe array (simultaneously in-plane/out-of-plane measurements at several locations) has been designed and is being built and tested.

APPROACH

- Use Laser Doppler interferometry (differential heterodyne) for high-resolution.
- Use optical fibers for rugged and compact probes.
- Use automated focusing and scanning around the shell for extensive data collection.
- Validate measurements on a simple structure.
- Perform digital FM demodulation of the signals.
Application

- Basic studies of propagation of elastic waves in submerged shells with structural discontinuities such as ribs and added mass

![Diagram of a fluid-loaded cylindrical shell with structural discontinuities such as ribs and added mass. The diagram includes symbols for evanescent waves and mode conversion.]

Fig. 1 A Fluid-Loaded Cylindrical Shell with Structural Discontinuities such as Ribs and Added Mass

- Active Vibration Control
  (← Wave Number Spectral Analysis)
Experimental Set-Up

Optical Set-Up

- Differential Laser Doppler Technique for Measurement of In-Plane Motion
- Heterodyne Detection Scheme (Using Bragg Cell)
- Compact Laser Probe
  - Projecting Optics — GRIN Lens & Single Mode Fiber
  - Receiving Optics — Multimode Optical Fiber
    (Core Diameter: 1000 μm)

Fig. 2 Optical Set-Up
Automatic Focusing of Two Laser Beams

Control of horizontal movement of a laser probe

Motor Controller
System
Computer
Interface Board (GPIB)

Oscilloscope
Ch 1.
Ch 2.

Acoustic Signal

FM Detection (PLL)

100 KHz FM Signal

Mixer
40 MHz FM Signal

40.1 MHz Signal

Photodiode

Crystal Oscillator

Stable Acoustic Signal

when an amplitude of 100 KHz FM signal obtained from an oscilloscope is largest
WAVE-VECTOR ANALYSIS OF SHELL VIBRATIONS
FROM LDV MEASUREMENTS OF IN-PLANE MOTION
AT \( f = 15,930 \) Hz. (PRONY & FFT ALGORITHMS)
MULTIPLE HEAD LASER PROBE FOR STRUCTURAL ACOUSTICS

40.1 MHz FM signal  
40.3 MHz FM signal  

40.0 MHz FM signal  

single mode optical fibers

Photodiode

large core multimode fiber

1D-alignment

Vibrating Surface
DATA ACQUISITION AND PROCESSING

receiving photodiodes

1
2
3
4

FM signals with carriers at 100, 200, 300 kHz

A/D
25 MHZ

COMPUTER

Digital Filtering

Hilbert Transform Demodulation

Output:

IN-PLANE MOTION
OUT-OF-PLANE MOTION

(separate each carrier)
ONR CONTRACT INFORMATION

Contract Title: Laser Doppler Interferometry for Structural Acoustics and Material Characterization

Performing Organization: Georgia Institute of Technology

Principal Investigator: Yves H. Berthelot and Jacek Jarzynski

Contract Number: N00014-91-J-1888 and N00014-94-1-0947

R & T Project Number: 432 p 006

ONR Scientific Officer: Dr. Ph. B. Abraham and Dr. G. L. Main

ONR FY94 End of Fiscal Year Letter
(01 Oct 1993 - 30 Sep 1994)
Enclosure (1)
A. Description of the Scientific Research Goals

The main objective of the research program is to develop a versatile laboratory for laser Doppler interferometric measurements relevant in structural acoustics. Particular attention is placed on the detection of in-plane vibrations because of the importance of shear and longitudinal waves on acoustic radiation from submerged structures as well as the importance of mode conversion at discontinuities such as bulkheads and end caps.

The applications of the research can be divided into two groups:

(1) direct imaging of elastic waves in complex structures to identify sources and sinks of structural acoustic energy;

(2) characterization of the response of some new materials of interest in naval applications.

The research effort is divided into four on-going projects (2 in each group):

(1a) wave propagation in cylindrical shells. Automated scanning laser system for detection of longitudinal, shear, flexural, and evanescent waves.

(1b) simultaneous measurements of in-plane and out-of-plane surface motion at 4 points. Time domain measurements and data analysis.

(2a) Characterization of the response of polyurethane films known to exhibit "giant electrostrictive" properties. (Partially supported by NSWC)

(2b) Characterization of the response of SHT coating materials. (Work done in conjunction with NRL-USRD)

Technical Approach

Project #1a:

A fully automated laser Doppler interferometer has been designed and built to measure in-plane surface motion on a cylindrical shell excited radially by a shaker. The laser probe makes use of optical fiber technology and allows for a compact head (about 4 x 3 x 0.5 inches) to be scanned at a distance of about one inch from the surface. The probe is designed to measure displacement amplitudes in the nanometer range at low to mid-frequencies (i.e., up to twice the ring frequency of the shell). The data is analyzed in the frequency-wavenumbers (axial and circumferential) domain by means of FFT's for the frequency and circumferential variables, and with the overdetermined modified extended Prony (OMEP) algorithm for the axial coordinate. The results allow one to separate longitudinal, shear, and flexural waves, and to observe evanescent waves near the shaker and mode conversion at the end caps.
Project #1b:

A 1 watt argon-ion laser is used in a fiberoptic interferometer to measure independently at 4 surface points both the in-plane and one of the out-of-plane components of the vector displacement at the surface. Time-domain analysis of the data is achieved either by phase-locked loop demodulation or, alternatively, by digital Hilbert transform. The system can be used to analyze wave propagation and mode conversion and, to some extent, to study power flow in complex elastic structures. The effects of surface roughness, laser polarization, and angles of incidence on the signal-to-noise ratio of the system have been studied in detail to improve detection and ease of operations.

Project #2a:

A laser Doppler interferometer has been designed and built to measure normal displacements simultaneously on both sides of freely suspended thin samples of polyurethanes (elastomers) known to exhibit the "giant electrostrictive effect". Measurements on both side allows one to subtract any bending motion that may occur in the sample and provide direct measurements of the $d_{33}$ piezoelectric coefficient. Phase-mixed and phase-separated samples obtained from NSWC are being tested.

Project #2b:

A laser system dedicated to material characterization is being built. It consists of 5 independent fiberoptic compact interferometers, each capable of measuring in-plane and out-of-plane motion. The system will be used to study material properties of SHT coating materials and experimental results will be compared with numerical modeling being done at NRL-USRD.

B. Significant Results in the Past Year

The most important aspect of this research effort is that the Georgia Tech group has developed some strong expertise in the area of laser interferometry for structural acoustics. Many graduate students have benefited from this exposure and some of them have joined NRL. But specifically, the significant results obtained to date can be grouped as follows.

Project #1a:

- Shear, longitudinal, and flexural waves have all been detected with the automated scanning and focusing laser system. The data shows that the LDV technique can directly measure all the waves which propagate in the shell. The measurements at Georgia Tech are the first direct noninvasive detection of the fast longitudinal and shear waves.

- Evanescent waves have been observed near the free ends for the circumferential mode $n = 3$, clearly indicating mode conversion. Also, a strong torsional mode ($n = 0$) has been observed. The LDV technique is the only method which can detect directly and noninvasively torsional motion.
Although all measurements have been made in air, the probe has also been tested in water.

Project #1b:

- The proof-of-concept for simultaneous measurements at several surface points of both in-plane and out-of-plane motion has been established. Measurements have been verified with a piezoelectrically excited cylindrical shell with known surface displacement amplitudes.

- Measurements on an L-shaped beam confirm that mode conversion between longitudinal and flexural waves occur at the bend. Estimates of power flow can be obtained from such measurements.

- The effect of surface roughness, laser beam polarization and angles of incidence on the signal-to-noise ratio are being quantified so as to improve the system.

Project #2a:

The electrostrictive thickness coefficient, \( d_{33} \), has been measured for polyurethane film samples with two different controlled material morphologies prepared at the NSWC (White Oak) laboratory. The large electrostrictive strains exhibited by the polyurethane films make these materials promising candidates for low frequency underwater sound projectors.

Project #2b:

None so far. (Project just started 07/01/94)

C. Plans for Next Year's Research

Project #1a: - Publish results in J. Acoust. Soc. Am. Terminate project.

Project #1b: - Set up system for real-time simultaneous detection of in-plane and out-of-plane displacements.
- Quantify the effects of surface roughness, laser polarization, and angles of incidence on the detectability of the signals.

Project 2a: - Set up system for real-time simultaneous detection on both sides of the sample.
- Measure \( d_{33} \) in various samples.

Project 2b: - Set up system and measure the response of SHT material between 100-3000 Hz.
D. LIST OF PUBLICATIONS/REPORTS/PRESENTATIONS (*)

1. Papers Published in Refereed Journals

None

2. Non-Refereed Publications and Published Technical Reports


3. Presentations

a. Invited

None

b. Contributed


4. Books (and sections thereof) None.

(*) Under this ONR contract

Enclosure (2)
### E. LIST OF HONORS/AWARDS

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Enclosure (3)
F. **Participants**

H. G. Kil, Ph.D. candidate  
M. Yang, Ph.D. candidate  
F. Guillot, M.S. candidate  
L. Willis, M.S. candidate

G. **Other Sponsored Research**

Title: Acoustic transduction with lasers  
Sponsor: National Science Foundation  
Amount: $50,000  
Charge Time: 50%, Summer '94. 0% AY '94  
Start-End: 1988-1994 (Presidential Young Investigator Award)
### SUMMARY OF FY94
**PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS/PARTICIPANTS**
(Number Only)

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Enclosure (4)
FY 95 Report to the Office of Naval Research

Title: Laser Doppler Interferometry for Material Characterization

P.I: Yves H. Berthelot, associate professor of mechanical engineering
    Tel (404) 894-7482 - Fax: (404) 894-7790 - e-mail: yves.berthelot@me.gatech.edu

Institution: Georgia Institute of Technology

Contract: N00014-91-J-1888 and N00014-94-I-0947 (+ AASERT support)

Scientific Officer: Dr. G. L. Main (Code 334)

1. Objectives and Rationale

The main objective of the research program is to develop a versatile laboratory for laser Doppler interferometric characterization of acoustic materials. It is critical to develop a reliable non-contact technique to measure accurately the dynamic response of complex materials such as microvoided polymers because standard measurements with miniature accelerometers have clearly shown the limitations imposed by such contact techniques. Optical interferometry offers the possibility of measuring accurately and noninvasively both in-plane and out-of-plane displacements at the surface of an acoustic material. The emphasis of the research is on obtaining reliable, consistent data on the linear response of soft acoustic materials. This experimental data will be used by the NRL-USRD team (headed by M. McCollum) to extract the complex frequency dependent properties of the samples. (See MIME FY95 progress report).

2. Technical Approach

A laser system dedicated to acoustic material characterization has been designed, built, and tested. It consists of 5 independent fiberoptic compact interferometers, each capable of measuring either in-plane or out-of-plane motion. The system is designed to measure the dynamic response of a sample simultaneously at 5 points in the 100 - 5000 Hz frequency range for CW excitation, (or up to 60 kHz, with transients excitation). The sample is excited by a shaker. The minimum detectable surface velocity is of the order of a few μm/s, while the maximum detectable surface velocity is in the range of a few cm/s so that, at present, the system is not designed to measure the response of very large strains occurring under shock conditions but instead in the linear regime of deformations.

The sample currently being tested is a nominally 1 in x 2 in x 3 in black polymer voided
with microspheres. The location of the 5 laser probes is shown in Figure 1. At present the shaker used is a small B&K 4810 shaker.

3. Significant results

Consistency between measurements taken over several days on a given sample has been established. Figure 2 shows on the left column experimental results measured with the 5 laser probes and, on the right column, numerical predictions based on the finite element model used at NRL-USRD with material parameters estimated with the DTRC model. The quantity plotted is, on a logarithmic scale, the velocity measured at each probe normalized by the measured based velocity as a function of frequency. The base velocity is measured with an accelerometer (Kistler 8616, 0.5g). The agreement between data and model is good, considering that the B&K shaker provided uniform base motion only to within ±1 dB below 2 kHz, and ±1.5 dB below 3.5 kHz.

The consistency of the data from one day to another under nominally similar conditions is encouraging. Three sets of data were recorded over a span of 3 days and in each case the temperature was recorded at the time of measurements. The temperature fluctuated by 0.2 °C. The experimental data shown in Fig 2 shows the results of the three data sets: solid line (22.4°C), dotted line (22.5 °C), and dashed line (22.6°C). The results are very consistent, except for probe number 4, where the signal to noise ratio was very low. (Probe 4 is not an interesting point to measure and will not be used in future experiments).

4. Plans for future work

- **painting the sample**: Currently, measurements are made using small pieces of reflective tape glued to the sample at each measurement point. Instead, we will paint the sample with a thin coating of Kilz (which has been shown to give good results in other LDV experiments) and repeat the experiment reported above.

- **measurements with the Ling shaker**: Plans are being made to use the Ling shaker (from USRD) instead of the B&K shaker. The Ling shaker has a very flat response below 3.5 kHz and, unlike the B&K shaker, it provides a nominally flat piton-like motion of the base. Replacing the shaker will require some changes in the arrangement of the optical bench, but the system should be in place and operational before January 96.

- **automated data analysis**: To analyze the data in a meaningful way for the MIME project (NRL-USRD), it is necessary to automate the data acquisition system so as to collect enough data to obtain a meaningful standard deviation.

- **relative humidity**: It appears that, at least on thin samples, relative humidity is an important factor in determining acoustical properties. Relative humidity (and temperature) will be monitored during all future experiments.
- **consistency between several samples**: Consistency between measurements taken over several days on a given sample has been established. The next step is to establish whether or not measurements taken from various samples are repeatable or not. Several samples will be provided by Tracor, through Walt Madigowski.

5. **Publications, Conference Presentations, and Research Dissemination**


6. **Personnel**

The DoD-AASERT grant is used to support graduate and undergraduate students in the area of laser interferometry for structural acoustics.

1) **material characterization project**
- R. L. Willis, MS student (expected graduation Fall 1995)
- T. S. Stone, Ph.D. student (Starting Fall 1995)
- A. Moore, M.S. student (starting Fall 1995)
- E. Hamilton, undergraduate (Minority student) (Fall 1995)

2) **other structural acoustics projects**
- M. Yang, Ph.D. student (female) (expected graduation Winter 1996)
- H-G. Kil, Ph.D, student (Graduated Summer 1995)
- Faculty supervision by Professors Berthelot and Jarzynski.
EXPERIMENTAL CONFIGURATION

location of the 5 laser probes

Voided polymer
H = 2.985 in
L = 1.970 in
W= 0.991 in

Aluminum base

accelerometer
Kistler 8616 (0.5 gram, 0.2 in diam)

shaker
B&K 4810

CW: f_0 (0.1 - 5 kHz)
or transients (tone bursts)

Measurement: \[ \frac{\text{rms - velocity at probe # (optical)}}{\text{rms - velocity at base (accelerom.)}} \]

Assumption: piston-like motion of the base
EXPERIMENTAL RESULTS and FEM MODEL (with DTRC properties)

Figure 2
SYSTEM PERFORMANCE

• PLL frequency response:
  flat from DC to 63 kHz (w/o 10 kHz LP filter)

• In-plane interferometers:
  - sensitivity: 23 (μm/s) / mv
  - minimum detectable velocity:
    65 μm/s w/o averaging
    5 μm/s with 64 averages
  - maximum detectable velocity:
    35 mm/s (speckle and surface dependent)

• Out-of-plane interferometers:
  - sensitivity: 11.5 (μm/s) / mv
  - minimum detectable velocity:
    35 μm/s w/o averaging
    3.5 μm/s with 64 averages
  - maximum detectable velocity: 12 mm/s

• Probe mounting resonance: ~30 Hz
The research performed under this contract is concerned with applications of laser Doppler interferometry for various measurements of interest in structural acoustics. Specifically, our contributions fall under three areas: (1) acoustic material characterization: simultaneous measurement of bulk and shear dynamic (complex) moduli of voided polymers with an experimental/numerical technique; (2) array of laser probes for vector measurements in structural acoustics: applications to the measurement of structural intensity and power flow, and to mode conversion in an L-shaped beam; and (3) automated scanning and focusing of a laser probe around a cylindrical structure: simultaneous measurements of longitudinal, shear, flexural, and evanescent waves. The main results of each area have been disseminated in the form of three refereed journal articles submitted to the Journal of the Acoustical Society of America.
LASER DOPPLER MEASUREMENTS FOR STRUCTURAL ACOUSTICS

P.I: Yves H. Berthelot, Professor of mechanical engineering  
Tel (404) 894-7482 - Fax: (404) 894-7790 - e-mail: yves.berthelot@me.gatech.edu

Institution: Georgia Institute of Technology

Grant: N00014-91-J-1888

R & T number: 432p006—17

Scientific Officer: Dr. G. L. Main (Code 334)

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3. Publications, Presentations, Graduate Students, and Patents ............. page 60
1. Executive Summary

The research performed under this contract is concerned with applications of laser Doppler interferometry for various measurements of interest in structural acoustics. Specifically, our contributions fall under three areas: (1) acoustic material characterization, (2) array of laser probes for vector measurements in structural acoustics, and (3) automated scanning and focusing of a laser probe around a cylindrical structure. The main results of each areas have been disseminated in the form of three refereed journal articles submitted to the Journal of the Acoustical Society of America.

**Paper #1:** As far as material characterization is concerned, an experimental/numerical technique has been implemented to determine simultaneously (and noninvasively) the bulk and shear dynamic moduli (and loss factor) of viscoelastic materials of arbitrary shape under low-frequency (0.5 - 3 kHz) harmonic excitation. The method consists in measuring the dynamic response of the sample at several surface points with a set of five independent interferometers, and matching the response with the predictions from a finite element code in which the two moduli are the adjustable parameters. Results indicate that the technique is versatile, robust, and accurate.

**Paper #2:** For the second research area, a single high-power argon:ion laser was used with a sophisticated network of optical fibers to build a system capable of measuring simultaneously at four points of a vibrating surface, two components (one tangential and one normal) of the vector velocity of the surface dynamics, directly in the time-domain. A U.S. patent was awarded for this design. The system has been used to study structural intensity and power flow in a beam supporting both flexural and longitudinal waves, and also to study mode conversion in an L-shaped beam.

**Paper #3:** The third area of research was a continuation of work initiated under N00014-91-J-1888 to study structural vibrations of a cylindrical shell with a scanning laser Doppler probe designed specifically to measure in-plane surface vibrations. Results obtained with this automated scanning, focusing, and data acquisition system show that it is capable of detecting simultaneously all types of waves present on the shell (e.g., longitudinal, shear, flexural, and even evanescent waves). Measurements were made both below and above the ring frequency and they are in excellent agreement with theoretical predictions.
2. Significant results

2.1. Material Characterization by laser interferometry

(paper to be submitted 12/96 to J. Acoust. Soc. Am.)

A laser-based experimental-numerical technique for evaluating the bulk and shear elastic moduli of acoustic materials.

R. Lance Willis, T. Shane Stone, and Yves H. Berthelot

Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405

Abstract

An experimental/numerical technique has been implemented to determine simultaneously the bulk and shear dynamic moduli (and loss factor) of viscoelastic materials of arbitrary shape under low-frequency (0.5 - 3 kHz) harmonic excitation. The method consists in measuring the dynamic response of the sample at several points on the surface with a set of five independent laser interferometers, and matching the response with the predictions from a finite element code in which the two elastic moduli are the adjustable parameters. Results are presented for measurements made in air, under standard pressure and temperature conditions.

PACS numbers: 43.35.M, 43.35.C, 43.20.J, 43.58.D
I. Introduction

Viscoelastic polymers have become widely used in many areas involving air-borne or water-borne transmission and attenuation of acoustic waves. In order to use these materials effectively, it is critical to determine accurately their complex, bulk and shear moduli. One of the most common and reliable experimental method is that proposed by Madigoski and Lee\textsuperscript{1,2} in which a longitudinal wave is transmitted down a narrow strip of material with accelerometers attached at either end. The complex modulus of elasticity can be calculated at each resonant frequency from the measured $Q$ of the resonance. The technique works throughout the audio range but it is limited to a specific one-dimensional sample geometry, a discrete set of frequencies, and measures only the complex Young’s modulus.

In this paper, we propose a new method for measuring the two complex elastic moduli simultaneously with a sample of arbitrary shape, over a continuous frequency range. The method consists in measuring the dynamic response of a sample with a set of five independent laser interferometers, and matching the response with the predictions from a finite element code in which the two complex elastic moduli are the adjustable parameters. The sensitivity of this material characterization inverse problem has been studied in details by McCollum, Black, and Siders\textsuperscript{3} at the Underwater Sound Reference Detachment in Orlando. The experimental arrangement is presented in Section II, the inversion procedure is described in Section III, the results are shown in Section IV and summarized in Section V.

II. Experimental System

For simplicity, measurements were made in air at ambient conditions with a sample of rectangular shape (width = 2.52 cm, length = 5.00 cm, and height = 7.58 cm) excited harmonically at the base by a shaker (Ling Dynamic Systems, model 556) between 500 Hz and 3,300 Hz. A block diagram of the experimental arrangement is shown in Figure 1. The sample is a black, hard polymer that was pressure-cycled. It contains closed cell, voided microspheres with nominal void fraction of about 40%. It has a total mass of about 60 grams. The sample is glued to an aluminum base with a cyanoacrylate bond. The base is designed to minimize the amount of wobble and the presence of flexural waves, so that the base motion can be modeled as uniform. Measurements on the unloaded base indicate that this assumption is valid (within 5%) in the frequency range of interest with the Ling dynamic shaker. (With a small B& K 4810 shaker, the nonuniformity of the base motion
reaches 10% in the 0.3 - 2 kHz range and 20% in the 2-4 kHz range).

The surface dynamics of the sample was measured in the time-domain, in real-time, simultaneously at four points and referenced (amplitude and phase) to the base motion. To do so, five independent laser interferometers were built, calibrated, and tested. Two of the interferometers were used in the out-of-plane configuration to measure the vertical component of the velocity at the top-center of the sample (point #5) and at the base (point #4). The other three interferometers were used in the in-plane configuration to measure the in-plane component of the surface velocity of the sample as indicated in Figure 1. Probe #1 and #2 measured in-plane surface velocity (horizontally polarized), close to the edge of the sample, at a height $z = H/3$ and $z = 2H/3$, respectively, where $H = 7.58$ cm is the sample height and $z$ is the vertical axis ($z = 0$ denotes the base). Probe #3 measured the in-plane surface velocity (vertically polarized) at a height $z = H/2$, on the edge of the sample. Each probe is very compact $4$ (about 3.2 cm x 1.5 cm x 1 cm).

Each interferometer was powered by a 10 milliwatt He:Ne laser. For convenience, single mode fibers were used to bring the light to the probe heads which were positioned close to the sample (1 cm for the in-plane configuration, and 4.4 cm for the out-of-plane configuration). Each probe was mounted on a small aluminum arm secured to a series of optical stages (2 translations and 1 rotation) for precise alignment. Each stage assembly was mounted on four vibration damping feet on an optical bench. (Two photographs of the experimental setup can be viewed on the web site http://www.me.gatech.edu/yves.berthelot/bothphotos.html).

The principle of laser interferometry for in-plane and out-of-plane measurements is well understood $5,6$. In our experiment, five optical benches with lasers and associated optics lead separately to the five optical probe heads. On each optical bench, the light from the laser passes through an acousto-optic Bragg cell driven by a precise quartz oscillator at $f_B = 40.0$ MHz. The Bragg cell splits the beam into two beams, one of which is frequency shifted by 40 MHz while the other remains unaffected at the optical frequency, $f_L$. Each beam is then launched into a single mode optical fiber. Figure 2 shows the geometry associated with either in-plane or out-of-plane detection. To detect the in-plane surface displacement, $u(t)$, the two arms of the interferometer illuminate the surface at an angle $\alpha$ (30°) symmetrically with respect to the normal to the surface. The two beams are focused to a tight spot ($\approx 75\mu m$) by two cylindrical rod lenses with graded index of refraction (GRIN lenses). At the surface, the two beams interfere at 40 MHz, and light is scattered in all direction because of surface
roughness. Some of it is collected by a large core (1000 μm diameter) fiber and fed to a sensitive avalanche photodiode. In this configuration, the detected signal is of the form \( \cos[2\pi f_B t + 2k \sin \alpha u(t)] \), where \( k \) is the optical wavenumber. To increase the amount of scattered light, a small patch (1 mm x 1 mm) of scattering film (retroreflective tape from 3M) is placed at the focusing point on the surface. In the out-of-plane configuration, only one beam illuminates the surface at an angle very close to the normal (< 3°). Some of the light is reflected to a large core multimode fiber (also 1000 μm diameter) and combined with the second arm of the interferometer (reference signal) to produce the interference signal at 40 MHz which, in this configuration, is of the form \( \cos[2\pi f_B t + 2k w(t)] \), where \( w(t) \) is the out-of-plane motion. To increase the amount of reflected light, a small patch (1 mm x 1 mm) of reflective mylar is placed at the focal point on the surface of the sample.

Each signal is then downshifted from a 40 MHz carrier to a 100 kHz carrier by mixing the output of the photodiodes with a 40.1 MHz signal generated by a very precise quartz oscillator. This allows one to use phase-locked loop (PLL) demodulation to extract the in-plane and out-of-plane signals. Basically, a PLL produces a voltage proportional to the instantaneous frequency deviation from the carrier frequency. Its output is therefore proportional to the instantaneous surface velocity components, \( \dot{u}(t) \) or \( \dot{w}(t) \), which are then displayed on a digital oscilloscope and transferred to a computer via a GPIB interface controlled by Labview. The PLL’s are designed to have a flat frequency response up to 63 kHz. In our design, each probe head can be configured either for in-plane or for out-of-plane detection. The system performance and characteristics are shown in Table 1.

Two 4-channel digital oscilloscopes are used to capture the 7 waveforms in real time: 5 surface velocities + 1 reference signal for each oscilloscope. (The reference signal for each oscilloscope is the output from the signal generator driving the shaker). The Labview software controls the data acquisition as follows. The waveform generator scans successively throughout the frequency range from 500 to 3,300 Hz, (usually by steps of 100 Hz). At each frequency, 7 waveforms of 5000 data points each are acquired 64 times successively, and converted to 64 files in ASCII format. At this point, the signals are processed under the Matlab environment as follows. The waveforms are slightly truncated to contain an integer number of cycles. A DFT is used to find the peak complex amplitude of each of the 7x64 waveforms at each frequency. (No special windowing is needed if one uses an integer number of cycles in the waveform). Then, each complex amplitude is multiplied by the appropriate calibration constant of each interferometer (PLL calibrations) and compensated for the
phase difference between the two reference signals of each scope (which may have slightly different trigger levels). All signals are referenced to the oscilloscope on which probe #4 (the base) is displayed. Subsequently, the mean and the standard deviation over the 64 samples (at each frequency) in amplitude and phase is calculated. These values characterize the average (and the spread) in the amplitude and the phase of the surface velocities at points #1, #2, #3, and #5, relative to point #4, at each frequency. As indicated in the next Section, the values can be compared with numerical predictions to infer the values of the elastic moduli of the sample.

III. Material Characterization

The viscoelastic material under investigation is usually assumed to be macroscopically homogeneous and isotropic\textsuperscript{7–10} in which case the dynamics is governed by the wave equation

\[(\lambda + \mu)\nabla \cdot \ddot{u} + \mu \nabla^2 \ddot{u} = \rho \frac{\partial^2 \ddot{u}}{\partial t^2}\]  

subject to the appropriate boundary conditions. Here, \(\lambda\) and \(\mu\) are the (complex) Lamé constants, \(\ddot{u}\) is the vector displacement, and \(\rho\) is the sample effective density. The Lamé constants are related to the shear and bulk moduli, \(G\) and \(K\), by \(G = \mu\) and \(K = \lambda + (2\mu/3)\). The lossy properties of the material are usually described by the loss tangents \(\tan \delta_K\) and \(\tan \delta_G\) defined by:

\[
\tan \delta_K = \frac{K''}{K'} \quad \text{with} \quad K = K' + iK'' = K'(1 + i \tan \delta_K) \tag{2a}
\]

\[
\tan \delta_G = \frac{G''}{G'} \quad \text{with} \quad G = G' + iG'' = G'(1 + i \tan \delta_G). \tag{2b}
\]

In our experiment, the boundary conditions are such that all faces of the sample are free except for the bottom face which is driven harmonically and uniformly. With such a simple geometry and boundary conditions, the problem is ideally suited for an efficient and accurate computation of the surface dynamics by the finite element method (FEM). Our numerical code (FAcoudl) uses the quarter symmetry of the sample geometry with 825 nodes and 36 elements: 6 along the height, 3 along the half-length, and 2 along the half-width. The algorithm used to extract the material properties from the data consists in minimizing the difference between the data and the predicted values. It turns out that, in practice, the imaginary part of the Poisson ratio is negligible. The finite element code is therefore written
in terms of only 3 material parameters: the Young modulus, \(E\), its loss factor, \(\eta\), and the Poisson ratio, \(\nu\). (As usual, the complex Young's modulus is defined as \(E(1 + i\eta)\) and the parameters \(E\) and \(\nu\) are related to the bulk and shear moduli by: \(E = \frac{9KG}{3K + G}\) and \(\nu = \frac{(3K - 2G)/(6K + 2G)}{,}\) or, conversely, \(G = E/[2(1 + \nu)]\) and \(K = E/[3(1 - 2\nu)]\).

The search for these 3 parameters \((E, \eta, \nu)\) is obtained by a three-dimensional direction set method (Powell’s method), a robust, classical method in optimization theory. The method requires initial estimates of the three parameters and a step size for the search procedure. The function that is minimized is the mean-square error defined by

\[
\Delta^2 = \sum_{i=1}^{4} \left[ \frac{\bar{X}_{data} - X_{FEM}}{X_{data}} \right]^2 + \left[ \frac{\bar{Y}_{data} - Y_{FEM}}{Y_{data}} \right]^2
\]

where \(X\) and \(Y\) are the real and imaginary parts, respectively, of the complex amplitude of the surface velocity of either the measured data or the FEM prediction. (The overbar indicates an average value over the 64 samples). The summation in equation (3) is over the 4 surface points \((\#1, \#2, \#3, \#5)\) normalized to point \#4 (the base) in both amplitude and phase. This function (and the minimization algorithm) is evaluated for one frequency at a time. The algorithm converges towards the optimum values of \(E, \eta,\) and \(\nu,\) from which one calculates the shear and bulk moduli (and also the Young modulus and Poisson ratio, the Lamé constants, and the shear and longitudinal sound speeds) at that frequency. The process is repeated over the frequency range of interest. A block diagram of the procedure is shown in Figure 3. The material inversion procedure takes typically 15 minutes per frequency on a Pentium Pro computer.

IV. Results

Figure 4 shows the amplitude and phase (normalized at each frequency to the amplitude and phase of the motion of the base at point \#4) measured at each of the 4 locations of interest, as a function of frequency. The amplitude is given in decibels referenced to the base velocity. The solid line represents the measurements obtained after 64 averages (mean value) while the two dotted line represent the mean value plus or minus one standard deviation, also after 64 averages. The chain-dotted line is the numerical prediction from the finite element model in which the material parameters were calculated from the mean values over 64 averages as a function of frequency.

Figure 5 shows the real and imaginary parts of the bulk and shear moduli as a function
of frequency. The solid line represents the “measurements” obtained after 64 averages (mean value) while the two dotted line represent the mean value plus or minus one standard deviation, also after 64 averages.

V. Summary

An experimental/numerical technique has been implemented to determine simultaneously the bulk and shear dynamic moduli (and loss factor) of viscoelastic materials of arbitrary shape under low-frequency (0.5 - 3 kHz) harmonic excitation. The method consists in measuring the dynamic response of the sample at several points on the surface with a set of five independent laser interferometers, and matching the response with the predictions from a finite element code in which the two elastic moduli are the adjustable parameters. Results are presented for measurements made in air, under standard pressure and temperature conditions.

Acknowledgements:

This work was supported by the Office of Naval Research, Structural Acoustics Program, Code 334. Dr. G. L. Main Scientific Officer. The authors would like to thank Steve Hahn for his help with the finite element code; and Michele McCollum, Steve Black, and Pat Klippel for many insightful discussions during the course of this project.

References:


Figure captions

- Figure 1: Experimental arrangement
- Figure 2: In-plane and out-of-plane configurations
- Figure 3: Block diagram of the experimental/numerical procedure for determining the material properties.
- Figure 4: Amplitude and phase at 4 points as a function of frequency
- Figure 5: Real and Imaginary parts of the bulk and shear moduli as a function of frequency.

List of Tables

- Table 1: System performances
Table 1: Typical performance and characteristics of the interferometers.
He-Ne lasers
Bragg cells
Fiber couplers

power supplies
Quartz oscillators
RF filters/ampl.

optical bench
for out-of-plane
detection

ampl.
filters
+40.1 MHz
mixers

PLL
demod.

LP
filters

5 optical
probe heads

5 avalanche
photodiodes
(APD)

APD bias

function gen.
0.3 - 3.3 kHz

4 ch. digital
scope

4 ch. digital
scope

Computer
automated data acq.(Labview)
signal processing (Matlab)
FEM code (Fortran)
Optimization

2 complex
elastic moduli

Figure 1: Experimental arrangement
Figure 2: In-plane and out-of-plane configurations
@ each frequency,

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MINIMIZATION (Powell's method)

FEM code Facoudl

\[
\Delta^2 = \sum_{i=1}^{4} \left[ \frac{X_{\text{data}} - X_{\text{FEM}}}{\bar{X}_{\text{data}}} \right]^2 + \left[ \frac{Y_{\text{data}} - Y_{\text{FEM}}}{\bar{Y}_{\text{data}}} \right]^2
\]

estimated \( E, \eta, \nu \) 
\( G, K, \lambda, \mu \)

+ standard deviation of estimates obtained from inversion of mean values ± stand. deviation of data

Tolerance met ?

\( E, \eta, \nu \) 
\( G, K, \lambda, \mu \)

Figure 3: Block diagram of the experimental/numerical procedure for determining the material properties.
Figure 4: Amplitude and phase at 4 points as a function of frequency.
Probe 5 Amplitude Response Relative to Base

Probe 5 Phase Response
Figure 5: Real and Imaginary parts of the bulk and shear moduli as a function of frequency.
Real Cs

Frequency

Meters/second

Imaginary Cs

Frequency

Meters/second
Real Cl

Imaginary Cl

Fig Ed
Measurement of longitudinal and flexural power flow in a beam by laser Doppler interferometry

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Abstract

A fiberoptic laser Doppler interferometer is described. It is designed to measure simultaneously at four locations the in-plane and out-of-plane surface velocity of a vibrating surface. The system has been tested with a thin beam in which both flexural and longitudinal waves were excited by a shaker driven at a single frequency. The power flow of both longitudinal and flexural waves was estimated by evaluating the spatial derivatives of the surface velocities with a finite difference technique and the results were compared with measurements obtained with an accelerometer.

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I. Introduction

The flow of mechanical power within a vibrating structure is a useful tool to locate sources and sinks of vibrational energy, and to characterize the conversion from one mode to another (e.g., flexural waves partially converted into longitudinal waves at an impedance discontinuity). Estimating the power in a structure is, however, challenging. First, sensors are in general limited to measurements on the surface of the structure. Second, measuring the power flow requires estimates of spatial derivatives of the vibrational field quantities. These derivatives are almost always calculated from a finite difference technique which requires measurements over a large surface area with, in general, a large array of accelerometers. A third difficulty in estimating the power flow is that the result depends critically on accurate phase information between the sensors. Although accelerometers are cheap, robust, and sensitive, measurement of power flow with a large array of accelerometers is somewhat impractical because of mass-loading of the structure and nonuniform phase response of each sensor. An attractive alternative is to use laser interferometry. McDewitt et al.\textsuperscript{1,2} have measured separately with two different experimental arrangements flexural and longitudinal power flow in a reverberant system supporting either only flexural waves or longitudinal waves.

In this paper, we describe a laser-based system capable of measuring simultaneously out-of-plane and in-plane surface velocities at four locations over a simple one-dimensional structure excited harmonically by a shaker. The experimental system is described in Section II. As an application, the system is used to measure simultaneously the flexural and longitudinal power flows in the beam. The estimates for the power flow are presented in Section III, and the results are given in Section IV, discussed in Section V, and summarized in Section VI.

II. Experimental System

Vibrating structure

An aluminum beam of length $L = 1.98$ m, width $b = 1.9$ cm, and thickness $h = 0.64$ cm is driven at one end by a shaker mounted at $45^\circ$ as shown in Figure 1 so as to excite both flexural and longitudinal waves in the beam. The driving frequency is set at 4 kHz. The longitudinal and flexural wavelengths are approximately 12.2 cm and 128 cm, respectively.
so that only plane waves propagate along the beam. An impedance head is mounted on the shaker to measure both the force and the velocity at the driving point, and therefore the total input power into the structure. To ensure that time-averaged mechanical power flows from the shaker to the other end of the beam, a sand box is placed at the end of the beam to provide attenuation and reduce reflections. One third of the length of the beam is buried in the sand. To gradually increase the mechanical damping in the sand, 12 metal strips of progressively longer length (from 2.54 cm to 11.43 cm) are glued to the side of the beam with an even spacing of 5 cm. Because of the additional surface area of the strips, sand adds progressively more friction along the x-axis to the rotational motion of the strips (and therefore to the flexural waves) and also to the translation of the strips (and therefore to longitudinal waves).

Optical System

The proposed detection system is shown in Figure 2. A 1 watt Argon-ion laser is used with fiberoptics to power 4 independent optical probe heads that can each measure simultaneously in-plane and out-of-plane surface motion at a point. The system has been described previously.³⁴ Because of the restricted number of filters available in the detection system, the results presented in this paper were obtained with a single probe head measuring simultaneously the in-plane and out-of-plane surface velocity at a point. The probe head was scanned along the length of the beam. However, no fundamentally different difficulty can be expected when using the complete system with the four optical heads. A probe head consists of 3 illuminating single mode fibers that focus 3 beams by means of miniature lenses with graded index of refraction (GRIN lenses) to an optical spot of about 75 μm diameter on the structure where the 3 beams overlap. Precise alignment is achieved by carefully machining the probe head and by making small adjustments of the GRIN lenses with 4 miniature screws. The three beams which illuminate the surface have actually been frequency shifted by three acousto-optic Bragg cells prior to being launched into the optical fibers. The frequency shifts of successive beam are 40.0, 40.1, and 40.3 MHz, respectively, so that interference between beams 1 and 2 occurs at \( f_{12} = 100 \text{ kHz} \), interference between beams 2 and 3 is at \( f_{23} = 200 \text{ kHz} \), and interference between beams 1 and 3 is at \( f_{13} = 300 \text{ kHz} \). Displacements of the surface result in a phase modulation of these three carriers. Alternatively, any surface velocity results in a Doppler frequency modulation of the carriers. The geometry of the probe head is such that beams 1 and 3 are symmetric with
respect to the normal of the surface, where the light scattered by the vibrating surface is
collected by a large core multimode fiber (1000 μm, numerical aperture of 0.37) and sent
to a photodetector. In this differential configuration, the 300 kHz carrier is modulated only
by the in-plane component of the surface motion while the other two carriers (100 and 200
kHz) are modulated by a combination of in-plane and out-of-plane motion. For each pair of
beams, the corresponding carrier frequency is modulated only by the surface displacement
component perpendicular to the bisector of the angle between the beams. As shown in Ref.
[1], the photodetector signal, \( v(t) \), is given by:

\[
 v(t) = V_{12} \cos(\omega_{12}t + \phi_{12}) + V_{23} \cos(\omega_{23}t + \phi_{23}) + V_{13} \cos(\omega_{13}t + \phi_{13}),
\]

(1)

where \( \omega_{ij} = 2\pi \times f_{ij} \) and where the amplitudes \( V_{ij} \) are proportional to \( \sqrt{I_i I_j} \), \( I_i \) being the
power of the \( i \)-th beam \( (i,j = 1,2,3) \). (The constant of proportionality is determined by
several factors, including the reflectivity of the surface and a fringe contrast factor associated
with the speckle pattern due to the surface roughness). The phase terms \( \phi_{ij} \) contain the
information about the surface displacement vector \( \xi(t) = u(t)\hat{i} + w(t)\hat{k} \) at the location being
probed. \( u(t) \) being the in-plane component and \( w(t) \) the out-of-plane component of the
surface displacement. The phase terms are given by:

\[
\phi_{12} = (\sin \alpha + \sin \beta)ku(t) + (\cos \beta - \cos \alpha)kw(t) \quad (2a)
\]

\[
\phi_{23} = (\sin \alpha - \sin \beta)ku(t) - (\cos \beta - \cos \alpha)kw(t) \quad (2b)
\]

\[
\phi_{13} = 2ku(t) \sin \alpha, \quad (2c)
\]

where \( k \) is the optical wavenumber, and the angles \( \alpha \) and \( \beta \) characterize the geometry of
the laser probe head. In our experiment, \( \alpha = 45^\circ \) and \( \beta = 10^\circ \). By filtering the signal with
sharp band-pass filters to separate the three FM signals with carriers at 100, 200, and 300
kHz, one can measure simultaneously the \( \phi_{ij} \)'s and deduce the \( u \) and \( w \) components of the
surface displacement. In our system, the demodulation is achieved with a combination of
filters and phase-locked loops (PLL) tuned around each carrier frequency. Basically, a PLL
produces a voltage proportional to the instantaneous frequency deviation from the carrier
frequency. Therefore, the putput of the PLL which demodulates the signal with phase term
\( \phi_{ij} \) is proportional to the time derivative \( d\phi_{ij}/dt \). From equations 2(a-c), it follows that
the putput of the PLL's is proportional to the instantaneous surface velocity components \( \dot{u} \)
and \( \dot{w} \) (and not displacements \( u \) and \( w \)) which are displayed on a digital oscilloscope and
transferred to a computer. For our application concerning the measurement of power flow
in a structure, phase information is critical so that each PLL (and its associated filter) was calibrated not only in amplitude but also in phase. The phase of each demodulated signal was always referenced to the signal driving the shaker.

The laser Doppler system was capable of measuring displacement amplitudes as low as 5 Å, when the signals were averaged 64 times on the digital oscilloscope. The low frequency limit of the present laser Doppler vibrometer is of the order of 100 Hz, set by environmental factors such as temperature fluctuations and random vibrations. For measurements at a single point, the high frequency limit of the optical system is dictated by the PLL demodulation chosen for our experiment: the SK3595 chip with a 10 kHz low-pass filter. However, for measurements of power flow, in which the spatial derivatives are approximated by finite differences (see section III below), the practical high frequency limit is determined by the minimum separation distance between two adjacent probes. These probes can be designed to be very compact, with a minimum separation distance of 4 cm.

Accelerometer

The optical measurements were compared with independent measurements made with a small cylindrical accelerometer (B&K 4375) designed to measure only the out-of-plane component of the surface vibration. The accelerometer was glued to the beam with a hard adhesive organic crystal. Although the accelerometer is not designed to measure in-plane motion along the beam, one can also estimate in-plane motion by gluing the accelerometer sideways along the beam with its most sensitive direction aligned with the x-axis of the beam and its most insensitive direction normal to the beam. This measurement is not expected to be very accurate especially if flexural waves dominate over longitudinal waves. Nevertheless, it provides a comparison with the optical measurements, and actually it emphasizes the benefits of using the optical method to determine power flow.

III. Estimates for the power flow in a beam

In principle, the power flow in the beam can be estimated from a limited set of in-plane and out-of-plane measurements at a few positions along the beam. If only harmonic plane longitudinal and flexural waves are present in the beam, the time-average power flow is given
by

\[ P(x) = S \ E \ \omega \left\langle \ddot{u} \frac{\partial u}{\partial x} \right\rangle_t + E \ I \ \omega \left\langle \ddot{w} \frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 w \ \partial \dot{w}}{\partial x^2} \right\rangle_t \]  

(3)

where \( \langle \rangle_t \) stands for time average, \( E \) is the Young modulus of the beam, \( I = bh^3/12 \) is its moment of inertia, \( S = bh \) is the cross-section, and \( \omega = 2\pi f \) is the angular frequency of the excitation. It is understood in Eq. (3) that \( u, \dot{u}, w, \) and \( \dot{w} \) are all functions of \( x \) and \( t \).

The first term on the right-hand side of equation 3 represents the power flow carried in the \( x \)-direction by longitudinal waves, and the second term represents that carried by flexural waves. The tilde indicates that the variable has been phase-shifted by \( \pi/2 \). (In general, equation 3 is written in complex notation with \( iu \) instead of \( \ddot{u} \), but equation 3 in its present form is closer to our actual processing of the measured signals). An important assumption has been made in the derivation of equation 3. It is assumed that most of the out-of-plane displacement is caused by the flexural wave and most of the in-plane displacement is caused by the longitudinal wave. This assumption is appropriate in our case because the flexural and the longitudinal wavelengths are much greater than the thickness \( h \) of the beam. (The ratio of the in-plane to out-of-plane displacements caused by the flexural wave is \( (h/2)k_F \) which is about 8% in our case; and the ratio of the out-of-plane to in-plane displacements caused by the longitudinal wave is \( \mu(h/2)k_L \) which is less than 0.1% in our case. Here, \( \mu \) is the Poisson ration, and \( k_F \) and \( k_L \) denote the wavenumbers of flexural and longitudinal waves, respectively).

To evaluate the power flow in the beam, it is necessary to estimate the spatial derivatives which appear in equation 3. The simplest way to do so is by a finite difference method where a function \( F \) is measured at several equispaced (spacing \( \delta \)) locations (denoted by the index \( n \)) and the derivatives are obtained from Taylor series expansions with the result that

\[
\frac{\partial F}{\partial x} \bigg|_n = \frac{1}{2\delta} (-F_{n-1} + F_{n+1}) + O(\delta^2) \quad (4a)
\]

\[
\frac{\partial^2 F}{\partial x^2} \bigg|_n = \frac{1}{\delta^2} (F_{n-1} - 2F_n + F_{n+1}) + O(\delta^2) \quad (4b)
\]

\[
\frac{\partial^3 F}{\partial x^3} \bigg|_n = \frac{1}{2\delta^3} (-F_{n-2} + 2F_{n-1} - 2F_{n+1} + F_{n+2}) + O(\delta^2) \quad (4c)
\]

Note that one could use the approximation \( F_n = (F_{n-1} + F_{n+1})/2 \) to estimate the power flow from measurements of in-plane and out-of-plane motion at only 4 surface point \((n - 2, n - 1, n + 1, \text{and } n + 2)\), i.e., with only four laser probe heads. In the experiments reported below, the derivatives were actually estimated from measurements at 5 points for better
An important parameter that determines the accuracy of the estimate is the separation distance $\delta$. If $\delta$ is too large, the finite differences are very imprecise estimates of the derivatives. If $\delta$ is too small, the finite differences are very small quantities, and the estimated derivatives are severely affected by the noise present in the data. A more detailed discussion of the errors associated with our measurement technique will be given in Section V.

IV. Results

Flexural power flow

The power flow estimated at different locations along the beam, in the farfield, should remain the same because absorption within the material is negligible over the length considered. The optimum value of the separation distance $\delta$ was determined experimentally by finding a value of $\delta$ for which the power flow remains fairly constant along the length of the beam. The data was taken in the farfield, away from the shaker (at $x \geq 63.5$ cm) to avoid unnecessary complications due to the possible presence of evanescent waves near the driving point. First, the data was taken from $x = 63.5$ cm to $x = 121.9$ cm, with a spatial resolution $\delta = 1.27$ cm (approximately equal to a tenth of the flexural wavelength). The amplitude of the out-of-plane displacements measured along a portion of the beam are shown in Figure 3 for two different instants separated by 0.12 ms (about half a period). Figure 3 confirms that the flexural wavelength is about 12 cm. The small difference between the amplitudes of the two wave shapes is possibly due to the presence of a standing wave (superposed to the propagating wave) caused by the imperfect anechoic termination. The two curves are out-of-phase because of the half-period delay between the two measurements. The power flow estimates along the beam are shown in Table 1 for two values of the separation distance $\delta$ which correspond to one tenth and one fifth of the flexural wavelength, respectively. For comparison, the power flow estimates obtained from measurements with the accelerometer are also shown in Table 1. Clearly, the separation distance of 1.27 cm is too small for a good evaluation of the power flow and, consequently, the power flow estimated from the finite difference scheme is corrupted by the noise in the system. However, when the separation distance is a fifth of the flexural wavelength, the power flow estimated along the length of the beam remains reasonably constant and both the optical sensor and the accelerometer yield values that are in close agreement ($15 \pm 2\mu W$ and $13 \pm 1\mu W$, respectively).
Longitudinal power flow

The instantaneous amplitude of the in-plane displacements measured along a portion of the beam are shown in Figure 4 for two different instants separated by 0.12 ms (about half a period). The solid and dashed curves are the data measured at $t_0$ and $t_0 + 0.12$ ms. Figure 4(a) shows the data measured with the accelerometer while Figure 4(b) shows the data measured with the optical probe. Unfortunately, the length of the optical fibers carrying the light from the optical bench to the probe head was too short to scan the probe over the entire length of the beam. This resulted in an optical data set limited to a fraction of the longitudinal wavelength. This explains the difference in scale for the abscissas of figure 4(a) and figure 4(b). Figure 4 shows that the measurements made with the accelerometer underestimate the data measured optically, a result which is not surprising because the accelerometer is not meant to be used to detect in-plane motion. Note also that the measurements made with the optical probe indicate the presence of a perturbation with a spatial periodicity that corresponds to that of the flexural wavelength. This is to be expected because the flexural wave with a 20 nm peak displacement produces a slight in-plane surface displacement which is detected by the optical probe. Because of the limited scan and the long wavelength of longitudinal waves, only a single estimate of power flow could be made from the optical data, and this estimate was for a separation distance of a tenth of the longitudinal wavelength, $\lambda_L$. The power flow of the longitudinal wave was estimated by the finite difference technique to be $130 \mu W$. (For comparison, the power flow of longitudinal waves determined from the underestimated measurements made with the accelerometer was found to be $59 \pm 13 \mu W$ for a separation distance of $\lambda_L/10$ and $50 \pm 10 \mu W$ for a separation distance of $\lambda_L/5$.)

The total power flow measured optically is therefore $15 \mu W$ for the flexural wave + $130 \mu W$ for the longitudinal wave, that is a total of $145\mu W$ of mechanical power. This value is in close agreement with the value of $150 \mu W$ calculated from the force and velocity input measured by the impedance head at the shaker. As indicated in the next Section, such a good agreement is somewhat fortuitous.

V. Discussion

Several authors have analyzed the sources of errors associated with intensity measure-
ments in air (with the two-microphone technique) or with the power flow in structures (measured with accelerometers). (See for instance references 6-8). The two most important sources of errors are (1) the error associated with amplitude and phase at each measurement point, and (2) the error associated with the finite difference approximation(s) of the spatial derivative(s). The total mean square error is the sum of the mean square errors of each type.

Consider first the longitudinal power flow which, in our case, is estimated from the in-plane displacement \( u \) measured at 3 points: \( x = \pm \delta \) to estimate the first spatial derivative, and \( x = 0 \) to measure \( \hat{u} \) (see Eq. (3)). The relative mean square error caused by the finite difference approximation is:

\[
\frac{(\Delta U')_{\text{rms}}}{U'_{\text{rms}}} = \frac{\frac{1}{T} \int_0^T [U' - \hat{U}']^2 \, dt}{\frac{1}{T} \int_0^T (U')^2 \, dt}
\]

where \( U' \) is the exact value of \( \partial u / \partial x \) at \( x = 0 \) and where \( \hat{U}' \) is the finite difference approximation. As a first approximation for the error analysis, we consider only the propagating wave and thus assume a perfectly anechoic termination. This is obviously an oversimplification, but it is nevertheless acceptable if one is primarily interested in orders of magnitude in the error analysis. In that case, \( u(x,t) = U_0 \sin(\omega t - kx) \), \( k \) being the longitudinal wavenumber, \( \hat{U}' = (U_0/2\delta)[- \sin(\omega t + k\delta) + \sin(\omega t - k\delta)] \), the wellknown result emerges that\(^6\text{--}^8\):

\[
\frac{(\Delta U')_{\text{rms}}}{U'_{\text{rms}}} = \left[ 1 - \frac{\sin(k\delta)}{k\delta} \right]^2.
\]

The sinc function is characteristic of the spatial window used to estimate the derivative. It reflects the fact that the system cannot resolve wavelengths smaller than \( \delta \) (unless \( k \) is known \textit{a priori}.) If \( k\delta \) is not small, the finite difference approximation introduces a large but easily quantifiable error. For \( k \delta = 2\pi/10 \), \( (\Delta U')_{\text{rms}} / U'_{\text{rms}} = 0.004 \).

Similarly, one can evaluate the relative mean square error caused by random errors in the amplitude of each measurement as:

\[
\frac{(\Delta U)_{\text{rms}}}{U_{\text{rms}}} = \frac{(U - \hat{U})_{\text{rms}}}{U_{\text{rms}}} = \frac{\frac{1}{T} \int_0^T [U_0 \sin(\omega t) - (U_0 + \Delta) \sin(\omega t)]^2 \, dt}{\frac{1}{T} \int_0^T [U_0 \sin(\omega t)]^2 \, dt} = \frac{\Delta^2_{\text{rms}}}{U_0^2} \quad \text{(7)}
\]

where \( \hat{U} = (U_0 + \Delta) \sin(\omega t) \), \( \Delta \) being a random variable with zero-mean and known variance \( \Delta^2_{\text{rms}} \). For the phase errors, one has

\[
\frac{(\Delta U)_{\text{rms}}}{U_{\text{rms}}} = \frac{(U - \hat{U})_{\text{rms}}}{U_{\text{rms}}} = \frac{\frac{1}{T} \int_0^T [U_0 \sin(\omega t) - U_0 \sin(\omega t + \phi)]^2 \, dt}{\frac{1}{T} \int_0^T [U_0 \sin(\omega t)]^2 \, dt} = \frac{\Phi^2_{\text{rms}}}{2}, \quad \text{(8)}
\]
where $\phi$ is a random variable with zero-mean and known variance $\Phi^2_{\text{rms}} \ll 1$. (It is indeed reasonable to assume that $\Phi_{\text{rms}}$ is small because each waveform has a common reference: the shaker driving signal.) It follows that the relative error (mean square) of the longitudinal power flow is:

$$\frac{\Delta P_L}{P_L} = 3 \left( \frac{\Delta_{\text{rms}}}{U_0^2} + \Phi^2_{\text{rms}} \right) + \left[ 1 - \frac{\sin(k\delta)}{k\delta} \right]^2. \tag{9}$$

In our experiment, we estimate that $U_0 \approx 20 \text{ nm}$, $\Delta_{pk} = \sqrt{2} \Delta_{\text{rms}} \approx 1 \text{ nm}$, $\Phi_{pk} = \sqrt{2} \Phi_{\text{rms}} \approx 0.1$ radian, and $k\delta = 2\pi/10$, which leads to $\Delta P_L/P_L \approx 0.023$ (mean-square), i.e., a 15% relative error in the amplitude of $P_L$.

Now we turn our attention to the flexural power flow which, in our experiment, is estimated from measurements of the out-of-plane displacement at 5 points, $(x = \pm 2\delta, x = \pm \delta, x = 0)$, where $\delta$ represents a fifth of the flexural wavelength. Let the out-of-plane amplitude be $W_0 \sin(\omega t - kx)$, where $k$ is now the flexural wavenumber. The relative mean square error is given by:

$$\frac{\Delta P_F}{P_F} = 5 \left( \frac{\Delta_{\text{rms}}}{W_0^2} + \Phi^2_{\text{rms}} \right) + \frac{(\Delta W')_{\text{ms}}}{W'_\text{ms}} + \frac{(\Delta W'')_{\text{ms}}}{W''_\text{ms}} + \frac{(\Delta W''')_{\text{ms}}}{W'''_\text{ms}}, \tag{10}$$

where the last three terms represent the mean square errors caused by the finite difference approximation on each of the first three spatial derivatives. It is straightforward to show that:

$$\frac{(\Delta W')_{\text{ms}}}{W'_\text{ms}} = \left[ 1 - \frac{\sin(k\delta)}{k\delta} \right]^2 \tag{11a}$$

$$\frac{(\Delta W'')_{\text{ms}}}{W''_\text{ms}} = \left[ 1 - \frac{1 - \cos(k\delta)}{\frac{1}{2}(k\delta)^2} \right]^2 \tag{11b}$$

$$\frac{(\Delta W''')_{\text{ms}}}{W'''_\text{ms}} = \left[ 1 - \frac{\sin(k\delta)}{k\delta} \times \frac{1 - \cos(k\delta)}{\frac{1}{2}(k\delta)^2} \right]^2. \tag{11c}$$

In our experiment, we estimate that $W_0 \approx 20 \text{ nm}$, $\Delta_{pk} = \sqrt{2} \Delta_{\text{rms}} \approx 1 \text{ nm}$, $\Phi_{pk} = \sqrt{2} \Phi_{\text{rms}} \approx 0.1$ radian, and $k\delta = 2\pi/5$, which leads to $\Delta P_F/P_F \approx 0.22$ (mean-square), i.e., a 47% relative error in the amplitude of $P_F$. (Most of the error is due to the poor approximation of the third derivative.) Combining the mean square errors for the longitudinal and flexural power flow leads to $\Delta P/P = 0.24$ (mean square error), i.e., a 50% relative error in the estimated amplitude of the power flow. In view of this result, it appears that the good agreement obtained between the measured and predicted power flows might be somewhat fortuitous.
VI. Summary

A fiberoptic laser Doppler interferometer has been used to measure simultaneously the in-plane and out-of-plane surface velocity along a thin beam in which both flexural and longitudinal waves were excited by a shaker driven at a single frequency. The optical probe was scanned along the beam axis but the laser-based system is designed to provide 4 independent optical heads for simultaneous measurements at different locations. The power flow was estimated by approximating the spatial derivatives with a standard finite difference technique. The results obtained for the flexural power flow indicate that a separation distance of a fifth of the flexural wavelength is a good compromise, both for the optical and the accelerometer-based measurements. With this separation distance, the flexural power flow was measured to be $15 \pm 2\mu W$ with the optical sensor and $13 \pm 1\mu W$ with the accelerometer. The longitudinal power flow was estimated to be $130 \mu W$ with the optical sensor from data taken with a separation distance of a tenth of the longitudinal wavelength. Because of practical (but not fundamental) limitations, no estimates could be made with a larger separation distance between the data points. The total power flow measured optically was therefore estimated to be $145 \mu W$, a value that compares very well with the total mechanical input power of $150 \mu W$ calculated from the force and velocity data measured by the impedance head at the shaker. However, an analysis of measurement errors shows that such a good agreement between measured and predicted power flow is somewhat fortuitous.

Acknowledgements:

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References:


Figure captions

- Figure 1: Mechanical setup for simultaneous measurements of flexural and longitudinal power flow. The damping fins in the sandbox are shown schematically. The actual orientation of the fins is the $x - z$ plane.
- Figure 2: Optical arrangement
- Figure 3: Instantaneous out-of-plane amplitude measured optically along the beam at $t = t_0$ (solid line) and at $t = t_0 + 0.12$ ms (dashed line).
- Figure 4: Instantaneous in-plane amplitude measured (a) with an accelerometer and (b) optically, along the beam at $t = t_0$ (solid line) and at $t = t_0 + 0.12$ ms (dashed line).

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<td>41</td>
<td>32 33 22 48 41 33 41 32 37 35 36 36 35 34 41 35</td>
<td>35</td>
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2.3: Automated scanning and focusing laser system for structural acoustics

(Paper to be submitted to J. Acoust. Soc. Am., 12/96)

Wave decomposition of the vibrations of a cylindrical shell with an automated scanning laser vibrometer

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Abstract

Elastic waves propagating in a cylindrical shell have been detected by an automated scanning laser vibrometer designed to record in-plane surface motion over the surface of the shell (32 points axially and 32 points circumferentially). The structure was freely suspended in air and excited radially by a shaker at a single frequency, either below or above the ring frequency of the shell. A wave-vector analysis of the data was performed with a fast Fourier transform and an overdetermined modified extended Prony method. The results clearly show the presence of longitudinal, shear, and flexural waves above the ring frequency. In addition, the Prony method reveals the presence of evanescent waves due to mode conversion of the propagating waves near the ends of the shell. Below the ring frequency, two types of in-plane waves and flexural waves were identified. The results are in excellent agreement with predictions from the dispersion curves for thin shells.

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I. Introduction

To fully understand the radiation and scattering of sound from complex elastic structures, it is important to decompose the vibrational wave field inside the structure into its basic components (e.g., flexural, extensional, shear, torsional, evanescent, etc...). This requires measurements over a large portion of the structure so as to establish the spatial periodicities that characterize each type of wave. It is in general impractical to load a structure with a large array of accelerometers and, in recent years, laser Doppler vibrometry has emerged as a useful alternative for imaging vibrating structures.\(^1\). Commercially available scanning vibrometers are designed to measure out-of-plane surface motion and, consequently, they are well suited for the detection of flexural waves but they are somewhat insensitive to longitudinal or shear waves.

In this paper, we report experimental results obtained with a fully automated scanning laser vibrometer designed to measure in-plane surface motion over a cylindrical shell. The objective of the research is to establish that longitudinal, shear, flexural waves, and even evanescent waves can be detected separately with a laser vibrometer. This work is an extension of the research presented in Ref. 2. The experimental system is described in Section II. The wave-vector analysis of the data is summarized in Section III, and the results are given in Section IV.

II. Experimental System

LDV System

The optical arrangement of the LDV system is shown in figure 1. A linearly polarized 10 mW He-Ne laser produces a coherent beam with a wavelength \(\lambda = 632.8 \text{ nm}\). A beam splitter divides the beam into two beams for interferometric measurements. For heterodyne detection, one of two beams is frequency-modulated by an acousto-optic Bragg cell driven by a quartz oscillator at a frequency of 40 MHz. Both beams are coupled into polarization maintaining single mode fibers, and carried to the laser probe head. Use of the fibers allows the probe head to be mechanically decoupled from the optical bench, and provides the flexibility needed for the scanning measurement technique. The probe head is designed to take full advantage of optical fiber technology. It is compact (about 4 cm \(\times\) 4 cm \(\times\) 1 cm ) and it can be scanned along the structure at a distance of about 1.5 cm. The probe head
consists of two graded-index (GRIN) cylindrical lenses as transmitting optics, mounted on a lexan plate. The angle between the incident beams is 22°. The two GRIN lenses produce two focused laser beams that are made to overlap on the vibrating surface. The transmitting fibers are oriented and positioned in front of the GRIN lenses so that the same polarization state is kept for the two incident beams. This increases the fringe contrast of the interference pattern and, therefore, the signal-to-noise ratio of the measuring system. A multimode fiber with a large core diameter of 1000 \( \mu m \) is placed at the bisector of the two incident beams to collect the scattered light from the vibrating structure. Thus, the multimode fiber forms a receiving probe without any collecting lens. The multimode fiber also transfers the scattered light from the vibrating surface to a photodetector (an avalanche photodiode). The signal output of the photodiode is an FM signal whose carrier frequency is 40 MHz and whose instantaneous phase is proportional to the in-plane displacement of the vibrating surface or, alternatively, whose instantaneous frequency deviation from the 40 MHz carrier signal is proportional to the instantaneous in-plane surface velocity of the vibrating structure. \(^2,^3\)

A schematic diagram for the detection system is shown in figure 2. After filtering the low frequencies, the signal is amplified and mixed with a 40.1 MHz signal generated by a crystal oscillator. Mixing the signals produces a down-shifted signal at 100 kHz that is still modulated by the phase variations due to the surface vibration. After amplification and filtering of the 100 kHz FM signal, the signal is demodulated by a calibrated phase-locked loop (PLL).^4 The output signal from the PLL is the vibration signal, proportional to the instantaneous surface velocity at the focal point where the two beams overlap. The vibration signal is displayed on a digital oscilloscope. A GPIB board is used to transfer the vibration signal data from the digital oscilloscope (Tektronix 2430A) to a system computer.

Typical operating conditions for the LDV system are as follows. The light power of each beam incident on the vibrating surface is 1.7 mW. Each beam is focused on the surface (which is rough on the scale of an optical wavelength) to a circular spot with about 50 \( \mu m \) diameter. Light is reflected diffusively in all directions by the surface. The average light power received by the photodiode is about 2.5 \( \mu W \). The fringe visibility factor \( F_0 \) was measured and found to be \( F_0 \approx 0.1 \) for a number of rough surfaces including lathe-finished metallic surfaces. The minimum detectable displacement amplitude was determined experimentally to be \( \sim 1 \) nm in the frequency range of 200 Hz - 20 kHz, with a variable bandwidth between 1 kHz and 40 kHz in the receiving electronics, and with averaging the signal 64
times to reduce noise. The accuracy of the displacement measurement was determined by comparing the LDV data with measurements made using a calibrated accelerometer. The LDV measurements agreed with the accelerometer to within $\sim 1\text{nm}$. The repeatability of the measurements was also found to be $\sim 1\text{nm}$. The dynamic range of the system, for displacement measurements, is 50 dB. The lower limit is the minimum detectable displacement of 1 nm. The upper limit is set by the maximum deviation ($\sim 40\text{ kHz}$) in frequency which the PLL can track. This corresponds to a displacement of $\sim 300\text{ nm}$ at 10 kHz. The frequency range of the system is set by the PLL demodulator. The response of the demodulator used was flat over the range 200 Hz - 20 kHz. However, the optical probe has a very wide response, extending from very low frequencies to above 100 MHz. At frequencies below 100 Hz environmental noise (temperature fluctuations and vibration) increases significantly in the system.

**Cylindrical Shell**

The experimental model is a thin cylindrical shell made of #304 stainless steel. The shell has a radius $a = 7.55\ cm$, a thickness $h = 0.15\ cm$ and a length $L = 93.39\ cm$. In order to approximate the free-free boundary conditions at both ends of the shell, the shell is held between two aluminum end caps with four uniformly spaced pieces of materials such as neoprene and corprene along each end of the shell, as shown in figure 3. The two end caps are attached to each other by means of 3 threaded long rods which passed inside shell. The brass rod is clamped into the center of the upper cap. It provides vertical suspension from the rotary table. The shell is excited by a piezoelectric shaker driven with a continuous harmonic signal. The shaker is mounted normally to the shell with a 4-40 stud located $30\ cm (0.32L)$ above the bottom of the shell.

**Automated Scanning and Focusing Algorithm**

Wave-vector analysis of structural vibrations requires in general extensive vibration field data over a large portion of the structure. For practical implementation, it is therefore important to design a laser vibrometer with automated scanning capability and automated alignment of the optical probe head at each data point. Scanning was accomplished by means of a computer-controlled, motor-driven vertical linear positioner and rotary table which provided two degree of the freedom for the cylindrical geometry as shown in figure 2. Motion of the probe head towards or away from the shell (horizontal axis) was accomplished
by a computer-controlled motor-driven positioner controlling the overlapping and focusing of the two beams on the surface of the shell. (The angles between the two beams is fixed at 22° and it is not an adjustable parameter in the focusing algorithm). This horizontal positioner has a fine enough (12.7 μm advance per step with accuracy within 0.0125 cm/m) to achieve focusing of the two beams.

The key element in the detection system is to obtain a stable FM signal. An FM signal with large and stable amplitude indicates that the two beams overlap well and produce a good fringe contrast, thus ensuring accurate demodulation of the signal to extract the surface velocity data. A good measure of the amplitude and stability of the FM signal is its rms value. Therefore, the automated focusing algorithm is based on measuring the rms value of the FM signal as the probe head moves towards the surface by means of a computer-controlled fine stepping motor. Figure 4 shows a typical pattern of the rms value of the 40 MHz FM signal as a function of the distance of the probe to the vibrating surface, clearly indicating the range where one should take the data, i.e. demodulate the FM signal. Alignment is performed manually for the initial data point of the scan over the cylindrical structure and this optimum distance is used as an assumed optimum distance when the probe is repositioned to the next consecutive data point in the scan. After the first point of the scan, the system is fully automated. The probe head moves by steps of 25 μm towards and away from the shell, in a prescribed range (e.g. ±800μm around the assumed optimum distance). At every step, the rms value of the 40 MHz FM signal is directly acquired from Channel 1 on the oscilloscope. The optimum distance is found by comparing the measured rms values and, after scanning the full range, the probe head moves back to the newly determined optimum distance for the probe head. The newly acquired rms value of the FM signal is compared with the previously acquired maximum rms value of the FM signal. If the new amplitude of the FM signal is within some range of the maximum value (e.g. 80%), the FM signal is “accepted” and demodulated by the phase-locked loop. The demodulated vibration signal is averaged to improve the signal-to-noise ratio at the oscilloscope and acquired on Channel 2 of the oscilloscope. The signal is then stored as a data file in the system computer. The same procedure is repeated at the next data point in the scan around the cylinder. All those procedures are controlled by a computer program except for the manual alignment at the initial data point.

Measurement Procedures
In the experiments reported in this paper, the shaker was driven with a continuous single frequency signal. In order to identify the low- and high-frequency characteristics of wave propagation on the shell, two frequencies were chosen, one below and one above the ring frequency of the shell. Those were 9.238 Hz and 18.275 Hz, respectively, because a good signal-to-noise ratio was observed at those frequencies which correspond to 0.84 fr and 1.65 fr, respectively, where fr is the ring frequency. The circumferential and axial components of the vector displacement were measured at f = 18.275 Hz by positioning the probe head either along a circumferential axis or along an axial line, i.e., by rotating the probe by 90°. Measurements were made over an area defined by z = 40.38 cm to z = L = 93.39 cm, and φ = 0 to 2π, where z = 0 refers to the bottom end of the cylinder, and (z = 30 cm, φ = 0) refers to the location of the shaker. The axial component of the vector displacement was measured over the surface of the shell, above the shaker, at f = 9.238 Hz. The spacing of the measurement points over the scanning area was determined so as to avoid spatial aliasing. It led to the choice of 32 points circumferentially and 32 points axially at both frequencies.

III. Wave-vector analysis

Elastic waves excited at a given frequency travel in the shell at different phase velocities and in different directions, according to the dispersion relation. Each wave belongs to one of three types: (1) a propagating wave with purely real wavenumber, (2) an evanescent wave with a purely imaginary wavenumber, (3) an oscillatory decaying wave with a complex wavenumber. The propagating waves include fast waves (longitudinal, shear) with low wavenumber and slow waves (flexural) with high wavenumbers. The wave-vector representation of the spatial motion allows to identify the wavenumbers and therefore to separate the contributions of each type of waves to the overall structural vibration.

Consider the displacement field defined by the axial, circumferential, and normal components of the displacement vector \((u(a, \phi, z, f), v(a, \phi, z, f), w(a, \phi, z, f))\) over the surface of the cylindrical shell of radius \(a\) for the cylindrical coordinates \((r, \phi, z)\) and with the suppressed time dependence \(e^{-i2\pi ft}\) at a frequency \(f\). The wave-vector decomposition is based on the Fourier transforms of the displacement components \(u, v\) and \(w\) into helical wave
components $U, V$ and $W$. For example, the Fourier transform of $u(a, \phi, z, f)$ is expressed as

$$U(n, k_z, f) = \frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dt \, e^{-i2\pi f t} \, u(\phi, z, t).$$

where $n$ is the wavenumber index in the circumferential direction and $k_z$ the wavenumber in the axial direction. The quantity $U$ physically represents the complex amplitude of a helical wave propagating in the direction defined by the wave-number vector $\vec{k} = k_z \hat{z} + (n/a) \hat{\phi}$. Once the spatial distribution of the displacement field $u(\phi, z, t)$ is measured, the complex amplitude $U(n, k_z, \omega)$ can be predicted by taking the spatial Fourier integral transform of $u(\phi, z, t)$. The three dimensional plot of the magnitudes of the complex amplitudes in the wave-number-plane gives the so-called helical wave spectrum from which the dominant waves can be identified.

The fast Fourier transform (FFT) algorithm is well suited to perform the wave-vector representation of Eq. (1) provided that the spatial sampling in the circumferential and axial directions is fine and long enough to detect the periodicities of interest on the shell without any spatial aliasing and with good resolution. In the experiments reported below, the FFT method was appropriate in the circumferential direction but it was inaccurate in the axial direction because of the relatively short length of the shell in terms of the longitudinal wavelength. Furthermore, the FFT algorithm fails to identify the imaginary wavenumbers (evanescent waves, or decaying waves) because the vibration signal is represented as a summation of harmonic contribution by propagating waves with real wavenumbers. As a substitute for the axial wavenumber decomposition, we have used the extended Prony method. The Prony method models the data with $p$ exponentials of arbitrary complex amplitudes (magnitudes and phases) and complex wavenumbers (wavenumbers and damping terms) which are found by a least-square minimization of the difference between the original and the reconstructed data sets. The detailed algorithm can be found in [6]. The method is well suited for cases where the data is indeed a sum of complex exponentials, and when the order of the model (i.e., the number of exponentials) can be correctly guessed. It is then a very effective algorithm, even if the number of original data points is relatively small. However, the method is very sensitive to noise in the data. The overdetermined modified extended (OME) Prony method has been developed precisely to increase the robustness of the method to noise inherent in any real data. The basic idea behind the OME Prony method is to perturb the system and observe the behavior of the poles (wavenumbers) in the complex plane. The physical poles remain at a constant location while the noise-induced
poles wander in the complex plane. The reader is referred to references [6-9] for a more elaborate discussion on the procedure.

IV. Results

Above the ring frequency

Figure 5(a) shows the helical wave spectrum \( U(n, k_z, f) \) obtained by the OME Prony decomposition of the axial displacement field \( u(r = a, \phi, z, t) \) measured with the laser vibrometer at the frequency 18.275 Hz, or 1.65 times the ring frequency. Similarly, Figure 5(b) represents the helical wave spectrum \( V(n, k_z, f) \) obtained from the measured circumferential displacements \( v(r = a, \phi, z, t) \) at that same frequency. Each peak on the helical wave spectrum is associated with the amplitude of a wave propagating in the direction defined by the corresponding wavenumber vector. Strong peaks appear in the low wavenumber region while weak peaks appear in the high wavenumber region. The strong peaks are associated with in-plane waves (longitudinal and shear waves) which dominantly excite the in-plane motion of the shell at that frequency. It should be noted that the OME Prony decomposition allows one to resolve effectively the low wavenumber peaks, whereas results based on the FFT decomposition (not shown here) were unable to do so. In order to relate the peaks of the helical wave spectra to a given type of wave, it is instructive to plot the predicted dispersion curve of the shell (based on Donnell's shell equations) and compare the result with the measured peaks of the helical wave spectra shown in Figure 5. The result is shown in Figure 6 in which the solid lines are the predictions with the outer most line representing the flexural waves, the middle one representing the shear waves, and the inner most one following the longitudinal waves. In figure 6, the small circles correspond to the measured wavenumbers which are in excellent agreement with the predicted values. Clearly, in-plane surface measurements obtained by laser vibrometry can be used to separate the contributions of longitudinal, shear, and flexural waves propagating in a cylindrical shell excited harmonically above its ring frequency.

Below the ring frequency

Figure 7 shows the helical wave spectrum \( U(n, k_z, f) \) of the measured axial displacement field at a frequency 9238 Hz (0.84 times the ring frequency of the shell). Four strong peaks
are present in the low wavenumbers region and weak peaks follow a "figure 8" pattern which is characteristic of waves excited below the ring frequency. Figure 8 shows a comparison of the predicted dispersion curves with the measured data. The elliptic curve around the origin of the wavenumber plane corresponds to in-plane waves (type I) whose characteristics change from longitudinal wave mode to shear wave mode as their propagation directions rotate from the axial direction ($n = 0$) to the circumferential direction ($k_z = 0$). The "figure 8" curve is associated with two different types of propagating waves. The top and bottom of the "figure 8" curve correspond to flexural waves whereas the dip near small values of $n$ is associated with in-plane waves (type II) whose phase velocities are much smaller than the speed of either longitudinal or shear waves propagating in the axial direction. Again excellent agreement is obtained between theoretical predictions and experimental values.

It should be noted that the characteristics of flexural waves are very sensitive to the thickness of the shell while those of in-plane waves have little dependence on it. Thus the dispersion curves of flexural waves are very sensitive to the variation of the thickness of the shell. Note that the actual thickness of the shell is within the permissible variation of $\pm 10\%$ relative to the thickness $h$, which has been provided by the manufacturer and satisfies the ASTM (American Society of Testing and Materials) standard. The average value of thickness measured at both ends of the shell was $0.89h$. In figures 6 and 8, the theoretical dispersion curves were evaluated with the thickness $0.9h$ to fit the experimental results at both frequencies.

**Evanescent Waves**

An interesting result occurs when the OME Prony method is used to analyze separately the axial wave field for $n = 3$ along the length of the shell, i.e., when evaluating $U(n = 3, z, f)$ (at $r = a$). The result is shown in Figure 9 where the horizontal axis is the axial location along the length of the shell and where the vertical axis is the normalized amplitude of the axial wave field for $n = 3$. The solid line represents the measured data from one end of the shell to approximately the center of the shell. It clearly reveals the presence of an evanescent wave that extends from one end of the shell to its mid-point. The OME Prony decomposition of the axial data shows the presence of a wavenumber such that $k_z a = -0.11 + 0.92i$. (The value obtained from the dispersion relation is $k_z a = 0.98i$.) These evanescent waves (large imaginary wavenumber) are the result of the mode conversion of the flexural waves into
evanescent waves at the end of the shell. The evanescent waves are usually generated and localized to the region close to the structural discontinuities or the source point. However, the result in Fig.9 shows that a strong evanescent wave field can be excited along about half the length of the shell.

V. Summary

An automated scanning laser vibrometer has been built and automated to measure the axial and circumferential components of the displacement field over the surface of a cylindrical shell freely suspended in air, radially excited by a shaker at a single frequency. The data was collected over a set of 32 points circumferentially and 32 points axially. A wave-vector analysis of the data was performed with a fast Fourier transform (for the circumferential direction) and an overdetermined modified extended Prony method (for the axial direction). The results clearly show the presence of longitudinal, shear, and flexural waves above the ring frequency. In addition, the Prony method reveals the presence of evanescent waves due to mode conversion of the propagating waves near the ends of the shell. Below the ring frequency, two types of in-plane waves and flexural waves were identified. The results are in excellent agreement with predictions from the dispersion curves for thin shells.

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References:


Figure captions

- Figure 1: Optical arrangement
- Figure 2: Detection and scanning
- Figure 3: Shell
- Figure 4: Depth of field
- Figure 5: (a) Helical wave spectrum $U(n, k_z, f)$ of the measured axial displacement field at $f = 18,275$ Hz, using the FFT for the circumferential detection and the OME Prony method for the axial direction. (b) Helical wave spectrum $V(n, k_z, f)$ of the measured circumferential displacement field at $f = 18,275$ Hz, using the FFT for the circumferential detection and the OME Prony method for the axial direction.
- Figure 6: Comparison between wavenumber reconstruction from the experimental data (circles) and the theoretical dispersion curves (solid lines) calculated at $f = 18,275$ Hz.
- Figure 7: Helical wave spectrum $U(n, k_z, f)$ of the measured axial displacement field at $f = 9,238$ Hz. using the FFT for the circumferential detection and the OME Prony method for the axial direction.
- Figure 8: Comparison between wavenumber reconstruction from the experimental data (circles) and the theoretical dispersion curves (solid lines) calculated at $f = 9,238$ Hz.
- Figure 9: Decomposition of the axial wave field for $n = 3$ from the experimental data by means of the OME Prony method to illustrate the presence of an evanescent wave.
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3. Publications, Conference Presentations, and Research Dissemination


- F. Guillot, J. Jarzynski, and E. Balizer, "A fiber optic dual-beam laser Doppler vibrometer for measurement of electrostrictive and piezoeactive response of thin films".
129th meeting of the Acoustical Society of America (Washington DC, June 1, 1995).


Students who have graduated with support from this contract:

- H. - G. Kil, Ph.D., August 1995
- R. L. Willis, M.S., December 1995
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