

**Final Report for Period:** 07/2007 - 06/2008**Submitted on:** 06/27/2008**Principal Investigator:** Etnyre, John B.**Award ID:** 0707509**Organization:** GA Tech Res Corp - GIT**Submitted By:****Title:**

CAREER: Knot Theory and Dynamics in Contact Geometry

**Project Participants****Senior Personnel****Name:** Etnyre, John**Worked for more than 160 Hours:** Yes**Contribution to Project:****Post-doc****Name:** Baker, Kenneth**Worked for more than 160 Hours:** Yes**Contribution to Project:**

What partially supported on the grant during the 2006-08 academic years. During this time he began working on a project with E. Grigsby and M. Hedden aimed at solving the Berge Conjecture using geometric and Heegaard Floer techniques. He also completed joint work with C. Gordon and J. Luecke relating tunnel number to Heegaard genus. Ken and I have also begun work with J. van Horn concerning cabling open books, contact structures and 'rational open books' and surgery on transverse knots in contact structures. Ken also delivered approximately nine talks exposing this work to mathematicians across the country. At the end of this support Baker received a tenure track appointment at the University of Miami.

**Graduate Student****Name:** Herreiros, Pillar**Worked for more than 160 Hours:** Yes**Contribution to Project:**

First year graduate student interested in geometry and topology. Made significant progress in the PhD program.

**Name:** Schak, Benjamin**Worked for more than 160 Hours:** Yes**Contribution to Project:**

First year graduate student interested in geometry and topology. Made significant progress in the PhD program.

**Name:** Fletcher, David**Worked for more than 160 Hours:** Yes**Contribution to Project:**

David Fletcher is a first year graduate student at the University of Pennsylvania. He made progress in the PhD program.

**Name:** Tripp, James**Worked for more than 160 Hours:** Yes**Contribution to Project:**

James Tripp completed his PhD thesis entitled 'Contact structures on open 3-manifolds' in the Spring of 2005. His thesis significantly advanced our understanding of various phenomena that can occur with contact structures on open manifolds.

**Name:** Schoenenberger, Stephan

**Worked for more than 160 Hours:** Yes

**Contribution to Project:**

Stephan Schoenenberger completed his PhD thesis entitled 'Planar open books and symplectic fillings'. The thesis provides large classes of contact manifolds that are supported by planar open books. In addition, it is shown how to use this fact to classify, up to diffeomorphism, the symplectic fillings of such contact manifolds.

**Name:** Vick, David

**Worked for more than 160 Hours:** Yes

**Contribution to Project:**

David Shea Vick is a second year graduate student at the University of Pennsylvania working with the PI. He completed oral exams and began studying contact geometry and low-dimensional topology this year.

Vick is now in his third year of graduate school at the University of Pennsylvania. He was supported for one semester on the grant. Using this support he spent the Fall semester with me at the Georgia Institute of Technology. During this time he made progress on his PhD thesis studying problems in Legendrian knot theory and Heegaard Floer theory.

In the fifth year of this grant Vick wrote two papers. One on higher dimensional linking integrals and one on the Heegaard Floer Legendrian knot invariants. He will finish his PhD next year.

**Name:** Bogdan, Elena

**Worked for more than 160 Hours:** Yes

**Contribution to Project:**

Elena Bogdan is a third year graduate student at the University of Pennsylvania working with the PI. She is currently working on problems involving transverse knots, braids and open books on general 3-manifolds.

Bogdan is now a fourth year student at the University of Pennsylvania. She has made significant progress on bringing techniques of braid theory to a general 3-manifold using open book decompositions.

In the fifth year of this grant Bogdan finished her PhD under the PI's supervision and took a postdoctoral position at Rice University.

**Name:** Silberman, Andrew

**Worked for more than 160 Hours:** Yes

**Contribution to Project:**

Andrew Silberman is a first year graduate student at the University of Pennsylvania interested in geometry and topology. He made significant progress in the PhD program this year.

**Undergraduate Student**

**Technician, Programmer**

**Other Participant**

**Research Experience for Undergraduates**

## Organizational Partners

### Other Collaborators or Contacts

#### Year One:

Ko Honda from USC. We are investigating contact structures on hyperbolic manifolds and studying Legendrian knots in dimension three.

Tobias Ekholm from USC and Michael Sullivan from UMass Amherst. We have established the foundations of contact homology of Legendrian knots in jet spaces (or more generally in exact symplectic manifolds crossed with the real line) and worked out orientation issues in contact homology.

#### Year Two:

Ko Honda from USC. We are investigating the classification of 'Chekanov-Eliashberg Knots'. These were the first examples of non-simple Legendrian knots, despite this there is very little known about their classification.

Tobias Ekholm from USC and Michael Sullivan from UMass Amherst. We finished the paper setting up contact homology for Legendrian submanifolds in jet spaces.

Lenny Ng from Stanford and the American Institute of Mathematics, Ekholm and Sullivan. We are linking Ng's invariants of knots to contact homology (of the co-normal lifts of the knot).

Terry Fuller from Cal. State, Northridge. We have been investigating achiral Lefschetz structures on 4-manifolds.

#### Year Three:

The collaborations with Ekholm, Honda, Ng, and Sullivan mentioned in year two are continuing this year. In addition, the PI has begun the following new projects:

The classification of Legendrian torus links with Lisa Traynor of Bryn Mawr College.

The classification of unknots in overtwisted contact structures with Thomas Vogel of the University of Pennsylvania.

The study of and construction of open books with Burak Ozbagci of Koc University, Istanbul, Turkey.

#### Year Four:

The collaborations with Ekholm, Honda, Ng, and Sullivan mentioned in year two are continuing this year.

The classification of Legendrian torus links with Lisa Traynor and Jennifer Dalton of Bryn Mawr College mentioned in year three continues.

The classification of unknots in overtwisted contact structures with Thomas Vogel of the University of Pennsylvania mentioned in year three continues. We are now trying to achieve the isotopy classification. We have also begun various projects involving confoliations.

In addition, the PI has begun the following new projects:

In work with Burak Ozbagci we have been investigating the construction of open books of small genus and invariants of contact structures derived from open books.

In collaboration with Jeremy van Horn and Ken Baker we have begun to understand cabling of open books, contact structures and 'rational open books' and surgery on transverse knots in contact structures.

In joint work with Jeremy van Horn we have answered a question of Matt Hedden concerning fibered knots and the Bennequin inequality in a contact structure (see below).

Year Five:

The collaborations with Baker, Ekholm, Honda, Ng, Sullivan and Van Horn mentioned in year two, three and four are continuing this year.

The classification of Legendrian torus links with Lisa Traynor and Jennifer Dalton of Bryn Mawr College mentioned in year three continues and should be completed by the end of the summer.

In addition, the PI has begun the following new projects:

In work with Ekholm and Sabloff the PI has investigated duality exact sequences in higher order Legendrian contact homology. In separate work with Sabloff the algebraic structures on Legendrian DGA's has been studied.

New collaboration with Baker on the structure of Legendrian knots has begun.

### Activities and Findings

#### **Research and Education Activities:**

Year One:

Research: I have been continuing my investigations into Legendrian knots in all dimensions, contact structures on various 3-manifolds and fluid dynamics. I have also been studying symplectic cobordisms and open book decompositions.

Education: Standard undergraduate teaching and a year long course in contact geometry at the graduate level. Ran the Geometry and Topology Seminar. Gave a mini course on contact geometry and open book decompositions at the Clay Institute Summer School in Budapest.

Year Two:

Research: I have been continuing my investigations into Legendrian knots in all dimensions, symplectic cobordisms and open book decompositions. In addition, I have begun investigating Lefschetz fibrations on 4-manifolds.

Education: I taught a Graduate Algebraic Topology course and ran the Geometry and Topology Seminar in the Fall of 2005. I wrote a set of lecture notes on open book decompositions that will be published a part of the proceedings of the Clay Institute Summer School in Budapest. I have also written a set of notes about convex surfaces. These have been made available on the web and have proven useful for a great number of graduate students and researchers.

Education and Research: I co-organized, with Yasha Eliashberg and Peter Ozsvath, a month long workshop concerning 'Holomorphic Curves in Low-dimensional topology' at the Institute for Advanced Study in Princeton. This workshop had many mini-course and research talks and was attended by graduate students from across the country and by researchers from around the world.

## Year Three:

Research: I have furthered my study of open book decompositions and contact geometry by defining various invariants associated to a contact structure via an open book. In addition various constructions of open books associated to simple contact manifolds have been derived. The program to show that Ng's combinatorial knot invariant is really contact homology has taken a significant step forward and should be completed this coming year.

Education: I am supervising two undergraduate students in advanced reading courses.

One in Riemannian geometry and the other in topics in point set topology and fractals. In addition I am supervising an REU student who has created very useful computer code for computing invariants associated to the differential graded algebra in contact homology. Moreover, we have found several interesting examples of Legendrian knots that indicate the relative strength of these invariants derived from the contact homology DGA. I also co-organized the Geometry and Topology seminar at Georgia Tech and started a monthly joint seminar between the University of Georgia, Emory and Georgia Tech. I am also advising two new graduate students, David Vick and Elena Bogdan, who are just beginning their study of contact geometry. Finally, I gave a series of lectures at the Park City Mathematics Institute in the summer of 2006, in which I exposed a large number of graduate students and researchers from across the country and around the world to the latest techniques by which contact geometry illuminates low-dimensional topology.

## Year Four:

Research: I have continued my study of invariants of embeddings using contact homology as well as started studying some of the underlying algebraic structures on contact homology in both 3 dimensions and higher dimensions. I have also begun work on studying invariants of contact structures derived from open books as well as the constructions of open books with various properties. While studying open books I have discovered that one can generalize the notion of open books to rational open books and then studied how they relate to standard open books. This study was prompted by investigations into cabling open books and surgery on transverse knots in contact structures. I have also begun to study the space of plane fields on a 3-manifold and in particular the relations between foliations and contact structures in the spirit of Eliashberg and Thurston.

Educations: I am supervising two undergraduate students in a reading course introducing them the ideas in point set topology and low-dimensional topology. I have also been advising two graduate students at the University of Pennsylvania. One of these students: Elena Bogdan has made significant progress on her PhD thesis concerning braiding in general 3-manifold. The other student has begun several problems one of which will eventually turn into his PhD thesis. I have also given a series of lectures at the Park City Mathematics Institute concerning foliations and contact geometry (this topic overlapped a bit with year three above). These lectures were well received and attended by approximately 150 graduate students and researchers. I also wrote a set of lecture notes to accompany the lectures and they will be published in the PCMI proceedings. At the Georgia Institute of Technology I have been running extra classes to attract students and post-docs to low-dimensional topology. Specifically, I have taught a class in symplectic geometry and in Morse theory. In addition I have been developing a first year graduate geometry-topology sequence of classes at the Georgia Institute of Technology. Such a course has been needed for many years and will definitely increase the visibility of geometry and topology at Georgia Tech.

## Year Five:

Research: I have continued my work with Ekholm, Ng and Sullivan on invariants of knots in Euclidean space using contact homology. I have further understood relations between

foliations and contact structures begun in Year Four. In joint work with Van Horn we have classified certain fibered knots with maximal self-linking number and understood how and when the famous Bennequin inequality is sharp. Various projects involving non-loose Legendrian knots, Stein fillings, the structure of Legendrian knots in Euclidean space and the algebraic structure of Legendrian contact homology DGA has been explored.

Education: I have been running a weekly seminar for graduate students to present material and for professor to explain basic material. I have also taught a graduate course on 4-manifold topology and an 'informal' extra course on Hodge theory for graduate students. The extra course and seminars I have run over the past few years at Georgia Tech has helped increased the number of graduate students working in geometry and topology from 0 to 5 or 6 with at least 2 to 3 more coming in this year. Also, the first year graduate course I have been helping to design has now been approved and will be taught for the first time next year. This should further increase the geometry topology group here at Georgia Tech. Elena Bogdan finished her PhD thesis under my supervision this year and another student Shea Vick wrote two papers and made significant progress on his thesis this year as well. I also informally supervised several students at the Georgia Institute of Technology and two visiting students from the Middle Eastern Technical Institute in Ankara Turkey.

### Findings:

#### Year One:

Better understanding of Legendrian knots under the operation cabling. In studying Legendrian cables a new invariant of knots was defined. This new invariant is called the width (or contact width) of a knot and is related to maximal Thurston Bennequin invariant and thus to slice properties of the knot.

Better understanding of symplectic fillings of contact structures.

Invariants of contact structures in terms of open books were defined and proved to be non-trivial. Specifically, the invariant is the minimal genus of an open book associated to the contact structure. The invariant seems to be surprisingly subtle.

A rigorous definition of contact homology for Legendrian submanifolds in high dimensional Euclidean space, jet spaces and exact symplectic manifolds times the real line was achieved. Orientations were understood so that a refinement of contact homology can be made. Using contact homology various double point estimates on exact Lagrangian submanifolds can be obtained.

#### Year Two:

Better understanding of the existence of achiral Lefschetz fibrations on 4-manifolds.

Better understanding of the relation between a combinatorial invariant of knots in Euclidean 3-space and invariants of such knots defined via contact homology.

Better understanding of symplectic fillings of contact structures.

Better understanding of invariants of contact structures that are defined in terms of open books.

Better understanding of contact structures on open 3-manifolds.

#### Year Three:

More invariants of contact structures were defined using open books. The genus of an open book was further studied and a conjecture concerning elliptic open books was

disproved. The 'binding number' (minimal number of binding components for a supporting open book with minimal genus) was defined and computed in many cases.

Various constructions of open books for contact structures obtained through plumbing spheres were obtained. In many cases these were of minimal genus.

Better understanding of Legendrian torus links and Legendrian knots in overtwisted contact structures was obtained, though this work is not yet complete.

Year Four:

Relations between contact structures and foliations were illuminated, including the result that all contact structures are deformations of foliations, not all foliations can be deformed into contact structures and that near foliations with Reeb components you can always find overtwisted contact structures.

Contact surgeries were studied and various conditions under which surgeries on distinct knots yield contactomorphic contact structures.

Better understanding of Legendrian torus links and Legendrian knots in overtwisted contact structures was obtained, though this work is not yet complete.

Rational open books were shown to support contact structures. Open books on manifolds obtained from surgery on transverse knots were constructed. Contact structures associated to cabled open books were also understood.

Relations between fibered knots realizing the Bennequin inequality and contact structures supported by the fibered knot were understood.

Year Five:

More progress has been made on understanding the relation between contact structures and foliations in dimension three.

Many new non-loose Legendrian and transverse knots with novel properties have been found.

A classification of transversal knots realizing the Bennequin bound has been obtained.

An 'duality' exact sequence for the linearized Legendrian contact homology has been found. This leads to improvements on our understanding of double points for exact Lagrangian submanifolds in some cases; in particular, in Euclidean space.

A structure theorem for Legendrian knots in Euclidean spaces was observed and in collaboration with Baker a proof is being worked out. This result says in a given knot type any Legendrian knot with sufficiently negative Thurston-Bennequin invariant destabilizes. This work also leads to finiteness results for Legendrian knots in a fixed knot type with fixed  $tb$  and  $r$ .

The existence of infinitely many simply connected Stein fillings for a given contact manifold was achieved.

### **Training and Development:**

Year One:

The graduate students Herrerros and Schak made significant progress in the PhD program

Year Two:

The graduate students Schoenenberger and Tripp finished their PhD theses. Fletcher made progress in the PhD program.

Year Three:

Andrew Silberman made progress in the PhD program at the University of Pennsylvania. The students David Vick and Elena Bodgan made studied low-dimensional topology and contact geometry and are making progress on their PhD's. Burak Ozbagci was a visiting faculty member under the PI's guidance. During this time he wrote four papers, two co-authored with the PI.

Year Four:

Elena Bogdan made significant progress on here PhD thesis concerning braids in an arbitrary 3-manifold. The graduate student Shea Vick began studying several problems concerning the contact homology and Heegaard Floer homology of various Legendrian knots. Post docs Ken Baker, Anar Ahmadov and Nathan Geer were mentored during this year. Several joint projects were begun with Ken and Anar. Help was given to Nathan for his search for a permanent job and NSF grant applications. They all have also been attending classes on symplectic geometry and Morse theory taught by the PI.

Year Five:

Elena Bogdan finished her PhD thesis on braiding about open books in a general 3-manifold. She will be starting a postdoctoral position at Rice University next year. Shea Vick made significant progress on this PhD thesis and wrote two papers this year. The first is on explicit linking integrals in higher dimensions and the second proves the Heegaard Floer transversal knot invariant is non-zero for bindings of open books and proves a vanishing result for this invariant in other cases. Post docs Ken Baker, Anar Ahmadov and Nathan Geer were mentored during this year. Ken received a tenure track position at the University of Miami, Anar received one at the University of Minnesota (but will spend a year at Columbia first) and Nathan received one at Utah State University (but will spend the year at the Max Plank Institute first). Anar and I finished a paper on Stein fillings. Ken and I finished a project on cablings of open books and rational linking numbers we also began other projects. I have also been informally advising five graduate students at early stages of their studies.

## Outreach Activities:

### Journal Publications

John B. Etnyre, "On Symplectic Fillings", *Algebr. Geom. Topol.*, p. 73, vol. 4, (2004). Published,

John B. Etnyre and Robert Ghrist, "Generic hydrodynamic instability of curl eigenfields", *SIAM J. Appl. Dyn. Syst.*, p. 377, vol. 4, (2005). Published,

John B. Etnyre, "Planar open book decompositions and contact structures", *International Mathematics Research Notices*, p. 4255, vol. 2004, (2004). Published,

John B. Etnyre and Ko Honda, "Cabling and transverse simplicity", *Annals of Math*, p. 1305, vol. 162, (2005). Published,

Tobias Ekholm, John B. Etnyre and Michael G. Sullivan, "The Contact Homology of Legendrian Submanifolds in  $\mathbb{R}^{2n+1}$ ", *Journal of Diff. Geom.*, p. 177, vol. 71, (2005). Published,

Tobias Ekholm, John B. Etnyre and Michael G. Sullivan, "Non-isotopic Legendrian Submanifolds in  $\mathbb{R}^{2n+1}$ ", *Journal of Diff. Geom.*, p. 85, vol. 71, (2005). Published,

Tobias Ekholm, John Etnyre and Mike Sullivan, "Orientations in Legendrian contact homology and exact Lagrangian immersions", *International Journal of Math*, p. 453, vol. 16, (2005). Published,

John B. Etnyre and Terry Fuller, "Realizing 4-manifolds as achiral Lefschetz fibrations", *International Mathematics Research Notices*, p. 1, vol. 2006, (2006). Published,

John B. Etnyre and Burak Ozbagci, "Invariants of contact structures from open books", *Transactions of the AMS*, p. 3133, vol. 360, (2008). Accepted,

John B. Etnyre and Burak Ozbagci, "Open books and plumbings", *International Mathematics Research Notices*, p. 1, vol. Art. ID, (2006). Published,

John B. Etnyre, "Book Review "Surgery on contact 3-manifolds and Stein surfaces" by Burak Ozbagci and Andras I. Stipsicz", *Bulletin of the American Mathematics Society*, p. 429, vol. 43, (2006). Published,

John B. Etnyre, "Contact structures on 3-manifolds are deformations of foliations", *Math. Research Letters*, p. 775, vol. 14, (2007). Published,

John B Etnyre, "On contact surgery", *Proc of the AMS*, p. , vol. , (2008). Accepted,

John B. Etnyre, "Contact perturbations and deformations of foliations", *Math Res Lett*, p. 775, vol. 14, (2007). Published,

Kenneth L. Baker, "Once-punctured tori and knots in lens spaces", [arxiv.org/abs/math/0612525](http://arxiv.org/abs/math/0612525), p. , vol. , ( ). preprint,

Kenneth L. Baker, Cameron Gordon and John Luecke, "Tunnel number v. Heegaard Genus", preprint, p. , vol. , ( ). preprint,

Anar Akhmedov, John B. Etnyre, Thomas E. Mark and Ivan Smith, "A note on Stein fillings of contact manifolds", Preprint, p. , vol. , (2007). Submitted,

John B. Etnyre and Jeremy Van Horn-Morris, "Fibered Transverse Knots and the Bennequin Bound", Preprint, p. , vol. , (2008). Submitted,

Tobias Ekholm, John B. Etnyre and Joshua Sabloff, "A Duality Exact Sequence for Legendrian Contact Homology", Preprint, p. , vol. , (2008). Submitted,

Kenneth L. Baker, J. Elisenda Grigsby, Matthew Hedden, "Grid Diagrams for Lens Spaces and Combinatorial Knot Floer Homology", preprint, p. , vol. , (2007). Submitted,

Kenneth L. Baker and J. Elisenda Grigsby, "Grid Diagrams and Legendrian Lens Space Links", Preprint, p. , vol. , (2008). Submitted,

### **Books or Other One-time Publications**

John B. Etnyre, "Legendrian and Transversal Knots", (2005). Book, Published

Editor(s): William Menasco and Morwen

Thistlethwaite

Collection: Handbook of Knot Theory

Bibliography: Elsevier B. V.

John Etnyre, "Lectures on open book decompositions and contact structures", (2006). Book, Published

Editor(s): Ozsvath, Stipsicz and Szabo

Collection: Floer homology, gauge theory, and low-dimensional topology

Bibliography: Clay Math. Proc., 5, Amer. Math. Soc., Providence, RI

Tobias Ekholm and John Etnyre, "Invariants of Knots, Embeddings and Immersions via Contact Geometry", (2005). Book, Published  
 Collection: Fields Inst. Commun., 47  
 Bibliography: Amer. Math. Soc., Providence, RI

John Etnyre, "Contact Manifolds", (2006). Book, Published  
 Collection: Encyclopedia of Mathematical Physics  
 Bibliography: Elsevier B. V.

John B. Etnyre, "Lectures on contact geometry in low-dimensional topology", ( ). Book, Accepted  
 Editor(s): P. Ozsvath and T. Mrowka  
 Collection: PCMI proceedings 2006  
 Bibliography: AMS

### Web/Internet Site

#### **URL(s):**

<http://www.math.gatech.edu/~etnyre/preprints/notes.html>

#### **Description:**

This is a web page containing my expository papers on contact geometry, symplectic geometry, and Legendrian knots. It also contains class notes on convex surfaces that should be a useful reference for students entering the field.

### Other Specific Products

### Contributions

#### **Contributions within Discipline:**

##### Year One:

In work with Honda the nature of Legendrian knots in dimension three has been greatly illuminated. We now have a much better understanding about what current invariants can and cannot tell us about Legendrian knots. We are also making progress understanding the nature of contact structures on hyperbolic manifolds, though it is too early in our investigation to say what specifically will come of it.

In work with Ekholm and Sullivan new invariants of Legendrian submanifolds in higher dimensions have been defined. Moreover orientations have been brought into the contact homology, providing a considerably finer invariant of Legendrian knots. We also use contact homology to derive bounds on the double points of exact Lagrangian submanifolds.

Studying open book decompositions of three manifolds has lead, via Giroux's correspondence with contact structures, to a better understanding of symplectic filling of contact structures. It also lead to the definition of a new invariant of contact structures. This invariant is the minimal genus of an open book related to a contact structure. The invariant seems to be surprisingly difficult to compute, but in recent work the PI showed that the invariant is non-trivial: that is that it is not always zero (but it is always zero for an overtwisted contact structure).

##### Year Two:

Lenny Ng has described a combinatorial invariant of knots in Euclidean 3-space. This invariant is inspired from a contact geometric construction coupled with Legendrian contact homology. In work with Ekholm and Sullivan we have shown that one can define an

invariant using Legendrian contact homology. In joint work with Ekholm, Ng and Sullivan have made great progress in showing that Ng's invariant is indeed the same as the contact homology invariant. This should lead to new insights to Ng's invariant as well as a better understanding of contact homology. We also expect to be able to generalize this to submanifolds in higher dimensional Euclidean space.

In joint work with Terry Fuller we have made progress in understanding when a 4-manifold admits an achiral Lefschetz fibration. Have such a structure should be useful in computing invariants from the algebraic, like signature, to the analytic, like Heegaard Floer invariants/Seiberg-Witten invariants.

In Stephan Schoenberger's PhD thesis, supervised by the PI, it was shown that a large class of contact manifolds are supported by a planar open book. For example, he can show that all contact structures on almost all Seifert fibered spaces are supported by a planar open book (this could not be true for all Seifert fibered spaces). He also showed how to use a planar open book to study the symplectic fillings of contact manifolds. In particular, he showed that a specific small Seifert Fibered space has one tight contact structure and a unique, up to diffeomorphism, symplectic filling. These techniques should be useful in studying the fillings of any contact structure supported by a planar open book.

In James Tripp's PhD thesis, supervised by the PI, he studied contact structures on open 3-manifolds. Before this work there was very little known. Eliashberg had classified contact structures on Euclidean 3-space (and implicitly shown that tight contact structures on an open manifold with only  $S^2 \times \mathbb{R}$  ends behave like tight contact structures on a compact manifold) and showed that the open solid torus has uncountably many tight contact structures. Tripp reproved this result and, moreover, showed that any irreducible open 3-manifold that is the interior of a compact 3-manifold with boundary with at least one boundary component of positive genus, has uncountably many tight contact structures. He also found contact structures on the open solid torus very different from Eliashberg's. In particular, he found an uncountable family that cannot be embedded in any larger tight contact manifold.

Year Three:

Continuing joint work with Ekholm, Ng and Sullivan, the PI has established that Ng's  $\mathbb{Z}/2\mathbb{Z}$  combinatorial invariant is the same as the invariants coming out of contact homology. We are currently trying to establish this over  $\mathbb{Z}\mathbb{Z}$ .

In joint work with Burak Ozbagci, the PI has studied several aspects of open books. First he defined invariants of contact structures from open books: the support genus, binding number of norm. The support genus was previously defined by the PI and studied by Ozsvath, Szabo and Stipsicz. It was shown that with the support genus is larger than zero there are obstructions to the existence of symplectic fillings with certain properties. There is currently no way to show an open book has genus larger than one. Ozbagci and the PI have shown that a conjecture concerning genus one open books is false. We have also studied the binding number of an open book and shown for overtwisted contact structures it is bounded in terms of the Euler class of the contact structure. It is somewhat surprising as, for example, it does not see much of the homotopy type of the contact structure on  $S^3$ . We also made a few observations concerning open books with hyper-elliptic monodromy and the relations of the norm to the support genus and binding number. Finally we also constructed open books supporting contact structures obtained via contact surgery on certain links. In particular, we construct small genus open books for many plumbings of disk bundles over spheres.

Year Four:

Continuing joint work with Ekholm, Ng and Sullivan, the PI has established that Ng's  $\mathbb{Z}\mathbb{Z}$  combinatorial invariant is the same as the invariants coming out of contact homology.

Answering a question of Eliashberg and Thurston the PI has shown that all contact structures on a closed oriented 3-manifold are  $\mathcal{C}^\infty$ -deformations of foliations. The PI has also shown that there are foliations on  $T^3$  that cannot be perturbed into a contact structure. This is quite surprising and shows that a result of Eliashberg and Thurston cannot be strengthened. Moreover, it was shown that in any  $\mathcal{C}^0$  neighborhood of a foliation with Reeb components there is an overtwisted contact structure. Progress was made on trying to show that any contact structure near a Reebless foliation is tight.

The PI has shown that  $S^1$ -contact surgery on distinct Legendrian knots frequently produces contactomorphic manifolds. It was also shown that there are examples where this happens for  $-1$ -contact surgery. These results explain various Heegaard Floer computations of Lisca and Stipsicz. As a corollary of this the PI finds an overtwisted contact structures that contain a large number of distinct Legendrian knots with the same classical invariants and tight complements.

In joint work with Jeremy van Horn the PI has shown that if a fibered knot has a transversal representative in a fixed tight contact structure that realizes the Bennequin bound then (modulo issues about incompressible tori) the contact structure the fibered knot supports is the given fixed contact structure. This explains a Heegaard Floer computation of Matt Hedden.

In joint work with Jeremy van Horn and Ken Baker the PI has begun to investigate how to associate contact structures to fibered knots that are not necessarily open books (ie the fibers of the fibration are not necessarily longitudes for the knot). We have also seen how to relate these 'rational open books' to open books. This work also allows one to understand how to construct open books for contact structures obtained by surgery on a transverse knot. Finally, we have also shown that if you positively cable the binding of an open book then the resulting open book supports the same contact structures as the original open book, but if one negatively cables, the the resulting contact structure is overtwisted.

Year Five:

In joint work with Ekholm and Sabloff the PI has established a long exact sequence for Legendrian submanifolds  $L$  in  $P \times \mathbb{R}$ , where  $P$  is an exact symplectic manifold, which admit a Hamiltonian isotopy that displaces the projection of  $L$  off of itself. In this sequence, the singular homology  $H_*$  maps to linearized contact cohomology  $CH^*$  which maps to linearized contact homology  $CH_*$  which maps to singular homology. In particular, the sequence implies a duality between the kernel of the map ( $CH_* \rightarrow H_*$ ) and the cokernel of the map ( $H_* \rightarrow CH^*$ ). Furthermore, this sequence leads to a refinement of the Arnold Conjecture for double points of an exact Lagrangian admitting a Legendrian lift with linearizable contact homology, first proved by the PI in collaboration with Ekholm and Sullivan.

In collaboration with Van Horn-Morris the PI proved strengthened their work of the previous year (by understanding issues about incompressible tori) and showed that a nicely fibered link in a tight contact manifold  $(M, \xi)$  with zero Giroux torsion has a transverse representative realizing the Bennequin bound if and only if the contact structure it supports (since it is also the binding of an open book) is  $\xi$ . This gives a geometric reason for the non-sharpness of the Bennequin bound for fibered links. The classification, up to contactomorphism, of maximal self-linking number links in these knot types was also achieved.

In many situations it has been shown that a contact manifold has a finite number of Stein fillings. A few years ago Ozbagci and Stipsicz proved this was not always the case but used

the fundamental group to distinguish different fillings leaving open the possibility that there are only a finite number of simply connected fillings. In collaboration Akhmedov, Mark and Smith the PI showed that this is not the case.

Further progress on showing that any contact structure near a Reebless foliation is tight was made. It is hoped that by the end of the summer this will be complete.

In collaboration with Baker the PI has begun the investigation of finiteness properties for Legendrian knots with fixed classical invariants. It is believed that we will be able to show that there is always a finite number of such knot. This greatly enhances our knowledge of the overall structure of Legendrian knots in Euclidean space.

### **Contributions to Other Disciplines:**

Year One:

The previously mentioned work on understanding symplectic cobordisms provided a key step in Kronheimer and Mrowka's program to answer the much studied question 'Does every nontrivial knot satisfy Property P?' in the affirmative. Moreover, Ozsvath and Szabo were able to use this result to provide a simpler proof Gordan's conjecture that the only knot on which  $p$  surgery produces the lens space  $L(p,1)$  is the unknot.

Year Four:

The above mentioned work with Ekholm, Ng and Sullivan should be useful to studying many questions about the topology of manifolds and embeddings of them into Euclidean space.

Year Five:

The PI in collaboration with Ekholm, Ng and Sullivan have mostly enhanced their previous work to take account of orientations thus providing a stronger invariant for studying embeddings of topological manifolds in Euclidean space (so far just knots in 3-space).

### **Contributions to Human Resource Development:**

Year One:

The PI taught a year long graduate course in contact geometry providing the graduate students and some faculty members with the understanding necessary to begin work in this field.

The PI also produces class notes for part of the above mentioned class. These notes are posted on the web and have been used by many people as an introduction to convex surface techniques in contact geometry.

The PI gave a mini-course at the Clay Institute Summer school in Budapest on contact geometry and open book decompositions.

The PI is supervising two graduate students. James Tripp has made significant progress on his PhD thesis and Stephan Schoenenberger is also make good progress in the PhD program.

The PI is also running, in conjunction with Herman Gluck, a program at the University of Pennsylvania for incoming graduate students to improve their teaching skills and make their first experience in front of a class a pleasant one.

Year Two:

The PI taught a graduate course in advanced algebraic topology.

The PI also taught an informal course in Heegaard Floer homology that was attended by

visiting faculty, post-docs and grad students.

The PI gave a mini-course in the Havarford/Bryn-Mawr seminar on contact geometry and open book decompositions. The course was attended by faculty and grad students from many universities around the Philadelphia area.

The PI is supervising two graduate students, James Tripp and Stephan Schoenenberger, who finished their PhD theses this year.

The PI mentored a post-doc, Connie Leidy, and a visiting faculty member, Terry Fuller.

The PI is also running, in conjunction with Herman Gluck, a program at the University of Pennsylvania for incoming graduate students to improve their teaching skills and make their first experience in front of a class a pleasant one.

Year Three:

The PI taught an advanced undergraduate algebraic topology course. The majority of students were engineers, so the PI was able to expose broad range of people to relatively sophisticated ideas in mathematics.

The PI ran a short seminar series at Georgia Tech concerning Heegaard Floer homology.

The PI is supervising two graduate students, Elena Bogdan and David Vick.

The PI is supervising an REU student, Gokhan Civan, who has written computer code to perform difficult calculations in contact homology and has found interesting examples of Legendrian knots that indicate relative strengths of invariants that come from contact homology.

The PI supervised visiting faculty member Burak Ozbagci. During the visit Ozbagci wrote four papers, two of which were co-authored with the PI and the other two were inspired by conversations with the PI.

The PI taught a mini-course at the Park City Mathematics Institute this year on 'Contact geometry and low dimensional topology' where many of the main ideas used to probe questions in low dimensional topology via contact geometry were explained. Rough lecture notes are currently available at the PCMI web site and extended lecture notes will be written shortly.

Year Four:

The PI taught a year of graduate algebra to incoming graduate students to prepare them for their future graduate studies.

The PI taught an 'informal' (ie no credit) class in symplectic manifolds. This was attended by graduate students and post docs and exposed them to all the basic theorems and constructions of symplectic manifolds with an emphasis on dimension four.

The PI taught an 'informal' course on Morse theory. Again it was attended graduate students and post docs. The course exposed the audience to standard Morse theory, handlebody theory, Morse homology, gradient flow trees, Novikov rings and a basic introduction to Floer theory.

The PI supervised two graduate students Elena Bogdan and David Vick.

The PI supervised two post docs Ken Baker and Anar Ahmadov. The PI met with these post

docs once a week, advised on their work as well as began various collaborations.

The PI supervised a post doc Nathan Geer. The PI helps the post doc in his job applications process and NSF grant proposal.

The PI has developed a first year graduate course in geometry and topology for incoming students.

The PI wrote a series of lecture notes to accompany the lecture series mentioned in the previous year. These notes are available on-line and will be published in the PCMI proceedings.

Year Five:

The PI taught a graduate course in the topology of 4-manifolds.

The PI taught an informal course on Hodge theory aimed at introducing the use of analysis to topological and geometric problems. This class was very well attended by students and postdocs. Lecture notes for this course were latexed and may be distributed (past the students in the class) at some time in the future.

The PI supervised two graduate students Elena Bogdan and Shea Vick and co-advised two others Sinem Celik and Bulent Tosun. He informally advised several students who are early in their graduate work.

The PI supervised two post docs Ken Baker and Anar Ahmadov. The PI met with these post docs once a week, advised on their work as well as began various collaborations. He also helped them with their job applications.

The PI supervised a post doc Nathan Geer. The PI helps the post doc in his job applications process.

The PI first year graduate course in topology and geometry the PI helped create has now been approved and will be taught for the first time this fall.

#### **Contributions to Resources for Research and Education:**

Several items mentioned above under 'Contributions to Human Resource Development' also fit in this section.

#### **Contributions Beyond Science and Engineering:**

#### **Categories for which nothing is reported:**

Organizational Partners

Activities and Findings: Any Outreach Activities

Any Product

Contributions: To Any Beyond Science and Engineering



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April 2, 2007

National Science Foundation  
Career Awards Program

Colleagues,

I am pleased to renew the commitment made by the Georgia Institute of Technology in support of Professor John Etnyre and the project goals outlined his NSF Career proposal—originally awarded when he was on the faculty of the University of Pennsylvania.

John is now in his second year of service at Georgia Tech, and we are especially pleased to play a supporting role—in partnership with the National Science Foundation—in his continued development as researcher, teacher and mentor. Starting from a very high level, he has done everything in the past two years we could possibly have asked, and more. The School of Mathematics is fortunate to have recruited John to our campus, and we look forward to supporting and encouraging his continuing career development in the years to come.

Sincerely,

*William T. Trotter*

William T. Trotter, Chair