A METHOD FOR COMPARING HEURISTICS WITH
APPLICATIONS IN COMPUTATIONAL DESIGN OF FLEXIBLE
SYSTEMS

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The Academic Faculty

by

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A METHOD FOR COMPARING HEURISTICS WITH APPLICATIONS IN COMPUTATIONAL DESIGN OF FLEXIBLE SYSTEMS

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LIST OF SYMBOLS

$a$ An Artifact
$A$ Set of All Artifacts
$\mathcal{A}$ Search Process for an Artifact
$b$ A Flexible Artifact
$\hat{\beta}_k$ Regression Coefficients
$C$ Concept
$C$ Covariance Matrix of Residuals
$CC$ Capital Cost
$C_{DP}$ Design Process Costs
$C_f$ Failure Cost
$C_m$ Material Cost
$C_{MC}$ Maintenance Cost Per Space
$C_O$ Computing Cost
$C_{OC}$ Operating Cost
$CO_2$ Carbon Dioxide
$\chi$ State of the World
$X$ Set of Possible States of the World
$\dagger$ Omniscient Supervisor
$\delta$ A Decision rule
$D$ A Subset of the Property Space Projection
$D_t$ Demand for Spaces
$e$ Stochastic Component of the State Space
$E$ Weld Efficiency
<table>
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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$f$</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>$F_v$</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>Borel Sigma-Algebra</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Radius</td>
</tr>
<tr>
<td>$h$</td>
<td>Step Size</td>
</tr>
<tr>
<td>$h_n$</td>
<td>Heuristic</td>
</tr>
<tr>
<td>$i$</td>
<td>A Contextual Situation</td>
</tr>
<tr>
<td>$I$</td>
<td>The Set of All Contextual Situations</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Applicability Context</td>
</tr>
<tr>
<td>$L$</td>
<td>Pressure Vessel Length</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>A Particular Time Period</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Set of Time Periods</td>
</tr>
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<td>$m$</td>
<td>Deterministic Component of the State Space</td>
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<td>$M$</td>
<td>State Transition Function</td>
</tr>
<tr>
<td>$\mu_{\sigma_s}$</td>
<td>True Mean of the Ultimate Strength</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of Vessels Sold</td>
</tr>
<tr>
<td>$n_{asp}$</td>
<td>Number of Available Spaces</td>
</tr>
<tr>
<td>$n_f$</td>
<td>Number of Vessels that Fail</td>
</tr>
<tr>
<td>$n_L$</td>
<td>Number of Levels</td>
</tr>
<tr>
<td>$n_{sp}$</td>
<td>Number of Spaces Per Level</td>
</tr>
<tr>
<td>$n_{st}$</td>
<td>Number of Strength Tests</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of Samples</td>
</tr>
<tr>
<td>$OM$</td>
<td>Operating and Maintenance Costs</td>
</tr>
<tr>
<td>$\omega$</td>
<td>An Outcome of a Probabilistic Experiment</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Sample Space of a Probabilistic Experiment</td>
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\begin{align*}
p & \quad \text{Property} \\
P & \quad \text{Property Space} \\
P_f & \quad \text{Per Unit Failure Cost} \\
P_m & \quad \text{Per Unit Material Cost} \\
P_o & \quad \text{Per Unit Computing Cost} \\
P_s & \quad \text{Selling Price} \\
P_{sp} & \quad \text{Price Per Space} \\
P' & \quad \text{Property Space Projection} \\
P & \quad \text{Search Process} \\
PM & \quad \text{Profit Margin} \\
P_{rf} & \quad \text{Probability of Failure} \\
\varphi & \quad \text{Internal Pressure} \\
\Phi & \quad \text{Adjusted True Strength} \\
\psi & \quad \text{Outcomes of a Design Search} \\
Q & \quad \text{Safety Factor} \\
r & \quad \text{Discount Factor} \\
R & \quad \text{Risk Tolerance} \\
Rv & \quad \text{Revenue} \\
\sigma_h & \quad \text{Hoop Stress} \\
\sigma_r & \quad \text{Radial Stress} \\
\sigma_t & \quad \text{Tangential Stress} \\
\sigma_v & \quad \text{von-Mises Stress} \\
\sigma_z & \quad \text{Longitudinal Stress} \\
s & \quad \text{A Design Site} \\
S & \quad \text{A Matrix of Design Sites} \\
S & \quad \text{Set of All Artifacts}
\end{align*}
\( S_\sigma \) Sample Standard Deviation
\( t \) A Design action
\( t_a \) Analysis action
\( t_e \) Enabling action
\( t_s \) Synthesis action
\( T \) Final Time
\( T \) Set of All Design Actions
\( T_n \) Applicable Action Set
\( \theta \) Parameters of a Decision Rule
\( \tau \) Thickness of a Pressure Vessel
\( T \) Set of Possible Thicknesses
\( U \) Utility
\( \bar{U} \) Average Utility
\( \text{var}_{\sigma t_s} \) True Variance of the Ultimate Strength
\( V \) Volume
\( V_\lambda \) Value of a System at a Particular Time
\( V_o \) Value of an Option
\( w \) Classifier Weight
\( \bar{x}_\sigma \) Sample Mean
\( X \) Concept Prediction
\( X_t \) Stochastic Process
\( \xi \) A Sequence of Scenarios of Uncertainty
\( \Xi \) The Set of Possible Sequences of Scenarios of Uncertainty
\( Y \) Function Value
\( Y_f \) Expected Future Function Value
\( Z \) All Remaining Information, Beliefs, and Preferences
# LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ADP</td>
<td>Approximate Dynamic Programming</td>
</tr>
<tr>
<td>BOP</td>
<td>Balance of Plant</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CRN</td>
<td>Common Random Numbers</td>
</tr>
<tr>
<td>DA</td>
<td>Day-Ahead</td>
</tr>
<tr>
<td>DBD</td>
<td>Decision-Based Design</td>
</tr>
<tr>
<td>DDFM</td>
<td>Design Decision Framing Model</td>
</tr>
<tr>
<td>DR</td>
<td>Demand Response</td>
</tr>
<tr>
<td>HES</td>
<td>Hybrid Energy System</td>
</tr>
<tr>
<td>ISO</td>
<td>Independent System Operator</td>
</tr>
<tr>
<td>NPV</td>
<td>Net Present Value</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PMF</td>
<td>Probability Mass Function</td>
</tr>
<tr>
<td>RBDO</td>
<td>Reliability Based Design Optimization</td>
</tr>
<tr>
<td>RDT</td>
<td>Rational Design Theory</td>
</tr>
<tr>
<td>ROA</td>
<td>Real Options Analysis</td>
</tr>
<tr>
<td>RT</td>
<td>Real-Time</td>
</tr>
<tr>
<td>SE&amp;D</td>
<td>Systems Engineering and Design</td>
</tr>
<tr>
<td>SMR</td>
<td>Small Modular Reactor</td>
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All designers must make tradeoffs when making a decision. Ideally, a designer makes the decision that maximizes their value. However, selecting the correct alternative may not be obvious. To help make decisions, designers may employ heuristics, rules of thumb that recommend actions. But even selecting the best heuristic may be challenging. To best select a heuristic requires that we be able to compare different heuristics. Currently, there are no formal methods to fairly compare different heuristics. This dissertation proposes a Design Decision Framing Model (DDFM) and method for comparing heuristics to enable designers to make better decisions by identifying more valuable heuristics.

To demonstrate the method we focus on one research area that would greatly benefit from better heuristics: the field of flexible design. Flexible design explicitly recognizes that after a particular decision is made, subsequent decisions will be made that influence the value of an artifact. As such, designers should consider the effect of these subsequent decisions to make the best current decision. However, analyzing subsequent decisions can be very challenging. To address this, different heuristics have been suggested, but it is difficult to know which heuristic is best. In all likelihood, different heuristics perform better than others under different conditions. Thus, part of the challenge is in identifying the conditions under which a particular heuristic is most preferred. The DDFM is applied to compare the performance of different flexible-design heuristics. This comparison suggests that the DDFM is useful and can be used with a research method to characterize heuristics. Further, this dissertation proposes a new heuristic for analyzing flexible systems. The new heuristic uses dynamic programming and surrogate modeling to efficiently analyze future decisions, as well as provide insight into the reasons why certain decisions were made. The new heuristic is investigated in a case study of a Hybrid Energy System (HES). HES are long-lived systems subject to large uncertainty, making them particularly challenging to
analyze, especially in a flexible design context. The results of this case study suggest that the new heuristic can be advantageous under certain conditions.
CHAPTER I

INTRODUCTION

This dissertation focuses on the topic of improving the design process by using heuristics and considering flexibility. The search for an artifact cannot and should not in general be solved mathematically rigorously. Instead, designers should use heuristics to aid in the design process and realize more valuable design artifacts as a result. However, use of heuristics is challenging, in part because of confusion over what a heuristic is and how it should be used. Another method to realize more valuable design artifacts is to recognize that most design processes consists of a series of different decisions. To best make a decision, subsequent decisions should be considered. When a system is designed which explicitly considers subsequent decisions that may materially alter the system then this system is called a flexible system. While explicitly considering future decisions can reveal more valuable alternatives, it can also be very costly. As a result, methods to simplify the analysis of such systems are necessary.

In this dissertation, a precise definition of heuristics is identified to enable further research on heuristics. Also, a novel method for analyzing flexible systems is introduced and evaluated. The needs for a precise definition of heuristics and a method for analyzing flexible systems are motivated in the remainder of this chapter. In section 1.2 the gap that this research hopes to fill is introduced. Section 1.3 discusses the research objectives and the approach to reach those objectives. Section 1.4 explains the contributions and impact of this work. Finally, section 1.5 presents the outline for this dissertation.

1.1 Context & Motivation

Economic, political, and especially technological advances can occur very quickly. Following the example argument in (de Weck, et al., 2004), if such advancements are not
properly considered the results can be disasterous, as was the case for Iridium Communications. In the early 1990s, expected demand for wireless phones was growing rapidly. Cellular phones had not yet dominated the market due to a lack of a robust network of cellular towers. The expected demand for wireless phones and lack of apparent competition from cellular phones encouraged developers such as Iridium to investigate low earth orbit constellations of communication satellites for satellite phones. But, during the development and prior to installation of such satellites, cellular networks had improved greatly and led to widespread adoption of cellular phones. When the satellites were operational, the demand for mobile satellite phones was much less than anticipated, resulting in Iridium filing for bankruptcy in August of 1999, only a year after introducing the phones (Lim, et al., 2005, Lutz, et al., 2012). Iridium had waited until it could deploy the full constellation of satellites based on an old design approach, while would-be consumers flocked to cellular technology (de Weck, et al., 2004).

Iridium could have avoided bankruptcy if it had adopted a new design approach based on flexibility. The traditional approach is to estimate the expected demand and then design the system to meet that expected demand. This is particularly problematic if the realized demand is significantly less than anticipated, as was the case for Iridium. Instead, systems could be designed to respond flexibly to their changing environment. If Iridium had phased its deployment, it could have reduced life cycle costs by more than 20% (de Weck, et al., 2004). Flexibility enables decision makers to defer upgrades or expansions until additional information is available, decreasing the probabilities of very poor outcomes. Similarly, considering flexibility can result in a higher probability of very favorable outcomes; e.g., if the additional information indicates that demand is higher than initially expected.

Making design decisions can be challenging. Designers have many different decisions that influence the final artifact such as deciding what alternatives to consider,
what analyses to perform, or even what assumptions to make for a design. Because the outcomes of these decisions may not occur until a final artifact is chosen, it may be difficult to make the best decision, as was the case for Iridium. Because of the difficulties associated with design decisions, many designers employ rules of thumb called heuristics. Heuristics provide guidance for choosing what action to pursue, given the current state of the design process. They enable decision makers to make satisfactory decisions quickly, as compared to optimal decisions very slowly.

Analyzing a flexible system can further increase the complexity of a design process. Designers must anticipate many different environments, and how these different environments influence the future decisions. If a system is long-lived and subject to large uncertainty, the number of different future environments and future decisions grows substantially. Increasing the number of future decisions exponentially increases the resources necessary to analyze a decision. Because of the difficulties associated with analyzing future decisions, flexible systems necessitate the use of heuristics.

Due to their influence over the decision making process, the choice of a good heuristic is paramount. An incorrectly applied heuristic can result in a poor design artifact, with similarly poor outcomes. Such bad design decisions are more likely when there is poor agreement over the definition of a heuristic and the conditions under which a heuristic should be used. In addition, to determine the conditions under which a heuristic should be used we must be able to evaluate different heuristics against each other, which itself is challenging.

1.2 Research Gap

Heuristics have long been studied in engineering systems. However, many research questions still remain to be answered within the field of heuristics. As the complexity of engineering designs increases, as is the case for flexible designs, the need for heuristics also increases.
The primary limitation to heuristics research is a lack of a formal definition of heuristics. Heuristics are widely used in a variety of disciplines, and similarly, the definitions of heuristics are just as varied (Fu, et al., 2015). Heuristics are generally referred to as rules of thumb, which give acceptable solutions instead of the best solutions (Li, et al., 1996). However, there is no clarification given for what an “acceptable” solution is. Yilmaz and Seifert (Yilmaz and Seifert, 2011) state that a heuristic “often leads to an acceptable solution.” Again, there is no clarity on how often a heuristic must result in an acceptable solution. Fu, Yang, and Wood (Fu, et al., 2015) propose their own definition after reviewing previous authors’ definitions of heuristics: “A context-dependent directive, based on intuition, tacit knowledge, or experiential understanding, which provides design process direction to increase the chance of reaching a satisfactory but not necessarily optimal solution.” This definition provides more clarity, but does not define important terms such as the context, directive, or satisfactory. A more formal and precise definition of heuristics is needed to allow for further research in this area.

In addition to definitions of heuristics, researchers have also investigated how heuristics should be selected to solve problems. Of the research that exists, much is in the field of computer science (Braun, et al., 2001, Carpenter and Cosares, 2002, Van Breedam, 2001). However, there have been attempts at comparing flexible-design heuristics (Cardin, et al., 2017). Much of the problem in comparing heuristics is a lack of an agreed upon metric with which to value heuristics. To determine such a metric, we must determine what makes a “good” heuristic. Simply comparing heuristics based on accuracy or resource consumption to compare mean or median performance does not necessarily indicate which heuristics are better. Instead, we should rank heuristics based on an agreed upon metric, where the most preferred heuristic is the one with the highest rank. To rank heuristics, Golden and Assad use a decision theoretic framework (Golden and Assad, 1984). However, this ranking is only useful for the particular case examined, not on a general set of cases.
A gap exists in identifying a metric, as well as a method to compare heuristics across a broad set of contextual situations. Such comparisons can lead to greater insights that yield new and better heuristics.

Currently, there are many different approaches to solve flexible designs, but limited information on which approaches should be used. There have been many proposed methods for analyzing flexible designs, including Monte Carlo methods (Cardin, et al., 2015) and dynamic programming methods. While there have been comparisons of methods for flexible design (Cardin, et al., 2017), the investigations have been narrow. The above comparison suggests that dynamic programming methods can be accurate, but also increase the computational complexity. To reduce the computational complexity of analyzing flexible designs, many tools exist, such as screening models (de Neufville and Scholtes, 2011) or using a small set of representative scenarios of uncertainty (Cardin, 2007). Depending on the combination of methods and tools, a great deal of different approaches to analyzing flexible designs exist. However, because of the added complexity in analyzing flexible systems, no fair, unbiased, and efficient methods exist for comparing methods that analyze flexible systems. A gap exists for a method of comparing flexible-design heuristics.

1.3 Research Objectives and Approach

The preceding motivations and identified research gap bring us to the following motivating question for this research:

Motivating Question: How should heuristics be used in design?

We will argue that the motivating question has a simple answer, namely, heuristics should be used such that they maximize value. However discovering such usage is a difficult task. In order to assist in addressing the motivating question, three research questions have been posed. The research questions are used to restrict the scope of this investigation and address our ability to identify, value, and practically select heuristics.
In order to address the motivating question, we must first be clear about what a heuristic is. This leads us to the first research question:

**Research Question 1: What is a heuristic?**

The first research question seems like a rather very simple question, but it currently does not have a precise answer. Many different authors propose their own definition of a heuristic, each with their own slightly different interpretations. Many different interpretations may exist because of the different kinds of heuristics, such as artifact, process, search, analysis, framing, and planning heuristics (Lee and Paredis, 2014). There have been attempts to produce a unified definition (Fu, et al., 2015), but there is still room for improvement by being more precise in our definition. A clear definition is particularly important to facilitate research on heuristics. If there is poor agreement on what a heuristic is, it is unlikely that meaningful research on heuristics can proceed. Only after we have properly defined a heuristic can we make strides in using heuristics in design. To more precisely define heuristics, we begin by investigating the literature in Chapter II. Using previous definitions, we compare with popular usage of the term heuristic to identify its characteristics and intended purpose. We then expand upon previous work in which precise definitions have been proposed for terms related to heuristics and their use by proposing our own definitions in Chapter III.

One challenging aspect of heuristics is determining the goodness of a heuristic. There are clearly many desirable qualities such as low cost and a high probability of excellent solutions. However, information on the cost or the effectiveness of the solutions is rarely if ever available. Even if we had access to this information, it is inevitable that we would encounter tradeoffs. We must then quantify, for example, how much additional expense we are willing to accept for an increase in the effectiveness of the solutions. This leads us to the second research question:
**Research Question 2: How should designers choose among heuristics?**

The second research question identifies the need for a metric of a heuristics goodness. This is particularly important because it enables us to characterize heuristics. Such a metric allows designers to select their most preferred heuristic for a given situation. If we can determine the situations under which one heuristic is more preferred than another, we can begin to define that heuristic’s set of contextual situations for which it should be used. To develop the metric, we consider the effect of using a heuristic on a designer’s value in Chapter III.

One of the purposes of a metric for evaluating heuristics is to compare different heuristics. To compare heuristics, we must first estimate the value of the outcomes associated with using heuristics. Estimation is necessary, in part, due to limited resources, and is further explained in Chapter III. But estimating the value of the outcomes can be challenging, especially if the heuristics compared use different assumptions. This difficulty leads us to the third research question:

**Research Question 3: How should researchers compare heuristics?**

The third research question is focused on how to compare between heuristics. This is done using the metric that results from the second research question. To actually perform the comparisons, we have developed a model, the design decision framing model (DDFM), which was developed by considering decision theory and value of information theory. This design decision framing model emphasizes the overall design process, as compared to an emphasis on the artifact. By comparing heuristics across many different contextual situations, we can also gain valuable knowledge about what influences the heuristic’s performance. Armed with this knowledge we can then develop new and better heuristics.

One field in need of better heuristics is that of the design of long-lived systems subject to large uncertainty. By their nature, these systems are typically capital intensive,
whose benefits are variable and occur further out in time. Because of this, analyzing such systems can also be computationally intensive and resource consuming. Determining which heuristic should be used for such an analysis is the subject of the third research question:

**Research Question 4: Which heuristic should be used for the design of long-lived systems subject to large uncertainty?**

The fourth research question focuses on applying the answers to the second and third research questions to a particularly challenging set of designs. Such long-lived systems subject to large uncertainty are potential candidates for flexible design. However, the analysis of flexible designs is challenging and hence requires heuristics. Because of the long lifetime and large uncertainty, the perceived value of such systems can vary substantially with changing conditions. A heuristic that balances the complexity and accuracy of the analysis is an important tool in valuing such systems. Examples of existing flexible heuristics include analysis methods that may simplify the model of future decisions or discretize the design space to consider fewer alternatives. Both of these may result in a decrease in the accuracy of the estimate of value for the heuristics, but with the much needed decrease in the time necessary to analyze the system. These unique challenges mean that flexible design requires very different heuristics than inflexible designs. We must evaluate flexible-design heuristics to gain insights as to what makes a good flexible-design heuristic. Answering this question requires a more empirical process, using computational experiments applied to a case study of such a system.

### 1.4 Contributions

The key research goals of this dissertation may be decomposed into the areas of *discovery of fundamental knowledge* and the *development of methods and tools*. Contributions to the discovery of fundamental knowledge include a definition of heuristics,
a metric for selecting heuristics, and a research model and method for comparing heuristics. Contributions to the development of methods and tools include flexible-design heuristics, an approach for interpreting the results of dynamic programs, and a high fidelity model for analyzing hybrid energy systems.

The first contribution adds to fundamental knowledge: a precise definition of heuristics. There have not been many advancements in the field of heuristics, despite its widespread usage. The precise definition of heuristics provides structure to the field and enables further research into heuristics, such as a classification or search system for identifying the best heuristics.

The second contribution also contributes to fundamental knowledge: a metric of the quality of heuristics. Building on normative decision theory we develop a metric for selecting the most preferred heuristic. This metric not only applies to a particular contextual situation, but also to an entire context, a set of contextual situations.

The third contribution further adds to fundamental knowledge: the DDFM. The DDFM is a research model and method that can be used to compare different heuristics. Performing this comparison across a range of contextual situations aids in identifying the conditions under which heuristics should be used. This enables designers to select better heuristics, which in turn result in better designs. In addition, the research method reveals information on why a given heuristic is more preferred. This enables better heuristics to be created and subsequently used in design.

The fourth contribution adds to the development of methods and tools: flexible-design heuristics. A new flexible-design heuristic was developed to enable designers to evaluate flexible systems more accurately. The flexible-design heuristic leverages the benefits of dynamic programming and surrogate modeling to avoid oversimplifying the future decision and closely approximate normative decision theory. The flexible-design heuristic can analyze complex systems with future decisions with high accuracy and
moderate computational costs. In addition, the information that results from the flexible-design heuristic can be used to analyze future decisions to gain further insight. This contribution enables designers to better consider their future decisions, and design systems that are more valuable as a result.

The fifth contribution adds to the development of methods and tools: an approach to interpret the results of a dynamic program. A design that uses dynamic programming heuristics contains information on future decisions that is not readily available. The approach represents the information as simple decision rules. The simple decision rules approximate the future decision model used by the dynamic program by using a linear classifier. This presents the information in a format where designers can more easily understand why certain future decisions were made.

The sixth contribution adds to the development of methods and tools: a model for analyzing hybrid energy systems. The model allows for a variety of different configurations of hybrid energy systems to be created and analyzed. The model is written in Modelica (Modelica, 2009), which allows for more complex analysis, including transient phenomena. This contribution aids the renewable energy community, which is searching for alternatives to enable higher renewable penetrations.

1.5 Outline

This dissertation continues in the next chapter with a review of the related work in Chapter II. In particular, we focus on decision-based design, value of information theory, real options, and hybrid energy systems.

Using the background identified in Chapter II, heuristics and relevant terms are defined in Chapter III. Also in this chapter, we introduce the DDFM. The DDFM combines decision-based design and value of information theory to permit researchers to compare design heuristics. The DDFM uses the expected utility of the net present value of the design process as the metric of comparison for heuristics. This metric considers not only
the artifact value, but the design process costs as well. In addition, the DDFM provides a fair unbiased evaluation of artifacts, to prevent different heuristics from biasing their own value.

In Chapter IV, we discuss flexibility in design and introduce a method for generating flexible-design heuristics. Flexible-design heuristics are distinguished by their explicit consideration of future decisions. Using the DDFM, we compare different flexible-design heuristics and identify characteristics of valuable flexible-design heuristics.

In Chapter V, we evaluate a hybrid energy system using flexible-design heuristics. This example is distinctly different from the example in Chapter IV due to its complexity. Again, we use the DDFM to compare and identify common characteristics of valuable flexible-design heuristics.

In Chapter VI, we conclude with a summary and review of the research questions. We identify the contributions of this work and their impact, as well as discussing the limitations of this work. We also discuss the potential for continued research in the field.

Readers who are primarily interested in comparing flexible-design heuristics may wish to read section 3.4 and then skip to Chapter V and Chapter VI. The readers may then return to Chapter II and Chapter III for more background and in depth discussion, in particular for the DDFM.
CHAPTER II

RELATED WORK

2.1 Overview

While much research has been performed in the field of decision-based design, there has been limited research into improving heuristics. Since the choice of a heuristic, and the choice of an action, are decisions, it seems sensible to relate heuristics in a decision-based design context. Because heuristics can reduce the computational complexity of analyzing systems, one of the potential applications of heuristics is in the field of real options. Current methods of analyzing flexible systems require substantial computational resources or may oversimplify a system. To best design such flexible systems we explore heuristics and their potential role in analyzing challenging systems. One such example of challenging systems are hybrid energy systems. Hybrid energy systems are long-lived systems subject to large uncertainty. Managing risk in such systems is a challenging task. We explore the potential of applying real options methods to hybrid energy systems to manage the risk of such systems.

In the remainder of the chapter, a review of the relevant literature on design, heuristics, real options, and HES is presented. We begin by framing design in a decision-making context. We then review current approaches to heuristics in design. The field of real options is discussed as it expands upon traditional normative decision theory. Finally, the particular challenges and opportunity for hybrid energy systems is presented.
2.2  A Framework For Design

2.2.1 Decision-Based Design

Decision-Based Design (DBD) considers the part decision making plays in design (Hazelrigg, 2012). In this context, a decision is “an irrevocable allocation of resources” (Howard and Abbas, 2015). In DBD, design is considered as a decision making process that results in an artifact. Consequently, good design is the result of good decision making. To make good decisions, DBD provides a formal framework for making decisions instead of ad hoc decisions, which often yield poor results (Pandey, 2014). However, DBD is not a single design theory, but a compilation of different methods.

Different researchers recommend different DBD approaches for design. A common approach is to take the perspective of an enterprise and choose design solutions that maximize a single-attribute utility function that depends on profit (Hazelrigg, 1998). This method is based on utility theory. From simple axioms of rationality, von Neumann and Morgenstern derived a normative theory which guarantees that decision makers make decisions consistent with their beliefs and preferences (2007). In utility theory, the best decision is one which maximizes the decision maker’s expected utility. Others extend utility theory to a multi-attribute utility function (Keeney and Raiffa, 1993), however, this requires additional axioms (Dyer, 2005). Both methods capture uncertainty explicitly using probability theory (Durrett, 2010). To estimate the expected utility, decision makers can use Monte Carlo Simulation within their optimization algorithm (Rubinstein and Kroese, 2011). These methods can require considerable computational resources.

Rather than framing the design problem as expected utility maximization, Reliability Based Design Optimization (RBDO) frames it as a maximization of performance while meeting a specified reliability. In general, RBDO suffers from similar computational complexity challenges as expected utility maximization. Researchers have therefore proposed simplifications, such as the first-order reliability method (Rackwitz and
Fiessler, 1978) and the second-order reliability method (Der Kiureghian, et al., 1987), which define performance functions and approximate those performance functions using first and second order Taylor expansions, respectively. In these approaches, accuracy of the analysis is sacrificed in favor of reduced computation time. Care must be used when the failure region is not well approximated by a first or second order equation.

Although DBD enables selection of artifacts, it is challenging to choose between different DBD methods. While DBD focuses primarily on selection artifacts, selection of DBD methods must clearly consider the process costs such as the time and resources spent during the design (Lee and Paredis, 2014). In (Thompson, 2011), Thompson introduces Rational Design Theory (RDT) as a normative framework for evaluating design methods. Thompson includes many definitions that will be expanded upon in this work to more precisely define heuristics in Chapter III. While uncertainty is explicitly considered in RDT, different design methods that are investigated use the same assumptions, e.g. using the same equation for probability of failure. Comparing the solutions using different assumptions can lead to inconsistent conclusions on the value of different design methods. Design methods need to be evaluated using similar assumptions. However, if one design method’s assumptions are used this is likely to bias the results. Comparing different design methods with dissimilar assumptions still remains a challenge.

The primary purpose of reviewing the DBD literature is to provide a foundation for discussing design and decision making. The goals of this work include comparing and valuing heuristics. To accomplish these goals we must be able to identify what makes a heuristic add value. Therefore, we require an understanding of how heuristics impact a design decision.

2.2.2 Value of Information Theory

Value of information theory is an extension of decision theory used to help evaluate when a decision maker should seek information (Howard, 1966, Lawrence, 1999). Its aim
is to determine the economic value of information and to provide guidance regarding the price one should be willing to pay to consult a source of information.

Information is valuable only if it may change the decision maker’s choice to a more valuable alternative. Any information which does not change the decision maker’s choice has no value, even if it provides a more accurate estimation of an artifact’s value. The value of a particular information source is the difference in expected payoffs for considering the information and choosing a different alternative, versus not considering the information (Brennan, et al., 2007). The value of information may be influenced by many factors, including the relevant informativeness and cost of the information (Hammitt, et al., 1991). Information allows decision makers to choose better alternatives by changing the decision maker’s expected payouts and can help the decision maker assess how likely the possible states of the world are. If the information precisely specifies the state of the world, that is called perfect information.

Perfect information is such that it informs the decision maker precisely which state the decision maker will be in prior to making a decision (Lawrence, 1999). Because perfect information reveals the future state precisely, once the information has been revealed there is no uncertainty to the decision maker. Thus, once the information has been revealed the decision maker selects the decision with the maximum expected utility. However, prior to the information being revealed, the decision maker must choose whether to consult the source of perfect information. The value of perfect information is then the expected payoffs if the information is consulted, less the expected payoffs if the information is not consulted. Lave (1963) investigates the value of perfect information for the raisin industry, estimating the value of perfect information of knowing the weather three weeks in advance at $90.95. Because the value of information depends on how it changes the decision maker’s decision and their utility, it is case specific. This is seen in (Baquet, et al., 1976), where the value of nightly forecasts of frost have different values to eight different
orchardists. Perfect information is an idealization of the more typical case where information better informs, but does not precisely specify the state of the world. As a result, the value of perfect information provides an upper limit on the value of imperfect information, also known as the value of sample information (Schlaifer and Raiffa, 1961).

For imperfect information, sources provide information which the decision maker uses to update their beliefs about the outcomes, but these outcomes remain uncertain. After the information has been revealed, the decision maker chooses the alternative that maximizes their expected utility based on their posterior beliefs. The decision maker can incorporate the information with their prior beliefs to form their posterior beliefs using Bayes Theorem (Bayes, et al., 1763). Again, the decision maker must evaluate for all possible messages in order to value the imperfect information. As additional (accurate) messages are received, the value of imperfect information approaches the value of perfect information. Davis and Dvoranchik (1971) demonstrate this for the value of information on the annual streamflow under a bridge. Dakins et al. (Dakins, et al., 1994) verify this by comparing the value of imperfect information to perfect information for the level of contamination in New Bedford Harbor, Massachusetts. Adams et al. show that the value of imperfect forecasts of El Nino weather conditions is $96 million, as compared to $145 million for a perfect forecast (Adams, et al., 1995). After a certain point, the marginal benefit of additional information may be less than the marginal cost of seeking that information, at which point a rational decision maker stops seeking information.

There are many costs to consider when choosing whether to consult a source of information. There may be a fixed cost associated with consulting a particular source of information. The cost of these sources may scale with how informative they are, with more informative sources being more expensive. Once a decision maker elects to purchase from a source of information, the decision maker does not receive information, but rather a message. A message is the output from an information or data source. Only once a message
has been incorporated in the decision maker’s knowledge has information been gained (Lawrence, 1999). For example, a meteorologist may say “there is a 60% chance of rain tomorrow” and, until used to update one’s beliefs, the message is not information. There may be substantial costs associated with incorporating messages into knowledge, especially for complex messages independent of the financial cost to receive the message. When all costs have been considered, it may not make economic sense to consult a source of information (Lave, et al., 1988). Reichard and Evans (1989) show that for the case of testing the drinking water in private wells the cost of the most informative source exceeds its value. Different information sources will have their value and costs vary greatly from one another.

Designers often must choose whether to refine artifacts or seek information about artifacts. In design, the performance of an artifact is often predicted using models. These models serve as sources of information that help inform the designer in his decisions (Nickerson and Boyd, 1980). Treated this way, the concepts of value of information theory can also apply to engineering models. Valuing information enables decision makers to rationally choose whether to gather additional information until more valuable alternatives present themselves. To decide which actions to take, in (Wood and Agogino, 2005), a method for conceptual design is prescribed in which the expected value of a refinement of the design space is compared with a value of information approach to specifying evaluation functions. Value of information theory has also been used to compare artifact refinement and analysis in design methods (Thompson and Paredis, 2010), allowing the decision maker to choose the next synthesis or analysis action based on the currently available information.

The primary purpose of reviewing value of information theory literature is to provide a metric of heuristics. Heuristics are tools that can be used to aid in a design and can influence design decisions, similar to information. Thinking of heuristics as sources of
information can help us develop a metric for valuing heuristics, but also aids us in developing a method to compare heuristics, similar to how one might compare sources of information.

### 2.3 Heuristics in Design

To understand heuristics, we must first identify why heuristics are used in design. Designers have long used heuristics to make decisions. Koen has gone so far as to say “all engineering is heuristic” (Koen, 1985). Heuristics are often necessary because of an individual’s limited cognition or the constraints of the problem (Simon, 1987). As a result, decision makers may instead search for satisficing solutions, solutions that allow for some level of satisfaction, as compared to optimal solutions that would result from rationality (Simon, 1956). Instead of trying for rationality, Gigerenzer and Selten recommend bounded rationality for individuals who are constrained by limited resources, which in fact applies to all decision makers (Gigerenzer and Selten, 2002). In this case, rationality is actually being confused. It is clearly rational to use a heuristic if it results in better outcomes. What the authors may mean is that it is irrational to devote unlimited resources (in a context where resources are limited) to try and develop a perfect solution. If a good solution can be determined with limited resources then the good solution clearly has better outcomes.

However, heuristics are not without their own challenges. As a result of limited cognition, individuals use heuristics for challenging tasks, such as estimating probabilities, that often introduce biases (Tversky and Kahneman, 1975). Thus, individuals make decisions that are inconsistent with utility theory (Allais, 1953, MacCrimmon and Larsson, 1979).

To describe this behavior, Kahneman and Tversky introduce prospect theory (Kahneman and Tversky, 1979). Prospect theory can be used to explain phenomena such as anchoring, a cognitive bias that describes how humans evaluate alternatives against an
“anchor” that typically is formed from initial information (Strack and Mussweiler, 1997). Even so, prospect theory has its limitations in explaining decision making behavior, prompting cumulative prospect theory (Tversky and Kahneman, 1992). Cumulative prospect theory still does not provide us a reliable way to consider other important aspects of decision making, such as the emotional state of the decision maker. Though our ability to describe heuristic decision making behavior has advanced, there is still room for improvement in understanding heuristics.

Our understanding of heuristics is still limited in part due to poor agreement over what is and is not a heuristic. To combat this, several authors have put forth their own definition of heuristics. Heuristics are sometimes referred to as rules of thumb, which give acceptable solutions instead of the best solutions (Li, et al., 1996). Authors have identified that these solutions are appropriate if the solution can be reached relatively quickly. However, they leave open to debate certain terms such as what an acceptable solution is. How close to the optimal solution is acceptable? Yilmaz and Seifert state that a heuristic “often leads to an acceptable solution” (Yilmaz and Seifert, 2011). This suggests that heuristics do not necessarily always produce an acceptable solution, but still leaves the question of how often a heuristic provides an acceptable solution in order to be called a heuristic. This is substantiated by Magee and Frey, who claim that a heuristic should be “generally reliable, but potentially fallible” (Magee and Frey, 2007). Other authors have honed in on other characteristics of heuristics. Pearl states that “The term ‘heuristic’ has commonly referred to strategies that make use of readily accessible information to guide problem-solving” (Pearl, 1984). Pearl alludes to the idea that the applicability of a heuristic can be determined using currently available knowledge. Pearl suggests that a heuristic is a strategy to guide problem solving, but this is vague. Fu, Yang, and Wood (2015) have already investigated a great many definitions and have proposed their own definition: “A context-dependent directive, based on intuition, tacit knowledge, or experiential
understanding, which provides design process direction to increase the chance of reaching a satisfactory but not necessarily optimal solution.” This definition adds clarity, but may be improved upon. The above definitions help shed light on what a heuristic is, but can benefit from being even more precise.

Precise definitions of heuristics are needed to properly compare heuristics. We wish to compare heuristics for a few reasons. First, we want to rank order heuristics so that designers can select which heuristic to use for a given contextual situation. Second, we want to be able to identify the conditions under which a heuristic should be used, which can be done by comparing heuristics across a set of contextual situations. Third, we want to identify common characteristics of heuristics that perform well to assist in identifying other good heuristics or generating new heuristics. But comparing heuristics is not simple. Assumptions and preferences that are implicitly included in heuristics mean that different heuristics may suggest different solutions for the same problem. This makes comparing heuristics particularly challenging, as it is not clear how to evaluate the different design actions with potentially inconsistent assumptions.

Further, it is not immediately obvious what metric should be used to compare or rank heuristics. Silver, Vidal, and de Werra have proposed a tutorial on heuristic methods (Silver, et al., 1980). To measure the quality of a heuristic, the group offers four properties of good heuristics. These properties focus on the heuristic using minimal effort, being reliable, rarely resulting in poor artifacts, and being intuitive to users. These properties give insight into what makes a heuristic desirable, however the value metric is not very precise. For example, how does one determine how intuitive a heuristic is? There are others that use more qualitative methods for determining valuable heuristics. Braun et al. (2001) investigate eleven heuristics using a scalar quantity, the makespan time, in order to rank the heuristics. The authors also explore different case studies in order to come to the conclusion that the best heuristic depends on the situation. While the makespan time may
make sense depending on the field, it is by no means a general metric for evaluating heuristics. However, this analysis prompts an important question to be made about the value metric for different situations: How does one measure the value of a heuristic across different contextual situations? This question is very challenging to answer for a variety of reasons. First, it is unclear how we determine the particular set of contextual situations across which to evaluate the heuristics. A heuristic may perform very well in specific contextual situations, but perform poorly in a large majority of contextual situations. Second, as mentioned previously, it is unclear how to evaluate a particular heuristic’s recommended design action, especially if different heuristics propose different actions. Third, it is unclear what metrics should be for evaluating the goodness of a heuristic or for evaluating the goodness of a heuristic for a set of contextual situations. Although challenging, if we can compare heuristics we may be able to develop new and better heuristics.

The primary purpose of reviewing literature on heuristics is to gain a better understanding of how and why heuristics are used in design. This review has also identified imprecise definitions of heuristics, but has provided much background. We have also identified a clear gap in metrics for valuing and methods for comparing heuristics.

2.4 Real Options Analysis

Heuristics are tools used in the design of artifacts. One such class of heuristics are methods for analyzing flexible systems. These methods explicitly consider future decisions, which influence design outcomes. As a result, a real options analysis can cause a decision maker to select a different design alternative than they would have if they ignored future decisions, thereby improving the design of that artifact.

A Real Options Analysis (ROA) can provide a methodical analysis of the outcomes of flexible design decisions by considering future opportunities (de Neufville, 2003). An ROA evaluates the outcomes associated not only with an immediate decision, but also for
subsequent decisions that may take place. Often, the future decisions are a result of an investment that allows for the future decision, an option, to be exercised based on an uncertain future. An option is the “right, but not the obligation, to change a project in the face of uncertainty” (Trigeorgis, 1996). An example of an option in design is oversizing structural supports of a building to permit expansion in the future. As the decision maker gains information, he may, in time, decide to exercise the option, e.g. expand the building. The benefit of the option is the value of the information gained while waiting to exercise the option. Meanwhile, the cost of the option is the additional expense of oversizing the structural supports. The option is valuable if it leads to better outcomes, the benefits exceed the costs. Because an options value is partially related to the information gained over time, an option’s value may increase with uncertainty over the future (Cardin, et al., 2007). In essence, the purchasers of an option actually benefit high levels of uncertainty compared to those who do not consider options.

Considering options can lead to more valuable designs. Considering an option can never lead to selecting a worse alternative as compared to ignoring an option (Binder, et al., 2017). Instead, options have the ability to greatly improve the value of a design. Considering options can reduce the probability of poor outcomes, while increasing the probability of good outcomes (Babajide, et al., 2009, Cardin, et al., 2008). In general, an analysis of an inflexible alternative will understate the value of the project compared to the value if an option to modify the system were considered (de Neufville and Scholtes, 2011, de Weck, et al., 2004, Neely III and De Neufville, 2001). Even with the potential benefits, ROA has not been widely adopted. Research has been done to investigate procedures that aid in considering flexibility (Cardin, et al., 2013). In (Cardin, 2014), Cardin proposes a taxonomy of procedures to further enable designers to consider flexibility.

However ROA has its own challenges (Eschenbach, et al., 2007), part of which stem from the computational complexity of considering future decisions (Lander and
Pinches, 1998). Analyzing an inflexible system involves evaluating all possible outcomes of the current decision. To evaluate the option, we must analyze the outcomes of all possible future decisions for all possible potential outcomes of the current decision. This is necessary because the value of the current decision depends on the outcomes associated with the future decisions. If decisions are treated as optimizations, then an ROA is a nested optimization. Depending on the number of possible outcomes and different future decisions, the analysis quickly becomes intractible.

The more common methods to compute the value of an option are the Black-Scholes model (Black and Scholes, 1973), binomial lattice model (Cox, et al., 1979), and Monte Carlo methods (Boyle, 1977). The Black-Scholes model was originally introduced in finance as continuous time-based model that yields a closed-form solution for the value of the option. This model is useful when the system being analyzed is relatively simple. The binomial lattice model is a discrete time-based model that assumes that the value of the uncertain state can only change to one of two possible values, whose probabilities are dictated by the binomial distribution. At each discrete time interval, another branch is created which, again, follows the binomial distribution until the final time is reached and all possible values of the uncertain state have been identified. Then, the value of the option can be found via backward induction, evaluating the value at the final time and working backwards. As the time interval decreases for the binomial lattice, the value of the options approaches that of the Black-Scholes model (Mun, 2002). The binomial lattice can also be generalized to include more than two changes in the uncertain state where appropriate (Madan, et al., 1989). The binomial lattice is a discrete time-based case of more general dynamic programming methods (Zhao, et al., 2016). However, evaluating the value of an option may be more computationally costly as compared to Monte Carlo methods (Cardin, et al., 2017). Monte Carlo methods use random sampling to simulate different environments and compute the results for each deterministic simulation. The Monte Carlo
method is simple to set up, however it may require a large number of simulations to provide reasonable estimates of the value of the option.

Designers do have many tools to deal with the challenges that analyzing flexibility introduces. To reduce the computational costs, designers may use screening models (de Neufville and Scholtes, 2011). Screening models are simple, computationally inexpensive models that can be used to quickly eliminate obviously poor design alternatives (Moore, et al., 2011), and therefore reduce the design space. To further simplify real options analyses, the future decisions are often modeled by simple decision rules that implement the option (de Neufville, et al., 2006). For instance, rather than deciding to exercise the option of building a chemical plant based on an analysis of the expected utility of net present value, one could simply trigger the option when the price of chemicals exceeds a threshold value. By modeling the embedded decision as a simple decision rule, the cost of an analysis can be greatly reduced. A simple decision rule avoids having to maximize the expected utility each time exercising the option is considered, which can be very costly. However, because it is difficult to know if a particular decision rule is closely approximating a decision made from maximizing the expected utility, care should be used in creating them. Decision rules are typically used in conjunction with Monte Carlo simulation to quickly evaluate the performance of a flexible system (Deng, et al., 2013). As an alternative, future decisions can be considered with dynamic programming, although at a higher computational cost (Cardin, et al., 2017).

The above approximations make ROA tractable at the expense of accuracy, but it is not clear how to compare and select amongst the approximations. Different approximations have been compared, but these comparisons are often limited to accuracy and computational time for a small number of cases (Cardin, et al., 2017). This provides little assistance for the wide variety of situations where flexibility may add value.
The primary purpose of reviewing literature on real options is to understand how current methods to analyze flexible systems are implemented. This review has identified a clear gap in unbiased methods for comparing heuristics that analyze flexible systems. Although methods currently exist, there is a clear opportunity for additional heuristics to analyze flexible systems.

2.5 Hybrid Energy Systems

We review hybrid energy systems in preparation of a hybrid energy system case study later in this work. Hybrid energy systems are long-lived systems subject to large uncertainty, making them particularly challenge to design and analyze. To aid in the design of such systems, we propose flexible-design heuristics in Chapter IV, and compare the proposed heuristics in Chapter V.

Human activity, and particularly the power generation industry, appear to play an important role in contributing to global climate change (Pachauri, et al., 2014) in part because a considerable portion of the US electricity is produced by burning fossil fuels (U.S. Energy Information Administration Office of Energy Statistics, 2015). As an alternative, greenhouse gas emissions could be reduced by producing electricity from clean sources including nuclear and renewable energy, such as wind or solar. Unfortunately, the use of renewables introduces new challenges. Because it is challenging to predict wind speed or cloud cover, for example, high levels of renewable penetration may make matching consumption with equal production difficult, potentially leading into grid instability.

There are multiple options available to deal with grid instability. A first way could be to better choose what types and quantities of generation are produced by co-optimizing generation and transmission objectives (Khodaei and Shahidehpour, 2013, Liu, et al., 2013). A second method could be to utilize storage resources (e.g., batteries) to store excess energy and discharge energy as needed. However, the current technology and economics
of electrical batteries, for example, make them unsuitable for most large systems (Dunn, et al., 2011). Another energy storage system, pumped hydroelectric, is currently used to provide generation level storage, but may be hindered by the limited availability of locations where it can be cost effective (Yang and Jackson, 2011). A third option is to use load following generation, but this has its own economic and environmental challenges. To match demand to availability, base load and peaking load units must be ramped up or down, an expensive and often polluting process (Denholm and Hand, 2011). Fourthly, energy users could be encouraged to change their electrical consumption through a variety of Demand Response (DR) measures (Albadi and El-Saadany, 2008). But this too has technological, economic, and social difficulties associated with it (Torriti, et al., 2010). Although the above alternatives may be appropriate for some situations, many times technical and economic limitations hinder their adoption.

As potential solutions to overcome these challenges may involve energy systems that incorporate multiple energy sources and/or multiple energy outputs, we explore the notion of Hybrid Energy Systems (HES). Traditional HESs, as illustrated in Figure 1, use multiple inputs to produce a single output, typically electricity. In contrast, an advanced HES also includes multiple outputs, such as thermal and chemical loads, to more efficiently and flexibly manage energy generation, conversion, and distribution. When excess electricity is available or when prices are low, the system can divert thermal energy to other loads, so that electrical generation can be quickly modulated. The end result is a more interconnected system of energy sources and loads that can better contribute to the stability of the electrical grid.

The idea of combining multiple inputs and multiple outputs to create an HES is not new (Nehrir, et al., 2011). The design of an HES heavily relies upon its intended use. HESs may be designed for remote locations, removed from a reliable electrical grid (Muselli, et al., 1999, Seeling-Hochmuth, 1997). One such example considers utilizing hydrogen fuel
cells in the design of a stand-alone HES (Hakimi and Moghaddas-Tafreshi, 2009). In addition, there are also HESs for use with the grid (Ekren and Ekren, 2010, Forsberg, 2013) with the aim to increase the penetration of renewable resources (Nehrir, et al., 2011). To investigate these challenges, HESs have been analyzed for their performance and economic viability using a dynamic analysis (Dakins, et al., 1994, Dempster, 1967). A dynamic analysis is necessary to predict the effect of varying electrical generation and load on the quality of the electricity. Many other studies have investigated the economic and environmental impacts of HESs in an attempt to determine their feasibility (Bekele and Palm, 2010, Deshmukh and Deshmukh, 2008, Khan and Iqbal, 2005, Türkay and Telli, 2011). This information helps decision makers identify risks associated with designing HESs. To further aid in designing HESs, optimization is often used. Many optimization techniques such as Particle Swarm Optimization and Simulated Annealing have been used to design HESs (Ekren and Ekren, 2010, Hakimi and Moghaddas-Tafreshi, 2009). Although these studies considered risk to some extent, uncertainty and risk management have not been comprehensively considered for the design of HESs.

**Figure 1:** An example of a traditional and an advanced Hybrid Energy System (HES).
But advanced HESs also introduce unique challenges. For example, co-locating non-renewable sources and plants to produce chemical products carries increased risk. Although certain advanced HESs may not be feasible presently, this may change as the political, regulatory, and technological context evolves. Efforts are already in progress to determine and assess the potential obstacles and paths forward to support such systems. For example, new Small Modular Reactors (SMR) are being investigated for industrial uses such as producing hydrogen, water desalination, and oil refining (IAEA, 2012, 2013, Ingersoll, et al., 2014, Ingersoll, et al., 2014).

Valuing an HES is also very challenging. The performance of an HES depends on so many variable factors that may change in time, including the very uncertain costs, demand, and price for its operation. HESs are also subject to technological and political changes, which can either benefit or harm performance. The result of any major changes in the environment may prompt changes in the configuration or operation of HESs. Thus, designers turn to ROA to value HESs systems (Davis and Owens, 2003). For HESs specifically, only improvements in operational flexibility have been studied (de Oliveira, et al., 2014).

The primary purpose of reviewing literature on hybrid energy systems is to provide an understand of the opportunities and challenges associated with valuing such systems. Options have been identified as a potential tool to help manage risk in such long-lived systems subject to large uncertainty. However, those same characteristics make a real options analysis that much more challenging. A gap exists in methods for identifying flexible-design heuristics for long-lived systems subject to large uncertainty.

2.6 Summary

In this chapter, the literature on decision making, real options analysis, heuristics, and hybrid energy systems is reviewed. The focus of this dissertation is on improving design through use of heuristics and considering flexibility. While there is substantial
literature on these topics, there is opportunity in being more precise about heuristics and in additional methods for analyzing flexibility.

In the first part of this chapter, the fields of decision-based design and the value of information are introduced as a framework for design. While decisions concerning an artifact are well defined, there is not as much guidance on comparing different decision making methods. We also discussed the need to consider information in our decisions, and the challenges in evaluating the value of information.

The second part of this chapter introduces heuristics as tools that decision makers use during design. Such tools are necessary in order to efficiently design artifacts, however there is poor agreement on how to use heuristics. Part of the confusion comes about from a lack of precise definitions. By defining heuristics, we can conduct research into better heuristics, and into methods of employing heuristics most effectively. There is also poor agreement on metrics for evaluating heuristics. To compare and learn about the characteristics of good heuristics, we must identify a useful metric of comparison.

The third part of this chapter introduces real options analysis, including state of the art approaches to analyzing flexible systems. Like evaluating the value of information, many challenges are present because of the expanding possible outcomes that come from considering decisions in the future. There are many methods that are currently used to analyze flexible systems, but there is little research into comparing the different methods, or identifying the conditions under which a given method should be used.

Finally, the fourth part of this chapter introduces HES as a long-lived system subject to large uncertainty and the challenge of designing such a system. The goal of improving such systems depends on our ability to analyze the system properly, in particular for real options. However, due to the duration and associated uncertainty, there are many challenges associated with analyzing this system. While the large uncertainty and duration
make it a potentially valuable target for a real options analysis, those characteristics also
make analyzing the system much more difficult.
CHAPTER III
A RESEARCH METHOD FOR COMPARING HEURISTICS

3.1 Introduction

This chapter focuses on enabling researchers to compare heuristics. To accomplish this, we begin by motivating a definition of heuristics in section 3.2. To define heuristics, we must first understand how design is performed and the role of heuristics in design, which is explained in 3.3. After providing this background, we propose a precise definition of heuristics as an association between a set of contextual situations and a set of design actions in section 3.4. Also included in this section are related terms that are necessary to precisely define heuristics. Having defined heuristics, we then discuss what makes a good heuristic and identify a metric for choosing heuristics in section 3.5. Using this metric, we discuss the challenges of comparing heuristics and propose a solution in the form of a design decision framing model and method that enables researchers to compare heuristics in a fair and unbiased way in section 3.6. To gather evidence in support or against the design decision framing model as a useful way of comparing design heuristics, we consider the motivating example of the design of pressure vessels in section 3.7. Finally, we conclude with a summary of this chapter in section 3.8. It is important to note that portions of section 3.2 (Lee, et al., 2017) and section 3.7 (Binder and Paredis) are in the process of being published.

3.2 Toward a Definition of Heuristics

To introduce the concept of heuristics and why they are an integral part of design, we start the story with Herbert Simon, who in his seminal work on the Sciences of the Artificial (Simon, 1996), indicates the key objective of design: “Everyone designs who
devises courses of action aimed at changing existing situations into preferred ones.” Framed slightly more strongly, we could rephrase this in the context of systems engineering and design (SE&D): Systems engineers and designers should strive to change an existing situation into the situation that is most preferred.

Building on the mathematics of decision or choice theory (Howard and Abbas, 2015, von Neumann and Morgenstern, 2007), the extent to which a situation is preferred, can be measured as value. If a situation A is more preferred to a situation B, it is assigned a higher value, so that the most preferred situation is the one that maximizes value. Decision theory clarifies further that one must also take into account the time and the risk preferences. Starting from four simple axioms, von Neumann and Morgenstern (2007) proved that a rational decision maker chooses the alternative that maximizes expected utility, where a utility is a nonlinear transformation of value constructed such that risk preferences are accounted for by taking the expectation. Time preference is captured mathematically using a discount function. Combined, this allows us to express the objective for systems engineering and design as the following equation (Lee and Paredis, 2014):

\[
S: \max_{s \in S} E[u\left(\text{NPV}(s, C_{DP}(S))\right)]
\]

In other words, a designer must search over the set of all artifacts, S, for the artifact, s, that maximizes the designer’s expected utility of the Net Present Value (NPV). Notice that the NPV depends not only on the value we expect to derive from using, trading, or selling the resulting artifact, but also on the time, t(S), and the cost, C_{DP}(S), needed for the search/optimization process, that is, the cost and the time of design and development.

The challenge with this framing of an SE&D problem is that the optimization problem in Equation (1) cannot (and should not) be solved in a mathematically rigorous sense, that is, by using optimization algorithms to find the mathematically guaranteed global optimum. The set of all artifacts would require an infinite number of parameters to
describe mathematically, and the analysis of all these artifacts would require an infinite amount of time. In addition, because the time and cost of searching affect the objective, the designer must carefully balance the value of the resources invested in the search process with the value of the artifact. At some point, continuing to search will cost more than it is worth.

In artificial intelligence and operations research, such computational complexity challenges are overcome by using heuristics. Heuristic search sacrifices guarantees of optimality and completeness of the solution set for increased solution speed (Moore, et al., 2014). Similarly, in design, a heuristic is a rule of thumb that provides guidance for choosing what action to pursue, given the current state of the design process. Design heuristics rely on experience and knowledge to suggest actions that provide a good tradeoff between the cost of the SE&D process and the value of the resulting artifact.

We use the term “heuristic” broadly here. For simple detailed design decisions, a heuristic may directly constrain the artifact alternative. For example: “When designing a sheet-metal hem, the hem length should be at least four times the sheet metal thickness.” For more important decisions that strongly affect the value of the artifact, a heuristic may specify a sequence of design steps for how to constrain the artifact alternative, where each step in the sequence involves additional heuristics. For instance, a heuristic may suggest framing the design decision as an optimization problem across a heuristically defined design-space parameterization and heuristically suggested analysis-model approximations and idealizations. Finally, heuristics could also embody planning guidance, as in a heuristic suggesting how to decompose a high-level goal into sub-goals. In all three cases, the heuristic knowledge reflects previous experiences regarding the value-of-information tradeoffs (Howard, 1966, Lawrence, 1999) between the accuracy and cost of approximating Equation (1) in the specific design context encountered. The resources
allocated to a particular design choice should be commensurate with the potential impact the choice has on the artifact value.

There is poor agreement over how humans actually use and select heuristics. This is often the case because heuristics are the result of experience, and users may use them without being consciously aware of the heuristics. Even for users that acknowledge their use of heuristics, describing the heuristics can be challenging. Individuals typically perform on a relatively closed set of examples, such as the design of pressure vessels. Those designers will likely use heuristics that may work in other scenarios, but because of their experience the designers cannot describe, or do not believe the heuristic applies in other scenarios. This presents a challenging task for research about how heuristics are currently employed.

3.3 Design as a Search Process

To explain the nature and importance of heuristics, we first need to provide a conceptual framework to think about design. We introduce a framework in which design is conceptualized very generically as an information-gathering search process. To make this search process more explicit, we reframe Equation (1) in terms of searching for a sequence of design actions:

\[
\mathcal{T}: \max_{t \in T} \mathbb{E}\left[u\left(\text{NPV}(s(t), C_{DP}(t))\right)\right]
\]

where the optimization occurs over the set of all sequences of design actions (i.e., design actions), \( t \in T \). The end result is still an artifact specification, \( s(t) \), but it is obtained implicitly as the consequence of following an SE&D process, \( t \), rather than explicitly through optimization over \( S \).

Although this reformulation of the design problem is equivalent to Equation (1), it reflects more directly that the irrevocable allocation of resources to which a designer commits (i.e., the design decision) is the allocation of resources needed for the subsequent
design actions (e.g., further analysis, artifact refinement, physical testing, design optimization at a certain level of abstraction, etc.). These process choices are truly the decisions made by designers, as opposed to artifact “decisions” that can always be reconsidered and reversed. Figure 2 shows an example of this decision process using a decision tree. Initially, the decision maker elicits four possible concepts and analyzes their relative value, deciding not to consider C1 or C4 and further specify C2. However, after further specifying C2 into C2.1, C2.2, and C2.3, the decision maker determines that the current concepts are not as valuable as initially expected, and the decision maker elicits and analyzes additional concepts to pursue. In this case, the decision maker believes concept C5.1.1 is the most worthwhile concept to explore and further specify.

When one briefly explores what would be involved in solving Equation (2) rigorously, the equation implies that one should search across all possible processes, \( t \), each consisting of a sequence of actions that lead to an artifact specification, \( s(t) \). One should choose the process, \( p \), that maximizes the expected utility reflecting the designer’s preferences. However, each design action, \( t \), results in new information and influences the best choice for subsequent actions. It is thus best to commit only to the first action, obtain the information it results in, and then consider subsequent actions. In addition, the information obtained from an action is not known in advance—it is uncertain. To determine even the best first action in a sequence is extremely challenging because it would require considering every possible outcome of that action and every possible outcome of each optimally chosen subsequent action—in essence, an infinitely deep nested decision tree. Solving such a decision tree is computationally intractable, and reliance on approximations and heuristics is thus the only alternative. In summary, the question is therefore not: “Should we use heuristics in design or not?” but “Which heuristics should we use in design?”
In (Moore, et al., 2014), Moore et al. introduced a greedy, myopic approach in which the space of design actions, $T$ in Equation (2), is explored just one design action deep. By applying value-of-information theory (Howard, 1966, Lawrence, 1999), they demonstrated that such a simple approximation of Equation (2) still leads to a very efficient/valuable search process, in which the quality of the designed artifact is explicitly balanced with respect to the cost of design. A similar perspective is adopted in the literature on optimal learning (Powell and Ryzhov, 2012). However, in most design contexts, even the application of a one-step, greedy search approach is often too costly, meaning that the expected value-of-information of such an approach would be smaller than its cost. In such cases, a simpler design heuristic is appropriate—a heuristic that may not provide as much information value, but is much less expensive to apply.
3.4 A Definition of Heuristics

To define design heuristics formally, we must first define other terms. Thompson has identified several definitions that are instrumental in defining heuristics (Thompson, 2011). To begin, we set the foundation for discussing design by defining artifacts, properties, and concepts in section 3.4.1. Then, we discuss how a decision maker’s beliefs influence design by defining concept predictions in section 3.4.2. By considering the available information to the decision maker, we define contextual situations, and how they play a role in a heuristic’s applicability context, which is also defined in section 3.4.3. Heuristics assist a decision maker in moving from a contextual situation to a more preferred contextual situation. The mechanism by which this is accomplished are called design actions, and we define how design actions play a role in guiding decision makers in section 3.4.4. Having defined contextual situations and design actions, we define heuristics as an association between a set of contextual situations, called an applicability context, and a set of design actions, called the applicable action set, in section 3.4.5.

3.4.1 Artifacts, Properties, and Concepts

At their core, heuristics are used to aid in decision making, where a decision is an “irrevocable allocation of resources (Howard, 1966).” In design, decisions are made with the goal of improving a contextual situation, typically by creating an artifact. An artifact is the product of “human intelligence and effort (Clark and Baldwin, 2000, Simon, 1996).” This means that artifacts can be physical objects, such as vehicles, or non-physical objects, such as software. Artifacts can be described by their properties, however, it is unrealistic for designers to specify every property of a potential artifact. Instead, decision makers are forced to consider only a subset of properties, an abstraction. An abstraction that is used as a specification for an artifact is called a concept. Figure 3 graphically depicts the following definitions from (Thompson, 2011):
DEFINITION 1  
*Artifact set, S,* is the set of all artifacts, past, present, and future.

DEFINITION 2  
*A property, p,* is a descriptor of an artifact. It is a function defined over the artifact set: \( p: A \rightarrow Y \) with \( A \) as the subset of \( S, A \subset S \), where \( A \) is the domain of the property and \( Y \) is the topological space that is its range.

DEFINITION 3  
*Property space, P,* is the collection of topological spaces of all properties. Mathematically, this space corresponds to the Cartesian product of all the property range spaces, \( Y \).

DEFINITION 4  
An *abstraction, P’,* corresponds to a property space projection.

*Figure 3:* A graphical illustration of artifacts and the property space (adapted from (Thompson, 2011))
DEFINITION 5  A concept, C, is a partial specification for a hypothetical artifact. Mathematically, a concept C is defined as a subset of a property space projection $C \subseteq P'$.

3.4.2  Concept Predictions

Designers develop concepts as partial specifications for a potential artifact. When executed, these specifications may result in artifacts which have different properties from those prescribed in the specification. This can be the result of many factors, such as manufacturing tolerances, modifications to the concept by others, etc. Good designers account for this in their concepts by forming beliefs about the artifact’s properties, given that the concept will be used as a specification. For example, a designer considers the manufacturing tolerances when designing a component that has to fit in a given space. The designer expresses his belief that the component will have a total length less than that of the space. That is, the probability that the actual artifact will have property values within a subset of the property space projection, $D$. This is called an event, where the Borel σ-algebra, $\mathcal{F}$, is taken as the set of events considered. Then $X: \mathcal{F} \rightarrow [0,1]$ is defined as mapping the events considered in $\mathcal{F}$ to a probability measure that represents the designer’s beliefs about whether the events will occur. Figure 4 graphically shows the definition of concept predictions from (Thompson, 2011):

DEFINITION 6  A concept prediction, $X$, for a concept, $C$, is a mathematical characterization of the designer’s beliefs about the properties of the artifact that will be realized when the concept $C$ is used as a specification.
3.4.3 Contextual Situations and Applicability Contexts

Up until this point we have used the term “contextual situation” without precisely specifying what it is. Informally, a contextual situation is all the information, beliefs, and preferences a designer is currently aware of. Although we do not provide a precise definition of a contextual situation, we do specify that a contextual situation at least contains the concepts and concept predictions the designer has ideated and analyzed. The remaining information is then the portion of the contextual situation which is not included by the concepts or concept beliefs. We make this distinction because concepts and concept predictions represent the nodes of a decision tree, such as illustrated in Figure 2. A set of

**Figure 4:** A graphical illustration of concepts and concept predictions (adapted from (Thompson, 2011))
contextual situations can form a heuristic’s applicability context, the set of contextual situations for which a heuristic should be considered. Figure 5 graphically illustrates the definitions of contextual situation and applicability context:

**Figure 5:** A graphical illustration of contextual situations.
DEFINITION 7  A contextual situation, \( i \), a tuple of concepts, \( C \), concept predictions, \( X \), and all remaining information, beliefs, and preferences a designer is aware of, \( Z \).

\[ i = ((C_1, C_2, ..., C_{n-1}, C_n), (X_1, X_2, ..., X_{n-1}, X_n), Z) \]

DEFINITION 8  An applicability context, \( I_n \), is a subset of the set of all possible contextual situations, \( I \), for which a heuristic should be considered, \( I_n \subseteq I \).

We can illustrate the idea of applicability contexts using an example. Consider the heuristic “When using a bolt connection, design it to have at least one and one-half turns in the threads” (adapted from (Koen, 1985)). In this case, the condition “When using a bolt connection” is the applicability context as it constrains the contextual situations for which the heuristic should be considered.

3.4.4 Design Actions and The Applicable Action Set

In the search process that is design, designers to move from one contextual situation to another. This is done with actions, which, when performed, move the designer into a new contextual situation. Such actions are called design actions. We have identified three design actions that are used to bring a decision maker from one contextual situation to another. To achieve the objective of improving a contextual situation, design provides a specification of an artifact with a concept. It is important to note that while we describe the different design actions mathematically, this is to provide a notional definition and should not be interpreted to be rigorous as we recognize that the situation surrounding design actions is more complex than we will describe and needs to be studied further in the future.

The different design actions explored are the analysis, synthesis, and enabling actions. Analysis actions are actions that update a designer’s predictions concerning a given
concept or concepts. Synthesis actions generate or refine a concept to yield new concepts. But there are related actions that do not specifically update concept predictions or the set of concepts, but rather enable those actions, enabling actions. Some specific examples of enabling actions would be planning or developing a model. Planning is the process of identifying sub-goals which then enable subsequent design decisions. Of course, planning must only be done when it adds value, that is, when it enables future decisions that add more value than the cost of planning. Similarly, developing a model should only be done when it adds value. In the context of design, models can be considered information sources. Therefore, developing a model is developing an information source which can be used in the future to enable subsequent actions, such as analyzing the system. Figure 6 graphically shows the definitions of analysis, synthesis, and enabling actions:

DEFINITION 9 A design action, \( t \), is a transformation from one contextual situation to another, \( t : I \rightarrow I \).

DEFINITION 10 An analysis action, \( t_a \), is the design action of applying an analysis which is expected to result in an updated concept prediction, \( X' \), which is a part of an updated contextual situation, \( i' \).
\[
t_a(i) = i' : i = ((C_1, C_2, ..., C_{n-1}, C_n), (X_1, X_2, ..., X_{n-1}, X_n), Z) : \\
i' = ((C_1, C_2, ..., C_{n-1}, C_n), (X'_1, X'_2, ..., X'_{n-1}, X'_n), Z') : \exists j: X_j \neq X'_j
\]

DEFINITION 11 A synthesis action, \( t_s \), is the design action of conceptualizing concepts or specifying additional constraints on one or more properties to develop a new concept, which is a part of an updated contextual situation, \( i' \).
\[
t_s(i) = i' : i = ((C_1, C_2, ..., C_{n-1}, C_n), (X_1, X_2, ..., X_{n-1}, X_n), Z) : \\
i' = ((C_1, C_2, ..., C_{n}, C_{n+1}), (X_1, X_2, ..., X_n, X_{n+1}), Z')
\]
DEFINITION 12  
An enabling action, $t_e$, is the design action of enabling other design actions by updating the remaining information, beliefs, or preferences, $Z'$, which are a part of an updated contextual situation, $i'$. 

$t_e(i) = i' : i = ((C_1, C_2, ..., C_{n-1}, C_n), (X_1, X_2, ..., X_{n-1}, X_n), Z)$:  

$i' = ((C_1, C_2, ..., C_{n-1}, C_n), (X_1, X_2, ..., X_{n-1}, X_n), Z'): Z \neq Z'$

We can illustrate the idea of design actions with the previous bolt example. In this case, the action of “design it to have at least one and one-half turns in the threads” is a synthesis action which refines the concepts to those with, at minimum, one and one-half turns in the threads. Synthesis actions, and in fact all design actions, are a combination of other actions.
In truth, heuristics cannot recommend a single action. To completely specify a single action requires specifying an infinite number of properties. Instead, heuristics recommend a set of design actions, the applicable action set:

**DEFINITION 13** The *applicable action set*, $T_n$, are actions that are recommended by a heuristic, a subset of the set of all possible design actions $T_n \subseteq T$.

Referring to our example heuristic, the applicable action set are the set of actions which result in concepts with at least one and one-half turns in the threads.

### 3.4.5 Heuristics

With the above definitions we can now precisely define heuristics. Referring back to the bolt heuristic, we have identified two properties of a heuristic. The first is the applicability context, which is a set of contextual situations for which the heuristic should be considered. The second is the set of recommended actions, the design actions, which move the decision maker from one contextual situation to another. Put together, a heuristic is then an association between the applicability context and the recommended actions. Figure 7 shows the relationship between the applicability context and the applicable action set for a heuristic:

**DEFINITION 14** A *heuristic* is a tuple of an applicability context and an applicable action set, $h_n = \{I_n, T_n\}$ where the applicability context is a subset of the set of contextual situations, $I_n \subseteq O$, and the applicable action set is a subset of the set of design actions, $T_n \subseteq T$.

Defined this way, a great many, if not all, decisions are heuristic in nature. Any decision that is of the form “if condition then action” is clearly a heuristic. Even complex design methods are really just sets of heuristics. The difference between many design methods is either in the applicability context or the applicable action space. Different methods may
have different applicability contexts, or differ in the fineness of which the applicability context is described. The smaller the applicability context, the more specific the heuristic, which presumably recommends better actions as the heuristics are specifically for the conditions in the smaller applicability context. Different methods may also recommend different applicable action sets, depending on which actions are believed to be the most valuable for the applicability context.

Note that heuristics do not recommend a single action, but a set. While they constrain the actions to be considered, the designer must still choose an action from the set of possible actions. Which particular action to perform is left as a choice to the designer. Heuristics are only suggestions that help the designer quickly home in on the most promising design actions to consider. Also, because heuristics are not actions themselves, a heuristic does not actually move the designer into a new contextual situation.

Figure 7: A graphical illustration of heuristics.
It may occur that multiple heuristics apply (i.e., that the current contextual situation satisfies the applicability condition for multiple heuristics). Often, these heuristics constrain different aspects of the design action to be taken, so that the actions to be considered are in the intersection of the action sets. “When designing a robot manipulator, start by specifying the kinematic structure” may be combined with “When selecting a kinematics structure for a mechanism, consider first how many degrees of freedom are needed,” leading the designer to analyze the required number of degrees of freedom for the robot manipulator being designed. However, it is also conceivable that two heuristics have overlapping applicability contexts, but non-overlapping action sets. In such a situation a designer must apply good judgment and choose the action she believes to be most valuable.

3.5 A Metric for Comparing Heuristics

The goal of this section is to propose a metric for comparing heuristics. To develop a metric for choosing heuristics, we must first start by determining what value a heuristic adds to a design. In general, design requires a great many heuristics. Thus, we must analyze how other heuristics may influence the value of a heuristic in a design. Because all heuristics involve the decision maker selecting design actions, we can use decision theory to help identify a heuristic’s value. Finally, we must consider that a heuristic may be used for a variety of different contextual situations, and not a single contextual situation.

Is it meaningful to say that heuristic A is good, or heuristic B is bad? What determines the “goodness” of a heuristic? What we ultimately care about is the expected value (or more precisely, expected utility) of the outcome as expressed in Equations (1) and (2). The “goodness” of a heuristic must therefore be tied to this same criterion. It should reflect the designer’s ability to achieve preferred, valuable outcomes through the application of the heuristic. To capture this more explicitly, we will use the term “value” rather than “goodness.”
Even with this clarification, it is still not clear what the precise meaning is of the value of a heuristic. Note that the outcomes, and thus the value, depend not only on one heuristic but also on any subsequent actions chosen by the designer. It is therefore not meaningful to refer to “value” as a property of an individual heuristic but only as a property of the set of all heuristics used by the designer. However, for the purposes of this work we will compare different sets of heuristics where the only difference is a replacement of one heuristic with another and will refer to the value of the different sets of heuristics as the value of those heuristics which are different.

Finally, because preference cannot be measured in absolute terms (von Neumann and Morgenstern, 2007), the value of a heuristic also is not an absolute measure. Rather than saying that “heuristic A is good,” or “heuristic B is bad,” one can only characterize A relative to B: “heuristic A is better than heuristic B.”

Next, we consider how to determine which heuristic is better. One perspective argued in the literature is that design practices (i.e., sets of heuristics) should be consistent with normative decision theory (Abbas, 2013, Collopy and Hollingsworth, 2009, Hazelrigg, 1998). Practices, such as the use of system requirements to define a systems engineering problem, have been critiqued as being irrational and inconsistent with the normative theory. However, we need to be careful not to jump to conclusions. In light of Equations (1) and (2), we need to recognize that the use of requirements impacts not only the artifact being designed, but also the communication and synchronization between teams of engineers inside a potentially very large organization or possibly even across multiple organizations. In addition, the communication and synchronization processes are performed by humans as cognitive, emotional and social agents. In other words, a set of heuristics includes heuristics regarding artifacts, processes and organizational design, and thus needs to be assessed according to its impact on the overall outcomes, not only on the artifact, but also on the design processes and the human organizations responsible for
executing these processes. In conclusion, the metric for comparing heuristics for a given contextual situation will be the expected utility of the design process, consistent with Equations (1) and (2).

However, we also need a metric for comparing heuristics across a set of contextual situations, a context. Heuristics may perform differently in different contextual situations if either heuristic’s outcomes change as a result. For example, a heuristic that has many design actions and requires a large quantity of time and effort may be inappropriate if there is limited time as the heuristic will be too costly or yield an artifact of low value. But it is not feasible to introduce specific heuristic for every possible specific situation. To aid in the selection of heuristics, it may be meaningful for a heuristic to be applicable for a set of contextual situations. Our metric for comparing heuristics must then also change as a set of contextual situations is considered. Assuming each contextual situation is equally likely, the metric for comparing heuristics then becomes the average expected utility. If a heuristic has the greatest average expected utility for a given set of contextual situations, it is preferred for this context.

3.6 A Computational Method for Comparing Heuristics

The goal of this section is to identify a method to compare heuristics using the metric identified in the previous section. To begin, we discuss the challenges of valuing heuristics using real-world examples. Although real-world examples introduce challenges, computational experiments introduce their own challenges. Even so, these challenges are more manageable. Considering the challenges of comparing heuristics, we propose the Design Decision Framing Model as a solution to the problem of comparing heuristics.

To choose the most preferred heuristics, we must first be able to compare heuristics. Real-world examples are quite limited for a variety of reasons. First, solving the same design problem with multiple sets of heuristics can take months or years using just one set of heuristics and is thus prohibitively expensive. Second, comparing the value of two
different final artifacts is difficult, again because of costs, but also because the two cannot fairly be placed in the same market environment without affecting each other. As a consequence, very few real-world comparisons are published in the literature, and, because context matters, any existing comparisons offer little help in valuing the heuristics.

Instead, researchers may use computational models and simulations to compare heuristics. Simulation is relatively inexpensive and allows thus for a broad comparison across many design problems and situations. However, comparing heuristics with simulations still presents many challenges. Different heuristics may recommend different actions, leading to different artifacts. The challenge is to evaluate the results of those different heuristics, the design artifacts and design process costs, in a fair and unbiased way. How should one compare two heuristics when one suggests a pressure vessel fails, while the other suggests the pressure vessel does not fail? Clearly, if one heuristic is used as the predictive analysis model then this will bias the comparison. Instead, the heuristics could be compared using an unbiased third heuristic. This is the basis for the Design Decision Framing Model (DDFM), which is used in the research method to compare heuristics. The intended use of this research method is to characterize the performance of different heuristics rigorously and to collect evidence in support of claims regarding the performance of different heuristics.

### 3.6.1 Design Decision Framing Model

Evaluating heuristics in the real world is very challenging. As stated previously, it is impossible to compare different heuristics in the real world due to high costs and potential bias. However, it is even more challenging to estimate the expected utility of a heuristic in the real world. First, many samples must be taken the same contextual situation to be confident in an estimate of the expected utility. This is challenging for cost reasons, as well as the fact that the world is always changing, preventing the exact same contextual situation from being present. This problem is exacerbated when the average expected utility is to be
considered. Now, the heuristics that must be evaluated many times at the same contextual situation, must also be evaluated across the set of contextual situations.

Instead, we use computational experiments to evaluate heuristics. Unlike real world experiments, computational experiments can repeat the exact same contextual situation, allowing for the expected utility to be calculated. Since the cost of computational experiments is relatively low compared to real world experiments, the entire context can be explored to determine the average expected utility. Furthermore, computational experiments can evaluate heuristics without the heuristics mutually influencing each other.

However, computational experiments introduce their own challenges. While real world experiments are evaluated by actual outcomes, computational experiments must use models of reality to predict the performance of a heuristic. But this presents a problem if heuristics utilize different assumptions or models. For example, consider two heuristics that recommend two different minimum thicknesses of a pressure vessel to avoid failure under the same load. For a given contextual situation, there is only one minimum thickness to avoid failure. Therefore, one or both of the heuristic must be incorrect in determining the minimum thickness to avoid failure. Further complicating the matter, heuristics may not clearly state their assumptions, making it difficult to take the different assumptions into account.

In order to evaluate heuristics fairly, we must be able to evaluate their outcomes using similar assumptions. However, if one of the heuristics were used to evaluate the outcomes, there would be a clear bias towards this heuristic. This problem arises in (Aughenbaugh and Paredis, 2006), where the concept of imprecise probabilities is considered and compared to a probabilistic characterization of uncertainty. A motivating example of a pressure vessel is used to show how both uncertainty representations affect the value of a decision. To compare the different heuristics, we introduce an “omniscient supervisor” who knows the (artificially generated) truth, controls how much of this truth is
revealed to the designer, and determines the value of artifacts and process costs resulting from each heuristic, so that the heuristics can be objectively compared. The omniscient supervisor removes the bias from choosing a particular heuristic’s computation of the artifact value.

The above ideas form the basis of the Design Decision Framing Model (DDFM). Figure 8 shows the structure of the DDFM, which can evaluate the value of heuristics for a given contextual situation. Part of the contextual situation is the information that the omniscient supervisor reveals to the decision maker. The decision maker selects an artifact using a heuristic or set of heuristics. While heuristics can recommend actions, it is ultimately up to the decision maker to select actions which result in a concept. The decision maker must make this decision while considering the design space, the concepts the decision maker is willing to investigate, the modeling assumptions, the assumptions the

**Figure 8:** The Design Decision Framing Model.
decision maker uses to analyze the concepts, and the search strategy, the strategy the
decision maker uses to move through the decision tree and select a particular artifact. The
omniscient supervisor then evaluates the “true” value of the artifact, as well as the design
process costs, to determine the design process value. We can expand this to cover a set of
contextual situations by evaluating the design process value for different contextual
situations. Figure 9 shows a method for evaluating the value of heuristics over a set of
contextual situations, the average expected utility. To do this, a researcher must first select
a contextual situation. Then, to consider different beliefs that the decision maker may have
the omniscient supervisor reveals limited information to the decision maker. Using the
available information and the contextual situation, the decision maker selects a design
frame (heuristic) which ultimate results in the decision maker selecting an artifact. Then,
using the value of the “truth”, the omniscient supervisor evaluates the artifact and design
process value. By repeating this process for different beliefs, the omniscient supervisor
evaluates the expected utility of the design process. But this assumes a particular value of
the “truth”, which is unknowable. Thus, to further reduce potential bias the researcher can
investigate different values of the “truth.” In Figure 9, this is equivalent to considering
different contextual situations. Thus, we can consider different values of the truth while
simultaneously exploring different contextual situations. The value of the heuristic is then
averaged over these different contexts, including different truths in the average expected
utility. The DDFM thus provides a fair and unbiased computational framework for
evaluating different heuristics.

The DDFM is limited by the assumptions that it relies upon. For one, future
decisions need to be either ignored or straightforward such that they can be evaluated by
the omniscient supervisor. If the decisions are complex, the research method may require
substantial computational resources. The DDFM also assumes that the values of the truth
reasonably approximate future states of the world. For unknown future states of the world,
as all are, these may be more easily included using probability distributions. The DDFM further assumes that the omniscient supervisor provides an evaluation of the decision maker’s decisions that are reasonably recognize the value of the decisions for the chosen truth model. These assumptions are a reasonable approximation to applying the heuristics in practice while being executed at a comparably smaller cost that is suitable for academic research.

3.6.2 Computational Considerations

Computational tools are considered to make the application of the DDFM computationally tractable. These tools can be used in computational experiments to compare different heuristics. These tools enable a reasonable approximation of the average expected utility, thus enabling researchers to more easily determine when one heuristic is more preferred than another.

Figure 9: The Design Decision Framing Method.
A heuristic is preferred if its average expected utility is greater than the alternative. We specify the average expected utility instead of expected utility specifically as a metric for comparing heuristics. The omniscient supervisor is not uncertain about the value of a heuristic, but may wish to aggregate the performance of a heuristic over a set of contextual situations. We choose this aggregation to be a simple average. Thus, for a set of contextual situations, the omniscient supervisor can determine average expected utility, $\overline{U^\dagger}$, of the NPV based on the outcomes of the decision maker’s search for an artifact, $\psi$:

$$\overline{U^\dagger} = \int_{i \in I_n} f(i)U^\dagger(\psi, i) \, di$$

(3)

where the $\dagger$ denotes that the function is evaluated by the omniscient observer, as compared to a decision maker. $i$ is a particular contextual situation, from the applicability context, $I_n$, and with a Probability Density Function (PDF), $f(i)$. Equation (3) requires that we know truth, and therefore the future. However, the future is unknowable, and we therefore do not know the truth. In Equation (2), this is accounted for in the expected utility since the truth is considered a part of the contextual situation, thus considering a set of truths as well.

The average expected utility could then be approximated by computing the expectation with the Monte Carlo Method (Fishman, 2013, Rubinstein and Kroese, 2011). This approximation discretizes Equation (3) and the accuracy depends on the number of samples, $m$:

$$\overline{U^\dagger} \approx \frac{1}{m} \sum_{k=1}^{m} U^\dagger(\psi, i_k)$$

(4)

where we replace the probability density function with $1/m$ since we are using a simple average. But we are not only interested in computing one average utility, as we are comparing potentially many different heuristics.

To improve the accuracy of the expectation estimate, we use common random numbers (Gal, et al., 1984, Kleinman, et al., 1999). By using the same samples to determine
both estimates, the variance of the difference in estimates is reduced when the estimates are correlated (Fishman, 2013, Kleinman, et al., 1999). The difference in the average expected utility, $\Delta \bar{U}$, is thus estimated as:

$$\Delta \bar{U} \approx \frac{1}{m} \sum_{k=1}^{m} U^\dagger(\psi_1, i_k) - U^\dagger(\psi_2, i_k)$$

where $\psi_1$ and $\psi_2$ refer to the outcomes associated with the first and second heuristics, respectively. When $\Delta \bar{U}$ is positive, the first heuristic is preferred, when negative, the second heuristic. This comparison can be expanded to however many heuristics are considered. For more than two heuristics, the difference must be made with respect to all other heuristics. Then, the heuristic whose value of $\Delta \bar{U}$ is strictly positive when performing the difference for all other heuristics is the most preferred heuristic. These computational tools allow for a computationally efficient determination of which heuristic is preferred for a given context.

To characterize the heuristics, we investigate sets of contextual situations, contexts. By comparing the heuristics over a set, we can identify how much the variables in the contextual situation influence the preference for each heuristic. We can analyze, for example, how a designer's beliefs influence the most preferred heuristic.

### 3.7 Design of Pressure Vessels

To explore the DDFM, an example study of a pressure vessel is investigated, based on the problem introduced in (Thompson and Paredis, 2010). The example is centered on a designer who must select a value for the wall thickness of a pressure vessel with otherwise predetermined geometry and dimensions. In this case, the decision maker is a seller of pressure vessels, and receives revenue for each pressure vessel, but incurs a cost for pressure vessels that fail prematurely as well as the manufacturing costs of the pressure vessels. The business averages approximately $3.3 million in revenues per year. The
nominal pressure, \( \varphi \), for this vessel operates at 1.4 MPa, with a radius, \( \gamma \), of 0.2286 m, and a length, \( L \), of 1.2 m. A selling price, \( P_s \), of $415 per pressure vessel was assumed for this nominal case, which correlates to the number of vessels sold, \( n = 8,000 \) vessels. The decision maker must deal with uncertainty in the material’s ultimate strength, choosing the thickness that maximizes his profit. It is assumed that each pressure vessel’s material is randomly drawn from a normal distribution, modeling different qualities of material one would expect from a vendor’s batch. To select the thickness, three heuristics are considered. To simplify the comparison between the three heuristics, we restrict our focus to one year of revenues. The first heuristic is an algebraic heuristic, based on the ASME pressure vessel code for thin walled pressure vessels and uses a factor of safety. The second and third heuristics are optimization heuristics, based on value-driven design, utility theory, and thin walled pressure vessel assumptions. The third heuristic, the expert optimization heuristic, is different from the second by introducing “experience” in the form of updated beliefs concerning the probability of failure.

### 3.7.1 Algebraic Heuristic

<table>
<thead>
<tr>
<th>Planning Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When designing a pressure vessel</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Select the minimum thickness using the ASME Pressure Vessel Code</td>
</tr>
</tbody>
</table>

The algebraic heuristic is a simplified form of (ASME, 2007), which calculates two thicknesses and selects the more conservative one, i.e., the thicker one:

\[
\phi = \sigma_{ts}/Q \quad (6)
\]

\[
\tau_a = \varphi \cdot \gamma / (\phi \cdot E - 0.6 \cdot \varphi) \quad (7)
\]

\[
\tau_b = \varphi \cdot \gamma / (2 \cdot \phi \cdot E + 0.4 \cdot \varphi) \quad (8)
\]

\[
\tau_{req} = max(\tau_a, \tau_b) \quad (9)
\]
where $\sigma_{ts}$ is the ultimate tensile strength of the material, $Q$ is the factor of safety, $\tau_a$ is the minimum required thickness at longitudinal seam welds, $\tau_b$ is the minimum required thickness at circular seam welds, $\varphi$ is the internal pressure, $\gamma$ is the radius of the spherical ends of the pressure vessel, $E$ is the weld efficiency of the seams, and $\tau_{req}$ is the minimum required thickness for the pressure vessel. A weld efficiency of 1 was assumed for all calculations. A factor of safety of 3.5 was used as per ASME standards when using the ultimate strength to determine the minimum required thickness (ASME, 2007). The algebraic heuristic’s applicable action set recommends the designer apply Equations (6)-(9), and considering pressure vessels whose thickness is thickness identified in Equation (9).

### 3.7.2 Optimization Heuristics

**Planning Heuristic**

*Applicability Context:* When designing a pressure vessel

*Applicable Action Set:* Select the minimum thickness using an optimization of expected utility

The decision maker’s utility depends on the profitability of the business, $Profit(n_f, \tau)$, which depends on the number of vessels sold, the material cost, and the failure cost:

\[
V(\tau) = \frac{4}{3} \pi \left( \gamma^3 - (\gamma - \tau)^3 \right) + \pi L \left( \gamma^2 - (\gamma - \tau)^2 \right)
\]

(10)

\[
C_m(\tau) = P_m \cdot V(\tau)
\]

(11)

\[
C_f(n_f) = P_f \cdot n_f
\]

(12)

\[
Profit(n_f, \tau) = n \cdot (P_s - C_m(\tau)) - C_f(n_f).
\]

(13)

where $V$ is the volume of material per vessel, $C_m$ is the cost of materials per vessel, $P_m$ is the material cost, $\tau$ is the thickness of the material, $G_f$ is the cost incurred from failed
vessels, $P_f$ is the per unit failure cost, $n_f$ is the number of vessels that fail, $n$ is the number of pressure vessels sold, and $P_s$ is the price of each pressure vessel sold. To determine the probability of a particular pressure vessel failing, $Pr_f$, the decision maker expresses his beliefs about the ultimate strength.

The decision maker forms his beliefs using strength tests of the material. The decision maker evaluates the sample mean, $\bar{x}_{\sigma ts}$, sample standard deviation, $S_{\sigma ts}$, and degrees of freedom, $v$, of the observed ultimate strength values and characterizes his beliefs using the Student’s t-distribution. Then, the probability of a particular pressure vessel failing is the probability that the ultimate strength is less than the peak stress: the t-distribution’s cumulative distribution function (CDF), $F_v$, evaluated at the peak stress. The decision maker assumes a thin walled pressure vessel, and so evaluates the CDF at the hoop stress, $\sigma_h$ (Shigley and Mischke, 2001):

$$\sigma_h(\tau) = \frac{\varphi \ast Y}{\tau}$$

$$Pr_f(\tau) = F_v\left(\frac{\sigma_h(\tau) - \bar{x}_{\sigma ts}}{S_{\sigma ts}}\right)$$

This is a particular heuristic, used to analyze a concept and provide information on its properties, an analysis heuristic.

<table>
<thead>
<tr>
<th>Analysis Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the probability of failure of a thin-walled pressure vessel</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Compare the ultimate strength to the hoop stress</td>
</tr>
</tbody>
</table>

The probability of a given number of failures is described by the Probability Mass Function (PMF) of the binomial distribution:

$$B(n_f; n, Pr_f(\tau)) = \binom{n}{n_f} Pr_f^{n_f} (1 - Pr_f)^{n - n_f}$$

Given the probability of each possible failure, the expected profit, $E[P(t)]$, is equal to:

$$59$$
Since the decision maker is risk averse, he does not want to make a decision based on the expected profit, but rather choose the thickness which maximizes his expected utility. Although any monotonically increasing function can be used, we use the following equation for utility, which assumes a constant risk tolerance, $R$:

$$
Utility(n_f, \tau) = R \left(1 - e^{-\frac{Profit(n_f, \tau)}{R}}\right).
$$

For businesses, a ratio of risk tolerance to sales of 0.064 has been commonly measured (Howard, 1988). Given that sales are approximately $3.3$ million, a risk tolerance of $212,000$ is assumed.

Risk preferences should be considered when a design contains uncertainty. In this case, the decision maker is uncertain about the material’s ultimate strength. Thus, the heuristic prescribes that the expected utility be maximized:

$$
E[U(\tau)] = \sum_{i=0}^{n} B(i; n, Pr_f(\tau)) * Utility(i, \tau)
$$

with optimal thickness, $\tau^*$:

$$
\tau^* = \arg \max_{\tau \in \mathcal{T}} E[U(\tau)]
$$

where $\mathcal{T}$ contains all positive real numbers less than the radius.

**Expert Decision Maker**

**Analysis Heuristic**

*Applicability Context:* When analyzing the probability of failure of a thin-walled pressure vessel

*Applicable Action Set:* Compare the ultimate strength to the hoop stress, explicitly accounting for model-form uncertainty by adding an uncertain bias term
The different optimization heuristics differ in their use of heuristics within the optimization. For the naïve optimization heuristic, failure is predicted by comparing the ultimate strength to the hoop stress in Equation (15). That is, it uses a heuristic of the form: when analyzing the probability of failure for a thin walled pressure vessel, compare the ultimate strength to the hoop stress. However, this comparison ignores the model form uncertainty. The hoop stress comparison to ultimate strength is typically used with factors of safety to account for uncertainty. When used for optimization, it is necessary to recognize the uncertainty implicit in this equation. One way to account for this uncertainty is to compare the predicted failure rate as prescribed by the naïve optimization heuristic with the true failure rate, and update the belief about probability of failure. This is what experts do: update their beliefs based on past experience. For the expert optimization heuristic, failure is predicted by comparing the ultimate strength to the hoop stress, modified to reflect their updated beliefs. In order to more accurately represent the probability of failure, the decision maker introduces the random variable, $\delta$, which is used as a multiplicative term on the hoop stress. Then, the decision maker’s probability of a particular pressure vessel failing is the expected value of the probability of failure, including the probability density function (PDF), $f$, of $\zeta$:

$$Pr_f(\tau) = \int_{-\infty}^{\infty} F_V\left(\frac{\sigma_h(\tau) \cdot \zeta - \bar{x}_{\sigma ts}}{S_{\sigma ts}}\right) \cdot f(\zeta) \, d\zeta. \quad (21)$$

To compare the effect of experience, we will test the performance of the optimization heuristics using both Equations (15) and (21). The non-expert optimization heuristic uses Equations (10)-(20) while the expert optimization heuristic uses Equations (10)-(14) and (16)-(21) to select the thickness of the pressure vessels.

3.7.3 Omniscient Supervisor

The omniscient observer must evaluate the value of the three heuristics by evaluating the pressure vessels. Therefore, the omniscient observer requires its own set of
assumptions to determine failure. To minimize bias, the omniscient supervisor should not use any of the heuristics being investigated. The omniscient observer evaluates alternatives similarly to the optimization heuristics, but is made more conservative by calculating the von-Mises stress from the tangential, radial, and longitudinal stress to reduce potential bias. That is, the omniscient supervisor uses a similar procedure as the optimization heuristics, but a different method to predict failure. The equations for the von-Mises, tangential, radial, and longitudinal stress are calculated in the more general thick walled pressure vessel case (Shigley and Mischke, 2001):

\[
\begin{align*}
\sigma_t &= \frac{\varphi \ast (y^2 + (y - \tau)^2)}{y^2 - (y - \tau)^2} \\
\sigma_r &= -\varphi \\
\sigma_z &= \frac{\varphi \ast (y - \tau)^2}{y^2 - (y - \tau)^2} \\
\sigma_v &= \sqrt{\sigma_t^2 + \sigma_r^2 + \sigma_z^2}.
\end{align*}
\]

where \(\sigma_t\), \(\sigma_r\), \(\sigma_z\), and \(\sigma_v\) are the tangential stress, radial stress, longitudinal stress, and von-Mises stress, respectively. In this case, the omniscient supervisor compares the von-Mises stress to the ultimate strength to determine the true probability of failure, \(Pr_f^+\), based on the ultimate strength’s true mean, \(\mu_{\sigma_{ts}}\), and variance, \(\text{var}\sigma_{ts}\):

\[
Pr_f^+(\tau) = F_v\left(\frac{\sigma_v(\tau) - \mu_{\sigma_{ts}}}{\sqrt{\text{var}\sigma_{ts}}}\right).
\]

Here the ultimate strength’s true mean and standard deviation are used as the omniscient supervisor has no uncertainty concerning the distribution of ultimate strengths. The omniscient supervisor evaluates the probability of failure based on the normal distribution’s CDF, which reflects the true distribution of ultimate strengths.

In addition to determining the probability of failure, the omniscient supervisor also evaluates the design process costs. The omniscient supervisor explicitly considers the additional computational cost, \(C_o\), based on the amount of time, \(time\), necessary to
determine $\tau^*$ for a given contextual situation, $i$, to determine the omniscient supervisor’s evaluation of profit, $\text{Profit}^+(n_f, \tau)$:

$$C_O = P_o \times \text{time}(i)$$ (27)

$$\text{Profit}^+(n_f, \tau) = n(P_s - C_m(\tau)) - C_f(n_f) - C_O.$$ (28)

Thus, for a particular contextual situation and a given thickness, the omniscient supervisor can determine the expected utility, $E[U^+(\tau)]$:

$$\text{Utility}^+(n_f, \tau) = R \times \left(1 - e^{-\frac{\text{Profit}^+(n_f, \tau)}{R}}\right).$$ (29)

$$E[U^+(\tau)] = \sum_{i=0}^{n} B(i; n, \text{Profit}^+(\tau)) \times \text{Utility}^+(i, \tau).$$ (30)

The omniscient supervisor uses Equations (22)-(30) to evaluate each of the heuristics: the algebraic, non-expert optimization, and expert optimization. By comparing the expected utility for each heuristic, the most preferred heuristic can be determined for a given contextual situation. However, we are concerned about more than one particular contextual situation.

In this computational experiment, we aim to determine which method is best across a given context—the range of contextual situations in which the heuristics are more preferred than the other. Table 1 shows the variables and their bounds delineating the context. The variable, $\gamma$, is the radius of the spherical ends of the cylindrical pressure vessel. The rated pressure, $\varphi$, is the specified internal pressure of the pressure vessel relative to the external pressure. The length, $L$, is the length of the cylindrical portion of the pressure vessel, excluding the spherical caps. To allow for a fair comparison between the methods, the market price is assumed to be independent of the method, but varying across the contextual situations. It is determined by applying a profit margin, $PM$, applied to the costs associated with the optimal thickness, $\tau^+$, as determined by the omniscient supervisor under perfect information:
Because only material and failure costs are considered, the lower and upper bounds of the profit margin are chosen to be quite high so all heuristics result in reasonable profits. The per unit, $P_m$, is the cost of a cubic meter of steel. The per unit failure cost, $P_f$, includes payment for expected damage to property and nearby individuals. The per unit computing cost, $P_O$, is included to account for the additional cost of the optimization heuristics over the algebraic heuristic. The costs for model and code development are ignored for all heuristics, and the time to run the algebraic heuristic is considered negligible. The optimization cost is based on the average cost of using Amazon’s EC2 m3.large and m3.2xlarge on demand computing services using Windows (Amazon). The true mean of the ultimate strength, $\mu_{\sigma_{ts}}$, and the true standard deviation of the ultimate strength, $\sigma_{\sigma_{ts}}$, are

$$P_s = \frac{Pr_f^*(\tau^*)P_f + C_m(\tau^*)}{1 - PM}$$

Table 1: The properties considered in the context for the pressure vessel

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.183</td>
<td>0.274</td>
</tr>
<tr>
<td>$L$</td>
<td>0.96</td>
<td>1.44</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>11.2</td>
<td>16.8</td>
</tr>
<tr>
<td>$PM$</td>
<td>64.8</td>
<td>73.4</td>
</tr>
<tr>
<td>$P_m$</td>
<td>4,040</td>
<td>6,060</td>
</tr>
<tr>
<td>$P_f$</td>
<td>80,000</td>
<td>120,000</td>
</tr>
<tr>
<td>$P_O$</td>
<td>0.518</td>
<td>0.777</td>
</tr>
<tr>
<td>$\mu_{\sigma_{ts}}$</td>
<td>340</td>
<td>510</td>
</tr>
<tr>
<td>$\sigma_{\sigma_{ts}}$</td>
<td>16.2</td>
<td>24.4</td>
</tr>
<tr>
<td>$n_{st}$</td>
<td>24</td>
<td>36</td>
</tr>
</tbody>
</table>
characteristics of the distribution of ultimate strengths in delivered steel. The values, and choice of normal distribution, are those of delivered high grade steel (Hess, et al., 2002). The decision maker is given information about this distribution in the form of strength tests that reveal the ultimate strength of a plate of steel, randomly selected from the true distribution. The number of strength tests, \( n_{st} \), reflects the amount of information available to the decision maker. The strength tests are samples of the ultimate strength from the true distribution. The decision maker then uses those samples and determines the sample mean, \( \bar{x}_{\sigma_{ts}} \), sample standard deviation, \( S_{\sigma_{ts}} \), and the number of degrees of freedom, \( \nu \), to use as his beliefs in Equation (15) for the non-expert optimization heuristic, and Equation (16) for the expert optimization heuristic. The algebraic heuristic uses only the sample mean as the ultimate strength in Equation (6).

To gain a deeper understanding of the difference in performance between the different heuristics, we performed a 10-level full factorial across the number of strength tests and the failure cost. We focus on these two variables because they emphasize the difference between the optimization and algebraic heuristics: implicitly versus explicitly accounting for uncertainty and risk. The number of strength tests determines the amount of information available to the decision maker and hence his or her uncertainty about the material properties while the failure cost strongly influences the risk faced by the designer.

Finally, the experiments are performed for both a risk neutral and a risk averse decision maker to investigate the effect of risk preferences on the choice of heuristics.

### 3.7.4 Results

We first focus on the comparison for the non-expert optimization heuristic and the algebraic heuristic. Figure 10 shows the difference in expected profit between the non-expert optimization and algebraic heuristics for the risk neutral and risk averse cases. It is important to note that positive values refer to the non-expert optimization heuristic being preferred. The algebraic heuristic is only preferred in two regions for the risk neutral case.
Figure 10: The difference in expected profit between the non-expert optimization and algebraic heuristics for the risk neutral (left) and risk averse (right) cases.

The first region is the area at the top of the figure, where the non-expert is well informed about the material strength. Counterintuitively, this suggests that a non-expert decision maker with available information may actually be worse off by using the optimization heuristic. As available information increases, uncertainty about the material strength decreases. Thus, to maintain a similar probability of failure, the thickness of the pressure vessel can be decreased. However, the non-expert optimization heuristic incorrectly characterizes the probability of failure as compared to the omniscient supervisor. By assuming that Equation (14) is a fair approximation of the max stress in the pressure vessel, the non-expert is too aggressive in sizing the pressure vessel, recommending overly thin pressure vessels. Aggressive sizing of the pressure vessels also occurs for the risk averse case, but the risk aversion causes the heuristic to be more conservative even in the high available information region. For this region, the uncertainty in the material strength is smaller than the error in judgment introduced by Equation (14). Thus, even with perfect information, the algebraic heuristic may be preferred for certain
per unit failure costs. If the optimization heuristic more accurately modeled the probability of failure, additional information would benefit the optimization heuristic. This is confirmed by Figure 11, which shows the comparisons of the expert optimization heuristic and the algebraic heuristic for the risk neutral and risk averse cases. Now, the expert optimization heuristic is always more preferred as additional information becomes available.

The results of Figure 11 also show that the expert optimization heuristic is preferred more as the per unit failure cost decreases. This is true even when considering risk aversion. The slope of the difference in expected profit as a function of available information decreases as compared to the risk neutral case. Conversely, the slope increases as a function of the per unit failure cost, is a maximum at the lowest per unit failure cost, where available information has little influence on the difference in expected profit. In the extreme, if the per unit failure cost was zero, then the problem becomes a minimization of material cost: a minimization of thickness, where knowledge of the material strength is irrelevant.
The algebraic heuristic recommends a thickness independent of information or failure cost. With sufficient Monte Carlo samples, the average thickness recommended by the algebraic heuristic is related to the true mean strength of the material, and thus approximately constant at 28 mm. The algebraic heuristic implicitly accounts for the probability of failure and the per unit failure cost in Equations (6)-(9), by using safety factors. If these implicit assumptions do not match the contextual situation well, the heuristic may not perform well, as is the general case. Typically, the algebraic heuristic is too conservative, recommending a pressure vessel that is too thick. Figure 12 shows the average thickness recommended by the expert optimization heuristic for the risk neutral and risk averse cases. Except in the region to the lower right, the thickness is less than the algebraic heuristic’s thickness of 28 mm. While being excessively conservative decreases the average performance of the heuristic by increasing the material cost, it prevents extremely poor performance. This allows the algebraic heuristic to outperform the non-expert and expert optimization heuristics when available information is limited and the per unit failure cost is high.

Figure 12: The expected thickness of the expert optimization heuristic for the risk neutral (left) and risk averse (right) cases.
In general, the optimization heuristic performs better than the algebraic heuristic, but can suffer from occasional poor performance. This is readily seen from Figure 11 in the lower right region. The cause for the poor performance is twofold: the optimization heuristic is overly conservative, and a series of failed pressure vessels increases the average failure cost. The optimization heuristic recommends on average an overly thick pressure vessel, but will occasionally recommend an overly thin pressure vessel. First, it can be easily seen from Figure 12 that the average thickness in the lower right is greater than that of the algebraic heuristic for both the risk neutral and risk averse cases. This occurs because the amount of information is so limited that the decision maker in the optimization heuristic must consider extremely low material strengths as reasonably likely. In turn, the optimization heuristic recommends very thick pressure vessels to reduce the probability of failure. Second, Figure 13 shows the average failure cost for the expert optimization heuristic, with a quickly rising peak occurring in the lower right region. This occurs because of the relatively limited number of strength test samples. While on average the expected thickness is greatest in this region, there are cases where a cluster of high strength

Figure 13: A contour plot of the expected total costs associated with failure for the expert optimization heuristic in the risk averse case
test samples misleads the decision maker into believing the ultimate strength has a comparatively higher mean and lower standard deviation than the true distribution would suggest. This belief will mislead a rational decision maker into choosing a very thin pressure vessel, and result in a high number of failures. The fewer strength test samples are taken, the more likely this type of event can occur. Thus, despite the thickness being higher on average, the occasional thin pressure vessel greatly increases the expected costs associated with failure. This phenomenon also appears for the algebraic heuristic, although not to the same degree because of the safety factor. As a result, expected costs associated with failure for the algebraic heuristic is very extremely close to zero for all contextual situations investigated. Clustering never benefits the decision maker, as even when strength test samples group on the lower end of the strength tests, the decision maker will be more conservative and choose a greater thickness than is truly necessary. Any deviation between the decision maker’s beliefs and the true distribution of the ultimate strength has a negative effect on profitability for all heuristics, but especially so for the optimization heuristics, and even more so in cases where the ultimate strength is overestimated.

3.8 Summary

In this chapter we investigate the role of heuristics in decision making in design. First, we introduce design as a search process where the designer must perform actions n order to determine the final specification for an artifact. Second, we define heuristics as an association between a set of contextual situations we call the applicability context and a set of actions we call the applicable action set. Third, we explore what makes a “good” heuristic and develop a metric for comparing the relative value of heuristics. Fourth, we develop a research method for comparing heuristics that is computationally tractable, allowing researchers to explore the characteristics of more preferred heuristics. Finally, we
use the metric and research method on a motivating example of a pressure vessel to compare three heuristics.

In the first part of this chapter we show how the problem of selecting a specification for an artifact is the same as selecting a search process for that same specification. We also focus on selecting the specification or search process that maximizes the design process value, not just the artifact value. Thus, a search that yields a good but not “optimal” artifact may be more preferred if it requires substantially less resources to search for the artifact. Together this frames design as a series of decisions, where subsequent decisions should be considered when choosing or evaluating an immediate decision. However, this may be challenging and is unlikely to be done in practice, necessitating heuristics.

In the second part of this chapter we introduce heuristics as tools that can be used to aid the decision making process and precisely define heuristics. Specifically, for a given contextual situation a heuristic recommends a set of actions, the applicable action set, that, when performed, move the decision maker into a new contextual situation. The contextual situation contains not only the concepts and concept predictions, but other relevant information, beliefs, and preferences of the decision maker. A set of contextual situations also forms a heuristics applicability context, which is the set of contextual situations for which the heuristic should be used. After potentially many heuristics are used, the actions recommended by the heuristics have sufficiently refined the concepts such that the designer selects a concept to use as a specification for the artifact.

In the third part of this chapter we discuss the qualities of more preferred heuristics and develop a metric for comparing heuristics. Because the outcomes of a given decision depend on subsequent decisions, the value of the initial decision depends on the subsequent decisions. Similarly, the value of a heuristic depends on the subsequent heuristics to be applied. As such, we can only compare sets of heuristics, or singular heuristics when no subsequent heuristics are used. The metric for comparing sets of heuristics is the average
expected utility of the design process value. The average expected utility is used to specifically differentiate between the expected performance for a given contextual situation, which is averaged for a set of contextual situations.

In the fourth part of this chapter we apply our metric for comparing heuristics in a research method for comparing heuristics. To evaluate the performance of different heuristics, we investigate the value of the design process, including the selected artifact. To objectively value the selected artifact we introduce the concept of an omniscient supervisor. The omniscient supervisor can evaluate the “true” value of a design process, again, including the artifacts selected by different heuristics. To avoid potential bias, a researcher can investigate a set of truths to yield the average expected performance for the different heuristics. We also offer computational tools to reduce the computational complexity of applying this research method.

In the fifth part of this chapter, we apply these computational tools with the research method to compare three different heuristics for the motivating example of designing a pressure vessel. We investigate two optimization-based heuristics and an algebraic-based heuristic and conclude that each heuristic may be more preferred given different contextual situations. The non-expert optimization heuristic performs more poorly than the algebraic heuristic when very little or very much information is available. However, when the expert optimization heuristic is compared to the algebraic heuristic, the expert optimization heuristic outperforms for most cases, only underperforming for extremely high failure costs and very limited available information. The solution to this for the expert optimization heuristic is to seek additional information when such limited information is available.
CHAPTER IV
FLEXIBILITY IN DESIGN

4.1 Introduction

This chapter focuses on one type of heuristics: those which allow decision makers to analyze flexible systems. First, in section 4.2 we define what the value of an option is. Because real option methods are heuristics, as defined in Chapter III, there will be many similarities seen in section 4.2 whilst defining the value of an option. Then, in section 4.3 we discuss different methods of analyzing flexible systems for low- and multi-dimensional cases to allow designers to analyze flexible systems of varying complexity. The one-dimensional method is used to analyze future decisions associated with the design of a parking garage in section 4.5. The multi-dimensional method is used to analyze future decisions associated with the design of a hybrid energy system in Chapter V. In section 4.4 we use the methods from section 4.3 to discuss a method researchers can use to gather additional domain knowledge by investigating why the future decisions were made. This research method allows researchers to more easily identify valuable heuristics and aid in generating better heuristics. An example of this research method is shown in Chapter V for the case of a hybrid energy system. Another example of this appears as part of section 4.5, in which we introduce a motivating example of a parking garage to investigate the performance of our analysis method and research method for a simple one-dimensional case. Finally, in section 4.6 we conclude with a summary of the chapter.

4.2 Evaluating the Value of Options

We can determine how much more preferred a real options analysis is over other analysis methods by evaluating the value of options. Because real options methods are
heuristics, this value of options represents how much more preferred a heuristic is than another. We begin this section by defining what the value of an option is. Then, we consider how to practically evaluate the value added by options. Finally, we discuss some limitations of assessing the value of an option.

4.2.1 Comparing Flexible and Inflexible Alternatives

To compare value, we must first define what we mean by the value of an option. We define the value of an option, $V_O$, as the maximum amount a rational decision maker would pay for the option (Cardin, et al., 2007). That is, the amount that would make the decision maker indifferent between considering the option and not. Assuming the decision maker is rational and the option is purchased now, this can be mathematically expressed as:

$$E[U(\text{NPV}_{inflex}(a, X_{\lambda}))] = E[U(\text{NPV}_{flex}(b, X_{\lambda}) - V_O)]$$

(32)

where $a^*$ is the most preferred inflexible alternative (i.e., the design alternative that maximizes the expected utility of net present value in the case where no future options are considered), $b^*$ is the most preferred flexible alternative, and $X_{\lambda}$ is a stochastic process representing how the state of the world may change over time.

Note that the definition of $V_O$ is implicit. $V_O$ cannot be expressed as a difference in expected utilities due to the nonlinearity of the utility function. An explicit equation for $V_O$ is possible only for the case of risk neutrality, where the utility function is linear.

Also note that Equation (32) includes two different expressions for net present value: $\text{NPV}_{flex}$ and $\text{NPV}_{inflex}$ for the cases in which we do and do not consider the possibility of exercising the option. $\text{NPV}_{inflex}(a, X_{\lambda}(\omega))$ is the net present value of the value flows resulting from the initial system alternative, $a$, for a given realization of the stochastic process, $X_{\lambda}(\omega)$, where $\omega \in \Omega$ is an outcome from the sample space of a probabilistic experiment. The value flows considered in $\text{NPV}_{flex}(a, X_{\lambda}(\omega))$ are the same.
as for $NPV_{inflex}(a,X_{\lambda}(\omega))$ until the moment the decision is made to exercise an option. Depending on the specific realization, $X_{\lambda}(\omega)$, the decision maker may choose to exercise different options or exercise the same option but at a different moment in time. $NPV_{flex}$ embodies this decision process of determining whether and how the system will be modified, and reflects the corresponding value streams.

For a rational decision maker, the consideration of an option will never lead to worse expected outcomes:

$$E \left[ U \left( NPV_{inflex}(a,X_{\lambda}) \right) \right] \leq E \left[ U \left( NPV_{flex}(a,X_{\lambda}) \right) \right]$$  \hspace{1cm} (33)

In all future option decisions, not modifying the system is always an alternative. If this alternative is chosen for all realizations, $X_{\lambda}$, then $NPV_{flex}(a,X_{\lambda})$ is the same as $NPV_{inflex}(a,X_{\lambda})$. Instead, if for any realization, $X_{\lambda}(\omega)$, exercising the option would result in a higher expected utility, then considering this option will improve the expected outcomes of $a$.

Similarly, considering an option is likely to cause a rational decision maker to choose a different alternative, $b^*$. Based on the definition that $b^*$ is most preferred, we have:

$$E \left[ U \left( NPV_{flex}(a^*,X_{\lambda}) \right) \right] \leq E \left[ U \left( NPV_{flex}(b^*,X_{\lambda}) \right) \right].$$  \hspace{1cm} (34)

If the right hand side of Equation (34) is strictly greater than the left hand side, then there will be a different most preferred alternative for the initial system configuration, $a^* \neq b^*$. By combining Equations (33) and (34), we obtain:

$$E \left[ U \left( NPV_{inflex}(a^*,X_{\lambda}) \right) \right] \leq E \left[ U \left( NPV_{flex}(b^*,X_{\lambda}) \right) \right].$$  \hspace{1cm} (35)

Substituting Equation (32) into Equation (35) leads to the conclusion that the value of an option is always non-negative:

$$V_0 \geq 0.$$  \hspace{1cm} (36)
4.2.2 Evaluating the Value of an Option

We now turn our attention to the practical computation of $V_o$ based on Equation (32). Specifically, computing $V_o$ requires determining the most preferred alternatives, $a^*$ and $b^*$, and computing $NPV_{flex}$ requires modeling the future option decisions. The future decisions can be modeled using simplifying assumptions. If simple decision rules are used, we can estimate the value of the option, but it is likely that we will underestimate the value of the option. This is because if the simple decision rules make suboptimal decisions, the expected value of the flexible alternative will be comparatively lower than if optimal decisions were made. Thus, we can be confident that any value of the option that we identify is merely a lower bound on the true value for our assumptions. Of course, if the model of the flexible alternatives is poor, then the value of the option that is evaluated using that model is suspect.

To compute the expected utility and determine the value of the option, the Monte Carlo Method (Rubinstein and Kroese, 2011) is applied. However, we really are only interested in the expectation of the difference between two utilities:

$$
E[U(NPV_{inflex}(a^*,X_\lambda)) - U(NPV_{flex}(b^*,X_\lambda) - V_o)] = 0
$$

(37)

so that Common Random Numbers (CRN) can be used to decrease the number of samples needed to maintain similar accuracies in the estimate (Gal, et al., 1984, Kleinman, et al., 1999). CRN reduces the computational effort needed to determine the difference estimate with similar accuracy, but still does not allow for an explicit equation for $V_o$. To estimate the value of the option, $\hat{V}_o$, root finding is used (Schilling and Harris, 1999):

$$
\hat{V}_o = \text{root of } \sum_{k=1}^{N} U(NPV_{inflex}(a^*,X_\lambda(\omega_k))) - U(NPV_{flex}(b^*,X_\lambda(\omega_k)) - V_o) = 0.
$$

(38)

While the non-linear utility function prevents us from explicitly determining $V_o$, there is an exception when the decision maker is risk neutral. When risk neutral, the expected utility
of NPV will rank alternatives the same as the expected NPV. Therefore, we can separate
the terms in Equation (36):
\[
V_o = E[NPV_{flex}(b^*, X_\lambda) - NPV_{inflex}(a^*, X_\lambda)]
\]  
(39)
Because considering future decisions are a particular type of heuristics, we can also apply
Equation (39) to determine the value of heuristics. The value of heuristics is simply the
difference in value between the recommended actions. Of course, because heuristics
recommend sets of actions the value of the design must be the average expected utility.

4.2.3 Limitations

Evaluating the value of an option is limited by the assumptions used to assess the
value of the option. To determine the value of an option exactly we must be able to compare
the value of an inflexible and a flexible alternative. However, in practice we will only be
able to estimate the value of such alternatives. In general we can estimate the value of the
option by using root finding, however this too requires accurate assessments of the value
of the inflexible and flexible artifacts. Further, if the utility function of a decision maker is
particularly complex it may be challenging to accurately identify an estimate of the value
of the option.

4.3 Analyzing Flexibility

In this section we investigate and develop methods to analyze flexible systems to
enable designers to analyze flexible systems better or more easily. First, we review popular
methods for analyzing flexibility to provide background for the methods. Then, we develop
two methods: one for the case of one-dimensionality in section 4.3.1 and the other for the
case of multi-dimensionality in section 4.3.2.

First, we discuss the more traditional rule-based heuristics. Rule-based heuristics
execute a particular decision (e.g. expand a plant by 20%) based on the current and past
states of the world (e.g. when demand has increased by 10%). Decision rules are mappings
from the power set of states of the world to the power set of actions. An example decision rule heuristic is: when analyzing the flexible-design of a chemical plant, expand the plant when demand has increased by 10% of the initial demand. Following the notation in (Cardin, et al., 2017), let $\delta_\theta(\xi)$ denote a decision rule which prescribes an action based on a sequence of scenarios of uncertainty, $\xi$, from the set of possible sequences, $\Xi$, and where $\theta$ is a vector of parameters. Let $\xi_\lambda$ represent a sequence of scenarios of uncertainty that are observed up to period $\lambda$ from the set of periods, $\Lambda$. For simplicity, we assume that $\Xi$ is finite $\{\xi_1, ..., \xi_N\}$ of size $N$ and associated probabilities $P^{r_k} \geq 0, \sum_{k=1}^{N} P^{r_k} = 1$. We further assume that the period of interest is finite. We will assume that the decision rule prescribes an action in the beginning of the period, before the scenario of uncertainty for that time period is revealed, denoted $\delta_\theta(\xi|\lambda)$. Such rule-based heuristics are appropriate if they approximate the decision making process a designer would have performed. Rule-based heuristics have the advantage of being extremely quick to implement and solve, which are both factors of the average expected utility, and therefore important to consider for real options heuristics. If decisions are modeled by optimizations, rule-based heuristics remove nested optimizations. But rule-based heuristics may not closely approximate the future decision maker’s selection. This can occur for various reasons, such as the creator of the decision rule not being familiar with the system, or the decision making process not being well characterized by such a simple relation. In cases where the failure of the decision rule is due to a lack of familiarity with the system, designers may expend additional resources to determine better decision rules. Because the value of a decision rule likely depends on the initial configuration, best choosing the decision rule for each initial configuration is important.

Instead of using a particular decision rule and choosing an initial configuration of the system, designers may also intelligently choose the decision rule in conjunction with the initial configuration of the system. If decisions are modeled by optimizations, decision
makers may determine the best characteristics of the decision rule, \( \theta^* \), by maximizing the expected value to the decision maker \( E[V] \) (Cardin, et al., 2017):

\[
E[V(\delta_0(\xi),\xi)] = \sum_{k=1}^{N} P_r^k \sum_{\lambda=1}^{A} V_\lambda(\delta_0(\xi^k_{(\lambda)}),\xi^k_\lambda)
\]  

(40)

\[
\theta^* = \arg \max_\theta E[V(\delta_0(\xi),\xi)]
\]  

(41)

where \( V_\lambda \) is the value to the decision maker in time \( \lambda \) and is a function of the prescribed decisions and sequence of scenarios of uncertainty. For simplicity we have used discrete state variables, however Equation (40) can be further generalized to consider continuous state variables if necessary. Using the above example, decision makers may optimize the size of the expansion, as well as the threshold demand. That is, an example heuristic is: when analyzing the flexible-design of a chemical plant, expand the plant when demand has exceeded the value found to optimize the utility of the chemical plant. Such optimization of rule-based heuristics have been successfully performed (Binder, et al., 2017). However, optimized decision rule heuristics may still undervalue a particular initial configuration if the form of the decision rule does not approximate the actual decision making process. No level of optimization will help a decision rule that, for example, is determining to expand a chemical plant based on how sunny a day it is. We can compare between decision rules to determine which are better, but decision rule heuristics will always be limited by the complexity of the situation.

Other methods can better approximate the decision making process a future decision maker would undergo, such as dynamic programming (Bertsekas, et al., 1995). Dynamic programming is a method of solving problems by solving many smaller subproblems, and storing solutions of those subproblems for reuse. Dynamic programming works by evaluating the performance of a system for a finite set of states of the world. Note that here the state of the world, \( \chi \), from the set of possible states of the world, \( X \), contains scenarios of uncertainties as well as other information such as a particular decision, \( a \), that
the designer makes. Then, a state of the world may change into a different state of the world, \( \chi' \), based on the state transition function, \( M \), which is the probability density function, \( f \), that action \( a \) in state at time \( t \) will lead to state \( \chi' \) at time \( t' \):

\[
M_a(\chi, \chi') = f(\chi_{\lambda+\Delta\lambda} | \chi_{\lambda} = \chi, a_{\lambda} = a)
\]  

(42)

A dynamic programming model determines the value of the system, \( V \), evaluated at the final time, \( T \), for all possible states of the world:

\[
V_T(\chi) = Y(\chi)
\]  

(43)

where \( Y \) is the value function that depends on the state of the world. A dynamic programming model then recursively evaluates the performance of all possible states in the time preceding the final time investigated, according to the Bellman equation (Bellman and Dreyfus, 2015):

\[
y_f(a, \chi, \chi') = M_a(\chi, \chi')V_{t+1}(\chi')
\]  

(44)

\[
Y_f(a, \chi) = \int_{\chi' \in \chi} y_f(a, \chi, \chi')d\chi'
\]  

(45)

\[
a^*(\chi) = \arg \max_a \left( Y(a, \chi) + \frac{1}{1+r} Y_f(a, \chi) \right)
\]  

(46)

\[
V_\lambda(\chi) = Y(a^*, \chi) + \frac{1}{1+r} Y_f(a^*, \chi)
\]  

(47)

where \( r \) is a discount factor if the decision maker has preferences over when a payout occurs and \( a^* \) is the best policy for a particular state of the world. If the states of the world are independent of the decisions then Equation (44) is simplified to:

\[
y_f(\chi, \chi') = M(\chi, \chi')V_{\lambda+1}(\chi')
\]  

(48)

That is, heuristics for dynamic programming are of the form: when analyzing flexible designs, assume future decisions are such that they maximize the forward value. Note that all value functions depend solely on the values of the immediate future states. This may be fine for linear value functions such as NPV, but prevents the value from being determined for non-linear utility functions, unless the utility function can be approximated by
memoryless value functions (Kreps and Porteus, 1979). For a risk-averse decision maker, the decision depends on the non-linear utility of the expected wealth and therefore the utility function must be a function of all possible future values, not just the expected value in the next year. For risk-neutral decision makers, the utility function is linear. Thus, designers who are risk neutral can use Equations (43)-(47) to solve dynamic programming problems. If problems are solved in this way, each decision is made by a rational decision maker using all of the available information, consistent with normative decision theory.

When the future outcomes and probability of the states are well behaved, we may be able to compute the integral in Equation (45) directly. This may be the case for a subset of the problem we are investigating. For example, if the probability of a future state is modeled with a normal distribution and the outcomes depends linearly on the uncertain future state, then the integral can be determined directly. However, if this is the case, it is likely to be the case only for the final year(s) of a system. In this period of time, there are likely to be no further decisions concerning the system as the system is unlikely to operate much longer. Future decisions are likely to change the outcomes of a design in a non-linear fashion, preventing the expected value of future value streams from being determined directly. Thus, we will not be able to compute the integral in Equation (45) directly as decisions change the shape of the future outcomes.

Evaluating Equation (45) exactly may be challenging. Instead, it may make sense to approximate the integral using numerical methods. To apply numerical methods we must first discretize the states of the world, but we must use a sufficient discretization in order to maintain an appropriate approximation to Equation (45). However, this may not be feasible for multi-dimensional problems. The number of states increases drastically with the dimensionality of the problem, known as the curse of dimensionality (Powell, 2007). The increase in the number of states makes it challenging to evaluate the performance at each state, approximate the integral in Equation (45), and determine the optimal decision
for each state. Without a solution to the curse of dimensionality, dynamic programming is only applicable for a limited number of problems. One potential solution is to use Approximate Dynamic Programming (ADP), which provides methods, that, for example, approximate to the value function (Powell, 2007). One such example is classical ADP, which uses basis functions and statistical methods such as regression to provide estimates of the value function. For example, some authors use Monte Carlo integration to estimate the value functions (Keane and Wolpin, 1994). Others handle the curse of dimensionality by taking advantage of a particular problem’s structure, or by making certain assumptions to simplify a problem (Rust, 1987).

We will consider different methods of approximating the integral in Equation (45) based on the complexity of the problem. First, we consider relatively simple systems, where the number of states of the world is limited. Second, we expand and consider many potential states of the world, where the curse of dimensionality would present an issue. In both cases we introduce a dynamic programming method for approximating the value of a flexible design.

4.3.1 Analyzing Flexibility for the One-dimensional Case

Systems are often sufficiently complex that we cannot determine the integral in Equation (45) exactly, even for the one-dimensional case. If we were to discretize the states of the world, we can then employ numerical methods to help approximate the integral. One such solution is to approximate this integral using Simpson’s rule, which requires evenly spaced states. For the one dimensional case and an even number of states, \( N \), this estimates the integral as (Schilling and Harris, 1999):

\[
h = \frac{x_N' - x_0'}{N}
\]  

(49)
\[ Y_f(a, \chi) \cong \frac{h}{3} \left[ y_f(a, \chi, \chi'_0) + 2 \sum_{k}^{N - 1} y_f(a, \chi, \chi'_k) + 4 \sum_{k}^{N} y_f(a, \chi, \chi'_k) + y_f(a, \chi, \chi'_N) \right] \] (50)

where \( \chi'_k \) is the \( k \)th possible future state of the world for \( \chi \) and \( h \) is the step size. This is only an example application which uses Simpson’s rule, which corresponds to the three-point Newton-Cotes quadrature rule (Milne, 2015). In general, other Newton-Cotes formulas can be used based on the complexity of the system being analyzed and how accurately Equation (45) must be approximated. Higher order formulas may be necessary as the complexity increases, as may be the case as the number of states, and therefore the number of dimensions, increases. At a certain point, it may not be feasible to approximate Equation (45) using Equations (49) and (50), and decision makers must model flexibility differently.

4.3.2 Analyzing Flexibility for the Multi-dimensional Case

In cases where the system being evaluated has a multi-dimensionality, approximating Equation (45) may not be simple. The traditional approach of increasing the number of samples to maintain accuracy greatly impacts the computational complexity to the point where it is infeasible. We propose a method for approximating Equation (45) which has unique properties that allow for a considerable reduction in computational complexity.

We propose a surrogate modeling technique, kriging modeling, to model future outcomes, \( V_{i+1} \). Kriging, or Gaussian process regression, is used to interpolate values of the future outcomes based on a comparatively more sparse discretization of the states of the world. Because we can accurately interpolate values, we do not need to sample as many states of the world. Kriging modeling has many desirable properties that make approximating Equation (45) reasonable for high-dimensions while maintaining accuracy. To explain why, we explore how kriging models are implemented.
Regression kriging uses a combination of linear regressions, the deterministic component, and kriging of the regression residuals, the stochastic component. Regression kriging assumes that the outcomes, $y$, for a state space can be decomposed into a deterministic component, $m$, and a stochastic component, $e$, as a function of the spatial location, $s$:

$$y(x) = m(x) + e(x)$$  \hspace{2cm} (51)

The deterministic component is used as a “global” model of the space, while the stochastic component encompasses “local” deviations from the global model. The regression kriging estimator, $\hat{y}$, of a particular location of the space, $x_0$, is then the sum of the estimated global model, $\hat{m}$, and estimated local deviation, $\hat{e}$:

$$\hat{y}(x_0) = \hat{m}(x_0) + \hat{e}(x_0)$$ \hspace{2cm} (52)

In this case, $x_0$ is a point in the state space, but not necessarily a point that has been sampled. In the dynamic program, we only sample $N$ realizations, organized into a matrix of the design sites, $S$, and measure the outcomes, $Y$. In this case, $s$, is an $N \times m$ where $m$ refers to the number of variables in the uncertain states. $\hat{m}$ is the fitted deterministic estimator, whose regression coefficients, $\hat{\beta}_k$, are typically found by generalized least squares (Lophaven, et al., 2002):

$$\hat{\beta} = (S^T CS)^{-1} S^T CY$$ \hspace{2cm} (53)

$$\hat{m}(x_0) = \hat{\beta} g(x_0)$$ \hspace{2cm} (54)

where $C$ is the covariance matrix of the residuals, and $g$ are combinations of the independent variables at the location $x_0$ for the generalized least squares. For the case where $\hat{m}$ is assumed to be a constant, $f$ is a vector of ones with a length of $N$. For the case where $\hat{m}$ is quadratic, as in Chapter V, $g$ is of the form (Lophaven, et al., 2002):

$$g_1(x) = 1$$

$$g_2(x) = x_1, \ldots, g_m(x) = x_m$$ \hspace{2cm} (55)
\[ g_{m+2}(x) = x_1^2, \ldots, g_{2m+1}(x) = x_1x_m \]
\[ g_{2m+2}(x) = x_2^2, \ldots, g_{3m}(x) = x_2x_m \]
\[ \ldots, g_p(x) = x_m^2 \]

where \( p \) is \( \frac{1}{2}(m + 1)(m + 2) \). The covariance matrix, \( C \), depends on the choice of correlation function that is used. In Chapter V we use a Gaussian correlation function, which is of the form (Lophaven, et al., 2002):

\[
\mathcal{R}(\theta, s_i, s_j) = \prod_{k=1}^{p} \exp(-\theta_k(s_{ik} - s_{jk})^2) \tag{56}
\]

\[
C_{ij} = \mathcal{R}(\theta, s_i, s_j), \quad i, j = 1, \ldots, N \tag{57}
\]

where \( k \) denotes the component of the sample point \( s_i \) or \( s_j \), and \( \theta_k \) is the correlation parameter used for the \( k \)th variable. \( \theta_k \) is found by performing a minimization (Lophaven, et al., 2002):

\[
\sigma^2 = \frac{1}{N} (S - G\hat{\beta})^T C^{-1} (S - G\hat{\beta}) \tag{58}
\]

\[
\theta_k = \arg\min_{\theta} \left\{ |R| \frac{1}{\sigma^2} \right\} \tag{59}
\]

where \( \sigma \) is the maximum likelihood estimate of the variance. Let \( G \) be an \( N \times m \) matrix with \( G_{ij} = g_j(s_i) \) (Lophaven, et al., 2002):

\[
G = [g(s_1) \ldots g(s_N)]^T \tag{60}
\]

Then we can determine the estimate of the local disruption, \( \hat{e} \) (Lophaven, et al., 2002):

\[
r(x) = [\mathcal{R}(\theta, s_1, x) \ldots \mathcal{R}(\theta, s_N, x)]^T \tag{61}
\]

\[
\hat{e}(x) = r(x) \ast \mathcal{R}^{-1}(Y - F(x)\hat{\beta}) \tag{62}
\]

The estimate of the outcomes approximates the future outcomes based on the state of the world:

\[
\hat{V}_{t+1}(\chi) = \hat{m}(\chi) + \hat{e}(\chi) \tag{63}
\]
Kriging models are particularly advantageous to the field of flexibility in design because they can be customized to particular approximations by changing the order of polynomials or different correlation functions. In the above case, we assume that measured points are known precisely, thus the kriging model will go through every sampled point and provide an exact interpolation of the data (Cressie, 2015). Although not shown above, kriging models can also provide inexact interpolations where measured values are not exact and therefore the model will not necessarily intercept every measured point (Cressie, 1986). Regression kriging allows us to effectively characterize the space of future outcomes with relatively few sample points (Eldeiry and Garcia, 2010, Triantafilis, et al., 2001). Of course, more complex systems will require additional sample points.

The main advantage of Equation (63) is that it allows us to perform the integration in Equation (45) efficiently and accurately and using less samples than would otherwise be necessary. Because of the structure of our estimate, we can break the integration into two very simple integrals for a variety of distributions that are followed by the probability density function of a state transition, \( f(\chi) \). Using our estimate of the future outcomes, Equation (45) becomes:

\[
Y_f(a, \chi) = \int_{\chi' \in \chi} M_a(\chi, \chi') \hat{\nu}_{t+1}(\chi) \, d\chi' \\
Y_f(a, \chi) = \int_{\chi' \in \chi} M_a(\chi, \chi') \hat{m}(\chi) \, d\chi' + \int_{\chi' \in \chi} M_a(\chi, \chi') \hat{e}(\chi) \, d\chi' 
\] (64) (65)

We will show the power of Equation (65) using an example. Assume that the probability of a state transition is modeled using a normal distribution with mean, \( \mu \), and standard deviation, \( \sigma \). We will also assume that we performed a regression kriging using a quadratic regression and using a Gaussian correlation function. Then, the first integral of Equation (65) is:
\[
\int_{\chi' \in X} M_a(\chi, \chi') \hat{m}(\chi) d\chi' = \int_{\chi' \in X} \frac{1}{\sqrt{2\sigma^2 \pi}} e^{-\frac{(\chi' - \mu)^2}{2\sigma^2}} (\beta_2 \chi'^2 + \beta_1 \chi' + \beta_0) d\chi'
\]  
(66)

Which can be simplified to:

\[
\int_{\chi' \in X} M_a(\chi, \chi') \hat{m}(\chi) d\chi' = \beta_2 (\mu^2 + \sigma^2) + \beta_1 \mu + \beta_0
\]  
(67)

That is, the integral is simply equal to the regression’s zeroth coefficient, the offset term, added to the first coefficient, the linear term, multiplied by the mean of the normal distribution. Such an integral can be performed extremely quickly once the regression coefficients have been found. A single regression kriging can be performed for all uncertain states of the world, meaning that the computational cost of this integral does not increase substantially as additional uncertain states of the world are added. We must still perform the second integral, however this too is simplified:

\[
\int_{\chi' \in X} M_a(\chi, \chi') \hat{e}(\chi) d\chi' = \int_{\chi' \in X} \frac{1}{\sqrt{2\sigma^2 \pi}} e^{-\frac{(\chi' - \mu)^2}{2\sigma^2}} r(\chi') * \mathcal{R}^{-1}(Y - F(\chi')\hat{\beta}) d\chi'
\]  
(68)

Note that \( \mathcal{R} \) for a gaussian correlation function assumes that the correlation kernel, which describes how the correlation changes with the distance between points, for sampled points is a diagonal matrix, that they are independent of each other. Also, because the Gaussian correlation function is a similar form to the pdf of a normal distribution, we can simplify Equation (68) to:

\[
\int_{\chi' \in X} M_a(\chi, \chi') \hat{e}(\chi) d\chi' = \sum_{i=1}^{N} \frac{1}{\sqrt{2(\sigma^2 + Var_i)\pi}} e^{-\frac{(\mu - \bar{x}_i)^2}{2(\sigma^2 + Var_i)}}
\]  
(69)

where \( \bar{x}_i \) and \( Var_i \) are the mean and variance that correspond to the ith sample point’s representative mean and variance for the local disruption. That is, the integral is equal to a summation of known terms that mimic the pdf of a normal distribution with mean \( \mu - \mu_i \) and variance \( (\sigma^2 + \sigma_i^2) \). This too can be readily computed, and scales with an increase in
the number of states better than for the standard dynamic programming case, which scales \(O(x^4)\).

The main advantage of the multi-dimensional case is that it greatly reduces the number of states that we must consider in order to evaluate the value of an option. This is because a kriging model can be specified for a set of states of the world, which is then interpolated as necessary. If the response surface is well modeled by the kriging surface, then we do not need many sample points to characterize the kriging model. Certain kriging models also offer simplifications for Equation (45), meaning we can compute the integral very quickly without a substantial loss in accuracy. The end result is a model that does not suffer as much from the curse of dimensionality, greatly expanding the application of dynamic programming, especially with reference to flexible systems.

4.3.3 Limitations

There are limitations to the low- and multi-dimensionality models for flexible systems. The one-dimensionality model assumes that the number of states to be considered is relatively small. However, it is difficult to say how few is appropriate as the answer is very case specific. For systems that can be analyzed very quickly, the acceptable number of states is larger. One of the advantages of the one-dimensionality model is that the approximation can be made more precise by selecting higher order Newton-Cotes quadrature formulas, thus making it more adaptable to particularly complex value streams. However, as the order increases, the number of states likely needs to increase. For a particularly complicated value stream surface, the multi-dimensionality method is unlikely to work as well. For the multi-dimensionality model, the surfaces that are modeled must be well approximated by the kriging model, or the error may increase. Another challenge for the multi-dimensionality model is that it may be challenging to determine how many sample points are needed for a given surface. Again, more complicated surfaces will require more samples, but the proper number of samples is going to be case specific and
require substantial knowledge about the system to be done efficiently. The multi-dimensionality model assumes that uncertain states are independent, that there is no correlation between uncertain states. This may not be true in the general case, however there are ways of solving such problems. For highly correlated uncertain states, new states could be determined that are orthogonal to each other and can be related to the old uncertain states.

4.4 An Approach for Interpreting Dynamic Program Results

In this section, we introduce an approach for interpreting the results of a dynamic programming analysis of flexible systems. We assume that decision makers are using one of the models from section 4.3.1 or 4.3.2 to model flexibility. In both cases, we take advantage that the dynamic program evaluates future decisions for every state considered. This provides the decision with much more information that simply an evaluation of the value of an artifact, it provides insight into how decision in the future should be made. This information can be used to directly create flexible-design heuristics.

The results of a dynamic program can be used to create a flexible-design heuristic which will suggest actions that are consistent with the assumptions of the dynamic program. What this means is that we can create a heuristic that suggests actions to perfectly mimic a rational decision maker, at least up to the assumptions made in the dynamic program. The dynamic program at every state is given a set of decisions to consider, and evaluates the value of each decision, selecting the best. We assume that one of the results of the dynamic program is the policy of future decisions for each state of the world, $a^*$. If we aggregate these future decisions, we can, for example, make a simple decision rule that will result in the same value as the dynamic program. This would be the best decision rule that could be identified using Equations (40) or (41). How to do this in practice depends on whether the model considered is the low- or multi-dimensional model. However, it should be noted that heuristics that are created from the following methods are of limited
use as heuristics since the decision problem must have been already solved by the dynamic program. Because the results are likely very case specific, a heuristic that is identified from the following methods is unlikely to be useful for dissimilar cases. Instead, the goal is to identify characteristics that influence future decisions more generally so that better heuristics can be made. This is done by investigating the properties of the resulting heuristics. For example, if a resulting simple decision rule does not depend strongly on a particular state, we can conclude that the particular state is not relevant to the future decision. As will be shown, many of the insights from analyzing the future decisions may be difficult to identify otherwise.

First, we must clarify what we mean when we say the best decision rule heuristic. When we say the best decision rule heuristic we refer to the heuristic which results in the best flexible alternative and makes the best future decisions such that value is maximized. In practice, this means that the best decision rule must result in the true value of the option. The true value of the option is necessarily the greatest value of the option identified in section 4.2 when comparing with the value of the option as suggested by alternative decision rules. Therefore, we can use this as a check that we do indeed identify better decision rules.

### 4.4.1 One-Dimensional Case

In the one-dimensional case, we can directly use the results of the dynamic program to determine the best decision rule heuristic. We assume that for every state considered we are also informed what the decision from the dynamic program was. Then, the best decision rule heuristic parameters are such that for the set of states in which the dynamic program makes a particular decision, the decision rule heuristic also recommends that same decision. In practice this can be done by identifying low and high thresholds for each of the states considered which result in a particular decision. For example, consider a very simple case where we must make a decision to build or not to build a product based on
initially uncertain demand. Then the best decision rule will identify the set of demands for which a dynamic program would recommend the product be built. This can be in the form of a lower and upper bound on demand. In this case, it can most likely be assumed that the upper bound is infinity, although the dynamic program can only identify the upper bound as the maximum value of demand that it considered. An example of the one-dimensional method is shown in section 4.5.

4.4.2 Multi-Dimensional Case

Unlike the one-dimensional case, the multi-dimensional case cannot simply use the results of the dynamic program directly. This is because the multi-dimensional case presumably uses a very sparse collection of states, and therefore the decision rule identified from this is likely to be very poor. Instead, we will use the resulting kriging models to develop the best decision rule heuristics with linear classifiers.

Decision rules can be made by comparing kriging models based on the decision criterion. The decision criterion is a maximization of value, stated in Equation (47). As identified earlier, the expectation of the future values is identified by interpolating kriging models. A different kriging model exists for each potential decision, at each possible state, excluding the uncertain states that are accounted for in the kriging model. In the dynamic program, at each state the different kriging models are interpolated and compared to determine which decision to make. After the dynamic program however, we can make this same comparison to identify the conditions under which a rational decision maker would make different decisions. For a given state we evaluate the performance of the system for a particular decision, , and the expected future values as interpolated with the kriging model for the same decision. The value is compared for all decisions to determine which decision results in the highest value. For systems with many potential uncertain states, characterizing the boundaries for the different decisions may be difficult.
To characterize the boundaries for the decision rule we can use linear classifiers. Linear classifiers are used in the field of machine learning to discern the identity of an object based on its features (Herbrich, 2001). By selecting a series of points to be interpolated by the kriging models and classifying them by the best decision, the linear classifier then describes the boundaries for which the given decisions should be made. This method is elaborated upon and performed in Chapter V for the case of a hybrid energy system.

4.5 Design of a Parking Garage

By investigating the conditions under which each decision was made, flexible-design heuristics can be generated. By keeping track of which decisions are made for every state, and for every time, we can identify the properties of the state that drive each decision. This identification may be trivial if the state is relatively limited. For example, if the only uncertainty in the problem is the demand, then the flexible-design heuristic can be to make a decision, as identified by the dynamic programming model, whenever the demand exceeds a particular value, as identified as the minimum demand the decision was made for. In effect, we are generating the “best” decision rules for our problem. To verify our results, we then compare with a Monte Carlo approach using the found decision rules. This becomes more challenging as the number of states grows, and is covered in Chapter IV.

To compare flexible-design heuristics, we modify the design decision framing method identified in section 3.6.1. Flexible-design heuristics already require substantial computational resources. To manage the available resources, we use a representative contextual situation, instead of a set of contextual situations to compare the heuristics. This contextual situation also assumes a particular value of the truth to be evaluated by the omniscient supervisor. Neither the truth nor the amount of information revealed to the decision makers was varied in order to reduce computational costs. Furthermore, the omniscient supervisor uses the same heuristic as the dynamic programming heuristic. The
reason this is acceptable is because the different heuristics being investigated use similar assumptions regarding the value of a design alternative, and therefore evaluate the same design alternative similarly. Thus, either of the heuristics could be used as the omniscient supervisor and yield similar estimates for the value of the design process. To compare the inflexible-design and flexible-design heuristics we assume that the dynamic programming heuristic is the closest approximation of reality and use it as the omniscient supervisor, recognizing that the costs of using the dynamic programming heuristic may still make it unsuitable in certain contextual situations. Figure 14 shows the modified design decision framing method for the parking garage case. Note that because only one contextual situation is considered and the information revealed to the decision makers is same, we no longer loop through the process multiple times per heuristic.

Figure 14: The Design Decision Framing Method for the parking garage case.
4.5.1 The Value of Flexibility

To explore the above method of generating flexible-design heuristics, we investigate the design of a parking garage, based on the problem introduced in (de Neufville, et al., 2006). The example focuses on a designer who must select how many floors a parking garage should be built with. The area of each floor and the number of parking spaces per floor has already been determined. The decision maker receives revenue for every parking space rented per day, but incurs the cost of building and maintaining the parking garage, as well as the cost of leasing the land. Each level in the parking garage has 200 spaces, $n_{sp}$, each of which generates $10,000 per year on average if it is used, and costs $3,000 per year in maintenance costs, $C_M$, even if unused. The cost to build the first two floors of the parking garage, is $17,000 per space, or $3.4 million per floor, $C_{bf}$. However, the construction costs increase by 10% per level, $C_{af}$, above the second floor to account for the increased weight. It is assumed that all construction can be performed within one year, and a 2% fee, $C_s$, is charged based on the construction cost for expansions that occur after the initial construction. The cost to lease the land, is $3.3 million per year, and is for 16 years, the total time investigated for the parking garage. The decision maker is faced with uncertainty in the number of parking spaces demanded, and so must take care to select the number of floors that maximize his profit.

The decision maker also has the opportunity to invest in making the parking garage flexible. In this case, the decision maker can invest in thicker supports to permit additional floors to be built in the future. The cost of building the thicker supports, $C_f$, is assumed to be 30% of the cost for building the first two floors, $2.04 million. It is assumed that if and when the decision maker decides to add additional floors, it does not materially interrupt the daily business. The additional floors can then be built using the cost structure identified previously. The decision maker uses a dynamic programming approach to determine the number of initial levels to build the parking garage with.
The decision maker selects the initial number of levels and whether to invest in making the parking garage flexible. First, we analyze the problem assuming the design is not flexible using the inflexible-design heuristic:

<table>
<thead>
<tr>
<th>Planning Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the inflexible-design of a parking garage</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Select the number of levels that maximizes the NPV, not considering future decisions</td>
</tr>
</tbody>
</table>

The decision maker generates revenue, $R_v$, based on the total number of available spaces, $n_{avsp}$, or the demand in a given year, $D_t$, whichever is lowest, with a price per unit occupied, $P_{sp}$:

$$n_{avsp}(n_L) = n_L n_{sp}$$  \hspace{1cm} (70)

$$R_v(n_L, D_t) = \min(n_{avsp}(n_L), D_t)P_{sp}$$  \hspace{1cm} (71)

where $n_L$ is the number of levels and $t$ is the time in years. In this case, the inflexible case, the number of levels does not change with time, however in the flexible case it will be allowed to vary in time based on the decision maker’s future decisions. In the inflexible case, only the demand changes in time, and is explained after we introduce how the profit is determined for the decision maker. Along with the revenues, the decision maker also incurs many costs. As mentioned previously, the decision maker has an operating cost, $C_O$, and capital costs, $CC$:

$$C_O(n_L) = n_L n_{sp} C_M$$  \hspace{1cm} (72)

$$CC(n_L) = \begin{cases} C_{bf} n_L, & n_L \leq 2 \\ C_{bf} \left(2 + \sum_{j=1}^{n_L-2} (1 + C_{af})^j\right), & n_L > 2 \end{cases}$$  \hspace{1cm} (73)

Using a discount rate, $r$, of 10%, and the leasing costs, $C_L$, the inflexible profit, $P_{if}$, and inflexible NPV, $V_{if}$, of the parking garage can be determined:
\[ P_{lf}(n_L, D_t) = f(x) = \begin{cases} \text{RV}(n_L, D_{st}) - C_O(n_L) - C_L, & t = 0 \\ -C_L - CC(n_L), & t > 0 \end{cases} \quad (74) \]

\[ V_{lf}(n_L, D_t) = -CC(n_L) + \sum_{t=1}^{T} \frac{P_{lf}(n_L, D_t, t)}{(1 + r)^t} \quad (75) \]

Since the decision maker has no future decisions, the Bellman equation for this problem is rather simple:

\[ V_{lfB_t}(n_L, D_t) = P_{lf}(n_L, D_t, t) + \int_{D_f \in D} f(D_f | D_t) V_{lfB_{t+1}}(n_L, D_f) \frac{1}{1 + r} dD_f \quad (76) \]

where \( D_f \in D \) is the future demand, from the set of possible demands, whose probability density depends on the current demand. In the original example, the demand was modeled by an exponential function that prevented the demand in any given year from decreasing, and often resulted in demand increasing substantially year over year. To keep in spirit with this, we model the probability of the future demand using a triangular distribution that prevents demand from decreasing from its initial value of 750 parking spots, \( D_0 = 750 \):

\[ f(D_f | D_t) = \begin{cases} \frac{(D_f - D_t)}{(2.58 \times 10^{-3} D_t)}, & D_t \leq D_f < 1.068 D_t \\ \frac{(1.076 D_t - D_f)}{(3.04 \times 10^{-4} D_t)}, & 1.068 D_t \leq D_f \leq 1.076 D_t \end{cases} \quad (77) \]

Note that for a given year, the demand ranges between last year’s demand, and 107.6% last year’s demand. After 15 years, this possible demands range from 750 to 2250 spaces. This is chosen purposefully to limit the possible states that are investigated in the dynamic programming model. Since we include a state for each integer demand, the number of different states in the final year is 1501 per level investigated, \( n_D \). Because we are using discrete states, we must modify Equation (76) to reflect this:

\[ V_{lfB_t}(n_L, D_t) = P_{lf}(n_L, D_t, t) + \sum_{j=1}^{n_D} f(D_{j+1} | D_t) V_{lfB_{t+1}}(n_L, D_{j+1}) \frac{1}{1 + r} \quad (78) \]
where $D_{t+1}$ refers to each possible demand for the next year. This allows us to recursively solve for the NPV for a given initial number of levels and compare that NPV with other levels to determine the best initial number of levels.

The problem changes substantially when we now consider flexibility. Instead of remaining with our initial number of levels, we can now make decisions to increase the number of levels for the following years, allowing the decision maker to only invest in expansion when necessary. The decision maker now adopts a future decision policy to intelligently select the number of levels for each year, $n_{Lt}$. Because the number of levels can change with each year, we must update how the construction costs are determined, as the costs depend on the number of current floors and additional floors to be built, as well as the cost of the thicker supports to enable the future construction, $C_f$:

$$ C_f = 0.3CC(2) \quad (79) $$

$$ CC_f(n_{Lt}, n_{Lt+1}) = f(x) = \begin{cases} 0, & n_{Lt} \geq n_{Lt+1} \\ CC(n_{Lt+1}) + C_f, & n_{Lt} = 0 \\ (n_{Lt+1} - n_{Lt})C_{bf}C_s, & 0 < n_{Lt} < n_{Lt+1} \leq 2 \\ \sum_{j=n_{Lt}}^{n_{Lt+1}-n_{Lt}} (1 + C_{af})^jC_{bf}C_s, & 2 < n_{Lt} < n_{Lt+1} \\ (CC(n_{Lt+1}) - C_{bf})C_s, & 0 < n_{Lt} < 2 < n_{Lt+1} \end{cases} \quad (80) $$

where the number of floors to be built in the first year is required to be a positive integer, $n_{Lt} > 0$. Because the decision maker can now build in any year, we must also update the profit and NPV to reflect the flexible decision making, $P_f$ and $V_f$, respectively:

$$ P_f(n_{Lt}, n_{Lt+1}, D_t) = Rv(n_{Lt}, D_t) - C_o(n_{Lt}) - C_b - CC_f(n_{Lt}, n_{Lt+1}) \quad (81) $$

$$ V_f(n_{Lt}, n_{Lt+1}, D_t) = \sum_{t=1}^{T} \frac{P_f(n_{Lt}, n_{Lt+1}, D_t)}{(1 + r)^t} \quad (82) $$
Since the decision maker may change the number of levels in each year, the Bellman equation must represent the decision making process by choosing the number of floors which maximizes the decision maker’s value:

\[ E[V_t(n_L, n_L+1, D_t)] = P_f(n_L, n_L+1, D_t) + \int_{D_f \in D} \frac{\text{Pr}(D_f|D_t) V_{fB_{t+1}}(n_{L+1}, D_f)}{1 + r} dD_f \]  

(83)

\[ V_{fB_t}(n_L, D_t) = \max_{n_{L+1}} (E[V_t(n_L, n_{L+1}, D_t)]) \]  

(84)

Again, because we have discrete states we must modify Equation (84) accordingly:

\[ E[V_t(n_L, n_{L+1}, D_t)] = P_f(n_L, n_{L+1}, D_t) + \sum_{j=1}^{n_D} \frac{\text{Pr}(D_{j+1} | D_t) V_{fB_{t+1}}(n_{L+1}, D_{j+1})}{1 + r} \]  

(85)

\[ V_{fB_t}(n_L, D_t) = \max_{n_{L+1}} (E[V_t(n_L, n_{L+1}, D_t)]) \]  

(86)

Equation (86) enables us to determine the value of a flexible parking garage. Thus, the flexible-design heuristic is:

**Planning Heuristic**

*Applicability Context:* When analyzing the flexible-design of a parking garage

*Applicable Action Set:* Select the number of levels that maximizes the NPV while considering future decisions

We can compare the inflexible and flexible parking garages to determine the value of including flexibility. Because in this example we are looking at risk neutral decision makers, we can use Equation (39) to determine the value of the option:

\[ V_O = V_{fB_0} (n_{L_f}^*, D_t) - V_{ifB_t} (n_{L_if}^*, D_t) \]  

(87)

where \( n_{L_f}^* \) is the optimal number of levels for the inflexible case and \( n_{L_f}^* \) is the optimal number of levels for the flexible case.

Table 2 shows the chosen number of floors assuming no option, inflexible, and with the option, flexible, as well as the NPVs as calculated by the inflexible and flexible models. When it is assumed that no decisions will be made in the future, best decision is to use 5
levels. Although this means many spaces are unoccupied initially, the expected greater demand in the later years results in an overall profitable parking garage. However, initially constructing 5 levels with the thicker supports results in an overall loss. The option costs 2.04 million dollars, while the expected NPV decreased by about 1.51 million dollars. This suggests that the added flexibility increased the profitability of the parking garage by 0.53 million dollars, which is much less than the cost of the thicker supports. Therefore, at 5 levels a rational decision maker would choose not to purchase the option. Even so, by considering the option the decision maker can still improve the parking garage by initially building 4 levels. This decision results in an expected NPV of 0.57 million dollars, which is greater than the decision to build 5 levels without the option. But, if the decision maker elected not to purchase the option and initially built 4 levels the designer would expect the NPV to be a negative 0.60 million dollars. This means that although the option cost an additional 2.04 million dollars, the cost was more than offset by increased expected revenues of about 3.21 million dollars. The impact of the option increases the expected NPV by 375%.

This results in an overall value of the option of approximately 0.45 million dollars, including the cost of the option.

4.5.2 Interpreting the Results of Future Decisions

We can also use the results of the flexible dynamic programming model to create a flexible-design heuristic. For a given number of levels, demand, and time, the model chooses the future number of levels to maximize the expected NPV. By identifying the
minimum demand for which a level is chosen in a given year, we can create a decision rule that uses the optimal demand to decide when and how much of an expansion should occur. To do this we use conditional-go decision rules of the form: when analyzing the flexible-design of a parking garage, upgrade to a particular number of levels if the demand is within a particular range in a particular year. In this case, if the demand for a given time is within a chosen criteria, $\phi_t$, then a given number of levels is used. In this case, $\phi_t$ can be represented by an $n_{t_{\text{max}}} \times n_{t_{\text{max}}} \times 2$ array where $n_{t_{\text{max}}}$ refers to the maximum number of floors considered. The first dimension corresponds to the number of levels that should be chosen, given the conditions in the second and third dimension are met. The second dimension corresponds to the current number of floors, while the third dimension corresponds with the minimum and maximum demand. For example, if the third dimension of $\phi_t$ at the $n^{th}$ horizontal and $m^{th}$ vertical is [200,500], this means that the decision maker should choose $n$ floors if the current number of floors is $m$ and the demand is between 200 and 500 spaces per year. To determine the parameters of $\phi_t$ we refer to the dynamic programming results and select the minimum and maximum demands for which a given number of levels is selected for a given year:

$$n_{L_{\text{fut}}}(n_{t_{\text{c}}}, D_t, t) = \arg \max_{n_{t_{\text{c}}+1}} \left( \mathbb{E}[V_t(n_{t_{\text{c}}}, n_{t_{\text{c}}+1}, D_t)] \right)$$  \hspace{1cm} (88)

$$\phi_t(i, j, 1) = \min d$$  \hspace{1cm} (89)

s.t. $n_{L_{\text{fut}}}(j, d, t) = i$

$$\phi_t(i, j, 2) = \max d$$  \hspace{1cm} (90)

s.t. $n_{L_{\text{fut}}}(j, d, t) = i$

To use $\phi_t$ to determine the number of levels, we consider a row vector of binary variables, $e_t$, where each variable is a represents a given number of levels:

$$e_t(n_{t_{\text{c}}}, D_t) = 1(D_t \in \phi_t)$$  \hspace{1cm} (91)

which is used in the follow decision rule, $\delta_0(n_{t_{\text{c}}}, D_t)$:
\[ \delta_\theta(n_{Lt}, D_t) = e_t(D_t) [1, 2, ..., n_{L_{\text{max}}} - 1, n_{L_{\text{max}}}]^T \]  
(92)

To verify the results the dynamic programming model, which uses Equation (86), we can use Equation (82) with the optimal decision rule to determine the number of future levels:

\[ V_f(n_{Lt}, D_t) = \sum_{t=1}^{T} P_f(n_{Lt}, \delta_\theta(n_{Lt}, D_t), D_t) \frac{1}{(1 + r)^t} \]  
(93)

whose value should agree with the results of applying Equation (86).

We can also evaluate the expected NPV using Monte Carlo simulations. Instead of evaluating Equation (93) for a single realization of the stochastic process, we can estimate the expected NPV by using many realizations:

\[ E[V_f] = \frac{1}{n_{sim}} \sum_{j=1}^{n_{sim}} V_f(n_{Lt}, D_{jt}) \]  
(94)

where \( n_{sim} \) is the number of simulations. We use 10,000 simulations to keep the variance of our estimate small. Even with a large number of simulations, the computational cost of estimating the expected NPV is presumed to be much lower than the expected NPV as evaluated using Equation (86).

We can now use the results of the flexible dynamic programming model to develop a simple decision rule. However, the results indicate that the structure of \( \phi_t \) can be further simplified. As it turns out, the decision to expand the number of floors is independent of the current number of levels, and instead only depends on the current demand and time. The reason for this is that the decision maker chooses the number of levels which maximize their expected NPV, which is the same as identifying the largest number of levels whose marginal benefit is equal to or greater than the margin cost of expansion. For this example, the cost of expanding from two levels to three levels to four levels is the same as expanding from two levels to four levels directly. The same is true for the benefits of expanding from two levels to four levels. Therefore, if the decision maker finds the situation advantageous to expand from three levels to four levels, the decision maker will also find the situation
advantageous to upgrade from two levels to four levels. Now $\phi_t$ can be an $n_{L_{\text{max}}}$ x 2 matrix where the rows correspond to the number of future levels, and the columns correspond to the minimum and maximum demands for which the decision maker should expand to the given number of levels. Even though the decision to upgrade does not depend on the current number of levels, the decision rule does, so we must update our equation for the decision rule:

$$\delta_0(n_{L_t}, D_t) = \max(n_{L_t}, e_t(D_t)[1, 2, ..., n_{L_{\text{max}}} - 1, n_{L_{\text{max}}}])$$

Using Equations (88)-(90), we can determine the parameters of the decision rule. Table 3 shows the values of the entries for the $\phi_t$ matrix for all values of time. Note that due to our assumptions, the demand is never below 750 spaces, and the maximum number of spaces demanded only grows at 7.6% per year. Also, there are no entries for years 11 to 13 as the dynamic program elected not to expand in those years. For those years, $\phi_t$ is such that the decision rule will inform the decision maker not to expand. It does not make fiscal sense to expand in those years as there is insufficient time to recoup the additional investment.
Table 3: Decision rule parameters from the dynamic program results

<table>
<thead>
<tr>
<th>φ</th>
<th>Minimum Demand (Spaces)</th>
<th>Maximum Demand (Spaces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ₁</td>
<td>750</td>
<td>807</td>
</tr>
<tr>
<td>φ₂</td>
<td>750  865</td>
<td>864  869</td>
</tr>
<tr>
<td>φ₃</td>
<td>750  865</td>
<td>864  935</td>
</tr>
<tr>
<td>φ₄</td>
<td>750  865</td>
<td>864  1006</td>
</tr>
<tr>
<td>φ₅</td>
<td>750  865  1060</td>
<td>864  1059  1082</td>
</tr>
<tr>
<td>φ₆</td>
<td>750  865  1060</td>
<td>864  1059  1164</td>
</tr>
<tr>
<td>φ₇</td>
<td>750  865  1061</td>
<td>864  1060  1253</td>
</tr>
<tr>
<td>φ₈</td>
<td>750  865  1061  1258</td>
<td>864  1060  1257  1348</td>
</tr>
<tr>
<td>φ₉</td>
<td>750  869  1070  1289</td>
<td>868  1069  1288  1450</td>
</tr>
<tr>
<td>φ₁₀</td>
<td>750  899  1123</td>
<td>898  1122  1561</td>
</tr>
</tbody>
</table>
Even if the initial number of levels is less than four, the decision rule that is applied in the first year results in parking garages that are, at minimum, four levels. To expand and include a fifth floor, the minimum amount of time required to recoup the investment is five years assuming every space is sold every year. Since the parking garage is leased for sixteen years (including the construction in year zero), after the eleventh year, corresponding to \( \phi_{10} \), it does not make economic sense to expand, no matter the level of demand. In the two years prior, \( \phi_9 \) and \( \phi_{10} \) show that the minimum demand threshold for each possible upgrade changes with time. The minimum demand increases in this period to match the breakeven demand for the limited remaining time. When there is very little time left, the minimum demand threshold, and therefore expected revenue, must increase to justify the capital expenditure, i.e. the capital cost of expansion. However, in the years preceding \( \phi_9 \) the minimum demand threshold does not change. When there is a lot of time remaining the minimum demand threshold must be such that the expected revenue offsets the operating costs and opportunity cost of waiting to build. The opportunity cost in waiting is apparent by recognizing the decision maker’s preferences for immediate outcomes over future outcomes, represented by their discount rate. The cost of upgrading does not change from year to year, but because of the discount rate the decision maker would prefer to spend capital in later years. The difference in expected profit for the following year must exceed the difference in the capital costs for building in one year as compared to the next:

\[
\int_{D_f \in D} \frac{f(D_f | D_t) (R_v(n_{l+1}, D_f) - C_O(n_{l+1}) - R_v(n_{lt}, D_t) - C_O(n_{lt}))}{1 + r} dD_f 
\geq CC_f(n_{lt}, n_{lt+1})(1 - \frac{1}{1 + r})
\]

which can be further simplified in our case by recognizing that for each of our potential upgrades, the minimum demand threshold exceeds the maximum capacity of the current number of levels. Since the demand is unable to decrease in this example, the predicted
profitability for the following year if we do not upgrade is known, and we can simply investigate the profitability of increasing the number of levels by one:

\[
\int_{D_f \in D} f(D_f | D_t) (Rv(1, D_f - n_{L_t} n_{sp}) - C_D(1)) dD_f \geq CC_f(n_{L_t}, n_{L_t} + 1)(r) \quad (97)
\]

The integral of the PDF of a triangular with a linear function, which is the left-hand side of Equation (97) is simply the average of the lower bound, mode, and upper bound of the triangular distribution multiplied with the slope of the linear function and added to the offset of the linear function. This gives us a closed-form equation for determining the minimum demand for a given upgrade, assuming that there is sufficient time for the increased profitability to offset the capital cost of the upgrade and the demand does not exceed capacity:

\[
(1.048D_t - n_{L_t} n_{sp}) * P_{sp} - n_{sp} C_M = CC_f(n_{L_t}, n_{L_t} + 1)(r) \quad (98)
\]

Applying Equation (98) yields the following minimum demands for the decision to upgrade from four to five, five to six, and six to seven floors: 864, 1059, and 1255 spaces, respectively. These results are in very close agreement with the results of the dynamic program.

Figure 15 graphically shows the range of demands that yield the corresponding decision to upgrade for each set of levels as well as the theoretical minimum demands identified from Equation (98). Again, the theoretical minimum demand thresholds match the constant demand region for their respective levels. It can easily be seen that all upgrades investigated deviate from their constant regions beginning in year nine. After year nine, the rate of change of demand changes changes very rapidly, beginning with the most levels. For a given year after nine, the larger the number of recommended levels the higher the rate of change of demand with time. This is because the capital cost of upgrading increases
with the number of levels, thus requiring much higher demands to justify the increased expense.

Figure 15 also shows that the decision maker never elected to upgrade to eight or nine levels despite the minimum threshold demand for these levels being possible. The reason the decision maker did not choose to upgrade is because the minimum demand thresholds were not encountered until after year ten, when the time to recoup the investment in upgrading is too limited to justify the expense. It should be noted that Figure 15 displays possible states that are not actually encountered by the most preferred alternative as selected by the dynamic program. For example, it is easily seen that any alternative that began with less than four levels was quickly upgraded to four levels in year one. Thus, the best alternative did not encounter a situation where it could upgrade to two levels as the decision rule suggests in year three. However, because the dynamic program does not know
this a priori, it must evaluate every possible state and thus is able to determine that if a decision maker were in this situation he should make the decision to upgrade to two levels.

To verify the performance of the decision rule, we compare with a Monte Carlo method with 10,000 samples that utilize the above decision rule for future decisions. The results agree extremely well with the dynamic programming results, with an error of less than one percent. Comparing with the decision rule used in (de Neufville and Scholtes, 2011), the decision rule from the dynamic program outperformed. The Monte Carlo method also simulated the decision problem much more quickly, taking 1.5 seconds, whereas the dynamic program 18.6 seconds on an Intel® Core™ i7-3770 3.40 GHz processor with 20 GB of RAM. Of course, performing the Monte Carlo with the best found decision rule was only possible after performing the dynamic program, so we do not want to suggest that the Monte Carlo was a fair alternative to the dynamic programming approach in this case.

4.5.3 Conclusions

We have investigated the flexible-design of a parking garage using a dynamic programming method and Monte Carlo method. The results indicate that a dynamic programming approach can efficiently determine the best initial decision of building four floors when future decisions are considered, instead of the five floors that would be perceived to be the best alternative if future decisions are ignored. Including the option to increase the number of floors in the future was found to have a value of $450,000 for a system that was initially valued at $120,000, resulting at an impressive final value of approximately $560,000.

The investigation also revealed many insights concerning the future decisions for this system that can be used by other designers in similar circumstances. Importantly, we identify that for cases similar to the parking garage, there are two regimes that require very different attention. The first is the earlier regime where changes in minimum demand
thresholds for upgrading do not vary substantially with time. The second is the later regime where the changes in minimum demand thresholds do change substantially with time as the decision makers must evaluate if the upgrades will will be worth their expense. This information could be used to select or develop better heuristics of similar systems. However it should be noted that these results are not the general case. Another case is considered in Chapter V where the form of the future decision rules are quite different, emphasizing the need to consider the characteristics of the system being investigated.

4.6 Summary

In this chapter we address methods of enabling flexible design. First, we discuss how considering future decisions can add value and what we mean when we say the value of the option. Second, we discuss methods of modeling future decisions, including the use of dynamic programming. Third, we introduce an approach for interpreting dynamic programming results to better inform decision makers. Fourth, we introduce a motivating example of a parking garage to demonstrate a method of modeling future decisions and an approach for interpreting the dynamic programming results.

In the first part of this chapter we discuss and define the value of an option. An option adds value by revealing better alternatives, whose value is properly revealed by considering how future decisions will modify the system. Then, the value of an option is the difference in value between the best alternative as selected considering future decisions and the best alternative as selected when ignoring future decisions. We also introduce methods for evaluating the value of an option, which cannot be evaluated explicitly except for special cases. However, these methods assume that the future decisions can be appropriately modeled.

In the second part of this chapter we discuss methods for modeling future decisions. This must be done efficiently to allow designers to analyze flexible systems. This must also be done accurately, as the effectiveness of the decision will be reflected in the system value.
and therefore on the selection of the best alternative and the estimation of the value of the option. We discuss simple decision rules as one alternative, however due to the challenges associated with determining effective decision rules we recommend two dynamic programming approaches. The dynamic programming approaches differ based on the number of states being evaluated. Surrogate modeling is used for multi-dimensional cases as the low-dimensional method is inappropriate due to the curse of dimensionality. We discuss some novel simplifications that are possible due to the surrogate modeling that enable efficient and accurate future decisions to be made.

In the third part of this chapter we discuss how the results of the dynamic programming approaches can also lead to additional insights about the system and its future decisions. Such insights can better inform decision makers, enabling the selection or creation of better heuristics. These insights are possible because the dynamic programming approach evaluates alternatives from the perspective of a rational decision maker, choosing the alternative that maximizes their value. Thus, the best possible decision is made, within the assumptions and accuracies of the dynamic program. By investigating the characteristics of the state, we can learn about the factors of the state that are most important in making subsequent decisions for the system in question. We can use this knowledge to select heuristics that recommend actions consistent with the insights gathered for similar systems.

In the fourth part of this chapter we apply the dynamic programming heuristic and the approach for interpreting the results to a motivating example of a parking garage. We find that by considering future decisions we can identify a better alternative, and thus the value of the option is positive. In fact, the value of the option is substantial in this case, increasing the value of the system by a factor of almost four. We also used the results to gather insights about the future decisions that would have been challenging to realize otherwise. We also compare the dynamic programming results to a Monte Carlo approach.
that utilizes the best found decision rule and found the results to be consistent, suggesting that for similar systems, similar heuristics can be used to approximate the dynamic programming results more quickly.
CHAPTER V

EVALUATION OF A HYBRID ENERGY SYSTEM

5.1 Introduction

This chapter focuses on design and evaluation of Hybrid Energy Systems (HESs) using different real options techniques. HESs are long-lived systems involving large capital investments and subject to large uncertainty — the characteristics under which real options are likely to be beneficial. This case study is much more in depth and complex than the motivating example in section 4.5.

First, we discuss the performance model that was used. It is a screening model that was developed from a more complex Modelica model. Screening models are simple models that quickly analyze a system. One such example of a screening model is one which is developed from a more complex model. Then, we discuss the economic model used, including the uncertain states of the world that contribute to the complexity of the problem. To evaluate the problem, we use different decision models, including an optimization of a simple decision rule as well as the multi-dimensional dynamic programming model. In this section we also discuss an approach for interpreting dynamic programming results. Next, we discuss the results of implementing the decision models with the performance and economic models, including an investigation into how uncertainty influences the value of options. Finally, we summarize the conclusions of this chapter. Portions of this chapter have previously been published in (Binder, et al., 2017) and portions of section 5.3.1 are derived from (Binder, et al., 2014).
5.2 Design of a Hybrid Energy System

We have previously introduced and motivated hybrid energy systems in Chapter II. We now tackle how decision makers can manage the risks inherent to HESs to determine their viability. Although we have already elaborated their potential benefits, these benefits must be compared with the potential costs of determining whether an HES should actually be built. To help mitigate these costs, decision makers may pursue different risk management strategies. Using a particular risk management strategy, we then discuss the design problem that will be solved.

One risk management strategy is to reduce the uncertainty. For instance, for some uncertain factors, the uncertainty can be reduced by performing additional experiments or gathering additional information. Unfortunately, this is not so simple for HESs, which are subject to many different uncertain factors such as the price of electricity and other commodities. These prices in turn are affected by so many factors that accurately predicting them tens of years into the future is not feasible. Therefore, other risk management strategies should be used in conjunction with uncertainty reduction.

A second risk management strategy is to manage the uncertainty. For instance, feedback control can be used to manage the impact of uncertainty. Certain HESs incorporate this risk management strategy by allowing more operational freedom. One such example is an HES which can divert steam from the non-renewable generation as a function of renewable generation. This strategy is limited in the sense that it only manages uncertainty at the operational level. As an alternative, one could also allow for configuration changes to manage the risk.

A third risk management strategy is to design the system to be robust to uncertainty. While change is guaranteed, it is often uncertain and unknowable which particular change will occur. To mitigate large uncertainty, a designer may choose to make the system insensitive or robust to uncertainty. The “cost” of robustness is reflected in reduced
performance under nominal conditions. Due to uncertainty, the actual conditions may differ greatly from the nominal conditions so that robust performance is better than optimized nominal performance. Rather than requiring a specific problem formulation as is suggested in the Robust Design literature (Phadke, 1989, Taguchi, 1986, Taguchi and Clausing, 1990), robustness is automatically obtained by maximizing the expected utility assuming risk aversion (Lee, et al., 2010). Still, there may be times when the world has changed so much that an inflexible but robust system would still be too risky and therefore undesirable.

A fourth risk management strategy is to design the system to have option-based flexibility, so that it can adapt to future events when additional information becomes available. For example, consider the design of a plant that uses a Rankine cycle and only produces electricity. One of the potential risks is that the price of electricity may decrease drastically going forward, resulting in substantial losses. To reduce the negative outcome of this event, the decision maker could decide to install a steam header that allows for some or all of the steam that is generated to be diverted elsewhere. Then, if the price of electricity does indeed decrease, the decision maker can consider additional components to use the steam more profitably, e.g., use the steam as process heat for a chemical plant.

The possibility of future change is an important assumption in determining what decision to make now. Without uncertainty, there would be no knowledge gained in the future and thus no benefit in waiting to make a decision. However, we do not know the future, so we must make decisions by weighing the value of outcomes with their likelihood. By building flexibility into the system, we are building options to reconfigure or enhance the system that can be exercised in the future.

The configuration of the HES determines both the design variables and the option to consider. Figure 16 shows the initial HES configuration for the inflexible and flexible design alternatives. The configuration includes renewable and non-renewable sources of energy, wind and natural gas, respectively. To allow the HES to respond to changes in

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availability, the HES produces gasoline in addition to electricity. In cases where the available electrical generation exceeds demand, steam is diverted to the chemical plant. This provides process heat to produce gasoline in the form of low cost steam from the natural gas heat source. The HES generates revenue from selling electricity on the Day-Ahead energy market and from selling gasoline.

While recognizing it is not feasible today, the option reflects the belief that implementing a nuclear source in the future, could provide additional economic benefits. Specifically, the option is to build an SMR to replace the natural gas unit as the primary heat source. To generate electricity or provide process heat, steam from a secondary loop is used, preventing radioactive contamination.

The design variables are for a high-level, conceptual design of the HES, and include the size of the natural gas heat source (MWt), the renewable penetration (%), the size of

**Figure 16:** An example MIMO HES.
the electrical battery (MWh), the size of the balance of plant (MWt), and the trigger value ($/kWe). The trigger value determines when the option is exercised, and is explained in more detail in the Design Decision Model section. The renewable penetration is the ratio of energy generated by renewables to the energy demanded, whereas the instantaneous renewable penetration is the ratio of the power generated by renewables to the power demanded (Weisser and Garcia, 2005). For the inflexible design case, the size of the balance of plant (BOP) is assumed to be equal to the size of the natural gas heat source. Similarly, for the flexible design case, the BOP is designed to match the size of the SMR. The BOP thus represents the option. An oversized BOP, relative to the initial heat source, allows for the installation of a larger SMR in the future, providing a more valuable outcome over the life of the system. To determine the expected NPV of the HES, we must first evaluate its physical properties as a function of the independent variables.

5.3 Performance Model

The performance model describes the physical properties of the HES, such as the production of electricity, gasoline, or carbon dioxide. The model predicts these quantities, given as inputs the size of the primary heat generator, the renewable penetration from wind, and the size of the electrical batteries. The primary power generator in this system is the Rankine cycle, which uses the primary heat source to convert thermal energy to electrical energy. A Modelica model defines these physical performance attributes as a system of differential algebraic equations. This system is then simplified to act as a screening model and enable flexible analysis of the system for the following heuristic:

<table>
<thead>
<tr>
<th>Analysis Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the flexible-design of an HES</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Analyze the physical performance by using simple algebraic relations to approximate the performance as a quasi-steady-state model</td>
</tr>
</tbody>
</table>
5.3.1 Modelica Model

The Modelica model of the HES uses models from ThermoPower (Casella and Leva, 2003), and is divided into four categories. The categories are: generation, distribution, controls, and loads. Figure 17 shows the top level model for the HES. The generation category has four main components: the nuclear reactor, the renewable generation in the form of a wind farm, and the auxiliary boiler. The distribution category is further

![Modelica HES performance model, displayed in Dymola.](image)

Figure 17: Modelica HES performance model, displayed in Dymola.
categorized into: thermal conditioning and thermal to electrical conversion. The thermal conditioning subcategory has four main components: a preheater, secondary boiler, condensation drum, and a steam header. The thermal to electrical conversion has two main components: a series of three turbines for the secondary loop, and an auxiliary turbine, both of which convert thermal energy from the steam into mechanical energy and then into electrical energy. For simplicity, there is a condenser included with the turbine models that conditions the outlet steam. The controls category has two main components: A governor for the three turbines and a controller for the pumps. The loads category has two main components as well: an electrical load and a chemical load. The electrical load is modeled in the component for the wind farm and electric battery. The chemical load is present in the form of a chemical plant.

The generation category includes models that add energy to the system from an outside source. In this case, the outside sources are nuclear fuel, wind, and natural gas, which are converted to thermal, electrical, and thermal energy by the nuclear reactor, wind farm, and auxiliary boiler, respectively.

The nuclear reactor’s purpose is to supply thermal energy to be distributed to the turbines and chemical plant to be converted to electricity and chemicals, respectively. Because we are primarily concerned with the dynamics of the interconnected components between the electrical and chemical components, the model for the nuclear reactor is greatly simplified. Figure 18 shows the model for the reactor, which is modeled using a heat source that is ramp-up from a nominal starting power to full load. This ramping-up process represents the start up of the nuclear reactor from a reduced load and allows the system to warm up to steady state operating conditions. The heat from the reactor is transferred through a heat exchanger that represents the secondary fluid that would not be irradiated by the nuclear reactor. This ramping-up process is used to allow the simulation to arrive at its initial operating conditions robustly.
The wind farm’s purpose is to supply electrical energy directly to the electrical load. The wind farm produces electrical power, $P$, based on the wind speed, $v$:

$$ P = \frac{1}{2} \rho A v^3 C_p $$

(99)

where $\rho$ is the air density, $A$ is the cross sectional area of the blades, and $C_p$ is the power coefficient. We use representative wind speed data for a location near Idaho Falls, Idaho from the Western Wind dataset, made available by the National Renewable Energy Laboratory (2006). It is assumed that the seasonal changes in wind speed are reasonably
approximated by this data, and that future wind profiles do not differ substantially. Figure 19 shows the model for the wind farm, which is combined with the model for the electrical storage and the load for the power grid. It is assumed that the HES is bidding on the day-ahead active power market such that the independent system operator is responsible for managing ancillary services. This is explained in more detail in section 5.4, but what this suggests from a modeling perspective is that we can make simplifications that allow us to

Figure 19: Modelica model for the wind farm, electrical battery, and grid storage, displayed in Dymola.
only account for the active power that is generated. In this context, the electrical battery acts as a filter to smooth the power generation from the wind farm. Then, the difference between the power demanded and the power generated from renewable sources must be satisfied by the turbines and is described in the turbine and governor models.

The auxiliary boiler’s purpose is to supply low-pressure steam to the steam header that feeds the chemical plant. Figure 20 shows the model for the auxiliary boiler. The auxiliary boiler is heated by burning natural gas, whose rate, again, depends on the quantity of steam necessary. Then, a

Figure 20: Modelica model for the auxiliary boiler, displayed in Dymola.
plant when the secondary loop does not produce sufficient steam through the secondary boiler. Then, a pressure control valve controls the outlet of steam to maintain the low-pressure header at a pressure of 4.24 MPa. As the secondary boiler only generates steam when the turbines do not consume as much steam, the secondary boiler increases steam production as the apparent demand for electricity to the turbines decreases. This occurs when demand is low, or when the wind farm is producing more power. Thus, the quantity of steam necessary from the auxiliary boiler increases as the power generated by the wind farm decreases. The distribution category is further divided into the subcategories of thermal conditioning and thermal to electrical conversion. The thermal conditioning is made up of a preheater, secondary boiler, condensate drum, and low-pressure steam header while the thermal to electrical conversion consists of an auxiliary turbine and three turbines.
The preheater serves multiple purposes, conditioning the steam for the nuclear reactor, turbines, and auxiliary boiler. Figure 21 shows the model for the preheater. The preheater has a heat exchanger that is controlled with a temperature control valve to ensure that water entering the nuclear reactor is at 215.6 degrees Celsius. The heat exchanger uses steam from the outlet of the nuclear reactor to heat this water, which comes from the condensate drum, as will be discussed later in this section. The preheater also uses a pressure control valve to ensure that the outlet pressure from the nuclear reactor, the inlet to the turbines, is maintained at 6.764 MPa. If the turbines are generating more power than
demanded, this pressure control valve will release steam to the secondary boiler, which supplies steam to the low-pressure steam header, for use by the chemical plant.

The secondary boiler converts thermal energy from the secondary fluid to thermal energy in the tertiary fluid, which satisfies the needs of the chemical plant. Figure 22 shows the model for the secondary boiler. Steam from the high pressure control valve in the preheater is used to produce steam at 4.4 MPa. The high pressure steam that is used to heat the secondary boiler then passes through a preheater which warms the low-pressure water that will be heated by the secondary boiler before being sent to a condensate drum. After the low-pressure water has been preheated and then heated to low-pressure steam, it is then sent to the low-pressure steam header through a series of pipes which can be seen in Figure 22: Modelica model for the secondary boiler, displayed in Dymola.
17, through which the pressure is expected to drop to 4.24 MPa. The chemical plant and nuclear plant must be co-located or relatively close such that the thermal losses through these pipes do not diminish the value of the secondary boiler.

The condensate drum collects the mixture of steam and water from the secondary boiler and temperature control valve from the preheater to separate the liquid water and vaporous steam. Figure 23 shows the model for the condensate drum. The liquid water is pumped to the high pressure of 6.764 MPa before being sent to the preheater to be warmed before going to the nuclear reactor and be converted to high pressure steam. The outlet from the condensate drum is actually combined with the outlet from the condenser before being preheated and sent to the nuclear reactor. To control how much water is sent to the nuclear reactor, we use a pump controller that attempts to maintain the temperature at the
reactor outlet to control the overall flowrate of water and controls the flowrate from the condensate drum and condenser to maintain minimum levels of water in both components.

The low-pressure steam header’s purpose is to supply low-pressure steam to the chemical plant to produce gasoline. Figure 24 shows the model for the low-pressure steam header. The low-pressure steam header sends cold water to the secondary boiler, to be heated and returns as steam, which is combined with steam from the auxiliary boiler. This combined steam is then sent to the chemical plant as needed to produce gasoline. The steam that returns from the chemical plant is combined with any excess steam that was not sent

Figure 24: Modelica model for the low-pressure steam header, displayed in Dymola.
to the chemical plant and sent to an auxiliary steam turbine, which powers the distribution of steam for the low-pressure steam header.

The auxiliary turbine’s purpose is to power the distribution of the low-pressure steam that is fed to the chemical plant. It is assumed that any power it produces is approximately equal to the power needed to operate this distribution, and thus does not contribute to the demand for electricity. Figure 25 shows the model for the auxiliary turbine. After the steam passes through the turbine it is sent to a condenser. The outlet of the condenser is cold water, which is reused to feed the secondary boiler and auxiliary boiler’s needs for cold water.

Figure 25: Modelica model for the auxiliary turbine, displayed in Dymola.
The three turbines from the secondary loop are expected to produce the majority of electricity for the HES. Figure 26 shows the model for the three turbines. Because the wind farm may cause the demand for electricity to change quickly, we use three turbines so that the turbines can run at near optimal efficiency even when the flowrate of steam varies from zero to 100% utilization. As such, the turbines are sized to account for different proportions of the expected power to be generated. The three sizes are ‘60%’, ‘30%’, and ‘15%’. The larger the percentage, the larger the turbine. Note that together the turbines actually exceed 100%. This is to allow for a small increase in power over the rated power of the system as needed. One potential application is if the pressure controls for the high pressure steam fail, the excess pressure could be routed through the turbines, which can also be disconnected from the grid if need be. The flowrate to the individual turbines are controlled.
by flow control valves. In turn, the flow control valves are controlled by the governor, which will be explained further in this section. The turbines are mechanically connected and controlled to operate at a constant rotational speed of 60 Hz for electricity generation. The torque is controlled such that at 60 Hz the turbines generate the required power that is demanded by the electrical grid. As such, the torques generated from each turbine are combined additively, and with the speed of rotation can be used to determine the power generation. It is assumed that the mechanical efficiency, which accounts for friction losses, is 98%. At the same time, we assume the isentropic efficiency, the thermal energy available to be converted as compared to the ideal quantity of thermal energy available to be converted is constant at 92%. To determine the extraction pressure for the turbines we use Stodola’s cone law (Cooke, 1983). After exiting the turbines, the steam enters a condenser. The condenser’s outlet is pumped back up to the high pressure of 6.764 MPa and sent to the preheater. Again, the flowrate through the pump is controlled by the pump controller.

There are two main control circuits: a controller for the pumps and a governor for the three turbines. The controller for the pumps controls the temperature of the outlet from the nuclear reactor and the flowrates out of the pumps for the condensate drum and condenser. The governor controls the flowrate of superheated steam to the three turbines based on the power demanded and speed of the turbines. The pump controller performs three main functions: maintaining the temperature of the steam exiting the nuclear reactor, maintaining the level in the condenser, and maintaining the level in the condensate drum. The logic to determine the flowrate from the condensate drum and condenser is primarily in the textual equations. While the preheater ensures that water entering the nuclear reactor is at a particular temperature, the pump controller determines the flow rate of water that will enter the nuclear reactor. Since we do not modulate the nuclear reactor, the quantity of heat exchanged is constant. Therefore, if the flowrate of water entering the nuclear reactor is low, the outlet temperature of the steam may be very high. To prevent this, the
pump controller determines the flowrate to prevent the steam from exceeding 311.4 degrees Celsius using a PID controller. The total flowrate is determined by how much water is coming from the condensate drum and from the condenser. If either of these components run out of water the pumps would be pumping steam, which requires substantially more power to pump as the density is much lower than that of liquid water. Therefore, the controller determines the appropriate flowrate from the individual components to prevent either level from dropping below 10% of its full capacity.

The purpose of the governor is to determine the flowrate of steam that should pass to each of the turbines. Figure 27 shows the model and logic of the governor. Omega and tieLinePower refer to the speed and electrical power that is not satisfied by the wind farm. The governor is a non-linear controller that balances two objectives: a desired speed of rotation for the turbines, and a desired power generation for the turbines. To increase the speed of the turbines assuming a constant power demanded the flowrate of steam to the
turbines must be increased. To increase the power generation of the turbines assuming a constant speed the flowrate of steam to the turbines must also be increased. Thus, when the speed and power generated by the turbines are too low, the objectives of the controller are aligned and the flowrates to the turbines are increased. However, it may be that the objectives are not aligned. That is, the speed is low while the power generated is too high, or the speed is high while the power generated is too low. These situations may be the result of a power demand profile that changes very quickly, and are accounted for with Equation (100) at steady state (Glover, et al., 2012):

$$\Delta p_m = \Delta p_{ref} - \frac{1}{R_p} \Delta f$$

where $\Delta p_m$ is the change in the turbines output power, $\Delta p_{ref}$ is a reference power, $R_p$ is a regulation constant, and $\Delta f$ is the change in frequency.

The loads category has two main components as well: an electrical load and a chemical load. We have already discussed the electrical load briefly while discussing the
wind farm and electric battery. The chemical load is present in the form of a chemical plant and requires further discussion.

The chemical plant uses thermal energy from available steam to convert natural gas into a combination of gasoline and liquid petroleum gas. Figure 28 shows the model of the chemical plant. The process for converting the natural gas is broken up into four processes: reforming steam, methanol synthesis, methanol purification, and the methanol to gasoline conversion. Because we are more concerned with the interactions between the power generating systems and chemical plant, we have abstracted these processes to a series of first order models which approximate the production of gasoline and liquid petroleum gas. The amount of gasoline and liquid petroleum gas produced depends on the natural gas consumed, as well as the quantity of low-pressure steam that is made available by the low-pressure steam header. Waste steam is returned to the low-pressure steam header to be sent to the auxiliary turbine.

The above is a general overview of the Modelica models that have been created to simulate the HES. To simulate two weeks of operating the HES takes 62 seconds on an Intel® Core™ i7-3770 3.40 GHz processor with 20 GB of RAM. While it may make sense to use these models to design a HES in practice, we wish to simulate the system much more quickly in order to evaluate the viability of a flexible HES. Ignoring future decisions and uncertainty, if we assume that identifying the best configuration of the HES will take 100 simulations we assume that each simulation takes 62 seconds, then the total time required will only be approximately 100 minutes. However, if we include uncertainty and further assume that 1,000 Monte Carlo simulations per design investigated is appropriate, the total simulation time is now approximately 72 days. If we wish to consider future decisions in this scenario, we then require another decision optimization, itself with more possible realizations. With a nested optimization it is easy to see how this problem becomes
unmanageable. We will reduce the computational complexity of this problem in multiple ways, the first of which is the development of a screening model.

### 5.3.2 Screening Model

To analyze the HES’s performance while considering future decisions requires a model that simulates very quickly. The Modelica model simulates too slowly for this purpose. The Modelica model also considers aspects that are not of much importance to the flexible analysis, such as being able to investigate the transient changes in the temperature of the superheated steam as the wind farm produces more electrical power. While this is important for the operation of the HES, if we are confident that such transients do not alter substantially the production of electrical power and chemicals, then it is not as important to a flexible analysis. Therefore, we can make simplifying assumptions without negatively impacting our ability to analyze the system for flexibility.

To reduce the computational complexity of this model, an algebraic screening approximation has been developed (Binder, et al., 2014). We have achieved this by making several simplifying assumptions and approximations, but have maintained reasonable accuracy. For instance, the controllers that regulate the system performance are made infinitely stiff so that they respond instantaneously to transient changes and can be described algebraically. This simplification is appropriate because the response of the HES to changes in demand and wind power is extremely fast (Garcia, et al., 2015). After having confirmed that transients do not substantially alter the production of electricity and chemicals, we instead consider the quasi-steady-state performance of the system. While we still care to investigate how the wind farm influences the production of electricity from the Rankine cycle and the chemical plant, we can simplify the model by investigating the impact on the overall power, instead of how the differences in wind speed influence the speed of the turbines.
For other aspects of the model, we have used the Modelica model results to make assumptions about the operation. For example, we can use the results of the Modelica model to determine the overall efficiency of this Rankine cycle. Then, instead of modeling the dynamics of the power generation, we can select the size of the plant and use the cycle efficiency to determine how much power can be generated. The resulting screening model is used in conjunction with the economic and decision models to estimate the performance of the HES for different scenarios and identify the best design alternative and is included in the appendix.

5.4 Economic Model

The economic model describes the economic properties of the HES. In doing so, an important element to consider is the particular electric market of interest. Although dependent on the particular zonal/regional market and Independent System Operator (ISO) under consideration, four main electric markets can be identified, namely, Day-Ahead (DA) market, Hour-Ahead market, Real-Time (RT) market, and Ancillary Services markets. The DA market in turn includes the DA Energy and Capacity markets. Similarly, Ancillary Services markets include the Regulation and Operating Reserves markets, providing multiple services such as regulation up / down, responsive (spinning) reserves, and non-spinning reserves. All these markets except the RT market are financial markets in the sense that delivery of power is optional. While a given HES may potentially participate in several of these markets, we here restrict its participation to the DA energy market for simplicity. Furthermore, it is assumed that the given HES always underbid in its DA energy market so that its bid is always accepted, similar to the practice often conducted by baseload utilities using nuclear and coal energy (Bower and Bunn, 2001). Thus, our economic model characterizes yearly cash flows based on economic data from (Garcia, et al., 2015). However, the price of electricity has been modified to an average of 10.86 cents per kilowatt-hour, the average US cost of electricity (U.S. Energy Information
Administration, 2015), and the capital and operational cost of the BOP is based on (U.S. Energy Information Administration, 2013, Veatch, 2012). To determine the yearly cash flows, the economic model uses the outputs of the performance model. The performance is aggregated to determine the cash flows of the HES over a total of eighty years. Eighty years is used to allow the option to trigger and generate cash flows from the upgraded system. For example, a system that triggers the option in year fifteen would not be operational until year twenty and the nuclear source may operate for an additional forty or sixty years (Yang, et al., 2008). In addition, the cash flows after eighty years are discounted so heavily that they are negligible. The heuristic for analyzing the economic model is then:

<table>
<thead>
<tr>
<th>Analysis Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the flexible-design of an HES</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Analyze the economic performance by investigating the NPV of the system over 80 years</td>
</tr>
</tbody>
</table>

The economic value of the system value may also be impacted by the evolution of prices outside the decision maker’s control.

The economic model describes the NPV of the HES as a function of its performance. The model is based in part on (Garcia, et al., 2015). The expressions for the cost parameters follow (Garcia, et al., 2015), shown in Table 4. We define the NPV, \( NPV \), based on the free cash flow, \( FCF \), through the life of the system in years, \( T_{max} \) (Garcia, et al., 2015):

\[
NPV = \sum_{k=0}^{T_{max}} \frac{FCF_k}{(1 + r)^k}
\]  

(101)

where \( r \) is the discount rate. The free cash flow then depends on the revenue, \( R_{vt} \), operating costs, \( OM_t \), depreciation, \( DA_t \), costs associated with producing \( CO_2 \), \( GHG_t \), and the capital expenditures, \( CC_t \), (Garcia, et al., 2015):
\[ FCF_k = (Rv_k - OM_k - DA_k(1 + i)^{-k})(1 - t\alpha) + DA_k(1 + i)^{-k} - GHG_k - CC_k \] (102)

The operating and maintenance costs can be further broken down into fixed, \( OM_{f,k} \), and variable costs, \( OM_{v,k} \):

\[ OM_k = OM_{f,k} + OM_{v,k} \] (103)

The fixed and variable operating and maintenance costs are subdivided for each component:

\[ OM_{f,k} = OM_{nuc,f,k} + OM_{ng,f,k} + OM_{aux,f,k} + OM_{bop,f,k} + OM_{wind,f,k} + OM_{batt,f,k} + OM_{chem,f,k} \] (104)

\[ OM_{v,k} = OM_{nuc,v,k} + OM_{ng,v,k} + OM_{aux,v,k} + OM_{bop,v,k} + OM_{wind,v,k} + OM_{batt,v,k} + OM_{chem,v,ng,k} + OM_{chem,v,w,k} \] (105)

where the subscript \( nuc \) refers to the nuclear reactor, \( ng \) refers to the natural gas heat source, \( aux \) refers to the auxiliary boiler, \( bop \) refers to the balance of plant, \( wind \) refers to the wind farm, \( batt \) refers to the battery, and \( chem \) refers to the chemical plant.

Taking into account that the unit costs for many of the variable costs are zero, Equation (105) simplifies to:

\[ OM_{v,k} = OM_{nuc,v,k} + OM_{ng,v,k} + OM_{aux,v,k} + OM_{chem,v,ng,k} + OM_{chem,v,w,k} \] (106)

We can similarly expand the capital costs:

\[ CC_k = CC_{nuc,k} + CC_{ng,k} + CC_{aux,k} + CC_{bop,k} + CC_{wind,k} + CC_{batt,k} + CC_{chem,k} \] (107)

Each of the components has an associated lifetime duration, \( T_i \), indicated in Table 4. After this time the component needs to be rebuilt. We also consider how long the nuclear and natural gas components will take to build, \( t_i \), as included in Table 4. It is assumed that the initial build times of the other components are less than or equal to the build time of the natural gas, which must be built before any revenues can be generated. This leads to the
following capital costs, which are distributed over the build time for the nuclear and natural gas:

\[ t_{\text{build}} = t^* + jT_{\text{nuc}} \] (108)

\[ CC_{\text{nuc},k} = \begin{cases} \frac{\alpha_{\text{nuc}}P_{\text{nuc},r}}{t_{\text{nuc}}}, & t_{\text{build}} \leq k < t_{\text{build}} + t_{\text{nuc}}; \forall j = 0, \ldots, \frac{T_{\text{max}} - t^*}{t_{\text{nuc}}} \\ 0, & \text{else} \end{cases} \] (109)

\[ CC_{\text{ng},k} = \begin{cases} \frac{\alpha_{\text{ng}}P_{\text{ng},r}}{t_{\text{ng}}}, & 0 + jT_{\text{ng}} \leq k < t_{\text{ng}} + jT_{\text{ng}}; \forall j = 0, \ldots, \frac{\min(T_{\text{max}}t^*)}{T_{\text{ng}}} \\ 0, & \text{else} \end{cases} \] (110)

\[ CC_{\text{aux},k} = \begin{cases} \alpha_{\text{aux}}P_{\text{aux},r}, & k = t_{\text{ng}} + jT_{\text{aux}}; \forall j = 0, \ldots, \frac{T_{\text{max}}}{T_{\text{aux}}} \\ 0, & \text{else} \end{cases} \] (111)

\[ CC_{\text{bop},k} = \begin{cases} \alpha_{\text{bop}}P_{\text{bop},r}, & k = t_{\text{ng}} + jT_{\text{bop}}; \forall j = 0, \ldots, \frac{T_{\text{max}}}{T_{\text{bop}}} \\ 0, & \text{else} \end{cases} \] (112)

\[ CC_{\text{batt},k} = \begin{cases} \alpha_{\text{batt}}P_{\text{batt},r}, & k = t_{\text{ng}} + jT_{\text{batt}}; \forall j = 0, \ldots, \frac{T_{\text{max}}}{T_{\text{batt}}} \\ 0, & \text{else} \end{cases} \] (113)

\[ CC_{\text{chem},k} = \begin{cases} \alpha_{\text{chem}}P_{\text{chem},r}, & k = t_{\text{ng}} + jT_{\text{chem}}; \forall j = 0, \ldots, \frac{T_{\text{max}}}{T_{\text{chem}}} \\ 0, & \text{else} \end{cases} \] (114)

where \( t^* \) is found by the different decision models, discussed in section 5.5, and \( P_r \) refers to the rated size of a particular component. The capital costs directly influence the depreciation of the assets. The depreciation is based on a particular rate set by the IRS depending on when an asset was completed (Internal Revenue Service, 2017). Thus, the depreciation is the depreciation rate for a given year, \( \rho_{\text{da},k} \), multiplied by the capital cost of the asset:

\[ DA_k = \sum \rho_{\text{da},k} \alpha_l P_{l,r} \] (115)

where \( \alpha_l \) and \( P_{l,r} \) are used to generally refer to the per unit capital cost and rated power for the \( l \)th component.

Some quantities are modeled as stochastic processes: the price of electricity, gasoline, natural gas, and the capital cost of nuclear. The variables that depend upon these
prices are: $R v_k$, $O M_{n g,v,k}$, $O M_{a u x,v,k}$, $O M_{c h e m,v,n g,k}$, and $C C_{n u c,k}$. The $C C_{n u c,k}$ is determined only when the option is exercised. Once the option is exercised, the capital cost of nuclear does not vary. However, the remaining variables do vary in time. The revenue for a particular year is the total revenue from selling electricity and gasoline (Garcia, et al., 2015):

$$R v_k = \int_{t \in T_k} P_e \beta_e + M_g \beta_g dt$$

(116)

where $T_k$ is the year long time period in year $k$, and $P_e$ and $\beta_e$ are the quantity of electricity sold and the price of electricity, respectively. This is to capture effect the daily or hourly variations in price may have on revenue. Similarly, $M_g$ and $\beta_g$ are the mass flow rate of gasoline sold and the price of gasoline, respectively. The prices of electricity and gasoline are from our stochastic process, described below, while the quantities of electricity and gasoline are determined by the performance model for a given input. The performance model also determines the rate that natural gas is consumed, either from the primary heat generator, $M_{n g,p h g}$, auxiliary boiler, $M_{n g,a u x}$, or chemical plant, $M_{n g,c h e m}$. These costs are the operating and maintenance costs associated with the natural gas, $O M_{n g,v,k}$, auxiliary boiler, $O M_{a u x,v,k}$, and chemical plant, $O M_{c h e m,v,n g,k}$ (Garcia, et al., 2015):

$$O M_{n g,v,k} = \int_{t \in T_k} M_{n g,p h g} \beta_{n g} dt$$

(117)

$$O M_{a u x,v,k} = \int_{t \in T_k} M_{n g,a u x} \beta_{n g} dt$$

(118)

$$O M_{c h e m,v,n g,k} = \int_{t \in T_k} M_{n g,c h e m} \beta_{n g} dt$$

(119)

To approximate the integrals in Equations (117)-(119) we use the trapezoidal rule (Milne, 2015).
The remaining variables are determined using the coefficients in Table 4 and the results of the performance model. The operating and maintenance costs of the nuclear can be determined based on the per unit cost and rated power, for years after the system is built (Garcia, et al., 2015):

\[ OM_{\text{nuc},f,k} = \beta_{\text{nuc},f}P_{\text{nuc},r} : k \geq t^* + t_{\text{nuc}} \]  \hspace{1cm} (120)

\[ OM_{\text{nuc},v,k} = \beta_{\text{nuc},v}P_{\text{nuc},r} : k \geq t^* + t_{\text{nuc}} \]  \hspace{1cm} (121)

For natural gas, we define the fixed costs similarly, but only for years when nuclear is not online:

\[ OM_{\text{ng},f,k} = \beta_{\text{ng},f}P_{\text{ng},r} : k < t^* + t_{\text{nuc}} \]  \hspace{1cm} (122)

The fixed operating and maintenance costs can be determined similarly, occuring once online (Garcia, et al., 2015):
Table 4: Overview of the nominal cost parameters for the HES
(in part based on (Garcia, et al., 2015))

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{\text{nuc}} )</td>
<td>4718</td>
<td>$/kW</td>
<td>(Du and Parsons, 2009, Shropshire, et al., 2009)</td>
</tr>
<tr>
<td>( \beta_{\text{nuc},f} )</td>
<td>27.91</td>
<td>$/MWh</td>
<td>(Ganda, 2014)</td>
</tr>
<tr>
<td>( \beta_{\text{nuc},v} )</td>
<td>2.14</td>
<td>$/MWh</td>
<td>(U.S. Energy Information Administration, 2013)</td>
</tr>
<tr>
<td>( t_{\text{nuc}} )</td>
<td>5</td>
<td>years</td>
<td>(U.S. Nuclear Regulatory Commission, 2016)</td>
</tr>
<tr>
<td>( T_{\text{nuc}} )</td>
<td>60</td>
<td>years</td>
<td></td>
</tr>
<tr>
<td>( \alpha_{\text{ng}} )</td>
<td>1057.44</td>
<td>$/kW</td>
<td>(U.S. Energy Information Administration, 2010)</td>
</tr>
<tr>
<td>( \beta_{\text{ng},f} )</td>
<td>3</td>
<td>$/MWh</td>
<td>(U.S. Energy Information Administration, 2010)</td>
</tr>
<tr>
<td>( \beta_{\text{ng},v} )</td>
<td>0.045</td>
<td>$/kg</td>
<td>(DOE, 2013)</td>
</tr>
<tr>
<td>( t_{\text{ng}} )</td>
<td>3</td>
<td>years</td>
<td>(U.S. Nuclear Regulatory Commission, 2016)</td>
</tr>
<tr>
<td>( T_{\text{ng}} )</td>
<td>30</td>
<td>years</td>
<td>(Northwest Power and Conservation Council, 2006)</td>
</tr>
<tr>
<td>( \alpha_{\text{bop}} )</td>
<td>719</td>
<td>$/kW</td>
<td>(Veatch, 2012)</td>
</tr>
<tr>
<td>( \beta_{\text{bop},f} )</td>
<td>10.5</td>
<td>$/MWh</td>
<td>(U.S. Energy Information Administration, 2013)</td>
</tr>
<tr>
<td>( \beta_{\text{bop},v} )</td>
<td>0</td>
<td>$/MWh</td>
<td></td>
</tr>
<tr>
<td>( T_{\text{bop}} )</td>
<td>40</td>
<td>years</td>
<td>(Quartz, 2013)</td>
</tr>
<tr>
<td>( \alpha_{\text{wind}} )</td>
<td>1728</td>
<td>$/kW</td>
<td></td>
</tr>
<tr>
<td>( \beta_{\text{wind},f} )</td>
<td>36.91</td>
<td>$/kW</td>
<td>(Tegen, et al., 2012)</td>
</tr>
<tr>
<td>( \beta_{\text{wind},v} )</td>
<td>0</td>
<td>$/kW</td>
<td></td>
</tr>
<tr>
<td>( T_{\text{wind}} )</td>
<td>20</td>
<td>years</td>
<td>(Wind Measurement International)</td>
</tr>
<tr>
<td>( \alpha_{\text{batt}} )</td>
<td>81.42</td>
<td>$/kWh</td>
<td>(Dakins, et al., 1994)</td>
</tr>
<tr>
<td>( \beta_{\text{batt},f} )</td>
<td>3</td>
<td>%</td>
<td>(Dakins, et al., 1994)</td>
</tr>
<tr>
<td>( \beta_{\text{batt},v} )</td>
<td>0</td>
<td>$/kWh</td>
<td></td>
</tr>
<tr>
<td>( T_{\text{batt}} )</td>
<td>15</td>
<td>years</td>
<td></td>
</tr>
<tr>
<td>( \alpha_{\text{chem}} )</td>
<td>4266129.1</td>
<td>$/(kg/s)</td>
<td>(Wood, et al., 2010)</td>
</tr>
<tr>
<td>( \beta_{\text{chem},f} )</td>
<td>12</td>
<td>%</td>
<td>(Wood, et al., 2010)</td>
</tr>
<tr>
<td>( \beta_{\text{chem},v,w} )</td>
<td>1.06e-3</td>
<td>$/kg</td>
<td>(City of Forth Worth, 2014)</td>
</tr>
<tr>
<td>( T_{\text{chem}} )</td>
<td>40</td>
<td>years</td>
<td></td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>( i )</td>
<td>3</td>
<td>%</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>( r )</td>
<td>5</td>
<td>%</td>
</tr>
<tr>
<td>Tax</td>
<td>( \text{tax} )</td>
<td>35</td>
<td>%</td>
</tr>
</tbody>
</table>
Other variables depend on the results of the performance model. The performance model also determines how much CO$_2$, $M_{c,k}$, is produced from burning or consuming natural gas. Combined with the per unit cost of CO$_2$, $\beta_c$, the cost associated with CO$_2$ is (Garcia, et al., 2015):

$$GHG_k = M_{c,k} \beta_c$$

(127)

although the per unit cost of CO$_2$ is constant, so we need only determine the total mass of CO$_2$ generated. Similarly, the performance model indicates the quantity of water that is consumed in the chemical reactions, $M_{w,k}$ (Garcia, et al., 2015):

$$OM_{chem,v,w,k} = M_{w,k} \beta_{chem,v,w}$$

(128)

Random variables and processes are introduced to model the uncertain beliefs a decision maker would have regarding future conditions of the HES and its (economic) environment. Over the eighty years we consider four uncertain variables that influence the profitability of the HES considerably. These quantities are the price of electricity, $\beta_e$, the price of natural gas, $\beta_{ng}$, the price of gasoline, $\beta_g$, and the capital cost of SMRs, $\alpha_{nuc}$. We model the price returns of electricity, natural gas, and gasoline based on Geometric Brownian Motion (GBM) diffusion models, as is generally accepted for commodities (Deng, et al., 2001, Osborne, 1959). The stochastic process, $X_t$, satisfies the stochastic differential equation (Merton, 1973):

$$dX_t = rX_t dt + sX_t dW_t$$

(129)
where $r$ is the percentage drift, $s$ is the percentage volatility, and $W_t$ is the Wiener process. However, GBM does not account for discrete changes such as price spikes in the price of electricity (Barlow, 2002). Price spikes, and seasonal changes in price, occur because electricity is not easily stored, and therefore must be used once generated (Geman and Roncoroni, 2006). The GBM model for electricity prices is updated to account for such spikes, as well as daily, weekly, and seasonal fluctuations in price based on historical data (ERCOT, 2015, Producers, 2015, U.S. Energy Information Administration, 2015). For simplicity, the process for the capital cost of nuclear is assumed to be modeled using GBM, but without considerations for seasonal changes. Table 5 shows the percentage drift and percentage volatility for the GBM processes. These values were determined based on historical data (U.S. Energy Information Administration, 2012) and include beliefs about how prices may change as a result of economic, environmental, and technological changes in the future. Then, the heuristic used to account for uncertainties in the system is then:

<table>
<thead>
<tr>
<th>Analysis Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the flexible-design of an HES</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Model uncertainty using the prices of electricity, natural gas, gasoline, and the capital cost of nuclear used a Geometric Brownian Motion model, updated to account for price jumps</td>
</tr>
</tbody>
</table>

### 5.5 Decision Model

In this section we describe two different methods for analyzing the decisions, including future decisions. The first is a Monte Carlo method which uses a simple decision rule to approximate the future decisions. The second is a multi-dimensional dynamic programming approach identified in section 4.3.2. To expand upon these results, we have also included a discussion of how we explore the results of the dynamic program to gather additional information, as described in section 4.4.2.
For both models the objective function is the same, namely, the expected utility of NPV. We assume constant risk tolerance, as expressed in an exponential utility function:

$$U(NPV) = 1 - e^{-\frac{NPV}{R}}$$

(130)

where $R$ is the risk tolerance. Since HESs require significant capital, we assume a large corporation with experience in energy generation. A company with sales of approximately $360$ billion and a ratio of risk tolerance to sales of 0.064 (Howard, 1988), has a risk tolerance, $R$, of $23$ billion. For the NPV of the HESs considered in this case study, this risk tolerance is close to risk neutrality. This enables us to use a dynamic program method which approximates the objective function with a risk neutral objective function.

To objectively compare the different flexible-design heuristics, we use the DDFM. However, we can again simplify the general DDFM outlined in section 3.6.1. As we did in section 4.5, we consider a representative contextual situation and do not vary the information revealed to the decision makers to reduce the computational costs of the comparison. When comparing the flexible-design heuristics, we use the dynamic programming heuristic for the omniscient supervisor. The main difference between the heuristics being investigated is how future decisions are made, but it is assumed that the dynamic programming heuristic necessarily includes more detail and is the closest approximation to reality and therefore acts as the omniscient supervisor. This still allows for a meaningful comparison of the heuristics as the dynamic programming heuristic uses more computational resources. These assumptions are the same as the previous chapters and therefore we use the same modified design decision framing method same as the one in section 4.5. Figure 29 shows the modified design decision framing method for the HES case.
5.5.1 Optimization of Decision Rules

The initial design decision is modeled using optimization. This optimization occurs for both the inflexible case and the flexible case, in which the future decisions are considered. The objective in each optimization is to maximize the expected utility of the NPV. To solve the optimization problem, MATLAB’s multi-start method with `fmincon` (MATLAB) is used, which provides a good balance between computational cost and the likelihood of identifying the global maximum.

To determine the most preferred flexible design, $b^*$, we model $NPV_{flex}$ based on the assumption that the future decisions are based on a simple decision rule. The decision rule is to exercise the option if the capital cost of nuclear drops below a threshold value. To best choose this threshold, we add a design variable, the trigger value, $\theta$, to the

---

**Figure 29:** The Design Decision Framing Method for the HES case.
optimization. It is assumed that executing the option at the trigger value is a reasonable approximation for a more rigorous decision analysis. That is, the heuristic is:

<table>
<thead>
<tr>
<th>Analysis Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context:</strong> When analyzing the flexible-design of an HES</td>
</tr>
<tr>
<td><strong>Applicable Action Set:</strong> Include in your model an upgrade to nuclear when the capital cost of nuclear has decreased below the value found to optimize the expected utility of the HES and select the initial sizing of the plant that maximizes the NPV</td>
</tr>
</tbody>
</table>

Computational experiments are performed to determine the most preferred HES for two cases. In these experiments, an optimization of the expected utility is performed to determine which design alternative is most preferred for each case. In the computational experiments, the stochastic processes are included using a Monte Carlo with ten-thousand realizations of the stochastic process. The samples of NPV are then used to evaluate and choose the most preferred design alternative. Experiment 1 compares the results of an optimization on both the inflexible and flexible HESs. Experiment 2 takes this further by modifying the percentage volatilities from Table 5 to investigate how the value of the option changes with increased uncertainty.

**Experiment 5.4.1.1**

Experiment 5.4.1.1 evaluates the relative merits of a real options analysis of the HES. Using Equation (130) as the measure of the decision maker’s utility, we use Equation (39) and identify the value of an option to build an SMR in place of a natural gas primary heat source for the HES. To approximate future decisions, we use a simple decision rule of the form “When the capital cost of nuclear decreases below the trigger value, $\theta$, build an SMR whose size matches the BOP.” The parameter of the decision rule, $\theta$, also called the trigger, has a length of one and is added to the optimization of the HES design alternatives. This allows us to define the time at which building the nuclear plant should begin. $t^*$ is the
first time that the capital costs of nuclear falls below the threshold value indicated by $\theta$, and indicates when the building of the nuclear should begin, $t_{\text{build}}$:

$$
t^* = \min_k (k: \alpha_{\text{nuc},k} \leq \theta)
$$

(131)

**Experiment 5.4.1.2**

Experiment 5.4.1.2 builds on experiment 5.4.1.1 but addresses the question of how uncertainty influences the value of an option. Because options give the decision maker the ability to make a more informed decision by waiting until additional information becomes available, one would expect that the more uncertain a decision maker is, the more valuable an option will become.

To examine if this is true, we use a computational experiment, in which the size of the uncertainty is characterized by the standard deviation of the stochastic process, $X_t$. For different values of this standard deviation, we use the same approach as in experiment 5.4.1.1 to determine the value of the option.

**5.5.2 Multi-Dimensional Dynamic Programming**

The problem of optimizing the HES is also solved using a dynamic programming approach. Due to the numerous uncertain states, we take the multi-dimensional approach laid out in section 4.3.2. The optimization of the HES is actually broken up into an optimization of the dynamic programming results to minimize the number of states the dynamic program must solve. The dynamic program must evaluate all possible states, including the inputs. Rather than solve the HES problem for all possible values of the inputs, we use *fmincon* again to locate the optimal HES configuration with fewer computational resources. The objective is a maximization of the dynamic program, which is only used to evaluate a given configuration. Note that we apply a different use case for the dynamic programming approach as compared to the Monte Carlo approach in section 5.5.1. Therefore, the results between the two are not directly comparable.
The design problem does change slightly for the dynamic program approach. The variables that are used are: the size of the natural gas heat source (MWt), the renewable penetration (%), the size of the electrical battery (MWh), the size of the balance of plant (MWt). Note that the dynamic program approach does not need to optimize the trigger, as the dynamic program will select the decision to upgrade to nuclear or not based on the expected NPV criterion. Thus, \( t^* \) is not relevant, as it is possible, and likely, to vary based on the policy adopted. That is, the dynamic programming heuristic is:

<table>
<thead>
<tr>
<th>Planning Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Applicability Context</strong>: When analyzing the flexible-design of an HES</td>
</tr>
<tr>
<td><strong>Applicable Action Set</strong>: Upgrade the plant such that it maximizes the NPV of the plant and select the initial configuration that maximizes the NPV.</td>
</tr>
</tbody>
</table>

The main problem for the dynamic program is to approximate the integral of future value as evaluated by Equation (45). We have previously shown how to estimate the future value:

\[
Y_f(a, \chi) = \int_{\chi' \in X} M_a(\chi, \chi') \hat{m}(\chi) d\chi' + \int_{\chi' \in X} M_a(\chi, \chi') \hat{e}(\chi) d\chi' \tag{132}
\]

But now we must determine the value of these two integrals for this more complicated case where there are four uncertain variables in the state. For a four dimensional quadratic, the estimate of the global trend, \( \hat{m} \), is still a summation of coefficients and values of sample points. We will use a quadratic polynomial again, and while we will have cross terms, the result is straightforward to calculate and analytically solvable. For the second integral of Equation (132), we follow a similar procedure to develop the simplified expression:

\[
\int_{\chi' \in X} M_a(\chi, \chi') \hat{e}(\chi) d\chi' = \prod_{j=1}^{4} \sum_{i=1}^{N} \frac{1}{\sqrt{2(\sigma_j^2 + Var_i)\pi}} e^{-\frac{(\mu_j - \bar{x}_i)^2}{2(\sigma_j^2 + Var_i)}} \tag{133}
\]
Again, although the added dimensions have complicated the expressions, it is still easily computed for each state. However, we have yet to discuss the number of states we will consider.

The accuracy of the surrogate model depends on the number of sample points used, $N$. To determine the appropriate number of samples requires knowledge of the system being investigated. For this problem we have investigated how the number of samples changes the expected NPV of the best alternative and have selected five points per dimension, resulting in an $N = 5^4 = 625$ samples for each year and option, a total of 100,000 sample points.

5.5.3 Interpretation of the Dynamic Programming Results

Interpreting the results of the dynamic program is more challenging when there are additional dimensions. The decision to build the nuclear reactor is represented as a surface on a 4-dimensional space. The challenge comes from representing this surface such that it is easily interpretable. We will identify whether a given combination of uncertain states leads the decision maker to upgrade to nuclear or not by using linear classifiers.

We must first identify when the decision maker would make the decision to upgrade and when he would not. Thus, we must compare the surrogate models. In our case, we will have two surrogate models to compare for each year: one for the case where the option is exercised and one for the case where the option is not exercised. Where the surrogate models intersect identifies the decision surface. We use a Naïve Bayes classifier to approximate the decision surface in homogenous coordinates by a hyperplane characterized by a 5-by-1 vector, $w$:

$$\hat{y}(x) = (\text{sign}(w^T x) + 1)/2$$

(134)

where a value of 1 indicates the option is exercised, and a value of 0 indicates the option is not exercised. The classifier can be made more accurate by considering higher order polynomials, expanding the coefficient vector as well. However, this may make
interpreting the results more difficult as it is more challenging to analyze the subsequent cross-terms as well as a mixture of quadratic and linear terms.

To easily interpret the results, a linear classifier is preferred. However, if a linear classifier is very inaccurate, it may be necessary to use higher order polynomials. To determine the accuracy, we can compare the decision surface as described by the classifier with the decision surface from the kriging results. Taking N samples across the design site space, we can identify the relative error, \( \eta \):

\[
\eta = \frac{\sum_{j=1}^{N} |\hat{y}(x_j) - y(x_j)|}{N}
\]  

(135)

If the relative error is unacceptably high for the linear case, higher order polynomials may be appropriate. The threshold will vary on a case by case basis and depend on the particular problem and level of insight desired.

We can use the identified coefficients to interpret the results more easily. One of the immediately obvious results will be the sensitivity of the decision surface to each variable. The larger the coefficient, the more that particular variable determines whether the decision is made. If a coefficient is zero, or close to zero, then the decision is approximately independent of that particular variable. We will demonstrate this and other methods of interpreting the results in section 5.6.3.

5.6 Results

In this section we review the results of using the two decision models: the simple decision rule and the dynamic program. We then also discuss the additional information that is gleamed from the dynamic program.

5.6.1 Optimization of a Decision Rule

Value of Flexibility
Table 5: Outcomes of the inflexible and flexible design alternatives for the HES

<table>
<thead>
<tr>
<th></th>
<th>Inflexible, a*</th>
<th>Flexible, b*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected NPV (Billions of Dollars)</td>
<td>6.81±0.09</td>
<td>7.02±0.09</td>
</tr>
<tr>
<td>Expected Utility (utils)</td>
<td>0.239±0.003</td>
<td>0.246±0.003</td>
</tr>
</tbody>
</table>

Table 5 shows the results of experiment 1, optimizing the inflexible and flexible HESs. It is clear that the flexible HES outperforms the inflexible HES with expected utilities of 0.243±0.003 utils and 0.236±0.003 utils, respectively. Since the expected utility of the flexible case is greater than the inflexible case, the value of the option is positive. \( \hat{V}_o \) was found to be 209±7 million, which is approximately the same as the increase in the expected NPV. When risk neutral, the value of the option is equal to the difference in expected NPV.

However, this comparison is not entirely fair. Although the inflexible alternative is designed under that assumption that it will operate in this configuration for 80 years, this is not likely to occur in practice. The constant configuration is only a simplifying assumption made for design purposes. To obtain a fairer comparison, the best inflexible design alternative, \( a^* \), determined by maximizing \( E[U(\text{NPV}_{\text{inflex}})] \), is evaluated using \( \text{NPV}_{\text{flex}} \), to estimate its NPV if the HES were reconfigured anyway. In this case, because \( a^* \neq b^* \), the flexible alternative remains $191±7 million more valuable.

There are several differences between the best found inflexible and flexible design alternatives. Table 6 shows the values of the design variables for \( a^* \) and \( b^* \). While the initial sizes of the natural gas plants are similar, the remaining design variables show clear
differences. The greatest change is in how large the BOP is to be built. In the inflexible case, the BOP is assumed to be the same size as the heat source. However, in the flexible case the BOP is initially oversized by 215 MWt to take advantage of the additional profit obtained from a larger SMR. The size difference is approximately the size of the auxiliary boiler for the chemical plant, suggesting that, once the option is exercised, the SMR replaces the primary heat source and the auxiliary natural gas boiler in the chemical plant. The decision maker then saves considerably when the costs associated with burning natural gas exceed that of using nuclear. However, this does not happen in every case investigated, so the decision maker must choose when to exercise the option. When the capital cost of nuclear falls below the trigger value of $4,260/kWe, the decision maker elects to build an SMR.

In addition to the difference in the BOP, the flexible design alternative also uses a considerably smaller electrical battery, even with comparable renewable penetration. Since the renewable penetration is based on the initial size of the heat source, the effective renewable penetration after the option is exercised is considerably smaller, which allows the battery to be smaller.

In addition to flexibility, there are other interesting results concerning the system itself. The renewable penetration plays a unique role in this particular HES. When renewables produce power, steam is diverted to the chemical plant, offsetting the amount of auxiliary heat needed. This suggests that the renewables provide a marginal benefit equal to the difference in cost between producing steam from the primary and auxiliary heat sources. This marginal benefit is approximately constant, as is the marginal cost of wind. If the marginal benefits are always greater than the marginal costs, the renewable penetration should be as large as possible. In our experiments, we limit the instantaneous renewable penetration to 80% to prevent the flow of steam through the turbines from falling too low. This causes a decreasing marginal benefit of larger wind resources in design.
alternatives with greater than 25.6% renewable penetration. Even so, the decision maker found it valuable to increase the installed renewable penetration above 25.6%. This reduces the required wind speed for 80% instantaneous renewable penetration, thus requiring less non-renewable sources for electricity production for a given wind speed. If the marginal cost, however, were to exceed the marginal benefit below the 80% instantaneous renewable penetration limit, then no renewables would be installed. This occurs when the capital cost of wind increases above $2,210 per kilowatt of installed capacity.

**Effect of Uncertainty on the Value of Flexibility**

Finally, the value of the option also depends on the decision maker’s uncertainty about the future. Table 7 shows the results from experiment 2, indicating how \( \hat{V}_o \) changes with the size of the uncertainty. Previously, the value of the option was $209 million. Instead of strictly increasing, the value of the option shows no discernable pattern when the relative standard deviation is increased.

<table>
<thead>
<tr>
<th>Multiples of the Percent Volatilities (-)</th>
<th>( \hat{V}_o ) (Millions of Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>209±7</td>
</tr>
<tr>
<td>1.05</td>
<td>195±6</td>
</tr>
<tr>
<td>1.10</td>
<td>202±7</td>
</tr>
<tr>
<td>1.15</td>
<td>199±7</td>
</tr>
<tr>
<td>1.20</td>
<td>199±7</td>
</tr>
</tbody>
</table>

5.6.2 Multi-Dimensional Dynamic Programming

The results of the dynamic programming approach are very similar to the simple decision rule in terms of the most preferred alternatives for the inflexible case, \( a^* \), and the flexible case, \( b^* \). This suggests that the value of the heuristic comparing the dynamic program and the simple decision is near zero for this case. This is because the optimal decision strategy as determined by the dynamic program is very similar to using the simple decision rule heuristic. As will be seen in the next section, one of the uncertain states has no influence
on the decision. While the remaining three uncertain states do influence the future decision, their influence is small. This is easily seen from Table 5, which shows that the option only adds approximately $200 million for a project that is initially valued at $6.81 million, less than a three percent increase in value. When the maximum value added by the option is small, then comparing different analysis methods may not be useful. Instead, comparing different methods of analyzing future decisions should be reserved for high option value targets. Even though there was no change in decision, and therefore no change in value, the next section reveals that the dynamic programming approach was still valuable because of the additional information it conveys over the Monte Carlo approach.

5.6.3 An Approach for Interpreting the Results from Dynamic Programming

Figure 30 shows the normalized coefficients of the linear classifier. The relative values show that the coefficient for the chemical price is two to three orders of magnitude
smaller than the other coefficients and oscillates between positive and negative values. This suggests that the decision to upgrade to nuclear is nearly independent of the price of chemicals. This makes sense when reviewing the economic model. Because the size of the chemical plant is held constant and the HES does not change its operation as a function of the price of chemicals, the impact of the price of chemicals is independent on the decision. If the decision maker does upgrade to nuclear, they may reduce the costs associated with generating chemicals, but since the quantity of chemicals remains constant this is still independent of the price of chemicals. Figure 30 also shows that the coefficient for the nuclear is a magnitude higher than for the price of electricity or the price of natural gas. This suggests that the price of nuclear is the primary factor in determining whether to upgrade or not and explains why the simple decision rule, which only used the price of nuclear was able to perform similarly to the dynamic program.

We could also gain information by comparing the coefficients using other methods. One could compare the individual coefficients assuming the other uncertainties remained constant to isolate the effects of one variable at a time. For $w^T x = 0$, this is the same as restricting the vector of $x$ to one for each entry except one:

$$x_{e,\text{cons},k} = -\frac{(w_{0,k} + w_{ng,k} + w_{\text{chem},k} + w_{\text{nuc},k})}{w_{e,k}}$$  \hspace{1cm} (136)

$$x_{ng,\text{cons},k} = -\frac{(w_{0,k} + w_{e,k} + w_{\text{chem},k} + w_{\text{nuc},k})}{w_{ng,k}}$$  \hspace{1cm} (137)

$$x_{\text{chem},\text{cons},k} = -\frac{(w_{0,k} + w_{e,k} + w_{ng,k} + w_{\text{nuc},k})}{w_{g,k}}$$  \hspace{1cm} (138)

$$x_{\text{nuc},\text{cons},k} = -\frac{(w_{0,k} + w_{e,k} + w_{ng,k} + w_{\text{chem},k})}{w_{\text{nuc},k}}$$  \hspace{1cm} (139)

where the subscripts $e$, $ng$, $\text{chem}$, and $\text{nuc}$, refer to the uncertainties and coefficients associated with the price of electricity, natural gas, gasoline, and nuclear, respectively. The subscript $\text{cons}$ indicates that the other coefficients were held constant. While this may add
information, it may be misleading. In the dynamic program, each of the uncertain variables have a drift associated with them. Therefore, it is expected that the variables will change in value substantially compared to the initial case. However, even the general trend of values may be misleading due to the effect of inflation. Inflation makes it appear as if the variables drift to much higher values, when in current dollars they may not change very much at all. Thus, it may be meaningful to compare the individual coefficients on an inflation adjusted basis:

\[
x_{e,i,k} = -\frac{w_{0,k}}{w_{e,k}(1+i)^k} - \frac{(w_{ng,k} + w_{chem,k} + w_{nuc,k})}{w_{e,k}}
\]

\[
x_{ng,i,k} = -\frac{w_{0,k}}{w_{ng,k}(1+i)^k} - \frac{(w_{e,k} + w_{chem,k} + w_{nuc,k})}{w_{ng,k}}
\]

\[
x_{chem,i,k} = -\frac{w_{0,k}}{w_{chem,k}(1+i)^k} - \frac{(w_{e,k} + w_{ng,k} + w_{nuc,k})}{w_{chem,k}}
\]

\[
x_{nuc,i,k} = -\frac{w_{0,k}}{w_{nuc,k}(1+i)^k} - \frac{(w_{e,k} + w_{ng,k} + w_{chem,k})}{w_{nuc,k}}
\]
Note that in Equations (140)-(143) that only the intercept coefficient is divided by the inflation. This is because the every other coefficients would have an inflation term that would cancel out with the coefficient that is being investigated. Figure 31 shows the results of comparing the coefficients using Equations (136)-(143). To be clear on what is being described, if the price of electricity in year 50 is ten times its current price while all other variables were held to their initial values, then the decision to upgrade would be made. However, if the price were only two times its current price while all other variables were held to their initial values, the decision to upgrade would not be made. If the price were approximately five times the current price while all other variables were held to their initial values, the decision maker would be indifferent. Because we are now investigating the features adjusted for their coefficient values that lower values now indicate a higher
dependency for classification and explains why the values of the chemical price are so much larger than the others. And because we are now investigating the features, we can make meaningful comparisons of the decision to upgrade to nuclear or not. For the electricity and natural gas, areas above the line indicate that the decision maker should upgrade. We can clearly see that although the values are different, the trend between the price of electricity and natural gas are virtually the same. They suggest that as the HES approaches the end of its life, higher and higher prices of electricity and natural gas are necessary to justify the additional expense of upgrading to nuclear. This is in line with expectations as higher electricity prices and higher natural gas prices both make the nuclear option more preferable. Because the decision maker can increase the quantity of electricity produced by executing the option, the higher the price of electricity the more revenue that can be generated. The nuclear option also replaces the natural gas as the primary heat generator. As such, part of the benefit is the reduction in operating costs associated with burning natural gas. This benefit is increased with an increasing price of natural gas. Figure 31 also shows that the price of chemicals must reach extremely high levels in order to impact the decision to upgrade, confirming that it has little impact. Interestingly, the figure shows an unexpected relationship for the capital cost of nuclear. For the cons case, where all other variables remain at their initial values, the capital cost of nuclear takes an unexpected turn towards the end of life and increases instead of decreasing. In this case, the area below the line is the region where the decision maker should upgrade to the nuclear option. This suggests that higher capital costs are necessary to justify building the SMR. Also note that the price of nuclear becomes negative. This suggests that the decision maker would have to be paid to build the nuclear reactor. These confusing results are explained by investigating the inflation adjusted case. Looking towards the end of life, the relationship is as expected, where smaller capital costs are necessary to justify building the small modular reactor. However, the relationship in the beginning is not expected, and suggests
that the capital cost of nuclear needs to increase in order to justify the expense as compared
to the years before. To explore this further we can compare with other use cases of the
HES. However, this does not explain the sharp increase in the capital cost of nuclear that
occurs at 20 years, nor the other spike that occurs at approximately 32 years. Both of these
are the result of our model assumptions. For the model, it was assumed that systems that
reached their end of life prior to 80 years were rebuilt, even if this rebuilding occurred very
late in the process. The SMR has a life of 60 years, which means that if the SMR is built
prior to year 20 that the SMR will have to be rebuilt. Even though this cost occurs much
further in the future, the cost of building a new SMR is substantial and negatively affects
the NPV of a system. As a result, sharply lower capital costs of nuclear are necessary up to
year 20 to justify building two SMRs when the decision maker could elect to build the
SMR in year 21 and not face rebuild costs. Oddly enough the rapid increase and decrease
of the capital costs of nuclear that occurs at year 32 is related to this same issue, but for the
natural gas primary heat generator. The natural gas has a life of 30 years. Combined with
a 3 year build time, this puts the rebuild at the location of this rapid change in the capital
cost of nuclear. The decision maker is aware that the natural gas heat source must be rebuilt
in this time period. To prevent this quickly approaching expenditure, the decision maker is
willing to pay a comparatively higher price for the SMR. After the rebuild of the natural
gas has occurred, year 33, this phenomena no longer influences the decision to upgrade to
nuclear and the capital costs of nuclear resumes its downward trend. In fact these
phenomena occurred for the nominal case as well, and for the prices of electricity and
natural gas, but the large high discount factor made them much less pronounced.

Another use case of the HES is the case of reduced discount factor. The reason why
this is important will become apparent below. Figure 32 shows the cons and inflation
adjusted feature vectors for the case of \( r = 3\% \). In this case, the price of nuclear to justify
upgrading decreases in the early years before spiking and then resuming its downward
trend. The reason for this change is that when the discount rate is high, the decision maker can actually realize a higher value by waiting to build, even at potentially higher prices as long as the change in prices do not outpace the discount factor. Building the SMR is very costly, while the benefits occur over a long period. Thus, the decision maker can benefit by deferring the cost to a period when the same, or higher, dollar value has less real value.

Although the phenomena seen from analyzing the future decision criterion make logical sense, they are not immediately obvious. Such insights would have been extremely challenging to identify without analyzing the results of the dynamic program, as suggested in section 4.4. Such insights can be used to assist designers in selecting better heuristics without formally comparing them, or in building new heuristics by closely following the results of the dynamic program for similar cases. Insights also can assist designers in

**Figure 32:** Classifier weights adjusted to maintain the initial values for the uncertain states for a 3% discount rate (top). Classifier weights adjusted to account for inflation for a 3% discount rate (bottom).
developing better models, or identifying how the assumptions of a model may influence or bias the results. One example from the HES is the bias against upgrading to an SMR in the years before year 20 because of the arbitrary rebuild that would not necessarily occur for a built system.

5.7 Summary

In this chapter we discuss a case study of an HES. First, we discuss the design problem and methods for improving the value of such long-lived systems subject to large uncertainty. Second, we introduce two performance models for the HES investigated. Third, we explore the economic model for the HES investigated. Fourth, we introduce two decision models to analyze flexibility in the case of the HES. Fifth, we discuss the results of the case study of HES and demonstrate an approach for interpreting the results for a multi-dimensional case.

In the first part of this chapter we discuss challenges and opportunities when designing HESs. Although HESs may allow for a higher quantity of renewables for generating electricity, they are associated with many risks as a result of their status as a long-lived system subject to large uncertainty. We then discuss a particular HES to investigate flexibility as a potential risk management tool.

In the second part of this chapter introduce two performance models for the HES. The first is a model built in Modelica, composed of differential algebraic equations and capable of simulating transient events. However, this model requires a substantial amount of time to simulate and includes more detail than is necessary for a flexible analysis. Thus, we introduce the second performance model, a screening model that greatly simplifies the Modelica model. The results of the Modelica model are used to confirm that many of the assumptions and simplifications are appropriate. Thus enabling a rigorous simulation of the performance of the HES with minimal computational resources.
In the third part of this chapter we discuss the economic model used for the HES. We greatly simplify the situation by assuming that we operate in the day ahead active power market. In this section we also introduce the stochastic process for the four uncertain variables, the price of electricity, natural gas, and chemicals in addition to the capital cost of nuclear. However, modeling commodities with price spikes is a unique challenge. We modify the stochastic process to include price spikes and seasonal changes in prices that are common for the commodities investigated.

In the fourth part of this chapter we discuss two decision models and elaborate on the approach for interpreting results that is to be applied. The first decision model uses a simple decision rule to determine when the option of upgrading the primary heat generator to a small modular reactor should occur. To make this decision rule as good as reasonable we optimize for the parameter value while optimizing the design of the HES. The second decision model uses the dynamic programming approach described in section 4.3.2 for the high dimensional case. We elaborate on how, even though the design opportunity is quite complex, we can greatly simplify the analysis using kriging modeling. As a result, we are able to flexible analyze the HES with comparatively few samples. We also elaborate on how to apply the approach for interpreting dynamic programming results for this more complicated case by introducing linear classifiers. Using the linear classifier we can gather more information about the system using its features.

Finally, in the last part of this chapter we review the results of implementing the two decision models and the approach for interpreting dynamic programming results. We gather insights about the system, including the idea that for this specific case the option investigated only marginally improves the value of the system. In part due to this, the two decision models recommend similar designs and therefore offer no immediate advantage over one another. However, the results of the dynamic programming approach are further investigated and lead to several unexpected insights that would be difficult to obtain
otherwise, emphasizing the value of further investigating the dynamic programming results.
CHAPTER VI
CONTRIBUTIONS, DISCUSSION, AND FUTURE WORK

In this chapter we reflect back on the research questions introduced in section 1.3 and summarize the answers identified in the previous chapters. The primary objective of this chapter is to emphasize the contributions of this dissertation. We also summarize the limitations of the methods and propose directions for future work.

6.1 Recapitulation

In Chapter I, we motivate this work by introducing an example of a poorly applied heuristics. Starting with the story of Iridium, we can easily see how a designer’s choice of heuristics can lead to bad outcomes, ultimately resulting in bankruptcy. To avoid this, designers should only use the best heuristics. But this is not trivial. There is much confusion concerning the use of heuristics in systems engineering and design, which is particularly problematic because of the strong influence of heuristics on the design outcomes. By briefly reviewing the related research, we quickly identified the gap that this work seeks to fill and leads to the motivating question for this research:

Motivating Question: How should heuristics be used in design?

The primary hypothesis of this work is that heuristics should be used such that they maximize value. It is argued in Chapter III that the value heuristics should maximize is the value to the designer for the entire design process, not just the artifact value. However, identifying how heuristics should be used is too general to address in this work alone.

Because the motivating question is too broad, four research questions were identified in section 1.3 to guide this research. The research questions are reiterated below
along with a critical review of how they were addressed in this work. This includes a summary of evidence used to support the answers that have been provided.

The first research question focuses on setting the foundation for research on heuristics. As a result of investigating the literature in section 1.2, a clear gap was identified in that there was a lack of an agreed upon definition for heuristics and the resulting confusion greatly hindered research. To perform meaningful research on heuristics, we must be clear and precise about what we are investigating:

**Research Question 1: What is a heuristic?**

We propose a definition of heuristics in section 3.4.5, namely, that a heuristic is an association between the applicability context of the heuristic and the recommended design actions. A heuristic is a tuple of an applicability context and an applicable action set. Although we aimed to be as precise as possible, we recognize that there still is ambiguity in defining the components of a contextual situation and in our understanding of how heuristics are involved in human cognitive processes. We leave this for future work, discussed in more detail in section 6.3.

The first research question is primarily addressed in Chapter III, although supporting literature is included in Chapter II. The related literature identified in Chapter II identified three main characteristics of heuristics. The first characteristic is the context dependency. Many authors refer to contexts that are associated with heuristics. Using the terminology from Chapter III, other authors have identified that heuristics should be applied if the heuristic’s applicability context contains the decision maker’s contextual situation. Heuristics would not be useful if each heuristic only applied for a particular contextual situation. Presumably, the recommended actions would be the best actions for the contextual situation, however we would need an infinite number of heuristics to cover the space of possible contextual situations, each heuristic being described by an infinite number of properties. This would make comparing, valuing, and selecting heuristics an
impossible challenge, and detracts from the design process value. Instead, it is more useful to describe heuristics by a finite number of properties, and to have a finite number of heuristics from which to select. The second characteristic is the satisficing property of heuristics. Instead of maximizing the value of an artifact, heuristics add value by enabling decision makers to more efficiently move to more preferred contextual situations. That is, heuristics are not necessarily used to design better artifacts, but rather to improve the design process. The third characteristic addresses how using heuristics allows a decision maker to move to more preferred contextual situations: the recommendation of design actions. Instead of proposing a particular action, heuristics identify the most valuable design actions.

The second research question focuses on the selection of heuristics. As a result of investigating the literature in section 1.2, we identified a clear gap in metrics for choosing the most preferred heuristics. To best use heuristics, we must identify a metric for selecting between heuristics:

**Research Question 2: How should designers choose among heuristics?**

We propose a metric in section 3.5, namely, that designers should select the heuristic which maximizes the designer’s average expected utility of the design process. This metric reflects the decision maker’s preferences for heuristics that perform well for their entire applicability context. As a result, different preferences may cause different designers to prefer different heuristics. A particularly risk averse designer may prefer a heuristic that consistently results in good outcomes, while a more risk neutral designer may prefer a heuristic that usually results in exceptional outcomes, but rarely results in poor outcomes.

The second research question is primarily addressed in Chapter III. Building upon normative decision theory that is reviewed in section 2.2, it is clear that when design is framed as a search process that designers should maximize the value of the design process,
and not just the artifact. However, heuristics are used for sets of contextual situations, their applicability context. As such, it is not meaningful to just evaluate the expected utility for one contextual situation. Instead, designers should consider the average performance of the heuristic over its applicability context: the average expected utility of the design process.

The third research question focuses on comparing heuristics. As a result of investigating the literature in section 1.2, we identified a clear gap in methods for comparing heuristics, a necessary step to select the most preferred heuristics. To best use heuristics we must be able to compare different heuristics:

**Research Question 3: How should researchers compare heuristics?**

The proposed answer is in section 3.6: the Design Decision Framing Model (DDFM), and a corresponding method for comparing heuristics. The DDFM can evaluate the metric identified from the second research question, the average expected utility of the design process, for different heuristics, allowing for the heuristics to be ranked. To compare the heuristics, the DDFM evaluates the recommended design actions from each heuristic. To make this evaluation fairly, the DDFM uses a neutral third party, the omniscient supervisor. The omniscient supervisor knows the “truth”, the future states of the world, and uses this information to evaluate the design actions that are recommended by the different heuristics. However, because the future is unknowable, simply assuming a particular value of the truth would allow for bias. To reduce the potential for bias, the DDFM investigates the performance of the heuristics for different values of the truth.

The third research question is primarily addressed in Chapter III. To investigate the effectiveness of the DDFM, we consider the motivating example of a pressure vessel in section 3.7. We were able to successfully compare three heuristics: a novice-optimization heuristic, an expert-optimization heuristic, and an algebraic heuristic. The application of the DDFM required substantial resources, supporting the notion that it is best used for research. Even though it required substantial resources, many meaningful insights about
the three heuristics were identified that would have been challenging to identify otherwise. This provides evidence that the DDFM can effectively be used as a research method for comparing heuristics.

The fourth research question focuses on identifying the best heuristic for a particular set of systems which are challenging to analyze, long-lived systems subject to large uncertainty. The design of such systems may be improved by considering future decisions. However, as a result of investigating the literature in section 1.2, we identified a clear gap in comparing real option analysis methods. The design of long-lived systems subject to large uncertainty represents a challenge to the design community and leads to the fourth research question:

**Research Question 4: Which heuristic should be used for the design of long-lived systems subject to large uncertainty?**

We answer this question in Chapter IV, recommending that long-lived systems subject to large uncertainty should be designed flexibly, and therefore should use heuristics for solving flexible design problems. By considering an option, or options, designers can potentially recognize large increases in value.

The fourth research question is addressed in both Chapter IV and Chapter V. First, heuristics which solve a motivating example of a parking garage inflexibly and flexible are compared in section 4.5. One of the heuristics investigated is also from Chapter IV, the one-dimensional dynamic programming heuristic, which indicates that the value of the system increased by almost 300% compared to the inflexible value. Then, in Chapter V, the multi-dimensional dynamic programming heuristic is compared to a simple decision rule heuristic and a heuristic which solves a case study of an Hybrid Energy System (HES) inflexibly. In this case, both heuristics that solve the flexible design problem added value compared to the heuristic which solved the inflexible design problem, further supporting
that long-lived systems subject to large uncertainty should use heuristics for solving flexible design problems.

6.2 Contributions

The key contributions of this dissertation may be decomposed into *fundamental knowledge* contributions and contributions to the *development of methods and tools*.

6.2.1 Fundamental Knowledge Contributions

There are three fundamental knowledge contributions: a definition of a heuristic, a research metric for selecting heuristics, and a research method and model to compare heuristics. These contributions are elaborated in Chapter III.

- **A Definition of a Heuristic** - After reviewing literature in Chapter II it became clear that although heuristics are important and omnipresent, there is poor agreement on what a heuristic is. In Chapter III we provide a more precise definition of heuristics and related terms. Heuristics are associations between a set of contextual situations, which we call the applicability context, and a corresponding set of recommended design actions. If decision makers find themselves in a contextual situation that is also in the applicability context of a heuristic, the decision maker may consider the heuristic and select from the recommended actions. This definition of a heuristic is proposed in the hope of providing structure to the field and enabling further research into heuristics.

- **A Research Metric for Choosing Heuristics** - Using the definition of heuristics in section 3.4.5 and building on normative decision theory; we develop a metric for selecting the most preferred heuristic in section 3.5. The metric, the average expected utility of the design process, not only applies to a particular contextual situation, but also to an entire context, a set of contextual situations. This metric is impractical for designers, and is recommended for researchers to investigate.
heuristics. Further research may result in more preferred heuristics, which in turn recommend better design actions and result in better designs.

- **A Research Model and Method to Compare Heuristics** - Using the definition of heuristics in section 3.4.5 and the metric in section 3.5, we develop the DDFM in section 3.6. The DDFM is a computational model and corresponding method that can be used to compare different heuristics that can be represented computationally. Performing this comparison across a range of contextual situations aids in identifying the conditions under which heuristics should be used. To make the DDFM as fair and unbiased as possible, we introduce the concept of an omniscient supervisor, who knows the (artificially generated) truth and evaluates the design actions recommended by the different heuristics. This enables researchers to compare different heuristics, which, in turn, can be used to inform and improve design practice. In addition, the research method reveals information on why a given heuristic is more preferred. This enables better heuristics to be created and subsequently used in design.

### 6.2.2 Contributions to the Development of Methods and Tools

There are also three contributions to the development of methods and tools: two flexible-design heuristics, an approach for interpreting the results of dynamic programs, and a high fidelity model for analyzing hybrid energy systems. These contributions are elaborated in Chapter IV and Chapter V.

- **Flexible-Design Heuristics** - New flexible-design heuristics were developed to enable designers to evaluate flexible systems more accurately and quickly in section 4.3. Two heuristics are proposed: a heuristic for one-dimensional cases, and another for multi-dimensional cases. The flexible-design heuristics leverage the benefits of dynamic programming and surrogate modeling to avoid oversimplifying the future decision and closely approximate normative decision theory. The flexible-design
heuristic can analyze complex systems with future decisions with high accuracy and moderate computational costs. The one-dimensional dynamic programming heuristic can be used to analyze one-dimensional continuous flexible-design problems. Due to its speed, it may be successfully applied to design problems with a large action space. However, the multi-dimensional heuristic only applies for a certain design problems. The multi-dimensional heuristic should only be used for design problems where uncertainties are reasonably modeled such that simplifying assumptions similar to those described in section 4.3.2 can be made. Further, the multi-dimensional heuristic is only appropriate for design problems with relatively small action spaces and one-dimensional state spaces. The heuristics are used for the case of a parking garage in section 4.5, and for the case of a hybrid energy system in Chapter V.

*An Approach for Interpreting the Results of Dynamic Programs* - Dynamic programming heuristics already employ a model of the decision maker for making decisions in the future. However, it is not immediately obvious why certain decisions were made. We introduce an approach for interpreting the results of dynamic programming heuristics in section 4.4. Specifically, the approach represents the information as simple decision rules. The simple decision rules approximate the decision model used by the dynamic program by using a linear classifier. This approach is applied for the case of a parking garage in section 4.5.2 and a hybrid energy system in section 5.6.3. In both cases, the approach reveals insights on the respective domains that increase understanding of the system and that can be used to develop better heuristics that would be challenging to recognize otherwise.

*A Model of Hybrid Energy Systems* - The sixth contribution adds to the development of methods and tools: a model for analyzing hybrid energy systems. The model is
made up of a performance model, described in section 5.3, and an economic model, described in section 5.4. The model allows for a variety of different configurations of hybrid energy systems to be created and analyzed. The model is written in Modelica (Modelica, 2009), which allows for more complex analysis, including transient phenomena. The economic model and a simplified performance model are investigated using different decision models in section 5.6. This contribution aids the renewable energy community, which is searching for alternatives to enable higher renewable penetrations.

6.3 Limitations and Future Work

As is clear from the results gathered, these contributions add value to the design community. However, there are many limitations that we have identified that will require future work to overcome. In this section, we make explicit the limitations of the proposed contributions and identify potential areas for future work.

6.3.1 Defining Heuristics

As discussed in Chapter III, there is still room for even more precise definitions surrounding heuristics. One of the terms that would benefit from more precision is that of the contextual situation. In section 3.4.3 we provide guidance by specifying components of a contextual situation, a set of concepts, and a set of concept predictions. But we also identify an imprecise component, simply labeled “remaining information.” It is clear to us that there is more to a contextual situation than the concepts and conceptual predictions, and that the remaining information may serve an important role for heuristics. Future work should focus on further specifying what is encompassed by the remaining information and precisely define contextual situations.

In this work, we have focused exclusively on design heuristics. To this end, we take the perspective that design is a search process. This may not be generally accepted, and
opinions may differ over the purpose of design or heuristics. Designers whose opinions are
different may develop entirely disparate definitions than the ones we have proposed.

In addition, there is still room for research into how designers use heuristics in
practice. One of the benefits of heuristics is that they recommend design actions based on
contextual situations, and do not require domain knowledge to understand why a design
action is valuable. Research into how heuristics are used in practice could lead to better
approaches to teach others how to use heuristics in design.

6.3.2 Comparing Heuristics

As discussed in Chapter III, there are many limitations to comparing heuristics
using the DDFM. By its nature, the research method requires that a design problem be
solved many different times by different heuristics. This is likely to consume a great many
resources and is unlikely to be practical for designers. As a result, the DDFM is a
computational method that can only aid in evaluating heuristics that can be represented
computationally. Heuristics that cannot be represented computationally may not be
evaluated in a straightforward fashion, and other methods would be needed to value such
heuristics. The purpose of the research method is for researchers to compare different
heuristics to either identify the applicability contexts, or to use the information gained from
applying the method to help designers select or generate better heuristics. However, the
results of the DDFM depend on many choices made by the researchers. These include the
set of contextual situations investigated, as well as the heuristic used for the omniscient
supervisor. Comparisons for a given set of contextual situations may not be extrapolated
to other contextual situations, or even heuristics that operate similarly to those investigated.

For the heuristics that are investigated, the resulting comparison depends on the choice of
omniscient supervisor. The model of the “truth” is known to the omniscient supervisor.
However, the truth is unknowable, thus requiring that heuristics be investigated for a set of
contextual situations where ranges on the truth are investigated. If the model of the truth
does not match reality then comparisons that used this false truth are likely to be inaccurate. In addition, the omniscient supervisor approximates the value of the heuristics as if they were applied in the real world. However, due to a variety of reasons including a lack of understanding and limited resources, the omniscient supervisor may not closely approximate real world conditions. In such a situation, the results of any analysis are suspect. In addition, there is the potential for bias if the model of the omniscient supervisor is poorly chosen. The omniscient supervisor closely mimics one of the heuristics being investigated, it is likely to evaluate this heuristic more favorably than otherwise.

Future research should further investigate the notion of the value of a heuristic. The value of an option has been used successfully in ROA, and since methods to analyze flexible-design problems are heuristics, a similar value of a heuristic could be computed. Such a value of a heuristic may be used to compare different heuristics and quantify how much more preferred a heuristic is.

Future research should also investigate additional heuristics. In this work, we focused largely on synthesis heuristics, heuristics that refine concepts. However, research should also look into other types of heuristics such as analysis heuristics, heuristics that refine concept predictions, typically, by gathering additional information. However, it is challenging to know when additional information should be sought. In addition, once the decision to gather information has been made, selecting from potentially many sources of information is yet another challenge. Such research would greatly benefit designers who are currently unsure about how to go about gathering information.

6.3.3 Analyzing Flexibility

As discussed in Chapter IV, there are many limitations to the proposed flexible-design heuristics. The flexible-design heuristics only apply for a subset of design problems. For both cases, designers must be reasonably risk neutral for the dynamic programming approaches to be applicable. While the one-dimensional case can be used to analyze
continuous state systems, it is unlikely to apply for most complex design problems. The multi-dimensional case also only applies to certain design problems. Unfortunately, the design problems must have a small action space and relatively few states for the heuristic to be computationally tractable.

In addition, the multi-dimensional case assumes that there are simplifications for the evaluating expected future values. There are many possible simplifications, but they rely on the modeling choices relating to the uncertainty and surrogate models. While we identify two such simplifications, there may not be simplifications for all state transition functions, regression functions, and correlation functions. For our examples, another important assumption that allowed the simplifications is that the modeled uncertainties are independent of each other. For correlated uncertainties, designers may be able to define an equivalent orthogonal basis with which to analyze the system, but additional research is required to investigate this.

Further, the NPV of the systems that are analyzed must be well approximated by the kriging model. This in part relies on the number of samples that are used to build the model and may make certain systems inappropriate for this method. Unfortunately, it may be challenging to identify if a given number of samples is appropriate for a given system and may require additional computational resources to determine an appropriate number of samples.

We briefly compared the proposed flexible-design heuristic to a simple decision rule for the case of an HES. This case was relatively limited, and only one option was considered, which was determined to add only a marginal amount of value to the system. Future work should consider additional case studies, as well as compare with additional flexible-design heuristics to identify the relative merits of the proposed heuristic.
6.4 Closing Remarks

In this dissertation, we advance the field of decision making in design by proposing a definition of heuristics, proposing a metric for selecting heuristics, proposing a method and model for comparing heuristics, developing methods for analyzing flexible systems, proposing an approach for interpreting the results of the methods for analyzing flexible systems, and developing models to analyze hybrid energy systems. The proposed definition of heuristics, metric for selecting heuristics, and the model for comparing heuristics all add to fundamental knowledge. The proposed method for comparing heuristics, methods for analyzing flexible systems, and models for analyzing hybrid energy systems all contribute to the development of methods and tools.

Although there are many limitations identified in section 6.3, the contributions of this research are substantial and novel in the context of decision making in design. The results and conclusions drawn through the experiments support that the above are advancements in the field.
APPENDIX A

HES Screening Model

function [ outputs, error ] = HES1( inputs, parameters )
%HES1 simulates the performance of a Hybrid Energy System with a
%reactor,
%BOP, wind turbines, batteries, and a chemical plant.

PHG = inputs(1);
RP = inputs(2);
size_battery = inputs(3);
size_BOP = inputs(4);
size_chemicals = inputs(5);

max_electrical = parameters(16);
max_chemical = parameters(17);
efficiency = parameters(18);

%efficiency = 0.30;
electrical_load = min(PHG*efficiency,max_electrical);

perTurbinePower = 3.7e6;
num_turbines = electrical_load*RP/(perTurbinePower);
radius = 45;
rho = 1.225;
A = pi*radius^2;
Cp = 0.35;
density_NG = 0.5*1.409;
max_operational_RP = 0.8;
nominal_chemical = 45.2748;
size_chemicals = min(size_chemicals, max_chemical);
max_aux_heat = 2.08/2.10*2.10e8*size_chemicals/nominal_chemical;
% CO2 produced per watt of heat from burning NG
CO2FromNG = 7.13377e-8;
% NG consumed per watt of heat from burning NG
NGFromBurn = efficiency*0.0101*28.317*density_NG/(1e3*3600);
NGFromBurn = 0.75*2.64423e-8;
% Water consumed per 45.2748 kg/s of chemicals
chem_water = 232.564*size_chemicals/nominal_chemical;

%wind_velocity =
xlsread('SITE_03247_wind_1year_wTail_interpolated.xlsx');
{%
hExcel = actxserver('excel.application');

excelFileHandle =
hExcel.Workbooks.Open('C:\Users\wbinder3\Documents\HESRealOptions\HESRea
lOptions Current\SITE_03247_wind_1year_wTail_interpolated.xlsx');
wind_velocity = excelFileHandle.Worksheets.Item('Sheet1').UsedRange.Value;

excelFileHandle.Close;

hExcel.Quit;

delete(hExcel)
size(wind_velocity)
wind_velocity = cell2mat(wind_velocity);
%
wind_velocity(:,1) = linspace(0,31536000,52561);
wind_velocity(:,2) = 3000*rand(length(wind_velocity),1);
%
data = load('wind_velocity_2week.mat','new_wind');
wind_velocity = data.new_wind;

wind_velocity = wind_velocity(2,1)-wind_velocity(1,1); 
wind_power = zeros(length(wind_velocity),1); 
wind_power((wind_velocity(:,2)<7812).*(wind_velocity(:,2)>13.5)==1) = 
min(wind_velocity((wind_velocity(:,2)<7812).*(wind_velocity(:,2)>13.5)==1),1372); 
wind_power = wind_power*num_turbines*rho*A*Cp;

tao_battery = size_battery/7.92e10*3600;
battery_power(1,1) = wind_power(1);
for i=1:length(wind_power)-1
  battery_power(i+1,1) = battery_power(i)*(1-
time_interval/(tao_battery+time_interval))+wind_power(i+1,1)*time_inter
val/(tao_battery+time_interval);
end
battery_power = 
min(electrical_load*max_operational_RP*ones(length(battery_power),1),ba
tery_power);

turbine_load = electrical_load - battery_power;
heat_to_chem = (PHG - turbine_load/efficiency);
aux_heat =
max(max_aux_heat*0.05/0.160*0.160*ones(length(battery_power),1),max_aux
_heat-heat_to_chem);
wasted_heat = heat_to_chem - min(max_aux_heat,heat_to_chem);
PHG_mod =
min(PHG*ones(length(battery_power),1),max_aux_heat+turbine_load./effici
ency);
%aux_heat =
max(max_aux_heat*0.05*ones(length(battery_power),1),max_aux_heat-
heat_to_chem);
% Mass flow rate
chemicals_generated = size_chemicals*ones(length(battery_power),1);
% Power
electricity_generated = electrical_load*ones(length(battery_power),1);

PHG_CO2 = CO2FromNG.*PHG_mod;
aux_CO2 = CO2FromNG.*aux_heat;
chem_NG =
52.6164*size_chemicals/nominal_chemical*ones(length(battery_power),1);
PHG_NG = NGFromBurn*PHG_mod;
aux_NG = NGFromBurn*aux_heat;
chem_water = chem_water*ones(length(battery_power),1);

outputs(:,1) = electricity_generated;
outputs(:,2) = chemicals_gene
outputs(:,3) = chem_NG;
outputs(:,4) = aux_NG;
outputs(:,5) = PHG_NG;
outputs(:,6) = chem_water;
outputs(:,7) = aux_CO2;
outputs(:,8) = PHG_CO2;
outputs(:,9) = aux_heat;

{%}
sum(PHG_NG)
figure
plot(wind_velocity(:,1),aux_heat)
figure
plot(wind_velocity(:,1),PHG_mod)
%
error = 0;
end
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