

Matrix product approximations to multipoint functions in two-dimensional conformal field theory

Robert Koenig (TUM) and Volkher B. Scholz (Ghent University)
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Goal

- Understand the entanglement structure of quantum field theories using tensor network methods

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- Tensor networks model the entanglement properties of many body systems and are successfully applied in condensed matter physics
- What about quantum field theories?
- States of the quantum field theory and tensor network states live in different Hilbert spaces: how to measure closeness?

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- Focus on physical quantities: correlation functions
- If tensor networks can approximately reproduce correlation functions of quantum field theories, then we can use them to understand the entanglement structure of quantum field theories.
- Start with simplest interesting class of quantum field theories: 1+1 dimensional unitary Conformal Field Theories (a quantum field theory defined on the circle with conformal symmetry)

Recap: Matrix product states

- Tensor network states for spin chains:



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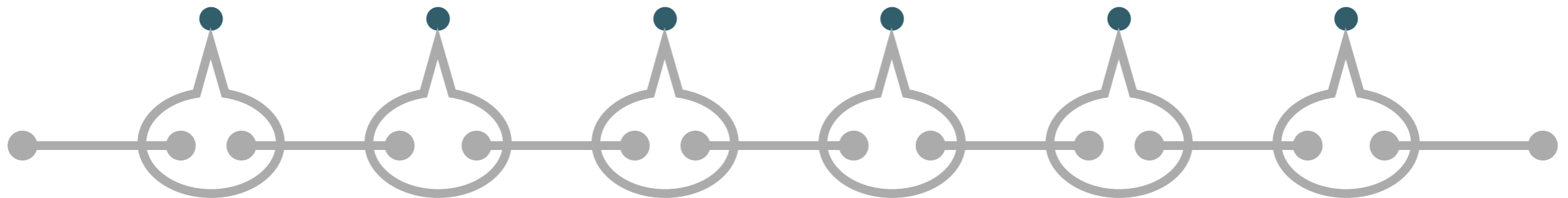
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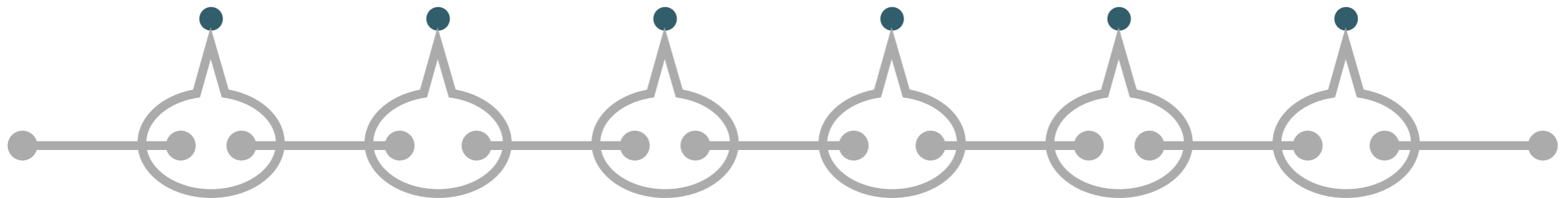
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- Correlation functions can be computed efficiently (in D) and reduce to the computation of a sequence of completely positive maps on matrices of dimension D

Main result

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Scaling of Parameters:

number of fields n , UV cutoff d (measured in terms of energy), approximation error ε , C constant depending on CFT (not necessarily central charge)

scaling of bond dimension	fixed n , UV cutoff d	fixed ε
$\log(D)$	$\sim \log(1/\varepsilon) C n/d$	$\sim \sqrt{C n}$

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- However, the parameter scaling is worse than would be expected from entropic arguments [Cardy&Calabrese, Holzhey et. al.,...]
- Uses the language of Vertex operator algebras: first introduced by Borcherds in his proof of the Moonshine conjecture

More Symmetries: Wess-Zumino-Witten models

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- In addition to conformal symmetries, WZW models possess an additional local symmetry given by an affine Lie algebra based on a simple compact Lie group
- These additional symmetries carry over to the MPS Tensors; leads to a group invariant MPS
- Moreover, the interactions (fusion rules) are completely described already in the lowest level; the higher order Tensors are only needed to model the conformal and affine symmetries

Proof sketch: regularization

- identify states with Hilbert-Schmidt operators on the chiral theory; field operators become linear maps: need to approximate by finite-dimensional ones

$$\langle \phi_1(x_1) \quad \phi_2(x_2) \quad \dots \quad \phi_n(x_n) \rangle$$

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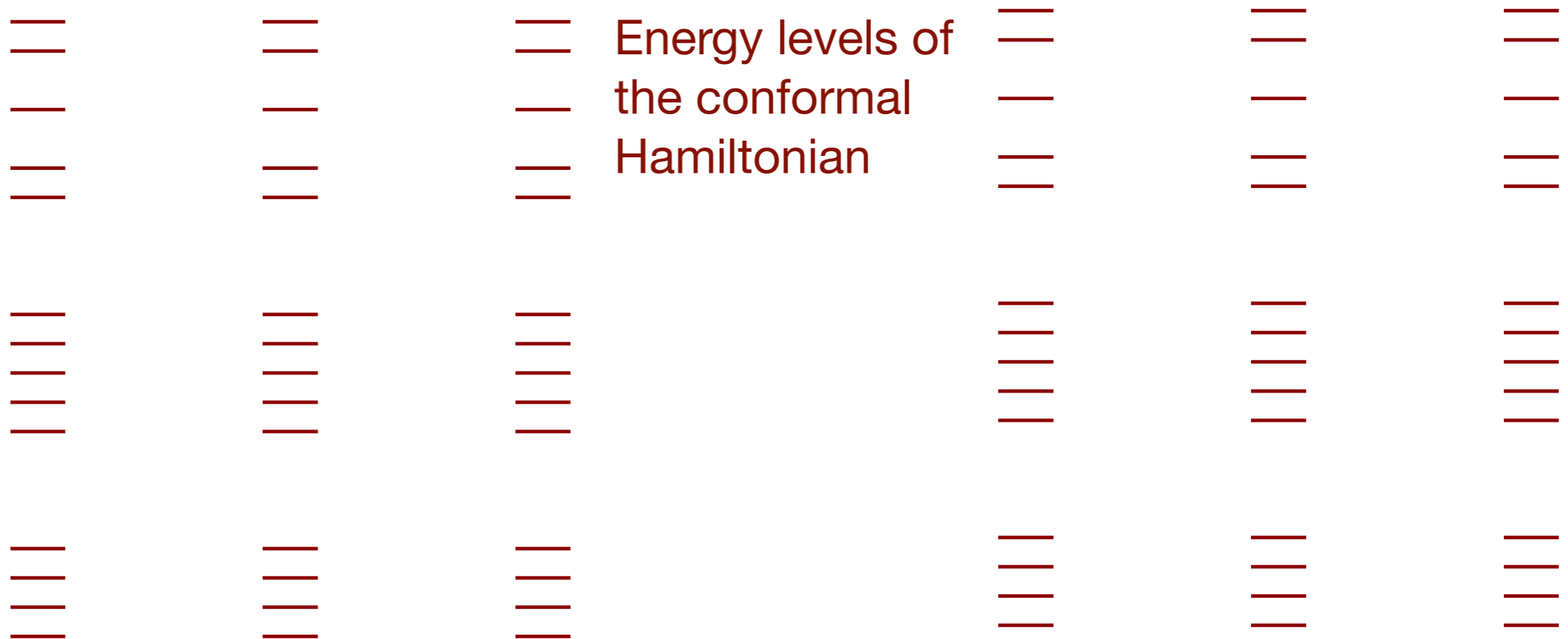
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- techniques: use results of Wassermann for WZW models (explicit bounds), and the existence of genus-1 correlation functions for general CFTs [Zhu, Huang]

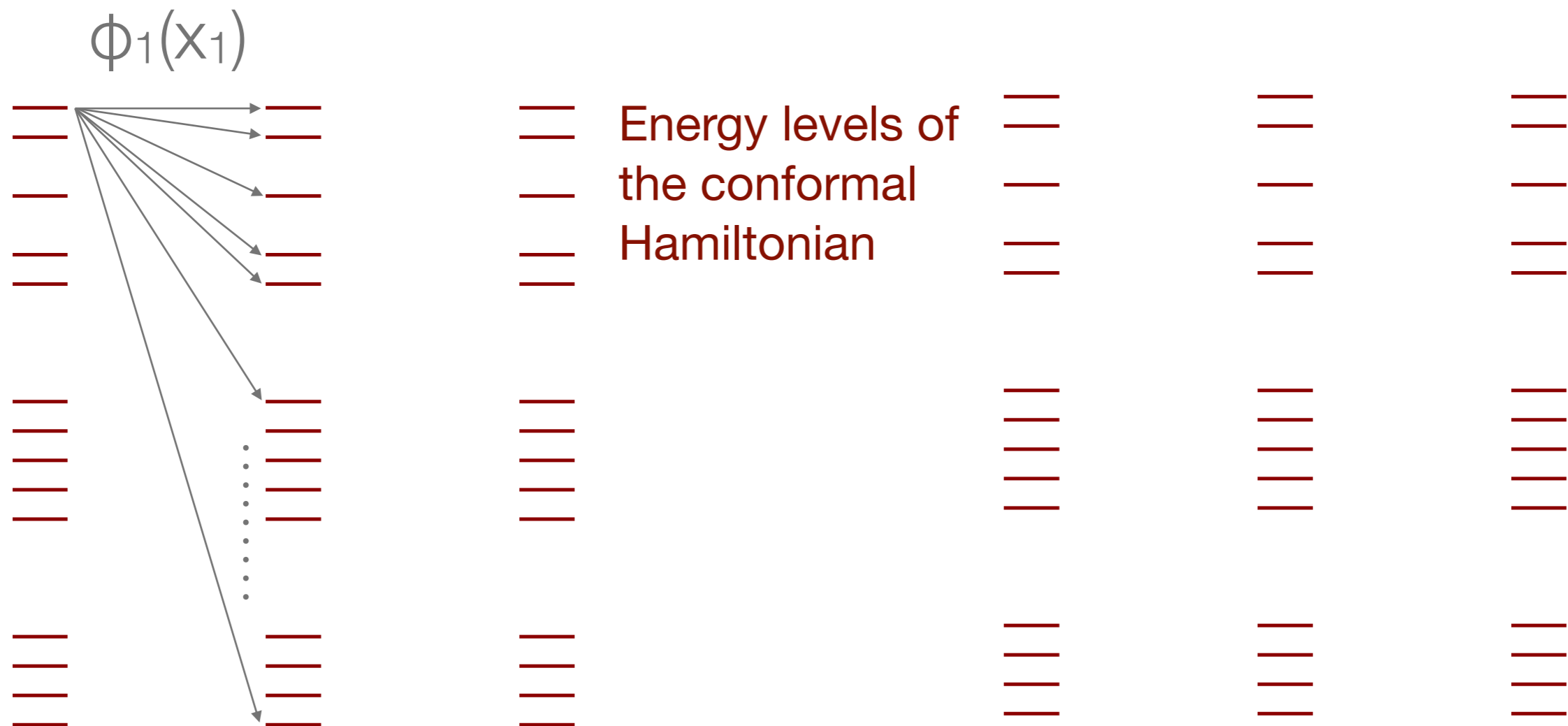
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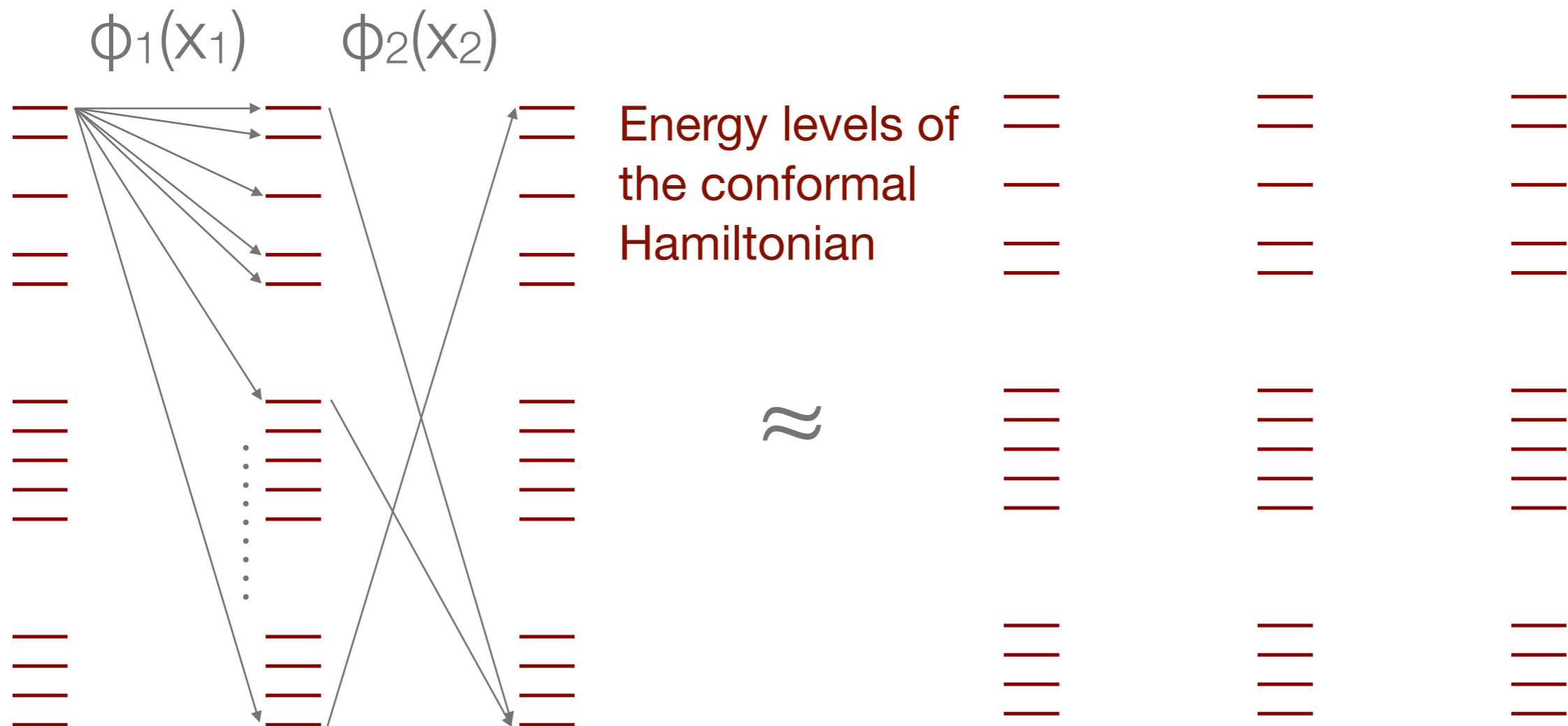
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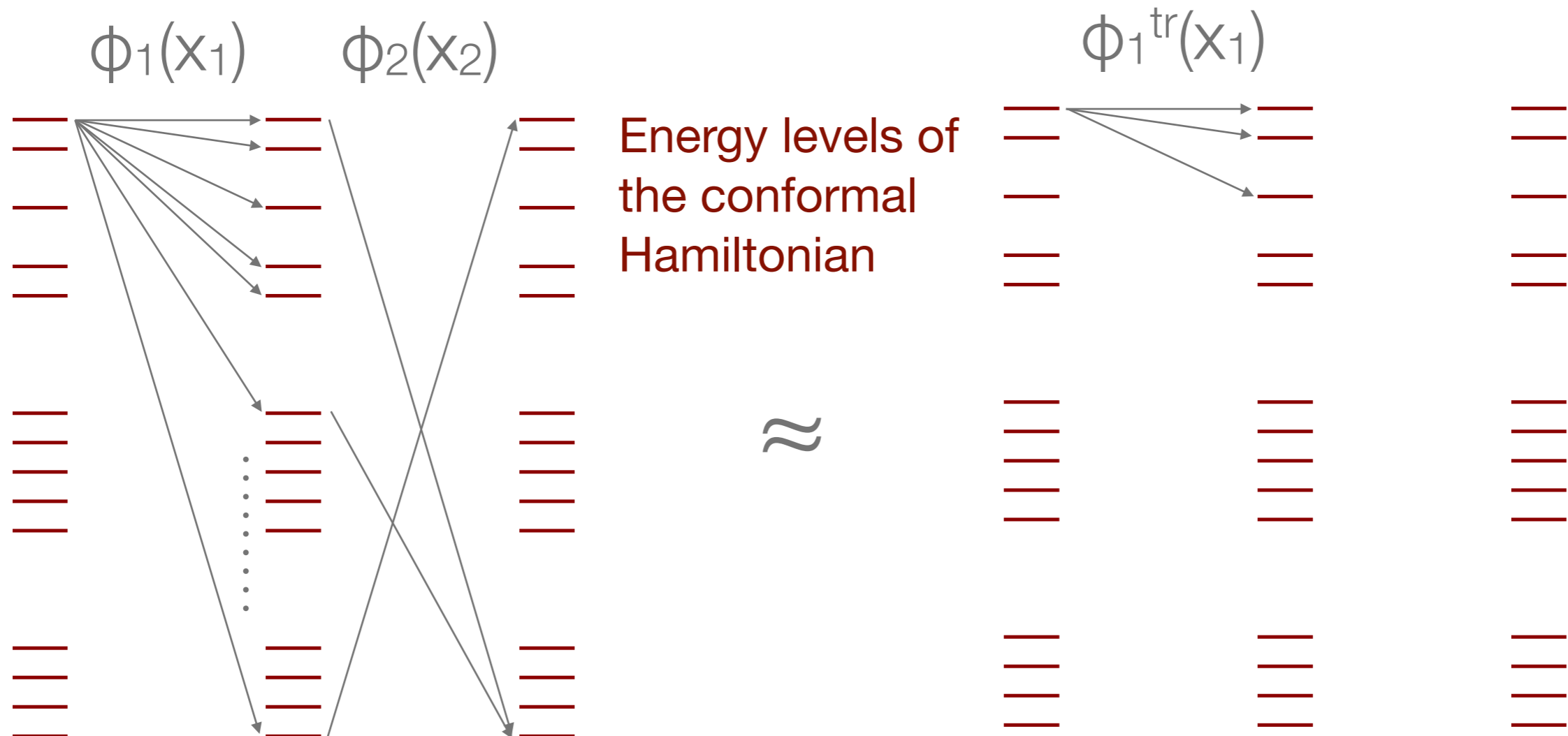
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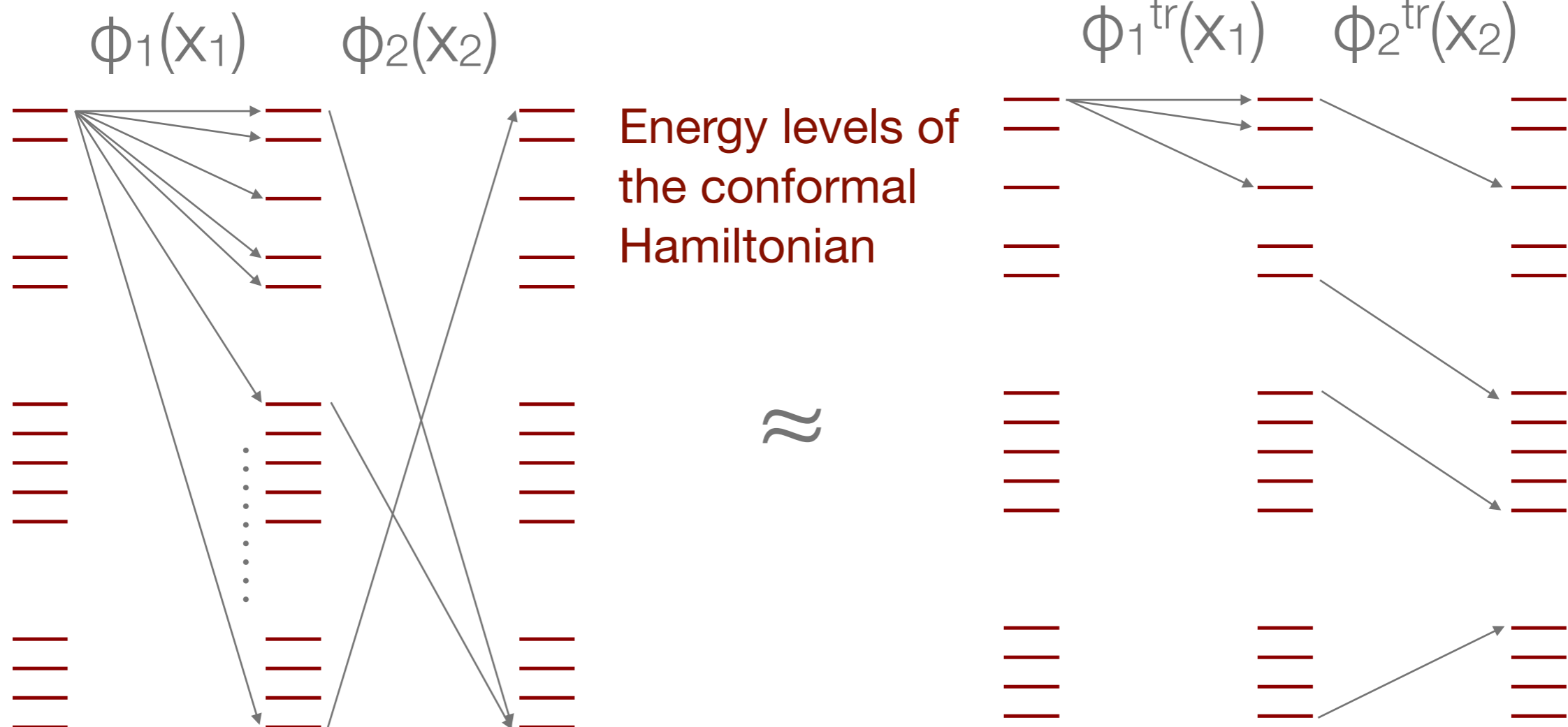
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- Our Approximations are constructive, provide rigorous error bounds and respect additional symmetries (WZW)
- Can be analysed further to understand low energy states of CFTs in quantum information theoretic terms (connection to quantum error correction?)
- Generalisation to MERA (multiscale entanglement renormalization Ansatz) seems possible and may provide better parameter scaling