GEOMETRY AND STRATIGRAPHY PARAMETERIZATION OF TOPOGRAPHY EFFECTS: FROM THE INFINITE WEDGE TO 3D CONVEX FEATURES

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GEOMETRY AND STRATIGRAPHY PARAMETERIZATION OF TOPOGRAPHY EFFECTS: FROM THE INFINITE WEDGE TO 3D CONVEX FEATURES

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To my family
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SUMMARY

Although the problem of seismic wave scattering by topographic irregularities has been studied for several decades, only recently it has attracted the attention of geotechnical earthquake engineering researchers. Macroseismic observations and recorded evidence from large earthquakes have highlighted that structural damage intensity is frequently higher on the surface of irregular topographies than on adjacent flat ground sites. Numerical and semi-analytical published studies have qualitatively corroborated these observations, but when compared to field recordings, have been shown to systematically underestimate the absolute level of topographic amplification up to an order of magnitude or more in some cases. This discrepancy between theory and observations has been attributed, at least in part, to idealizations of the above studies such as the assumptions of 2D geometry, homogeneous medium, linear elastic response, and monochromatic or narrowband ground shaking. In this research, we bridge the quantitative gap between previous theoretical studies and observations by systematically studying the role of geometry, stratigraphy, and ground motion characteristics through a series of elaborate numerical analyses. We specifically start from the topographic amplification caused by a 2D infinite wedge on the surface of a homogeneous elastic halfspace, and extend the state-of-the-art understanding of wave focusing and scattering by this fundamental block of irregular ground surface geometries. From there, we gradually increase the geometric and stratigraphic complexity up to a 3D convex layered topographic feature, identifying in each level the controlling factors of topographic amplification. Our results provide new insights into the effects of surface topography and its nonlinear coupling with subsurface soil
layering, and suggest that in real conditions, topographic amplification can only be quantitatively captured when geometry and stratigraphy of the site are simultaneously accounted for in theoretical predictive models.
CHAPTER 1
INTRODUCTION

1.1. Preface

The term local site conditions refers to the mechanical properties of near-surface geological formations and the geometry of the ground surface and subsurface. Local site conditions can significantly alter the characteristics of seismic waves that travel from the deeper layers of the crust compared to what the ground motion would have been on the surface of a flat homogeneous linear elastic half-space. This altering is known as ‘site effects’, and includes phenomena such as large amplification, frequency content shifts and significant spatial variability of seismic ground motion, all of which are very important for the assessment of seismic risk, in microzonation studies, and in the seismic design of important surficial and subterranean facilities.

This dissertation focuses on the modification of ground motion by the ground surface geometry that is known as topography effects. Topography effects are associated with the presence of strong topographic relief (hills, ridges, canyons, cliffs, and slopes), complicated subsurface topography (sedimentary basins, alluvial valleys), or geological lateral discontinuities (ancient faults, debris zones), and have been shown to significantly affect the intensity, frequency content and duration of ground shaking during earthquakes.

Documented observations from strong seismic events have shown that structures on the tops of surface irregularities had suffered greater damage than similar structures at the hill bases or on level ground. Numerous semi-analytical and numerical studies have been aimed to capture these effects within a theoretical framework. Although they are able to qualitatively reproduce the observed amplification response of different topography features, the calculated amplification factors are consistently smaller than field recordings.
The most prominent sources of the discrepancy between theory and observation that have been identified are: (a) the focusing of seismic rays in 3D topographic features, (b) the reverberations and scattering of seismic waves in stratified, heterogeneous soil formations, and (c) the assumption of linear elasticity. This dissertation addresses the first and second elements through dimensional analysis and an extensive parametric study of the effects of 3D topographies on layered ground subjected to seismic ground motion.

1.2. **Objective and Scope of the Research**

We seek to reduce the gap between theoretical studies and field observations by systematically studying the effects of 3D ground geometry coupled to soil stratigraphy, and parameterizing these effects in terms of the soil properties and ground motion characteristics. The research plan to achieve this goal consists of the following steps:

i. Developing tools for the discretization, absorbing boundary conditions and loading angle of incidence for implementation in 2D and 3D numerical models of convex surface topographies using a finite difference scheme;

ii. Validating the model for elastodynamics problems (e.g. stability and boundary conditions) by comparison with the analytical solution of infinite wedge;

iii. Conducting a systematic analysis on the role of geometry, material and excitation in 2D topography effect that includes the dimensions and slope of the topographic feature, the soil stratigraphy and incident wave characteristics (type, direction, frequency content);

iv. Performing an analogous parametric study on 3D convex features to demonstrate the additional effects of off-plane scatterers;

v. Studying the coupling of soil and topography effect through a systematic analysis and few case studies.
1.3. Organization of the Work

Besides the introductory and concluding chapters, this thesis is composed of six main chapters with the following content:

In chapter 2, we present an extensive review of previous theoretical and experimental studies on the basic element of topographic discontinuity i.e. the infinite wedge. Then, we use our numerical model to solve two canonical wedge problems: Rayleigh wave scattering by a right angle wedge and amplification of plane waves at the wedge tip, both having traction free boundaries. For the former problem, which has no closed form solution, we verify our results with existing theoretical and experimental values. For the problem of tip amplification, we first check our solution with available analytical solutions (for two special cases) and then extent the solution to the wider range of wedge angles and several values of Poisson’s ratio.

Chapter 3 contains the next level of geometric complexity in the study of topography effects. We first review the previous theoretical and instrumental studies as well as field observations. Then, we perform a set of systematic analysis to find the effects of geometry and excitation characteristics on the topographic amplification. A dam type geometry, as an idealized form of surface topography, is subjected to vertically propagated $SV$ wave. For different dimensionless width (normalized by incident wavelength), the problem is divided into three groups namely single slope, wedge and dam. For each case, amplification factors are calculated as a function of slope and dimensionless height. The effect of dimensionless width is also investigated for the dam topography.

In chapter 4, we study the effects of surface topography on the aggravation of seismic motion for oblique $SV$ wave. We first develop a novel numerical technique to model the propagation of oblique plane waves. The model is then used to obtain the combined topography-incidence effects for a single slope geometry.

Chapter 5 presents the parametric study on 3D topography effects using normalized geometric parameters. We use an idealized 3D convex feature of square frustum form to
study the additional effects of third dimension. Similar to the case of 2D topography, the problem is split into three categories based on the normalized width. For each case, the results are compared with corresponding 2D case.

In chapter 6, we demonstrate that topography and stratigraphy effects are coupled, and that in order to adequately capture these effects in real conditions, they need to be considered simultaneously in predictive models.
CHAPTER 2
INFINITE WEDGE

2.1. Introduction

Wedge models have been traditionally used in wave propagation studies as fundamental elements of geometric discontinuities. There are different wedge shaped features in various fields of science and engineering like continental margin, mountain root, and crustal discontinuity in geophysics and seismology, ground surface topography in earthquake engineering and surface defects and cracks in non-destructive testing (NDT). In fact, there is a common mathematical problem behind all these field of studies, which describes the propagation of incident wave in a semi-bounded medium. An important distinction between them, however, is the ratio of the incident wavelength relative to the characteristic size of the feature aka dimensionless wavelength. This ratio ranges from hundreds in geophysics and seismology to hundredths in NDT.

There are two general classes of wedge problem i.e. scalar and vector according to existing wave types and consequent displacement field. The scalar problem is usually encountered in electromagnetics and acoustics while the wedge problem in elastodynamics is of vector type. Sommerfeld (1896) studied the diffraction of electromagnetic wave by perfectly conducting half-plane and obtained the first solution for scalar wedge problem. The solution has been presented as a superposition of plane waves propagating in all (including complex) directions aka the Sommerfeld integral. The scattering of vector elastic waves by a wedge presents a much more difficult set of problems in elastodynamics compared to its scalar analogue. It involves complex phenomena like mode conversion at the boundary, diffraction from the wedge tip and surface waves. The major difficulty arises from the inextricable coupling between compressional and shear waves (known as P and S
in seismology) at the free boundary. The problem has been also of long standing interest in mathematics since no standard technique has been able to solve it. The rigorous analytical solution for the scattering of elastic waves by a traction free wedge is only available for few limited cases like half-space (Lamb, 1904) and half-plane (Friedman, 1949, Fillipov, 1956 and Maue, 1955), which may be considered as degenerate wedges of 180° and 360° angles, respectively. Due to the absence of mode converting reflection and tip diffraction, there also exist an analytical solution for plane SV wave (out-of-plane polarization) incident along the diagonal of a right angle wedge. The vector wedge problem seems unlikely to have a closed form solution beyond these simple cases. Therefore, there have been several attempts to solve the problem using semi-analytical and numerical techniques.

In seismology and geotechnical earthquake engineering, the solution of wedge problem is an important step in understanding the behavior of seismic waves at wedge shaped obstacles. When the incoming seismic wave, which is generated by various mechanisms (e.g. the causative fault), propagates through the medium and hits the wedge shaped scatterer, its characteristics (e.g. amplitude, frequency and duration) can significantly change because of such geometrical heterogeneity (Figure 2.1). That is the reason why simple theories cannot adequately explain the recorded wavefield. The wedge solution also helps to describe the scattering of elastic waves from surface topographies with traction free boundary and within dipping layers with mixed boundary conditions. Real surface and sub-surface topographies can be modeled as simplified convex and concave geometries and then analyzed as a sum of several wedge parts (as a first order approximation). Figures 2.2 shows the interaction of scattering fields generated by different wedge parts of a dam type topography. The interaction results in an amplification and deamplification patterns depicted by red and green arrows. Overall, to study the problem of elastic wave scattering by topography, it seems quite natural to start with such a simple model studied extensively through theoretical and experimental approaches. Theoretical
studies consists of analytical and semi-analytical methods mainly applied to in-plane polarized shear wave (SH) as well as numerical schemes (mostly finite difference) for all wave types. Experimental studies, however, are limited to reflection, transmission and conversion of Rayleigh waves passing a wedge tip. Figure 2.3 depicts different parts of scattered wavefield for incoming Rayleigh wave which is generated using our numerical model.

**Figure 2.1.** Scattering of incoming wave by wedge shaped heterogeneity

**Figure 2.2.** Scattering field around various wedge parts of a surface topography
Figure 2.3. Reflection, transmission and conversion of Rayleigh wave by a 90° wedge
In this chapter, we consider the initial-boundary value problem of elastic wave scattering by an infinite wedge. In the next section, the mathematical formulation of the problem is presented in terms of governing differential equations and associated boundary, radiation and tip conditions. In section 2.3, we review the previous studies (analytical/semi-analytical, experimental and numerical) on the wedge problem in details. It helps us to understand and summarize what has been accomplished so far and to identify the gaps in the solution of wedge problem. In the following two section, a finite difference (FD) model is used to simulate the propagation of elastic waves in an infinite wedge. The model is verified for two different problems i.e. the scattering of Rayleigh wave by a right angle wedge and amplification of in-plane shear wave at the wedge tip. We extend the solution of the latter problem to a broader range of wedge angles.

2.2. Problem Statement

Consider a two-dimensional (2D) infinite wedge of internal angle $\theta$ made of homogeneous isotropic materials as shown in Figure 2.4. An elastic wave of arbitrary type (Rayleigh, $P$ or $S$) and form (plane or cylindrical) is propagating toward the wedge tip at an angle $i$ ($i=0$ for Rayleigh wave).

Figure 2.4. General configuration of the wedge problem
The linear theory of elastodynamics expresses the propagation of elastic waves in a solid in terms of displacement vector $\mathbf{u}$ and stress tensor $\sigma$ as the following set of equations of motions:

$$\nabla \cdot \sigma + \rho \mathbf{f} = \rho \ddot{\mathbf{u}}$$ (2.1)

where nabla symbol $\nabla$ shows the vector differential operator, $\rho$ is the mass density of the medium and $\mathbf{f}$ is a resultant body force per unit mass. Constitutive equations could be used to express the stress tensor in terms of displacement vector:

$$\sigma = C(\nabla \mathbf{u} + \mathbf{u} \nabla)/2$$ (2.2)

where $C$ denotes the constitutive operator aka the matrix of material rigidity. For homogeneous isotropic elastic media with Lamé’s constant $\lambda$ and $\mu$, equation (2.2) is reduced to the following form:

$$\sigma = \lambda (\nabla \cdot \mathbf{u}) \mathbf{I} + \mu (\nabla \mathbf{u} + \mathbf{u} \nabla)$$ (2.3)

where $\mathbf{I}$ is the unit dyadic. Replacing the stress tensor of equation (2.1) with displacement form of equation (2.3), the Navier-Cauchy equation of motion is obtained as:

$$(\lambda + \mu)(\nabla \mathbf{u} + \mathbf{u} \nabla) + \mu \nabla \mathbf{u} + \rho \mathbf{f} = \ddot{\mathbf{u}}$$ (2.4)

Elastic waves are composed of compressional and shear parts with longitudinal and transverse particle motion, respectively. These waves ($P$ and $S$) propagates respectively with speeds $c_p$ and $c_s$ which are related to the material constants as:

$$c_p^2 = (\lambda + 2\mu)/\rho, \quad c_s^2 = \mu/\rho$$ (2.5)

Replacing Lamé’s constants by in the following form of elastic wave equation:

$$c_p^2 \nabla \nabla \cdot \mathbf{u} - c_s^2 \nabla \times \nabla \times \mathbf{u} + \rho \mathbf{f} = \ddot{\mathbf{u}}$$ (2.6)

In the absence of body force, the displacement vector can be expresses in terms of scalar and vector wave potentials (Helmholtz theorem):
\( u = \nabla \cdot \varphi + \nabla \times \Psi \) \hfill (2.7)

Replacing equation (2.7) into equation (2.6) and reducing the results for harmonic wave, a set of Helmholtz equations are obtained as:

\[
(\nabla^2 + k_p^2)\varphi = 0 \\
(\nabla^2 + k_s^2)\psi = 0 \hfill (2.8)
\]

where \( k_p \) and \( k_s \) are compressional and shear wavenumbers defined as:

\[
k_i = \frac{\omega}{v_i} = \frac{2\pi}{\lambda_i} \quad (i = p, s) \hfill (2.9)
\]

The problem of wave scattering by wedge is more challenging than usual initial-boundary value problems because it has different boundary conditions (BC1 and BC2) on two intersecting faces that share a common endpoint. These boundary conditions could be free (zero traction), rigid (zero displacement), impedance (interface of two media) or a combination of them (mixed condition). For surface excitation, the boundary condition along the loading face is modified accordingly. Solution of the problem should completely define the scattered wavefield generated by reflection (possibly multiple) from both sides as well as tip diffraction. Since the infinite wedge has no characteristic length, it is expected that the properties of consequent scattered wavefield are independent of the incident wavelength (\( \lambda_i, i=p,s,R \)). To ensure the unique mathematical solution, one needs to satisfy two auxiliary conditions i.e. the Sommerfeld’s radiation condition at infinity and the tip condition near the corner. The radiation condition implies that in the absence of energy source at infinity only outgoing scattered waves can exist. The tip condition verifies the absence of energy source at the corner and guarantees the strain energy density is bounded in any compact region near the vertex. Gangi (1960) demonstrated that the tip condition could be expresses in terms of the asymptotic behavior of displacement vector:

\[
u = O(r^a) \quad a \geq 1/2 \hfill (2.10)
\]
Finite energy is equivalent to zero displacement at the vertex and it does not necessitate finite stress at that point.

2.3. **Previous Studies**

Scattering of elastic waves by wedges has been the subject of numerous theoretical and experimental studies for about fifty years. Because of its applications in geophysics and seismology, most of these works have been concerned with Rayleigh waves. They deal with the partitioning of incident energy to transmitted and reflected surface waves as well as converted body waves. Furthermore, there exists a decent number of studies on scalar wedge problem i.e. the scattering of $SH$ wave. However, the vector wedge problem of elastodynamics, which considers the scattering of $P$ and $SV$ waves, has been receive much less attention because of its complex mathematical formulation. In this section, we present a detailed summary of existing works on the elastodynamic wedge problem. They are categorized in three groups of analytical and semi-analytical, experimental and numerical techniques. Furthermore, the results of Rayleigh wave scattering by a right angle wedge is presented in terms of reflection and transmission coefficients for each class of methods. As the most common wedge problem, it provides us with a good measure of comparison and validation.

2.3.1. **Analytical and Semi-analytical Techniques**

As opposed to numerical and experimental techniques that present the results applicable to a particular set of material and geometric parameters, analytical and semi-analytical methods are more general. Their solutions could be used to provide physical insight into the mechanism of resulting wavefield and to validate complex numerical techniques. Nevertheless, the analytical methods have limited applicability in practical problems as they oversimplified the material properties and the geometry of problem.
Before going through the different analytical and semi-analytical techniques for the solution of wedge problem, it is helpful to have a quick introduction of early methods presented for scalar wave equation. Gangi (1960) and Knopoff (1969) presented two excellent reviews on the available methods for scalar wedge problem and concluded that none of them could be extended to the vector case. Next, we present a summary of analytical and semi-analytical studies on the elastic wedge problem in Table 2.1. It could be used as a quick guide to find the detailed exposition of each study in the text. In addition, the analytical results of Rayleigh wave coefficients is presented in Table 2.2 for the canonical case of right angle wedge.

Gangi (1960) presented a summary of methods that have been used to solve the wedge problem for both scalar and vector waves. They treated the scattering of waves with different wavefronts i.e. plane, cylindrical and spherical corresponding to planar, line and point sources. Furthermore, both rigid and weak boundary conditions have been considered in these techniques. There are different techniques that are originally proposed for electromagnetic and acoustic wedge problems including Sommerfeld’s multivalued functions (method of images), integral transforms (Kontorovich-Lebedev), method of characteristics aka dynamic self-similarity and integral equation approach (Green’s theorem). Among them, he applied both Kontorovich-Lebedev transform and method of characteristics to a scalar elastodynamic problem of \(SH\) wave incident upon a rigid wedge. A case of reentrant angle \((\theta \geq \pi)\) has been considered to avoid multiple reflections between wedge faces. He then tried to solve the corresponding vector problem of incident \(P\) wave and discussed subsequent difficulties for vector problem. The presence of two wave types \((P\) and \(S)\) and their mode conversion at boundaries accounted for these difficulties. The Kontorovich-Lebedev transform turns the original partial differential equation into a simple ordinary differential equation. A Kernel of the transform, which is the modified Bessel function, contains wave velocity in its argument. In the vector wedge problem, two different kernels corresponding to compressional and shear potentials should be applied.
Therefore, the boundary conditions, which are differential equations of both compressional
and shear potentials, cannot be simply transformed. In the method of characteristics, the
incident field is assumed a homogeneous function of degree zero with respect to time and
radial coordinate. The assumption is valid for the wedge problem subjected to a plane wave
inasmuch as there is no characteristic length in the geometry. Using the normalized
distance from the wedge tip (η) as a characteristic variable, the polar coordinate system
(r,θ) is transformed into a new plane (η,θ). For different values of η, namely less than, equal
to or greater than unity, the original differential equation is transformed into a hyperbolic
(1D wave), parabolic or elliptic (Laplace) equations. In the (η,θ) plane, outside, perimeter
and inside a unit circle centered at the vertex correspond to hyperbolic, parabolic and
elliptic domains. The scattered field in the hyperbolic region of scalar equation is well
known. For η ≥ 1, a couple of transformations, which map the interior region of unit circle
to the lower half of the complex plane, is used along with the analytic continuation to solve
the Laplace equation. Using two characteristic variables corresponding to P and S wave
velocities, he obtained a solution for the vector wedge problem in the hyperbolic region.
This is essentially the elastodynamic part of the total scattered wavefield that exists outside
the diffracted zones. However, he found that analytic continuation into the elliptic region
is not possible without too many assumptions some of them are unjustified.

Knopoff (1969) reviewed the existing methods for the scalar wedge problem and
demonstrated why they cannot be generally extended to the corresponding vector problem.
He presented solutions of different scalar problems using method of images, integral
equation and representation theorem, Kontorovich-Lebedev integral transform and self-
similarity. He also showed that the iterative method could be a possible solution to the
vector problem at least for the geometric elastodynamic part of the total wavefield. For the
method of images, he pointed out that expanding the incident vector wavefield in a series
of images (cylindrical wave functions) that satisfy two simultaneous boundary conditions
is impossible. Additional terms in the expansion, which arise from mode conversion at the
boundaries and lead to nonphysical images, have been accounted for these difficulties. There are two options for applying Kontorovich-Lebedev transform to the vector wave equation. The vector equation can be replaced by two scalar wave equations using potential functions corresponding to $S$ and $P$ waves. He showed that if we apply transform with the same kernel on both scalar equations, one of them, which has different wavenumber, is no longer reduced to the simple ordinary differential equation. On the other hand, if we use two transforms with corresponding kernels, the transformation of boundary conditions would be a new challenge. We have the same problems when applying self-similarity method to the vector problem. As for the application of representation theorem in the vector problem, he pointed out that resulting dual integral equations could only be decoupled in the special case of half-plane.
### Table 2.1. Summary of analytical and semi-analytical studies on elastic wedge

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lapwood (1961)</td>
<td>R</td>
<td>Iterative Method – 1st Order</td>
<td>$T^a (90^\circ)$ – Defined by a different path (RPR)</td>
</tr>
<tr>
<td>Hudson (1963)</td>
<td>SH</td>
<td>Wave Expansion - Hankel Function</td>
<td>Mixed BC (Dipping Layer) – Introducing Diffracted Part of Total Wavefield – No Diffraction for $\theta = \pi/m$</td>
</tr>
<tr>
<td>Kane and Spence (1963)</td>
<td>R</td>
<td>Iterative Method – 1st Order</td>
<td>$T (120^\circ$-$240^\circ)$ – Effect of $\nu$</td>
</tr>
<tr>
<td>Sato (1963)</td>
<td>SH</td>
<td>Wave Expansion - Bessel Function</td>
<td>Complete Wavefield for Non-acute Wedges</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>No Diffraction for $\theta = \pi/m$</td>
</tr>
<tr>
<td>Hudson and Knopoff (1964a)</td>
<td>L</td>
<td>Iterative Method – 1st Order</td>
<td>$R$ for layered wedge (0-180°)</td>
</tr>
<tr>
<td>Hudson and Knopoff (1964b)</td>
<td>R</td>
<td>Iterative Method – 1st Order</td>
<td>$TR (0-180/360^\circ)$ – Effect of $\nu$</td>
</tr>
<tr>
<td>Kane and Spence (1965)</td>
<td>$R, L$</td>
<td>Variational Principle</td>
<td>$TR (R: 0-360^\circ, L: 110^\circ-250^\circ)$ – Symmetrization</td>
</tr>
<tr>
<td>Fuchs (1966)</td>
<td>P</td>
<td>Geometric Method</td>
<td>GEF$^b$ for Sharp wedges (5°, 10°)</td>
</tr>
<tr>
<td>Mal and Knopoff (1966)</td>
<td>R</td>
<td>Iterative Method – 2nd Order</td>
<td>$TR (0-180^\circ)$ – Contribution of Inward $R$</td>
</tr>
<tr>
<td>Kraut (1968a,b)</td>
<td>P</td>
<td>Wiener-Hopf Decomposition</td>
<td>Considering 2D Wedge in 3D – Unknown Stress Jump Assuming a Factorization for Matrix Kernel</td>
</tr>
<tr>
<td>Achenbach (1969)</td>
<td>UST$^c$</td>
<td>Dynamic Self-similarity</td>
<td>Closed Form Solution for Resulting Shear Stress</td>
</tr>
<tr>
<td>Knopoff (1969)</td>
<td>$S, P, R$</td>
<td>Review on Wedge Problem</td>
<td>Difficulties on Extension to Vector Problem</td>
</tr>
<tr>
<td>Ishii and Ellis (1970a,b)</td>
<td>SH</td>
<td>Geometric Method</td>
<td>Mixed BC (Dipping Layer) – Neglecting Tip Diffraction</td>
</tr>
</tbody>
</table>

$^a$ $T$, $R$ and $C$ stand for Transmission, Reflection and Conversion, respectively

$^b$ Geometric Elastodynamics Field

$^c$ Uniform Surface Traction
Table 2.1. Continued.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viswanathan et al. (1971)</td>
<td>$R$</td>
<td>Iterative Method – 2nd Order</td>
<td>TR ($23^\circ$-$122^\circ$) – Critical Region of Rayleigh Wave</td>
</tr>
<tr>
<td>Forristall and Ingram (1971)</td>
<td>UST</td>
<td>Kontorovich-Lebedev Transform</td>
<td>Solution in Complex Form w/o Physical Interpretation</td>
</tr>
<tr>
<td>Abo Zena and King (1973)</td>
<td>$SH$</td>
<td>Kontorovich-Lebedev Transform</td>
<td>GEF in Series Form, Diffracted Field in Integral Form No Diffraction for $\theta=\pi/m$</td>
</tr>
<tr>
<td>Viswanathan &amp; Roy (1973)</td>
<td>$R$</td>
<td>Iterative Method – $n^{th}$ Order</td>
<td>TR ($20^\circ$-$180^\circ$) – Finite Source Distance</td>
</tr>
<tr>
<td>Kapustianskii (1976)</td>
<td>$SV, P$</td>
<td>Dynamic Self-similarity</td>
<td>Mixed BC (Normal Stress and Tangential Displacement are Zero) – Closed form Solution for $P$ and $S$ Potentials</td>
</tr>
<tr>
<td>Achenbach and Khetan (1977)</td>
<td>USP</td>
<td>Dynamic Self-similarity</td>
<td>Stress Intensity Factor for Reentrant Wedge</td>
</tr>
<tr>
<td>Hong and Helmberger (1977)</td>
<td>$SH$</td>
<td>Generalized Ray Theory</td>
<td>Mixed BC (Dipping Layer) – Neglecting Tip Diffraction</td>
</tr>
<tr>
<td>Miklowitz (1982a,b)</td>
<td>USP</td>
<td>Double Laplace Transform</td>
<td>Complete Scattered Wavefield ($90^\circ$)</td>
</tr>
<tr>
<td>Momoi (1980, 1985)</td>
<td>$R$</td>
<td>Fourier Transform</td>
<td>Complete Scattered Wavefield ($90^\circ$,$270^\circ$) – Introducing Energy Reservoir Zone (Breathing Zone)</td>
</tr>
<tr>
<td>Fujii et al. (1984)</td>
<td>$R$</td>
<td>Fourier Transform</td>
<td>TR ($72^\circ$-$118^\circ$) – Effect of $\nu$</td>
</tr>
<tr>
<td>Krylov and Mozhaev (1985)</td>
<td>$R$</td>
<td>Wentzel-Kramers-Brillouin Approximation - 1st Order</td>
<td>TR ($15^\circ$-$180^\circ$) – Best Applicable for Sharp Wedges ($\theta&lt;30^\circ$) – Capture Oscillatory Behavior</td>
</tr>
</tbody>
</table>
### Table 2.1. Continued.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gautesen (1985a, 2002a)</td>
<td>$R$</td>
<td>Fourier Transform – Extinction Thm.</td>
<td>$\text{TR} \ (90^\circ)$ – Effect of $\nu$ – Symmetrization – FFSP$^c$</td>
</tr>
<tr>
<td>Gautesen (1985b)</td>
<td>$P$</td>
<td>Fourier Transform – Extinction Thm.</td>
<td>FFSP (90°) – Diffracted $R$ on Both Faces – Effect of $\nu$</td>
</tr>
<tr>
<td>Sánchez-Sesma (1985)</td>
<td>$SH$</td>
<td>Wave Expansion- Bessel Function</td>
<td>General Solution for Arbitrary $\theta$ – Peak AF=$360/\theta$</td>
</tr>
<tr>
<td>Gautesen (1986)</td>
<td>$R$</td>
<td>Fourier Transform – Extinction Thm.</td>
<td>$\text{TRC} \ (3D \ 90^\circ)$ – FFSP</td>
</tr>
<tr>
<td>Gautesen (1987)</td>
<td>$R$</td>
<td>Fourier Transform – Extinction Thm.</td>
<td>$\text{TR} \ (45^\circ$-$135^\circ)$ – Effect of $\nu$</td>
</tr>
<tr>
<td>Nakano et al (1988)</td>
<td>$R$</td>
<td>Fourier Transform</td>
<td>$\text{TR} \ (250^\circ$-$290^\circ)$ – Effect of $\nu$</td>
</tr>
<tr>
<td>Sánchez-Sesma (1990)</td>
<td>$SV, SH$</td>
<td>Geometric Method</td>
<td>$SV$: $\theta=$90°, (120°, $\nu=0.25$)</td>
</tr>
<tr>
<td>Li et al. (1992)</td>
<td>$R$</td>
<td>Representation Theorem – BIE$^d$</td>
<td>$\text{TR} \ (3D \ 90^\circ)$ – Oblique Incidence – Effect of $\nu$ (2D)</td>
</tr>
<tr>
<td>Fujii (1994)</td>
<td>$R$</td>
<td>Fourier Transform</td>
<td>$\text{TRC} \ (36^\circ$-$180^\circ$, 200°-$340^\circ)$ – Effect of $\nu$ Original Integration Path – Oscillation at Small $\theta$</td>
</tr>
<tr>
<td>Budaev and Bogy (1995)</td>
<td>$R$</td>
<td>Sommerfeld-Maliuzhinetz</td>
<td>$\text{TR} \ (95^\circ$-$240^\circ)$ – Symmetrization</td>
</tr>
<tr>
<td>Budaev and Bogy (1996)</td>
<td>$R$</td>
<td>Sommerfeld-Maliuzhinetz</td>
<td>$\text{TR} \ (30^\circ$-$355^\circ)$ – Valid for $\theta&lt;180^\circ$</td>
</tr>
<tr>
<td>Gautesen (2001)</td>
<td>$R$</td>
<td>Fourier Transform – Extinction Thm.</td>
<td>$\text{TR} \ (189^\circ$-$327^\circ)$ – Incoming (Incidence and Reflected) – $\theta&lt;180^\circ$</td>
</tr>
</tbody>
</table>

$^c$ Far-field Scattering Pattern (Diffraction Coefficients of GTD)  
$^d$ Boundary Integral Equations
<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Remarks</th>
</tr>
</thead>
</table>
Table 2.2. Analytical results of Rayleigh wave coefficients for right angle wedge

<table>
<thead>
<tr>
<th>Reference</th>
<th>ν</th>
<th>Reflection</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Amplitude</td>
<td>Phase (°)</td>
</tr>
<tr>
<td>Hudson and Knopoff (1964)</td>
<td>0.008</td>
<td>0.00</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>0.253</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.498</td>
<td>0.13</td>
<td>0</td>
</tr>
<tr>
<td>Kane and Spence (1965)</td>
<td>0.25</td>
<td>0.03</td>
<td>-280</td>
</tr>
<tr>
<td>Mal and Knopoff (1966)</td>
<td>0.25</td>
<td>0.19</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>0.25a</td>
<td>0.29</td>
<td>–</td>
</tr>
<tr>
<td>Viswanathan et al (1971)</td>
<td>0.25b</td>
<td>0.14</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>0.25c</td>
<td>0.32</td>
<td>55</td>
</tr>
<tr>
<td>Viswanathan &amp; Roy (1973)</td>
<td>0.25</td>
<td>0.32</td>
<td>5</td>
</tr>
<tr>
<td>Momoi (1980)</td>
<td>0.25</td>
<td>0.10</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.06</td>
<td>-71</td>
</tr>
<tr>
<td>Fujii et al. (1984)</td>
<td>0.234</td>
<td>0.09</td>
<td>-43</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.10</td>
<td>-41</td>
</tr>
<tr>
<td>Krylov and Mozhaev (1985)</td>
<td>0.35</td>
<td>0.60</td>
<td>–</td>
</tr>
<tr>
<td>Gautesen (1985a, 1987, 2002a,c)</td>
<td>0.17</td>
<td>0.07</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.10</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>0.16</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>0.07</td>
<td>53</td>
</tr>
<tr>
<td>Li et al. (1992)</td>
<td>0.25</td>
<td>0.10</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
<td>0.16</td>
<td>38</td>
</tr>
<tr>
<td>Fujii (1994)</td>
<td>0.25</td>
<td>0.10</td>
<td>-41</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.17</td>
<td>-30</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>0.27</td>
<td>-20</td>
</tr>
<tr>
<td>Budaev and Bogy (1996)</td>
<td>0.25</td>
<td>0.06</td>
<td>–</td>
</tr>
<tr>
<td>Kamotski et al. (2006)</td>
<td>0.234</td>
<td>0.10</td>
<td>-45</td>
</tr>
</tbody>
</table>

a First order coefficients recalculated from Hudson and Knopoff (1964)
b Second order coefficients recalculated from Mal and Knopoff (1966)
c Modified second order coefficients
2.3.1.1. Iterative Method

In this technique, one first finds an integral solution to the elastic wave equation using 2D elastodynamic representation theorem (Green’s function) or integral transforms. The representation theorem, which describes the mathematical formulation of Huygens’ principle\(^1\) in elastodynamics, is a relation between generated displacement field and its source i.e. body force or surface traction and displacement. The integral solution involves successive reflections between two faces of the wedge, which may be very complex. Therefore, the solution of integral equations on each side could be obtained by estimating scattered wavefield on the other boundary side. This approximate solution of scattered wavefield will be then substituted in the integral equations to find next order approximate solution on the other side and the procedure continues. Since the boundary conditions at two sides are not satisfied simultaneously, the convergence of this method is not guaranteed. The scattered wave considered in this method is geometric elastodynamic field and does not include tip diffraction.

Lapwood (1961) solved the classical Lamb (1904) boundary value problem of travelling Rayleigh pulse over a homogeneous isotropic elastic half-space (2D part) for a quarter-space. The goal was to find the form of transmitted Rayleigh pulse in terms of scattered wavefield.

\(^{1}\) It should be reminded that by Huygens’ principle, we mean its modified version as for propagation direction and interference. The original geometrical construction gives both forward and backward propagated wavefront while the latter does not really exist. Therefore, only forward propagated envelope should be considered. Furthermore, the original form cannot explain diffraction unless it is combined with Fresnel interference principle. From the mathematical point of view, in 3D space Huygens’s principle reads as the solution to the wave equation at any observation point at any instant only depends on the initial data in an infinitesimal neighborhood of the sphere centered at observation point with radius proportional to wave speed. Such wave has a sharp leading and trailing edge. This is also true for all odd dimensional spaces greater than three. However, in 2D and in all even dimensions the solution depends on the entire disk and fades away gradually. Therefore, it is not true as a Huygens phenomenon in even dimensions (Folland, 1995).
incident Rayleigh wave components generated by a point force \( p(x,t) = -Q\delta(x-a)\Phi(t). \) He found that the potential method used by Lamb (1904) is not an appropriate approach as compressional and shear displacement potentials vary exponentially with depth at different rates. It means that finding potential functions that satisfy boundary conditions at two faces is an intricate task. Therefore, using Fourier (time) and Laplace (space) transforms, he converted the strong form differential equations of wave motion to a weak form integral equations. Then he used an iterative method with different approximation order on each wedge sides to find both the incident and transmitted Rayleigh pulse. He found the zeroth order approximation on each face by neglecting the contribution of other face in the integral solution. In the first order approximation, which includes the effects of both faces, the incident Rayleigh wave travels on the primary face then at some point it is converted to body waves. These body waves are converted to a Rayleigh wave (transmitted) on the opposite face. It is noteworthy that this definition of transmitted Rayleigh wave is somehow different from other works in this field of study where it is defined as a part of incident Rayleigh wave that turns around the corner (no body wave conversion). The solution was verified through comparison of incident Rayleigh pulse (first approximation on the incident face) with the Lamb’s solution. The components of incident pulse parallel and normal to the wedge face have the same temporal variation as \( \Phi(t) \) and its Hilbert transform \( \Phi'(t) \), respectively. However, the components of transmitted Rayleigh pulse are expressed as a linear combination of these two functions i.e. \( a\Phi(t) + b\Phi'(t) \) where \( a \) and \( b \) are functions of \( P, S \) and Rayleigh wave velocities.

Hudson and Knopoff (1964a) studied the reflection of Love wave incident upon the corner of a 2D layered convex wedge \((\theta \leq 180^\circ)\). They found complex reflection coefficients (magnitude and phase) as a function of wedge angle and wavelength of incoming Love wave. The representation theorem has been used in the form of Helmholtz integral solution to the scalar \((SH\) displacement only) wave equation. They then applied
this integral solution to each layer separately and combined the results to eliminate the interface contribution and find a representation theorem for the entire domain. They considered the total displacement field as a sum of incident and scattered parts. The scattered wave field composed of reflection from boundaries, refraction at interface and diffraction by vertex. Applying zero stress boundary condition and radiation condition to corresponding integral contours, they found an exact integral solution for scattered wavefield at interior points. However, it requires exact scattered wavefield on the receiving wedge side. They disregarded the multiple reflection between wedge sides and interface as well as diffracted body waves and simply chose the specular reflection on the receiving side as a first order approximation of scattered wavefield. Using a modal form of incident Love wave and its derivative along with the Green’s function for layered half space, they calculated reflected wave through integration over Love wave contributions. Their approximation give rise to exact solution for two wedge angles i.e. complete reflection for right angle wedge and pure transmission for half space. An important point about first order approximation is that it results in symmetric amplitude and antisymmetric phase about $\pi/2$ which physically is not correct as we will see later. It is because of the fact that in the formulation, the reflection coefficients of $\theta$ and $\pi-\theta$ are complex conjugate.

In the second part of their study, Hudson and Knopoff (1964b) investigated the transmission and reflection coefficients of Rayleigh wave in a 2D homogeneous elastic wedge of different angle. They used the same technique i.e. representation theorem with appropriate Green’s function and approximate scattered wavefield on the boundaries. As for the representation theorem, Knopoff (1956) derived Kirchhoff type solution for transient acceleration vector. The solution involves boundary values of time retarded displacement vector and its divergence and curl. de Hoop (1958) found representation theorem for displacement vector instead of acceleration. His solution consists of boundary values of displacement vector and normal traction. To derive expressions for reflected and transmitted Rayleigh waves from de Hoop’s integral solution, they used the half space
Green’s function for primary and secondary faces, respectively. These integral solutions could be more simplified by applying boundary values and radiation condition on the corresponding integral paths. The exact solution involves unknown scattered field on a segment of integration path (primary or secondary face). They neglected multiple reflection between two sides and diffraction by the vertex. As a first order approximation of transmitted wave, they assumed zero scattered field on the corresponding wedge face. For reflected wave, they used as an unknown scattered wave on the secondary face, the specular reflection obtained by potential method. Finally, they found displacement based transmission and reflection coefficients from approximate scattered wave solution by replacing Green’s function with its Rayleigh wave component. These coefficient have been presented as a function of wedge angle (0-2π) and Poisson’s ratio. The reflection coefficients are limited to wedge angles less than π because they assumed no reflection would occur beyond this range. A computational error in the magnitude of reflection coefficient has been detected and fixed by Viswanathan et al (1971). While Poisson’s ratio has no detectable effect on the transmission phase, its variation produces conspicuous but still small change on the amplitude. It also clearly affects both the amplitude and the phase of reflection coefficient. Similar to other first order approximations, the results are symmetric for amplitude and antisymmetric for phase.

Kane and Spence (1963) applied the iterative method to displacement potentials of wave equation in order to derive transmission coefficient for Rayleigh wave propagating in an elastic wedge. It is discussed that finding potential wave functions that simultaneously satisfy boundary conditions at both wedge faces is not a straightforward task. As a first order approximation, they added a set of scalar and vector potentials (scattered field) to the incident Rayleigh potentials to satisfy boundary conditions on the secondary face. In fact, these unknown wave functions are response of the secondary face to the incident wave. They used Cauchy integral formula along with the analytic continuation to represent the scattered field in the integral form. Applying zero stress boundary conditions to the total
wavefield, they found amplitude of unknown potential functions. Residue of the resulting integral solution at Rayleigh pole represents the contribution of transmitted Rayleigh wave to the total field. The resulting transmission coefficients, which are frequency independent, have been presented for a limited range of wedge angle \(120^\circ \leq \theta \leq 240^\circ\) and different values of Poisson’s ratio. They pointed out the negligible effect of Poisson’s ratio on both the amplitude and the phase of transmission coefficient.

Mal and Knopoff (1966) extended the first order approximate solution of Hudson and Knopoff (1964b) for transmission and reflection coefficients of Rayleigh wave incident upon a wedge to the higher order. They were the first to treat a wedge as a basic element of topography and to recognize the relation between scattering of elastic wave in wedges and topographic features (Mal and Knopoff, 1965). They first found stresses along the secondary face \(S_2\) due to incident Rayleigh wave on the half-space with primary face \(S_1\) as a boundary. Then, they satisfied boundary condition on \(S_2\) through applying a set of virtual sources on this side and solving Lamb’s problem for \(S_2\) half-space (neglecting the other face). The resulting representation theorem has been solved for Rayleigh poles to obtain first order surface wave on \(S_2\). Solutions of Rayleigh wave are complex conjugate and represent waves propagate both toward and away from the vertex. In fact, their contribution to the problem was to consider further transmission and reflection of these inward Rayleigh waves. To find higher order reflection and transmission coefficients, they used the same technique as Hudson and Knopoff (1964b) i.e. representation theorem but with different integral contours and different estimates for scattered wave. Regarding the reflected waves on \(S_1\), they considered three terms for scattered field on \(S_2\) i.e. outward and inward Rayleigh waves as well as body waves. Instead of calculating branch line integrals, they used an equivalent displacement for body waves (specular reflection). This representation theorem for \(S_1\) face with appropriate scattered field on \(S_2\) results in a relation between second order transmission and reflection coefficients. For transmitted wave on \(S_2\), they used the same representation theorem with half-space Green’s function of this face.
and found another relation between these two coefficients. Finally, they solved the resulting system of algebraic equations to find second order reflection and transmission coefficients as a function of first order coefficients and amplitude of secondary Rayleigh wave on $S_2$. Phase and magnitude of coefficients have been presented for $\theta = 0$-$180^\circ$ and Poison’s ratio of $\nu = 1/4$.

Viswanathan et al (1971) summarized the previous studies of Hudson and Knopoff (1964b) and Mal and Knopoff (1966) on the transmission and reflection of Rayleigh wave in a wedge. They detected some nonphysical features in the previous solutions like occurrence of minimum reflection magnitude at $\theta = 90^\circ$ and attributed them to computational errors. They recomputed questionable coefficients and presented corrected numerical results. In addition, they used the same technique i.e. representation theorem for primary and secondary sources on each face to derive a new second order approximate solution. To see whether the outgoing Rayleigh wave on each face affects the other face or not (applicability of Huygens’s principle) they checked the critical regions for Rayleigh waves in Lamb’s problem. Rayleigh waves generated by a point source on the half-space cannot exist in two nested conical regions occupied by $P$ and $S$ wave. The boundaries of these regions construct critical angles $\theta_a$ and $\theta_b$ with the free surface. Therefore, the outgoing Rayleigh wave on each face intersects other face only for sufficiently small wedge angle. When the face lies outside the shear cone ($\theta \leq \theta_b$), both parts of Rayleigh wave i.e. shear and compressional contributions have been taken into account. That is, the solution of Mal and Knopoff (1966) did not changed in this range. However, for the range of wedge angles where the face lies between two cones, they neglected the shear potential contribution. They modified second order coefficients and presented the numerical results for the range of wedge angles where outgoing Rayleigh wave only has compressional contribution ($\theta_b \leq \theta \leq \pi - \theta_a$). Compared to previous studies, their results have a closer qualitative agreement with experimental data and are able to capture more experimental features e.g. the peak of reflection magnitude at $\theta = 60^\circ$. 
Viswanathan et al (1971) studied the wedge problem for the case of Rayleigh wave incidence from infinity. In the second part of their work, Viswanathan and Roy (1973) solved the problem in more details for general case of Rayleigh source lies at a finite distance from the apex. They also took into account the effect of critical regions for both outgoing and incoming Rayleigh waves. Further, the effect of corner waves was somewhat included in their solution through modifying boundary conditions. They first excluded contributions of compressional and shear potentials in the incident Rayleigh wave outside $P$ and $S$ wave conical regions via multiplying them by Heaviside function. For infinitely distant source, points of interest are outside these regions and no modification is required. Transmission and reflection coefficients have been divided into three parts based on causative factors i.e. incoming and outgoing Rayleigh waves along with corner body waves. To obtain the contribution of inward/outward Rayleigh waves, they modified first order coefficients and the induction factor of Mal and Knopoff (1966) for the finite distant source. Considering a sequence of interaction of Rayleigh waves with wedge faces, they presented general higher order coefficients in the form of integral equations. They found that integral equations of incoming Rayleigh wave behave like Volterra, Fredholm and their mixture for lower, higher and intermediate ranges of wedge angles, respectively. The integral equations of outgoing waves are of Fredholm type for all wedge angles. In preceding studies, at each step of iterative method, Lamb’s problem with a series of virtual sources was solved for the half-space where the wedge face constructs part of its boundary. For the other half i.e. the extension of that face, stress free condition was assumed. Regarding the share of corner waves, they modified this conventional boundary condition to reduce stress jump over the wedge tip. For wedge angles $\theta=45^\circ-135^\circ$ where this effect is significant, they applied the same stress free boundary condition on the extension lines. The contribution was only obtained for transmission part up to the first order which is the same as the induction coefficient of incoming Rayleigh wave. They pointed out the fact that coupled integral equations obtained for higher order coefficients represent a physical
process i.e. interaction between two faces. Therefore, instead of usual elimination, they should be solved by method of successive substitution. For small wedge angles ($\theta \leq 45^\circ$ for Poisson material), the magnitude of induction factor is greater than unity and successive substitution does not converge to the same result as that of elimination method. For comparative purpose, they presented numerical results of nth order transmission and reflection coefficients due to Rayleigh source at infinity. They also calculated the first order coefficients for finite distant Rayleigh source. The results have been presented as a function of wedge angle and source distance normalized by Rayleigh wavelength. The normalized distance has considerable effects for values less than unity.

2.3.1.2. Wave Expansion and Geometric Methods

Hudson (1963) investigated the scattering of cylindrical $SH$ wave by a wedge having mixed free/rigid boundary conditions along its upper/lower faces. Following the method of Oberhettinger (1954), he represented the incident and scattered displacement fields in terms of infinite integrals containing the Hankel functions of the second kind (expansion of cylindrical waves). He obtained unknown functions in the scattered field representation by applying zero stress and displacement boundary conditions. He then transformed the integral solution to a form useful for physical interpretation. For the special case where the wedge angle is an integral fraction of $\pi$, he simplified the resulting integral to a series of wavefield due to image sources. He demonstrated that there is no diffraction term in this case. Then, he reformulated the integral representation of total displacement field for arbitrary wedge angle using the same concept of image sources. The total wavefield has been divided into four parts for different combinations of wedge, source and observation angles. He simplified each term as a sum of geometric field and diffracted wave from the vertex. The geometric elastodynamics field is defined as a superposition of wavefields due to a finite number ($\sim 2\pi/\theta$) of image sources (multiple reflections from the boundaries). He applied the Fourier integral to the harmonic solution in order to obtain the
impulse response of the wedge. For an incident cylindrical pulse, he obtained a simplified form of diffracted displacement field. He observe that the amplitude of diffracted pulse decreases as $(rr')^{-1/2}$ where $r$ and $r'$ are distances of source and observation point from the apex. In addition, the amplitude vanishes for sufficiently short duration pulse. He used the $SH$ solution of the wedge to study the characteristics of Love wave propagating over the surface of a dipping layer far from the source. These waves are generated due to constructive interference of images i.e. when the phase difference of reflected waves is an integral multiple of $2\pi$.

Sato (1963) used the method of wave function expansion to study the scattering of plane $SH$ wave in non-acute elastic wedges. He defined three different geometry-excitation scenarios based on the wedge angle and the direction of incident wave. For each case, he split up the wedge medium along the boundary of reflection zone and specified the corresponding components of the solution in either region. Generally, the displacement field consists of known incident and reflected waves as well as unknown diffracted part. He then expressed the diffracted wavefield as a sum of cylindrical waves (Bessel functions) with unknown coefficients. These coefficients were obtained by applying zero stress boundary conditions and continuity along the dividing line. Finally, the diffracted displacement field has been represented in an integral form. Using the residue theorem and the Neumann expansion of exponential function, he recast the resulting improper integrals to a series expansion in terms of Bessel functions. To describe the characteristics of diffracted wavefield, he obtained the impulse response of the wedge using the Fourier inverse formula. It consists of separate expressions for incident, reflected and diffracted displacements. He showed that for wedge angles of the form $\pi/m$ where $m$ is an integer, the expression of diffracted wave vanishes. He presented the numerical results for normalized amplitude of diffracted wave as a function of wedge angle. He found a singular behavior of diffracted wave at wedge angles close to diffraction free wedge ($\theta = \pi/m$). He also calculated the normalized amplitude of diffracted wave for a number of wedges as a
function of observation azimuth angle. He observed similar singular behavior near the boundary of incident/reflected shadow zone. Furthermore, he obtained a schematic profile of diffracted wave for a reentrant wedge at different observation angles.

Using a simplified geometric method, Fuchs (1966) obtained the synthetic seismograms of $P$ waves propagating in a sharp wedge with traction free faces. Tip diffraction has been disregarded in his calculations. The method is, therefore, valid up to the arrival time of diffracted waves. Given the reflection coefficients of plane waves in half-space, he obtained the components of multiple reflected $S$ and $P$ impulses. The total displacement field has been presented as a sum over the (finite) number of reflections, which depends on the wedge angle, the angle of incidence and material properties. He presented an iterative procedure to calculate the amplitude and direction of multiple reflections. Then, he obtained the geometric elastodynamics field of the wedge for an arbitrary shaped incident wave using its impulse response and convolution. He considered two sharp wedges of angles $5^\circ$ and $10^\circ$. For a Gaussian shaped plane $P$ wave, incident along the bisectrix, he obtained the synthetic seismograms on the wedge face. He also presented the particle motion at discrete points along the face near the wedge tip. He observed that, close to the vertex, multiple reflections interfere with the primary $P$ wave and deform it. Comparing the theoretical velocity of the wavefront (phase velocity) with the measured velocity of the signal front (group velocity), he concluded that the signal is dispersed near the vertex. He found that in this region the signal velocity is very close to the velocity of Lamb wave. That is a sharp wedge behaves like a plate with parallel boundaries. He defined a transition zone (about a wavelength long) near the vertex within which all changes in wave characteristic occur. It involves the sense of particle motion and the dominant frequency of the wave in addition to the signal velocity. Elliptical particle motion shows a switch in polarization from prograde to retrograde similar to the surface Rayleigh wave. Furthermore, the dominant frequency of the signal is suddenly decreased passing the transition zone.
Ishii and Ellis (1970a) used the geometric ray method to investigate the effect of dipping layer on the surface response of incident plane $SH$ wave. The dipping layer has been simplified as a wedge overlaying an elastic medium. They first obtained the reflection and refraction coefficients of $SH$ wave incident on the dipping interface from either side. These are functions of incidence and dip angles as well as material properties. Then, they successively applied these coefficients to a plane $SH$ wave incident on the interface from the underlying elastic medium to construct the geometric elastodynamics field in the wedge. They showed that the total number of multiple reflections inside the wedge is finite. In addition, there is a displacement discontinuity in the last reflected ray of geometric solution along a certain dipping angle. They ascribed such discontinuity to the absence of diffracted term in the total wavefield. They computed the amplitude of displacement discontinuity as a function of dip angle for several incident directions. The results show that under down-dip incidence, the diffraction effect could be controlling even for small dip angles. They also presented the surface spectral amplitude as a function of dip angle for two types of incidence i.e. up-dip and down-dip. In the latter, they observed a rapid change in amplitude with frequency at several dip angles. Again, it demonstrates the lack of diffraction term in the total wavefield for this configuration. The amplitude of discontinuity (diffraction) is large because the down-dip reflected wave hits the boundaries only few times before leaving the wedge. Generally, the magnitude of discontinuity provides a measure for applicable regions of ray theory.

Ishii and Ellis (1970b) used the plane wave solution of Ishii and Ellis (1970a) to study the scattering of cylindrical $SH$ wave by a dipping layer. The diffracted part is again neglected from the total wavefield. Such assumption gives rise to an adequate approximate solution for stations far from the vertex. The method is thus applicable to the initial part of recorded seismograms where the diffracted waves have not arrived yet. They repeated the plane wave procedure and obtained the solution for harmonic cylindrical wave by integrating the resulting expressions over the entire incidence angles. They evaluated the
consequent integral for the first series terms corresponding to waves once reflected from the interface. They deformed the original path of integration and calculated the contributions from saddle and branch points respectively define the reflected and refracted (head) waves. The travel time of reflected and head waves has been obtained using the geometry to verify the physical interpretations of corresponding terms in the displacement integrals. They also obtained displacement expressions for incident, reflected and head waves due to a smooth varying impulse. They pointed out that the displacement discontinuity and corresponding jump in stress is due to the lack of tip diffraction in the total wavefield. However, for two special cases of surface source and free/rigid interface, they obtained geometric conditions where the total cancelation occurs (no diffraction). For specific cases of wedge angle and source position, they obtained synthetic seismograms by combining the displacement components of incident, first reflected and head waves.

Hong and Helmberger (1977) studied the propagation of cylindrical $SH$ wave in a dipping layer using the generalized ray theory. They calculated the displacement response of a single wedge with mixed free-rigid boundary conditions (the same problem as Hudson, 1963) in terms of ray integrals. Then, they extended the results to a wedge overlying an elastic half-space. Starting with the wave equation for Laplace transformed displacement, they applied the Oberhettinger transformation (Oberhettinger, 1954) to obtain the total wavefield in an integral form. They simplified the solution for surface observation points and a short period impulsive source. It consists of the geometric elastodynamics part and the tip diffraction. They neglected the diffraction part by removing the apex from the dipping layer model. As for the geometric elastodynamics solution, they utilized the Cagniard-de Hoop technique (Cagniard, 1939 and de Hoop, 1960) to evaluate the inverse Laplace transform by deforming the integration path. The displacement field has been presented in terms of generalized rays. To analyze the characteristics of the generalized rays, they used the first motion approximation for a particular ray. In this method, the leading part of the wave near the arrival time of ray is evaluated. They extended the solution
to a dipping layer with elastic boundary by modifying the reflection-transmission coefficients in the ray integral. There are three simplifying assumptions in their approximate model: neglecting diffracted waves, high frequency approximation for the Bessel functions and using elastic reflection/transmission coefficients along the interface. To check the accuracy, they compared the results with those of finite element method and exact generalized ray theory (as opposed to first order approximation). They presented the synthetic seismograms of SH wave propagating in a dipping layer and compared the results with those of the horizontal layer.

Pao and Ziegler (1982) presented a generalized ray solution for $SH$ waves scattering in a dipping layer over an elastic half-space. They relaxed two out of three simplifying assumptions made by Hong and Helmberger (1977) i.e. using high frequency approximation of Bessel functions and replacing the rigid coefficients with elastic ones. However, they still neglected the tip diffraction from the total wavefield. They defined two coordinate systems with associated wavenumbers (slowness) along the wedge boundaries. Elements of generalized ray integrals i.e. source and phase functions as well as reflection coefficients have been obtained in terms of two local wavenumbers. They assumed that source and receiver points are both located inside the dipping layer and hence only reflection coefficients are incorporated in ray integrals. They solved the scalar wave equation for Laplace-Fourier transformed displacement field and applied the inverse Fourier integral to obtain an integral representation for the Laplace transformed displacement. The solution, which is an outgoing cylindrical wave, represents the superposition of plane waves passing through the source in all directions. Considering the receiver point in the same layer, it also denotes the source ray integral. Incorporating the reflection coefficients of plane wave, they obtained multi-reflected ray integrals for upward and downward source rays in terms of two local wavenumbers. They transformed all wavenumbers in ray integrals to a common wavenumber based on the invariance of the phase function. Then, they synthesized these ray integrals to find the Laplace transform of
geometric elastodynamics field in terms of the common slowness. They presented a geometric interpretation of phase functions for reflected rays and showed how to determine these functions in the imaging space. Finally, they used the Cagniard method to evaluate the inverse Laplace transform of generalized ray integrals. The Cagniard method has been applied in its original form (Cagniard, 1939) which maps the integration variable of the phase function (wave slowness) onto another variable (time) in a complex plane. They presented a first motion approximation for direct rays.

Ziegler and Pao (1984) used the same technique as their earlier work (Pao and Ziegler, 1982) to investigate the scattering of transient cylindrical $P$ and $SV$ waves in a dipping layer. This is the vector analog of the previous scalar wedge problem. Although the tip diffraction is still neglected, the mode conversion at the boundaries makes the ray integrals more complex compared to the scalar problem. They expressed the vector wave equation in terms of two scalar equations for displacement potentials as usual. Applying the same procedure as in the SH case for each scalar component, they found two source ray integrals for compressional and shear potentials. The integrals again represent outgoing cylindrical waves in unbounded medium as a superposition of plane waves propagating in all directions. In addition to source rays, they found generalized ray integrals up to order 3 for multi-reflected $P$ and $SV$ waves. Both mode preserving and mode converting reflections have been considered. Similar to the scalar case, the ray integrals associated with interface reflection have been expressed in terms of transformed wave slowness. In the vector problem, there exists a pair of wave slowness for $P$ and $SV$ waves. Therefore, it is difficult to find a general expression for phase function in terms of a common wave slowness like the scalar case. The results have been presented up to the generalized ray integrals for Laplace transformed displacement potentials. As for the application of the Cagniard method, they briefly discussed the mapping of phase functions and arrival time of direct and refracted rays.
Following MacDonald's (1902) formulation for electromagnetic waves, Sanchez-Sesma (1985) obtained an analytical solution for the scattering of $SH$ wave in elastic wedges. For a source of harmonic cylindrical $SH$ wave inside the wedge, an inhomogeneous Helmholtz equation governs the resulting motion. He expressed the total displacement field as a series expansion of Bessel functions. Unknown coefficient could be obtained through boundary conditions. The consequent displacement field contains incident, reflected and diffracted parts. Then, he normalized the displacement by the free-field amplitude of incident motion to obtain an expression for amplification factor. It is a function of wedge angle, direction of incident wave and normalized (by wave number) distance from the wedge tip. Considering the asymptotic behavior at infinity, he calculated the amplification factors for incident plane wave. In addition, a similar expression has been obtained for the case of fixed boundary conditions. As for the numerical results, he presented the surface amplification factors of plane $SH$ wave for different wedge angles (obtuse and reentrant) and incidence directions. Reentrant wedge angle corresponds to the valley geometry where deamplification occurs at the vertex. In general, the amplification factor of $2/\nu$ has been found at the vertex of wedge subtends an angle $\theta = \nu \pi$. He pointed out that the results are also given in space and frequency domains due to self-similarity. That is at a given station they are transfer function of the wedge. He also depicted the spatial variation of the motion for a fixed frequency. For the right angle wedge with all incident directions and the $120^\circ$ wedge under vertical incidence, he showed that the diffraction term vanishes and the geometric elastodynamics field completely describes the total wavefield. For inclined incidence, he found both amplification and deamplification patterns on the illuminated face due to constructive and destructive interferences. Using inverse Fourier transform, he also obtained the time history of surface displacement for a set of stations along the wedge faces. The time domain results show that higher amplification factor could be expected along the illuminated face due to the constructive interference of reflected and incident waves.
Sanchez-Sesma (1990) used the geometric approach to study the response of two particular wedges subjected to vertically incident SH and SV waves. He obtained the geometric elastodynamics solution for zero traction wedges of 90° and 120° internal angles. For these geometries and under vertical incidence, the total wavefield does not include the tip diffraction. That is the geometric wavefield is continuous throughout the wedge. For 120° wedge under incident SV wave, the diffraction free condition holds only for Poisson’s material. In other cases, mode conversion at the free surface and presence of diffracted and Rayleigh waves make the problem much more complex. At the tip of 90° wedge, he found horizontal amplification factor (normalized by incident wave) of 4 and 0, respectively for SH and SV waves. The incidence angle of SV wave with respect to the face normal is 45° and therefore total reflection occurs along the face. In this case, the surface displacement field is perpendicular to the wedge face. For 120° wedge, the tip amplification factors corresponding to SH and SV waves are 3 and 4, respectively. There is a total mode conversion for the vertical SV wave impinging the wedge face at 30°. That is the reflection coefficients for mode preserving and mode converting waves are 0 and 1. An interesting feature of this case is that the displacement field is purely horizontal. Using the amplification factors at these two angles along with the doubling factor of half-space, he presented a likely variation of amplification factor with respect to wedge angle.

2.3.1.3. Method of Characteristics (Dynamic Self-Similarity)

The method of characteristic has been widely used in supersonic aerodynamics where it is called the method of conical flow (Busemann, 1935). According to the wedge geometry and boundary conditions, there is no characteristic length in the wedge problem. Therefore, it is possible to consider the scattered wavefield as a homogeneous function of normalized distance from the wedge tip. That means the dimension of the problem could be reduced by one. Applying an appropriate mapping, the reduced wave equation is transformed into the Laplace equation whose solution is a harmonic function.
Miles (1952) used the method of characteristics to study the diffraction of acoustic pressure pulse by an infinite rigid wedge. Hyperbolic characteristic of the governing equation results in a conical region of influence for the edge. Outside the cone, the equation has rectilinear characteristics tangent to its surface. Among these two, which constitute the boundaries of disturbance area, the conical diffraction region is of major interest. Because of such conical field and the geometry of the wedge, the solution could be considered as a homogeneous function of dimensionless distance to wedge tip (normalized by a travelled distance). He then used the Chaplygin’s transformation to map the conical region of influence onto the unit circular cylinder and to rewrite the governing wave equation as a Laplace equation. He mentioned that for a plane wave incident, the resulting equation in transformed polar coordinates system is no longer of Laplace type and therefore the method of Duhamel superposition should be used. In order to apply boundary conditions in the new system, he considered two different scenarios of incident angle. For the case that involves geometric shadow zone, the incident wave will be reflected only from the illuminated face. Outside the unit cylinder, there is no interaction between two faces of the wedge. The radiation condition of outgoing scattered wave is implied in the transformed boundary conditions and it only needs to apply edge condition. He then found a solution of dimensionless scattered wave field for both shadow and light cases. For the half plane problem subjected to a normal incidence, he presented a reduced form of the solution and compared with the previous results of Friedlander (1946).

Keller and Blank (1951) studied the scattering of plane scalar waves (electromagnetic and acoustic) by a wedge subjected to both Dirichlet and Neumann boundary conditions. The interior and exterior incidences have been referred to as corner and wedge problems in their study. To formulate the problem, they considered a solution to the governing wave equation that has jump discontinuity due to pulse excitation. Then, they applied the equation of wave front or Eiconal equation to this surface. As a basic element of geometrical optics, the Eiconal equation implies the construction of wavefront
surface by Huygens’ principle, the propagation along the normal (ray) direction and the specular reflection from the boundary. The jump remains unchanged until the pulse hits the wedge and generates reflected (one or two planes) and diffracted (a circular cylinder) discontinuity surfaces. At each instant after the contact, these surfaces can be acquired from the initial configuration using the Huygens’ principle (similar form with different scale). There is a reverse jump across the reflected plane(s) and zero on the diffracted cylindrical surface. Therefore, the solution is known everywhere except within the circular segment of the cylinder bounded by the wedge which comprises all diffraction effects. To express the initial and boundary conditions, they divided the problem into four categories based on incidence and wedge angles. Two cases (with and without shadow zone) involve the initial reflection pulse while the other two do not. They presented the solutions for the initially reflected cases because the other two problems have the same solutions, respectively. The solution is independent of the coordinate along the edge because of the geometry and the boundary conditions (i.e. its dimension could be reduced by one). Hence, they were able to transform the conical domain of diffracted wavefield into a unit circle and rewrote the wave equation as a Laplace equation. It is known from the complex analysis that the real and imaginary parts of any analytic function are harmonic conjugate functions i.e. solutions to the Laplace equation. Accordingly, they expressed the solution as the imaginary part of an analytical function and recast the problem as to find such an analytic function in the unit circular sector. As for the corner geometry, they included the resulting multiple reflections in the initial conditions. There are two types of discontinuity surfaces similar to those of the wedge geometry i.e. plane reflected and cylindrical diffracted surfaces. Except within the semi-cylindrical sector, the solution is piecewise constant and known all over. They divided the incident rays into two types according as they reach the upper or lower face first and found the direction and total number of reflections for each type. Based on the direction of last reflected ray, they categorized the corner problem into four cases. For each case, the diffracted semicircular region has piecewise constant boundary values bounded
by its intersection with reflected waves. They presented the solutions for all four configurations each for two boundary conditions (free and rigid). They found that the solutions of corner problem are also valid for the wedge geometry if the angle is replaced with its supplement. No diffraction condition for which the solution is constant inside the circle has been obtained as \( \theta = \pi/m \). They used Duhamel’s theorem to derive the solution for harmonic incident wave from the available pulse solution in the form of Fourier integral. In the last part of their study, they considered the general incidence of discontinuity surface where the wavefront is not parallel to the edge. In this case, it hits the edge at all times and consequently a new set of plane reflected and conical diffracted surfaces is generated at any instant. These new discontinuity surfaces should be considered in the initial conditions. They used a new spatiotemporal coordinate system which is moving along the edge with projected wave velocity. All discontinuity surfaces are stationary in this system and the same reduced wave equation as in parallel case can be applied and solved.

Kostrov (1966) studied the diffraction of \( S \) and \( P \) waves by a friction free (smooth) rigid wedge embedded in elastic medium and presented closed form solution for incident plane wave of Heaviside type. Boundary conditions include zero normal displacement and shear stress on wedge faces. Under these assumptions, no mode conversion occurs at the plane boundary and there are two uncoupled boundary conditions for \( P \) and \( S \) waves. He showed, however, that the edge condition (conservation of energy) could not be met using single wave type and therefore there exist both \( P \) and \( S \) waves diffracted from the wedge tip (difference between acoustic and elastic wedge problems). Therefore, he considered the total wavefield as a sum of acoustic and elastic parts where the latter is associated with the edge condition.

Graggs (1959) used self-similarity method to find scattered wavefield in a half-space subjected to two different boundary conditions i.e. a constant traction over the finite length of the boundary and a transient concentrated line load. He found analytical solutions
for different regions bounded by diffracted $S$ and $P$ waves and head wave. He also presented the numerical results for uniform step loading on a half space of Poisson material.

Kapustianskii (1976) studied the diffraction of plane $S$ and $P$ waves by an elastic wedge of internal angle greater than $\pi$ (reentrant angle). He used the same approach as Kostrov (1966) for a different boundary condition of zero normal stress and tangential displacement on the wedge faces. He satisfied the edge conditions by adding a term of unknown coefficient to the scalar solution. Considering the bounded displacement near the tip, he found closed form solutions of total longitudinal and shear potentials for incident wave of Heaviside type.

Achenbach (1969) studied the response of elastic wedge subjected to uniform shear stress on its face(s) using the method of characteristics. Both Dirac delta and Heaviside step functions have been considered as time dependency of input motion. He found a closed-form expression for the resulting shear stress in the wedge of $\theta \geq \pi/2$ and obtained the form of stress singularity near the wedge tip. According to his findings, stress singularity only occurs in the wedges of reentrant angle. He presented the general solution for case of delta function and some special solutions for step function consist of quarter space, half space and half plane. He used the last solution to find the shear stress due to the sudden opening of semi-infinite slit in an unbounded medium and its associated singularity.

Achenbach and Khetan (1977) tried to solve the vector wedge problem using the self-similarity method. They considered a wedge of reentrant angle subjected to normal stress and free of shear stress along its faces. The resulting wavefield consists of $P$, $SV$ and head waves. First, the governing wave equation has been decomposed into two potential equations. Then, they applied the usual change of variable along with the Chaplygin’s transformation to reduce these uncoupled equations to a set of Laplace equations. Defining two analytic functions whose real parts are solutions to the Laplace equations and applying conformal mapping, they came up with a system of equations. They converted the resulting system to a system of linear algebraic equations by expanding in Chebyshev polynomials.
Numerical results have been presented for the stress intensity factor and singularity coefficients of normal stress near the vertex.

Similar to the work of Achenbach and Khetan (1977), Vojcik (1977) used the method of self-similarity to solve the vector wedge problem. He sought the elastodynamic solution for a wedge subjected to uniform normal and shear tractions along either face. He decomposed the original problem into the symmetric and antisymmetric cases based on the even and odd load components. Expressing the potential forms of wave equation in terms of self-similar variables, he identified three distinct regions in the resultant wavefield. In the elliptic region, which is a circular sector bounded by $S$ wavefront, after applying the Chaplygin’s transformation, the wave equation is reduced to a set Laplace equations. Solution to these equations (analytic functions of complex characteristics) has been obtained up to the Fredholm integral equation of the second type. In the hyperbolic region, which is beyond the $P$ wavefront, one need to solve a 1D wave equation. The third zone i.e. the composite region is located between $S$ and $P$ wavefronts. He presented the hyperbolic solution of the wave equation in this region, which is known as head wave.

2.3.1.4. The Kontorovich-Lebedev Transform

The basic concept of the method is similar to other integral transform methods i.e. to simplify the governing equation by replacing the unknown function with its integral transform. After solving the transformed boundary value problem, the original unknown could be recovered through an inversion formula. Kontorovich and Lebedev (1934) studied the diffraction of a plane electromagnetic wave by a conducting half-plane. He introduced an integral transform whose kernel is Hankel function of the second kind with arbitrary imaginary order. Applying the radiation and edge conditions and considering the asymptotic behavior of Hankel function for large argument, the proposed integral transform turns the scalar wave equation into an inhomogeneous harmonic equation whose solution is trivial. The particular term represents the incident wave (Green’s function) while
the homogeneous term denotes the scattered wave. Using the inverse transform, which is a contour integral in the complex plane, one can find the resultant scattered wavefield.

Gangi (1960) used the modified version of Kontorovich-Lebedev transform to study the diffraction of cylindrical \( SH \) wave by a rigid wedge of reentrant angle. The kernels of transform pair are modified Bessel functions of complex order. He applied the Kontorovich-Lebedev transform to the governing Helmholtz equation i.e. the Laplace transform with respect to time of the original wave equation. Solving the resulting harmonic equation and applying inversion formula, the Laplace transform of the scattered wavefield has been obtained. Instead of taking inverse Laplace transform of the last result, he used asymptotic behavior of the contour integral to find the time solution of scattered wave in the neighborhood of its wavefronts. Then, he examined whether the method is applicable to the corresponding vector problem of incident \( P \) wave. He applied two Kontorovich-Lebedev transforms with kernels of different arguments to obtain two harmonic equations for \( S \) and \( P \) wave potentials. Similar to the scalar problem, the general solution of these equations can be simply obtained. However, transforming the boundary conditions, which are presented in the form of differential equation, will result in a more complicated integral equation with no available solution. On the other hand, applying the same Kontorovich-Lebedev transform to both equations, we will have a differential-integral equation of unknown solution. Therefore, he concluded that the method is not applicable to the vector wedge problem.

Forristall and Ingram (1971) used the Kontorovich-Lebedev technique to study the response of elastic wedge under normal surface tractions. They decomposed the vector wave equation for compressional and shear potentials and took the Laplace transform with respect to time to obtain modified Helmholtz equations. They applied the Kontorovich-Lebedev transform to scalar wave equations and found homogeneous differential equations with complex characteristic roots. The transformed scattered potentials (solutions) have been obtained as a linear combination of even and odd unknown functions. Then, he
expresses stress components in terms of transformed potentials and applied boundary conditions to reformulate the problem as singular integral equations. The resulting integrands contain Bessel functions of different order and arguments ($P$ and $S$ wavenumbers). Therefore, they were not able to convert integral equations into algebraic system using an inverse transform. As an alternative approach, they used a series expansion of Bessel functions and a substitution of unknown function that eliminates a singularity. Thereby, the integral equations have been reduced to a set of functional equations. They presented a singular integral equation with given algorithm of solution whose solution satisfies the resulting functional equations. They proved that any bounded solution of this integral equation is the Kontorovich-Lebedev transform of wave potential that satisfies the boundary conditions. The singular integral equation has been further reduced to a non-singular Fredholm integral equation of the second type. Finally, the wave potentials could be obtained by numerical integration of the inverse Kontorovich-Lebedev formula. Although the proposed semi-analytical technique is quite straightforward, the final expressions are very complex and numerical evaluation does not provide physical interpretation of involved processes.

Abo-Zena and King (1973) used the Kontorovich-Lebedev transform to obtain a semi-analytical solution for $SH$ wave scattering in an elastic wedge of arbitrary angle. The incident wave is generated by an impulsive point source applied on the wedge face. He first took the Laplace transform of the scalar wave equation with respect to time. Then, he reduced the resulting equation to an inhomogeneous harmonic equation by applying the Kontorovich-Lebedev transform to an appropriate form of displacement field. The solution of the latter is defined up to unknown coefficients of the homogeneous part. These functions could be determined through applying the boundary conditions. Using the Kontorovich-Lebedev inversion formula he found the Laplace transformed displacement field. He then applied boundary conditions, expressed the resulting improper integrals in series form, and used the inverse Laplace transform to find the displacement field in time
domain. After some simplifications, he expressed the total displacement field as a piecewise function defined separately for the wedge tip and the rest of wedge medium. The solution contains expressions for the incident and reflected waves in a series form and the diffracted waves in an integral form. They could be identified in the total wavefield based in their travel time. He demonstrated that for special cases where the wedge angle is integer fraction of $\pi$, e.g. half-space and quarter space, there is no tip diffraction and the total displacement could be described as geometric elastodynamics field. To describe the characteristics of the resultant wavefield, he presented synthetic seismograms for specific obtuse and reentrant wedges. Considering the symmetry and using superposition, he also obtained the solution for mixed boundary condition where one wedge face is fixed.

2.3.1.5. The Fourier Transform

Zemell (1975) used the Fourier transform technique to study the diffraction cylindrical (near-field) $P$ and $SV$ waves by a smooth rigid wedge. For such boundary condition i.e. zero normal displacement and shear stress, in contrast to rigid or stress free boundaries, there is no coupling between two wave types at the boundary. However, edge condition requires both wave type exist near the tip because of the law of conservation of energy. Therefore, and in accordance with Kostrov (1966), he divided the total wavefield into the known acoustic part and unknown elastic part due to edge condition. He presented the closed form solution for both steady and transient excitation. He verified the result with plane (far-field) wave solution of Kostrov (1966) by moving the source to infinity. In general, the elastic wavefield generated from the vertex exists throughout the domain. However, he pointed out two exceptions of the $P$ wave line source along the bisector (no edge effect) and the half-plane (constant elastic term only between $P$ and $S$ wavefronts).

Momoi (1980) presented a semi-analytical technique to obtain the complete scattered wavefield generated by incident Rayleigh wave in a right angle wedge. He ascribed the discrepancies between experimental and theoretical results to the lack of
appropriate representation of tip diffraction in approximate theories. He proposed a new method based on the Fourier transform to remedy the issue. He defined the incident Rayleigh wave in a form that satisfies the Helmholtz equations ($P$ and $S$ wave potentials) and the boundary condition of incident face. Then, he superimposed scattered waves onto the incident motion to satisfy the traction-free boundary conditions. He represented the scattered displacement filed along either face in terms of Fourier sine and cosine integrals with unknown coefficients. Then, applying the boundary condition and taking inverse transform, he obtained a set of functional equations for unknown Fourier coefficients. Since the resulting integral equations are too complex and not tractable to analytical solution, he solved them numerically. He expressed the integral equations in discrete form to obtain a system of algebraic equations. However, integrands rapidly change over the original integration path in the complex plane (real axis). That is to maintain the accuracy, the number of equations increase considerably. The key step of his semi-analytical technique is to deform the integration path so that the integrands vary smoothly. Replacing the Fourier coefficients into the integral representation of displacement field, he found expression for displacement components on each face. Considering the pole contribution of resulting integrals, he obtained displacement components of transmitted and reflected Rayleigh wave. As for the scattered body waves, he found expressions of radial and tangential displacements at far field corresponding to $P$ and $S$ wave. He also calculated the normalized energy flux for different components of the total wavefield. For transmitted and reflected Rayleigh waves, the normalized energy is equal to the square of corresponding coefficient. He presented the numerical values of normalized energy for transmitted and reflected Rayleigh waves as well as diffracted $P$ and $S$ waves over the range of Poisson’s ratio 0-0.45. He found that the energy of transmitted Rayleigh wave, which contains the largest portion of scattered energy, is almost constant over the range of Poisson’s ratio. As for the transmitted and reflected Rayleigh coefficients, he presented the phase shift and compared with available experimental results. The amplitude of scattered
wave along the wedge faces has been shown to converge to that of Rayleigh wave at large distance from the apex. Using the particle motion along the wedge face, he demonstrated that the transmitted Rayleigh wave takes its complete form earlier than the reflected part. In addition, the far-field directivity patterns of $S$ and $P$ waves have been presented for several values of Poisson’s ratio. They show typical features of directivity patterns like vanishing $S$ diffraction coefficient along the bisectrix. To explain the behavior of scattered wavefield near the vertex, he presented the spatial variation of energy density and phase of $S$ and $P$ waves. He referred to a double source effect near the wedge tip. One apparent source resulting from the wave trapping in the corner (resonance) and one real source due to the incident Rayleigh wave. This phenomenon gives rise to a vortex like pattern of $S$ and $P$ wave phases near the corner.

Momoi (1985) used the same technique as what he devised in his earlier work (Momoi, 1980) to solve the scattering problem of Rayleigh wave in a different geometry i.e. three quarter space. He divided the medium inside the wedge into three regions along the wedge faces. He expressed displacement in terms of Fourier sin and cosine integrals and applied boundary and continuity conditions to consequent stress and displacement fields. Again, he came up with integral equations for unknown Fourier coefficients from which displacement components could be recovered. Deforming the integration path and discretizing its different parts with various intervals, he reduced the integral equations to a system of algebraic equations. He presented the numerical results for the same parameters as in the quarter space problem. The normalized energy flux of different wave components has been obtained over the same range of Poisson’s ratio (0-0.45) in each region. It shows that most of the incident Rayleigh wave is converted to $S$ waves in the same direction as the incident wave. Far-field directivity patterns of $S$ wave also renders the maximum energy density along the extension of incident face. The higher the Poisson’s ratio goes, the larger the amplitude of $S$ wave becomes. Spatial variation of energy density and phase for scattered $S$ and $P$ waves have been presented to explain the behavior of wavefield
around the corner. Again, the dominant feature is the large S wave energy in the direction of incident wave. Around the vertex, compressional and shear parts of the wavefield have very large amplitude and inverse phases. Therefore, the superposition of them gives rise to small amplitude waves.

In the theoretical part of their work, Fujii et al (1984) extended the semi-analytical solution of Momoi (1980) to the more general case of arbitrary wedge angle. They defined two coordinate systems along the wedge faces and split the scattered displacement field accordingly. Either part of scattered field has been expressed in terms of Fourier sin and cosine integrals. They substituted the total displacement field in the zero traction boundary conditions and reformulated the problem as a set of functional equations. They solved the consequent integral equations numerically (Simpson’s method) to determine the unknown Fourier coefficients. They aimed to supplement the measured Rayleigh coefficient with theoretical results. Therefore, they extracted contributions of Rayleigh pole from the integral representation of scattered field and obtained both transmitted and reflected Rayleigh waves. As for the non-physical poles of integrands, they deformed the integration path to avoid these singularities and maintain sufficient accuracy. They found a restriction condition for wedge angles so that extra poles are excluded from the path and theoretical results are valid. For Poisson materials, the condition leads to the applicable range of $90^\circ \pm 18^\circ$. They compared experimental coefficients with theoretical results and found a good agreement over the common range of wedge angles. They also investigated the effect of Poisson’s ratio on the transmission and reflection coefficients. At larger Poisson’s ratios, the results show more transmission and less reflection amplitudes for acute wedge angles while a reverse trend occurs for obtuse angles.

Takeuchi et al. (1984) used the semi-analytical technique developed by Fujii et al (1984) to investigate the behavior of scattered wavefield inside a wedge. They presented a physical interpretation for the sudden change of Rayleigh coefficients near the wedge angle $\theta = 80^\circ$ (For Poisson’s materials). They extended the evolution process of scattered
Rayleigh wave, originally presented by Momoi (1980), to a wedge of arbitrary angle. At large distance from the wedge tip, the total wavefield could be decomposed into the Rayleigh waves (transmitted and reflected) as well as converted S and P body waves. In the intermediate domain, however, there is a transition zone between the scattered S wave and undeveloped (incomplete) Rayleigh waves. This zone, referred to as the breathing zone by Momoi (1980), is extended along the bisectrix of the wedge. They presented the spatial variation of energy density and contours of constant phase for several acute and obtuse wedges within a range where the theory is applicable (72°-108°). The domain of different components of scattered wavefield has been clearly shown in these plots. In the generation of Rayleigh wave, they demonstrated that the shear part of scattered wavefield contributes more than the compressional part. Decreasing the wedge angle below 90° down to 83°, the reflected Rayleigh wave is decreasing and transmitted wave is consistently increasing. The change in energy is associated with narrower reflected and wider transmitted regions. Below 82°, where the reflection is minimized, the region of incomplete Rayleigh wave region begins to gain more energy from the breathing zone while the transmitted part loses its energy.

Similar to the work of Fujii et al (1984) that extended the method of Momoi (1980) to the wider albeit limited range of wedge angles, Nakano et al. (1988) studied the scattering of Rayleigh wave in a reentrant wedge by generalizing the technique of Momoi (1985). They partitioned the wedge region into three parts with boundaries perpendicular to the free surfaces and defined two coordinate systems along them. Applying zero stress boundary conditions on wedge faces together with continuity conditions along interfaces, they found a set of functional integral equations for unknown Fourier coefficients. The original path of integration has been distorted to exclude the singularities. Similar to the case of quarter plane, it poses some restriction on the valid range of wedge angles. They then discretized the new path and recast the integral equations into a system of algebraic equations. In the integral representation of displacement field along wedge faces, they
calculated residues at Rayleigh poles to obtain reflected and transmitted coefficients. They presented these complex valued coefficients for the range of wedge angles $270^\circ \pm 20^\circ$ and Poisson’s ratio 0.15-0.45. The results show a more smooth variation of Rayleigh coefficients with wedge angle compared to the case of $\theta < 180^\circ$. To study the behavior of body waves, they presented the spatial variation of shear displacement for three different wedge angles. Most of the incident energy is converted to the scattered $S$ wave toward the transmitting region. Using the same procedure as Fujii et al (1984), they also conducted an experimental study and measured Rayleigh wave coefficients over the range of $190^\circ - 330^\circ$.

Fujii (1994) modified his earlier semi-analytical technique (Fujii et al, 1984) to solve the problem of Rayleigh wave scattering over a wider range of wedge angles. Deviation from the original work starts at the numerical solution of integral equations. Assuming a complex value for the angular frequency, he separated the poles from the original path of integration i.e. the real axis. Therefore, he was able evaluate the singular integrals of Fourier coefficients numerically without distorting the path. The degree of separation is specified by the ratio of imaginary and real parts of complex frequency. Regarding the stability of computation, he found that lower/higher values of separation parameter are required for smaller/larger wedge angles. He determined the unknown Fourier coefficients by discretizing the real axis and solving the consequent algebraic system of equations. The transmission and reflection coefficients have been presented over the range of wedge angles $36^\circ - 180^\circ$ and for three different values of Poisson’s ratio (0.25, 0.35, 0.45). Using the same technique, he also extended the solution of Nakano et al. (1988) for reentrant wedges to a broader range of $200^\circ - 340^\circ$. The theoretical results have been validated with the experimental data of his earlier studies (Fujii et al, 1984 and Nakano et al. 1988) for Poisson’s ratio $\nu = 0.234$. He pointed out the oscillatory behavior of Rayleigh amplitudes at small wedge angles, similar to the observation of Krylov and Mozhaev (1985). He also presented the variation of scattered Rayleigh energy normalized to the total incident energy over the same range of wedge angles. While the normalized Rayleigh
energy shows monotonic increase beyond the minimum point of $\theta = 120^\circ$, it has multiple oscillations for smaller wedge angles. Accordingly, the maximum conversion of incident energy to body waves occurs at the apex of $120^\circ$ wedge. He investigated the behavior of scattered body waves for several obtuse and acute wedges by presenting the displacement vector field. In obtuse wedges, the major part of scattered wave energy (both S and P) lies along the transmitting face. In acute wedges, on the other hand, the patterns is much more complex. He demonstrated that the concept of energy reservoir zone (breathing zone), originally proposed for right angle wedge, holds over the new wider range.

In a series of studies (Gautesen, 1985a,b, 1986, 1987, 2001, 2002a,b,c), Gautesen presented a new semi-analytical technique for elastic wedge problem. The method consists of four general steps. He used the generalized Green’s theorem aka extinction theorem to represent the displacement field in the form of a single layer potential (Neumann boundary value problem). This is equivalent to the superposition of cylindrical wavefields generated by point sources located on the wedge faces. As opposed to the representation theorem, the extinction theorem is limited to the wedge boundaries and is valid in the entire plane. He obtained a reduced form of extinction theorem by narrowing the entire domain (2D) to the line parallel to and just below the wedge face (1D). Next, he symmetrized the problem through dividing the total displacement field into symmetric and antisymmetric fields with respect to bisectrix. This reduces the number of unknown displacements on the wedge faces by half. However, the problem should be solved twice for symmetric and antisymmetric cases and the results need to be added to find the total unknown wavefield. He then applied the full Fourier transform to the reduced form of extinction theorem. It gives a set of functional equations for unknown one-sided Fourier transform of displacement field along the wedge face. These equations are too complex for analytic solution and should be reformulated in a form convenient for numerical solution. He decomposed the total wavefield into its components and replaced the corresponding Fourier transform in the functional equations to find a set of Fredholm integral equations of the second type. He
used different numerical schemes to solve these equations and recovered the wavefield characteristics (transmission and reflection of Rayleigh wave as well as body wave diffraction) from the solutions.

Gautesen (1985a) studied the scattering of harmonic Rayleigh wave propagating along the face of a traction-free right angle wedge. He divided the total wavefield into the incident and the scattered fields. He then expressed the governing differential equation and boundary conditions for scattered wavefield. Using the free space Green’s function and applying the divergence theorem, he derived the extinction theorem. After applying the Fourier transform to the extinction theorem, he took the even part (cosine transform) of the resulting integral equations. For the right angle wedge, it brings forth to a set of decoupled integral equations corresponding to tangential and normal displacements. He decomposed the Fourier transform of total displacement field into the Rayleigh surface wave and the diffracted body wave. He used this expression to reformulate the integral equations in terms of unknown Rayleigh coefficients and diffracted wave components. Evaluating these equations at Rayleigh wave number where the Rayleigh function is zero and diffracted waves are bounded, he found a relation between Rayleigh wave coefficients and diffracted wave. Replacing this relation into the functional equations, he obtained two uncoupled integral equations for diffracted wave components, which has to be solved numerically. Recursively, he found transmission and reflection coefficients of Rayleigh wave by adding symmetric and antisymmetric parts. He presented the amplitude and phase of transmission and reflection coefficients for a wide range of Poisson’s ratio (0.025-0.475). Each coefficient has been calculated based on normal and tangential displacement component to check the numerical accuracy of the results. He also validated his results by comparing the amplitude of transmission and reflection coefficients with those of previous experimental and numerical studies at three different Poisson’s ratio. He observed that changing the Poisson’s ratio has negligible effect on the transmission coefficient. Finally, he obtained
the far-field directivity patterns, which are equal to diffraction coefficients of $P$ and $S$ waves.

Gautesen (1985b) solved the diffraction problem for an elastic wedge subjected to incident plane $P$ wave using the same semi-analytical technique as Gautesen (1985a). He decomposed the Fourier transform of displacement field on the wedge face into three terms: the geometric elastodynamic field, the scattered Rayleigh wave propagating along the wedge face, and diffracted body waves. He solved both the resulting integral equations (normal and tangential components) numerically and computed the complex valued diffraction coefficients of Rayleigh wave on either face as a function of Poison’s ratio for a range of incidence angle. In addition, he obtained the asymptotic behavior of the reduced form of extinction theorem to define the far-field directivity patterns for diffracted $S$ and $P$ waves. These scattering patterns, which represent diffraction coefficients in geometrical theory of diffraction, vanish on the wedge free boundaries. He obtained the singular points for diffraction coefficients of $S$ wave and observed its vanishing values around the bisectrix.

Extending his earlier work (Gautesen, 1985a) to 3D space, Gautesen (1986) studied the scattering of Rayleigh wave obliquely incident on the edge of elastic quarter-space. He obtained a reduced form of the extinction theorem by narrowing the entire 3D domain to the plane of wedge face (2D). Taking 2D Fourier transform of the reduced extinction theorem, he reformulated the problem as a set of functional equations. He then took the even and odd parts of these equations to find a decoupled system of equation for three components of Fourier transform of displacement field. He decomposed the Fourier transform of displacement field into the surface and body waves and replaced it in the corresponding functional equation. He numerically solved the resulting equations to obtain the transmission and reflection coefficients of Rayleigh wave. Using the asymptotic behavior of extinction theorem at large distance from the edge, he also obtained the far-field directivity patterns for $SV$, $SH$ and $P$ waves. He observed that on the traction free
surfaces, $P$ and $SV$ diffraction coefficients as well as the normal derivative of $SH$ diffraction coefficients vanish. In addition, for angle of incidence below the critical angle, no body wave exists in the scattered wavefield. In this case, the total normalized surface energy defined as the sum of square amplitudes of Rayleigh coefficients is equal to unity. He presented the complex valued Rayleigh coefficients for the range of incidence angles 0-90° and two values of Poisson’s ratio (1/4 and 1/3). Similar to Poisson’s ratio (Gautesen, 1985a), changing the angle of incidence has negligible effect on the transmission phase. He also presented the $P$, $SV$ and $SH$ diffraction coefficients (portion of incident surface energy converted to body waves) for different angles of incidence and two Poisson’s ratios. He validated the results for the case of normal incident with 2D results of Gautesen (1985a).

Gautesen (1987) generalized the solution of Rayleigh wave scattering by an elastic right angle wedge (Gautesen, 1985a) to a wide range of wedge angle. Replacing the decomposed Fourier transform of total displacement field (surface + body) into the functional equations and taking the even part, he found two equations for unknown Rayleigh coefficients and diffracted waves. It is noteworthy that taking the Fourier cosine transform does not decouple the functional equations for the general wedge angle. He presented the amplitude and phase of Rayleigh coefficients for wedge angles 45°-135° for two different values of Poisson’s ratio i.e. $\nu = 1/4$ and 1/3. He superimposed the results obtained from tangential and normal displacement components to show the numerical accuracy. He also presented the variation of Rayleigh coefficients with Poisson’s ratio at two specific wedge angles. Analogous to the right angle wedge, transmission coefficient is almost constant over the range of Poisson’s ratio.

Gautesen (2001) modified his semi-analytical technique (Gautesen, 1985a) to solve the problem of Rayleigh wave scattering in a wedge of reentrant angle. Although he presented the results for Rayleigh wave incidence, the proposed method works for incident $P$ and $S$ waves as well. Using the reduced form of extinction theorem, he computed the tractions, dilation and rotation on a line parallel to the free surface. The Fourier transform
of these quantities give rise to a set of functional equations in terms of one-sided Fourier transform of displacement along the wedge face. Then, he replaced the unknown displacement by its components i.e. incident wave, geometric elastodynamics field and diffracted waves. Computing the jump across the branch cut (Hilbert transform) of Fourier transform of displacement, he found a set of Fredholm integral equations of the second type suitable for numerical solution. The Fourier transform of unknown traction, dilation and rotation do not appear in the final Fredholm integral equation because they are continuous across the branch cut. These functions are analytic in the lower half of complex plane while the Fourier transform of displacement is analytic in the upper half (Contribution from these terms vanish). The results of transmission and reflection coefficients are presented for wedge angles 189°-327° and Poisson’s ratio 1/4. He pointed out the numerical instability beyond this range where there is a pole close the contour of integration. It is shown that the results approaching the known limiting values of half-space and half-plane.

Gautesen (2002a) used the same modified semi-analytical technique as Gautesen (2001) to find a more accurate solution for the problem of Rayleigh wave scattering in an elastic wedge. He numerically solved the resulting set of decoupled Fredholm integral equations to find the Hilbert transform of diffracted displacement. He then calculated the amplitude and phase of Rayleigh coefficients for the range of Poisson’s ratio 0.1-0.4. Compared to his earlier work (Gautesen, 1985a), the maximum numerical error, defined as the difference in magnitude of Rayleigh coefficients obtained from normal and tangential displacements, is much smaller (2e-5 vs. 0.03). He also validated the results with those of Fujii (1994) and Li et al. (1992). Again, he observed a nearly constant transmission coefficient over the Poisson’s ratio range.

Gautesen (2002b) used the new numerical scheme of Gautesen (2001) to solve the problem of Rayleigh wave scattering in an elastic three-quarter space (a wedge subtends the angle of $\theta=270^\circ$). He presented the variation of transmission and reflection coefficients
over the range of Poisson’s ratio 0.1-0.4. The results show that the transmission coefficients linearly varies with Poisson’s ratio while the reflection coefficient is almost constant.

Gautesen (2002c) used the semi-analytical technique of Gautesen (2001) to revise his earlier study on the scattering of Rayleigh wave in a wedge of obtuse angle (Gautesen, 1987). He decomposed the Fourier transform of displacement field on the free surface into the surface waves, geometric elastodynamics field and diffracted waves. He calculated the geometric elastodynamics field up to the second order reflections (re-reflection) due to numerical instability. This put a lower limit on the applicable range of wedge angles. The complex valued transmission and reflection coefficients have been presented for wedge angle 63°-180° and Poisson’s ratios 1/4 and 1/3. He validated the results with those of Fujii (1994), Li et al. (1992) and Budaev and Bogy (2001).

Gautesen and Fradkin (2010) presented a semi-analytical solution to the problem of diffraction of plane waves by a traction free elastic wedge. They used the extinction theorem (generalized representation theorem) to express the total displacement field in the form of integral equations. In the representation theorem, the wavefield inside the body is represented by a superposition of cylindrical waves radiated from imaginary point sources along the boundaries. The extinction theorem states that the incident wave within the medium (inside or outside the wedge) is extinguished by the induced waves at the boundaries. In fact, the extinction theorem combines the governing elastodynamic equation with zero traction boundary condition. Adding radiation and tip conditions ensures the unique solution for the problem. The resulting integral equations consist of four unknowns, normal and tangential displacements on either face of the wedge. They applied rotation and dilation operators (curl and divergence) on the simplified version of integral equations to separate longitudinal and transverse displacements. Similar to other works of Gautesen, they split the original problem into independent symmetric and antisymmetric sub-problems. Using extinction theorem instead of representation theorem enabled them to apply the full Fourier transform on the integral equations and to reformulate them as matrix
functional equations. This gives a relation between one-sided (physical range) Fourier transform of displacement vector on lower face and that of dilatation and rotation on the upper face. To describe different parts of the scattered wavefield, they determined the singularities of transformed displacement vector. Incident and diffracted Rayleigh poles as well as $P$ and $S$ wave branch cuts constitute the singularities. Therefore, the total displacement field may be decomposed to the geometric elastodynamics field (specular reflections of body waves from boundaries), Rayleigh wave and diffracted body waves (together is defined as scattered). Substituting this decomposition into the functional equations and rearranging for incident poles, they obtained an iterative procedure for the geometric elastodynamic field. Determinant of the coefficient matrix of functional equations is the Rayleigh function with simple roots as Rayleigh poles. Next, they replaced the unknown diffracted displacement vector (one-sided Fourier transform) by its Hilbert transform defined as the jump across the branch cut. They regularized the resulting ill-posed integral equation by eliminating unknown Rayleigh coefficients to obtain a well-posed vector integral equation. The resulting Fredholm integral equations of the second kind may be solved numerically. They presented expressions for Rayleigh waves propagating on either face of the wedge. As for the tip diffraction, they obtained asymptotic expression for far-field diffraction coefficients or directivity of $P$ and $S$ waves. They used two different benchmarks to verify the numerical accuracy of their code: zero diffraction coefficients outside the wedge where there is no energy and the reciprocity principle for diffraction coefficients. They also extended the Sommerfeld-Maliuzhinetz-Budaev technique to find the far-field diffraction coefficients for validation purpose. The results of diffraction coefficients are valid for wedge angles $25^\circ/35^\circ$ to $180^\circ$, respectively for incident $P$ and $S$ waves. One can also use their code to calculate transmission and reflection coefficients of Rayleigh wave incident upon a corner.
2.3.1.6. The Sommerfeld-Maliuzhinetz Technique

Budaev and Bogy (1995) studied the scattering of Rayleigh wave by a traction free elastic wedge using Sommerfeld-Maliuzhinetz technique. This method, which is based on the Sommerfeld’s integral representation of harmonic wavefields, reduces the diffraction problem to a set of functional equations in complex plane (Maliuzhinetz, 1958). They used the method of Budaev (1995) to study the resultant functional equations. Using longitudinal and shear wave potentials, they decoupled the original wave equation into a set of Helmholtz equations. Then, they represented the solutions of Helmholtz equations i.e. the unknown total wavefields in the form of Sommerfeld integrals. In other words, the total wavefield is replaced by the superposition of plane waves propagating in complex directions (Sommerfeld, 1896). The amplitudes of these plane waves are analytical functions called Sommerfeld amplitudes. Considering the region bounded by Sommerfeld’s contours and their steepest descent paths, one can describes different components of the scattered wavefield as follows. While the singularities within the region describe the surface (pole) and geometric body waves (branch cut), the integrals along the steepest descent paths represent the diffracted waves from the wedge tip. Replacing Sommerfeld integrals into the boundary conditions, they obtained a set of functional equations for unknown Sommerfeld amplitudes. Applying the radiation and tip conditions, they formulated the analytical properties of amplitude functions. The tip condition determines the asymptotic expansion of potential functions at imaginary infinity and guarantees the convergence of Sommerfeld integrals (Kamotski et al., 2006). They also used the symmetrization of displacement field with respect to the wedge bisectrix to split the functional equations into two independent sets. They showed that if amplitude function are found in a sufficiently wide vertical strip, then the solution may be analytically continued to the entire complex plane. The unknown functions have been decomposed into a known term associated with geometric elastodynamics field and an unknown term.
corresponding to Rayleigh wave and tip diffraction. Substituting this decomposition into
the functional equations gives rise to a system of functional equations for new unknowns.
Then, they applied a Hilbert type transform to reduce the Maliuzhinetz-like functional
equations into Fredholm integral equation of the second type on the real axis, which is
convenient for numerical evaluation. The solution of integral equations has been converted
to wave characteristics using a recursive procedure. They presented the numerical results
for transmission and reflection coefficients as a function of wedge angle in the range 95°-
240°. Rayleigh coefficients are defined as a linear combination of symmetric and
antisymmetric parts. They verified the results with the experimental coefficients of Pilant
et al. (1964). Finally, they presented a step-by-step algorithm for calculation of Rayleigh
coefficients which is useful for coding. Their method for solution to the wedge diffraction
problem is hereafter referred to as the Sommerfeld-Maliuzhinetz-Budaev technique.

Budaev and Bogy (1996) tried to extend the limits of wedge angle for which
Rayleigh coefficients are calculated by improving the numerical procedure of their method.
The numerical scheme has two major steps i.e. solving the system of singular integral
equations and transforming the solution to wave characteristics. The first step involves
more complexity. The first step, which involves more complexity, has been modified in
two ways. First, they recalculated the geometric elastodynamics part of functional
equations which had shown increasing numerical error for smaller angles. Decreasing the
wedge angle, the number of poles and the range of their residues increase which give rise
to numerical instability. The next improvement has been done on the regularization scheme
of the singular integral equations. They presented the transmission and reflection
coefficients for wedge angles 30°-355°. However, as they acknowledged later in Budaev
and Bogy (2001, 2002), the Rayleigh coefficients are inaccurate for wedge angles larger
than 180°. If the wedge angle exceeds 180°, the new calculation scheme causes the
geometric elastodynamics part of Sommerfeld amplitudes to have extraneous poles in the
prescribed vertical strip, which results in numerical errors. As for the lower angle limit of
their results i.e. 30°, it is attributed to processing capacity of that time. In fact, the increasing number of involved terms (poles) and their oscillatory behavior for small wedge angles magnifies the computational cost. As an offset, they supplemented their results with those of Krylov and Mozhaev (1985) for smaller wedge angles. They also described a close relation between steps of their approach and those of the Wiener-Hopf technique for half-plane problem.

Kamotski et al. (2006) used the Sommerfeld-Maliuzhinetz-Budaev technique to study the scattering of Rayleigh wave by a traction free elastic wedge. Following the original method, they recast the diffraction problem as a set of functional equations and came up with singular integral equations in two unknowns. A constant that comes from the tip asymptotic expression and a function, which is related to the diffracted Sommerfeld amplitude via a Hilbert type transform. They presented a new semi-analytical scheme to evaluate the unknowns of singular integral equations. The diffracted Sommerfeld amplitudes may be recovered from the unknown functions using singular integral transforms. They also obtained approximate quadrature formulae for these recovering integrals. In addition to the numerical improvement, they clarified the theoretical concepts of Sommerfeld-Maliuzhinetz-Budaev technique through a more transparent and detailed description. Some verification tests have been used to check if the amplitude functions obtained from functional equations are the solution to the original physical problem. They presented the results of transmission and reflection coefficients for the wedge angles 45°-178° and validated them with numerical and experimental results of Budaev and Bogy (2001), Fujii (1994) and Gautesen (2002). They pointed out that at small wedge angles their numerical code becomes unstable because of multiple reflections (poor agreement at this range).
2.3.1.7. Other Techniques

Kane and Spence (1965) used a variational technique to study the scattering of Raleigh and Love waves in elastic wedge. Symmetry of the problem has been taken into account to reduce the number of unknowns by a factor of two. Considering the bisectrix of wedge as a plane of symmetry, they broke down the main problem into a pair of even (symmetric) and odd (asymmetric) problems. Since the problem is linear, the total unknown field can be expressed as a sum of even and odd sub-fields. For the vector problem, they replaced the initial condition, a unit Rayleigh wave on the loading face \( S_1 \) along with no excitation on the other \( S_2 \), by equivalent partial waves of half amplitude. The new condition comprises two partial waves of the same sign on \( S_1 \) and another two with opposite sign on \( S_2 \). In the even problem, the wedge is excited by a pair of identical partial waves on either face. In this case, compressional and shear potentials are respectively even and odd about the plane of symmetry. They simplified these demanding conditions to even displacement field with no normal component along the bisectrix. The new condition reduces the problem to a bisected wedge with zero normal displacement along the new face (bisectrix) as well as zero stress on the old face \( S_1 \) or \( S_2 \). The odd problem that contains odd displacement field could be reduced likewise with zero tangential displacement along the new face. For each minor problem, they defined a partial reflection coefficient along the old face. The reflection and transmission coefficients of the main problem were then expressed as a sum and difference of even and odd partial reflected coefficients, respectively. They considered total displacement field along the plane of symmetry as a trial function of variational procedure. It consists of contributions from incident Rayleigh wave as well as partial reflected wave of unknown amplitude. For the even and odd problems, they respectively minimized normal and tangential displacements in the mean square sense to find unknown coefficients. They presented the complex coefficients for the complete range of wedge angles \((0-2\pi)\). The normalized reflected and
transmitted energy can be obtained by squaring the magnitude of associated coefficient. Subtracting the sum of these energies from unity, they found the residual energy. They attributed the residual energy to both approximation error and converted body waves. Considering the S-Rayleigh conversion, they modified the coefficient of shear potential in Rayleigh wave by an angle dependent factor. It resulted in non-symmetric coefficients that capture more features of experimental data. Using the same technique with different trial functions, they solved the scalar problem of Love wave incident upon a layered wedge (where it can exist) for the first two modes of propagation. The magnitude of transmission and reflection coefficients have been presented as functions of wedge angle (symmetric range of 110°-250°), dimensionless thickness of superficial layer and rigidity ratio.

Kraut (1968a,b) investigated the diffraction of plane P wave by a rigid right angle wedge using the Wiener-Hopf technique. He decomposed the total wavefield into incident and scattered (reflected and diffracted) parts. Then, combining the representation theorem for scattered field with boundary conditions (extinction theorem), he recast the original problem in a set of singular Fredholm integral equations of the first kind. The unknown are stress discontinuity across the plane of the wedge in third dimension. Considering the 2D wedge problem in 3D space, which is the key step of the proposed method, enabled him to reduce the resulting integral equations to a system of Wiener-Hopf equations in two complex variables. To fulfill such reduction, he applied the 2D double-sided Laplace transform to the integral representation of scattered displacement. Then, he used the convolution theorem to express the transformed scattered field as a product of a known matrix kernel and an unknown function of two complex variables (2D Laplace). According to integral limits, he expressed the Laplace transformed scattered field as a sum of four double integrals. Using zero displacement boundary condition, one of these integrals is defined in terms of the incident wave and the rest remain unknown. Repeating the solution of the single variable Wiener-Hopf problem presented by de Hoop (1958) for the rigid half-plane diffraction, Kraut tried to construct an analogous solution for the wedge diffraction.
problem. However, for the general case of wedge angle, the Wiener-Hopf method reduces the governing equations to a coupled system of functional equations. The crucial step of the Wiener-Hopf technique is product factorization of the kernel through specific contour integrals. While the existence of such factorization for a general matrix has been proved for a long time (Gohberg and Krein, 1960), its procedure is currently available only for special cases (Khrapkov factorization). Therefore, without explicitly defining the factors, he assumed the kernel (Laplace transform of elastic Green’s function) as a product of four matrices and presented the scattered displacement in terms of them. He also obtained a condition for matrix factorization that ensures the uniqueness of the solution. Using the Cagniard-De Hoop technique (integration along the steepest descent path), he evaluated the resulting integral representation of scattered wavefield. He obtained expressions for various parts of the scattered displacement field i.e. diffracted and reflected $P$ and $S$ waves in terms of Laplace transformed stress jump and Green’s function. Abrahams (2002) used the Pade approximation (rational function) to present explicit approximate factors for a general matrix kernel. He solved the subsequent functional equations of several canonical problems in elastodynamics without recourse to any integral equation.

Alsop et al. (1974) used the concept of inhomogeneous waves to study the transmission and reflection of Rayleigh wave. In the plane wave solution of the Helmholtz equation, they considered a general direction of propagation through a complex wavenumber vector. In contrast to the real wavenumber, which results in the usual homogeneous plane wave, the solution with complex direction of propagating defines an inhomogeneous wave. In homogeneous plane waves, planes of constant amplitude and phase are parallel. For inhomogeneous plane waves, on the other hand, directions of attenuation and propagation are perpendicular. That is the imaginary and real parts of wavenumber vector are orthogonal. Generally, inhomogeneous waves are non-allowed solutions to the wave equation as they contravene the boundary conditions at finite points. However, a combination of inhomogeneous waves can satisfy certain boundary conditions.
For the case of elastic half-space, they demonstrated that constructive interference (same frequency and phase velocity) of inhomogeneous $P$ and $SV$ plane waves propagating along certain directions gives rise to the Rayleigh wave that satisfies zero stress boundary condition. They also showed that while two inhomogeneous components have prograde orbital motion, the resultant Rayleigh wave is retrograde. The inhomogeneous $P$ wave decays in depth faster than the $SV$ wave because of smaller wavenumber. Therefore, the overall motion at depth would be mainly $SV$ with prograde motion again in accordance with Rayleigh characteristics. For geometries where inhomogeneous waves are non-allowed, they showed that these waves could be used to approximate the real solution. Ignoring the diffracted waves and mode conversion at the boundary, they obtained a simple geometric solution for transmission and reflection of inhomogeneous plane $P$ and $S$ waves. The results have been used to construct a solution for transmission and reflection of Rayleigh wave. They obtained Rayleigh wave coefficients for two different models i.e. a jointed quarter space and a step topography (three-quarter space).

Considering the wedge as a set of two coupled waveguides for surface waves, Krylov and Mozhaev (1985) presented an approximate solution for the problem of Rayleigh wave scattering. As opposed to the most wedge solutions of that time, which are valid for wedge angles about $\theta=180^\circ$, their method is best applicable to sharp angles ($\theta<30^\circ$). The Rayleigh source is located at sufficiently large distance from the edge of the wedge where the local thickness is larger than Rayleigh wavelength (far-field). They represented the displacement field of incident Rayleigh wave by a symmetric and an antisymmetric mode of the waveguide system. For small wedge angles, these modes can be approximated by the first symmetric (longitudinal) and antisymmetric (flexural) Lamb modes in a plate whose thickness is equal to the local depth of the wedge. Then, they used the first order approximation of Wentzel-Kramers-Brillouin (WKB) method to express the displacement components in exponential forms. At far-field, symmetric and antisymmetric wavenumbers are equal to the Rayleigh wavenumber and two modes are in phase. Near the
edge, however, different symmetric and antisymmetric wavenumbers result in a phase difference between two modes. They ascribed the oscillatory behavior of Rayleigh coefficients to such $\theta$–dependent phase difference between the longitudinal and the flexural plate modes. The explanation is based on the assumption that the conversion of surface wave to body waves at the edge is negligible. It could be justified by total reflection and transmission of Rayleigh wave at small wedge angles (Rayleigh coefficients are equal to unity). Considering the phase shift, they modified the symmetric and antisymmetric displacements and therefrom obtained expressions for reflected and transmitted Rayleigh waves at far-field. For larger wedge angles where the phase shifts deviate from the values of plate theory, they assumed a linear variation between $\theta = 0$ (plate) and $\theta = 180^\circ$ (half-space). They calculated the transmission and reflection coefficients for the range of wedge angles $\theta = 15^\circ$-180$^\circ$ and Poisson’s ratio $\nu = 0.35$. The results show that maxima in the transmission coefficient correspond to minima in the reflection coefficient and vice versa. In addition, they found that for wedge angles less than 30$^\circ$, the coefficients oscillate at cycles about 3$^\circ$. For larger wedge angles where the normalized energy of Rayleigh wave is less than unity (conversion to body waves), the results are still in good qualitative agreement with experimental results of Victorov (1958). For oblique incidence of Rayleigh wave upon a wedge of fixed angle (general 3D case), they observed that transmission and reflection coefficients oscillate with the incident angle.

Miklowitz (1982a, b) used a double Laplace transform technique to analyze the scattering of transient elastic waves by a right angle wedge. The wedge is subjected to normal stress of step type and zero shear stress along one of its faces while the other is traction free. He applied double Laplace transform with respect to time and space to the equation of motion. It gives rise to a system of second order ordinary differential equations. The solution i.e. double transformed displacement field involves unknown functions along the loading face. He simplified the solution through applying the radiation condition to branch functions. Applying the boundary conditions to both homogeneous and particular
solutions of the system, he found the final form of double transformed displacement components. The traction free boundary condition along the free face of the wedge results in the Rayleigh function with a set of real roots. The poles corresponding to unbounded and inadmissible (Rayleigh wave speed greater than $P$) solutions are non-physical. Therefore, the residue (contribution) of these poles should be excluded from the solution. Setting residue equal to zero gives rise to a set of coupled integral equations in terms of loading face unknowns (time-transformed displacement components and their gradients). The general form of time-transformed $P$ and Rayleigh displacements are known from waveguide and half-space problems, respectively. Substituting these forms, he reduced the integral equations into a set of algebraic equations for unknown coefficients. He replaced the boundary functions back into the solution and found the time transformed displacement field in the form of inverse integral. Applying the Schwarz principle of reflection in the complex analysis, he reduced them to a set of real equations. He then obtained the inverse Laplace transform of displacements by Cagniard-de Hoop method. In the second part of his study, he derived the wavefront for all scattered field components i.e. $P$, $S$, $PS$ and Rayleigh waves both inside the wedge and along its faces.

Li et al. (1992) studied the scattering of plane Rayleigh wave obliquely incident upon a quarter space using both theoretical and experimental methods. In the theoretical part of their work, they presented a new semi-analytical technique for calculating the transmitted and reflected Rayleigh fields. He expressed the incident Rayleigh wave as a linear combination of plane $P$ and $S$ waves propagating toward the edge of the wedge under complex direction. For a quarter space elongated along its edge subjected to an incident plane wave, the resulting wavefield is translationally invariant with respect to the edge coordinate (out-of-plane). They obtained displacement and stress Green’s functions for a spatially harmonic line load by integrating the corresponding functions due to a point load. Using free-space Green’s functions and considering translationally invariance of the problem, they reduced the problem to a set of singular boundary integral equations along
in-plane coordinates. The usual procedure has been used to construct the representation theorem for scattered displacement field. Taking the limit of consequent integrals over the boundary and applying the zero traction boundary conditions, they obtained boundary integral equations for scattered field. Far from the edge, where the scattered body waves diminish due to geometrical attenuation, the total scattered wavefield can be solely expressed as a sum of reflected and transmitted Rayleigh waves. They truncated the limits of boundary integrals to those corresponding to vanishing body waves. They finally used the boundary element method to solve the consequent system of proper integral equations for transmitted and reflected displacements. For different values of Poisson’s ratio, they calculated the reflection and transmission coefficients of normal incident Rayleigh wave (2D case). Also presented the variation of Rayleigh wave coefficients as a function of incidence angle (3D case).

Croisile and Lebeau (1999) used the spectral function method to study the diffraction of plane sound waves ($P$) by an immersed elastic wedge. They represented the displacement field as a sum of two single layer potentials corresponding to two faces. The Fourier transforms of the potentials densities are called spectral functions. Applying boundary conditions, they reduced the problem to a system of singular integral equations in terms of spectral functions. They derived a recursive equation to obtain the unknown spectral functions. The existence and uniqueness of the “outgoing” solution has been proved for non-grazing incoming waves (incident or reflected). By outgoing, they meant a solution that satisfies a form of limiting absorption principle at infinity and the finiteness of energy near the vertex. They excluded the grazing wave because it cannot be represented by spectral functions. Kamotski and Lebeau (2006) used the same technique to prove the solvability of diffraction problem in the traction free elastic wedge. Although they removed the upper limit of applicable wedge angle ($\pi$) from the original method, the constraint on the incidence angle still existed. They expressed both the internal (bulk wave) and surface (Rayleigh wave) radiation conditions in an integral form. These physically consists forms
have been used to formulate and prove of a general uniqueness theorem. Kamotski (2004) showed that the theorem is also valid for the critical incidence and consequent grazing plane wave propagating toward the wedge tip. He split the problem over the regions far from and near the wedge tip. He used the same technique of spectral functions to solve the far field problem. Near the wedge tip, on the other hand, he nullified the reflected wavefield and presented the solution in terms of the Green tensor of elastic wedge. The case of grazing incidence along the wedge face was still excluded from the solution.

2.3.2. Experimental Studies

Mathematical difficulties involved in the problem of Rayleigh wave scattering by a wedge results from converted $P$ and $S$ waves. Complex coupling between different wave types has rendered the general analytical solution arduous if not impossible and motivated many researchers to solve the problem experimentally. Experimental techniques usually apply a normal excitation on a free surface of the model to generate different wave types including Rayleigh wave. The $P$ wave grazing along the boundary continuously generates a shear wave that connects $P$ and $S$ wave heads (Fig. 2.3). This head wave (Von Schmidt wave) along with transmitted and reflected Rayleigh and converted $P$ and $S$ body waves form a complete scattered wavefield. The Huygens’ principle has been used to explain how these components of wavefield are generated (Knopoff and Gangi, 1960; Pilant et al., 1964). The incident Rayleigh wave would trigger the apex as a new source of radiation. The latter would then excite the edges of the wedge and the medium by surface and body waves, respectively. In order to interpret the elastic wave propagation by Huygens’ principle where there are two wave velocities, two different types of secondary sources (time retardation in the integral solution) should be recognized. Secondary sources in continuous medium will emit the same wave type while those on the surface of discontinuity may generate both types. Therefore, surface of discontinuity is responsible for mode conversion.
There are two general groups of experimental studies based on recording quantity. The point recording method puts limited number of point receivers on the boundary of model to record time history of generated waves while in the plane recording technique the complete wavefield is captured at each time step. Table 2.3 shows a summary of experimental studies on Rayleigh wave scattering by wedge. It also contains the complex transmission and reflection coefficients for right angle wedge.

**Table 2.3. Summary of experimental studies on elastic wedge**

<table>
<thead>
<tr>
<th>Reference</th>
<th>Material Type</th>
<th>( \nu )</th>
<th>( \theta ) [°]</th>
<th>Step</th>
<th>Coefficients (90° Wedge)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Trans.</td>
</tr>
<tr>
<td>de Bremaecker (1958)</td>
<td>Polystyrene</td>
<td>0.17</td>
<td>0-150</td>
<td>10</td>
<td>0.40</td>
</tr>
<tr>
<td>Victorov (1958)</td>
<td>Duralumin</td>
<td>0.35(^a)</td>
<td>15-165</td>
<td>5</td>
<td>0.49</td>
</tr>
<tr>
<td>Knopoff and Gangi (1960)</td>
<td>Aluminum</td>
<td>0.266</td>
<td>45-135</td>
<td>5</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230-300</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Pilant et al. (1964)</td>
<td>Aluminum</td>
<td>0.25</td>
<td>30-170</td>
<td>10</td>
<td>0.45</td>
</tr>
<tr>
<td>Bond (1979)</td>
<td>Aluminum</td>
<td>0.34</td>
<td>90</td>
<td>–</td>
<td>0.36</td>
</tr>
<tr>
<td>Kinra and Vu (1983)</td>
<td>Aluminum</td>
<td>0.34</td>
<td>90</td>
<td>–</td>
<td>0.36</td>
</tr>
<tr>
<td>Fujii et al. (1984)</td>
<td>Acrylite</td>
<td>0.234</td>
<td>39-165</td>
<td>1</td>
<td>0.46</td>
</tr>
<tr>
<td>Li et al. (1992)</td>
<td>Aluminum</td>
<td>0.33</td>
<td>90</td>
<td>–</td>
<td>0.45</td>
</tr>
<tr>
<td>Lewis and Dally (1970)</td>
<td>Epoxy</td>
<td>0.30(^a)</td>
<td>40-180</td>
<td>10</td>
<td>0.51(^b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.66(^c)</td>
</tr>
<tr>
<td>Henzi and Dally (1971)</td>
<td>Polymer</td>
<td>0.33</td>
<td>90</td>
<td>–</td>
<td>0.44</td>
</tr>
</tbody>
</table>

\(^a\) Typical value (Not mentioned in the text)
\(^b\) Based on stress
\(^c\) Based on displacement
2.3.2.1. Point Recording Method

Most of the physical models used in the point recording method are based on the 2D experimental model of Oliver et al. (1954). Trying to eliminate difficulties and limitations associated with 3D models, they utilized a thin plate (in-plane wave propagation) for construction of desired configurations. When the wavelength of input motion is large enough compared to the thickness of wedge, wave motion in thin plate can be considered as 2D. de Bremaecker (1958) and Victorov (1958) attempted the first experimental study on the problem of Rayleigh wave propagation in a wedge of arbitrary internal angle. The former study was performed on a thin plate 2D seismic model while the latter used a rectangular metal bar of unknown thickness. Knopoff and Gangi (1960), Pilant et al. (1964), and Fujii et al. (1984) used the same 2D seismic model of single wedge geometry. Kinra and Vu (1983) built a plane strain 2D model using a thick metal plate (about 10 times larger than the wavelength of input motion). Such difference in the model types and therefore in the wave propagation patterns must be taken into account when comparing the results. Material types and Poisson’s ratios used in different studies are listed in Table 2.3. de Bremaecker (1958) used the same test arrangement to measure the attenuation of the medium. Fujii et al. (1964) canceled the effect of attenuation by arranging receivers on the edges of model such that the incident, reflected, and transmitted waves have the same propagation path length. In other works, this effect was assumed negligible compared to the accuracy of the test. The range of studied wedge angle together with its resolution is also presented in Table 2.3. In addition to this range, Victorov (1958) evaluated the effects of a slit as an extreme case of wedge angle i.e. \( \theta = 360^\circ \) on the propagation characteristics of Rayleigh wave. For this case, he used dimensionless depth of the cut (normalized by wavelength) as an independent variable. The relative arrangement of source and receiver on the model’s geometry determines the type of recorded quantity i.e. reflected, transmitted and converted wave amplitude. Figure 2.4 illustrates two different
test configurations which have been used in point recording studies to evaluate transmission and reflection or conversion.

\textit{a. Transmission and Reflection Test}

General layout of the transmission and reflection test consists of a source and two receivers. The source, placed on the flat boundary, generates Rayleigh wave along with other wave types and receivers, each on the two opposite sides of the wedge, record the transmitted and reflected waves (Figure 2.5a). Both the amplitude and the phase of incident wave are prone to change as it passes over or reflects from the apex. Normalizing by input motion, change in the amplitude is expressed in terms of dimensionless transmission and reflection coefficients (Victorov, 1958; Pilant et al., 1964; Kinra and Vu, 1983; Bond, 1979; Fujii et al., 1984). The energy percentage of surface wave transmitted or reflected over the corner is proportional to the square of these coefficients. In order to check the validity of measurements, the principle of conservation of energy could be applied between initial and final wavefield. Therefore, and in accordance with de Bremaecker (1958) and Knopoff and Gangi (1960), the energy definition of transmission and reflection coefficients is presented here. Among all wedge angles, the measured coefficients for 90°, as a common geometry between different studies, are compared in Table 2.3.

Taking Fourier transform of recorded time histories, Pilant et al. (1964) expressed transmission and reflection coefficients as complex numbers, whose modulus and argument are equal to amplitude ratio and phase shift, respectively. However, these coefficients must be independent of wavelength or frequency inasmuch as the infinite wedge geometry has no characteristic length. They defined frequency independent coefficients as the average amplitude over the frequency range and the \(y\)-intercept of phase shift spectrum. Kinra and Vu (1983) and Fujii et al. (1984) used the same method to obtain frequency independent coefficients.
Figure 2.5. Schematic illustration of wedge model and arrangement of transducers: (a) transmission (T) / reflection (R) test, and (b) conversion test
Results of different studies show that the Rayleigh wave turns around the corner on the range 130°-180° where the transmission coefficient increases monotonically with almost no reflection. As the geometry of wedge model approaches the half-space, all the incident energy transmitted through the corner. For smaller wedge angles, however, the transmission and reflection coefficients have clear peaks and troughs. Except for wedge angle about 110°, where they have the same value, maximum transmission occurs at minimum reflection and vice versa. In addition, while the first order approximation of reflection results in symmetric amplitude about 90°, experimental data are non-symmetric and have a minimum at 80°. For slit model, Victorov (1958) found an oscillatory trend of reflection coefficient whose average at larger depth approaches the corresponding value of right angle wedge. With regard to the transmission coefficient, for slit cut deeper than 1.5λ (influence depth of Rayleigh wave), it takes a constant value. In this case, the Rayleigh wave simply turns around the cut. To study the effect of material type, Victorov repeated the wedge experiment with a steel bar. He obtained essentially the same results with slightly change in the position of extrema and therefore postulated the same transmission and reflection characteristics for other elastic wedges.

de Bremaecker (1958) also studied the effect of wedge corner on the waveform change and found the 90° phase shift of transmitted wave upon the incidence on right angle wedge. Knopoff and Gangi (1960) presented normal component of wave motion along the edges for some wedge angles to show such effect. Comparing with the amplitude spectrum of input motion, Pilant et al. (1964) showed how the incident pulse degrades when scattered on a wedge. They found 75° and 20° phase shifts respectively on transmission and reflection at the right angle wedge. The nonzero phase shift on reflection is in accordance with the observation of Knopoff and Gangi (1960).

To interpret the complex variation of amplitude coefficients and phase with wedge angle, Pilant et al. (1964) asserted that the transmitted and reflected waves – just like incident Rayleigh wave – is coming from pole singularities (frequency independent).
Wedge angle accounts for such complicated behavior of residues at these poles. Knopoff and Gangi (1960) tried to interpret the experimental data through the response of two extreme cases of wedge angle i.e. slit and half-space to a simple input motion. For incident delta function upon a slit cut, they obtained scattered wave of step function form i.e. the edge of the cut acts as a 2D line source. For half-space subjected to Rayleigh pulse of delta function form, the only nonzero part i.e. transmitted wave is of the same form. Therefore, the resulting Rayleigh wave was assumed as a sum of two terms: the normal transmission and reflection without waveform change and a wave resulting from a line source at the apex. In other words, they presented the transmitted and reflected Rayleigh waves as a sum of unknown dimensionless coefficients of delta and step functions. They approximated the wavelet of input motion by a set of Gaussian functions and used the convolution theorem to express transmitted and reflected wave in terms of Gaussian function and convolution integral. Then, a procedure was presented for the calculation of coefficients from experimental data. They claimed that the calculated coefficients are valid for any shape of input motion if the pulse duration is short compared to the travel time.

In the experimental part of their study, Li et al. (1992) put forward a self-calibrated technique to investigate the transmission and reflection characteristics of a quarter space. Performing the test on thick block instead of thin plate enabled them to study the effect of off-plane incident angle. However, it was impractical to reproduce the same coupling between the transducers and the wedge faces for different angles. Such variable coupling brings about inconsistent incident waves and response functions. They used a new configuration consists of four transducers i.e. a set of transmitter and receiver on each face to remove these effects. For every incident angle, they triggered transmitters consecutively and found two different expressions for each coefficient. The response functions of transducers and propagation paths were then canceled out in the coefficients’ ratio. Therefore, such self-calibrated recording technique works only for measuring the ratio of transmission and reflection coefficients, rather than their magnitudes itself. It should be
noted that the values presented in Table 3 are based on the comparison of their theoretical and experimental results.

b. Conversion Test

Wedge models used in the conversion test have a circular boundary to send incident or receive converted body waves. Normal excitation on the wedge face was recorded radially on the round edge and vice versa (Figure 2.5b). In addition to transmission and reflection, de Bremaecker (1958) and Victorov (1958) studied the conversion of Rayleigh wave to body waves at the wedge tip. Such energy transformation exists because the set of incident, transmitted and reflected Rayleigh waves cannot satisfy the stress-free condition on the edges of the wedge. They presented the percentage of total surface energy converted into longitudinal and transverse waves as a function of wedge angle. de Bremaecker (1958) showed that this energy portion is reasonably constant (~50%) for wedge angles 10°-100°. He also recorded both $P$ and $S$ waves on the perimeter of a quarter-circle model excited by Rayleigh wave on its plane edge. Distribution of Rayleigh to $P$ wave ($RP$) and Rayleigh to $S$ wave ($RS$) conversion coefficients were then presented along the circular edge. For right angle corner, the total converted energy is split almost equally between compressional and shear modes.

Gangi (1967) and Gangi and Wesson (1978) utilized the conventional 2D seismic model to study energy conversion at the wedge corner. They formed several concave wedge models ($\theta > 180^\circ$) by cutting a circular aluminum plate. Owing to such geometry, converted wave at the wedge tip has the same travel time for different positions of source/receiver along the circular edge. Placing both the transmitter and the receiver on the circular edge, Gangi (1967) calibrated the model and obtained the measure of amplitude errors (~10%). He constructed travel time curves for all arrivals in the model using measured velocities of direct $P$, $S$ and Rayleigh waves. The measured amplitudes of direct $P$ and $S$ waves as well as reflected/diffracted $P$ and converted/diffracted $S$ waves are then
compared with calculated values. Reflection and conversion of incident body waves occur along the wedge boundaries while the apex is responsible for diffraction. Then, he found $RP$ conversion coefficients by moving the source to the flat edge and normalizing the measured amplitudes to the incident motion at the corner. According to the principle of seismic reciprocity (Knopoff and Gangi, 1959), these coefficients are essentially the same as $P$ wave to Rayleigh wave ($PR$) conversion coefficients. To have a direct measurement of $PR$ coefficients, Gangi and Wesson (1978) switched the position of transducers on their model. Using the travel time curves, they found a range of incident angle at which the target event i.e. $PR$ is well separated from other wave types. Both studies presented the variation of $RP/PR$ conversion coefficient with incidence angle for different wedge models. The curves show two broad peaks near the normal incidence on the wedge faces and a trough for parallel incidence. Placing the source at the wedge base (grazing incidence) results in local minimums. In accordance with zero conversion of half space, lower wedge angles take smaller coefficients. Symmetry about zero incidence angle for larger wedge angles is another characteristic of conversion curves.

2.3.2.2. Plane Recording Method

The second group of experimental studies on wedge problem is based on dynamic photoelasticity. This technique provides visual information representing the complete wavefield i.e. incident, transmitted, reflected and scattered over the entire domain. Dynamic photoelastic measurements are in the form of light-field fringe patterns for each time step. Although a single test gives a wealth of data, difficulties associated with data reduction process and appropriate material procuring have limited its application. Lewis and Dally (1971) utilized this method to find the transmission and reflection coefficients of wedges. Results show the same general characteristic as previous studies i.e. a rapid and complex change of coefficients with wedge angle. For wedge angles above 130° the incident Rayleigh wave passes the corner without considerable change. That is the
magnitude of reflected wave from such near half-space configuration is too low so that it cannot be detected in fringes. For smaller wedge angles, however, more portion of incident surface energy reflects back the corner and converts to body waves. In addition, decreasing the wedge angle increases the complexity of the problem due to the interaction between two close edges. For example, the reflecting shear wave generated by incident P wave upon the free edge has more complicated interference with incoming head wave (PS) for smaller wedge angle (Figure 2.3). As it is shown for stress based coefficients of 90° wedge in Table 2.3, some of their results violate the energy conservation and therefore could not be reliable.

Henzi and Dally (1971) studied the wave propagation problem in a right angle wedge using the same technique. Before the incident P wave arrives at the apex, wedge creates the same wavefield as the half-space i.e. P, PS (Von Schmidt), S and Rayleigh waves (Fig. 2.3). They considered the elastodynamic response of wedge as the reaction of its corner and unloaded edge to this wavefield. As for the interaction of Rayleigh wave with apex, they defined transmission and reflection coefficients based on calculated surface energy. The energy portion of compressional reflection and shear conversion of incident P wave were evaluated as well. They also referred to the attenuation of Rayleigh wave with depth by presenting subsurface stresses distribution for incident, reflection and transmission. Maximum stress of incident surface wave diminishes by 1/3 factor along the transition layer of depth \(0.3\lambda_R\). Such transition layer, characterized by complex variation of maximum stresses, also exists for transmitted wave. Energy associated with transmitted wave at deep layers is conspicuously greater than incident wave. Attenuation of reflected Rayleigh wave with depth is fast and continuous with no transition later.
2.3.3. Numerical Methods

Although various numerical techniques have been presented for the scattering of elastic wave by wedges, the major part of them are based on finite difference (FD) scheme. The basic idea of the FD method is to discretize the domain (space and time) into a set of grid points and then to approximate the continuous governing equation over them. In its explicit form, it will result in an equation that gives the displacement of specific node at next time level based on its displacements at current and previous time levels as well as displacement of neighboring points at the current step. Traction free boundary conditions give the displacement components along the fictitious lines of grid points (pseudo nodes). Having them, the explicit difference scheme could be used for all points inside the medium and along the boundary. Ilan et al. (1975) presented a FD scheme of higher order for boundary nodes. They replaced the normal derivatives in the equation of motion, which can be approximated only by one-sided difference scheme, with tangential derivatives using boundary conditions. The standard central difference formula is then used to approximate the resulting equations. Since the basic idea is the same among various works, only a selected number of pioneering studies is presented in this section. To compare the order of accuracy with other theoretical and experimental techniques, results of Rayleigh coefficient are summarized in Table 2.4.

Following the work of Alterman and Karal (1968), Alterman and Rotenberg (1969) presented an explicit FD scheme to investigate the scattering of cylindrical $P$ wave in a right angle wedge. To model an infinite wedge using a finite numerical model, they placed virtual truncating boundaries far from the observation point such that the spurious reflections do not distort the scattered wavefield. The impulsive point source is placed along the diagonal of the wedge whose faces are both traction free. Therefore, the resulting wavefield is symmetric with respect to the bisectrix and computational cost is reduced by half. The incident wave function used in their computations has singularity in the neighborhood of the source. Therefore, the FD formulation cannot be directly used in this
Table 2.4. Numerical results of Rayleigh wave coefficients for right angle wedge

<table>
<thead>
<tr>
<th>Reference</th>
<th>ν</th>
<th>Reflection</th>
<th></th>
<th>Transmission</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(Amplitude</td>
<td>Phase [°]</td>
<td>(Amplitude</td>
<td>Phase [°]</td>
</tr>
<tr>
<td>Alsop and Goodman (1972)</td>
<td>0.25</td>
<td>–</td>
<td>–</td>
<td>0.42</td>
<td>–84</td>
</tr>
<tr>
<td>Munasinghe and Farnell (1972)</td>
<td>0.245</td>
<td>0.13</td>
<td>25</td>
<td>0.42</td>
<td>–90</td>
</tr>
<tr>
<td>Munasinghe and Farnell (1973)</td>
<td>0.245</td>
<td>0.13</td>
<td>38</td>
<td>0.41</td>
<td>–79</td>
</tr>
<tr>
<td>Cuozzo et al. (1977)</td>
<td></td>
<td>0.17</td>
<td>0.07</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.07</td>
<td>0.52</td>
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<tr>
<td></td>
<td></td>
<td>0.34</td>
<td>0.18</td>
<td>0.52</td>
<td></td>
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<tr>
<td></td>
<td>0.24b</td>
<td>0.15</td>
<td></td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.24b</td>
<td>0.18</td>
<td></td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Bond (1979)</td>
<td></td>
<td>0.29</td>
<td>0.31</td>
<td>0.22</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>0.34a</td>
<td></td>
<td>0.24</td>
<td>0.41</td>
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<tr>
<td></td>
<td>0.34b</td>
<td>0.22</td>
<td></td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Blake and Bond (1990)</td>
<td></td>
<td>0.29</td>
<td>0.12</td>
<td>0.46</td>
<td>–</td>
</tr>
<tr>
<td>This Study</td>
<td></td>
<td>0.25</td>
<td>0.10</td>
<td>0.47</td>
<td>–85</td>
</tr>
</tbody>
</table>

*a Pseudonode Model

*b Second order Scheme

Nevertheless, the scattered wave obtained by subtracting the incident part from the total wavefield is no longer singular. They calculated the scattered (bounded) and the total displacement fields respectively inside and outside the singular region using FD scheme. Knowing the incident field everywhere in the wedge, the corresponding total and scattered displacements could be obtained for interior and exterior regions. These values provide boundary conditions for the FD formulation of the opposite region. That is, the computed scattered displacement in the exterior region is served as boundary value for the FD calculation of scattered field in the singular interior region. They approximated the second order derivatives (time and space) of governing differential equations of motion by central difference scheme. Rearranging the results, a set of recursive equations have been obtained for displacement components. To express the boundary conditions in terms of difference
equations, they added fictitious lines of grid points beyond the actual free surfaces. Then, the normal and tangential derivatives of boundary conditions have been approximated using respectively backward and central differences. To satisfy the boundary condition at the vertex, they derived an additional equation through approximating the sharp tip by a circular curve. The results have been presented in terms of synthetic seismograms of horizontal and vertical displacements depicting different parts of the total wavefield. It consists of direct, reflected and diffracted body waves as well as Rayleigh surface waves. They showed the reflected waves up to the order two beyond that the amplitude becomes very small. Both mode preserving and mode changing reflections have been considered in their analysis. The diffracted $P$ and $S$ waves generated at the wedge tip cannot be easily identified. To distinguish different wave components in the synthetic seismograms, they used a relatively sharp impulse as an incident wave. This will result in diffracted waves of small amplitudes (Hudson, 1963). The other factor that makes the recognition of diffracted wave difficult is the simultaneous arrival with some reflected waves. For stations near the surface, they identified two types of Rayleigh wave at the end of synthetic seismograms. The Rayleigh wave resulted from the tip diffraction arrives slightly earlier. The next one, which is generated at the wedge boundary, corresponds to the incidence beyond the critical angle. It gives rise to transmitted and reflected Rayleigh waves at the wedge corner. They also presented the particle motion for points near the boundary to show the change in polarization. The motion at the wedge corner shows a single peak resulted from the constructive interference of direct $P$ wave and surface waves generated near the tip right after the direct pulse. Comparing the displacement amplitude at the vertex with that of free-field motion, they found the amplification factor of three for $\nu=0.35$. For stations away from the vertex, the amplification factor suddenly decreases to two. They presented an explanation for the sharp change in amplification factor based on the superposition of direct and reflected waves.
Alterman and Loewenthal (1970) used the same technique as Alterman and Rotenberg (1969) to study the scattering of cylindrical $P$ wave in traction free wedges. As an extension to the previous work, the location of impulsive point source was no longer restricted to the diagonal. In addition, they obtained the wavefield in three-quarter space and analyzed the effect of elastic properties on the tip amplification. Adding a small perturbation to the solution of wave equation, they obtained the stability condition in terms of maximum time step to grid spacing ratio. The numerical accuracy has been checked by comparing the results for several grid sizes. For impulsive source located along the bisectrix (symmetric case), they presented the synthetic seismograms at the wedge corner for different shear to compressional velocity ratios. The results show that larger Poisson’s ratio results in smaller/larger amplitude at the quarter/three quarter vertex. Comparing with the free-field motion, they obtained amplification factors of 3 and 1.5 respectively for quarter and three-quarter spaces at Poisson’s ratio $\nu = 0.35$. They showed that the total wavefield of the quarter space could not be obtained by the superposition of two perpendicular half-space. The separation of results starts at the arrival time of diffracted and double reflected waves. Unlike the quarter space, there is no multiple reflection in three-quarter space inasmuch as the diffracted waves could be identified more easily. Decomposing the diffracted wavefield of three-quarter space into its components along polar coordinates, they found that $P$ and $S$ waves are mainly changing in the radial and tangential directions, respectively. They compared the displacement components of surface waves in quarter/three-quarter spaces with that of half-space. The results show that the normal/tangential components are the same for the case of quarter/three-quarter space while the opposite components are considerably different. In addition, they found that surface waves have largest amplitude in quarter space and smallest in three-quarter with intermediate values for half-space. Finally, they presented the synthetic seismograms for several off diagonal sources and showed that the amplitude at the wedge tip is independent of the source location.
Alterman and Loewenthal (1971) modified the FD scheme of Alterman and Rotenberg (1969) to obtain the wavefield generated by a surface impulse in a right angle wedge. As opposed to the previous numerical scheme, which converges slowly for sources near the free surface, the modified technique has fast convergence even for a surface source of excitation. They obtained synthetic seismograms of displacement components at the wedge tip. Different parts of the wavefield i.e. $P$, $S$ and Rayleigh waves have been recognized in these seismograms. The particle motion at the vertex shows an elliptical polarization whose major axis makes an angle of 45° with respect to wedge faces. They identified two different types of Rayleigh waves in seismograms and particle motions. The first one, which is generated by the point source, propagates toward the wedge corner where it is subsequently reflected/transmitted. The other surface wave is resulted from the diffraction of $P$ and $S$ waves at the wedge tip. On the loading face, the horizontal component of the Rayleigh wave has the same time dependency as the applied force. The Hilbert transform of this function define the form of vertical components. Passing the wedge corner, the form of Rayleigh components will be reversed that shows a 90° phase shift. The phase shift is more obvious for observation points far from the vertex. They also obtained the results for an impulsive tangential source and compared them with the analogous problem of shear crack in half-space.

Alsop and Goodman (1972) presented a hybrid finite element – finite difference technique to solve the problems involving Neumann boundary conditions on irregular geometries. This is feasible because with linear approximation functions on triangular finite elements whose nodes are grids of a rectangular FD mesh, both techniques result in the same difference formulas. In fact, they added the flexibility of finite element method in modeling complex boundary conditions to the simplicity of FD formulation. They started with a rectangular grid and transformed it into triangular elements by splitting along the diagonals. Then, they applied the usual finite element approach with piecewise linear approximation functions (tent functions). For two different problems of anti-plane and
plane strain, they obtained stiffness matrix using strain and stress multiplier matrices. Extracting the contribution of central node of tent function from the stiffness matrix, they derived central difference formulas for each case. Along the boundary, where the tent function is crossed, only the interior segments contribute to the stiffness matrix. For dynamic problems, they used the principle of conservation of energy to derive an explicit central difference approximation for time derivative. As an application example, they considered the scattering of Rayleigh wave by a right angle wedge. The problem involves the Newman boundary condition (traction free) on the irregular geometry. They used the formulation of plane strain problem to obtain the difference equations for acceleration components along the boundaries and at the vertex. They presented the displacement components of propagating Rayleigh waves at several time steps. They also calculated the amplitude and phase of transmission coefficient and compared with available experimental results.

Sato (1972) investigated the wavefield generated by a surface normal stress in a right angle wedge using the FD method. He used the central difference scheme to approximate both time and space derivatives in the equation of motion. Stress free boundary conditions have also been expressed in difference form. He presented a new technique to satisfy the boundary conditions at the wedge tip. Instead of applying all four conditions (two components for each side) for the point of intersection, he only used zero normal stress along each free surface. As for the excitation, he considered a bounded pulse with half sine time dependency and cosine spatial distribution. The results have been presented in the form of displacement vector field within the wedge at successive times of propagation. He designated different wave components in the total wavefield consist of direct $P$, $S$ and Rayleigh waves, head wave, transmitted and reflected Rayleigh waves. These wave parts have been also identified in the wave profiles along both wedge faces. He obtained the travel time curve for each pulse and compared with their theoretical velocities. The particle motion of direct and reflected/transmitted Rayleigh waves show a
distortion in the incident wave passing the wedge corner. He further verified the numerical results with field properties whose theoretical values are known. They consist of $P$, $S$, and Rayleigh wave velocity and the orbital shape of Rayleigh wave.

Munasinghe and Farnell (1973) presented an explicit FD scheme to solve the problem of Rayleigh wave scattering at discontinuous boundaries. Two different geometries of right angle wedge and step change has been considered in the study. They approximated the time and space derivatives in the equation of motion by the central difference formula. The stability condition same as Alterman and Loewenthal (1970) has been used to obtain a maximum allowable ratio of time step to grid spacing. Similar to other numerical techniques for the wave simulation in unbounded media, they reduced the actual semi-infinite medium to a truncated (finite) model. The length of wedge sides was assumed sufficiently large inasmuch as reflected and transmitted Rayleigh waves reach their complete (steady state) form. At depth $3\lambda_0$, where $\lambda_0$ is the dominant wavelength of the incident wave, the amplitude of Rayleigh wave is near zero. Therefore, they put the bottom boundary below this depth and set the displacements equal to zero. The zero displacements have also been applied to vertical boundaries. To avoid spurious reflection from fixed ends, they put the truncating boundaries far from the corner (source of scattering) and iterated the model up to the arrival time of these unwanted waves. Regarding the actual boundaries with traction free condition, they used fictitious lines of grids (pseudonodes) outside them. They presented a procedure for calculating the displacements of pseudonodes satisfying boundary conditions. In contrast to previous studies, they approximated both normal and tangential derivatives of boundary conditions by the central difference formula. The only exception was the wedge corner where they used the method of Alterman and Loewenthal (1970) with slight modification. The procedure takes displacement of interior grids and gives the exterior displacements at the same time level. Then, the explicit scheme could be used to compute the displacement field at the next time step. They used a narrow band Ricker wavelet as an initial pulse to see
whether the results are frequency dependent or not. The numerical scheme has been applied to the canonical problem of Rayleigh wave propagation in half-space to verify the model (it is expected to propagate unchanged along the boundary). They found an applicable frequency range about central frequency of incident wave for which the error of spectral amplitude is negligible. For right angle wedge, they computed the complex reflection and transmission coefficients as a function of frequency. They demonstrated that these coefficients are constant over the applicable range of frequency as expected. In addition, they showed that transmitted wave reaches its final form before the reflected part. The results of amplitude and phase for transmitted and reflected waves along with the energy portion converted to body waves have been compared with available experimental data. For the step discontinuity, they obtained the variation of transmission and reflection coefficients with normalized height (divided by the wavelength).

Munasinghe and Farnell (1972) used the FD method to study the scattering of Rayleigh wave by irregular geometries. They examined the problem for three different types of scatterer namely quarter space, three-quarter space and step discontinuity. The incident Rayleigh wave is generated by a Ricker normal pulse applied on the free surface. They used the same technique as Munasinghe and Farnell (1973) to satisfy traction free boundary conditions along the wedge faces and at the corner. The numerical accuracy of the scheme has been checked for the Rayleigh wave propagating on a half-space. They presented the amplitude and phase of Rayleigh reflection and transmission coefficients for both wedges and compared them with existing theoretical and experimental data. In addition, the portion of incident energy diffracted to body waves has been calculated. They also obtained the relative distribution of converted energy along the azimuth direction for three-quarter space. For the step change geometry, which possesses a characteristic length, Rayleigh coefficients have been presented as a function of normalized height. At large normalized heights, the transmission coefficient approaches the product of corresponding values for quarter and three-quarter spaces.
Alterman and Nathaniel (1975) extended the FD technique of Alterman and Rotenberg (1969), which has been presented for right angle wedge, to the case of arbitrary angle \((0 < \theta < 180^\circ)\). Similar to the original problem, an impulsive point source of \(P\) wave is located along the diagonal of traction free wedge. They expressed the equations of motion in a distorted coordinate system whose axes extended along the wedge faces. Then, they approximated the consequent differential equations by central difference formula over a bounded parallelogram. Again, due to spurious reflections from truncated boundaries, the solution is valid (not polluted) up to the arrival time of these noises. They demonstrated that the solution of FD scheme converges to the real solution of governing equation by proving the consistency and stability conditions. The consistency condition requires the discrete scheme to be sufficiently close to the continuous operator while the stability shows that the solution is bounded. To verify the numerical accuracy in practice, they calculated the tip displacement using different mesh sizes for two different wedge angles. The results show that the solution converges as the grid spacing decreases. They presented the tip amplification factor for several Poisson’s ratio over the range of wedge angles \(60^\circ–180^\circ\). It shows that the larger the wedge angle and the Poisson’s ratio, the smaller the tip amplitude. They also presented synthetic seismograms for stations along the wedge bisectrix where displacement components are equal (symmetry). They calculated the theoretical arrival time of different wave components and designated them on the seismograms. To study the scattering of surface waves, they excited the wedge by a surface normal pulse. They modified difference equations according to the new boundary conditions along the loading face. The resulting surface waves have been divided into direct, reflected/transmitted and diffracted waves. They obtained particle motion at a point far from the corner for several wedge angles. It shows the elliptical polarization of Rayleigh waves with major axis perpendicular to the boundary. Therefore, they deduced the phase shift of transmitted Rayleigh wave as \(180^\circ-\theta\). At the corner, the major axis is perpendicular to the wedge bisectrix. Computing the tip amplitude for several locations of point source,
they concluded that the amplification factor does not depend on the source location (similar to the right angle wedge).

Cuozzo et al. (1977) used the FD method to study the scattering of Rayleigh wave in domains with irregular boundary. Two elementary geometries of quarter and three-quarter spaces (plane strain) have been considered to explain the formulation and the characteristics of resulting wavefield. They used the same technique as Alterman and Loewenthal (1970) and Munasinghe and Farnell (1973) to express the governing equations of motion and boundary conditions in terms of difference formula. The input motion in their analysis was a sinusoidal wave train rather than an impulse. Applying the stability condition with fixed grid spacing, they calculated the maximum allowable time step for several Poisson’s ratio. They showed that different wave types are generated upon the incidence of Rayleigh wave on the wedge corner consist of transmitted and reflected Rayleigh waves as well as converted body waves. Based on the exponential decay of Rayleigh wave amplitude with depth, they divided the quarter space into four regions each contains different wave types. The explicit difference scheme has been iterated until the scattered wave reaches the boundary of corresponding region. They calculated the reflection and transmission coefficients at surface stations far from the wedge tip, where converted body waves fade out and Rayleigh wave reaches its steady form. The numerical results have been verified with experimental data obtained from several studies. For a range of Poisson’s ratio, they presented the amplitude and phase of Rayleigh coefficients in both quarter and three-quarter wedges. The portion of incident surface energy converted to body waves has been presented as conversion coefficient. They observed that the transmission coefficient of right angle wedge is larger than reflection coefficient while the latter changes with Poisson’s ratio more rapidly. On the other hand, the Rayleigh coefficients of three-quarter space are both very small with slight change over the range of Poisson’s ratio. Therefore, the conversion coefficient of three-quarter wedge is much larger than right angle wedge. Then, they extended the elementary solutions to more complex discontinuities like
upward and downward step changes, ridge and trough. As a first order approximation, they replaced the step geometries by a sequence of wedges. The distance between two wedges (characteristic length) has been assumed large enough such that no secondary Rayleigh wave arises from converted body waves. They obtained periodic patterns for reflection and transmission coefficients over the range of characteristic length $0-\frac{\lambda}{2}$ where $\lambda$ is the incident wavelength. They also calculated the response of step change of small height directly from the numerical scheme to demonstrate that the approximate method is not valid for distances smaller than a wavelength.

In the theoretical part of his study, Bond (1979) used the numerical scheme of Alterman and Loewenthal (1971) to study the scattering of Rayleigh wave by surface irregularities. He satisfied the traction free boundary conditions using both the first (pseudonodes) and the second order (eliminating tangential derivatives) schemes. Snapshots of deformed grid showing the evolution of wavefield has been presented for several geometries (right angle wedge, step change and trough). He used them to designate various components of the scattered wavefield in each case. For several Poisson’s ratios representing different materials, he computed the reflection and transmission coefficients of Rayleigh wave in the quarter space model. The numerical results, including the converted energy portion, have been compared with available experimental and theoretical data.

Fuyuki and Nakano (1984) used the FD technique to calculate the transmission coefficient of Rayleigh wave incident on an upward step discontinuity. For large dimensionless heights, there is enough spatiotemporal distance between various components of transmitted wave such that they were able to distinguish them and to find dominant one. Among these components are the diffracted body waves emitted from two virtual sources i.e. tip and toe of the step change. The former is expected to be negligible when the small transmission coefficient of quarter space multiplied by the small conversion of quarter space (Cuozzo et al. (1977). For the latter, they calculated free-field (isolated)
displacement along the upper face of the step using the corresponding internal line in a three-quarter space. For small height of the step, they compared the results with those of Mal and Knopoff (1965) (based on representation theorem) to see the effect of missing toe diffraction. They found that for very small heights, superposing the toe diffraction to the analytical results is not sufficient and tip diffraction has to be considered as well. As for the finite numerical model, they used the absorbing boundary condition (Clayton and Engquist, 1977) along truncated boundaries instead of simplifying fixed condition.

Blake and Bond (1990) presented a hybrid finite element – finite difference technique to investigate the scattering of Rayleigh wave by 2D irregular geometries. The basic finite element formulation derived from the variational technique could be expressed as the matrix form of Newton’s second law in which the force term is a product of stiffness matrix and displacement vector. They presented the difference equations for interior nodes based on the rectangular finite elements with bilinear approximation functions. As for the surface nodes along the inclined boundary, a different set of equations have been obtained using linear triangular elements. The finite element method does not require additional treatment for the boundary conditions since they are inherent part of the formulation. Along the boundary and at the corner, only the elements inside the domain contributes to the mass and stiffness matrices. For comparison purpose, they also presented the explicit difference formulation of wave equation based on the pure FD method. While both schemes use the same neighboring points, the finite element equation involves more displacement components. They used central difference formula to approximate the time derivatives in either technique and presented the stability conditions for them. To check the accuracy of proposed numerical methods, the steady propagation of Rayleigh wave along the half-space has been simulated. They found that increasing the size of grids or elements results in quadratic growth of error. The results show that the hybrid scheme is much more accurate compared to conventional FD method. In addition, the stability analysis reveals that the new method is stable at larger time step. For each geometry, they presented the
displacement vector filed along with seismogram synthetics of surface displacements to explain various components of scattered wavefield. Several wedge models (obtuse and reentrant) have been used to describe basic wave elements i.e. transmitted and reflected Rayleigh waves, converted $P$ and $S$ waves and von Schmidt head wave and their characteristics. The head wave emerges from the reflection of grazing $P$ wave. Although there are similar scattered pulses in all wedge models, the energy partitioning among wave components is quite different. These elementary wave components could be used to explain the scattering of Rayleigh wave in more complex geometries. They also calculated the amplitude of reflection and transmission coefficient for these wedge models and compared them with other theoretical results. They obtained an amplification factor of $\sim 3$ at the corner of right angle wedge. Increasing the wedge angle reduces the tip amplitude insofar as amplification factors become less than unity in reentrant wedges. In addition, in reentrant wedges, most of the incident energy is converted to body waves and only a small portion is reflected back from the corner. Furthermore, the transmission coefficient approaches the unity as the reentrant wedge reduces to half-space. For step discontinuity, they categorized the problem into three different cases based on normalized height (divided by fundamental wavelength of incident pulse). For small normalized height or low frequency case, the scattered energy is small as deviation from the half-space is minor. For high frequency incidence, two corners of the step act separately. For step height comparable to incident wavelength, the interference of resulting scattered wavefield from two corner gives rise to oscillatory Rayleigh coefficients. They analyzed normal and inclined downward steps with normalized height of unity and discussed the characteristics of resulting wavefield. They designated various parts of the scattered wavefield generated by different frequency range of incident pulse. For these two geometries, the reflected and transmitted amplitudes are presented over a range of normalized height. In the second part of their study (Blake and Bond, 1990), they obtained the wavefield generated by a Rayleigh pulse incident upon an upward step. Then, the results have been combined with those of downward step to study
the scattering characteristics of troughs. They showed the contribution of these two parts to the total scattered wavefield of troughs at different frequency ranges.

2.4. Rayleigh Wave Scattering by Right Angle Wedge

Rayleigh wave can occur over a wide range of wavelengths. Therefore, the scattering of Rayleigh wave by wedge has application in various fields of study. The theory of Rayleigh wave scattering by wedges helps researchers in seismology and geotechnical earthquake engineering to explain the phenomena involved in surface wave propagation along surface/subsurface discontinuities. It could also be used in ultrasonic NDT techniques to characterize the size of surface defects. Achenbach et al. (1980) showed that for sufficiently deep surface breaking cracks (high frequency solution), the scattering coefficients of Rayleigh wave could be obtained by synthesizing the corresponding values of right angle wedge and half-plane. The incidence of Rayleigh wave upon the corner of a traction free wedge gives rise to scattered wavefield consists of reflected and transmitted surface waves as well as body diffracted ($P$ and $S$) and head waves. While body waves attenuate with distance to the wedge tip, the scattered Rayleigh waves propagate with constant amplitude. Therefore, at sufficiently large distance from the vertex, the total scattered wavefield is dominated by reflected and transmitted Rayleigh waves propagating along wedge faces. In this section, we use our numerical model to simulate the scattering of Rayleigh wave by a right angle wedge and calculate the corresponding reflection and transmission confidents. The results presented in Table 2.4 shows a good agreement with other theoretical and experimental studies.

2.4.1. Numerical Model and Excitation

Nucleation, propagation and scattering of elastic waves within an infinite wedge is simulated using a finite numerical model. In addition to the actual traction free surfaces, this model contains virtual truncating boundaries on its peripherals. A nonreflecting
boundary condition need to be applied along them to avoid spurious reflections. We used the finite difference code FLAC2D™ (Itasca, 2008) for all simulations of this study. Figure 2.6 shows the numerical model we use to simulate the scattering of Rayleigh wave by right angle wedge. As an input motion on the wedge face, we used the Ricker wavelet of unit amplitude. Analogous to the function \( I_2 \) in Ricker (1940), this narrowband pulse is defined as:

\[
f(t) = (1 - 2a)e^{-a}, \quad a = (\pi f_p (t - t_0))^2
\]  

(2.11)

where \( f_p \) denotes the central frequency in the Fourier spectrum and \( t_0 \) is an appropriate time shift from 0. The Ricker wavelet and its smooth Fourier spectrum are shown in Figure 2.7.

**Figure 2.6.** Configuration of the numerical model for Rayleigh wave scattering

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2 In fact, FLAC is based on the finite volume technique of Wilkins (1964) which presents a difference scheme for arbitrary shaped grids.
The quiet (absorbing) boundary condition inbuilt in FLAC is used along the truncating boundaries to minimize unwanted reflections. A point source of normal excitation placed far from the wedge corner to ensure the generation of complete incident Rayleigh wave ($R_{Inc}$). Furthermore, recording stations of reflected ($R_{Ref}$) and transmitted ($R_{Trans}$) Rayleigh waves located at sufficiently large distance from the tip (~10$\lambda$). It allows the scattered Rayleigh waves to receive enough energy from the breathing zone (Momoi, 1980) and to acquire their developed forms.

2.4.2. Reflection and Transmission Coefficients

Snapshots of the total wavefield are presented in Figure 2.3 at two stages of propagation. Figure 2.3a shows the resulting wavefield due to point source before the Rayleigh wave hits the corner (Half-space response). Turning round the corner, the incident
Rayleigh is split into surface (transmitted and reflected) and body (converted) parts. In addition, initial P, S and head waves propagating ahead of Rayleigh wave are reflected from the second face. These two sets of waves form a complex scattered wavefield as shown in Figure 2.3b. Vanishing S and P waves respectively along the wedge diagonal and faces could be checked in this figure. The evolution of scattered wavefield in time could be better explained by time histories of surface recordings. Figure 2.8 shows the seismogram synthetics of normal velocity augmented by designation of various parts of wavefield. Among all features we can interpret from this figure, the following are more important:

i. Transmitted Rayleigh wave has larger amplitude compared to the reflected part;

ii. Transmitted wave gain its complete form earlier than reflected part as it does not interact with the reflected S wave;

iii. Initial body waves i.e. cylindrical S and P wave have curved surface traces connecting by a straight head wave;

iv. The amplitude of grazing S wave is larger than corresponding part of P wave.

As reviewed in section 2.3, the main aim of all studies on Rayleigh wave scattering is to obtain the partitioning of Rayleigh wave energy upon its incidence on the corner. Of special importance are complex valued transmission and reflection coefficients. Figure 2.9 shows the time history of normal displacement for various parts of scattered Rayleigh wave. The amplitudes of incident, reflected and transmitted parts are respectively 3.41, 1.09 and 2.35 in length unit. Since the incident Rayleigh wave constitutes a portion of initial nucleation, its amplitude is less than that of incident displacement (integral of velocity pulse). Squaring the ratios we obtain the energy based reflection and transmission coefficients equal to 0.10 and 0.47, respectively. They are compared with the results of other numerical studies in Table 2.4. In addition, Tables 2.2 and 2.3 could be used to see where these numbers stand among other theoretical and experimental values. Furthermore, the phase shift of reflected and transmitted Rayleigh pulses with respect to central frequent have been calculated using
Figure 2.8. Seismogram synthetics of normal velocity

Figure 2.9. Normal displacement of incident, reflected and transmitted Rayleigh waves
the Hilbert transform. Another point of interest in Rayleigh wave scattering by wedges is the tip amplification factor. Comparing incident and tip displacements amplitude, we obtain an amplification factor of 2.5 (Figure 2.10) which is in agreement with literature.

Finally yet importantly, the polarization of particle motion at two stations on the wedge faces is presented in Figure 2.11. All parts of scattered Rayleigh wave have elliptical polarization with major axis perpendicular to the surface. The relative size of ellipses provide us with a qualitatively measure of energy partitioning. It is noteworthy that while the incident and transmitted Rayleigh waves have retrograde particle motions, the motion of reflected wave is prograde.

2.5. Vertical Plane SV Wave in Traction Free Wedges

In contrast to Rayleigh wave, the scattering of in-plane $SV$ waves by an infinite wedge has received very little attention, despite its relevance to seismological and earthquake engineering problems. The only available closed form solution is presented by Sanchez-Sesma (1990), who studied the wedge problem using the geometric method for two diffraction-free cases. For particular configurations of geometry, material and excitation, the geometric elastodynamics field completely describes the resulting scattered wavefield. In this section, we use our numerical model to obtain the amplification at the wedge tip for these simplified problems. Then, we extend the solution to a wide range of wedge angles where other wave components also exist in the total wavefield. In addition, the effect of Poisson’s ratio is investigated by computing the results for several material properties.

2.5.1. Numerical Model for Symmetric Problem

All simulations are based on elastic wave propagation in homogeneous medium. Therefore, the only source of scattering are irregular boundaries and the total scattered wavefield theoretically consists of specular reflection from wedge sides and tip diffraction.
Figure 2.10. Normal displacement of incident and tip Rayleigh waves

Figure 2.11. Elliptical particle motion of scattered Rayleigh waves
The infinite extension of actual wedge problem cannot be captured in the numerical model. Therefore, we will have a parasitic wavefield generated by truncating boundary that does not included in the real scattered field. Although the absorbing boundary condition can remove or at least reduce the spurious reflection from artificial boundaries, it does not work for unwanted diffraction from new corners. To solve this problem, we set the side length of model greater than $10\lambda$ thereby we can separate the effect of direct incident wave and that generated by spurious diffraction. Furthermore, the resulting wavefield is symmetric with respect to the wedge bisectrix because of geometry and excitation (vertical plane wave). The computational cost could be reduced accordingly by modeling half of the problem. The total wavefield has no vertical component along the new boundary (diagonal) and we could fix it in this direction. Figure 2.12 shows the configuration of the numerical model used in our analyses. In this figure, red bulb denote the spurious diffraction generated by artificial corner.

**Figure 2.12.** Configuration of the numerical model for SV wave propagation
2.5.2. Amplification at the Wedge Tip

The recorded amplitude at the vertex normalized by input motion (amplification factor) is obtained for two diffraction free wedge of 90° and 120° (latter made of Poisson’s material). The velocity time histories at the wedge tip are presented for these problems in Figure 2.13. They show the amplification factors of $4e-3$ and 4.002 for 90° and 120° wedges, respectively, which agrees quite well with the analytical solutions of 0 and 4 (Sanches-Sesma, 1990). The arrows in this figure depict the point of measurement i.e. when two direct arrivals reach the wedge tip. To interpret the characteristics of other wave components in the time history, we need to add a space dimension to this temporal plot. The result is seismogram synthetics that shows the complete scattered wavefield within the wedge. Figures 2.14 shows such plot for 90° wedges superimposed by designation and path of different wave types. All the components with $Toe$ subscript are parasitic waves generated by nonrealistic corner. There are two $P_{Toe}$ waves with straight and curves paths corresponding to the grazing part on one face and cylindrical wavefront on the opposite side. The grazing $P_{Toe}$ wave is reflected from the tip and propagating down the wedge face. The grazing $S$ wave ($S_{Toe}$) which has a very small amplitude is masked by stronger Rayleigh wave ($R_{Toe}$) propagating behind the shear wave. A more comprehensive explanation of different wave parts could be obtained through the snapshots of total wavefield. Figure 2.19 shows the evolution of wavefield within the 90° wedge using such plots. The head wave ($SP_{Toe}$) connecting diffracted $P$ and $S$ waves, which has no surface manifestation, is depicted in Figure 2.19a. It also shows the specular reflection of incident $SV$ wave from the free surface. The second snapshot corresponds to the interference of direct waves and shows the maximum surface motion normal to the wedge face. In the last snapshot, we can check that there is no diffraction from the wedge tip. In the velocity time history of 90° wedge, the point of measurement corresponding to the right end of blue dashed line in Figure 2.14 occurs around $t = 3.75s$. The zero amplitude at the vertex is a clear evidence of destructive interference of two upcoming $S$ waves. However, there are two nonzero
pulses around this point showing the constructive interference. Referring to the seismogram synthetics, the left pulse corresponds to the superposition of diffracted $P$ waves. The other bump in the time history, which has much larger peak around 5.3s, shows the interaction of upcoming surface waves (yellow dashed line in Figure 2.19a) generated from the corner.

Figure 2.13. Velocity time history at the wedge tip

For the second simplified problem i.e. 120° wedge, we can see a case of constructive interference at the tip where the input motion is amplified by a factor of 4. More details of the resulting scattered wavefield are presented in synthetic seismograms and snapshots of Figures 2.15 and 2.20. At the first look, it seems that there is no evidence of spurious diffraction in the seismogram synthetics. However, it exists and needs a meticulous look to be identified. This part of the wavefield is superimposed by larger amplitude $SV$ wave reflected from the free face. The first snapshot of Figure 2.20 shows all three components of diffracted wave more clearly. There are two more pulses in the
time history of 120° wedge tip, one of them merges to the main pulse. Similar to right angle wedge, they are created by superposition of diffracted $P$ and Rayleigh waves. Looking at the seismogram synthetics, we can see that the diffracted $P$ wave has smaller slope compared to the direct $S$ wave. The seismogram synthetics provide us with a powerful tool for understanding the wave mechanism. Nevertheless, they could be misleading if we do not consider their surface nature. In fact, what we measure from these figures as a velocity of wave is apparent velocity of surface manifestation. Similar to the right angle wedge, there is no diffraction from the tip of 120° wedge. However, in contrast with 90° wedge, the surface motion at the vertex is horizontal. This is in accordance with Figures 2 and 4 of Sanches-Sesma (1990).

After validating the numerical model, we extend the solution to a broader range of internal angles. One of the most interesting cases is the vertically propagated $SV$ wave incident on the wedge face at the critical angle. For $SV$ wave reflecting from the free boundary of elastic medium, the critical angle is defined as:

$$
\theta_{cr} = \sin^{-1}(c_s/c_p) = \sin^{-1}\left(\sqrt{(0.5 - \nu)/(1 - \nu)}\right)
$$

(2.12)

Hence, for Poisson material ($\nu = 0.25$), the corresponding wedge angle (critical wedge angle) is obtained as 109.5°. Beyond the critical angle, the mode converting part of reflection propagates along the wedge face as a surface wave. This is important for our analysis because the maximum tip amplification occurs just below this wedge angle (except for $\nu = 0.49$). The scattered wavefield of this geometry is more complex because of additional diffracted $P$, $S$ and surface waves from the tip. Figures 2.16 and 2.21 show the seismogram synthetics and wavefield snapshots for 109° wedge. In addition to grazing direct wave and diffracted Rayleigh, there is a secondary wavefield generated by tip diffraction (all waves with subscript Tip). The path designated by $P'_{Toe}$ shows the toe $P$ wave reflected at the opposite face and grazing down the face. The superposition of surface waves that contain most of the incident wave energy result in such a high peak amplitude.
(amplification factor of ~9). Different parts of the total scattered wavefield along with the extraordinary large surface motion at tip are better represented in the snapshots.

The complex wavefield of 109° wedge could be even more involved if we reduce the wedge angle. Figures 2.17 and 2.22 show seismogram synthetics and snapshots for the wedge angle 45°. In this case, direct $P$ waves generated at toe have a very small amplitude and it is hard to identify their paths on both wedge sides. On the other hand, diffracted $S$ and Rayleigh waves are stronger with clear traces on the seismogram synthetics ($S_{Toe}$ and $R_{Toe}$). Reflection of the direct $SV$ wave, which has the diffracted $S$ wave as its tail, propagating upward until it hits the opposite face. Upon incidence, it generates both mode preserving (stronger $SV$) and mode converting reflections. The interaction of this grazing reflected wave ($S'$) with the direct $SV$ wave is responsible for high amplification at the tip (amplification factor of ~5.5). The body part of reflection collides the opposite side and manifested at the surface as a curves path ($S''$). The last path in the seismogram synthetics describes the diffracted Rayleigh wave that reflected back from the wedge tip ($R'_{Toe}$). There are several other paths between $S''$ and $R'_{Toe}$ correspond to the direct wave reflecting back from the corner and tip diffracted waves. There is a very large peak in the time history of tip station corresponding to constructive interference of upcoming diffracted Rayleigh waves. This high amplitude pulse, which is an artifact because of its generators, should be excluded from the range of interest.

As a last example, we consider the propagation of vertical $SV$ wave in reentrant wedge. Seismogram synthetics and snapshot of total wavefield in a 270° wedge are presented in Figures 2.18 and 2.23. The resulting wavefield is much simpler compared to acute and obtuse wedge due to lack of multiple reflections. In addition, it is self-similar until the $P$ wavefront hits the truncating boundary (the reason for presenting single snapshot).

Finally, the variation of tip amplification factor with wedge angle (40°-260°) and Poisson’s ratio (0.15-0.49) is presented in Figure 2.24. There is some threshold angle in
acute wedges (ranging from $45^\circ$-$50^\circ$ for different Poisson’s ratios) below that the resulting scattered wavefield is very complex due to multiple reflections. It gives rise to an oscillatory variation of amplification factor with wedge angle. On the other hand, increasing the wedge angle beyond this threshold up to $90^\circ$, the amplification factor decreases almost linearly. In addition, the effect of Poisson’s ratio is negligible in this range and geometry controls the results. In fact, the linear decrease in the amplification factor is because of linear change in the wedge angle (geometry). The right angle is a checkpoint where the wedge is supposed to be diffraction free regardless of its material. Beyond this angle, we have increase in the amplification factor up to its maximum value. The maximum amplification occurs at wedge angle below the critical wedge angle. However, the difference between two cases is an exponential function of Poisson’s ratio. It ranges from less than $1^\circ$ for $\nu=0.49$ to $35^\circ$ for $\nu=0.49$. Furthermore, increasing the Poisson’s ratio gives rise to a smaller tip amplification factor and a more smooth variation around the peak. After the peak, the amplification factor monotonically decreases to 2 for half-space and then to lower values for reentrant wedges.
Figure 2.14. Seismogram synthetics of horizontal velocity – 90° wedge

Figure 2.15. Seismogram synthetics of horizontal velocity – 120° wedge
Figure 2.16. Seismogram synthetics of horizontal velocity – 109° wedge

Figure 2.17. Seismogram synthetics of horizontal velocity – 45° wedge
Figure 2.18. Seismogram synthetics of horizontal velocity – 270° wedge
Figure 2.19. Snapshots of total wavefield (2.5s, 3.75s, 4s) – 90° wedge
Figure 2.20. Snapshots of total wavefield (2s, 2.5s, 3s) – 120° wedge
Figure 2.21. Snapshots of total wavefield (2.5s, 2.9s, 3.5s) – 109° wedge
Figure 2.22. Snapshots of total wavefield (4s, 5.2s, 6s) – 45° wedge
Figure 2.23. Snapshots of total wavefield (2.5s) – 270° wedge

Figure 2.24. Tip amplification factor vs. wedge angle for incident $SV$ wave
2.6. Conclusions

In this chapter, we revisited the scattering of elastic waves by an infinite wedge otherwise known as the wedge problem. The solution to this canonical problem could be considered as a fundamental step toward the understanding of more involved problems in elastodynamics. According to the motion components existing in the total wavefield, the wedge problem takes either scalar or vector form. The former whose solution dates back to the beginning of last century is more common in electromagnetics and acoustics. The vector wedge problem, on the other hand, has remained unsolved in its general form. The major difficulty arises from the coupling of different wave types at the boundary. In attempt to solve reduced forms of the problem, several theoretical and experimental studies have been conducted so far. We presented a detailed review of these works in three different categories i.e. analytical and semi-analytical, experimental and numerical. Generally, each review contains the problem statement (geometry, initial/boundary conditions and simplifying assumptions), the method of solution, the form of results and selected concluding remarks. We then compared the results for the common problem of Rayleigh wave scattering by a right angle wedge. While the early studies are quite scattered, the more recent semi-analytical result are converging and could be used for validation purpose. Having an overview on the wedge problem and the gaps in its solution, we proposed a FD numerical model to extend the solution. We checked the applicability of numerical model in elastic wave simulation using two standard problems. Then, we used the model to solve the vector wedge problem of $SV$ wave scattering by a traction free wedge. The results presented in terms of tip amplification factor for a wide range of wedge angles and Poisson’s ratios. The variation of wedge angle considerably alters the tip amplification especially in non-reentrant wedges. There are three turning points between them the factor varies monotonically. They describe the threshold of oscillatory behavior, maximum peak and zero amplification. While the first two points are functions of material properties, the last one, which represents the diffraction free case, is constant among all Poisson’s ratios.
Furthermore, by increasing the Poisson’s ratio, the peak of amplification curve is shifting toward larger angles while its amplitude is decreasing passing its maximum value (9.1) at $\nu = 0.25$. For selected wedge angles, we presented seismogram synthetics and wavefield snapshots to explain underlying wave mechanism in details. The basic illustration could be used in the next chapter when we are dealing with a next level of complexity in geometric scattering i.e. the finite wedge. This simple scatterer is included in basic forms of surface topography like single slope, wedge and dam.
CHAPTER 3

TWO-DIMENSIONAL TOPOGRAPHY EFFECTS

3.1. Introduction

The term “local site conditions” describes the mechanical properties of near-surface geological formations and the geometry of surface irregularities (topography effects). Because of these effects, the properties of seismic wave recorded on the ground surface could be considerably different from the case of flat homogeneous half-space. These conditions may induce large amplification and significant spatial variability to the incoming seismic ground motion, and are therefore of particular significance in the assessment of seismic risk, microzonation studies, and the seismic design of important surficial and subterranean facilities. Topography effects in particular, are associated with the presence of strong topographic relief (hills, ridges, canyons, cliffs, and slopes), complicated subsurface topography (sedimentary basins, alluvial valleys), or geological lateral discontinuities (ancient faults, debris zones), and have been shown to significantly affect the intensity, frequency content and duration of ground shaking during earthquakes.

Documented observations from strong seismic events have shown that structures on the tops of hills, ridges, and canyons had suffered greater damage than similar structures at the hill bases or on level ground. Among several recorded motions that provided evidence on topographic amplification, three striking examples are worth noting:

a. The peak ground acceleration ($PGA = 1.93g$) recording of the hilltop Tarzana station during the 1994 Northridge Earthquake (Graizer, 2009);

b. The Pacoima Dam ($PGA = 1.25g$) recording during the 1971 San Fernando earthquake (Boore, 1972);
c. The recent extraordinary ground motion \((\text{PGA} = 2.74\text{g})\) recorded at K-Net station MYG004 during the 2011 Tohoku Earthquake on the crest of a 5m high, steep man-made slope (Nagashima et al., 2012).

Various field records, summarized by Bard (1999), confirm the macroseismic observations, indicate systematic amplification of seismic motion over convex topographies such as hills and ridges, deamplification over concave topographic features such as canyons and hill toes, and complex amplification and deamplification patterns on hill slopes that result in significant differential motion. Even though these studies offer valuable insight on the nature and significance of topographic effects, their limited number does not allow advancements in the understanding, parameterization, and successful simulation of topographic amplification to be made based on recorded data alone.

The problem of scattering and diffraction of seismic waves by topographic irregularities has also been studied analytically and numerically, and published results have focused, for the most part, on the response of 2D idealized geometries of isolated ridges or depressions on the surface of homogeneous, linearly elastic half-spaces. A comprehensive review of such analyses is given in a study by Assimaki (2004). In a comparative study of observations and predictions of topographic effects, Geli et al. (1988) showed that topographic amplification ratios typically range from 2 to 10, whereas events have also been recorded with spectral amplifications on the order of 20 or more. Later, Bard (1999) summarized the findings by Geli et al. (1988) as follows:

i. There exists a qualitative agreement between theory and observations on ground-motion amplification at ridges and mountaintops, and de-amplification at the base of hills. However, there exists a clear quantitative discrepancy between theory and observations.

ii. The observed or computed amplification is first-order related to the “sharpness” of the topography: the steeper the average slope, the higher the peak amplification.
iii. Topography effects are frequency-dependent; the stronger effects correspond to wavelengths comparable to the horizontal dimension of the topographic feature.

The most prominent sources of this discrepancy are: (a) the focusing of seismic rays in 3D topographic features, (b) the reverberations and scattering of seismic waves in stratified, heterogeneous soil formations, and (c) the assumption of linear elasticity.

Before considering complex configurations, which are more realistic, we need a fundamental understanding of topography effects at its basic level. It is defined as an idealized 2D surface irregularity in isotropic homogeneous elastic material subjected to vertical excitation. Generally, we can ascribe topography effects to constructive and destructive interferences of various parts in the scattered wavefield. Figure 3.1 shows such patterns in a single slope topography as a first order of geometric complexity. While the usual doubling effect occurs at the free boundary (green), focusing and defocusing of resulting wave components give rise to lower (blue) and higher (red) surface recordings (time histories are scaled). There are other important features in this simple case like the tailing parts of time histories correspond to diffracted body and surface waves and the occurrence of maximum amplification behind the crest (not at the crest).

![Figure 3.1. Scattering of seismic waves by surface irregularity: constrictive and destructive interferences](image-url)
The goal of this chapter is to explain these features in more details through presenting a systematic analysis of 2D topography effects. In the next section, we briefly review the previous theoretical investigations as well as field observations and instrumental studies on the topography effects. It is noteworthy that only surface topographies, which are related to this study, are considered in this review. In section 3.3, the current practice for incorporating topography effects in seismic designs is examined. In the following section, we use our numerical model to investigate the problem of 2D topographic amplification in a series of parametric studies. The results contain the effects of geometry (shape) as well as the frequency of excitation. The latter is represented as the characteristic length of the topographic feature normalized by the wavelength of incident wave. It is reminded that the simple 2D topographies considered in this chapter could be considered as extensions to infinite wedge in the sequence of geometric complexity.

3.2. **Background**

Considering the medium through which the wave propagates as isotropic homogeneous material, geometric discontinuity would be the only source of scattering. Under this simplifying assumption, the problem of wave scattering by surface irregularity has more complex configuration and thence more involves scattered wavefield compared to the wedge problem. The situation becomes even worse for surface topographies in geomaterials where the variations of material properties also needs to be considered. Therefore, laboratory experiments on topography effects, which are involving simplified material properties and boundary conditions, have attracted less interest from people in seismology and earthquake engineering. Two exceptions are Hudson et al. (1973) and Rogers and Bennett (1974). Hudson et al. (1973) measured the PR conversion coefficient for a triangular corrugated free surface to find the valid range of perturbation theory. Rogers and Bennett (1974) used the 2D seismic model of Oliver et al. (1954) to study the scattering of P wave by an isolated wedge shaped ridge. They obtained the peak spectral
amplification of 2 at the wedge tip. As opposed to scarce laboratory experiments, several field observations and instrumental studies have been conducted in attempt to capture fundamental aspects of the phenomenon. As for the theoretical studies, there are numerous semi-analytical and numerical approaches proposed for the problem. In this section, we review some of the investigations on the problem of seismic wave amplification by surface topography.

3.2.1. Field Observations and Instrumental Studies


There are also several instrumental evidence that surface topography affects the amplitude and frequency contents of the motion. These studies use dense instrumental arrays to measure amplification factors at the crest compared to the base (chosen free-field). Although they provide a decent tool to verify macroseismic observations (Celebi, 1987), they are limited to low amplitude recordings of aftershocks and microtremors. Reviews of such instrumental studies and results can be found in Geli et al (1988), Faccioli (1991) and Finn (1991). Among others, recorded motions on a steep site in Southern Alps studied by Bard and Meneroud (1987) and Nechtschein et al (1995) revealed crest to base
spectral ratios as high as 20, and the topographic spectral amplification observed in the Tarzana station during the 1994 Northridge Earthquake, which was approximately equal to 5 (Celebi, 1995; Bouchon & Barker, 1996; Graizer, 2009).

3.2.2. Theoretical Studies

We have seen in chapter 2 that there is no analytical (closed form) solution for the wedge problem except for few degenerate cases. Therefore, for the problem of surface topography with more involved geometries, the solution should be sought through other theoretical methods. There are numerous semi-analytical and numerical techniques proposed for various 2D configurations of topography problem. Generally, we can divide them into surface and subsurface problems according to the boundary conditions (traction free or impedance). In this study, we only deal with surface topographic features of convex form. We can further categorize surface topographic irregularities into acute (piecewise linear) and curved features. Examples of the former are V-shaped canyon and wedge shape (Λ) ridge, which is essentially a union of infinite wedge and half-space. Semi-circular and semi-elliptical canyon and ridge form curved surface topographies. It is evident that ray focusing would be more pronounced in acute topographies. As for the 3D case, there are very few studies mainly on smooth features like semi-ellipsoidal ridge. Tables 3.1 and 3.2 lists a summary of previous studies on different topographic features using semi-analytical and numerical techniques, respectively. It shold be noted that boundary methods i.e. boundary integral equations (semi-analytical) and bounadry element methods (numerical) have the same general formulation. However, we use definition of Bouchon and Sánchez-Sesma (2007) to split them according to the type of domain discretization (points or elements).
Table 3.1. Summary of Analytical and Semi-Analytical studies on surface topography effects

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Configuration – Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gilbert and Knopoff (1960)</td>
<td>P, SV, R</td>
<td>Perturbation Method</td>
<td>General 2D Irregularity – Small Height and Slope(^a) ((\alpha)) Replacing Irregularity by Equivalent Stress Distribution</td>
</tr>
<tr>
<td>Abubakar (1962a,b, 1963)</td>
<td>SV, P</td>
<td>Perturbation Method(^b)</td>
<td>Isolated Gaussian Canyon, Sinusoidal Surface Effect of Normalized Characteristic Length ((\eta))</td>
</tr>
<tr>
<td>Hudson (1967)</td>
<td>P</td>
<td>Perturbation Method</td>
<td>General 3D Surface Scatterer (Shape and Material)</td>
</tr>
<tr>
<td>Trifunac (1973)</td>
<td>SH</td>
<td>Wave Expansion – Bessel Function</td>
<td>Semicircular Canyon – Effects of Incident Angle ((i)) and (\eta)</td>
</tr>
<tr>
<td>Bouchon (1973)</td>
<td>P, SV, SH</td>
<td>Aki-Larner Method(^c)</td>
<td>General 2D Irregularity – Small Slope – Effects of (i) and (\eta)</td>
</tr>
<tr>
<td>Wong and Trifunac (1974)</td>
<td>SH</td>
<td>Wave Expansion – Bessel Function</td>
<td>Semielliptical Canyon – Effects of (i) and (\eta)</td>
</tr>
<tr>
<td>Neerhoff (1975)</td>
<td>SH, L</td>
<td>Boundary Integral Equation (BIE)</td>
<td>General 2D Canyon with Loaded Boundary Far-field Scattering Pattern (FFSP)</td>
</tr>
<tr>
<td>Wong and Jennings (1975)</td>
<td>SH</td>
<td>BIE – Representation Theorem</td>
<td>General 2D Canyon – Effects of (i) and (\eta) Topography Effects on Broadband Motion</td>
</tr>
</tbody>
</table>

\(^a\) Hudson et. At (1973) validated the theory with experimental data and found a maximum applicable angle of 25°
\(^b\) Modification of Rice Perturbation Method (Rice, 1951)
\(^c\) A Discrete Wavenumber (DWN) Technique presented by Aki and Larner (1970) – It is assumed that the irregularity is periodic.
Table 3.1. Continued.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Configuration – Remarks</th>
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<tbody>
<tr>
<td>Singh and Sabina (1977)</td>
<td>P</td>
<td>Wave Expansion – Bessel Function</td>
<td>2D/3D Circular Canyon/ Basin – Applicable for $\nu=0.5$</td>
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<tr>
<td>Sills (1978)</td>
<td>SH</td>
<td>BIE – Representation Theorem</td>
<td>General 2D Irregularity – Effects of $i$ and $\eta$</td>
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<tr>
<td>Sánchez-Sesma and Rosenblueth (1979)</td>
<td>SH</td>
<td>Indirect Boundary Integral Equation (IBIE)$^d$ – Point Sources Method$^e$</td>
<td>General 2D Canyon – Effects of $i$ and $\eta$</td>
</tr>
<tr>
<td>Wong (1979, 1982)</td>
<td>P, SV, R</td>
<td>IBIE – Point Sources Method</td>
<td>General 2D Canyon – Effects of $i$ and $\eta$</td>
</tr>
<tr>
<td>England et al. (1980)</td>
<td>SH</td>
<td>IBIE – Point Sources Method</td>
<td>General 2D Canyon – Effects of $i$ and $\eta$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Effects of Geometry (H/W Ratio), $\nu$, $i$ and $\eta$</td>
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<tr>
<td>Bouchon (1985)</td>
<td>SH</td>
<td>Modified Aki-Larner Method</td>
<td>General 2D Irregularity – No Restriction on Slope$^g$</td>
</tr>
</tbody>
</table>

$^d$ Applications of direct/indirect BIE methods in elastodynamics is reviewed in Bouchon and Sánchez-Sesma (2007)
$^e$ Otherwise known as Ohsaki method (Ohsaki, 1973)
$^f$ Originally presented by Herrera (1980)
$^g$ It is, however, an order of magnitude slower due to more wavenumber samples (Axilrod and Ferguson, 1990)
Table 3.1. Continued.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Configuration – Remarks</th>
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<tr>
<td>Geli et al. (1988)</td>
<td>SH</td>
<td>Modified Aki-Larner Method</td>
<td>Single/Multiple Layered Gaussian Shaped Ridge</td>
</tr>
<tr>
<td>Geli et al. (1988)</td>
<td>SH</td>
<td>Modified Aki-Larner Method</td>
<td>Single/Multiple Layered Gaussian Shaped Ridge</td>
</tr>
<tr>
<td>Luco et al. (1990)</td>
<td>P, SV, SH</td>
<td>IBIE – Point Sources Method 2.5D Green’s Function</td>
<td>3D Response of 2D Canyon in Layered Viscoelastic Medium – Effect of Azimuthal Angle of Incidence, i, η</td>
</tr>
<tr>
<td>Pederson et al. (1994a)</td>
<td>P, SV, SH, R</td>
<td>IBEM – 2.5 Green’s Functions</td>
<td>3D Response of 2D Canyon</td>
</tr>
<tr>
<td>Ashford et al. (1997)</td>
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b When the boundary is discretized to a set of elements (instead of points), BIE and IBIE are referred to as BEM/IBEM

g They applied amplitude threshold which results in sparse coefficient matrix
<table>
<thead>
<tr>
<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
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Table 3.2. Summary of numerical studies on surface topography effects

<table>
<thead>
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<th>Reference</th>
<th>Incident Wave</th>
<th>Technique</th>
<th>Configuration – Remarks</th>
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<tr>
<td>Boore (1972)</td>
<td>$SH$</td>
<td>Finite Difference Method (FDM)</td>
<td>A Ridge – Effect of $\alpha$ and $\eta$ – Pseudonodes on Boundary</td>
</tr>
<tr>
<td>Ilan et al. (1979)</td>
<td>$P$</td>
<td>FDM</td>
<td>Rectangular Trench – Effect of $i$ and $\eta$</td>
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<tr>
<td>Ilan and Bond (1981)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Fuyuki and Matsumoto (1980)</td>
<td>$R$</td>
<td>FDM</td>
<td>Rectangular Trench – Absorbing Boundary$^b$</td>
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<tr>
<td></td>
<td></td>
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<td>Effect of $\eta$ on Transmission and Reflection Coefficients</td>
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<tr>
<td>Zhenpeng et al. (1980)</td>
<td>$SV$</td>
<td>FDM</td>
<td>Truncated Conical Hill/Basin (Axisymmetric → 2D)</td>
</tr>
<tr>
<td>Boore et al. (1981)</td>
<td>$SV, P$</td>
<td>FDM</td>
<td>Single Slope (45°,90°) – Spectral Amplification Factor</td>
</tr>
<tr>
<td>Ohtsuki and Harumi (1983)</td>
<td>$SV, R$</td>
<td>FEM + Particle Model$^c$</td>
<td>Single Slope w/ and w/o Soft Layer on Toe/Slope</td>
</tr>
<tr>
<td>Ohtsuki et al. (1984)</td>
<td></td>
<td></td>
<td>Effect of Geometry and $\eta$ – Absorbing Boundary$^d$</td>
</tr>
<tr>
<td>Zahradnik and Urban (1984)</td>
<td>$SH$</td>
<td>FDM – Vacuum Formalism</td>
<td>2D Mountain Model – Impulse and Broadband Responses</td>
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<td>Moczo et al. (1997)</td>
<td>$P, SV$</td>
<td>DWN + FDM + FEM (Hybrid)</td>
<td>General 2D Irregularities – Viscoelastic Medium</td>
</tr>
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</table>

$^a$ A Nonreflecting Boundary presented by Smith (1974)

$^b$ According to Clayton and Engquist (1977)

$^c$ Analogous to FDM but involves different nodes in the approximate scheme

$^d$ According to Kunar and Rodriguez-Ovejero (1980)
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\textsuperscript{e}Topographic Aggravation Factor
3.3. Current Practice

The scarcity of well-documented field studies on topographic amplification and the quantitative discrepancy between observations and predictions on topography effects have hindered their incorporation in the majority of contemporary seismic code provisions and microzonation studies. Two notable exceptions are highlighted below:

1. The European Seismic Code (CEN, 2000) proposes an empirical correction factor for both cliff and ridge type topographies, as a function of the height $H$ and the slope inclination $i$ (Figure 3.2a). Topography effects are quantified by means of a frequency-independent aggravation factor $F_{\text{topo}}$, defined as $a_{2D} = F_{\text{topo}}.a_{1D}$, and are considered negligible for either cliff height $H < 30$ m or slope inclination $i < 15^\circ$.

2. The 1995 French Seismic Code (AFPS, 1995) proposes a similar aggravation factor $S_T$ to account for 2D amplification on cliff-type topographies (Figure 3.2b), as a function of the height $H$ of the cliff and the slope inclination $i$. In this case, $F_{\text{topo}}$ varies from 1.0 to 1.40, and the minimum slope inclination below which topographic effects are neglected is $i = \tan^{-1}(0.4) = 22^\circ$.

Nonetheless, these empirical factors: (i) have been developed based on limited recorded data on topography effects and simplified predictions for idealized configurations; (ii) they do not account for the effects of ground motion frequency (i.e. wavelength compared to the feature size) and intensity, soil layering, and material softening during strong ground shaking; and (ii) they are limited to 2D configurations, while it has been shown that topography effects are even stronger for 3D features.

Case studies, however, such as the one of Tarzana Hill in Los Angeles (Bouchon & Barker, 1996; Graizer, 2009) demonstrate that extremely high acceleration levels (1.93g) could take place despite the small height ($H = 18$ m) and low slope angle ($i = 10^\circ$) of the configuration. This shows that there exist additional factors defining the lower bounds in terms of height and inclination, such as the frequency dependence of topographic
aggravation, and the contribution of soil layering and nonlinearity as shown by Assimaki et al. (2005b).

\[
\begin{align*}
\text{max } F_{\text{topo}} &\geq \left\{ \begin{array}{ll}
1.4 & i > 30^\circ \\
1.2 & 15^\circ < i < 30^\circ \\
1 & H < 30m \text{ or } i > 15^\circ 
\end{array} \right. \\
a &= H/3 \\
b &= \min\{ (H+10)/4, 20\kappa \} \\
c &= H/4 \\
\text{max } F_{\text{topo}} &= 1 + 0.8 (\kappa - \mu - 0.4) \\
1 \leq \text{max } F_{\text{topo}} \leq 1.4
\end{align*}
\]

\textbf{Figure 3.2.} Seismic Code provisions for topographic aggravation factor, (a) European code (CEN, 2000), and (b) French code (AFPS, 1995), (After Assimaki et al., 2005b)

\section{3.4. Parametric investigation of 2D Topography Effects}

In this section, we systematically investigate the response of idealized 2D convex topographic features (Figure 3.3) to vertically propagating seismic waves. We use SV Ricker wavelet as an excitation, and parameterize the problem as a function of the slope angle ($\alpha$), the dimensionless height ($\eta = H/\lambda$) and dimensionless width ($\zeta = D/\lambda$). All the simulations on topography effects are performed for Poisson’s material ($\nu = 0.25$). Figure 3.3 shows the numerical model used in our simulations. In order to minimize the spurious reflections from truncating boundary, a combination of quiet absorbing boundary and free-
field zones has been used. The aim of free-field boundary of FLAC is to simulate the free-field motion i.e. the motion that would exist in the absence of scatterer (surface topography in our problem). The scattered wavefield resulted from various parts of two adjacent slopes (toe, face and tip) interact with each other. The extent of this interaction is determined by the width distance. We can define three different problems for three different scenarios of total wavefield:

i. Single slope topography ($\zeta \rightarrow \infty$): There is no theoretical interaction between the wavefields generated from two sides. Therefore, we can attribute all components of the total wavefield to a single scatterer;

ii. Wedge topography ($\zeta = 0$): Maximum interaction occurs between two sides. For small slope angle ($\alpha < 45^\circ$), however, this process does not necessarily result in maximum amplification;

iii. Dam topography ($\zeta < \zeta_{thld}$): there is a threshold width distance beyond that the interaction of two sides has no effect on the peak amplification. It only creates minor components in the total wavefield.

For each sub-problem, we present the variation of peak amplification with problem parameters. In addition, plots of seismogram synthetics and snapshots of wavefield at different time steps are presented for discussion purpose.

![Figure 3.3. General configuration of the numerical model for 2D topography problem](image-url)
3.4.1. Single Slope Topography ($\zeta \to \infty$)

Investigating the scattering mechanism by a single slope geometric discontinuity is a critical step toward comprehending the topography effects for more complex features. Nonetheless, it does not necessarily mean that we can superpose the consequent wave components to obtain the total wavefield (except for sufficiently far slopes or as a first order approximation). A set of parametric studies are conducted on the effects of slope angle as well as frequency of excitation on the topographic amplification. In all cases, peak acceleration recorded at or behind the crest normalized with respect to the free-field (1D) response is defined as the amplification factor. It helps us to illustrate the additive role of surface geometry in amplifying the ground motion intensity in the vicinity of topographic irregularities. A complete range of practical slope angles ($\alpha = 0 - 90^\circ$), which contains all cases from half-space to vertical cliff, is considered in the study. Furthermore, we repeat the numerical simulations for several dimensionless slope heights that reflect the role of frequency ($\eta = 0.125 - 4$). Smaller values correspond to incident waves of longer wavelength and lower frequencies.

Figure 3.4 shows the variation of horizontal amplification factor with slope angle and frequency. We can interpret several important features from this figure as follows:

i. All the curves are located above the amplification factor of unity. It means that the presence of single slope topography aggravates the free-field motion behind its crest for all configurations;

ii. For excitations of lower frequency, the variation of amplification factor with slope angle is relatively smooth and almost monotonic. For such small characteristic lengths of topography, the diffracted wavefield at the toe, which is merged into the grazing part of direct wave, is in its initial stages of mobilization. The interaction of this spatially dense wavefield with the direct $SV$ wave behind the crest gives rise to smooth variation of amplification.
iii. Increasing the frequency, we can identify two characteristic slope angles that show a peak and a trough in the slope response. Generally, they correspond to constructive and destructive interferences of direct vertical and upcoming surface waves. For the case of $\eta = 1$, where the wavelength of incident wave is equal to characteristic length, these two angles are equal to $39^\circ$ and $60^\circ$, respectively. We will explain the resulting wavefield for this case in the next part of this section.

iv. The first peak of amplification factor at lower slope angles becomes larger and narrower for higher frequencies. In addition, its location shifts toward the critical angle ($\alpha_{cr} = 35.25^\circ$). For large characteristic length, the diffracted wave has enough spatiotemporal distance to develop its components. Therefore, the upcoming surface wave, which has its maximum amplitude around the critical angle, contributes more to the peak amplification. The limiting case is similar to the infinite wedge where a very large amplitude occurs over a narrow range of angles. In fact, a single slope
with sufficiently large character length subjected to vertical wave could be considered as an infinite wedge under oblique incidence ($\alpha/2$ with respect to the wedge bisectrix).

It should be reminded that these results are obtained for Poisson’s material. For other Poisson’s ratios, we expect a change in value and location of peak amplification factor similar to the case of infinite wedge. Now, let us take a closer look at scattered wavefield for characteristic angles of $\eta=1$ case. Figures 3.5 and 3.6 show seismogram synthetics of horizontal and vertical acceleration for slope angles 39° and 60°, respectively. Designation of various wave components help to understand their interference at each case. It contains direct $S$ wave, forward and backward diffracted Rayleigh wave from slope toe and tip ($R_{Toe}$ and $R_{Tip}$), and the grazing part of diffracted $P$ waves ($P_{Toe}$ and $P_{Tip}$). The vertical acceleration component is parasitic in the sense that is generated due to scattering and is not present in the input motion. There is a second peak in the amplification factor at $\alpha=84°$ with larger amplitude. The grazing $S$ wave, which contains most of the energy along the slope, interacts with direct $S$ wave constructively and gives rises to such high amplitude. Figure 3.7 shows the same components seismogram synthetics for this slope angle. In order to understand the scattering mechanism better, snapshots of total wavefield are presented in Figures 3.8 to 3.10 for slope angles 39°, 60° and 84°, respectively. Furthermore, snapshots of 84° slope for $\eta=2$ is presented in Figure 3.11. It provides additional insight into the concept of wave separation. Different parts of scattered wavefield are more separated compared to Figure 3.10. Therefore, they have a different interference patterns and consequently result in a different amplification factor.

Figure 3.12 shows the spatial variation of horizontal and vertical amplification factors for all three angles. The vertical amplification factor is obtained through dividing the vertical acceleration by the same free-field horizontal acceleration (the vertical component of free-field response is trivially zero). We can see the amplification and deamplification patterns respectively behind the crest and in front of the toe. Furthermore,
a rapid complex change occurs along the slope face. For cases of constructive interface, larger peak amplification is associated with narrower influence width. This important factor should be taken into consideration for the seismic design of structures located on top of the cliffs. There is another interference between direct and scattered wavefield in front of the slope toe. It gives rise to a pseudo-peak with almost the same amplitude and form among all slopes. We can simply explain such behavior by looking at involved wave components. As opposed to wave interaction behind the crest, the grazing part of direct wave (projection along the slope) is absent. Therefore, there is no slope dependent factor and we obtain same amplitude and shape for all cases. This is clearly shown in the first snapshot of Figures 3.8 to 3.10. Finally, the spatial variation of vertical component helps to understand the energy partitioning of incident wave. In other word, the presence of surface topography change the distribution of incident energy along the surface (amplification and deamplification) as well as within its components (horizontal and vertical).
Figure 3.5. Seismogram synthetics of acceleration for single slope (↑: H, ↓: V) \( \alpha = 39^\circ \)
Figure 3.6. Seismogram synthetics of acceleration for single slope (↑: H, ↓: V) – $\alpha = 60^\circ$
Figure 3.7. Seismogram synthetics of acceleration for single slope (↑: H, ↓: V) – $\alpha=84^\circ$
Figure 3.8. Snapshots of total wavefield (↑: 1s, ↔: 1.17s, ↓: 1.4s) – $\alpha = 39^\circ$
Figure 3.9. Snapshots of total wavefield (↑: 1.0s, ↔: 1.23s, ↓: 1.4s) – $\alpha=60^\circ$
Figure 3.10. Snapshots of total wavefield (↑: 1s, ↔: 1.2s, ↓: 1.4s) – $\alpha = 84^\circ$
Figure 3.11. Snapshots of total wavefield (↑: 0.9s, ↔: 1.05s, ↓: 1.3s) – α = 84°, η = 2
Figure 3.12. Spatial variation of amplification factor for single slope (↑: H, ↓: V)
3.4.2. Wedge Topography ($\zeta=0$)

We next simulate the effects of geometry and excitation characteristics for the wedge type topography. The general structure of the resulting wavefield and hence the amplification factor is similar to the case of single slope. However, a distinguishing feature is the reaction of either side to the wavefield generated by the opposite side. Here again, the critical angle of incidence plays a decisive role in the amplification pattern along the wedge, with surface waves generated on both sides of the topographic feature, traveling uphill and interfering with the upgoing primary waves at the vertex. This phenomenon gives rise to excessive ground motion amplitude at the top of the feature and strong differential displacement along the sides. Figure 3.13 shows that how the tip amplification factor varies with the wedge angle and the frequency of excitation. It is possible to compare the amplification factor of wedge topography with that of infinite wedge (the $\nu=0.25$ curve in Figure 2.24). However, we should keep in mind the major difference between finite and infinite wedge models i.e. the side length. While we have sufficiently large distance (practically infinite) to separate the effect of toe diffraction (artificial) and direct incidence along the face of infinite wedge, this distance is controlled by the finite characteristic length in the wedge topography. As a result, the peak amplification of wedge topography is a mixture result of diffracted and direct waves. At low frequency range, the amplification factor monotonically varies with the wedge angle like the single slope topography. For higher frequency incidences (i.e. wavelengths substantially shorter than the size of the topographic feature), on the other hand, we could see an amplification response similar to the infinite wedge (the $\eta=4$ curve in Figure 3.13). In this case, a larger peak occurs at wedge angles closer to critical value ($\theta_{cr}=109.5^\circ$). There is another characteristic wedge angle where the response behind the crest is of particular interest. This angle, which reflects the destructive interference of wave components, is about $90^\circ$ for all frequency ranges. Thus, we can assert that the diffraction free wedge angle $\theta=90^\circ$ is a special case where the
destructive interference occurs independent of material (Figure 2.24) and excitation (Figure 3.13) characteristics.

For the case of \( \eta = 1 \), characteristics angles correspond to peak and trough of amplification factor are respectively 124° and 92°. In the following, we present a more detailed representation of the resulting wavefield at these two angels. Furthermore, to compare with the single slope topography, similar results for the wedge angle 60° are included as well. Figure 3.14 to 3.16 show seismogram synthetics of horizontal and vertical acceleration for wedge angles 124°, 92° and 60°, respectively. Compared to the single slope, we have a more complex wavefield associated with larger amplitude. There are several Rayleigh waves generated at the vertex due to the interaction of two wedge faces. The number of Rayleigh waves increases with decreasing the wedge angle (It is clearer in vertical plots where the effects of body waves are almost excluded). Corresponding snapshots of total wavefield are presented in Figures 3.17 to 3.19. In all cases, the middle plot shows the time of maximum horizontal amplification. Among all instructive features in these figures, we could mention the pure horizontal wavefield along the face of 124° wedge (close to 120°), the deamplification of ground motion at the tip of 92° wedge and the substantial amplification in 60° wedge due to the superposition of upcoming surface waves. Figure 3.20 shows the spatial variation of horizontal and vertical amplification factors for all wedge angles of interest. Again, for cases of constructive interference, we have larger amplitudes over a narrower region centered at the wedge tip. Deamplification troughs and pseudo-amplification peaks in front of the wedge toe could also be seen in the horizontal plot. As for the vertical component, a very large parasitic amplification (even larger than incident horizontal amplitude) occurs for smaller wedge angles (92° and 60°). For 124° wedge, however, most of the incident energy is transformed into horizontal components and hence the vertical amplification is generally negligible.
Figure 3.13. Horizontal amplification factor vs. wedge angle and frequency – Wedge
Figure 3.14. Seismogram synthetics of acceleration for Wedge (↑: H, ↓: V) – $\theta = 124^\circ$
Figure 3.15. Seismogram synthetics of acceleration for Wedge (↑: H, ↓: V) – $\theta = 92^\circ$
Figure 3.16. Seismogram synthetics of acceleration for Wedge (↑: H, ↓: V) – $\theta=60^\circ$
Figure 3.17. Snapshots of total wavefield (↑: 0.9s, ↔: 1.08s, ↓: 1.4s) – $\theta = 124^\circ$
Figure 3.18. Snapshots of total wavefield (↑: 0.9s, ↔: 1.35s, ↓: 1.4s) – θ=92°
Figure 3.19. Snapshots of total wavefield (↑: 0.9s, ↔: 1.21s, ↓: 1.4s) – $\theta=60^\circ$
Figure 3.20. Spatial variation of amplification factor for wedge (↑: H, ↓: V)
3.4.3. Dam Topography ($\zeta < \zeta_{\text{thld}}$)

The final configuration studied is the dam-type geometry, an intermediate case between the single slope and the wedge topographies. Before we go through the results, let us speculate about the dam response based on our knowledge for single slope and wedge cases. If we freeze the slope angle in our parametric analysis on dam topography, dimensionless height and width will affect the response. In fact, we have two characteristic length within the problem each normalized by incident wavelength (frequency). As for the normalized height, we generally expect to see oscillations of larger amplitude for higher frequencies (similar to single slope and wedge). That is the dimensionless height determines whether the incident wave ‘see’ the topography or not. Normalized width, on the other hand, is related to the interaction of two sides in terms of constructive and destructive interferences. Therefore, we should see a wedge and single slope types of behavior for sufficiently small and large dimensionless width, respectively. In between, there may be a couple of peaks and troughs due to the interaction of surface waves. It is noteworthy that the practical large distance, which gives rise to single slope response, is much smaller than the theoretical value ($\zeta \rightarrow \infty$). Thus, we can define a threshold distance beyond that the additive effect of either side on the response of other side is negligible ($\zeta_{\text{thld}}$). Figures 3.21 to 3.26 depict peak amplification curves as a function of normalized height and width for several slope angles from $15^\circ$ to $75^\circ$. It is aimed to cover the whole range of angles from gentle to steep slopes. The case of critical incidence is also included to show the interaction of surface waves containing a major part of incident energy. Roughly speaking, the amplification curves are self-similar for each slope angle. It demonstrates that the same wave mechanism constructs the peak amplitude (fixed slope angle). Increasing the slope angle, we can see the same trend on the ensemble of amplification curve as what we had for single slope and wedge. That is an increase up to the critical angle followed by a decrease corresponding to destructive interference and then another increase of higher rate. Furthermore, we have generally larger number of
oscillations for steeper slopes. In the next part of this section, we will identify characteristic width for each slope angle and investigate the resulting wavefield in more details.

Figure 3.21 shows that gently sloped topographies whose characteristic lengths are comparable to the incident wavelength have largest amplification factors ($\eta = 1$ curve). This is not a case for steeper slopes where the interaction of two sides plays a more decisive role. There are three dimensionless widths where the response of dam topography shows a drastic change (peak and trough), namely $\zeta = 0.03, 0.59$ and 1.13. Seismogram synthetics of horizontal and vertical accelerations are presented for these points in Figures 3.27 to 3.29. Increasing the width distance, two slope crests act as separate sources of diffraction. Therefore, we can see the traces of additional diffracted waves ($P$ and $R$) at larger dimensionless widths. Furthermore, the corresponding spatial variation of amplification factors are compared in Figure 3.30. The plot of horizontal component clearly shows the separation process of two adjacent slopes as the width increases. The single strong peak at the center of small widths is gradually transformed to a couple of isolated weaker peaks near the crests. Similar pseudo-peaks occurs in front of the slope toe just like the single slope case. The vertical component again help us to interpret the process in the context of energy conservation. Figures 3.31 to 3.38 show the similar plots for slope angle angles 30° and 36°. The larger the slope angle, the more involved the resulting wavefield and the amplification pattern. As it is expected, the number of reflected Rayleigh waves travelling down the slope also increases for steeper angles. The curves of slope angle 36°, which corresponds to the critical incidence, show the largest amplification factors as expected. Figure 3.39 displays the resulting complex wavefield at this angle for the case of $\eta = 1$ and $\zeta = 0.69$ (peak amplification). Beyond the critical angle, there is a descending trend in the amplification factor down to the slope angle 60°. Figures 3.40 to 3.49 present the same form of the results for slope angles 45° and 60°. While the former shows an even more complex wavefield due to the diffracted field generated at the toe, the latter poses a rather simple structure of multiple reflected Rayleigh waves. Another important feature of 60°
slope is extremely large amplitude at very small widths (wedge type behavior). This amplitude suddenly falls to smaller values (similar to those of gentle slopes) within a short dimensionless width. Finally, the results of slope angle 75° are presented in Figures 3.50 to 3.53. Compared to 60 slope, we have a larger amplitude at small width, more reflected Rayleigh waves and a distinguishing pattern for spatial variation of vertical amplification. The latter shows a very large vertical component near the crests of double-cliff topography.
Figure 3.21. Horizontal amplification factor vs. normalized width – $\alpha = 15^\circ$

Figure 3.22. Horizontal amplification factor vs. normalized width – $\alpha = 30^\circ$
Figure 3.23. Horizontal amplification factor vs. normalized width – $\alpha = 36^\circ$

Figure 3.24. Horizontal amplification factor vs. normalized width – $\alpha = 45^\circ$
Figure 3.25. Horizontal amplification factor vs. normalized width – $\alpha = 60^\circ$

Figure 3.26. Horizontal amplification factor vs. normalized width – $\alpha = 75^\circ$
Figure 3.27. Seismogram synthetics of acceleration for Dam ($\uparrow$: H, $\downarrow$: V) – $\theta=15^\circ$, $\zeta=0.03$
Figure 3.28. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – θ=15°, ζ=0.59
Figure 3.29. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – \( \theta = 15^\circ, \zeta = 1.13 \)
Figure 3.30. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – $\theta = 15^\circ$
Figure 3.31. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=30^\circ, \zeta=0.03$
Figure 3.32. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=30^\circ, \zeta=0.97$
Figure 3.33. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=30^\circ, \zeta=1.44$
Figure 3.34. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – θ=30°
Figure 3.35. Seismogram synthetics of acceleration for Dam ($\uparrow$: H, $\downarrow$: V) – $\theta=36^\circ, \zeta=0.69$
Figure 3.36. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=36^\circ$, $\zeta=1.19$
Figure 3.37. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=36^\circ$, $\zeta=1.69$
Figure 3.38. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – θ=36°
Figure 3.39. Snapshots of total wavefield (↑: 0.9s, ↔: 1.15s, ↓: 1.4s) – $\theta = 36^\circ$ – $\zeta = 0.69$
Figure 3.40. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=45^\circ, \zeta=0.88$
Figure 3.41. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta = 45^\circ, \zeta = 1.38$
Figure 3.42. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=45^\circ, \zeta=2.09$
Figure 3.43. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – θ=45°
Figure 3.44. Snapshots of total wavefield (↑: 0.9s, ↔: 1.23s, ↓: 1.4s) – \( \theta = 45^\circ \) – \( \zeta = 0.88 \)
Figure 3.45. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=60^\circ$, $\zeta=0.06$
Figure 3.46. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=60^\circ, \zeta=0.53$
Figure 3.47. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta = 60^\circ, \zeta = 1.5$
Figure 3.48. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – θ=60°
Figure 3.49. Snapshots of total wavefield (↑: 0.9s, ↔: 1.15s, ↓: 1.4s) – $\theta=60^\circ$ – $\zeta=0.53$
Figure 3.50. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – θ = 75°, ζ = 0.06
Figure 3.51. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta=75^\circ, \zeta=0.53$
Figure 3.52. Seismogram synthetics of acceleration for Dam (↑: H, ↓: V) – $\theta = 75^\circ, \zeta = 1.5$
Figure 3.53. Spatial variation of amplification factor for Dam (↑: H, ↓: V) – $\theta=75^\circ$
3.5. Conclusions

In this chapter, we use the fundamental understanding of infinite wedge problem developed in the previous chapter to investigate the scattering of elastic waves by more complex geometries. In particular, we were interested in the amplification of seismic waves by surface irregularities. We first briefly reviewed the previous studies on the effects of surface topography. It consists of numerous semi-analytical and numerical techniques as well as several field observations and instrumental studies. In summary, they show that the theoretical approaches cannot capture the field observations. Then, we presented the current practice of topography effects in seismic design codes (topographic factor). In order to fill the gaps between theory and observation and to present more elaborated topographic factor, which considers more parameters, we proposed a systematic analysis. A dam type geometry with piecewise linear boundaries has been considered as the idealized form of 2D topography. We defined a set of normalized parameters (by incident wavelength) that reflect the effects of geometry and excitation. The same finite difference numerical scheme has been used to model the scattering of elastic waves by surface irregularity. We divided the general problem into three categories based on the normalized width of the dam (distance between two adjacent slopes). At very large distance, two single slope components of the dam respond to incoming wave separately. It forms the single slope topography whose simple solution could be used to understand the problems of more involved geometries. For very small width, on the other hand, we have maximum interaction between two slopes. We demonstrated that this interaction does not necessarily results in maximum amplification. Finally, intermediate dimensionless width defines a dam type topography for which the resulting wavefield is more complex (additional source of scattering). For the first two problems, we presented the results as a function of slope angle and dimensionless height (frequency). Several characteristic angles corresponding to constructive and destructive interferences have been identified in the amplification response. The wave mechanism at these angles has been explained using seismogram
synthetics, spatial variation of amplification factors and wavefield snapshots. As for the dam topography, we investigated the additional effect of width by freezing the slope angle. The results have been calculated for several slope angles covering the practical range. The curves show an oscillatory behavior due to interference of different wave components (generated at toe/tip of each side). For intermediate dimensionless width (not too close to the wedge), the critical slope angle shows the maximum amplification factors at each frequency level. All the simulations performed in this chapter correspond to the case of vertical incidence. In the next chapter, we will see the additive effects of oblique incidence on the topographic amplification.
CHAPTER 4

TOPOGRAPHY EFFECTS FOR OBLIQUE INCIDENCE

4.1. Introduction

Compared to the vertical incidence, oblique waves lead to the generation of surface waves upon incidence on flat ground, an effect further aggravated by the presence of irregular surface topography. Generally, two different excitation-medium scenarios could result in oblique motion. The first type refers to the near-field of an oblique-slip fault (Figure 4.1a). In this case, generated body waves due to the fault rupture do not have enough space to complete the refraction process. Therefore, when the waves reach the ground surface, they still have some inclination angle with respect to global vertical direction. Another case for oblique incidence is attributed to the change in subsurface material properties and geometry (Figure 4.1b). Passing through horizontal layers of gradually decreasing stiffness, original wave is bent to the vertical direction. However, local patch near the surface topography with different properties and/or specific geometry could alter the direction of wave again and cause it to hit the ground surface in an oblique sense. To capture the effects of incident angle, we present a novel numerical scheme using an explicit finite difference method. The concept of the scheme is to decompose the particle motion into horizontal and vertical components and then applying these projections along the truncating boundaries of numerical model. When the consequent oblique wave hits the corner of model i.e. the intersection of artificial vertical boundary with actual horizontal surface, it gives rise to a set of spurious waves. This is analogous to the toe effects in the infinite wedge model. Since we were only interested in the tip amplification (single point), we ignored the byproducts of numerical model. For inclined wave simulation, however, we need an isolated direct wave over the whole width of our topography model (including
sufficiently long distance from truncating boundaries). Therefore, we propose a technique to remove the corner diffracted waves and to find such isolated wave just like actual half-space. The accuracy of numerical model is checked with the available analytical solution of half-space. Then, we use this numerical model to study the effects of surface topography on the aggravation of seismic motion for non-vertical $SV$ waves. The results are compared with our previous findings on vertically propagating seismic wave amplification in the vicinity of topographic features. Finally, we investigate the effect of incident angle for a strong motion station located on the more realistic surface topography.

Figure 4.1. General scenarios of oblique incidence: (a) Near-field seismic source, (b) Change in the subsurface material and geometry
4.2. Oblique Incidence on Flat Ground - Analytical Solution

The effect of incidence angle on the flat ground response is well known. Analytical solution for elastic half-space subjected to oblique body waves has been proposed by Knopoff at al. (1957). Figure 4.2 shows the horizontal and vertical amplification factors versus incident angle for $SV$ wave. The results are normalized with the response of unit vertical wave on the half-space. We can find a close relation between the response of half-space subjected to $SV$ wave at angle $i$ and that of infinite wedge of angle $\theta = \pi - 2i$ under vertical incidence (Figure 2.24). Nevertheless, it is trivial that one cannot consider the wedge problem as a superposition of two inclined half-spaces. There are two characteristic features in the response of half-space. The first one refers to the peak horizontal amplitude and corresponding zero vertical amplitude around the critical incident angle, which is clearly depicted for various Poisson’s ratios. For the case of $\nu = 0.49$, the difference between critical and peak angles becomes significant as it is discussed in section 2.5.2. The zero horizontal amplitude at $i = 45^\circ$ for all Poisson’s ratios is another characteristic feature, which can be considered as analogy to zero vertical amplitude in vertical incidence. The particle motion is pure vertical at this angle and no energy is transformed to the horizontal component. Therefore, the vertical amplification factor is obtained as $\sqrt{2}/2$ in accordance with the conservation of energy. For larger Poisson’s ratios, the share of vertical motion in the total energy increases as expected.

4.3. Numerical Model for Oblique Incidence

In this section, we present a numerical model to simulate the propagation of oblique wave in 2D case. It is based on the decomposition of incident wave into two perpendicular components and applying them along truncating boundaries with appropriate time delay. The resulting wavefield contains both direct $SV$ wave and spurious diffracted waves from the corner of model. We take advantage of explicit scheme of FLAC to remove these unwanted waves through a nullification process.
Figure 4.2. Amplification Factor vs. Incident Angle $i$ for flat ground ($\uparrow$: H, $\downarrow$: V)
4.3.1. Time Marching Approach (Fix-Release)

In order to simulate an oblique plane wave, we first decompose the input motion into two horizontal and vertical components. Projected waves along the lower and left truncating boundaries are shown in Figure 4.3. The apparent wave velocity is then utilized to calculate the delay time for successive points. Two sets of such so called delayed time histories are applied on the boundaries. We keep the usual quiet boundary along the right end of the model to avoid fictitious wave focusing at this corner. It is worth pointing out that instead of applying transformed stress (flexible) on the boundaries of the numerical model, we used delayed velocity components (rigid) of the inclined waves. The importance of this choice becomes more evident when the material undergoes large strain over small stress change (close to plastic). It is useful for future application of this technique for material of more complex response. The associated problem with prescribed velocity is the spurious reflection from the source boundary (Figure 4.4a). This situation will be treated using a time-marching scheme in prescribing boundary conditions hereby referred to as ‘fix-release’.

![Figure 4.3. Definition of delayed time histories](image)
Consider two ‘dead’ points A and B on the input Ricker pulse where the amplitude is practically zero beyond them. The projected influence width of input motion is therefore defined over the lower source boundary as A′B′. We could counteract fixity induced reflection by releasing the source boundary when the influence width of input motion is traveled (Figure 4.4b). The nodes, on which velocity boundary conditions are initially imposed, are relaxed shortly after the incident wave passage ($l_{Disturb}$) to allow far-field attenuation of the reflected waveforms (radiation condition). The fix-release process continues until the last point of disturbance reaches the right end of the model. For large incident angles (the extreme case would be the horizontally propagated wave), the scattered wave may reach the fixed boundary before we release it. Therefore, it seems necessary to
provide large distance between the source of scattering e.g. slope toe and the horizontal bottom line to prevent another reflection. Nonetheless, the amplitude of such reflected body waves on the ground level (our point of interest) is negligible even for depths of few wavelengths because of the geometric attenuation.

4.3.2. Sponge Boundary

To avoid reflection from the bottom source boundary, releasing fixed nodes is necessary but not sufficient. Actually, downward coming wave cannot pass through the vacuum and additional region should be provided for its transmission. Another source of spurious reflection is the upper right corner where the usual quiet boundary does not seem enough to absorb scattered field, especially the constant amplitude Rayleigh wave. Uniformly varying absorbing layers based on the Rayleigh damping model are therefore used outside the truncated numerical domain to trap any backward scattered waves. These layers which are referred to as sponge boundaries are shown in Figure 4.5 (gray regions). The damping parameters and the width of sponge boundary are selected according to the incident angle of input wave and the amplitude of consequent wavefield. Finally, another set of quiet boundary is defined at new borders of the model (red dashed line) to remove any possibly remaining waves.

![Figure 4.5. Sponge boundary](image-url)
4.4. Oblique Incidence on Flat Ground - Numerical Solution

In order to check the numerical accuracy of our model, we simulate the propagation of inclined $SV$ wave along the half-space and compare the results with analytical solution. Figure 4.6 shows the seismogram synthetics of velocity components for angle of incidence $i = 15^\circ$. It depicts three clear traces corresponding to the grazing direct $SV$ wave and spurious diffracted waves ($P$ and Rayleigh) from the corner. One could see the constant amplitude of Rayleigh wave and the vanishing surface manifestation of cylindrical $P$ wave. In the next section, when we put this oblique wave on the topographic feature, diffracted wave will be removed to see the net effect of $SV$ wave. Figures 4.7 to 4.9 show the same plots for incident angles $i = 30^\circ, 45^\circ, 60^\circ$. The general structure is the same among all angles and the only difference if the order of wave components. While the Rayleigh wave velocity is constant (with lowest speed) in all cases, the apparent shear wave velocity decreases from $i = 15^\circ$ to $i = 60^\circ$. Thus, it rotates around the $P$ wave trace (also constant in all cases) in counterclockwise direction. The energy partitioning between horizontal and vertical components is also clear in these figures, especially for $i = 45^\circ$. Figure 4.10 shows the horizontal and vertical amplification factors along the surface of half-space. In analogous with Figure 4.2, the results are normalized by the surface response of vertical incidence. In the first few wavelengths from the corner (left end), the merging wave components give rise to bump shaped features. Sufficiently far from the corner, which is a function of incident angle, they separate from each other due to speed difference. The asymptotic values at the right side of plots show the amplification factors. For horizontal component of $i = 45^\circ$, where the direct wave is masked by larger amplitude Rayleigh wave, we could directly check the time history (Figure 4.11). The largest numerical error of our model, which corresponds to the horizontal amplification factor of $i = 45^\circ$, is equal to 0.3%.
Figure 4.6. Seismogram synthetics of velocity for incident angle $i=15^\circ$ (↑: H, ↓: V)
Figure 4.7. Seismogram synthetics of velocity for incident angle $i=30°$ (↑: H, ↓: V)
Figure 4.8. Seismogram synthetics of velocity for incident angle $i=45^\circ$ (↑: H, ↓: V)
Figure 4.9. Seismogram synthetics of velocity for incident angle $i=60^\circ$ ($\uparrow$: H, $\downarrow$: V)
Figure 4.10. Spatial variation of amplification factor for half-space (↑: H, ↓: V)
Figure 4.11. Velocity time history at $x/\lambda = 15$
4.5. Amplification of Oblique Plane SV Wave by a Single Slope Topography

We next use the numerical model to investigate the effect of inclination angle on topographic amplification for single slope geometry. Based on the relative alignment of slope and wave direction, two distinct configurations are considered as shown in Figure 4.12. The ‘forward’ case refers to the wave propagation along the toe-crest upward direction while the ‘backward’ implies the opposite course. Generally, the backward case results in more aggravation because of the energy focusing at the crest. Analysis of oblique incidence is performed for slopes of unit dimensionless height ($\eta = 1$). Before presenting the results, it is worth mentioning the elimination process of unwanted diffracted waves, which is referred to as the wave nullification. At sufficiently large distance from the corner, wave components are well separated from each other. On the other hand, the explicit capability of FLAC allows us to control the material and excitation characteristics at any time step during the simulation. Therefore, we could identify the spatiotemporal extent of unwanted waves (diffracted P and Rayleigh) within a medium and replace the region with vacuum (using null mechanical model of FLAC). Although the process seems very simple, the appropriate selection of space and time of nullification is a tedious task. It could not be performed only based on surface recordings i.e. synthetic seismograms. Instead, it requires a complete picture of resulting scattered wavefield within a medium.

![Figure 4.12. Forward and backward incidence](image-url)
Figures 4.13 to 4.20 show the seismogram synthetics of horizontal and vertical accelerations for $\alpha=15^\circ$ single slope subjected to a $SV$ wave under various angles. Among all features we could interpret from these figures, the following are more important:

i. The general structure of scattered wavefield is the same as that of vertical incidence. It consists of diffracted $P$ and Rayleigh waves propagating on both sides of the slope. However, differences in the amplitude and phase of various wave components result in a different interference patterns and thence a different amplification factor.

ii. A distinct feature of oblique incidence is nonzero vertical component of free-field motion, which results in larger vertical amplification factor for such small slope angle. This is not necessarily true for steeper slopes where the larger portion of incident energy is transformed to the vertical motion.

iii. Another feature of oblique incidence is a zero horizontal motion for $i=45^\circ$ incidence (figures 4.17 and 4.18). Analogous to vertical motion of normal incidence, we could consider the resulting scattered field as parasitic.

iv. We can split the incident energy into two parts, one reflected back from the slope and the other transmitted in the direction of incidence. For small slope angle where the focusing effect at the crest is minor, both backward and forward incidences give rise to larger transmitted energy. The effect is more pronounced for larger incident angles (Figures 4.19 and 4.20).

v. Decreasing the apparent velocity of grazing shear wave with increasing the incident angle is also depicted by bending the branch of direct $SV$ wave toward diffracted $P$ wave.

The corresponding plots of amplification factors for $\alpha=15^\circ$ single slope are presented in figures 4.21 and 4.22. In these figures, the horizontal and vertical amplitudes are normalized by horizontal response of flat ground under vertical incidence. Therefore, the oblique curves show the combined effects of incidence angle (free-field) and topographic feature. To have a better measure for the effect of incident angle, the amplification curves
of vertical incidence is also added (forward and backward cases are symmetric with respect to the center point of slope). The following features are worth mentioning in the amplification curves:

i. For either case of incident direction (forward and backward), the horizontal amplification factor decreases with increasing the incident angle. The case of \( i = 15^\circ \) is an exception where the vertical curve placed between lower backward and upper forward curves (upper plots in Figures 4.21 and 4.22). It could be explained by the direction of particle motion with respect to the slope face. Under backward incidence, the particle motion is parallel to the slope and results in minimum surface waves. Forward incidence, on the other hand, has the largest deviation of particle motion and gives rise to maximum surface waves. The vertical amplification curves have a reversed order i.e. they are moving upward for larger incident angles.

ii. The horizontal amplification curves are clustered in two groups for incident angles below and above the critical value due to significant difference in free-field values.

iii. Forward and backward incidences give rise to the same amplification patterns with respect to the free-field motion (the net topography effect). For \( i = 15^\circ, 30^\circ \) (in accordance with vertical incidence) we have amplification behind the crest and deamplification in front of the toe. The case of \( i = 60^\circ \), however, has a reversed pattern. Furthermore, \( i = 45^\circ \) is an exception as it only amplifies the zero far-field motion.

iv. The net topography and combined topography-inclination effects are clearly depicted in these figures. For example, compare them for \( i = 15^\circ \) and \( 45^\circ \) in the upper plot of figure 4.21. While the \( 45^\circ \) incidence results in higher topographic amplification compared to \( 15^\circ \) (1.56 vs. 1.32), its overall amplification factor is much less (0.59 vs. 1.29).

v. In addition to the amplification factor, the extent of influence region should be taken into consideration in seismic design. For example, although the forward incidence of
$i=30^\circ$ gives rise to a lower amplification factor compared to the peak value of vertical case, it affects a broader range behind the crest (Upper plot in figure 4.21).

Figures 4.23 to 4.32 show the same set of seismogram synthetics and amplification factors for $\alpha = 30^\circ$. We could see the same general structure of scattered wavefield as that of $i=15^\circ$ with diffracted waves of larger amplitude. Furthermore, wave focusing at the slope crest is stronger for this slope angle and a larger part of incident energy is reflected back (Figure 4.24). Increasing the incident angle, this effect becomes less dominant and again we have more transmitted energy (Figure 4.30). Regarding the amplification factor curves, there is a similar descending/ascending trend for horizontal/vertical components with increasing incident angle. While the net topography effect is larger compared to the case of $\alpha = 15^\circ$, the combined topography-inclination effect controls the overall amplification values. The seismogram synthetics of $\alpha = 45^\circ$ show a similar scattering mechanism for various incident angles. Again, backward incident results in larger reflected energy at smaller incident angles. As a new feature, we could see larger number of diffracted $P$ and Rayleigh waves on both sides of the slope. This is because of larger projected distance between the toe and the crest of slope that separates their surface manifestations.

The spatial variation of amplification factors for $\alpha = 45^\circ$ are presented in Figures 4.41 and 4.42. While the horizontal component of forward incidence shows the same decreasing trend with increasing incident angle, the variation of vertical component is more complex with a very large amplification for $i=30^\circ$ (the vertical case contains the larger topography effect but its overall amplification is smaller). The backward case shows a huge horizontal amplification for $i=45^\circ$ where the particle motion is parallel to the slope surface. The overall amplification factor for this slope-inclination configuration reaches as large as 1.23 in spite of minor free-field value (0.1). The vertical component is also interesting where we have ascending amplification factors with increasing incident angle (1.17 for $i=60^\circ$).
Finally, Figure 4.43 to 4.52 show the seismogram synthetics and amplification factor for steepest slope in our study i.e. $\alpha = 60^\circ$. Compared to smaller slope angles, seismogram synthetics show stronger reflected waves for backward case (due to wave focusing) even for larger incident angles (Figure 4.50). The amplification curves are even more complex at this slope angle. For horizontal component, the vertical component is no longer the leading curve. We remember from section 3.4.1 (Figure 3.4) that it forms a trough in the amplification-slope angle curve. Instead, the $i=30^\circ$ incidence has the largest amplification factor for both forward and backward incidences. The particle motion perpendicular to the slope surface in forward case and propagation along the bisectrix of corner wedge (maximum interaction between faces) are contributing factors to such high amplification. For vertical component of forward case, the topography effects could be expressed as general amplification and deamplification respectively for incident angles below and above the critical value. Therefore, the vertical incidence shows the maximum amplification factor despite the zero background value. For backward case, on the other hand, we could see an opposite trend as higher amplitudes occur for larger incidences because the net effect of topography is amplification.

Freezing the incident direction and angle, we could also find the angle at which maximum amplification occurs. For vertical incidence, the peak amplification corresponds to slope angle $\alpha = 39^\circ$ (Figure 3.4), which is closer to $\alpha = 45^\circ$ in our discrete set of slope angles. As an example, we find the slope angle of peak amplification for forward oblique incidence. These angle are obtained as $\alpha = 30^\circ$, $60^\circ$, $45^\circ$ and $45^\circ$ respectively for $i = 15^\circ$, $30^\circ$, $45^\circ$ and $60^\circ$. Such scattered pattern could be explained by different constructive and destructive patterns which itself resulted from amplitude and phase differences of various wave components.
Figure 4.13. Seismogram synthetics of acceleration for $\alpha=15^\circ$ slope (↑: H, ↓: V) – $i=15^\circ F$
Figure 4.14. Seismogram synthetics of acceleration for $\alpha = 15^\circ$ slope (↑: H, ↓: V) – $i = 15^\circ$ B
Figure 4.15. Seismogram synthetics of acceleration for $\alpha=15^\circ$ slope (↑: H, ↓: V) – $i=30^\circ$F
Figure 4.16. Seismogram synthetics of acceleration for $\alpha=15^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=30^\circ$B
Figure 4.17. Seismogram synthetics of acceleration for $\alpha = 15^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 45^\circ$F
Figure 4.18. Seismogram synthetics of acceleration for $\alpha=15^\circ$ slope (↑: H, ↓: V) $-i=45^\circ$B
Figure 4.19. Seismogram synthetics of acceleration for $\alpha = 15^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 60^\circ$F
Figure 4.20. Seismogram synthetics of acceleration for $\alpha=15^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=60^\circ$B
Figure 4.21. Spatial variation of amplification factor for $\alpha=15^\circ$ slope (↑: H, ↓: V) – FW
Figure 4.22. Spatial variation of amplification factor for $\alpha = 15^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – BW
Figure 4.23. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope (↑: H, ↓: V) – $i = 15^\circ$F
Figure 4.24. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 15^\circ$ B
Figure 4.25. Seismogram synthetics of acceleration for $\alpha=30^\circ$ slope (↑: H, ↓: V) – $i=30^\circ$F
Figure 4.26. Seismogram synthetics of acceleration for $\alpha=30^\circ$ slope (↑: H, ↓: V) – $i=30^\circ$B
Figure 4.27. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 45^\circ$F
Figure 4.28. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 45^\circ$B
Figure 4.29. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 60^\circ$F
Figure 4.30. Seismogram synthetics of acceleration for $\alpha = 30^\circ$ slope ($\uparrow$: H, $\downarrow$: V) $-i = 60^\circ$B
Figure 4.31. Spatial variation of amplification factor for $\alpha=30^\circ$ slope (↑: H, ↓: V) – FW
Figure 4.32. Spatial variation of amplification factor for $\alpha = 30^\circ$ slope (↑: H, ↓: V) – BW
Figure 4.33. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope (↑: H, ↓: V) – $i=15^\circ$F
Figure 4.34. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=15^\circ$B
Figure 4.35. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=30^\circ$F
Figure 4.36. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=30^\circ$B
Figure 4.37. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=45^\circ$F
Figure 4.38. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope (↑: H, ↓: V) – $i=45^\circ$B
Figure 4.39. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i=60^\circ$F
Figure 4.40. Seismogram synthetics of acceleration for $\alpha=45^\circ$ slope (↑: H, ↓: V) – $i=60^\circ$B
Figure 4.41. Spatial variation of amplification factor for $\alpha=45^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – FW
Figure 4.42. Spatial variation of amplification factor for $\alpha=45^\circ$ slope (↑: H, ↓: V) – BW
Figure 4.43. Seismogram synthetics of acceleration for $\alpha = 60^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 15^\circ$ F
Figure 4.44. Seismogram synthetics of acceleration for $\alpha = 60^\circ$ slope ($\uparrow$: H, $\downarrow$: V) $\sim i = 15^\circ$ B
Figure 4.45. Seismogram synthetics of acceleration for $\alpha = 60^\circ$ slope (↑: H, ↓: V) – $i = 30^\circ$F
Figure 4.46. Seismogram synthetics of acceleration for $\alpha = 60^\circ$ slope ($\uparrow$: H, $\downarrow$: V) $- i = 30^\circ$B
Figure 4.47. Seismogram synthetics of acceleration for $\alpha = 60^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 45^\circ$F
Figure 4.48. Seismogram synthetics of acceleration for $\alpha=60^\circ$ slope (↑: H, ↓: V) – $i=45^\circ$B
Figure 4.49. Seismogram synthetics of acceleration for $\alpha=60^\circ$ slope (↑: H, ↓: V) – $i=60^\circ$F
Figure 4.50. Seismogram synthetics of acceleration for $\alpha = 50^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – $i = 60^\circ$B
Figure 4.51. Spatial variation of amplification factor for $\alpha = 60^\circ$ slope (↑: H, ↓: V) – FW
Figure 4.52. Spatial variation of amplification factor for $\alpha=60^\circ$ slope ($\uparrow$: H, $\downarrow$: V) – BW
4.6. Case Study: MYG004 Station (Japan)

To explore the topographic amplification and the role of incident angle in actual cases, we consider a more realistic surface feature in this section. Figure 4.53(a) shows the event map for Tohoku earthquake with a large rupture zone including the main shock and aftershocks. The strong motion station MYG004, which is selected for this study, is depicted by a green triangle. The Google map view of selected topographic feature is shown in Figure 4.53(b). The large green arrow points to the wave propagation direction with respect to the topography section. In Figure 4.53(c), the recorded NS horizontal acceleration is highlighted having the peak of 2.74g. We extract a representative 2D section to simulate the propagation of $SV$ wave and to obtain amplification factors. Three different scenarios have been considered for incident direction i.e. $i=0$, $30^\circ$ and $45^\circ$. Density, shear wave velocity and central frequency of Ricker wavelet are $\rho=2200$ kg/m$^3$, $V_s=620$ m/s and $f_0=10$Hz, respectively. We use the technique of section 4.3 to apply oblique incidence.

Figures 4.54 to 4.56 show seismogram synthetics of acceleration components for three different incident angles. Typical diffracted $P$ and Rayleigh waves propagating on both sides of the feature construct the total scattered wavefield in each case. Compared to idealized geometries like wedge and dam, however, there exist several more diffracted waves generated at surface discontinuities. The feature is more pronounced in the vertical components of normal incidence where the free-field motion is zero (Figure 4.54↓). The amplification and deamplification of free-field motion is clearly depicted in cases of oblique incidence. The spatial variation of amplification factor along the surface topography is presented in Figure 4.57. The recording station MYG004, which is located at the crest of $35^\circ$ slope, is shown by a green triangle. Similar to the idealized geometry, the background motion has a considerable effect on the overall amplification. For example, when the horizontal topographic amplification of $i=45^\circ$ case combines with its negligible free-field motion, the overall amplification becomes the least among all cases. The $30^\circ$ incidence gives rise to the largest horizontal amplification at the recording station.
Furthermore, one could see a shift in the amplification pattern by changing the incident scenario. While the vertical input motion is demplified at the recording station, the $30^\circ$ incidence shows amplification at the same location. For vertical amplification factor, on the other hand, the case of $i=45^\circ$ has largest topographic and background values which results in much more amplification compared to the vertical case. The change of amplification pattern due to incident angle is also clear in this case.

Figure 4.53. (a) Event map for Tohoku earthquake, (b) Google map view of the selected topography, (c) NS Horizontal acceleration time history
Figure 4.54. Seismogram synthetics of acceleration for MYG004 (↑: H, ↓: V) – i=0
Figure 4.55. Seismogram synthetics of acceleration for MYG004($\uparrow$: H, $\downarrow$: V) – $i=30^\circ$
Figure 4.56. Seismogram synthetics of acceleration for MYG004 (↑: H, ↓: V) – $i=45^\circ$
Figure 4.57. Spatial variation of amplification factor for MYG004 (↑: H, ↓: V)
4.7. Conclusions

In this chapter, we studied the effects of surface topography on the aggravation of seismic motion for non-vertical $SV$ wave incidence (e.g. in the near-field of an oblique-slip fault), and compared the results to our previous findings on vertically propagating seismic wave amplification in the vicinity of topographic features. Inclined incidence, as opposed to vertically propagated wave, gives rise to surface waves upon incidence on flat ground. The resulting wavefield, which has zero vertical component, is further altered by topographic irregularity. Depending on the relative alignment of slope and inclination angle, the combined effect could be amplification or deamplification with respect to free-filed response of vertical motion. To capture these effects, we developed a novel numerical simulation scheme using the explicit finite difference scheme. Instead of applying transformed stress on the boundaries of the numerical model, we define an internal (truncated) domain and employ delayed velocity components of the inclined waves on its boundaries. The nodes on which velocity boundary conditions are initially imposed, are relaxed shortly after the incident wave passage to allow far-field attenuation of the reflected waveforms, a time-marching scheme in prescribing boundary conditions hereby referred to as ‘fix-release’. Uniformly varying absorbing layers (referred to as sponge boundaries) are used outside the truncated numerical domain to trap any backward scattered reflections. Using this numerical model, multiple slope geometries were subjected to direct $SV$ wave incidence, which is isolated from the surface wave reflections through a virtual obstacle, and the ground response components were computed for various incidence configurations. Results show that the topography feature changes the free-field motion of oblique incidence in a different amplification and deamplification pattern compared to the vertical incidence. Although the net topography effect is much larger for some incidence angles, the background amplification correspond to free-field motion determines the overall amplification in most cases. Except for slope angle $\alpha = 60^\circ$, where we have a trough in horizontal amplification factor due to destructive interference, the vertical incidence could
be considered as a controlling case. For vertical component, on the other hand, the combination of large free-field amplifications of oblique incidences \( (i = 45^\circ \text{ and } 60^\circ) \) and topographic amplification result in overall peak amplification for most slope-incident scenarios. We investigated the effects of various geometry and excitation parameters on the topographic amplification for 2D geometries. Now, we are ready to add another level of geometric complexity by introducing the new constraint in third dimension. In the next chapter, we will present a similar systematic analysis for 3D convex topographic features.
CHAPTER 5
THREE-DIMENSIONAL TOPOGRAPHY EFFECTS

5.1. Introduction

A common practice in engineering analysis is to reduce a complex 3D problem into a 2D case using simplified assumptions for geometry and excitation. However, such simplification brings about an error (either underestimate or overestimate) that could be significant in some cases. For problems in elastodynamics, where we are dealing with the propagation and scattering of elastic waves, the consequences of this reduction are more evident. For example, while diffracted body waves from a point of discontinuity are cylindrical waves of $r^{-1}$ geometric attenuation for 2D problem, they are spherical waves with $r^{0.5}$ attenuation in 3D case ($r$ defines the distance from the source). In chapters 2 to 4, we studied the controlling geometry and excitation factors on the topography effect for 2D case. Although the investigation provides us with insight into fundamental characteristics of underlying wave mechanism, it lacks some real features. The next step towards the more realistic configuration in the analysis of topographic effects is to consider the 3D geometry. We replace the 2D model, which represents the plane strain condition for in-plane polarized $SV$ wave, with an idealized convex 3D model. Two sets of perpendicular constraints (in-plane and out-of-plane) are responsible for several diffracted wavefields that interact with each other to generate a more complex pattern of constructive and destructive interference. Generally, we expect to see a larger number of oscillations in the response associated with higher peaks and lower troughs. In the next section, we define the general configuration of our numerical model. It describes a combination of four mutually perpendicular slopes located at some specific distance from each other. We categorize the problem into three groups based on this distance and present the amplification response for each case.
5.2. Parametric Investigation of 3D Topography Effects

In this section, we systematically investigate the response of idealized 3D convex topographic features (Figure 5.1) to vertically propagating $SV$ waves. Similar to the 2D case, we parameterize the problem as a function of the slope angle ($\alpha$) and dimensionless width ($\zeta = D/\lambda$) which is assumed to be the same in both horizontal directions. All the simulations of this chapter are performed for Poisson’s material ($\nu = 0.25$) and unit dimensionless high ($\eta = H/\lambda = 1$). Figure 5.1 shows the numerical model used in our simulations. In order to minimize the spurious reflections from truncating boundary, a combination of quiet absorbing boundary and free-field zones has been used. The scattered wavefield resulted from various parts (toe, face and tip) of four mutually perpendicular adjacent slopes interact with each other. The extent of this interaction is determined by the width distance. We can define three different problems for three possible scenarios of total wavefield:

i. Single corner topography ($\zeta \to \infty$): There is no theoretical interaction between the wavefields generated from two opposite sides. Therefore, we can attribute all components of the total wavefield to a single 3D scatterer;

ii. Pyramid topography ($\zeta = 0$): Maximum interaction occurs between all four sides. For some slope angles (close to $\alpha = 45^\circ$), however, this process does not necessarily result in maximum amplification;

iii. Incomplete pyramid (square frustum) topography ($\zeta < \zeta_{thld}$): there is a threshold width distance beyond that the interaction of two opposite sides has no effect on the peak amplification. It only creates minor components in the total wavefield.

For each sub-problem, we present the variation of peak amplification with problem parameters. In addition, plots of seismogram synthetics and snapshots of wavefield at different time steps are presented for discussion purpose.
5.2.1. **Single Corner Topography** ($\zeta \rightarrow \infty$)

A single corner, which could be considered as a 3D analogue of single slope, is a common topographic feature in real world. Hills with the wide top width provide an appropriate area for building structures. Such topographic irregularities could be adequately described using the single corner model. In this section, we conduct a set of parametric studies on the role of side slope angle in topographic amplification. A complete range of practical slope angles ($\alpha = 0 - 90^\circ$), which contains all cases from flat ground to vertical cliff, is considered. Figure 5.2 shows the variation of horizontal amplification factor with slope angle for both 2D slope and 3D corner. The response of single corner is generally similar to the single slope because of same interference patterns (scattering mechanism). Increasing the slope angle, we have two cases of constructive interference at $\alpha = 50^\circ$ and $73^\circ$ each shifted with respect to the single slope ($\alpha = 39^\circ$ and $84^\circ$). We could ascribe the absolute value of the angle shift ($11^\circ$) to the angle difference between slope side and edge. In the 2D single slope, there is only one angle controls the generation and propagation of surface waves. For the single corner, on the other hand, side and edge angles

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**Figure 5.1.** General configuration of the numerical model for 3D topography problem
are different (For square frustum they are related by a factor of $\sqrt{2}$). The results show that the latter angle controls the consequent surface waves and thence the amplification factor. Destructive interference, on the other hand, occurs almost at the same slope angle i.e. $\alpha = 50^\circ$. It demonstrates the dependency of deamplification mechanism to the corner toe rather than its crest. In addition, the net 3D effect, which could be defined as a difference between two curves, is larger at steeper slopes. It is expect because of more severe wave focusing within the corner at this range of slope angles. We could explain these feature better using seismogram synthetics and spatial variation of amplification factor. Figures 5.3 to 5.5 show seismogram synthetics of horizontal and vertical acceleration for slope angles $50^\circ$, $59^\circ$ and $60^\circ$, respectively. The 3D scattered wavefield has a similar structure as 2D with higher amplitude at some points. This amplitude difference becomes more evident at larger slope angles. A new feature of 3D seismogram synthetics is a little depression around the crest (red oval in figure 5.3) that reflects the 3D nature of diffracted body waves. We will see a more transparent trace of this feature in the case of pyramid and incomplete pyramid. Figure 5.6 shows the spatial variation of horizontal and vertical amplification factors for all three angles. They contain a typical amplification and deamplification patterns respectively behind the crest and in front of the toe. Therefore, the discussion of wave mechanism presented in chapter 3 is valid for this case as well. Figures 5.7 to 5.9 compare the responses of 2D slope and 3D corner at each specific angle. Again. We could see a similar structure with peaks of higher amplitudes. Pseudo-peaks with exactly the same shape are also depicted in this figure (contribution of toe diffracted waves). A clear difference is a residual horizontal amplification on the right side (upper flat ground) due to wave trapping in the numerical model.
Figure 5.2. Horizontal amplification factor vs. slope angle—2D slope vs. 3D corner—$\eta = 1$
Figure 5.3. Seismogram synthetics of acceleration for single corner (↑: H, ↓: V) – $\alpha=50^\circ$
Figure 5.4. Seismogram synthetics of acceleration for single corner (↑: H, ↓: V) – $\alpha = 59^\circ$
Figure 5.5. Seismogram synthetics of acceleration for single corner (↑: H, ↓: V) – $\alpha = 73^\circ$
Figure 5.6. Spatial variation of amplification factor for single corner (↑: H, ↓: V)
Figure 5.7. Spatial variation of amplification factor for $\alpha = 50^\circ$ – 2D vs. 3D (↑ H, ↓ V)
Figure 5.8. Spatial variation of amplification factor for $\alpha=59^\circ$ – 2D vs. 3D ($\uparrow$: H, $\downarrow$: V)
Figure 5.9. Spatial variation of amplification factor for $\alpha = 73^\circ$ – 2D vs. 3D (↑: H, ↓: V)
5.2.2. Pyramid Topography ($\zeta=0$)

We next consider the amplification of a plane shear wave by the pyramid topography where the maximum interaction occurs between opposite sides. Although the pyramid has very little application in the study of surface topography, its response is important from the theoretical point of view. The results, which reflects multiple reflections between sides as well as extreme superposition of upgoing surface waves, could be considered as an upper limit for the problem. The variation of horizontal amplification factor with wedge angle is presented in figure 5.10 for both 2D and 3D cases. Again, the 3D response shows the similar structure as that of 2D wedge with an angle shift in peak and trough. At large wedge angles, where the wave focusing near the tip is not severe, two models result in almost the same amplification curve. Decreasing the wedge angle beyond the point of destructive interference ($\theta=98^\circ$), the role of side slopes and therefore the 3D effect becomes more pronounced. For example, at wedge angle $\theta=60^\circ$, the 3D feature gives rise to 220% more amplification compared to the 2D wedge. For two characteristic angles $\theta=120^\circ$ and $98^\circ$, where the pyramid response shows a peak and a trough, seismogram synthetics of horizontal and vertical accelerations are presented in figures 5.11 and 5.12, respectively. The case of $\theta=60^\circ$ is also shown in figure 5.13 as a representative point on the ascending branch of amplification curve. Dominant features are typical diffracted waves on both sides as well as multiple reflections whose number increases with at smaller wedge angles. We could also see the trace of 3D diffracted body waves (red oval in figure 5.12), which does not exist in the 2D case. Figure 5.14 compares the components of amplification factor between these characteristic angles. Clear constructive and destructive interferences respectively at $\theta=120^\circ$ and $98^\circ$ and a huge vertical amplification at $\theta=60^\circ$ are among the important features of this figure. Comparing the lower plot of

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1 In fact, we cannot define a single tip wedge angle for the pyramid. However, we used the same definition as that of infinite and finite wedges ($\theta=\pi-2\alpha$) to provide a more meaningful comparison.
figure 5.14 with that of figure 3.20, we could find the same 3D effect on the vertical motion as that of horizontal component (220% increase).

![Figure 5.10](image)

**Figure 5.10.** Horizontal amplification factor vs. slope angle – wedge vs. pyramid – $\eta = 1$
Figure 5.11. Seismogram synthetics of acceleration for pyramid (↑: H, ↓: V) – $\theta = 120^\circ$
Figure 5.12. Seismogram synthetics of acceleration for pyramid ($↑$: H, $↓$: V) – $\theta = 98^\circ$
Figure 5.13. Seismogram synthetics of acceleration for pyramid (↑: H, ↓: V) – $\theta=60^\circ$
Figure 5.14. Spatial variation of amplification factor for pyramid (↑: H, ↓: V)
5.2.3. Incomplete Pyramid (Square Frustum) Topography ($\zeta < \zeta_{thld}$)

The last 3D convex feature we study in this chapter is the incomplete pyramid of equal width i.e. the square frustum. This geometry describes an intermediate case between the single corner and the pyramid topographies. Similar to the case of dam topography, the practical large distance, which gives rise to single corner response, is much smaller than the theoretical value ($\zeta \rightarrow \infty$). Therefore, we can define a threshold distance beyond that the additive effect of opposite sides is negligible ($\zeta_{thld}$). Figure 5.16 shows the peak amplification curve as a function of normalized width for slope angles $\alpha = 15^\circ$. In the corresponding response of dam topography (figure 3.21), each amplification factor represents the maximum absolute value along the surface of topography (2D section). For 3D model, on the other hand, we have several 2D sections defined along the out-of-plane coordinate. Generally, each point in figure 5.16 correspond to a different 2D section of 3D topography. The location of such ‘peak amplification section’ are depicted in figure 5.16 for the same slope angle. Since the problem is symmetric, the results are presented for the half normalized width. For lower normalized widths, where the geometry is close to the pyramid, maximum amplification occurs at the central 2D section (the first four dots). Increasing the width, the location of peak station moves toward the edge of frustum ($D/\zeta = 0.5$). If we ‘roughly’ split the amplification curve into several pieces of the same ascending and descending trends, we could see that for each segment the peak amplification occurs at the same station. For the case of $\alpha = 15^\circ$, while the peak amplification of descending part ($\zeta < ~0.5$) happens at the centerline, it occurs at the edge for ascending segment. Refereeing to figure 5.15 again, we could find a self-similarity between 2D and 3D responses. It shows that the same wave mechanism brings about the amplification patterns. In addition, the threshold width is almost the same for two cases ($\zeta_{thld} = 2$). Figures 5.17 to 5.24 show the same set of plots for slope angles $30^\circ$ to $75^\circ$. For all cases, the 3D curve shows a similar variation with normalized width, which is shifted upward to higher values. Plots of peak
amplification sections also demonstrates the existence of different wave mechanism for
different parts of amplification curve.

To complete the discussion, we pick several characteristic points from the
amplification curve of each slope angle and present more details about the consequent
wavefield. Figures 5.25 to 5.28 show the component of seismogram synthetics and spatial
variation of amplification factor for slope angle \( \alpha = 15^\circ \) at normalized widths \( \zeta = 0.06, 0.5 \)
and 1.13 where it has either a peak or a trough. The switch between the extreme responses
of pyramid (\( \zeta = 0.06 \)) and single corner (\( \zeta = 1.13 \)) is clearly depicted in this figure. Figures
5.29 and 5.30 show the snapshots of total wavefield for the first two normalized widths.
The middle plots, which correspond to the time of maximum amplification, show the
constructive and destructive interferences of direct and scattered (from all four sides)
wavefields. Figures 5.31 to 5.50 show the same set of plots for other slope angles.
Generally, the complexity of the resulting wavefield increases at steeper slopes where we
have surface waves of larger amplitude as well as multiple reflections between adjacent
sides. For small width, the interference of these surface waves gives rise to extreme
amplitude components around the tip (figure 5.46).
Figure 5.15. Horizontal amplification factor vs. normalized width – $\alpha = 15^\circ$ – 2D vs. 3D

Figure 5.16. Location of peak amplification section with respect to center line – $\alpha = 15^\circ$
Figure 5.17. Horizontal amplification factor vs. normalized width – \( \alpha = 30^\circ \) – 2D vs. 3D

Figure 5.18. Location of peak amplification section with respect to center line – \( \alpha = 30^\circ \)
Figure 5.19. Horizontal amplification factor vs. normalized width – $\alpha=45^\circ$ – 2D vs. 3D

Figure 5.20. Location of peak amplification section with respect to center line – $\alpha=45^\circ$
Figure 5.21. Horizontal amplification factor vs. normalized width – $\alpha = 60^\circ$ – 2D vs. 3D

Figure 5.22. Location of peak amplification section with respect to center line – $\alpha = 60^\circ$
Figure 5.23. Horizontal amplification factor vs. normalized width – $\alpha=75^\circ$ – 2D vs. 3D

Figure 5.24. Location of peak amplification section with respect to center line – $\alpha=75^\circ$
Figure 5.25. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=15^\circ, \zeta=0.06$
Figure 5.26. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=15^\circ$, $\zeta=0.50$
Figure 5.27. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=15^\circ, \zeta=1.13$
Figure 5.28. Spatial variation of amplification factor for IP (↑: H, ↓: V) – $\alpha=15^\circ$
Figure 5.29. Snapshots of total wavefield ($\uparrow$: 0.9s, $\leftrightarrow$: 1.02s, $\downarrow$: 1.1s) – $\alpha=15^\circ$ – $\zeta=0.06$
Figure 5.30. Snapshots of total wavefield (↑: 0.9s, ↔: 1.02s, ↓: 1.1s) – $\alpha = 15^\circ$ – $\zeta = 0.50$
Figure 5.31. Seismogram synthetics of acceleration for IP ($\uparrow$: H, $\downarrow$: V) $-\alpha=30^\circ, \zeta=0.56$
Figure 5.32. Seismogram synthetics of acceleration for IP ($\uparrow$: H, $\downarrow$: V) -- $\alpha=15^\circ$, $\zeta=1.0$
Figure 5.33. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha = 15^\circ, \zeta = 1.5$
Figure 5.34. Spatial variation of amplification factor for IP (↑: H, ↓: V) – $\alpha=30^\circ$
Figure 5.35. Snapshots of total wavefield ($\uparrow$: 1.0s, ↔: 1.11s, ↓: 1.2s) – $\alpha=30^\circ$ – $\zeta=0.56$
Figure 5.36. Snapshots of total wavefield (↑: 1.0s, ↔: 1.09s, ↓: 1.2s) – $\alpha = 30^\circ$ – $\zeta = 1.50$
Figure 5.37. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=45^\circ, \zeta=0.88$
Figure 5.38. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha = 45^\circ$, $\zeta = 1.56$
Figure 5.39. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) $\alpha=45^\circ$, $\zeta=2.13$
Figure 5.40. Spatial variation of amplification factor for IP (↑: H, ↓: V) – $\alpha = 45^\circ$
Figure 5.41. Snapshots of total wavefield (↑: 1.0s, ↔: 1.23s, ↓: 1.3s) – $\alpha=45^\circ$ – $\zeta=0.88$
Figure 5.42. Snapshots of total wavefield (↑: 1.0s, ↔: 1.44s, ↓: 1.3s) – $\alpha = 45^\circ$ – $\zeta = 2.13$
Figure 5.43. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=60^\circ, \zeta=0.50$
Figure 5.44. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha=60^\circ, \zeta=0.81$
Figure 5.45. Spatial variation of amplification factor for IP (↑: H, ↓: V) – α=60°
Figure 5.46. Snapshots of total wavefield (↑: 1.1s, ↔: 1.29s, ↓: 1.4s) – $\alpha=60^\circ – \zeta=0.06$
Figure 5.47. Snapshots of total wavefield (↑: 1.1s, ↔: 1.17s, ↓: 1.3s) – $\alpha = 60^\circ$ – $\zeta = 0.5$
Figure 5.48. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) – $\alpha = 75^\circ, \zeta = 0.69$
Figure 5.49. Seismogram synthetics of acceleration for IP (↑: H, ↓: V) $\alpha = 75^\circ, \zeta = 1.13$
Figure 5.50. Spatial variation of amplification factor for IP (↑: H, ↓: V) – $\alpha = 75^\circ$
5.3. Conclusions

In this chapter, we added the next level of geometric complexity to the problem of topography amplification. An idealized 3D convex feature has been considered in the form of square frustum. We used the same numerical scheme as 2D case to simulate the propagation of vertical shear wave within this geometry. Based on the normalized distance between two opposite slopes, the general problem has been divided into three different cases i.e. single corner, pyramid and square frustum. In the first two cases, we obtained the variation of peak amplification factor on the surface of topography with the slope angle (for the fixed dimensionless height $\eta = 1$). The amplification response of single corner geometry, which is a 3D analogue of the single slope, has the same structure as 2D case. It contains two peaks and a trough respectively corresponding to constructive and destructive interferences. However, due to additional out-of-plane constraint, stronger wave focusing occurs in this case that results in larger peak amplitudes. In addition, there is an angle shift at peak values, which could be attributed to different angles of slope side and edge. We observed the same relation between 2D and 3D responses of the pyramid topography. However, because of interaction between two close opposite sides, additional 3D effects (at larger slope angles) becomes more pronounced. For both single corner and pyramid, we supplemented our discussion through a set of seismogram synthetics and amplification curves for characteristic angles. The third geometry i.e. square frustum shows an intermediate amplification response between two previous extreme cases. In this case, an additional parameter controls the intensity and the shape of consequent scattered wavefield i.e. normalized width. To see the effects of this parameter, we froze the slope angle at several representative values (covering the applicable range) and obtained the amplification curve for each case. The results show a clear similarity between 2D and 3D responses with higher amplitudes at the latter case. In 3D case, the station at which maximum amplification occurs belong to a 2D domain (surface of topography). Therefore, when we are comparing the results with 2D case (1D boundary line), it is important to
locate the section of peak amplification. Generally, for geometries close to the pyramid, peak amplification occurs at the centerline where we have maximum interference of two upgoing surface waves with direct incidence. For large normalized distances, which show a single corner behavior, we could observe the peak amplification near the edge of topography feature. Thus far, we investigated the effects of geometry and excitation parameters on the topographic amplification. In the next chapter, we will incorporate the material type scattering by adding stratigraphy to our models.
CHAPTER 6
SOIL-TOPOGRAPHY COUPLING EFFECTS

6.1. Introduction

The material and geometric properties of near surface layers, which is known as local site condition, can significantly change the characteristics of incoming seismic waves compared to the case of flat homogeneous linear elastic half-space. The effects of geometric discontinuities, from the simple case of 2D infinite wedge to more complex 3D convex features, have been studied in chapters 2 to 5. The results, which are presented in the form of topography amplification patterns, could be used to explain field observations in a more realistic way. Nonetheless, a complete site response analysis for strong ground motion should also account for the change in material properties. In this chapter, we investigate the effect of surface topography when it is combined with subsurface soil stratigraphy as a simple case of material discontinuity. We first perform a systematic analysis on the soil-topography coupling effects for 2D convex features. Two different types of near surface layering has been added to the general form of dam-type topography. As it is discussed in chapter 3, the model could be reduced to either wedge or single slope at two extreme values of width. Controlling parameters, i.e. the thickness of upper layer and the stiffness contrast between two layers have been defined in dimensionless form as usual. Furthermore, and to have a better understanding of coupling effects in actual cases, two case studies are presented and discussed in the next part. The first one is MYG004 strong ground motion station that we used in chapter 4 to see the effect of oblique incidence. Now, we add a surface layer of lower stiffness and compare the amplification factors with homogeneous case for vertical incidence. The next example is CI-MSC strong motion station in California. We extract the middle 2D section in both NS and EW
directions and obtain the amplification curves for different scenarios of soil stratigraphy. Furthermore, we use a layered 3D model to see the coupled soil-topography effects in more realistic condition.

6.2. Parametric Study of 2D Soil-Topography Coupling Effects

In this section, we systematically investigate the soil-topography coupling effects for the 2D dam-type topography. Figure 6.1 shows the configuration of two layered models subjected to vertically propagating SV wave. In addition to dimensionless parameters of single layer model i.e. $\eta$ and $\zeta$, we need to define a set of new parameters accounting for upper layer geometry and material. They consist of dimensionless free-field thickness of upper layer ($\xi = h/\lambda$) and the ratio of shear wave velocities ($Q$) which is less than unity for

![Figure 6.1. Configurations of layered models used for investigating the coupling effect](image-url)

$$\xi = \frac{h}{\lambda}$$

$$Q = \frac{V_{s,bot}}{V_{s,top}}$$
near surface soft layer. All the simulations are performed for slope angle $\alpha=45^\circ$, Poisson’s material ($\nu = 0.25$) and unit dimensionless high ($\eta = H/\lambda = 1$). We present the variation of horizontal peak amplification with the dimensionless width for several combinations of $Q$ and $\xi$. Furthermore, plots of seismogram synthetics, spatial variation of amplification factor and snapshots of wavefield at different time steps are presented for discussion purpose.

Before we go through the results, let us speculate about the response of layered models based on our previous findings for dam-type topography (chapter 3). For the layered model with horizontal interface, henceforth denoted by M1, we solve a similar problem as in the case of single layer dam (no refraction for vertical incidence). However, there are two differences between M1 and the original problem. First, we have incoming wave of less energy depending on the stiffness contrast. For stiffness ratios considered in this study, the soil amplification (1D site response) controls the overall behavior at far-field and gives rise to larger amplification factors. Another deviation of M1 from the original problem is its shallower depth. This results in multiple reflections between two layers (the stiffness contrast controls the number of reflections) and higher order diffracted waves (both body and surface) from tips and toes. The layered model of parallel interface, referred to as M2, has more sources of scattering (tips and toes of interface) and narrower surface region for trapping the incident energy. Therefore, we expect to see a larger number of reflections between two layers compared to model M1. Figure 6.2 shows the peak amplification curve as a function of normalized width for both layered models and several sets of $Q$ and $\xi$. In each curve, the blue curve denotes the response of single layer topography ($\eta = 1$ curve in Figure 3.24). As we expected, the amplification curves of layered models are similar to that of single layer. The peak amplification factor occurs at the same dimensionless width as in the single layer case ($\zeta = 0.88$). Furthermore, they are clustered in three different categories (black, red and blue) based on the stiffness contrast with higher values at larger contrast. The dimensionless thickness of upper layer, on the other hand, has little effect on the amplification response. This is also expected as the
constructive interference of first arrivals (direct and diffracted waves) gives rise to peak amplification. The lower plot of Figure 6.2 shows a different wave mechanism for model M2. First, they are no longer similar to the single layer model inasmuch as peaks and troughs occur at different dimensionless widths. In addition, there is no clustering based on stiffness contrast. Instead, the thickness of upper layer play a more important role in the form of amplification curves. One could see that the curves of same line type and different colors are almost similar. This is because the free surface boundary and interface form a single scatterer whose characteristic length (thickness) controls the consequent wavefield. Finally, the threshold dimensionless width, beyond that the dam response turns into that of single slope, is much larger than model M1. For example, the amplification curve of $Q=4$ and $\xi=1/4$ is still changing at $\zeta=4.0$. The energy trapped in the upper layer, which reflects back and forth between two boundaries, accounts for such oscillatory behavior.

To complete the discussion, we pick several characteristic points from the amplification curves of each layered model and present more details about the consequent wavefield. Figures 6.3 and 6.4 show the component of seismogram synthetics at the peaks of model M1 ($\zeta=0.88$, $\xi=1/4$) respectively for stiffness contrast 2 and 4. They have the same general structure (diffracted surface and body waves from tips and toes on both sides) as that of single layer case (Figure 3.40). A distinguishing feature, which is more pronounced in larger contrast (Figure 6.4), is the reverberation of incoming wave in the upper layer. Traces of low amplitude diffracted Rayleigh waves are masked by those of body waves. Figure 6.5 compares the spatial variation of amplification factor of these layered models with that of single layer. We could check the nonlinear coupling between soil and topography amplifications in this figure. The blue curve shows the pure topography amplification factor of 2.05. Red and black curves show soil amplification (free-field response) of 1.49 and 2.03 for $Q=2$ and 4, respectively. We can also extract the total (coupled) amplification factors from these curves as 2.83 and 3.38. It is evident that
we cannot obtain the total amplification by multiplying topography and soil amplification factors. We can draw the same conclusion from the vertical component. Figures 6.6 and 6.7 show the snapshots of total wavefield for model M1 with $Q=2$ and 4. The middle plots, which correspond to the time of maximum amplification, show the constructive interferences of direct and diffracted wavefields (first arrivals). The third plots show the part of incident energy that is trapped in the upper layer and generates the subsequent multiple reflections. Figures 6.8 to 6.12 and 6.13 to 6.17 show the same set of plots for model M2 ($\zeta=1/4$ and $Q=2,4$) at two characteristic width. The first set correspond to $\zeta=0.88$, where both single layer and M1 models have a peak amplification. It shows that model M2 has a trough rather than a peak due to destructive interference at this width. At larger width ($\zeta=1.84$), however, diffracted surface waves constructively interfere with the transmitted part of incoming wave to form the peak horizontal amplification. This is clearly shown in the middle plots of total wavefield snapshots (Figures 6.16 and 6.17). Comparing the results of model M2 with those of M1, we could interpret the following features:

i. Larger number of multiple reflections because of shallower depth in the middle region (between two toes) and more points of scattering;

ii. Faster attenuation of the scattered wavefield due to more energy leakage within the middle region;

iii. More regular structure of scattered wavefield since the upper layer acts as a unit;

iv. More complex coupling between soil and topography amplifications and hence clearer evidence of its nonlinearity in the middle region (Figure 6.10↑);

v. Having a bi-material medium of irregular interface, both types of surface waves i.e. Rayleigh and Stoneley exist in model M2.
Figure 6.2. Horizontal amplification factor vs. normalized width – (↑: M1, ↓: M2)
Figure 6.3. Seismogram synthetics of acceleration, M1 (↑: H, ↓: V) – $\zeta = 0.88$, $\xi = 1/4$, $Q = 2$
Figure 6.4. Seismogram synthetics of acceleration, M1 (↑: H, ↓: V)–ζ = 0.88, ξ = 1/4, Q = 4
**Figure 6.5.** Spatial variation of amplification factor for M1 (↑: H, ↓: V) – $\zeta=0.88$
Figure 6.6. Snapshots of total wavefield (↑: 0.7s, ↔: 0.98s, ↓: 1.5s) – $\zeta = 0.88$, $\bar{\zeta} = 0.25$, $Q = 2$
Figure 6.7. Snapshots of total wavefield (↑: 0.5s, ↔: 0.85s, ↓: 1.2s) – $\zeta=0.88$, $\bar{\zeta}=0.25$, $Q=4$
Figure 6.8. Seismogram synthetics of acceleration, M2 (↑:H, ↓:V) – $\zeta = 0.88$, $\xi = 1/4$, $Q = 2$
Figure 6.9. Seismogram synthetics of acceleration, M2 (↑: H, ↓: V) – $\zeta = 0.88$, $\xi = 1/4$, $Q = 4$
Figure 6.10. Spatial variation of amplification factor for M2 (↑: H, ↓: V) – ζ=0.88
Figure 6.11. Snapshots of total wavefield (↑: 0.7s, ↔: 0.85s, ↓: 1.5s) – $\zeta=0.88$, $\xi=0.25$, $Q=2$
Figure 6.12. Snapshots of total wavefield ($\uparrow$: 0.5s, ↔: 0.72s, ↓: 1.2s) – $\zeta$ = 0.88, $\xi$ = 0.25, $Q$ = 4
Figure 6.13. Seismogram synthetics of acceleration, M2 (↑: H, ↓: V) – $\zeta = 1.84, \bar{\zeta} = 1/4, Q = 2$
Figure 6.14. Seismogram synthetics of acceleration, M2 (↑: H, ↓: V) – $\zeta = 1.84$, $\zeta = 1/4$, $Q = 4$
Figure 6.15. Spatial variation of amplification factor for M2 (↑: H, ↓: V) – $\zeta = 1.84$
Figure 6.16. Snapshots of total wavefield (↑: 0.7s, ↔: 0.84s, ↓: 1.5s) – $\zeta=1.84$, $\bar{\zeta}=0.25$, $Q=2$
Figure 6.17. Snapshots of total wavefield (↑: 0.5s, ↔: 0.67s, ↓: 1.2s) – $\xi = 1.84, \zeta = 0.25, Q = 4$
6.3. Case Studies

In this section, we investigate the coupling effects of surface topography and soil layering for two actual sites. The first one is MYG004 strong motion station that we used in chapter 4 to analyze the topography effects for non-vertical incidences. Here, we add a surface layer of lower stiffness and calculate the response under vertical incidence to see the coupling effects in a more realistic material and geometry case. Another case study is CI-MSC strong motion station in California. We extract the middle 2D section in both NS and EW directions and obtain the amplification curves for successive levels of material complexity. Furthermore, we use a layered 3D model to study the coupled soil-topography effects in even more realistic setting.

6.3.1. MYG004 Station (Japan)

We transform the homogeneous model of section 4.6 into a layered model by adding a shallow crust of soft material. This layer has a uniform thickness of 6m (1/4 of slope height) and its shear wave velocity is 1/4 of underlying medium (155 m/s). A Ricker type vertical SV wave of the same central frequency is used as an input motion. Figures 6.18 shows the components of seismogram synthetics for the layer models. As compared to the homogeneous case (Figure 4.54), reverberation of incident wave within the upper layer gives rise to a more complex scattered wavefield. Each time the upcoming wave hits the hill (free surface) and diffracted from its geometric discontinuities, a new set of diffracted waves (body and surface) is generated. Energy leakage to the lower layer results in higher order reflections of lower amplitude. Figure 6.19 compares the spatial variation of amplification factors between two models. Again, the nonlinear coupling effect could be checked by comparing the pure topography, soil and overall amplification factors. While the homogeneous model shows no amplification at the recording station, the layered model amplifies the free-field motion by a factor of two. A large vertical motion recorded at the same location renders the role of surface layering more critical.
Figure 6.18. Seismogram synthetics of acceleration for MYG004 – layered (↑: H, ↓: V)
Figure 6.19. Spatial variation of amplification factor – homogen. vs. layered (↑: H, ↓: V)
6.3.2. CI-MSC Station (Southern California)

In this section, we study the effects of surface topography on the aggravation of seismic motion in the vicinity of a strong motion station site at Southern California (SC). Simulations are conducted for idealized surface topography models consist of both homogeneous and stratified soil formations. Results revealed the additive effects of topography and soil response and the role of ground motion characteristics in site amplification near irregular topographic features. Comparing the results of 3D model with those of 2D sections highlights the additional effects of out-of-plane scattering. For this site, which has three layers characterized by a stiffness contrast of approximately 10, the observed amplification cannot be explained by the effects of topography or stratigraphy alone. Coupling of the two effects leads to amplification patterns that could be underestimated or overestimated by a homogeneous topography model assumption, depending on the site response characteristics and ground motion frequency content. A parametric study is conducted to investigate the effects of successively more complex layering scenarios in the overall amplification. All models are subjected to vertically propagated $SV$ waves of Ricker type with various central frequencies. Seismogram synthetics and spatial variation of amplification factors are presented to illustrate the relative contribution of soil and topography effects in the overall ground motion amplification.

6.3.2.1. Geometry and Material

The bird-eye view of station stations CI-MSC is shown in Figures 6.20(a) for stations CI-MSC. The site is characterized by low sediment site with $V_{S30}=377 \text{ m/s}$ while the later is a hard rock site with $V_{S30}=1029 \text{ m/s}$. To put within the framework of our general 3D model, we approximate the real irregular surface topography by a simplified regular 3D geometry shown in Figures 6.20(b). Figures 6.20(c,d) show a scaled view of EW and NS cross section geometries along with the stiffness of each layer. As we can see in Figure
6.20(b,d), long distance between two slopes in the NS direction causes the feature to behave as 2D in the EW direction. The NS section, however, will be affected by 3D topography characteristics.

**Figure 6.20.** Station location and idealized numerical model at CI-MSC: (a) Bird-eye view, (b) Simplified model, (c) EW section, (d) NS section
6.3.2.2. Homogeneous vs. Layered

In order to study the effect of soil layering (material) when it is coupled with surface topography (geometry), we considered three different scenarios for the EW section of CI-MSC i.e. entirely homogeneous, two layered and truly layered (Figures 6.21 to 6.23). The 2-layers model is similar to real 3-layers model with the upper low velocity sediment replaced by middle rock. All 2D sections are subjected to a series of vertically incident $SV$ Ricker pulses with central frequencies of $f_0 = 3, 7$ and $15 \text{ Hz}$. Results of spatially distributed peak acceleration values normalized by input motion are shown at the bottom of these figures. The normalized acceleration for homogeneous case clearly shows the doubling effect associated with free surface. It also demonstrates that the aggravation of motion with wavelength of the order of feature characteristic length is more pronounced (60% for 3Hz case). Accounting for surface layering in the model, we can see a significant amplification at far-field (1D soil response) and different amplification pattern near the feature (shifting the location of the peak from the vertex to the vicinity of the toe, similarly to a case of basin-edge effects). To have a better quantitative assessment of soil-topography coupling effect, we could represent the results in a different framework by freezing the input frequency. Figures 6.24 shows the amplification ratio for 3 different models subjected to incident motion of $7 \text{ Hz}$. Simple math at the bottom simply shows that the soil-topography coupling effect cannot be obtained by superposition of 1D site response and geometric amplification. The substantial difference in predicted amplification between pure topography case (homogeneous) and soil-topography coupling case (layered) can be attributed to the complexity of the wavefield in two models. Figure 6.25 shows wave propagation snapshots of two models subjected to the same input at the same instant.
Homogeneous, $V_s = 1400$ m/s, Height = 42 m
Top Width = 10 m

**Figure 6.21.** Spatial variation of peak acceleration for homogeneous model
Figure 6.22. Spatial variation of peak acceleration for 2-layers model
Figure 6.23. Spatial variation of peak acceleration for 3-layers model
Topography Effect: $2.6/2 = 1.3$
Layering Effect (2L): $3.5/2 = 1.75$
Layering Effect (3L): $5.3/2 = 2.65$
Layering (2L) $\times$ Topography = 2.8 $\neq$ 3.7
Layering (3L) $\times$ Topography = 3.5 $\neq$ 7.6

Figure 6.24. Soil-topography coupling effect
Figure 6.25. Snapshots of total wavefield: (a) Homogen., (b) Layered; The latter yields 2.2 times higher PGA
6.3.2.3. Additional Effects of 3D Configuration

3D response of strong motion station site subjected to EW incident motion is the topic of our next simulations. The components of seismogram synthetics at the middle section are compared with those of 2D model (Figures 6.26 and 6.27). Furthermore, results of spatially distributed peak acceleration values normalized by rock outcrop motion are presented in figure 6.28 for quantitative comparison purpose. They clearly prove our initially hypothesis about 2D behavior of CI-MSC in NW direction. As a summary, time histories of horizontal acceleration at different points of 1D/2D/3D models are compared in Figure 6.28. From top to bottom, they consist of the following records:

i. Homogeneous 1D: The free-field response of single layer model that used in the definition of amplification factor (normalization propose);
ii. Layered 1D: The free-field response of 3-layers model that shows the 1D site response (soil amplification);
iii. Homogeneous 2D – Crest: The response of 2D homogenous model at its crest that shows the pure topography amplification;
iv. Layered 2D – Crest: The response of 2D layered model at its crest that shows the coupled amplification factor and is different from ii × iii;
v. Layered 3D – Crest: The response of 3D layered model at its crest that shows additional effects of out-of-plane scattering (maximum among all cases);
vi. Homogeneous 2D – Toe: The response of 2D homogenous model at its toe that shows geometric deamplification;
vii. Layered 2D – Toe: The coupled amplification factor of 2D layered model at its toe (vi < vii < iv);
Figure 6.26. Seismogram synthetics of horizontal acceleration – EW (↑: 2D, ↓: 3D)
Figure 6.27. Seismogram synthetics of vertical acceleration – EW (↑: 2D, ↓: 3D)
Figure 6.28. Spatial variation of amplification factor – EW (↑: H, ↓: V)
Figure 6.29. Time histories of horizontal acceleration at crest/toe of 2D/3D model
6.4. Conclusions

In this chapter, we investigated the coupling effects of surface topography and soil stratigraphy through a series of systematic analysis and case studies. The former has been fulfilled using two general configurations of layered model. These models, which are built upon the framework of 2D dam-type topography, reflect two common scenarios of subsurface layering i.e. horizontal layering and surface layer of constant thickness. We used the same numerical scheme as homogeneous case to simulate the propagation of vertical shear wave within these models. Similar to the dam-type topography, we could extract the responses of layered wedge and single slope at extreme values of dimensionless width. In addition to previously defined parameters $\eta$ and $\zeta$, the thickness of upper layer and the stiffness contrast between two layers have been also defined in dimensionless form. To keep the total number of possible cases within a practical range, we performed the analysis using fixed slope angle ($\alpha=45^\circ$) and dimensionless height ($\eta=1$). For each layering scenario, amplification factors have been presented as a function of dimensionless width at several sets of dimensionless thickness and stiffness contrast. For the layered model with horizontal interface (M1), the scattered wavefield is similar to the homogeneous case as no scattering occurs at the interface. However, there exists a lower boundary at a finite depth bringing about multiple reflections. Since the amplitude of these reflections are not dominant (energy leakage at the interface), the form of amplification pattern is similar to that of homogenous case. Furthermore, the peak amplification factor of all parameter sets occurs at the same dimensionless width as in the homogeneous case ($\zeta = 0.88$). As for the effects of stiffness contrast and thickness, the former cluster the curves in distinct categories while the latter shows negligible impact. The layered model of parallel interface (M2) contains several more sources of scattering and narrower surface region for trapping the incident energy compared to M1. Therefore, it involves a different wave mechanism and consequently a different amplification pattern (peaks and troughs are no longer coincide with homogeneous case). Furthermore, the thickness of upper layer has more influence on the form of amplification curves. It could be
explained as the free surface boundary and interface act as a single scatterer whose characteristic length (thickness) controls the overall behavior. As for the single slope response, we observed that the threshold dimensionless width shifts toward larger values. The reflection of transmitted energy between two boundaries (free surface and interface) accounts for such oscillatory behavior. The spatial variation of amplification factors at selected characteristic point clearly show the nonlinear coupling between soil and topography amplification. Finally, and to have a better understanding of soil-topography coupling effects in more realistic settings, two case studies have been presented and discussed. The results show that 3D layered model, which contains out-of-plane scattering effects in addition to 2D soil-topography effect, gives the maximum amplification factor.
CHAPTER 7
CONCLUSIONS AND FUTURE WORK

7.1. Summary and Conclusions

Goal of this dissertation was to reduce the quantitative gap between previous theoretical findings on topography effects and field observations, through a more systematic analysis of the governing parameters of the problem. To this end, we performed a series of parametric studies on the ground motion modification by topography features, where we increased gradually the complexity of the analyses to understand the additive mechanisms of 2D, 3D scattering and focusing of seismic waves and their coupling to the effects of stratigraphy. We started from the topographic amplification caused by a 2D infinite wedge on the surface of a homogeneous elastic halfspace, and extended the state-of-the-art understanding of wave focusing and scattering by this fundamental block of irregular ground surface geometries. From there, we gradually increased the geometric and stratigraphic complexity up to a 3D convex layered topographic feature. The research findings at each level are summarized below:

Infinite Wedge

We found two characteristic wedge angles on the amplification response of an infinite wedge. The former, which is related (but is not equal) to the critical angle of incidence shows the maximum amplification due to the constructive interference of upgoing surface waves. By increasing the Poisson’s ratio, the peak of horizontal amplification curve is shifting toward larger angles while its amplitude decreases passing its maximum value (~9) at $\nu=0.25$. Other characteristic angle ($\theta=45^\circ$), which shows zero amplification, corresponds to the diffraction free case and is
independent of material properties. Since the wedge is infinite, there is no characteristic length involved and the amplification response does not depend on the frequency of excitation.

**2D Convex Topography**

The results demonstrate that the amplification response of general dam type topography primarily depends on its dimensionless width. When the width is very large, two single slope components of the dam respond to incoming wave separately. For very small width, on the other hand, we have maximum interaction between two slopes. The amplification response has been obtained for these cases as a function of slope angle and dimensionless height. Several characteristic angles corresponding to constructive and destructive interferences have been identified. Below the angle of trough, single slope and wedge of unit dimensionless height have peak amplifications of 1.6 and 2.0 respectively at \( \alpha = 39^\circ \) and 28°. While the single slope shows another peak beyond that angle, the wedge amplification factor monotonically increases. For higher frequencies, the peak amplification of single slope shows a uniform increase (2.3 for \( \eta = 4 \)) while the wedge response is more complicated. As for the dam topography, we investigated the additional effect of width by freezing the slope angle. The curves show an oscillatory behavior due to interference of different wave components (generated at toe/tip of each side). For intermediate dimensionless width (namely configurations distinct from the wedge), the slope angle close to the critical value shows the maximum amplification factors at each frequency level (3.3 for \( \eta = 4 \)).

**Oblique Incidence**

Changing the angle of incidence \( (i) \), we found that the topography feature alters the free-field motion of oblique incidence in a different amplification and deamplification pattern compared to the vertical incidence. Although the net topography effect is much larger for some incidence angles, the background
amplification of the free-field motion determines the overall amplification in most cases. Except for slope angle $\alpha = 60^\circ$, where we have a trough in horizontal amplification factor due to destructive interference, the vertical incidence could be considered as a controlling case. For vertical component, on the other hand, the combination of large free-field amplifications of oblique incidences ($i = 45^\circ$ and $60^\circ$) and topographic amplification result in overall peak amplification for most slope-incident scenarios.

### 3D Convex Topography

Similar to the case of dam topography, the results of square shows the controlling role of the dimensionless width. For very small and very large (above the threshold value) dimensionless width, the general problem reduced to that of a single corner and complete pyramid i.e. 3D variants of single slope and wedge. The amplification response of single corner and pyramid geometries follow the same trends as their 2D counterparts. However, due to the additional out-of-plane constraints, stronger wave focusing occurs that results in larger peak amplitudes. For single corner we observed 24% and 32% increase in the amplification factor at $\alpha = 39^\circ$ and $83^\circ$ (below and above the trough angle) compared to the single slope. Pyramid topography shows a significantly different behavior on two sides of trough angle i.e. 14% at $\alpha=39^\circ$ and 220% at $\alpha=60^\circ$ and the 3D effect is still increasing at steeper angles. In addition, there is an angle shift at peak values between 2D and 3D cases, which could be attributed to different angles of slope side and edge. To see the effect of dimensionless width for intermediate case of 3D topography, we froze the slope angle at several representative values and obtained the amplification curve for each case. The results show a clear similarity between 2D and 3D responses with higher amplitudes at the latter case. We observed 25% increase for slope angle $\alpha = 39^\circ$ and unit dimensionless height. According to our previous findings on the
effect of slope angle and frequency, we expect to see even larger 3D effects for edge slope angles close to critical values and higher frequency incidences.

**Soil-Topography Coupling Effect**

Comparing the combined effects of surface topography and soil stratigraphy with those of pure topography and stratigraphy shows that the phenomena are nonlinearly coupled, that is new effects emerge from the combination of stratigraphy and topography that would not have been present otherwise. The results suggest that in real conditions, topographic amplification can only be quantitatively captured when geometry and stratigraphy of the site are simultaneously accounted for in theoretical predictive models.

In conclusion, to capture the quantitative effects of topography that have been measured in field studies and inferred from intensity measurements, we need predictive models with detailed considerations of geometry, excitation and material properties such as the ones presented at the top level of this study (Chapter 6). Specifically for the geometry, the 3D predictive model should be accurately depicted on the scale that is comparable to the incident wavelength, where the effects of topography are the strongest.

### 7.2. Broader Impacts

The outcome of this research will allow the geotechnical earthquake engineers and earth scientist to understand the complexity of 3D site effects (geometry and layering) and recognize that these effects should be accounted for in simulated ground motions and in performance-based design procedures and building code provisions. Our work will contribute in the long run to hybrid and synthetic hazard maps and attenuation relations, particularly for extreme event scenarios where 3D effects will not only contribute to ground motion amplification, but will also encapsulate the inability of near surface soils to transmit stress pulses that exceed the material strength thus bounding ground motion parameters for rare events.
7.3. **Recommendations for Future Research**

The insights provided by this work suggest the following areas of potential future works:

i. Developing a general 3D layered model and performing a systematic analysis to see the additional soil-topography coupling effects in 3D case;

ii. Studying the effect of incident angle on topographic amplification in 3D where it has additional component in azimuth direction;

iii. Investigating the effects of more realistic broadband excitations in both 2D and 3D;

iv. Increasing the level of material complexity to obtain the nonlinear amplification response and ultimately seismic slope stability factors of safety and slope failure displacements;

v. Developing a site-specific simplified amplification factor to incorporate topography effects in ground motion prediction equations;

vi. Combining our understanding of topography effects to landslide hazard maps and displacement risk on a regional scale, and propose a framework to capture these effects in a unified way similar to what was shown in this dissertation, namely as a function of the geometry, stratigraphy and ground motion intensity.
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