Quantum Music: Applying Quantum Theory to Music Theory and Composition

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QMATH 13, Georgia Tech, October 10, 2016
Music is a collection of sounds, or *tones*, that feature pitch, beat, and volume. Music consists of 3 elements:

- **Melody**: A set of musical notes, or tones, that are played in succession.
- **Harmony**: A set of musical notes that are played simultaneously.
- **Rhythm**: Duration of these sounds (or lack of sounds) in time.
Traditionally, sounds called *whole tones* are represented as follows:

\[
\begin{align*}
&\text{C} \quad \text{D} \quad \text{E} \quad \text{F} \quad \text{G} \quad \text{A} \quad \text{B} \\
&\text{C} \quad \text{C}^\# \quad \text{D} \quad \text{D}^\# \quad \text{E} \quad \text{F} \quad \text{F}^\# \quad \text{G} \quad \text{G}^\# \quad \text{A} \quad \text{A}^\# \quad \text{B}
\end{align*}
\]

For a single octave, all of the sounds used in music are represented by the *Chromatic scale*:
Two elements of music notation that are used to denote pitch are $b$, the *flat* and $\#$, the *sharp*.

An alternate way to represent tones is by using the number 0 to represent no sound, and the numbers 1 – 12 as follows: $C \rightarrow 1$, $C\# \rightarrow 2$, $D \rightarrow 3$, ..., $A\# \rightarrow 11$, $B \rightarrow 12$
How to quantize Music?

A quantum tone can be represented be a quantum musical state $|\psi\rangle$. A quantum musical state would be defined as:

$$|\psi\rangle = \alpha_0 |\psi_0\rangle + \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle + \ldots + \alpha_{12} |\psi_{12}\rangle$$  \hspace{1cm} (1)

With $|\psi\rangle \in \mathbb{C}^{13}$ and $\sum_i |\alpha_i|^2 = 1$. Here, $|\psi_i\rangle$ are the orthogonal unit vectors:

$$|\psi_0\rangle = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad |\psi_1\rangle = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \quad \ldots, \quad |\psi_{12}\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$
We can also define a flat and sharp operators such that:

\[
\hat{b} |\psi_i\rangle = \psi_{i-1} |\psi_{i-1}\rangle \quad (2)
\]

\[
\hat{\sharp} |\psi_i\rangle = \psi_{i+1} |\psi_{i+1}\rangle \quad (3)
\]
Melody: A simple example

We start by constructing a simple system of two quantum tone states

\[ |\psi\rangle_1 = \frac{1}{\sqrt{2}} |\psi_0\rangle + \frac{1}{\sqrt{2}} |\psi_1\rangle \]  \hspace{2cm} (4)

\[ |\psi\rangle_2 = \frac{1}{\sqrt{2}} |\psi_0\rangle + \frac{1}{\sqrt{2}} |\psi_1\rangle \]  \hspace{2cm} (5)
The system would have the following 4 possible outcomes, with equal probabilities:

```
\begin{music}
\begin{notation}
\begin{notation}
\begin{notation}
\end{notation}
\end{notation}
\end{notation}
\end{music}
```
Similarly for the following system quantum tone states,

\[ |\psi\rangle_3 = \frac{1}{\sqrt{3}} |\psi_0\rangle + \frac{1}{\sqrt{3}} |\psi_1\rangle + \frac{1}{\sqrt{3}} |\psi_6\rangle \]  \hspace{1cm} (6)

\[ |\psi\rangle_4 = \frac{1}{\sqrt{3}} |\psi_0\rangle + \frac{1}{\sqrt{3}} |\psi_1\rangle + \frac{1}{\sqrt{3}} |\psi_6\rangle \]  \hspace{1cm} (7)
The system would have the following 9 possible outcomes, with equal probabilities:
An advantage of this representation is that melodies can be composed by applying transformations, to tone states. In the previous example,

$$|\psi\rangle_4 = \hat{1} |\psi\rangle_3$$  \hspace{1cm} (8)
Quantum Harmony can be thought as an entangled state of more than one quantum note, and could be represented using quantum chords.
Challenges and Future Work

- Incorporating rhythm.
- Incorporating volume.
- Finally, how to implement all of these ideas?
Introduction: What is Music?
How to quantize Music?
Melody: A simple example
Harmony
Challenges and Future Work

Bibliography

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