Effects of spin-orbit coupling on the BKT transition and the vortex-antivortex structure in 2D Fermi Gases

Carlos A. R. Sa de Melo
Georgia Institute of Technology

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Main References for Talk

Ultra-cold fermions in the flatland: evolution from BCS to Bose superfluidity in two-dimensions with spin-orbit and Zeeman fields

Li Han and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
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Effects of Spin-Orbit Coupling on the Berezinskii-Kosterlitz-Thouless Transition and the Vortex-Antivortex Structure in Two-Dimensional Fermi Gases

Jeroen P. A. Devreese,1,2 Jacques Tempere,2,3 and Carlos A. R. Sá de Melo1
1School of Physics, Georgia Institute of Technology, Atlanta 30332, USA
2TQC, Universiteit Antwerpen, B-2610 Antwerpen, Belgium
3Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
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Li Han

Ian Spielman

Jacques Tempere

Jeroen Devreese
Outline

1) Introduction to 2D Fermi gases.

2) Creation of artificial spin-orbit coupling (SOC).

3) Quantum phases and topological quantum phase transitions of 2D Fermi gases with SOC.

4) The BKT transition and the vortex-antivortex structure.

5) Conclusions
Conclusions in words

• Ultra-cold fermions in the presence of spin-orbit and Zeeman fields are special systems that allow for the study of exciting new phases of matter, such as topological superfluids, with a high degree of accuracy.

• Topological quantum phase transitions emerge as function of Zeeman fields and binding energy for fixed spin-orbit coupling.
Conclusions in words

• The critical temperature of the BKT transition as a function of pair binding energy is affected by the presence of spin-orbit effects and Zeeman fields. While the Zeeman field tends to reduce the critical temperature, SOC tends to stabilize it by introducing a triplet component in the superfluid order parameter.

• In the presence of a generic SOC the sound velocity in the superfluid state is anisotropic and becomes a sensitive probe of the proximity to topological quantum phase transitions. The vortex and antivortex shapes are also affected by the SOC and acquire a corresponding anisotropy.
Conclusions in Pictures

Change in topology

TRANSITION FROM GAPLESS TO GAPPED SUPERFLUID
BKT transition and vortex-antivortex structure
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Condensed Matter meets Atomic Physics

In optical lattices many types of atoms can be loaded like bosonic, Sodium-23, Potassium-39, Rubidium-87, or Cesium-133; and fermionic Lithium-6, Potassium-40, Strontium-87, etc...

In real crystals electrons or holes (absence of electrons) may be responsible for many “electronic” phases of condensed matter physics, such as metallic, insulating, superconducting, ferromagnetic, anti-ferromagnetic, etc...

Neutral atoms (bosons or fermions)

Electrons of holes (fermions only)
How atoms are trapped?

- Atom-laser interaction
- Induced dipole moment.
- Trapping potential

\[ V(r, t) = -d \cdot E(r, t) \]
\[ d = -\alpha(\omega)E(r, t) \]
\[ V(r, t) = -\alpha(\omega)[E(r, t)]^2 \]
Atoms in optical lattices

\[ V(r) = -\alpha(\omega) < [E(r,t)]^2 > \]

\[ V(r) = -\frac{1}{2} \alpha(\omega) [E(r)]^2 \]
How optical lattices are created?

- **a** Laser beam
- **b** Laser standing wave
- **c** Potential well
- **d** ...
Single plane excitations

Vortex-antivortex pairs

BKT transition:
Physics of 2D XY model
Critical Temperature

Pairing Temperature

Fermi Liquid

Bose Liquid

0.125

BCS-Bose Superfluidity in 2D
2D Fermi gases with increasing attractive interactions, but no SOC.

Vortex-Antivortex Lattice in Ultracold Fermionic Gases

S. S. Botelho and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
(Received 14 September 2005; published 3 February 2006)

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Observation of a Two-Dimensional Fermi Gas of Atoms

Kirill Martiyanov, Vasilyi Makhalov, and Andrey Turlapov

Institute of Applied Physics, Russian Academy of Sciences, ul. Ulyanova 46, Nizhniy Novgorod, 603000, Russia
(Received 20 May 2010; published 15 July 2010)
Outline

1) Introduction to 2D Fermi gases.

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5) Conclusions
Raman process and spin-orbit coupling

\[
\left( \frac{(k - k_R)^2}{2m} + \frac{\delta}{2} \right) + \frac{\Omega}{2} - \frac{(k + k_R)^2}{2m} - \frac{\delta}{2} \right)
\]

\[
\hbar \omega_Z
\]

\[
| -1 \rangle = | \downarrow \rangle
\]

\[
| 0 \rangle = | \uparrow \rangle
\]

18
SU(2) rotation to new spin basis:

\[ \sigma_x \rightarrow \sigma_z; \quad \sigma_z \rightarrow \sigma_y; \quad \sigma_y \rightarrow \sigma_x \]
Spin–orbit–coupled Bose–Einstein condensates

Y.-J. Lin¹, K. Jiménez–Garcia¹,² & I. B. Spielman¹

\[
\begin{align*}
\left(\frac{k^2 + k_R^2}{2m} + \Omega \right) - i\left(\frac{\delta}{2} - \frac{k_R}{m} k_x\right) - i\left(\frac{\delta}{2} - \frac{k_R}{m} k_x\right)
\end{align*}
\]

spin-orbit

detuning

Raman coupling
Hamiltonian with spin-orbit

\[ H = \sum_{k,s} \varepsilon(k) c_{ks}^+ c_{ks} - \sum_{k,s} h_{s',s}(k) c_{ks'}^+ c_{ks} \]
Parallel and perpendicular fields

\[ h_{\parallel}(k) = h_z(k) \]

\[ h_{\perp}(k) = h_x(k) - ih_y(k) \]

\[ H_0(k) = \begin{pmatrix} \varepsilon(k) - h_{\parallel}(k) & -h_{\perp}(k) \\ -h^*_\perp(k) & \varepsilon(k) + h_{\parallel}(k) \end{pmatrix} \]
Hamiltonian in terms of $k$-dependent magnetic fields

Hamiltonian Matrix

$$ H_0(k) = \varepsilon(k)1 - h_x(k)\sigma_x - h_y(k)\sigma_y - h_z(k)\sigma_z $$

Momentum Space Two-Level System

in a momentum dependent magnetic field

$$ h(k) = [h_x(k), h_y(k), h_z(k)] $$
Eigenvalues

\[ \varepsilon_{\uparrow} (k) = \varepsilon(k) - |h_{\text{eff}} (k)| \]

\[ \varepsilon_{\downarrow} (k) = \varepsilon(k) + |h_{\text{eff}} (k)| \]

\[ |h_{\text{eff}} (k)| = \sqrt{\left| h_x (k) \right|^2 + \left| h_y (k) \right|^2 + \left| h_z (k) \right|^2} \]
Rashba Spin-Orbit Coupling

\[ H_R(k) = v_R \begin{pmatrix} 0 & k_y + i k_x \\ k_y - i k_x & 0 \end{pmatrix} \]
Equal-Rashba-Dresselhaus (ERD) Spin-Orbit Coupling

\[ H_R(\mathbf{k}) = v_R \begin{pmatrix} 0 & k_y + ik_x \\ k_y - ik_x & 0 \end{pmatrix} \]

\[ H_D(\mathbf{k}) = -v_D \begin{pmatrix} 0 & k_y - ik_x \\ k_y + ik_x & 0 \end{pmatrix} \]

\[ H_{ERD}(\mathbf{k}) = v \begin{pmatrix} 0 & ik_x \\ -ik_x & 0 \end{pmatrix} \]
Energy Dispersions in the ERD case

Simpler case:

\[ h_x(k) = 0 \]
\[ h_y(k) = \nu k_x \]
\[ h_z(k) = 0 \]
\[ \varepsilon_{\uparrow}(k) = \varepsilon(k) - |\nu k_x| \]
\[ \varepsilon_{\downarrow}(k) = \varepsilon(k) + |\nu k_x| \]

\[ \varepsilon(k) = \frac{k^2}{2m} \]
Energy Dispersions and Fermi Surfaces

\[ \varepsilon_\alpha(k) = \frac{k^2}{2m} \pm |v k_x| \]
Momentum Distribution (Parity)

\[ h_x(k) = 0 \]
\[ \frac{h_y(k)}{\varepsilon_F} = 0.71 \frac{k_x}{k_F} \]
\[ \frac{h_z(k)}{\varepsilon_F} = 0.05 \]
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Bring Interactions Back (real space)

\[ \mathcal{H}(r) = \mathcal{H}_0(r) + \mathcal{H}_I(r) \]

\[ \mathcal{H}_0(r) = \sum_{\alpha\beta} \psi_\alpha^\dagger(r) \left[ \hat{K}_\alpha \delta_{\alpha\beta} - h_i(r) \sigma_{i,\alpha\beta} \right] \psi_\beta(r) \]

- Kinetic Energy
- Spin-orbit and Zeeman

\[ \mathcal{H}_I(r) = -g \psi_\uparrow^\dagger(r) \psi_\downarrow^\dagger(r) \psi_\downarrow(r) \psi_\uparrow(r) \]

Contact Interaction
Bring Interactions Back
(momentum space)

\[ \mathcal{H}_I = -g \sum_q b^\dagger(q) b(q) \]

\[ b^\dagger(q) = \sum_k \psi_k^\dagger(k + q/2) \psi_k^\dagger(-k + q/2) \]

\[ \Delta_0 = -g \langle b(q = 0) \rangle \quad \text{and} \quad \Delta_0^* = -g \langle b^+(q = 0) \rangle \]
Bring interactions back:
Hamiltonian in initial spin basis

\[ H_0 = \begin{pmatrix}
\tilde{K}_{\uparrow}(k) & -h_\perp(k) & 0 & -\Delta_0 \\
-h_\perp^*(k) & \tilde{K}_{\downarrow}(k) & \Delta_0 & 0 \\
0 & \Delta_0^\dagger & -\tilde{K}_{\uparrow}(-k) & h_\perp^*(-k) \\
-\Delta_0^\dagger & 0 & h_\perp(-k) & -\tilde{K}_{\downarrow}(-k)
\end{pmatrix} \]

\[ \tilde{K}_s(k) = \varepsilon(k) - \mu - sh_z(k) \]
Bring interactions back: Hamiltonian in the generalized helicity basis.

\[ \tilde{H}_0 = \begin{pmatrix} 
\xi_{\uparrow}(k) & 0 & \Delta_T(k)e^{-i\varphi_k} & -\Delta_S(k) \\
0 & \xi_{\downarrow}(k) & -\Delta_T^*(k)e^{-i\varphi_k} & \Delta_S(k) \\
\Delta_T^*(k)e^{i\varphi_k} & -\Delta_S^*(k) & -\xi_{\uparrow}(k) & 0 \\
\Delta_S^*(k) & -\Delta_T(k)e^{i\varphi_k} & -\xi_{\downarrow}(k) & 0 
\end{pmatrix} \]

\[ \varphi_k = \text{Arg} [h_\perp(k)] \]
Order Parameter: Singlet & Triplet

\[ \Delta_S(k) = \Delta_0 \frac{h_x(k)}{|h_{\text{eff}}(k)|} \]

\[ \Delta_T(k) = \Delta_0 \frac{h_y(k)}{|h_{\text{eff}}(k)|} \]

\[ |\Delta_T(k)|^2 + |\Delta_S(k)|^2 = |\Delta_0|^2 \]

\[ h_\perp(k) = \nu k_x \quad h_z(k) = h_z \]

\[ h_{\text{eff}}(k) = (0, \nu k_x, h_z) \]

\[ h_{\text{eff}}(k) = \sqrt{\nu^2 k_x^2 + h_z^2} \]
Excitation Spectrum

\[ E_1(k) = \sqrt{\left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right)^2 + \left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(k)|^2} + |\Delta_T(k)|^2, \]

\[ E_2(k) = \sqrt{\left( \frac{\xi_{\uparrow} - \xi_{\downarrow}}{2} \right)^2 + \left( \frac{\xi_{\uparrow} + \xi_{\downarrow}}{2} \right)^2 + |\Delta_S(k)|^2} + |\Delta_T(k)|^2, \]

\[ E_3(k) = -E_2(k) \]

\[ E_4(k) = -E_1(k) \]

\[ \xi_{\uparrow}(k) = K_+(k) - |h_{\text{eff}}(k)| \]

\[ \xi_{\downarrow}(k) = K_+(k) + |h_{\text{eff}}(k)| \]
Excitation Spectrum

Making singlet and triplet sectors explicit

\[ E_2(k) \leftrightarrow E_-(k) \quad E_1(k) \leftrightarrow E_+(k) \]

\[ E_{p\pm}(k) = \sqrt{\left(E_S(k) \pm |h_{\text{eff}}(k)|\right)^2 + |\Delta_T(k)|^2} \]

\[ E_S(k) = \sqrt{|K(k)|^2 + |\tilde{\Delta}_S(k)|^2} \]

singlet sector
Excitation Spectrum (ERD)

\[ \Delta_T(k) = \Delta_0 \frac{|h_{\perp}(k)|}{|h_{\text{eff}}(k)|} = 0 \]
Lifshitz transition

Change in topology
Topological invariant (charge) in 2D

\[ \hat{m}(k) = (m_x, m_y) \]

\[ N_w = (2\pi)^{-1} \int d\ell \hat{z} \cdot \hat{m} \times d\hat{m}/d\ell \]

\[ m_x(k) = \left[ E_S(k) - |h_{\text{eff}}(k)| \right] / E_{p-}(k) \]

\[ m_y(k) = \Delta_T(k) / E_{p-}(k) \]
Vortices and Anti-vortices of $m(k)$

\[
\frac{h_z}{\varepsilon_F} = 0.2 \\
US - 0
\]

\[
\frac{E_b}{\varepsilon_F} = 1.0
\]

\[
\frac{h_z}{\varepsilon_F} = 1.5 \\
US - 1
\]
For $T = 0$ phase diagram need chemical potential and order parameter

\[ \Omega_0 = V \frac{|\Delta_0|^2}{g} - \frac{T}{2} \sum_{k,j} \ln \{1 + \exp[-E_j(k)/T]\} + \sum_k \tilde{K}_+ , \]

\[ \tilde{K}_+ = \left[ \tilde{K}_{\uparrow}(-k) + \tilde{K}_{\downarrow}(-k) \right] / 2 \]

- Order Parameter Equation: \[ \frac{\delta \Omega_0}{\delta \Delta_0} = 0 \]
- Number Equation: \[ N_+ = -\frac{\partial \Omega_0}{\partial \mu_+} = 0 \]
T = 0 Phase Diagram in 2D

a) Normal

b) US-1

c) ERD SOC $v/v_F = 0.8$

d) US-1

$E_b/\epsilon_F$

$P_{\text{nd}}$

Superfluid

Inaccessible

Normal
Momentum distributions in 2D

FIG. 3: (color online) The momentum distributions $n_s(k_x, k_y)$ for ERD SOC $v/v_F = 0.8$ and $E_b/\epsilon_F = 0.1$ at $T = 0$, where $s = \uparrow (\downarrow)$ for upper (lower) panels. (a)(d) i-US-0 phase with $h_z/\epsilon_F = 0.2$; (b)(e) US-2 phase with $h_z/\epsilon_F = 0.4$; (c)(f) US-1 phase with $h_z/\epsilon_F = 1.0$. The color coding varies continuously from purple ($n_s = 0$) to red ($n_s = 1$).
Thermodynamic signatures of topological transitions
$T = 0$ Thermodynamic Properties in 2D

\begin{itemize}
  \item $E_b/\epsilon_F = 0.1$ (solid line)
  \item $E_b/\epsilon_F = 0.2$ (dashed line)
  \item $E_b/\epsilon_F = 0.5$ (dotted line)
  \item $E_b/\epsilon_F = 1.0$ (dot-dashed line)
\end{itemize}

ERD SOC $v/v_F = 0.8$
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Hamiltonian in Real Space

\[ \mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r}) \]

\[ \mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}) \left[ \hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_\beta(\mathbf{r}) \]

- Kinetic Energy
- Spin-orbit and Zeeman

\[ \mathcal{H}_I(\mathbf{r}) = -g \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi_\downarrow(\mathbf{r}) \psi_\uparrow(\mathbf{r}) \]

Contact Interaction
Effective Action at finite $T$

\[\psi_{r,s} \rightarrow \psi_{r,s}e^{i\theta_r/2}\]

\[\Delta_r = |\Delta_r|e^{i\theta_r}\]

\[S = -\frac{1}{2} \text{Tr} \left\{ \ln \left[ \beta \left( \begin{array}{c|c} A_+ & D_+ \\ \hline D_- & A_-^* \end{array} \right) \right] \right\} - \frac{\beta L^2 |\Delta|^2}{g} - \frac{\sum_{k,s} (i\omega_n + k^2 - \mu_s)}{2} + \frac{1}{8L^2} \int dr \sum_k [\nabla_r(\theta_r)]^2.\]
Effective Action at finite $T$

\[ S = S_{sp} + S_{fl} \]

\[ S_{sp} = -\frac{1}{2} \text{Tr}\{\ln[\beta M_k(0,0)]\} + \frac{\beta}{2} \sum_{k,s} \left(-i\omega_n + k^2 - \mu_s - \frac{\beta L^2|\Delta|^2}{g}\right) \]

\[ S_{fl} = \frac{1}{2} \int dr \left( A \left( \frac{\partial \theta_r}{\partial \tau} \right)^2 + \sum_{\nu = \{x,y\}} \rho_{\nu\nu} \left( \frac{\partial \theta_r}{\partial \nu} \right)^2 \right) \]
BKT Transition Temperature

\[ h_z = 0.0 \quad \nu_R = 1.0 \]

\[ h_z = 0.2 \quad \nu_R = 1.0 \]

\[ T_{\text{BKT}} \]

\[ E_b \]

- \( \nu = 0 \)
- \( \nu_D = 0 \)
- \( \nu_D = 0.5 \)
- \( \nu_D = 1.0 \)
Beyond the Clogston Limit
Full Finite Phase Diagram
Anisotropic speed of sound

\[ \widetilde{\omega}^2 - (q_x \quad q_y) \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{xy} & \rho_{yy} \end{pmatrix} \begin{pmatrix} q_x \\ q_y \end{pmatrix} = 0 \]
Vortex-Antivortex Structure

(a) RASHBA

(b) ERD

(c) ERD

(d) ERD
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Rashba

ERD
THE END