

**PREDICTIVE ANALYTICS AND OPTIMIZATION FOR  
IMPROVED ELECTRIC POWER NETWORK  
RELIABILITY AND OPERATION**

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The Academic Faculty

by

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**PREDICTIVE ANALYTICS AND OPTIMIZATION FOR  
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RELIABILITY AND OPERATION**

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*To my beloved parents, Güzin and Alkan, my brother Burak,  
and all the beautiful people in my life...*

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## SUMMARY

This dissertation presents a general framework for modeling and controlling degradation processes in complex energy networks. We develop novel statistical and optimization methods that exploit real time sensor information to derive predictive failure risk assessments and decision models to enhance reliability and profitability in electric power systems.

At a first step, we focus on developing and solving large-scale optimization models to compute sensor-driven optimal operational and maintenance decisions for a fleet of power plants. Operational decisions relate to the well-known unit commitment problem, which identifies dispatch and commitment profiles that satisfy demand requirements, yet are optimized against real-time degradation levels of each power plant. Maintenance decisions focus on deriving optimal fleet level condition-based maintenance schedules that exploit potential economic and stochastic dependence existing among the individual generating units. The decisions are performed while adhering to constraints, such as generation and ramping limits of the power plants, capacities of transmission lines, network reliability, etc. Due to the scale and complexity of the problem, direct solution methods fail in most practical networks. Hence, we propose a solution methodology, where the idea is to start by using a relaxation to find good initial solutions, and then to add iterative integer cuts to refine our solution to account for the non-convexities. We also present the finite convergence and  $\epsilon$ -optimality of the proposed solution algorithm.

On a parallel line of research, we present a maintenance and operations scheduling policy specifically for wind farms. Maintenance considerations in this problem

differ significantly from the previous model. For instance, regardless of the number of turbines scheduled for maintenance, when crews visit an offshore location, they incur significant costs due transport of workboats and helicopters. In this work, we therefore consider the trade-off between sensor-driven optimal maintenance decisions for single-turbine systems, and the significant cost reductions arising from grouping the turbine maintenances together. The effectiveness of our approach, and the impact of electricity price and crew deployment cost are illustrated in extensive experiments. Recently, there has been a growing interest in sensor-driven maintenance policies for single-turbine systems. We show that for most practical cases, these policies perform poorer than the traditional time-based fleet maintenance policies. Our findings clearly illustrate that to obtain the full benefit of sensor information, a policy should integrate the dynamics within the maintenance and operations of the wind farm as a whole.

Lastly, we consider the interaction between the operational load on the power plants, and their corresponding rate of degradation. This interaction is particularly important since it significantly affects the remaining life of the power plants and the optimal maintenance decisions. Also, by deciding on the dispatch level of the power plants, a maintenance planner can intentionally alter the optimal maintenance time (i.e. by lowering the load on the cheap power plants and postponing their preventive maintenance, or by increasing the load on the cheap power plants and using more capacity before early maintenance). We propose optimal maintenance and dispatch decisions for power plants operating in this environment, and show that considering the load dependency can provide significant savings in both maintenance and operations cost, while ensuring a more reliable electricity system.

# CHAPTER I

## INTRODUCTION

Effective generation maintenance policies play a pivotal role in ensuring reliability and profitability of electric power systems. In simple terms, the objective of these policies is to determine the optimal maintenance schedule for a fleet of power plants in an effort to i) minimize the instances of unexpected failures, ii) extend the equipment lifetime, iii) reduce the number of unnecessary/early maintenances, and iv) alleviate the effects of maintenance on the power generation and operations [40].

Traditionally, maintenance activities have been scheduled at regular intervals using engineering expertise, manufacturing specifications, and failure statistics. While constructing these schedules, the main focus is on analyzing failure time data and drawing inferences across the entire population of a particular type of power plant. These properties are called the *population-specific* characteristics. Although this analysis helps quantify uncertainty and variability in failure processes for a particular type of power plant, it does not capture the degradation characteristics unique to each generation asset. Evidently, maintenance policies driven by population characteristics often recommend frequent unnecessary maintenance routines otherwise they run high risks of unexpected failures. Even in conservative maintenance operations, however, unexpected failures remain an inevitable eventuality. On the other hand, condition monitoring (CM) processes collect sensor data (such as temperature, pressure, vibration, noise, etc.) only from a specific power plant to estimate its current state of health, and do not necessarily capture the variances across a population of power plants. Our objective is to i) provide a dynamic, sensor-driven prediction of failure by leveraging on the real time CM sensor data, and ii) to use this information to construct a maintenance scheduling policy for a fleet of power plants.

In this dissertation, we thus provide a unified framework that uses low-level real-time sensor information to learn *unit-specific* failure characteristics of the power plants in field, and link this dynamic learning process with high-level operational and maintenance decisions for power plant fleets. We propose new mixed-integer optimization models for generation maintenance scheduling, which effectively incorporate the dynamic sensor information. In the first model, we focus primarily on the maintenance problem. We arrive at an optimal fleet-level condition based maintenance (CBM) schedule that accounts for optimal asset-specific CBM schedules driven by the condition monitoring data. We also consider maintenance dependencies between power plants, such as the limit on the number of simultaneous maintenances, inclusion/exclusions, etc. In the second model, we augment the first model by integrating operations. The operational decisions identify the optimal commitment and dispatch profiles that satisfy the demand and network feasibility requirements. The problem considers the economic and stochastic dependence between these decisions and proposes a schedule that provides significant improvements in cost and reliability. In the third model, we provide optimal sensor-driven maintenance and operations planning in wind farms. We consider the trade-off between sensor-driven optimal maintenance decisions for every turbine, and the significant cost reductions arising from grouping the turbine maintenances together - a concept called opportunistic maintenance. We present a solution that benefits from opportunistic cost and market-level operational savings while ensuring a certain level of reliability. Lastly, in the fourth model, we couple the loading condition on the power plants with the maintenance and operations scheduling. This framework provides a more accurate characterization of the remaining life distribution subject to the loading condition, and gives flexibility to the maintenance planner for changing the optimal time of maintenance for a fleet of power plants.

## ***1.1 Literature Review***

In this section, we provide a brief introduction to the fundamental works in failure prediction and maintenance scheduling models for power plants. For the ease of exposition, power plants and generators will be used interchangeably for the remaining of this dissertation.

### **1.1.1 Traditional Maintenance Models**

Maintenance scheduling for complex machinery has been one the first active areas of research in reliability engineering [61]. Failure distributions such as Weibull, Normal, Exponential and Gamma distribution characterized the failure time distribution of these engineering systems [28, 42, 101]. These failure distributions were used to construct maintenance policies in a number of different settings, including i) single-unit age replacement models [21, 29, 49], multi-component block replacement models [6, 60, 78], and opportunistic replacement models [30, 107, 108]. A review of maintenance and replacement can be found [98].

In the generator maintenance literature, most of the techniques also employ traditional maintenance models. In particular, they employ a periodic maintenance policy, whereby the maintenances for each generator are conducted within allowed maintenance windows, typically in a yearly maintenance schedule [12]. Some approaches consider additional maintenance dependencies between generators, such as priorities, exclusions, and separations between consecutive maintenances [24]. Much work has been focused on operational and market-related challenges such as the interaction between generation companies and independent system operators [7, 24, 43], the consideration of operational uncertainties in load forecast, price, water inflow levels [102–104], and the integration with the transmission maintenance [35, 44, 72]. Shahidehpour and Marwali provide a coherent review of the problems in generator maintenance in [85]. Recently, [2] integrated the failure distribution of the generators

into the maintenance scheduling problem. To do so, the paper used an approximation of a Weibull distribution to represent the failure rate and maintenance dependency. This technique is called the reliability-based maintenance approach since it captures general failure behaviors of the generator type, but does not consider any unit specific information.

Limitations of traditional maintenance models: Traditional models do not consider the condition or degradation characteristics of an individual equipment. Practitioners of maintenance believe that periodic policies are too conservative [56]. In fact, it is estimated that only 10% of the equipments replaced according to time based maintenance policies had to be replaced at the time of planned schedule [83]. Perhaps more important than the excessive conservatism, is the reduced ability of these policies to avoid unexpected failures. Generators are complex electrical equipments, and their degradation is not only a function of their age. Manufacturing variations, environmental factors, and the complex failure mechanisms together affect the lifetime of a particular generator [76, 83].

### **1.1.2 Condition Monitoring**

Generators degrade over time with accumulation of damage due to wear and tear. Different types of failure processes in generators manifest themselves in sensor readings. This sensor data can be captured through the use of integrated condition monitoring (CM) systems. Three main monitoring techniques are common in practice: Mechanical, Electrical, and Chemical [50, 58, 64, 70, 74, 82, 91, 92, 95]. Among these monitoring techniques, mechanical analyses provide the most sophisticated tools available to the operators [92]. Depending on the location of the vibration sensor, and the dominant frequencies of the acquired vibration signals, operators can automatically detect the severity and isolate the root cause of the ongoing degradation processes such as i) short circuited turns in rotor windings, ii) bearing problems, iii) generator and turbine shaft

misalignment, fatigue and torsion, iv) rotor cracking or v) end winding deterioration in the main unit. Along with the main generator unit, vibration analyses also provide information on a large number of critical components connected to the generator such as the turbine structures, and the motors for compressors and pumps [92]. Electrical analysis mainly focuses on characterizing the gradual deterioration in the stator windings through partial discharge (PD) techniques. PD techniques have been used for 50 years; however there is still need for more research to improve the predictive quality through better techniques for noise isolation and better interpretation of the complex PD patterns. Important work in this field can be found in [50, 58, 66, 70, 91]. Lastly, chemical techniques involve analyses such as the detection of gases and pyrolysed matter within the cooling hydrogen to detect overheating of insulation [95]. In large generators high penalties associated with unexpected failures makes even complex CM systems profitable in operation [83]. The increasing use of CM technologies is a result of profitability even for single generator systems since it provides more confidence to the asset managers regarding the time of the generator failures [56]. [83] shows that at least 1% of the capital value of the plant being monitored should be invested in CM technologies, while the realistic figure would be around 5%. Real time CM investments and technologies are expected to grow further over the next decades [83].

CM is actively used for many wind turbines as well. In contrast to the conventional generators, wind turbines experience more frequent failures. This is a result of the unique structural constraints on the wind turbines, as well as the highly irregular loading from unstable wind conditions, and a drastically increased frequency of start-stop incidences. Main faults within these systems occur due to i) imbalance, fatigue and impending cracks in the rotor, ii) eccentricity of tooth wheels and tooth wear in gear boxes, iii) overheating and electrical asymmetries in generators, iv) resonance, cracks and fatigues in the tower structures, and v) wear, pitting, deformation

and impending cracks of shafts in bearings [52]. Similarly, CM techniques for wind turbines can be summarized under three main categories. Mechanical monitoring provides the most mature technology in wind turbine CM and involves vibration analysis for analyzing the wheels and bearings in gearbox, bearings of the generator and the main bearing, crack detection methods, strain measurements, and acoustic monitoring. Electrical monitoring mainly concentrates on partial discharge analysis. Oil contamination and change in concentration characteristics can indicate wear in the components. [25] provides a survey of available commercial CM technologies for wind turbines, and [75, 105] studies the profitability of installing CM on wind turbines. In fact [75] states that most of the new wind turbines come with integrated CM capabilities.

Limitations of condition monitoring: CM systems collect sensor data from a specific generator to estimate its current state of health, but do not necessarily capture the variances across a population of generators. Another major concern with the CM techniques is that they define failure predictors based on deviations from normal conditions. However, such deviations occur frequently due to external and internal operational effects, such as the dispatch level of the generator, and weather conditions.

### **1.1.3 Condition Based Maintenance Models: Single Generator Systems**

CBM in conventional generators typically involve sensor-driven alarm systems. These systems are established by major original equipment manufacturers (OEMs) through use of long-term service agreements with the utility companies. OEMs remotely monitor the generation assets for potential faults. Typically, sensor data from various generators are transmitted to a centralized hub where conventional classifiers and control limit based techniques are used to trigger alarms. A number of case studies have been published on the implementation of condition monitoring guided maintenance in medium sized combined-cycle power plants [23], gas turbine engines [69],

and nuclear power plant components [109].

A parallel line of research on wind turbines use condition monitoring information and focus on optimal maintenance policies for single turbine systems [18, 19]. These models do not capture the dependencies between different turbines. Recently, [97] has proposed a maintenance scheduling policy that considers opportunistic maintenance for turbines subject to condition monitoring. In this work, the authors suggested a two-threshold policy, whereby a strict failure threshold applies to the first turbine to be maintained, and a more conservative failure threshold is imposed on the remaining turbines in an effort to group them with the first turbine. Although this work proposes an opportunistic policy, it does not necessarily consider the complex economic, and maintenance interdependencies between the turbines.

Limitations of the single generator CBM policies: The main drawback of single generator CBM policies is that they do not consider the complex interdependencies between the generators. In the maintenance side, there are coupling constraints between generators with respect to the inclusion/exclusion of the generators, maintenance crew capacity, etc. In the operational side, economic dependencies significantly affect the reliability and profitability of the network operations. Second drawback of these policies is that the decisions taken by the CBM policies are typically restricted to imminent repairs with limited advance warning capability. However, considering the time-sensitive nature of the decision making processes, e.g., unexpected shut down of a power plant, any viable maintenance policy must provide ample response time.

## ***1.2 Contributions***

In this section we provide an outline of the dissertation, and summarize the key contributions of each chapter.

In Chapter 2, we provide a general degradation modeling framework for generators in service. Unlike most approaches in the literature, we present a sensor-driven

method that combines the population degradation data with unit specific information to further refine the failure probability and the RLD. Using this estimate, we generate dynamic maintenance cost functions for every generator. This function considers the trade-off between the risks associated with unexpected failures and the cost of preventive maintenances in each generator.

In Chapters 3 and 4, we introduce adaptive maintenance and operations scheduling problem for conventional generators. Main contribution of these chapters can be listed as follows:

1. We propose two sensor-driven adaptive maintenance scheduling models:
  - (a) We first consider a fleet maintenance model that provides a generator fleet maintenance schedule subject to limited labor resources and no operational constraints.
  - (b) In the second approach, we expand the previous model to consider the effects of maintenance on network operation by coordinating generator maintenance schedules with the unit commitment (UC) and dispatch decisions.

Proposed models differ significantly from the existing models due to two main reasons: i) they incorporate the dynamic sensor information into the optimization model, and ii) they allow the optimization model to determine the number of maintenances to be scheduled within the planning horizon.

2. We provide a novel two-stage reformulation for the second maintenance model and an effective solution algorithm to solve large-scale instances. In particular, the proposed maintenance model can be viewed as an MIP with integer recourse variables (the UC decisions). The reformulation relaxes the integer recourse but effectively compensates for the cost difference between the original and

the relaxed models so that the exact cost of the maintenance is recovered. This reformulation inspires a two-level algorithm which essentially decomposes the maintenance and operation decisions and iteratively searches for the best maintenance solutions.

3. We construct a platform on which extensive experiments are conducted using real-world physical degradation signals. In particular, the predictive analytics module acquires vibration signals from rotating machinery in a laboratory experiment to emulate generator degradation signals. Extensive tests on the IEEE 118-bus system show that the proposed maintenance model significantly outperforms the traditional periodic maintenance and reliability based maintenance models in key metrics such as the number of unexpected failures, the frequency of scheduled maintenances, the effectiveness in the use of equipment life, and operation costs. These metrics coincide with the objectives presented in [40].

Contribution 1-a is addressed in Chapter 3, while contributions 1-b, and 2 are discussed in Chapter 4 of this dissertation. Contribution 3 is presented separately for each chapter.

In Chapter 5, we provide an adaptive maintenance scheduling model for wind farm maintenance and operations. The operational decisions identify the dispatch profiles. Maintenance decisions, on the other hand, involve two types of decisions. Preventive maintenance decisions focus on arriving at an optimal fleet-level condition based maintenance (CBM) schedule. The reactive maintenance decisions, on the other hand, identify the optimal time to repair the failed turbines. The problem also takes into account the significant cost reductions arising from grouping turbine maintenances together. The economic and stochastic dependence between operations and maintenance decisions are also considered. Experiments conducted using a 100-turbine

case, and a 200-turbine case study involving 3 wind farm locations demonstrate the advantages of our proposal.

Lastly, in chapter 6, we consider the interaction between the operational loading on the generators, and their corresponding rate of degradation. This interaction is particularly important since it significantly affects the remaining life of the generators and the optimal maintenance decisions. Also, by deciding on the operational profile of the generators, a maintenance planner can intentionally alter the optimal maintenance time (i.e. by lowering the load on the cheap generators and postponing their preventive maintenance, or by increasing the load on the cheap generators and using more capacity before early maintenance). We propose optimal maintenance and dispatch decisions for generators operating in this environment. Our experiments suggest that the load dependent framework provides considerable advantages over conventional models.

## CHAPTER II

### PREDICTIVE ANALYTICS

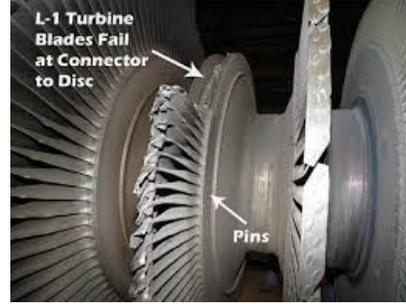
Generators experience physical wear and tear in their critical components due to aging. For example, in a wind turbine, degradation would be wear and spalling of the main bearings, teeth wear and breakage of gears in the gearbox attached to the turbine generator. In conventional generators, degradation can be the wear of bearings that carry the main rotor shaft, cracking of the main shaft, rotor and/or stator deformation, or loosening of the windings. For instance, in stators, loose end-winding support structures can cause significant damage to the mechanical integrity of the stator conductor bars and other winding components. This loosening develops gradually due to high AC electromagnetic fields inside the generator. Degradation processes are also important for assets supporting the generator operations, such as gas turbines. Gas turbines consist of a compressor section that has up to 15 successive rotating discs, with each disc consisting of rows of blades that are mounted on huge rotating discs. Often these blades crack and can be dislodged. The compressor often rotates at relatively high speeds. The dislodged blade can fly through the entire compressor section causing millions of dollars of damage, see Figures 1, and 2 [64].

In maintenance scheduling, we focus on failures that result from this gradual and irreversible accumulation of damage, called a *degradation process*.

It is often very difficult to have direct observations of the degradation processes. Fortunately, these processes manifest themselves through tractable measures that can be analyzed using sensor-based CM systems. Typically, raw degradation data coming from CM systems can be transformed into *degradation signals* through appropriate

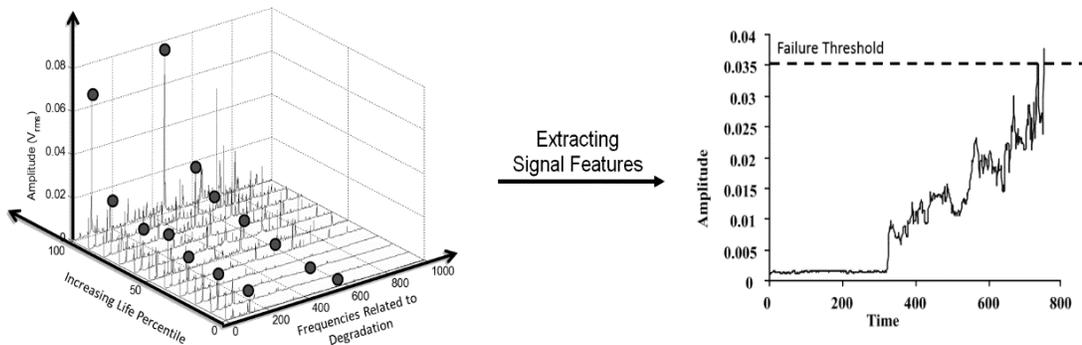


**Figure 1:** New Compressor Blades



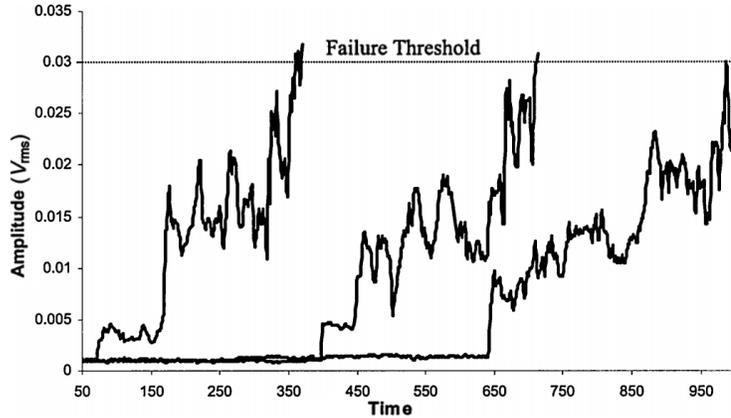
**Figure 2:** Broken Compressor Blades

data manipulation. The resulting signals can be correlated with underlying degradation, and can be used to predict failure and RLD (*prognosis*). For example, Figure 3 presents the vibration spectra of a rotating machinery application from its brand new stage to its failure. Highlighted points in the spectra correspond to the defective frequencies related to the degradation signal. As the system degrades, defective frequencies become more prominent. Upon suitable feature extraction, the resulting degradation signal can show the progression of the degradation signal throughout the asset lifetime.



**Figure 3:** Vibration Spectra and its Degradation Signal Transformation

As outlined in Chapter 1, there are many different manifestations of physical degradation. In many cases, some of the critical ones can be measured using a variety of sensors. Examples of such sensors include: accelerometers that measure all kinds of vibrations in rotating equipment, acoustic sensor that measure noise levels both at low and high (ultrasonic) frequencies, thermometers and infra-red imaging cameras that measure temperature variations, and many others. The raw sensor data from



**Figure 4:** Vibration Based Degradation Signals for Rotating Machines

these sensors can be transformed into degradation signals using engineering domain knowledge and expertise as well as knowledge of the physical nature of degradation. For more examples of degradation and condition monitoring in generators, we refer the reader to the monitoring and diagnostics chapter in [64].

Degradation signals capture the current degradation state of the generators and provide information about how that state is likely to evolve in the future. Typically, a set of similar generator components exhibit a common functional form for their degradation signals, i.e., degradation signals following an increasing exponential trend over time. The shape of the degradation function is typically driven by the underlying physical degradation processes. However, although the functional forms may be identical, there is still significant variation in terms of the degradation rates of identical machines. For example, Figure 4 shows 3 degradation signals from 3 identical rotating machines. Failure time is the time at which the degradation signal crosses a prespecified failure threshold. As apparent in the figure, identical machines may still experience different degradation rates, and hence different failure times. This variability is due to numerous sources that include non-homogeneity in manufacturing, materials used, etc. Capturing this variability plays a pivotal role in our framework.

In this chapter, we develop a general parametric degradation function to characterize a population of generation assets. The term degradation signal refers to our

sensor-based inference on the condition of the turbine, whereas *degradation modeling* refers to the parametric framework whereby we model the progression of this signal for predictive purposes. The degradation model mimics the degradation shape associated with the generator, and builds a parametric model composed of deterministic and stochastic parameters. Deterministic parameters are assumed to be known and constant throughout the generator’s life. Stochastic parameters require a more thorough analysis. Typically these parameters, such as the rate of degradation, follow some distributional form across all the generation assets of the same type, with the parameters of the individual generators being random ‘draws’ from this population distribution. We first use practitioner expertise, OEM recommendations, and reliability analysis to characterize the population distribution. Condition monitoring that forms the next stage of our approach, streams the continuous sensor information from an operational generator in order to improve the predictions on the stochastic parameters for that particular generator, which results from the aforementioned ‘draw’. More specifically, real-time Bayesian updating is used to combine the population parameters with the sensor-driven information coming from the specific generator. Evidently, this sensor observation is especially useful in detecting unit to unit variances among the particular generators being operated. Inherent factors such as error terms, measurement errors, and signal transients are also considered in this stochastic degradation model.

## ***2.1 Degradation Modeling and the Bayesian Framework***

In this section, we develop a parametric model to characterize generator degradation. Our approach revolves around modeling the degradation signal as a continuous-time continuous-state stochastic process. The basis of this approach is the degradation modeling framework proposed by [47] where a parametric stochastic model is used to model degradation signals from a population of generators. The model consists

of deterministic and stochastic parameters. Deterministic parameter is used to capture fixed degradation attributes that are constant across the generator population. Stochastic parameter is assumed to follow a known distribution and capture the unit-to-unit variability among the individual generators. Specifically, stochastic parameter is used to capture the variability in the degradation rates. We represent the observed degradation signal from generator  $i$ , or its suitable transformation, as follows:

$$D_i(t) = \phi_i(t; \kappa, \theta_i) + \epsilon_i(t; \sigma), \quad (1)$$

where  $D_i(t)$  is a continuous-time stochastic process representing the generator degradation measure observed through sensors,  $\phi_i(t; \kappa, \theta_i)$  is a general parametric degradation function, whose specific form depends on generators, and  $\epsilon_i(t; \sigma)$  is the error term defined through the variance parameter  $\sigma$ . In (1),  $\kappa$  characterizes the deterministic population-specific degradation parameter common to all generators of the same type, and  $\theta_i$  represents the stochastic degradation characteristics unique to generator  $i$ .

We define the *time of failure*  $\tau_i$  of generator  $i$  as the first time that the degradation signal  $D_i(t)$  crosses the failure threshold  $\Lambda_i$ , namely:

$$\tau_i = \min\{t \geq 0 \mid D_i(t) \geq \Lambda_i\}. \quad (2)$$

Given the degradation model parameters  $\kappa$ ,  $\sigma$  and  $\theta_i$ , the probability that generator  $i$  survives until time  $t$  can be found as follows:

$$\begin{aligned} P(\tau_i > t \mid \theta_i) &= P\left(\sup_{0 \leq s \leq t} D_i(s) < \Lambda_i \mid \theta_i\right) \\ &= P\left(\sup_{0 \leq s \leq t} \{\phi_i(s; \kappa, \theta_i) + \epsilon_i(s; \sigma)\} < \Lambda_i \mid \theta_i\right). \end{aligned}$$

In most cases, the stochastic parameter  $\theta_i$  may be unknown. We assume that it

follows a certain prior distribution  $\pi_i(\theta_i)$ . This prior distribution reflects the engineering knowledge, manufacturing specifications, and studies on failure statistics. In cases where the degradation data from other generators are available,  $\pi_i(\theta_i)$  can also be estimated.

The unconditional probability that generator  $i$  survives until time  $t$  can then be presented as follows:

$$\begin{aligned} P(\tau_i > t) &= \int P\left(\sup_{0 \leq s \leq t} D_i(s) < \Lambda_i | \theta_i\right) \pi_i(\theta_i) d\theta_i \\ &= \int P\left(\sup_{0 \leq s \leq t} \{\phi_i(s; \kappa, \theta_i) + \epsilon_i(s; \sigma)\} < \Lambda_i | \theta_i\right) \pi(\theta_i) d\theta_i. \end{aligned}$$

Observed degradation signals allow us to improve our estimation on the parameter  $\theta_i$ . More specifically, conditioning on the degradation signal observations, we can update the prior parameter distribution  $\pi_i(\theta_i)$  to the posterior distribution  $v_i(\theta_i)$  via Bayesian learning.

To accomplish that, for generator  $i$ , we observe the degradation signals  $\mathbf{d}_i^o = (d_i^1, \dots, d_i^{t_i^o})$  at times (in terms of the generator's age)  $\mathbf{t}_i = \{t_i^1, \dots, t_i^o\}$  such that  $t_i^1 < t_i^2 < \dots < t_i^o$ . We consider the observations from working generators. The conditional joint density function of  $\mathbf{d}_i^o = (d_i^1, \dots, d_i^{t_i^o})$  given the parameter  $\theta_i$  can be represented as follows:

$$P(\mathbf{d}_i^o | \theta_i) = \prod_j P(D_i(t_j) = d_i^j | \theta_i, A_j),$$

where  $A_j$  denotes the condition that  $D_i(t_k) = d_i^k$  for all  $t_i^k \in \mathbf{t}_i^o$  such that  $t_i^k < t_i^j$ . Given the observations  $\mathbf{d}_i^o$ , the posterior distribution of the parameter  $\theta_i$  is given as follows:

$$v(\theta_i) = P(\theta_i | \mathbf{d}_i^o) = P(\mathbf{d}_i^o | \theta_i) \pi_i(\theta_i) / P(\mathbf{d}_i^o).$$

The denominator  $P(\mathbf{d}_i^o)$  does not need to be computed since it is a normalization factor. If an appropriate conjugate pair can be found for the particular parameter distributions, the posterior distribution  $v(\theta_i)$  might have a closed form expression. In

other cases, calculating the posterior estimate involves evaluation of high-dimensional integrations. Solving these integrals analytically might be very difficult, and one might instead use sampling methods to estimate the posterior distribution [48].

## 2.2 *Estimating the Remaining Life - Prognosis*

For a partially degraded generator  $i$ , once the distribution of the degradation parameter  $\theta_i$  is updated, the next challenge is to estimate the distribution of its remaining life  $R_{t_i^o}^i$  at observation time  $t_i^o$ :

$$P(R_{t_i^o}^i > t) = P(\tau_i > t + t_i^o | \mathbf{d}_i^o).$$

In other words, we estimate the distribution of the remaining life  $R_{t_i^o}^i$  of generator  $i$  at observation time  $t_i^o$ , given the posterior distribution  $v(\theta_i)$  as follows:

$$P(R_{t_i^o}^i > t) = \int P\left(\sup_{t_o \leq s \leq t_o + t} D_i(s) < \Lambda_i | \theta_i\right) v(\theta_i) d\theta_i. \quad (3)$$

In some cases, a closed form solution can be acquired for this expression, e.g, linear models with normal i.i.d. error, and Brownian models with constant drift [47]. For other models, sampling methods may be needed [48]. A simulation process can be implemented as follows:

- S.0. Given the posterior distribution  $v_i(\theta_i)$ , get a sample of the parameter  $\theta_i$ . In this context,  $n^{th}$  realization is denoted by  $\tilde{\theta}_{i,n}$ .
- S.1. For each realization, simulate the continuous stochastic degradation function  $S_{i|\tilde{\theta}_{i,n}}(t)$  for all  $t > t_o$ , until the realization  $\tilde{s}_{i|\tilde{\theta}_{i,n}}(t)$  reaches the failure threshold  $\Lambda_i$ . Register this time  $t$  as the time of failure for the  $n^{th}$  simulation, and let the realization of remaining life as  $\tilde{r}_{t_o,n}^i$ .
- S.2. Use the realizations  $\tilde{r}_{t_o,n}^i$  from all the simulations, to estimate the distribution of  $R_{t_o}^i$ .

### 2.3 Special Case: Exponential Degradation Function

We next present a special case of the degradation framework to be used for analyzing the condition monitoring data from our rotating machinery application.

Recall that we define  $D_i(t)$  as the amplitude of the degradation signal of generator  $i \in \mathcal{G}$ , at time  $t$ . For the degradation data we consider, exponential degradation function provides the best fit. We represent this function,  $D_i(t)$  as follows:

$$D_i(t) = \phi + \theta_i e^{\beta_i t + \epsilon_i(t) - \frac{\sigma^2 t}{2}} = \phi + \theta_i e^{\beta_i t - \frac{\sigma^2 t}{2}} e^{\epsilon_i(t)}, \quad (4)$$

where  $\phi$  and  $\sigma$  are constant deterministic parameters,  $\theta_i$  and  $\beta_i$  are random variables, and  $\epsilon_i(t)$  is a Brownian motion [47]. We focus on the log exponential degradation function denoted by  $L_i(t) := \ln(D_i(t) - \phi)$ ,

$$L_i(t) = \theta'_i + \beta'_i t + \epsilon_i(t) \quad (5)$$

where  $\theta'_i = \ln(\theta_i)$  and  $\beta'_i = \beta_i - (\sigma^2/2)$  are assumed to follow prior normal distributions  $\pi(\theta'_i)$  and  $\pi(\beta'_i)$ , with means  $\mu_0$  and  $\mu_1$ , and variances  $\sigma_0^2$  and  $\sigma_1^2$ , respectively.

We use a two-stage method to estimate the population prior distributions,  $\mu_0, \mu_1, \sigma_0, \sigma_1$ . In stage 1, we develop estimates for  $\theta'_i$ , and  $\beta'_i$  for each specimen. The resulting estimates are used in stage 2 to evaluate the prior distributions. We denote the log degradation function amplitude at observation time  $t_k$  as  $\ell_i(t_k)$ , and assume that we monitor  $\ell_i(t_k)$  at times  $t_1, t_2, \dots, h_i$ , where  $t_1 < t_2 < \dots < h_i$ . In our experiment, the sensor data is observed with constant intervals.

*Stage 1 Estimate.* In this stage, we estimate the component specific degradation parameters  $\theta_i$ , and  $\beta_i$ , based on data acquired from one tested specimen. We require that the error term  $\epsilon_i(0) = 0$ , thus  $L_i(0) = \hat{\theta}_i$ . Since the error increments in Brownian motion are i.i.d, we use the incremental values to estimate  $\beta_i$  as follows:

$$\hat{\beta}_i = \frac{1}{h_i} \sum_{k=1}^{h_i} \frac{\ell_i(t_k) - \ell_i(t_{k-1})}{t_k - t_{k-1}}$$

where  $h_i$  is the time of last observation before generator  $i$  fails. Once  $\hat{\beta}_i$  is obtained, we can estimate  $\sigma_i^2$  as follows:

$$\hat{\sigma}_i^2 = \frac{1}{(h_i - 1)} \times \sum_{k=1}^{h_i} \frac{\left( \ell_i(t_k) - \ell_i(t_{k-1}) - (t_k - t_{k-1})\hat{\beta}_i \right)^2}{(t_k - t_{k-1})}$$

since the term  $\left( \ell_i(t_k) - \ell_i(t_{k-1}) - (t_k - t_{k-1})\hat{\beta}_i \right)$  is normally distributed with mean 0, and variance  $\sigma^2(t_k - t_{k-1})$ .

*Stage 2 Estimate.* In this stage, we use the estimates in Stage 1 from a number of components to obtain the estimates for the population degradation parameters. We use the sample mean of  $\hat{\theta}_i$  and  $\hat{\beta}_i$  for components  $i \in \{1, 2, \dots, G\}$ , to find the estimates  $\hat{\mu}_0$  and  $\hat{\mu}_1$ . We use the corresponding sample variances to acquire the estimates  $\hat{\sigma}_0^2$  and  $\hat{\sigma}_1^2$ . Lastly, we obtain the estimate  $\hat{\sigma}^2$  from  $\{\hat{\sigma}_1^2, \hat{\sigma}_2^2, \dots, \hat{\sigma}_G^2\}$ .

Degradation signals are acquired during operation of the generator. Using this data, the degradation parameters can be updated in a Bayesian manner. Given that the observed logged degradation signal  $\{\ell_i(t_1), \dots, \ell_i(t_k)\}$  at times  $t_1, \dots, t_k$  from a particular generator  $i$ , the posterior distribution of the degradation parameters  $(\theta'_i, \beta'_i)$  can be estimated as a bivariate normal distribution with means  $(\mu_{\theta'_i}, \mu_{\beta'_i})$ , variances  $(\sigma_{\theta'_i}, \sigma_{\beta'_i})$  and correlation coefficient  $\rho_i$  [47]:

$$\begin{aligned} \mu_{\theta'_i} &= \frac{(\ell_{i,1}\sigma_0^2 + \mu_0\sigma^2t_1)(\sigma_1^2t_k + \sigma^2) - \sigma_0^2t_1\left(\sigma_1^2\sum_{e=1}^k\ell_{i,e} + \mu_1\sigma^2\right)}{(\sigma_0^2 + \sigma^2t_1)(\sigma_1^2t_k + \sigma^2) - \sigma_0^2\sigma_1^2t_1} \\ \mu_{\beta'_i} &= \frac{\left(\sigma_1^2\sum_{e=1}^k\ell_{i,e} + \mu_1\sigma^2\right)(\sigma_0^2 + \sigma^2t_1) - \sigma_1^2(\ell_{i,1}\sigma_0^2 + \mu_0\sigma^2t_1)}{(\sigma_0^2 + \sigma^2t_1)(\sigma_1^2t_k + \sigma^2) - \sigma_0^2\sigma_1^2t_1} \\ \sigma_{\theta'_i}^2 &= \frac{\sigma^2\sigma_0^2t_1(\sigma_1^2t_k + \sigma^2)}{(\sigma_0^2 + \sigma^2t_1)(\sigma_1^2t_k + \sigma^2) - \sigma_0^2\sigma_1^2t_1} \\ \sigma_{\beta'_i}^2 &= \frac{\sigma^2\sigma_1^2(\sigma_0^2 + \sigma^2t_1)}{(\sigma_0^2 + \sigma^2t_1)(\sigma_1^2t_k + \sigma^2) - \sigma_0^2\sigma_1^2t_1} \\ \rho_i &= -\frac{\sigma_0\sigma_1\sqrt{t_1}}{\sqrt{(\sigma_0^2 + \sigma^2t_1)(\sigma_1^2t_k + \sigma^2)}}, \end{aligned}$$

where  $\ell_{i,e} = \ell_i(t_e) - \ell_i(t_{e-1})$ .

The failure time  $\tau_i$  of generator  $i$  is defined as the first time that the logged degradation signal  $L_i(t)$  crosses failure threshold  $\Lambda$ . More specifically,  $\tau_i = \inf(t : t > 0, L_i(t) = \Lambda)$ .

A conservative estimate for the probability of failure can be presented as the boundary crossing probability of the Brownian motion process [38]. In this context, the failure time  $\tau_i$  follows an Inverse Gaussian distribution with mean parameter  $\chi = \frac{\Lambda - \ell_i(t_k)}{\mu_{\beta'}}$  and shape parameter  $\gamma = \frac{(\Lambda - \ell_i(t_k))^2}{\sigma^2}$ , that is:

$$P\{\tau = t | \ell_1, \dots, \ell_k\} = f_{t_k}(t) = \sqrt{\frac{\gamma}{2\pi t^3}} \exp\left\{-\frac{\gamma(t - \chi)^2}{2\chi^2 t}\right\}.$$

## 2.4 Dynamic Maintenance Cost

The predictive framework introduced in this chapter is tightly integrated with our optimization models. This is achieved through a dynamic cost function that translates the RLD of generators into a degradation-based function of cost over time. More specifically, the dynamic maintenance cost function quantifies the tradeoff between the cost of preventive action and the risk of unexpected failures by defining their corresponding probabilities through the sensor-updated remaining life estimates. In this function, we use renewal theory to characterize the long-run average maintenance cost

$$C_{t_i^o, t}^{d,i} = \frac{c_i^p P(R_{t_i^o}^i > t) + c_i^f P(R_{t_i^o}^i \leq t)}{\int_0^t P(R_{t_i^o}^i > z) dz + t_i^o}, \quad (6)$$

which is the cost rate associated with conducting generator maintenance  $t$  time periods after the time of observation  $t_i^o$ ;  $c_i^p$  and  $c_i^f$  are the costs of planned maintenance and failure replacement, respectively;  $c_i^f$  is typically higher than  $c_i^p$ , since unexpected failures require maintenance to be conducted on demand without prior planning. This leads to increased costs in materials and labor. Additionally, any unexpected failure might lead to a series of damages to the generator subcomponents, further increasing the cost of maintenance. The probability  $P(R_{t_i^o}^i > t)$  in this function is derived from the RLDs evaluated by expression (3). In essence, the dynamic cost functions are

directly related to the RLDs and hence the degradation states of each generator.

Certain generators might be scheduled more than once. Thus it would be beneficial to characterize the associated maintenance cost of a new generator that has just completed its maintenance. For a new generator, the maintenance cost function  $C_t^{n,i}$  takes the following form:

$$C_t^{n,i} = \frac{c_i^p P(\tau_i > t) + c_i^f P(\tau_i \leq t)}{\int_0^t P(\tau_i > z) dz}. \quad (7)$$

The dynamic cost functions help identify the optimal time to repair a generator based on their most recently updated RLD. Our goal is to optimize these decisions across *all the generators*. In the following chapters, we discuss four different scheduling policies. All of these policies will propose mixed-integer optimization models that will integrate this dynamic cost function into their objective function.

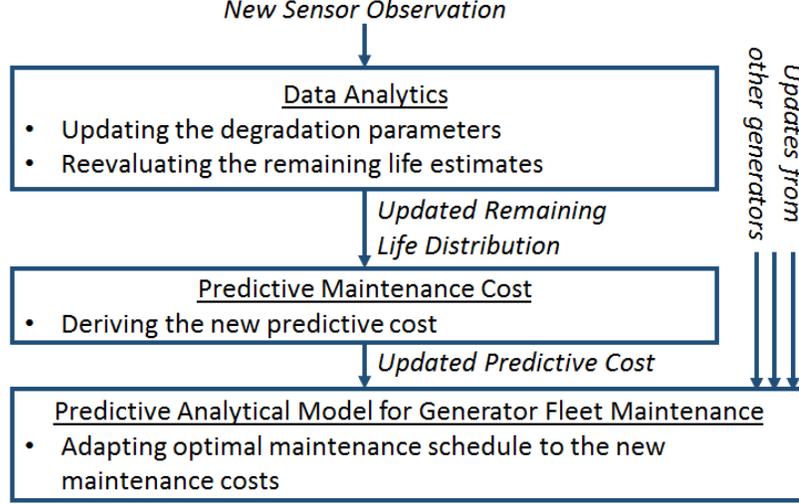
## CHAPTER III

# SENSOR-DRIVEN CONDITION-BASED GENERATION MAINTENANCE SCHEDULING: MAINTENANCE PROBLEM

### *3.1 Introduction*

In this chapter, we propose a new framework for generation maintenance scheduling that combines state-of-the-art sensor-data analytics and mixed-integer programming techniques to construct sensor-driven condition-based maintenance scheduling models. Figure 5 presents the structure of the framework, which consists of two modules: the predictive analytics module and the optimal scheduling module. The predictive analytics module employs Bayesian prognostic techniques to dynamically estimate the remaining life distribution (RLD) of generators from sensor data and update the dynamic maintenance cost for each generator. The optimal scheduling module incorporates the sensor analytics results into a mixed-integer programming (MIP) model that coordinates the maintenance and operation decisions in a generation fleet.

The remainder of the chapter proceeds as follows. Section II introduces detailed formulation for the basic adaptive maintenance model. In Section III, we present the degradation framework used as the basis for the experiments. We first present a method to estimate the population parameters of the degradation signals using real world data. We then present an experimental framework that uses this degradation database to study a number of test cases. We show the effectiveness of our model, and the impact of the maintenance updating frequency on the maintenance performance. In section IV, we conclude this chapter with some closing remarks.



**Figure 5:** Set of Changes Triggered by a New Sensor Information

### 3.2 Adaptive Predictive Maintenance Model I

In this section, we present the first adaptive predictive maintenance model (APMI). In this model, the decision maker leverages the condition monitoring information coming from generation assets to decide on both the *time* and the *number* of maintenances to be scheduled within the planning horizon. We assume the operational decisions such as unit commitment and dispatch are not significant, therefore they are ignored in the APMI model. This assumption is applicable to problems where the outage of an individual generator does not necessarily cause significant impact on the system operations. For example, in a fleet maintenance scheduling problem of a wind farm composed of a large number of wind turbines, the outage of one wind turbine has limited impact on the overall wind farm operation.

#### 3.2.1 Decision Variables

Before introducing the objective and constraints, we first use a simple example to illustrate the meaning of the decision variables  $\mathbf{z}$  and  $\boldsymbol{\nu}$ . To ease exposition, we define  $\boldsymbol{\nu}_{:,i,k} = \{\nu_{1,i,k}, \dots, \nu_{H,i,k}\}$  and  $\mathbf{z}_{:,i,k} = \{z_{1,i,k}, \dots, z_{H,i,k}\}$ . In this example, there are 14 maintenance epochs, each corresponding to a week. Consider the following

schedule:

$$\boldsymbol{\nu}_{:,i,1} = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$\boldsymbol{\nu}_{:,i,2} = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]$$

$$\boldsymbol{\nu}_{:,i,3} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$$\boldsymbol{z}_{:,i,1} = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], z_{i,1}^o = 0$$

$$\boldsymbol{z}_{:,i,2} = [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0], z_{i,2}^o = 0$$

$$\boldsymbol{z}_{:,i,3} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0], z_{i,3}^o = 0.$$

In this schedule, the first maintenance of generator  $i$  starts at week 3. The following maintenances start at weeks 8 and 14, respectively.  $\boldsymbol{\nu}_{:,i,k}$  indicate these starting times.  $\boldsymbol{z}_{:,i,1}$  is defined identical to  $\boldsymbol{\nu}_{:,i,1}$ . The remaining  $\boldsymbol{z}_{:,i,k}$ 's indicate the time difference between two maintenances. For instance, the time difference between the first and the second maintenance is 5 weeks, and this difference is captured by  $\boldsymbol{z}_{:,i,2}$ .

Unique to our modeling is the predetermined input  $M_i$  defined as the maximum number of maintenances to be scheduled on generator  $i$  within the planning horizon  $H$ . Given  $M_i$ , the model dynamically decides how many maintenances to schedule. For this particular example we allow the model to schedule up to 4 maintenances for generator  $i$ . In this example, 3 maintenances are scheduled within the planning horizon of 14 weeks. Therefore,  $\boldsymbol{z}_{:,i,4}$  is a zero vector, and the corresponding vector of  $\boldsymbol{\nu}_{:,i,4}$  is identical to that of the third maintenance which is the last scheduled maintenance.

$$\boldsymbol{\nu}_{:,i,4} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$$

$$\boldsymbol{z}_{:,i,4} = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], z_{i,4}^o = 1.$$

Since the first three maintenances are scheduled for the generator,  $z_{i,k}^o = 0, \forall k \in \{1, 2, 3\}$ . The fourth maintenance is not scheduled, therefore,  $z_{i,4}^o = 1$ .

### 3.2.2 Objective Function

The objective in the APMI model is to minimize the total dynamic maintenance cost of the generator fleet:

$$\sum_{i \in \mathcal{G}} \sum_{t=R_i+1}^H z_{t,i,1} \cdot C_{t_i^o, t-R_i}^{d,i} + \sum_{i \in \mathcal{G}} \sum_{t=Y_i+1}^H \sum_{k=2}^{M_i} z_{t,i,k} \cdot C_{t-Y_i}^{n,i}. \quad (8)$$

Recall that the binary variable  $z_{t,i,k} = 1$  if the  $k$ -th and the  $(k-1)$ -th maintenances of generator  $i$  are separated by  $t$  maintenance epochs.  $\mathcal{G}$  denotes the set of generators. The constants  $H$ ,  $R_i$ ,  $M_i$ , and  $Y_i$  refer to the planning horizon in terms of maintenance epochs, the remaining time required for maintenance of generator  $i$  at the start of the planning period, the maximum number of maintenances to be scheduled for generator  $i$  within the planning horizon, and the maintenance duration for generator  $i$ , respectively.

The objective function evaluates the dynamic costs associated with the first and the consecutive maintenances separately. The first maintenance might benefit from sensor information, whereas the consecutive maintenances are conducted based on new generator costs.

For the first maintenance, we consider two cases: 1) If  $R_i = 0$ , then partially degraded generator  $i$  is operational at the time of planning  $t_p$ . In these cases, the cost function for the generator  $i$  is determined using the sensor updated RLDs. The age of generator  $i$  at  $t_p$  is  $t_i^o$ . For generator  $i$ , sensor observations until time  $t_i^o$  change the estimate on the degradation parameters  $\psi_i$ , and therefore the estimate on  $P(R_{t_i^o}^i > t)$ . Since the dynamic maintenance cost  $C_{t_i^o, t-R_i}^{d,i}$  depends on this estimate, the objective function of APMI also adapts to this update. Otherwise, 2) if  $R_i > 0$ , then generator  $i$  has an ongoing maintenance at the time of scheduling and a new generator will be available at time  $R_i + 1$ . For generator  $i$ , we cannot observe any sensor information, therefore, the dynamic cost for these cases will correspond to a time shifted cost function of a new generator, namely,  $C_{\cdot, t-R_i}^{d,i} = C_{t-R_i}^{n,i}$ .

Certain generators might be scheduled for more than one maintenance. We assume that when a generator is maintained, it starts a new degradation cycle. In other words, the generator becomes as good as new. For these generators, the variable  $\mathbf{z}$  indicates the time difference between the start of two consecutive maintenances. To find the generator age at the time of maintenance, we simply shift the time in  $\mathbf{z}$ , by the duration of maintenance for generator  $i$ , namely  $Y_i$ . When estimating the remaining life distribution of these new degradation cycles, we use only the prior estimations since no other information is revealed to the decision maker at the time of planning.

We next introduce the model constraints.

### 3.2.3 Constraints

#### 3.2.3.1 Maintenance time limits

- Constraint (37) ensures that the first maintenance occurs within  $\zeta_i^d$  maintenance epochs, where  $\zeta_i^d$  depends on the RLD of unit  $i$ . Depending on the application,  $\zeta_i^d$  can be set to a limiting period, when the updated cumulative failure probability exceeds a specific control threshold. Similarly, constraint (10) limits the duration between the start times of two consecutive maintenances using the threshold  $\zeta^n$ .

$$\sum_{t=1}^{\zeta_i^d} \nu_{t,i,1} \geq 1, \quad \forall i \in \mathcal{G}. \quad (9)$$

$$\sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k-1} \leq \zeta^n, \quad (10)$$

$$\forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}.$$

where  $\mathcal{T}$ , and  $\mathcal{K}_i$  refer to the sets of maintenance epochs within the planning horizon, and possible maintenances for generator  $i$ , respectively.

### 3.2.3.2 Maintenance coordination

- APMI allows a number of maintenances to be scheduled within the planning horizon. Constraint (11) ensures that for every such maintenance, a start time is selected.

$$\sum_{t \in \mathcal{T}} \nu_{t,i,k} = 1, \quad \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i. \quad (11)$$

- Constraint (12) controls two factors. Firstly, for generator  $i$ , it dictates whether the  $k^{\text{th}}$  maintenance is scheduled within  $H$  (namely,  $z_{i,k}^o = 0$ ) or is projected to take place beyond  $H$  ( $z_{i,k}^o = 1$ ). Secondly, for any maintenance that is scheduled within  $H$ , it ensures that a certain time is selected to register the difference between two consecutive maintenances.

$$z_{i,k}^o + \sum_{t \in \mathcal{T}} z_{t,i,k} = 1, \quad \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i. \quad (12)$$

- Constraint (13) ensures the  $k$ -th maintenance is scheduled only if the  $(k-1)$ -th maintenance is scheduled.

$$z_{i,k}^o \geq z_{i,k-1}^o, \quad \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}. \quad (13)$$

- Constraint (14) ensures the  $k$ -th maintenance cannot be scheduled before the  $(k-1)$ -th maintenance.

$$\sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} \geq \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k-1}, \quad (14)$$

$$\forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}.$$

- Constraint (15) stipulates that if the  $(k-1)$ -th maintenance is scheduled within  $\zeta^n$  periods from the end of the planning horizon, then the  $k$ -th maintenance cannot be scheduled after the  $(k-1)$ -th maintenance. Therefore, constraints (14)-(15) together ensure that if the  $(k-1)$ -th maintenance is scheduled within

$\zeta^n$  periods from the end of the planning horizon, then the  $k$ -th maintenance is not scheduled.

$$\begin{aligned}
& \sum_{t=1}^{H-\zeta_i^n} H \cdot \nu_{t,i,k-1} + \sum_{t=H-\zeta^n+1}^H t \cdot \nu_{t,i,k-1} \\
& \geq \sum_{t=1}^{H-\zeta^n} H \cdot \nu_{t,i,k} + \sum_{t=H-\zeta^n+1}^H t \cdot \nu_{t,i,k}, \\
& \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}.
\end{aligned} \tag{15}$$

- Constraints (16) and (17) couple the  $\mathbf{z}$  and  $\boldsymbol{\nu}$  variables. For the first maintenance,  $\mathbf{z}$  and  $\boldsymbol{\nu}$  variables are identical as in constraint (16). For the remaining maintenances,  $\mathbf{z}$  captures the time difference of two consecutive maintenances as in constraint (17).

$$z_{t,i,1} = \nu_{t,i,1}, \quad \forall t \in \mathcal{T}, \quad i \in \mathcal{G}. \tag{16}$$

$$\begin{aligned}
\sum_{t \in \mathcal{T}} t \cdot z_{t,i,k} &= \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k-1}, \\
& \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}.
\end{aligned} \tag{17}$$

- The following set of constraints ensure that a unit maintenance cannot be started if there is an ongoing maintenance. Constraints (18) and (60) represent this relationship for the first maintenance and the consecutive maintenances, respectively.

$$\sum_{t=1}^{R_i} \nu_{t,i,1} = 0, \quad \forall i \in \mathcal{G}. \tag{18}$$

$$\begin{aligned}
H \cdot z_{i,k}^o + \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k-1} &\geq Y_i + 1 \\
& \forall i \in \mathcal{G}, \quad k \in \mathcal{K}_i \setminus \{1\}.
\end{aligned} \tag{19}$$

### 3.2.3.3 Maintenance capacity

- The following constraints (44) ensure that the number of ongoing maintenances at time  $t$  does not exceed a limit  $L$ , e.g., a limit on the available labor capacity.

Such constraints have been proposed in literature for problems considering one maintenance per generator [85]. Since our model allows a flexible number of maintenances, we need to consider three cases separately: 1) if  $t \in \{1, \dots, H - \zeta^n\}$ , we need to check for every maintenance  $k$  (constraint (20a)); 2) if  $t \in \{H - \zeta^n + 1, \dots, H - \zeta^n + Y_i - 1\}$ , we check all maintenances scheduled up to time  $H - \zeta^n$  and then check only the last maintenance afterwards (constraint (20b)); 3) if  $t \in \{H - \zeta^n + Y_i, \dots, H\}$ , we only check the last maintenance to eliminate double counting (constraint (20c)).

$$\sum_{i \in \mathcal{G}} \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{Y_i-1} \nu_{t-e,i,k} \leq L \quad \forall t \in \{1, \dots, H - \zeta^n\} \quad (20a)$$

$$\sum_{i \in \mathcal{G}} \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{J}_i^1(t)} \nu_{t-e,i,k} + \sum_{i \in \mathcal{G}} \sum_{e \in \mathcal{J}_i^2(t)} \nu_{t-e,i,M_i} \leq L \quad (20b)$$

$$\forall t \in \{H - \zeta^n + 1, \dots, H - \zeta^n + Y_i - 1\}$$

$$\sum_{i \in \mathcal{G}} \sum_{e=0}^{Y_i-1} \nu_{t-e,i,M_i} \leq L \quad \forall t \in \{H - \zeta^n + Y_i, \dots, H\} \quad (20c)$$

where the sets  $\mathcal{J}_i^1(t) = \{t-H+\zeta^n, \dots, Y_i-1\}$  and  $\mathcal{J}_i^2(t) = \{0, \dots, t-H+\zeta^n-1\}$ .

### 3.2.4 APMI Model

In summary, the APMI model is given as

$$\begin{aligned} (APMI) \quad & \min_{\boldsymbol{\nu}, \boldsymbol{z}} \quad (36) \\ & \text{s.t.} \quad (37) - (44) \\ & \quad \{\boldsymbol{z}, \boldsymbol{\nu}\} \in \mathcal{F}^m. \end{aligned}$$

where  $\mathcal{F}^m$  is defined as:  $\mathcal{F}^m = \{\boldsymbol{z}, \boldsymbol{\nu} \mid z_{t,i,k}, \nu_{t,i,k}, z_{i,k}^o \in \{0, 1\} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \forall k \in \mathcal{K}_i\}$ .

### ***3.3 Experiments***

In this section we present the design of our experiments and the results for APMI. We first use a special case of the degradation model introduced in Section III to model real world degradation data. We then show how we use this data to conduct our experiment. Finally, we present the experimental results to show the performance of the proposed models.

In this chapter, we use vibration data acquired from a rotating machinery application; namely rolling element bearing degradation captured through condition monitoring. Rolling element bearing is chosen for several reasons: i) In condition monitoring of generating units, mechanical methods constitute the most mature branch of technologies used in industry practice [92]. ii) Rolling element bearings are typical examples of components that experience degradation during operation [55].

We use the degradation from bearings as representative of the degradation observed in the generating units. An experimental setup is used to observe the degradation of bearings from brand new state until their failure. Details of this setup can be found in [47].

#### **3.3.1 Experimental Implementation**

In order to test our model, we design an experimental framework. In this framework i) we first solve the maintenance problem to determine the maintenance schedule, and then ii) we execute the chain of events during a freeze period. Based on what happens during this period, we update the operating environment and resolve the maintenance problem. This procedure exhibits a rolling horizon fashion.

We present the two main modules of the experimental procedure as follows:

1. Optimization module: Given dynamic maintenance costs and remaining maintenance downtimes for each generator, this module solves APMI.

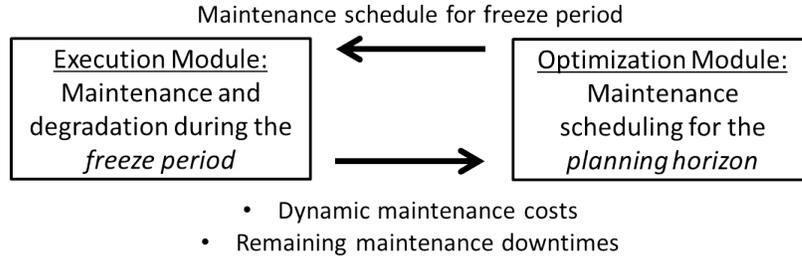
2. Execution module: Given the maintenance plan, this module mimics the system behavior for the duration of the freeze period. More specifically, it uses the degradation database from the rotating machinery application to represent the degradation processes in each generator. For every maintenance epoch during the freeze period, the module checks if any of the generators are experiencing a maintenance downtime, a scheduled preventive maintenance, or an unexpected failure. To detect failure, the module checks if the degradation signal associated with the generator exceeds the failure threshold. This process is repeated for every maintenance epoch within the freeze period. For any failed generator, the module keeps the asset under maintenance for a specified duration. Then, a new degradation signal from the database is chosen to represent the degradation of the new generator after maintenance. Once the execution module reaches to the end of the freeze period, it updates the dynamic maintenance costs for each generator based on the most recent sensor observations. More specifically, the execution module utilizes the observations from the degradation signals of the generators, and derives new RLD and dynamic maintenance cost estimates following the procedure in Chapter 2. The module also takes account of the generators that have undergoing maintenances.

During the execution module process, the key metrics such as the number of unexpected failures & successful preventive maintenances, and the unused life of every generator that experiences preventive maintenance, is computed to present the effectiveness of the current maintenance policy.

Figure 6 presents this experimental framework.

### **3.3.2 Experimental Results**

In this section we present a series of studies to show the performance of APMI. In our analyses, we use a 54 generator system. We obtain the age of generators at the



**Figure 6:** Experimental Framework

start of the experiments by running the generators for a warming period. In all our studies, we set the preventive maintenance cost  $c^p = \$200,000$  and the failure cost  $c^f = \$800,000$ . In order to ensure a fair comparison, we repeat every scenario ten times with different generator ages, and take the average of these experiments. All the models are solved using Gurobi 5.6.0 [51].

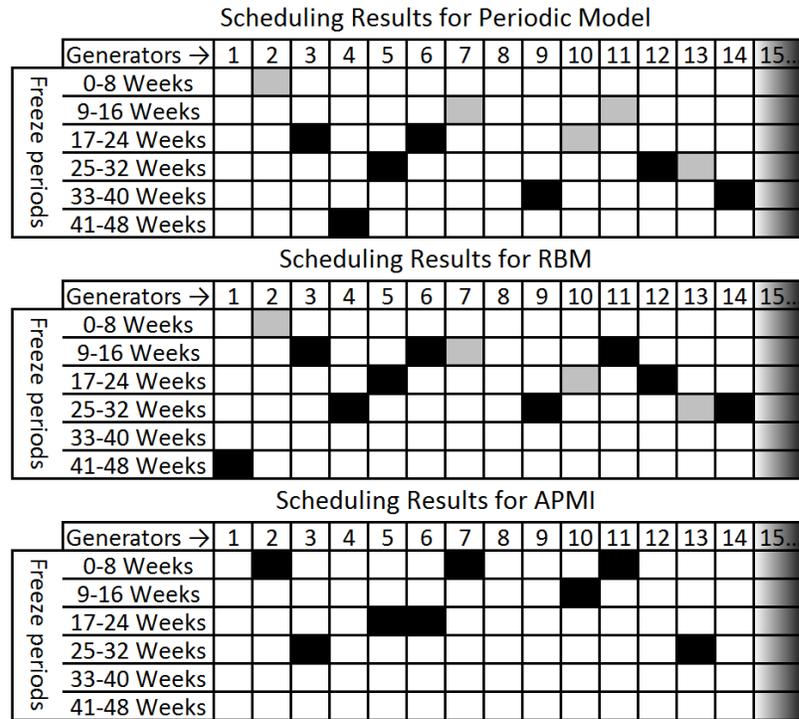
For the purposes of our analysis, the generator maintenance decisions are weekly as suggested by [12], and the system level generator maintenance scheduling is updated according to the specified freeze period  $\tau_R$ . Planning horizon for every optimization model is 110 weeks. Depending on the type of generator and the comprehensiveness of the maintenance study, different periods can be considered for the maintenance decision blocks and the updating frequency.

All experiments involve executing the maintenance framework introduced in the previous subsection. More specifically, to test the performance of a maintenance policy, we first solve the maintenance problem, then run the execution module, which i) mimics the system behavior during the freeze period, and ii) collects the important performance metrics for the analysis. We repeat this process in a rolling horizon fashion.

### 3.3.2.1 Comparative Study on APMI

In this study, we perform a benchmark test for APMI. To do so, we compare the performance of APMI with two policies: periodic maintenance and reliability based

maintenance (RBM). In the periodic maintenance policy, we modify the existing APMI model as follows: i) we let the dynamic maintenance cost be zero, that is  $C_{t_i^o, t}^{d,i} = C_t^{m,i} = 0 \quad \forall i \in \mathcal{G}, \forall t \in \mathcal{T}$ , and ii) we include an additional constraint to ensure that maintenance is conducted when the generator's age is between 66 and 69 weeks. This period is obtained by using the traditional approach proposed by [5]. The problem solves as a feasibility problem with labor capacity constraints. For the RBM case, we use the exact optimization model of APMI, however the cost function for this scenario is derived using a Weibull distribution. We first derive a Weibull estimate using the failure times from the rotating machinery application  $F_W(t)$ , and then condition this distribution on the time of survival to estimate the remaining life distribution and the associated maintenance costs.  $F_W(t)$  in this model, provides the best available prediction of the remaining life distribution without condition monitoring [47]. We let the freeze period  $\tau_R = 8$  weeks, and solve the maintenance problems in a rolling horizon fashion to cover a period of 48 weeks.



**Figure 7:** A Scheduling Plan from Comparative Studies of APMI

In Figure 7, we illustrate different maintenance policies in one of the scheduling scenarios obtained during the comparative studies. Note that the maintenance decisions are weekly. For the sake of illustration, we present the maintenance schedules using time blocks of 8 weeks. We also present the schedule for 14 generators only. A black box indicates a preventive maintenance, and gray box indicates a failure.

We first note that APMI detects when the generator’s condition becomes critical, and conducts a preventive maintenance. For instance, APMI schedules a maintenance between week 25 and 32 for generator 13. This maintenance was not conducted by the periodic model or the RBM model. Therefore, both of them incurred an unexpected failure. In some cases, APMI required maintenance to be conducted at earlier time blocks. For instance, APMI conducts maintenance for generator 7 in the first 8 weeks, otherwise the generator would have failed between the weeks 9 and 16. This means that APMI conducts the maintenance of the generator earlier in order to decrease the risks of failure. This leads to the concept of unused life. Unused life is defined as the time difference between the time of maintenance, and the failure time of the generator under no maintenance regime. This metric quantifies how much of the generator’s available life is sacrificed by the maintenance policy. Evidently, as this value decreases, the risk of failure increases. If the maintenance scheduler would have infinite labor crew resources and perfect information about the component’s failure time, the maintenance would be conducted right before failure. This forms a theoretical bound on the maintenance performance. Since this is not the case in any practical scenario, any additional information helps the policy use more of generators’ useful life. For instance, generator 9 was put under schedule by the periodic and the RBM, although it could survive the 48-week period. Sensor information provided this insight for APMI policy, and thus a maintenance was not scheduled.

We next analyze the results of the comparative study as shown in Table 1. The

**Table 1:** Benchmark for APMI

<b>APMI with <math>\tau_R = 8</math></b>	<b>Periodic</b>	<b>RBM</b>	<b>APMI</b>
<b># Preventive</b>	23.5	33.4	26.6
<b># Failures</b>	13.7	9.6	1.5
<b># Total Outages</b>	37.2	43.0	28.1
<b>Unused Life (weeks)</b>	908.6	1409.3	309.5
<b>Maintenance Cost</b>	\$15.66 M	\$ 14.36 M	\$ 6.52 M

comparative study involves running ten instances of the 48-week experimental implementation for each method. In other words, the results in Table 1 come from 30 experiments, and every presented metric is obtained by taking the average of ten experiments.

The first three metrics relate to the average number of preventive maintenances, failures and total outages observed during these studies. Unused life refers to the average number of sacrificed weeks among all generators. Given the same information, a scheduling model that increases the number of preventive maintenances is expected to create a more conservative maintenance policy, and therefore incur less number of unexpected failures, and sacrifice more lifetime. In our experiment, RBM policy is more conservative, scheduling more preventive maintenances (33.4 v.s. 23.5) than periodic, and consequently incurring a decreased number of unexpected failures (9.6 v.s. 13.7), and sacrificing more weeks of generator lifetime (1409.3 v.s. 908.6 weeks). In terms of the maintenance cost, however, RBM provides significant benefits.

APMI, on the other hand, utilizes the sensor information to improve upon both of these benchmark policies. APMI conducts slightly more preventive maintenances than the periodic model, while incurring significantly less unexpected failures (1.5 for APMI v.s. 13.7 for Periodic) and saving substantial unused lifetime (34.1% of that of the periodic model).

The maintenance cost presented in table is calculated by multiplying the average number of successful preventive maintenances and unexpected failures by  $c^P$ , and

$c^f$  respectively, and then by calculating the total cost incurred. The cost of APMI is 41.6% of the cost in the periodic model. Compared to the RBM model, APMI conducts less preventive maintenances and incurs significantly less failures and unused lifetime. This shows that the maintenance schedule of APMI is superior to that of the periodic and RBM models in terms of both reliability and cost.

### 3.3.2.2 Impact of the Freeze Time on Maintenance Schedules

**Table 2:** Impact of the Freeze Time on APMI

	$\tau_R = 8$	$\tau_R = 6$	$\tau_R = 4$	$\tau_R = 2$
<b># Preventive</b>	26.6	27.2	26.9	26.8
<b># Failures</b>	1.5	1.1	0.7	0
<b># Total Outages</b>	28.1	28.3	27.6	26.8
<b>Unused Life (wks)</b>	309.5	306.9	255.2	187.7
<b>Maintenance Cost</b>	\$6.52 M	\$6.32 M	\$5.94 M	\$5.36 M

Having a flexible maintenance crew that can adapt to more frequent changes in the maintenance schedule might be a feasible economic option for the fleet maintenance for generators of smaller capacities. Since APMI model mainly considers this type of generator fleets, it might be beneficial to study the effect of the freeze time  $\tau_R$  on the maintenance performance. In this study we compare the performance of the maintenance models when the freeze period: i)  $\tau_R = 8$  weeks, ii)  $\tau_R = 6$  weeks, iii)  $\tau_R = 4$  weeks, and iv)  $\tau_R = 2$  weeks. Table 2 presents the results.

As the freeze time decreases, in other words, as the updates in the maintenance schedule become more frequent, APMI can learn more about the generator’s degradation characteristics before making the final maintenance plan. This corresponds to a better understanding if a maintenance can be postponed (thus getting more out of the available resources), or scheduled to an earlier time (thus decreasing the risks of failure). We note that the average costs of maintenance decreases as the maintenance schedule is updated more frequently. Thus, it would be reasonable to invest up to

\$200,000 to improve the maintenance crew flexibility to be capable of  $\tau_R = 6$  weeks, as opposed to  $\tau_R = 8$  weeks. Additional investment of up to \$380,000 can be made to further improve the flexibility so that the crew can respond to monthly changes in maintenance.  $\tau_R = 2$  follows a similar pattern.

### ***3.4 Conclusion***

In this chapter, we proposed a mathematical framework that incorporates the sensor-driven predictive analytics that estimates the remaining life distribution of generators, into the maintenance scheduling optimization problem. To do so, we proposed an innovative mixed-integer optimization model for the fleet maintenance problem. Experimental results indicate that using our method provides significant advantages in both cost and reliability. More specifically, APMI significantly reduces the number of unexpected failures by  $\geq 84.37\%$ , the unused life by  $\geq 65.93\%$ , and the maintenance cost by  $\geq 54.59$  (See Table 1). We also note that APMI favors flexible maintenance workforce. As the maintenance crew's ability to adapt to changes in the maintenance schedule increases, the APMI model allows observation of more sensor information before making decisions, therefore improving the quality of the maintenance schedule (See Table 2).

In the following chapter, we expand the model presented herein to consider the effects of maintenance on network operation by coordinating generator maintenance schedules with the unit commitment (UC) and dispatch decisions. We also propose an effective solution algorithm to solve the new model, and conduct computational experiments.

## CHAPTER IV

# SENSOR-DRIVEN CONDITION-BASED GENERATION MAINTENANCE SCHEDULING: INCORPORATING OPERATIONS

### *4.1 Introduction*

In this chapter, we expand on the adaptive predictive generator maintenance model introduced in Chapter 3 by incorporating unit commitment and dispatch decisions. We present a solution methodology to solve this extended scheduling problem in large cases. Finally, we run a series of experiments to present the performance of the proposed model. The results indicate that the use of adaptive predictive model provides considerable improvements in both cost and reliability as identified by the IEEE task force [40].

The chapter is organized as follows. In Section II we propose a new adaptive predictive maintenance model (APMII) that considers unit commitment and dispatch decisions. In Section III, we reformulate APMII as a two-stage mixed integer problem, and also introduce its relaxation. In Section IV, we propose a new reformulation of the APMII model, which has a relaxed subproblem structure but the objective is augmented so that it exactly recovers the true cost of the APMII model. In Section V, we propose an exact algorithm to solve a reformulation of this problem, which is particularly useful for solving large-scale cases of the APMII model. In Section VI, we present an experimental framework that uses this degradation database to study a number of test cases. We show the effectiveness of our model through extensive comparative studies. In section VII, we conclude this chapter with some closing remarks.

## 4.2 Adaptive Predictive Maintenance Problem II

We start by providing a list of decision variables, sets and constants used in APMII.

### Decision Variables:

$\nu_{t,i,k} \in \{0, 1\}$        $\nu_{t,i,k} = 1$  iff the  $k^{th}$  maintenance of generator  $i$  starts at maintenance epoch  $t$ . If a certain maintenance  $k$  is not scheduled, then  $\nu_{t,i,k} = \nu_{t,i,k_\ell}$  for all  $t \in \mathcal{T}$ , where  $k_\ell$  is the last scheduled maintenance.

$z_{t,i,k} \in \{0, 1\}$        $z_{t,i,k} = 1$  iff the duration between the start of the  $k^{th}$  and the  $(k-1)^{th}$  maintenances of generator  $i$  is  $t$  maintenance epochs.

$z_{i,k}^o \in \{0, 1\}$        $z_{i,k}^o = 0$  iff the  $k^{th}$  maintenance is scheduled for generator  $i$  within the planning horizon.

$x_{s,i}^t \in \{0, 1\}$        $x_{s,i}^t = 1$  iff generator  $i$  is committed in hour  $s$  within maintenance epoch  $t$ .

$\pi_{s,i}^{U,t} \in \{0, 1\}$        $\pi_{s,i}^{U,t} = 1$  iff generator  $i$  starts up in hour  $s$  within maintenance epoch  $t$ .

$\pi_{s,i}^{D,t} \in \{0, 1\}$        $\pi_{s,i}^{D,t} = 1$  iff generator  $i$  shuts down in hour  $s$  within maintenance epoch  $t$ .

$y_{s,i}^t \in \mathbb{R}_+^n$       Generation output of generator  $i$  in hour  $s$  within maintenance epoch  $t$ .

$\psi_{s,p}^{DC,t} \in \mathbb{R}_+^n$       Demand curtailment in hour  $s$  within maintenance epoch  $t$  at demand bus  $p$ .

$\psi_{s,\ell}^{TL,t} \in \mathbb{R}_+^n$       Transmission line slack variable in hour  $s$  within maintenance epoch  $t$  at line  $\ell$ .

**Sets:**

$\mathcal{V}$  Set of loads.

$\mathcal{G}$  Set of generators.

$\mathcal{K}_i$  Set of possible maintenances for generator  $i$ .

$\mathcal{L}$  Set of transmission lines.

$\mathcal{S}$  Set of hours within one maintenance epoch.

$\mathcal{T}$  Set of maintenance epochs within the planning horizon.

**Constants:**

$B_{s,i}^t$  Generation cost of generator  $i$  in hour  $s$  within maintenance epoch  $t$ .

$C_{t_i^o,t}^{d,i}$  Cost of maintenance for a partially degraded generator  $i$ , when the maintenance is scheduled to  $t$  maintenance epoch after the time of observation  $t_i^o$ .

$C_t^{m,i}$  Cost of maintenance for a new generator  $i$ , when the age of the generator at the time of its maintenance is  $t$  maintenance epochs.

$Y_i$  Maintenance duration for generator  $i$ .

$H$  Planning horizon in terms of maintenance epochs.

$L$  Maximum number of generators that can be under maintenance simultaneously.

$M_i$  Maximum number of maintenances to be scheduled for generator  $i$  within the planning horizon.

$P_{DC}$  Penalty cost for unit unsatisfied demand.

$P_{TL}$  Penalty cost for unit overload on a transmission line.

- $R_i$  Remaining time required for maintenance of generator  $i$  at the start of the planning horizon.
- $U_{s,i}^{U,t}$  Start-up cost of generator  $i$  in hour  $s$  within maintenance epoch  $t$ .
- $U_{s,i}^{D,t}$  Shut-down cost of generator  $i$  in hour  $s$  within maintenance epoch  $t$ .
- $V_{s,i}^t$  No-load cost of generator  $i$  in hour  $s$  within maintenance epoch  $t$ .
- $\zeta_i^d$  Period within which at least one maintenance should be scheduled to start for degraded generator  $i$ .
- $\zeta^n$  Period within which at least one maintenance should be scheduled to start for a new generator.

In this Chapter, we expand our analysis to consider generation commitment and dispatch in the optimal maintenance scheduling problem. The key balance in APMII is between explicit and implicit costs of maintenance. We continue leveraging the results of the predictive analytics to ensure an adaptive characterization of the costs of maintenance, but at the same time, we now consider the impact of maintenance on operations, such as the overall production cost and network feasibility. The main intuition behind APMII model is that, in most practical applications, it would be preferable to deviate from the pure maintenance optimal policy (APMI policy presented in Chapter 3) in an effort to decrease the unit commitment and dispatch cost. Utility companies put great emphasis on the forecasted demand while deciding on the maintenance schedules. APMII provides a model that considers the consequences of every maintenance action on the operational side, while benefiting from the adaptive predictive estimates on the generator failure risks. The maintenance variables are identical as in APMI, besides we also have commitment variable  $\mathbf{x}$  and dispatch variable  $\mathbf{y}$ .

### 4.2.1 Objective Function

The objective is to minimize the maintenance and operations cost:

$$\begin{aligned}
& \xi_m \left[ \sum_{i \in \mathcal{G}} \sum_{t=R_i+1}^H C_{t_i^o, t-R_i}^{d,i} \cdot z_{t,i,1} + \sum_{i \in \mathcal{G}} \sum_{t=Y_i+1}^H \sum_{k=2}^{M_i} C_{t-Y_i}^{m,i} \cdot z_{t,i,k} \right] \\
& + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} \sum_{s \in \mathcal{S}} \left( V_{s,i}^t \cdot x_{s,i}^t + U_{s,i}^{U,t} \cdot \pi_{s,i}^{U,t} + U_{s,i}^{D,t} \cdot \pi_{s,i}^{D,t} + B_{s,i}^t \cdot y_{s,i}^t \right) \\
& + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \left( \sum_{p \in \mathcal{D}} (P_{DC} \cdot \psi_{s,p}^{DC,t}) + \sum_{\ell \in \mathcal{L}} (P_{TL} \cdot \psi_{s,\ell}^{TL,t}) \right), \tag{21}
\end{aligned}$$

where  $\xi_m$  is the maintenance criticality coefficient. The objective function (53) consists of two components: dynamic maintenance cost (the first line) and operational cost including UC, dispatch, and penalty costs (the second and third lines). For the explanation on the dynamic maintenance cost in the first line of the objective function, we refer the reader to Section 2 in Chapter 3. The remaining cost factors are typical in UC literature.

### 4.2.2 Constraints

The cost (53) is minimized subject to some of the constraints defined in Chapter 3. More specifically, APMII is subject to:

1. *Maintenance time limits:* This set refers to the restrictions on the time of the first maintenance, and the time between consecutive maintenances (37,10).
2. *Maintenance coordination:* These constraints i) impose logical restrictions such as maintenance durations, ii) allow flexible number of maintenances within the planning horizon, and iii) ensure a mapping between the time of maintenance, and the age of the generator at the time of maintenance (11)-(19).
3. *Maintenance capacity:* This set of constraints ensure that the number of ongoing maintenances at any time  $t$  does not exceed a prespecified limit  $L$  (20).

We consider two additional sets of constraints for coupling of maintenance and operations, and unit commitment (UC).

#### 4.2.2.1 *Coupling of maintenance and operations*

- In cases where a certain generator is under maintenance at the start of the planning horizon, the corresponding commitment variable  $\mathbf{x}$  is set to zero.

$$x_{s,i}^t = 0 \quad \forall i \in \mathcal{G} \text{ and } R_i > 0 \quad (22)$$

$$\forall s \in \mathcal{S}, t \in \{1, \dots, R_i\}.$$

- In the following set of constraints, we couple the maintenance decision variable  $\nu$  with the commitment variable  $\mathbf{x}$ . Constraint (45) ensures that if a unit is under maintenance during maintenance epoch  $t$ , it cannot be committed in any of the hours within that epoch.

$$x_{s,i}^t \leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{Y_i-1} \nu_{t-e,i,k} \quad (23a)$$

$$\forall i \in \mathcal{G}, t \in \{1, \dots, H - \zeta^n\}, s \in \mathcal{S}$$

$$x_{s,i}^t \leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{J}_i^1(t)} \nu_{t-e,i,k} + \sum_{e \in \mathcal{J}_i^2(t)} \nu_{t-e,i,M_i} \quad (23b)$$

$$\forall i \in \mathcal{G}, t \in \{H - \zeta^n + 1, \dots, H - \zeta^n + Y_i - 1\}, s \in \mathcal{S}$$

$$x_{s,i}^t \leq 1 - \sum_{e=0}^{Y_i-1} \nu_{t-e,i,k} \quad (23c)$$

$$\forall i \in \mathcal{G}, t \in \{H - \zeta^n + Y_i, \dots, H\}, s \in \mathcal{S},$$

where the sets  $\mathcal{J}_i^1(t) = \{t - H + \zeta^n, \dots, Y_i - 1\}$  and  $\mathcal{J}_i^2(t) = \{0, \dots, t - H + \zeta^n - 1\}$ .

#### 4.2.2.2 Unit commitment

- The UC problem includes constraints on i) commitment status: such as minimum up/down, and start-up/shut-down, ii) dispatch level: such as energy balance, transmission limit and ramping, iii) commitment coupling: such as minimum and maximum dispatch levels for each generator based on the commitment status. In its compact form, we represent this set of constraints as follows:

$$\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{y} \leq \boldsymbol{\ell} \quad (24)$$

where  $\mathbf{x}$  includes the generator commitment, start-up, and shut-down variables, and  $\mathbf{y}$  includes generation dispatch, demand curtailment, and line slack variables.

#### 4.2.3 APMII Model

In summary, the APMII model is given as:

$$\begin{aligned}
 (APMII) \quad & \min_{\mathbf{z}, \boldsymbol{\nu}, \mathbf{x}, \mathbf{y}} \quad (53) \\
 \text{s.t.} \quad & \{ (37)-(44) \text{ from Chapter III} \}, \\
 & (46) - (24) \\
 & \{\mathbf{z}, \boldsymbol{\nu}\} \in \mathcal{F}^m, \\
 & \mathbf{x} \in \{0, 1\}^{3|\mathcal{G}| \times H \times |\mathcal{S}|}, \mathbf{y} \in \mathbb{R}_+^{(|\mathcal{G}|+J) \times H \times |\mathcal{S}|},
 \end{aligned}$$

where  $J = |\mathcal{V}| + |\mathcal{L}|$ . It turns out that we can relax the  $z_{t,i,k}$  variables to be continuous and still obtain a binary optimal solution for both APMI and APMII, as shown in the following lemma.

**Lemma 1.** *If the binary variables  $z_{t,i,k}$ 's in (APMII) are relaxed to be continuous, then the relaxed problem still has a binary optimal solution, which is thus optimal for (APMII). The same statement also holds for APMI introduced in Chapter III.*

*Proof.* See Appendix A. □

### 4.3 Two-Stage Reformulation of APMII

The APMII model has a natural two-stage structure, namely the first stage makes the maintenance decision, while the second stage deals with the UC problem based on the maintenance decision.

#### 4.3.1 APMII Reformulation

The APMII model can be written in the following compact form:

$$\min_{\mathbf{z}, \boldsymbol{\nu}, \mathbf{x}, \mathbf{y}} \quad \mathbf{c}^\top \mathbf{z} \quad + \quad \mathbf{v}^\top \mathbf{x} \quad + \mathbf{b}^\top \mathbf{y} \quad (25a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} \quad + \mathbf{K}\boldsymbol{\nu} \quad \leq \mathbf{g} \quad (25b)$$

$$\mathbf{B}\boldsymbol{\nu} \quad + \mathbf{E}\mathbf{x} \quad \leq \mathbf{h} \quad (25c)$$

$$\mathbf{F}\mathbf{x} \quad + \mathbf{G}\mathbf{y} \quad \leq \boldsymbol{\ell} \quad (25d)$$

$$\{\mathbf{z}, \boldsymbol{\nu}\} \in \mathcal{F}^m, \mathbf{x} \in \{0, 1\}^{3|\mathcal{G}| \times H \times |\mathcal{S}|}, \mathbf{y} \in \mathbb{R}_+^{(|\mathcal{G}|+J) \times H \times |\mathcal{S}|}.$$

where  $\mathbf{z}, \boldsymbol{\nu}$  are the maintenance variables,  $\mathbf{x}$  is the generator commitment, start-up, and shut-down variables, and  $\mathbf{y}$  includes generation dispatch, demand curtailment, and line slack variables. Here,  $\dim \mathbf{g} = 8 \cdot \sum_{i \in \mathcal{G}} M_i \cdot |\mathcal{G}| + 3 \cdot H \cdot |\mathcal{G}| + H - 4 \cdot |\mathcal{G}|$ ,  $\dim \mathbf{h} = H \cdot |\mathcal{S}| \cdot |\mathcal{G}|$ , and  $\dim \boldsymbol{\ell} = H \cdot |\mathcal{S}| + H \cdot |\mathcal{S}| \cdot |\mathcal{G}| + 2 \cdot H \cdot |\mathcal{S}| \cdot |\mathcal{L}|$ .

In this formulation, the objective function is identical to (53). The constraint (25b) corresponds to maintenance decisions, such as the maintenance labor capacity constraints and the interaction between different maintenance variables, namely constraints (37)-(44) in Chapter III}. Constraint (25c) couples the maintenance and the unit commitment variables, so that a generating unit is not committed, if a maintenance activity is still being conducted on that particular unit. They correspond to the constraints in (45) and (46).

The APMII model (25) can be decomposed into a two-stage program, where the maintenance problem resides in the first stage, and UC given maintenance decisions constitutes the second-stage problem as follows,

$$\min_{\mathbf{z}, \boldsymbol{\nu}} \quad \mathbf{c}^\top \mathbf{z} + q(\boldsymbol{\nu}) \quad (26a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} \leq \mathbf{g} \quad (26b)$$

$$\{\mathbf{z}, \boldsymbol{\nu}\} \in \mathcal{F}^m.$$

where  $q(\boldsymbol{\nu})$  denotes the UC problem given maintenance decision  $\boldsymbol{\nu}$ . We consider minimum up/down, and ramping constraints within the hours of the same maintenance epoch (e.g. a week). In this way, once the maintenance decision  $\boldsymbol{\nu}$  is fixed, the unit commitment decisions for any maintenance epoch  $t \in \mathcal{T}$ , namely  $\{\mathbf{x}^t, \mathbf{y}^t\}$ , become independent. Thus the subproblem  $q(\boldsymbol{\nu})$  can be further decomposed into different maintenance epochs,  $q(\boldsymbol{\nu}) = \sum_{t=1}^N q^t(\boldsymbol{\nu})$  where  $q^t(\boldsymbol{\nu})$  is given by:

$$q^t(\boldsymbol{\nu}) = \min_{\mathbf{x}^t, \mathbf{y}^t} \quad (\mathbf{v}^t)^\top \mathbf{x}^t + (\mathbf{b}^t)^\top \mathbf{y}^t \quad (27a)$$

$$\text{s.t.} \quad \mathbf{E}^t \mathbf{x}^t \leq \mathbf{h}^t - \mathbf{B}^t \boldsymbol{\nu}, \quad (27b)$$

$$\mathbf{F}^t \mathbf{x}^t + \mathbf{G}^t \mathbf{y}^t \leq \boldsymbol{\ell}^t \quad (27c)$$

$$\mathbf{x}^t \in \{0, 1\}^{3|\mathcal{G}| \times |\mathcal{S}|}, \mathbf{y}^t \in \mathbb{R}^{(|\mathcal{G}|+J) \times |\mathcal{S}|}.$$

Even in large cases, it is not computationally expensive to solve  $q^t(\boldsymbol{\nu})$  for a given  $\boldsymbol{\nu}$ .

### 4.3.2 Relaxation for APMII (R-APMII)

We next define a relaxation for the APMII problem, namely R-APMII, by relaxing the binary UC variables to continuous variables so that  $\mathbf{x}^t \in [0, 1]^{3|\mathcal{G}| \times |\mathcal{S}|} \forall t \in \mathcal{T}$ . This new model can be decomposed into a master maintenance problem and a linear relaxation of the UC subproblem in a similar manner. We denote the objective of the relaxed UC subproblem in R-APMII as  $\tilde{q}(\boldsymbol{\nu})$ , and its cost for any maintenance

epoch  $t$  as  $\tilde{q}^t(\boldsymbol{\nu})$ . That is,  $\tilde{q}(\boldsymbol{\nu}) = \sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu})$ . Then, R-APMII can be represented as follows:

$$\min_{\mathbf{z}, \boldsymbol{\nu}} \quad \mathbf{c}^\top \mathbf{z} + \sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}) \quad (28a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} \leq \mathbf{g} \quad (28b)$$

$$\mathbf{z}, \boldsymbol{\nu} \in \mathcal{F}^m.$$

We note that this new formulation provides a lower bound for the APMII problem. It is considerably easier to solve through Benders' Decomposition, since the subproblems are non-integer. However, there is a need to link the solution of this relaxed formulation to the APMII problem.

#### 4.4 *Alternative Formulation (AF) for APMII*

In this section, we construct an alternative formulation (AF) that can recover the true cost of the APMII problem with a subproblem structure identical to the relaxed model R-APMII.

We start with the observation that the interaction between the maintenance and the unit commitment variables can be completely characterized through generator's maintenance status. That is, for any maintenance epoch  $t$ , the UC cost can be determined if we know which generators have an ongoing maintenance. For the sake of clarity, we define an additional variable  $m_i^t$  that takes the value  $m_i^t = 1$  if generator  $i$  is undergoing maintenance at maintenance epoch  $t$ , and  $m_i^t = 0$  otherwise. We note that  $m_i^t$  is uniquely determined by the maintenance variables  $\boldsymbol{\nu}$ . In particular, generator  $i$  would have an ongoing maintenance at maintenance epoch  $t$ , if its maintenance has started during  $\{t - Y_i + 1, \dots, t\}$ , in other words, if  $\sum_{k \in \mathcal{K}_i} \sum_{e=0}^{Y_i-1} \nu_{t-e,i,k} \geq 1$ . The idea is that to find the cost for a certain maintenance status  $\hat{\mathbf{m}}$ , we can solve the relaxed model, R-APMII, and then add the difference  $\sum_{t \in \mathcal{T}} q^t(\hat{\mathbf{m}}^t) - \tilde{q}^t(\hat{\mathbf{m}}^t)$  back to the objective cost of the relaxed model R-APMII. In this way, the cost of the true

model APMII is recovered.

We assume for the time being that we can enumerate all possible maintenance statuses for every  $t \in \mathcal{T}$ . This complete set is denoted by  $\bar{\Omega} := \{\bar{\Omega}^1, \dots, \bar{\Omega}^H\}$  with cardinality  $H \times 2^{|\mathcal{G}|}$ . We let  $\hat{\mathbf{m}}_h^t$  denote one of these statuses at  $t$  with a corresponding status index  $h \in \bar{\Omega}^t$ . In what follows, we show how we can i) create an additional variable and constraint to check if the maintenance solution  $\boldsymbol{\nu}$  corresponds to the particular maintenance status  $\hat{\mathbf{m}}_h^t$ , and ii) recover the true UC cost when the maintenance solution  $\boldsymbol{\nu}$  implies  $\hat{\mathbf{m}}_h^t$ .

We start with the first objective. For generator status  $h$  at  $t$ , we define a binary variable  $\eta_h^t$  subject to:

$$\eta_h^t \geq \left( \sum_{i \in \mathbb{K}(\hat{\mathbf{m}}_h^t)} m_i^t - \sum_{i \in \mathbb{F}(\hat{\mathbf{m}}_h^t)} m_i^t - |\mathbb{K}(\hat{\mathbf{m}}_h^t)| + 1 \right), \quad (29)$$

where the index set  $\mathbb{K}(\hat{\mathbf{m}}_h^t) := \{i | \hat{m}_{h,i}^t = 1\}$ , and  $\mathbb{F}(\hat{\mathbf{m}}_h^t) := \{i | \hat{m}_{h,i}^t = 0\}$ . This constraint ensures that the binary variable  $\eta_h^t \geq 1$  when  $\mathbf{m}^t = \hat{\mathbf{m}}_h^t$ , and  $\eta_h^t$  is not bounded otherwise. This claim holds since: i) when  $\mathbf{m}^t = \hat{\mathbf{m}}_h^t$ , the right hand side equals 1, ii) otherwise, if there is at least one  $i$  where  $m_i^t \neq \hat{m}_{h,i}^t$ , then the right hand side becomes less than or equal to zero. Constraint (29) is presented for the sake of clarity. In reality, we need to link the solution  $\boldsymbol{\nu}$  to  $\hat{\mathbf{m}}_h^t$ . We note again that given the maintenance start variables  $\boldsymbol{\nu}$ , maintenance status variables  $\mathbf{m}$  can be obtained in a straight forward way. We also note that if the  $k$ -th maintenance is not scheduled for generator  $i$ , then  $\nu_{t,i,k} = \nu_{t,i,k-1} \forall t \in \mathcal{T}$ . In order to eliminate double-counting of maintenance instances, the constraint (29) can be constructed using  $\boldsymbol{\nu}$  as follows:

$$\eta_h^t \geq \sum_{i \in \mathcal{G}} \left( R_{h,i}^t \cdot \left( \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{Y_i-1} \nu_{t-e,i,k} \right) - U_{h,i}^t \right) + 1 \quad (30a)$$

$$\forall t \in \{1, \dots, H - \zeta^n\}$$

$$\eta_h^t \geq \sum_{i \in \mathcal{G}} R_{h,i}^t \cdot \left( \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{J}_i^1(t)} \nu_{t-e,i,k} + \sum_{e \in \mathcal{J}_i^2(t)} \nu_{t-e,i,M_i} \right) - \sum_{i \in \mathcal{G}} U_{h,i}^t + 1 \quad (30b)$$

$$\forall t \in \{H - \zeta^n + 1, \dots, H - \zeta^n + Y_i - 1\}$$

$$\eta_h^t \geq \sum_{i \in \mathcal{G}} \left( R_{h,i}^t \cdot \sum_{e=0}^{Y_i-1} \nu_{t-e,i,M_i} - U_{h,i}^t \right) + 1 \quad (30c)$$

$$\forall t \in \{H - \zeta^n + Y_i, \dots, H\},$$

where  $R_{h,i}^t = 1, U_{h,i}^t = 1$  if  $\hat{m}_{h,i}^t = 1$ . Otherwise,  $R_{h,i}^t = -1, U_{h,i}^t = 0$ . Note that the term with  $R_{h,i}^t$  corresponds to the difference of summations in (29). The second term with  $U_{h,i}^t$  provides the cardinality of the set in (29).  $\mathcal{J}_i^1(t)$  and  $\mathcal{J}_i^2(t)$  are defined similarly in [106, Eq.(18b)-(18c)]. We denote the constraint (30) for maintenance epoch  $t$  and the binary variable  $\eta_h^t$  in its compact form as:  $(\mathbf{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \geq u_h^t$ .

Define the cost  $e_h^t$  associated with the  $h$ -th maintenance status  $\hat{\mathbf{m}}_h^t$  as the difference between the true and relaxed costs of the UC subproblem at time  $t$ , namely,

$$e_h^t = q^t(\hat{\mathbf{m}}_h^t) - \tilde{q}^t(\hat{\mathbf{m}}_h^t). \quad (31)$$

where  $q^t(\hat{\mathbf{m}}_h^t)$  is the solution  $q^t(\boldsymbol{\nu})$  in (27) for any  $\boldsymbol{\nu}$  that implies  $\hat{\mathbf{m}}_h^t$ .

We can repeat this process for all  $h \in \Omega_h^t$ . Then, the following holds for any  $\boldsymbol{\nu}$ :

$$q^t(\boldsymbol{\nu}) = \tilde{q}^t(\boldsymbol{\nu}) + \min_{\boldsymbol{\eta}} \left\{ \sum_{h \in \bar{\Omega}^t} \eta_h^t e_h^t : \eta_h^t \geq u_h^t - (\mathbf{r}_h^t)^\top \boldsymbol{\nu}, \forall h \in \bar{\Omega}^t \right\}. \quad (32)$$

In fact, the solution for this problem is clear, that is, only for one  $h \in \bar{\Omega}^t$ ,  $u_h^t - (\mathbf{r}_h^t)^\top \boldsymbol{\nu} = 1$  is true. We denote this term by  $h^*$ . Then:  $q^t(\boldsymbol{\nu}) = \tilde{q}^t(\boldsymbol{\nu}) + e_{h^*}^t$ .

We next use this observation to reformulate the APMII problem, by replacing  $q(\boldsymbol{\nu})$  with its equivalent in (32). The following AF problem can attain the optimal objective cost and maintenance decisions of APMII:

**AF Problem:**

$$\min_{\mathbf{z}, \boldsymbol{\nu}, \boldsymbol{\eta}} \quad \mathbf{c}^\top \mathbf{z} + \mathbf{e}^\top \boldsymbol{\eta} + \sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}) \quad (33a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} \leq \mathbf{g} \quad (33b)$$

$$(\mathbf{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \geq u_h^t \quad \forall t \in \mathcal{T}, \forall h \in \bar{\Omega}^t \quad (33c)$$

$$\mathbf{z}, \boldsymbol{\nu} \in \mathcal{F}^m, \boldsymbol{\eta} \in \{0, 1\}^{H \times 2^{|\mathcal{G}|}}. \quad (33d)$$

In the following, we develop an iterative algorithm to solve this AF problem.

#### 4.5 Solution Algorithm for APMII

APMII is a computationally expensive problem to solve. Therefore, it is important to design an efficient algorithm to solve large-scale APMII models. In this section, we present an exact solution algorithm that uses the special structure of APMII to intelligently reconstruct the elements of (33), in an attempt to find the optimal solution for APMII. We have two observations to motivate the algorithm at this point: i) the cost  $\tilde{q}^t(\boldsymbol{\nu})$  can be recovered through Benders' decomposition, ii) more importantly, it would be sufficient to incorporate subset of maintenance statuses  $\Omega \subseteq \bar{\Omega}$  in the AF problem to recover the true UC cost. This set is typically small due to the following properties of the maintenance problem:

1. The minimizer of the maintenance cost term  $\mathbf{c}^\top \mathbf{z}$  is an important factor for determining the time of maintenance. The APMII's optimal solution typically does not schedule maintenance very far from this minimizer.
2. The difference between the total cost of the relaxed formulation R-APMII and the APMII problem is small. Therefore, when one considers the maintenance cost and relaxed UC cost, it suffices to check only a number of different points before the true costs from these points reside below the lower bounds of conducting maintenance in other time epochs.

In line with these claims, for a given set of generator maintenance statuses  $\Omega := \cup_{t \in \mathcal{T}} \{\Omega^t : \Omega^t \subseteq \bar{\Omega}^t\}$  and a set of Benders' optimality cuts  $\mathcal{BD} := \cup_{t \in \mathcal{T}} \{\mathcal{BD}^t\}$ , a restricted master problem  $\mathbf{RMP}(\Omega, \mathcal{BD})$  can be presented as follows:

**RMP**( $\Omega, \mathcal{BD}$ ) Problem:

$$\min_{\mathbf{z}, \boldsymbol{\nu}, \boldsymbol{\eta}, \boldsymbol{\varphi}} \quad \mathbf{c}^\top \mathbf{z} + \mathbf{e}^\top \boldsymbol{\eta} + \sum_{t \in \mathcal{T}} \varphi^t \quad (34a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} \leq \mathbf{g} \quad (34b)$$

$$(\mathbf{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \geq u_h^t \quad \forall t \in \mathcal{T}, \forall h \in \Omega^t \quad (34c)$$

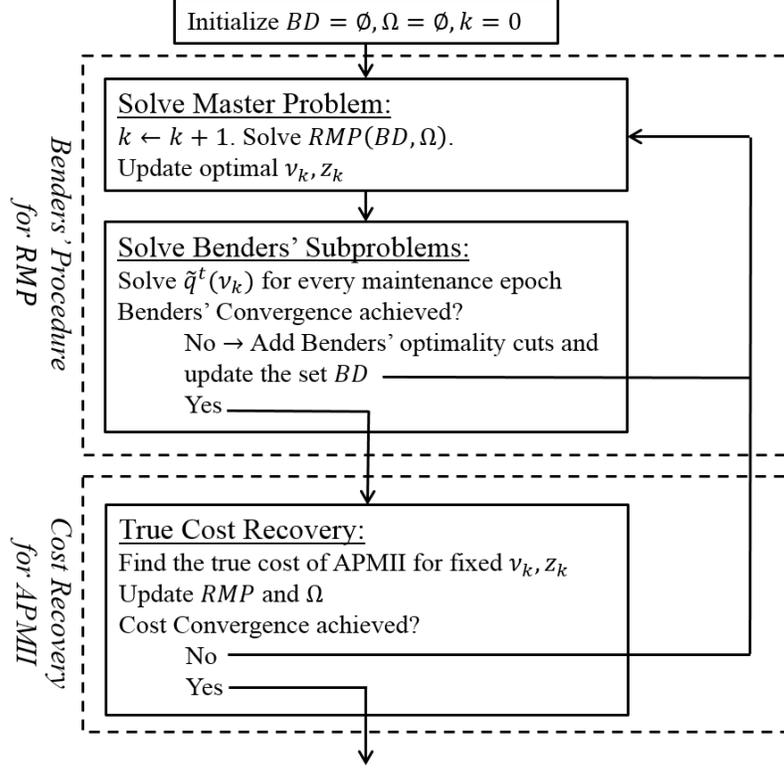
$$(\boldsymbol{\alpha}_k^t)^\top (\mathbf{h}_k^t - \mathbf{B}_k^t \boldsymbol{\nu}_k) + (\boldsymbol{\beta}_k^t)^\top \boldsymbol{\ell}_k^t \leq \varphi^t \quad (34d)$$

$$\forall t \in \mathcal{T}, \forall k \in \mathcal{BD}^t$$

$$\mathbf{z}, \boldsymbol{\nu} \in \mathcal{F}^m, \boldsymbol{\eta} \in \{0, 1\}^{|\Omega|}.$$

The Benders' optimality constraints (34d) will be discussed in the algorithm description. When  $\Omega^t = \Omega_c^t$  for all  $t$ , and the set  $\mathcal{BD}$  ensures Benders' convergence so that the optimal  $\varphi^{t*} = \tilde{q}^t(\boldsymbol{\nu}^*)$  for all  $t$ , the optimal maintenance decisions  $\{\mathbf{z}^*, \boldsymbol{\nu}^*\}$ , and the objective total cost becomes identical in APMII, AF, and RMP. This simple observation comes from Eq. (32). As we noted previously, only a subset of these generator availability vectors may be needed to recover the optimal cost and maintenance decisions  $\{\mathbf{z}^*, \boldsymbol{\nu}^*\}$  for APMII. This observation provides a claim parallel to the findings of [67].

Due to the two-stage nature of this problem, we propose a two-level algorithm to solve APMII. In the upper level, the algorithm solves the restricted master problem  $\mathbf{RMP}(\Omega, \mathcal{BD})$  iteratively to generate Benders' optimality cuts for every maintenance epoch. The Benders' optimality cuts are appended to the set  $\mathcal{BD}$ , and the algorithm repeats the Benders' process until convergence. Then the current solution is used to generate variables and constraints as in Eq. (30) in order to recover the true cost of APMII. We append the maintenance scenario of the current solution to the set  $\Omega$ ,



**Figure 8:** Flowchart of the Proposed Algorithm for Solving APMII.

then check for convergence in terms of true cost recovery. Repeat the process if cost convergence criteria is violated; otherwise, terminate. Flowchart of the algorithm is illustrated in Figure 8 and the method is formally presented in Algorithm 1. The following theorem proves the convergence of the algorithm.

**Theorem 1.** *Algorithm 1 with tolerances  $\epsilon^b$  and  $\epsilon^c$  terminates in a finite number of steps, and returns an  $\epsilon$ -optimal maintenance solution  $\{\mathbf{z}^*, \boldsymbol{\nu}^*\}$ , i.e.  $\rho^* \leq \rho(\mathbf{z}^*, \boldsymbol{\nu}^*) \leq \rho^*(1 + \epsilon)$ , where  $\rho^*$  is the optimal cost of APMII,  $\rho(\mathbf{z}^*, \boldsymbol{\nu}^*) = \mathbf{c}^\top \mathbf{z}^* + q(\boldsymbol{\nu}^*)$  in (26), and  $\epsilon = (1 + \epsilon^b)(1 + \epsilon^c) - 1$ .*

*Proof.* See Appendix A. □

Remark: Note that the current form of the algorithm has slack variables in the demand balance and line flow constraints in the unit commitment subproblem, so it remains feasible for any maintenance decision  $\boldsymbol{\nu}$ . We can also remove these slack variables and incorporate Benders' feasibility cuts to  $\mathbf{RMP}(\Omega, \mathcal{BD})$ .

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**Algorithm 1:** Solution Algorithm for APMII
 

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1 Let  $\mathcal{BD} \leftarrow \emptyset$ ,  $\Omega \leftarrow \emptyset$ ,  $k \leftarrow 0$ , and  $h \leftarrow 0$ .
2 Denote the tolerance levels of Benders' decomposition and total cost as  $\epsilon^b \geq 0$ 
  and  $\epsilon^c \geq 0$ , respectively. Define the corresponding convergence flags as
  Benders' decomposition convergence ( $BDC$ ), and total cost convergence
  ( $TCC$ ). Let  $\epsilon \leftarrow (1 + \epsilon^b)(1 + \epsilon^c) - 1$ ,  $BDC \leftarrow 0$ , and  $TCC \leftarrow 0$ .
3 while  $TCC = 0$  do
4    $h \leftarrow h + 1$ 
                                     /* Start Benders' for current RMP */
5   while  $BDC = 0$  do
6      $k \leftarrow k + 1$ 
7     Solve RMP( $\Omega, \mathcal{BD}$ ). Denote its optimal solution as  $\{z_k, \nu_k, \eta_k, \varphi_k\}$ 
      and optimal cost as  $\rho_k^*$ .
8     for  $t \in \mathcal{T}$  do
9       Solve the dual of  $\tilde{q}^t(\nu_k)$ :
          
$$\tilde{q}^t(\nu_k) = \max_{\alpha^t, \beta^t} (\alpha^t)^\top (\mathbf{h}^t - \mathbf{B}^t \nu_k) + (\beta^t)^\top \ell^t$$

          s.t.
          
$$(\mathbf{E}^t)^\top \alpha^t + (\mathbf{F}^t)^\top \beta^t \leq \mathbf{v}^t$$

          
$$(\mathbf{G}^t)^\top \alpha^t \leq \mathbf{b}^t$$

          
$$\alpha^t \leq \mathbf{0}, \beta^t \leq \mathbf{0}$$

          Denote optimal solution as  $\{\alpha_k^t, \beta_k^t\}$ .
10    end
11    if  $\sum_{t \in \mathcal{T}} \tilde{q}^t(\nu_k) > (1 + \epsilon^b) \cdot \sum_{t \in \mathcal{T}} \varphi_k^t$  then
12      for  $t \in \mathcal{T}$  do
13        if  $\tilde{q}^t(\nu_k) > \varphi_k^t$  then
14          Generate a Benders' optimality cut
          
$$(\alpha_k^t)^\top (\mathbf{h}_k^t - \mathbf{B}_k^t \nu) + (\beta_k^t)^\top \ell_k^t \leq \varphi_k^t$$
. Add this cut to the list
           $\mathcal{BD}^t$ .
15        end
16      end
17    else
18       $BDC \leftarrow 1$ 
19    end
20  end
                                     /* End Benders' for current RMP */ Execute
       $\text{TCR}(\text{RMP}(\cdot), \nu_k, \rho_k^*, (\tilde{q}^t(\nu_k))_{\forall t}, \epsilon^c, \Omega, h)$  /* Run the TCR procedure */
21 end
22  $z^* \leftarrow z_k$  and  $\nu^* \leftarrow \nu_k$ .
Output: Maintenance solution  $\{z^*, \nu^*\}$ .

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**Procedure True Cost Recovery(TCR) for APMII**

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**Input:**  $\text{RMP}(\cdot), \boldsymbol{\nu}_k, \rho_k^*, (\tilde{q}^t(\boldsymbol{\nu}_k))_{\forall t}, \epsilon^c, \Omega, h$

- 1  $\delta^h \leftarrow 0.$
- 2 **for**  $t \in \mathcal{T}$  **do**
- 3     Find the generator statuses at  $t$  corresponding to the solution  $\boldsymbol{\nu}_k$ , namely  $\hat{\mathbf{m}}^t.$
- 4     **if**  $\hat{\mathbf{m}}^t$  is not contained in the list  $\Omega^t$  **then**
- 5          $\Omega^t \leftarrow \Omega^t \cup \{\hat{\mathbf{m}}^t\}.$  Solve  $q^t(\boldsymbol{\nu}_k)$  in model (27).
- 6          $e_h^t \leftarrow q^t(\boldsymbol{\nu}_k) - \tilde{q}^t(\boldsymbol{\nu}_k),$  and  $\delta^h \leftarrow \delta^h + e_h^t.$
- 7         Add variable  $\eta_h^t,$  cut  $(\mathbf{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \geq u_h^t$  and objective cost  $\eta_h^t e_h^t$  to  $\text{RMP}(\cdot).$
- 8     **end**
- 9 **end**
- 10 **if**  $\rho_k^* + \delta^h \leq \rho_k^*(1 + \epsilon^c)$  **then**     /\* If current RMP cost is sufficiently close to its corresponding true cost \*/
- 11      $TCC \leftarrow 1, BDC \leftarrow 1$
- 12 **else**
- 13      $TCC \leftarrow 0, BDC \leftarrow 0$
- 14 **end**

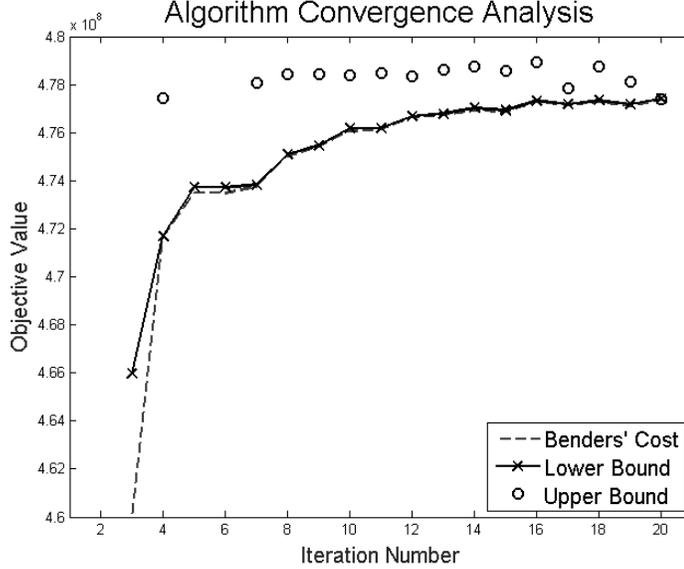
**Output:**  $\text{RMP}(\cdot), \delta^h, \Omega, BDC, TCC$

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## 4.6 Experiments

In this section we present the experimental results for APMII. We first provide a convergence analysis for the solution of APMII using the algorithm introduced in Section V. We then briefly introduce the experimental procedure, and use this procedure to conduct two comparative studies on APMII. The first study considers the basic case, where we assume that handling a failed generator takes the same amount of time as conducting preventive maintenance. The second study considers a more realistic case where the failure interruption takes twice as long as a planned maintenance interruption. We will illustrate the effectiveness of our approach in each study.

In all of our analyses, we use the IEEE 118-bus system. The system has 54 generators, 118 buses, and 186 transmission lines. We obtain the age of generators at the start of the experiments by running the generators for a warming period. We set the maintenance decisions weekly, and operational decisions hourly. Planning horizon for each problem is set at 110 weeks. We set the preventive maintenance



**Figure 9:** Convergence Analysis for APMII.

cost  $c_p = 200,000$  and the failure cost  $c_f = 800,000$ . In all our experiments, we use Gurobi 5.6.0 [51].

To highlight the performance of the algorithm, we first provide a convergence analysis using one of the instances of APMII used in our case studies. Direct solution of this problem with the state-of-the-art MIP solver such as Gurobi proves to be problematic with the solver quickly running into memory problems on our computer with 8GB RAM. However, we can solve this problem to 0.3% optimality gap using the proposed algorithm in 20 Benders' iterations ( $k = 20$ ) and 15 cost recovery iterations ( $h = 15$ ). The total running time is 121 minutes.

For any iteration  $k$ , the dashed line in Figure 9 indicates the solution of  $\mathbf{RMP}(\Omega, \mathcal{BD})$ , namely  $\rho_k^*$  in Algorithm 1, line 7. The solid line denote the corresponding dual solution of the current RMP problem, namely  $\mathbf{c}^\top \mathbf{z}_k + \mathbf{e}^\top \boldsymbol{\eta}_k + \sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}_k)$ . This provides a valid lower bound as indicated in the proof of Theorem 1. For the iterations where the Benders' convergence is attained, we also calculate the upper bound ( $\rho_k^* + \delta^h$  in TCR Procedure, line 10). We note when  $\epsilon^b = 0$ , the lower bound is monotonically increasing, while the corresponding feasible solution ( $\rho_k^* + \delta^h$ ) does not exhibit

a monotone behavior.

In our experimental studies, we use a degradation analysis procedure similar to Chapter III. More specifically, we take the vibrational data from rolling element bearings as representative of generator degradation. To model the degradation of the bearings, we use the exponential degradation function with Brownian error. We refer the reader to [106] for details on the estimation of the prior estimates, and the real-time Bayesian updates of the degradation parameters.

In order to test the effectiveness of APMII, we design an experimental framework consisting of two main modules: i) optimization module, and ii) execution module. In the optimization module, given dynamic maintenance costs and remaining maintenance downtimes for each generator, we solve APMII. Then in the execution module, we fix the maintenance schedule during the freeze time, and execute the chain of events during a freeze period. Experimental implementation for the APMII is similar to that of APMI, except that in the implementation for APMII, for every time period, we determine which generators are available (not failed, or undergoing maintenance), and solve a unit commitment problem with the available generators. This allows us to calculate the resulting operational costs for each week. We let the freeze period  $\tau_R = 8$  weeks, and solve the maintenance problems in a rolling horizon fashion to cover a period of 48 weeks.

In order to ensure a fair comparison, we repeat this implementation spanning 48 weeks, using generator with different ages. We take the average of these experiments to obtain any of the metrics we present.

#### **4.6.1 Comparative Study on APMII**

In this section, we consider the fleet maintenance scheduling of conventional generators. For these generators, the effect of any outage on the operational costs is significant. In this comparative study, we set the generator maintenance downtime

ratio to be 1 : 1, meaning that conducting preventive maintenance takes the same amount of time as handling a failure. We use the algorithm presented in section IV to solve APMII with optimality gap of 1%.

We show the superiority of the maintenance scheduling of APMII by comparing it with the periodic, RBM and APMI models. The periodic model has the same modifications imposed on APMII. As a result, the periodic model conducts maintenance at a maintenance epoch  $i$  between the 66<sup>th</sup> and 69<sup>th</sup> weeks with the objective of minimizing the total operational cost. Therefore, the periodic model for this study is a cost minimization problem. For the RBM case, we use the exact optimization model of APMII, however the cost function for this scenario is derived using a Weibull distribution. We first derive a Weibull estimate using the failure times from the rotating machinery application  $F_W(t)$ , and then condition this distribution on the time of survival to estimate the remaining life distribution and the associated maintenance costs. We also evaluate the performance of the APMI model in this study. To find the resulting schedule, we first implement the APMI model during the freeze period. Based on the fixed schedule, we find the resulting operational (UC) costs. We continue this process in a rolling horizon fashion.

Table 5 presents the reliability and cost metrics for the four policies considered in the comparative study. We first compare APMII with the periodic model and RBM. We note that RBM remains a conservative policy in comparison to the periodic model, since it schedules more preventive maintenances (24.9 v.s. 23.9), incurs less number of unexpected failures (12.3 v.s. 13.7) and sacrifices more lifetime (1019.6 v.s. 943.6 weeks).

APMII, on the other hand, benefits from the additional sensor data to learn more about the ongoing degradation in the generators. Consequently, APMII decreases the number of unexpected failures by 86.9% compared to the periodic model, and by 85.4% compared to the RBM model. Considering the total useful life unused among

the 54 generators, we see that APMII provides significant improvements, i.e., unused life of APMII is only 31.9% of the periodic model, and 29.5% of RBM, respectively.

We note that periodic and RBM policies have comparable maintenance costs, with periodic policy incurring an additional maintenance cost of \$0.9M on average. APMII incurs a smaller maintenance cost that constitutes 42.31% of the periodic, and 44.94% of the RBM policy. The periodic maintenance policy results in significantly higher operational costs. This is because the periodic policy enforces a maintenance window that limits the flexibility of the maintenance policy to adapt to the demand profile.

The operational costs of the RBM and the APMII shows an interesting pattern. This pattern reflects the trade-off between the minimization of the operations cost, and the maintenance cost. RBM and APMII uses the same problem structure, however, since the remaining life estimates of RBM is not as accurate as APMII, the dynamic cost function of RBM is more flat. This, in turn, allows more flexibility for RBM to further minimize the operational cost, at the expense of increased risks of unexpected failures. We note that the flat dynamic maintenance cost function of RBM generates a slightly lower operational cost, but increases the maintenance cost so much that the total cost of RBM exceeds that of APMII.

Another interesting pattern can be recognized between APMI and APMII. We note that APMI minimizes the maintenance cost without considering the impact on operational costs. APMII, on the other hand, optimizes the maintenance schedule to minimize the total cost, thus deviating from the optimal maintenance cost (provided by APMI), to ensure more gains from the operational cost. This makes APMI marginally more reliable, yet significantly more expensive than APMII.

In terms of the total cost, we see that the RBM policy performs better than the periodic policy. This is due to the considerable difference in the operational costs of the periodic policy and the other policies. But APMII achieves the smallest total cost among four policies with savings of \$12.6M, \$7.9M, and \$1.2M compared to the

periodic, RBM, and APMI, respectively.

**Table 3:** Benchmark for APMII - Maintenance Downtime Ratio 1:1

	Periodic	RBM	APMI	APMII
# Preventive	23.9	24.9	26.6	26.1
# Failures	13.7	12.3	1.5	1.8
# Total Outages	37.6	37.2	28.1	27.9
Unused Life (wks)	943.6	1019.6	309.5	300.7
Maintenance Cost	\$15.74 M	\$ 14.82 M	\$6.52 M	\$6.66 M
Operations Cost	\$ 188.19 M	\$ 184.35 M	\$185.98 M	\$184.62 M
Total Cost	\$ 203.92 M	\$ 199.17 M	\$192,50 M	\$191.28 M

**Table 4:** Benchmark for APMII - Maintenance Downtime Ratio 1:2

	Periodic	RBM	APMI	APMII
# Preventive	24.0	25.3	26.6	25.7
# Failures	13.7	12.2	1.5	1.9
# Total Outages	37.7	37.5	28.1	27.6
Unused Life (wks)	950.1	1012.9	309.4	295.6
Maintenance Cost	\$15.76 M	\$14.82 M	\$6.52 M	\$6.66 M
Operations Cost	\$191.24 M	\$186.54 M	\$ 186.09 M	\$185.08 M
Total Cost	\$207.00 M	\$201.36 M	\$ 192.61 M	\$191,74 M

#### 4.6.2 Comparative Study on APMII with Realistic Failure Recovery Times

In the previous section, we assumed that conducting a preventive maintenance takes the same amount of time as handling an unexpected failure. In reality, when a generator fails, maintenance practitioners need significantly more time to put the generator back online, since: i) an unexpected failure can cause other subcomponents to fail as well, increasing the scope of inspection and maintenance, ii) full inventory of the needed maintenance equipment and crew would not be ready to start the maintenance immediately. To model this realistic scenario, we set the failure recovery time twice as long as a preventive maintenance duration, thus using maintenance

downtime ratio 1 : 2.

Table 4 presents the reliability and cost metrics in this scenario. Reliability results are comparable to the previous section. We note that since APMI and APMII incurs less number of failures, they are effected only minimally by the set of changes introduced in this section. However, we observe a significant effect of these changes on the operational costs of the periodic and RBM policies. Introducing realistic failure recovery increases the operational cost of APMII by \$0.46M, while the periodic and RBM policies incur an additional operational cost of \$3.1M and \$2.2M, compared to the corresponding numbers in Table 5, respectively. This effect is due to the significant number of failures experienced by the periodic and RBM policies.

Table 4 also shows that APMII provides significant savings on operational cost and total cost. In particular, the operational cost of APMII is \$6.2M, \$1.5M, and \$1.0M lower than the that of the periodic, RBM, and APMI policies, respectively. Correspondingly, the total cost of APMII is \$15.3M, \$9.6M, and \$0.9M lower.

### 4.6.3 Discussion on the Results

The results show that the proposed framework has significant advantages in terms of maintenance and operational costs and system reliability over the traditional approaches. More specifically, comparing to the best performance of the periodic and RBM policies, Table 4 shows the following advantages of our approach:

- *APMI/II significantly reduce the number of unexpected failures:* In all our experiments, we observe that our models provide significant improvements in terms of the unexpected failures. Comparing to the best among the periodic and RBM policies, APMI and APMII only have 12.3% and 15.6% of the unexpected failures, respectively.
- *APMI/II extend the equipment lifetime:* Using the additional sensor observations allow our policy to utilize more of the generator lifetime. This is because

our approach can reason through predictive analytics when a maintenance might not be necessary. We observe that the unused lifetimes of APMI and APMII are 32.6% and 31.1% of the best among the periodic and RBM policies, respectively.

- *APMI/II require less outages:* Compared to the benchmarks, our approach always required less interruptions to the generator’s dispatch schedule, i.e. the total outages of APMI and APMII are 74.9% and 73.6% of the best among the periodic and RBM policies, respectively.
- *APMI/II significantly reduce the maintenance costs:* Our approach incurs less than 44.9% of the maintenance costs associated with the periodic and RBM policies.
- *APMII significantly reduce the total cost:* In terms of the total cost, APMII outperforms all three other models, with savings of \$15.3M, \$9.6M, and \$0.9M comparing to the periodic, RBM, and APMI policies.

## 4.7 Conclusion

In this chapter, we presented an extended model on the unified framework that links low-level performance and condition monitoring data with high-level operational and maintenance decisions for generators. The operational decisions identify the optimal commitment and dispatch profiles that satisfy the demand and network feasibility requirements. Maintenance decisions focus on arriving at an optimal fleet-level sensor-driven schedule that accounts for optimal asset-specific schedules driven by the condition monitoring data. We provided an effective solution algorithm to solve large instances of APMII, and show the effectiveness of our approach. To conduct the computational studies, we implement an experimental framework that integrates the dynamic information obtained from sensor measurements and predictive analytics with the proposed maintenance scheduling module. Extensive computational

experiments are conducted on this platform. In particular, real-world degradation data collected from sensor measurements of rotating bearings are used in the experiments. The experiments compare the proposed sensor-driven condition-based generation maintenance approach (APMII) with the traditional periodic and reliability-based approaches, and the APMI model introduced in Chapter 3.

In what follows, we will present an integrated adaptive maintenance policy specifically for wind farms.

## CHAPTER V

# SENSOR-DRIVEN CONDITION-BASED OPPORTUNISTIC WIND FARM MAINTENANCE AND OPERATIONS

### *5.1 Introduction*

Increasing societal concerns on sustainability of power systems has resulted in economic incentives for promoting investment in both on-shore and off-shore wind assets. Evidently, global wind investments have been growing steadily in recent years. Maintenance operations, which constitute approximately 20-25 percent of the total levelized cost per kWh [36], therefore became a sector on its own right. However, this growing sector needs to adapt to the maintenance concerns of wind farms that differ dramatically from conventional power systems. Firstly, wind turbines are much more prone to failure [4], however their relatively simple mechanical construction makes it easier to monitor these failure processes via integrated sensors [52]. Secondly, wind farm operators are more interested in profitability of their wind farms as opposed to the prioritized reliability policies of individual wind turbines. This is in sheer contrast with conventional power systems that impose redundancies to eliminate the risks of any asset failure. Evidently, an integrated maintenance framework that i) effectively harnesses the sensor information to predict the remaining lifetime of the turbines, and ii) considers the interdependencies between the maintenance and operations of all the turbines within a wind farm, can provide significant benefits.

The focus of our work is integrating this sensor information into wind farm maintenance and operations planning problem. Maintenance decisions focus on optimizing repair schedules to ensure maximum profitability and reliability without violating

maintenance constraints. These constraints include i) crew and material constraints that puts a limit on the number of turbines to be maintained simultaneously, as well as ii) weather constraints that halts all maintenance activities during harsh weather conditions. Traditionally, integrated maintenance and operational decisions for wind farms are based on a combination of i) reactive policies (*repair it after it fails*), or ii) fixed time-based periodic schedules [80]. Recently, there has been a growing interest in single-turbine sensor driven maintenance models [18, 19]. However, these models do not necessarily capture the complex interdependencies between the turbines.

In this chapter, we propose a novel wind farm maintenance and operations scheduling methodology consisting of two key components, a predictive degradation model and an optimization model. First, a predictive stochastic model is used to characterize how the sensor signals evolve over time in order to predict remaining life distributions of the wind turbines. A Bayesian framework is used to incorporate real-time signals from each turbine in order to revise its RLD based on the most recent degradation state of that turbine. These dynamically evolving RLDs are transformed into *dynamic cost functions* that balance the expected cost of repair versus the cost of unexpected failure. The cost functions act as a key link between the predictive model and the optimization framework. Next, the dynamic cost functions are incorporated into a mixed integer optimization model and they are used to derive cost-optimal operational and maintenance decisions for each wind turbine. Our goal is to optimize these decisions based on the degradation states and predicted RLDs of all the wind turbines. To do so, we develop a novel integrated maintenance optimization model that provides a maintenance schedule for a fleet of turbines based on their individual degradation states and subject to limited labor resources and weather conditions. We also consider the effects of maintenance on electricity production by coordinating wind turbine maintenance schedules with the turbine dispatch. We evaluate the performance of our approach through an extensive set of experiments on 100 and

200-turbine systems, where we emulate turbine degradation using real-world vibration sensor signals from a rotating machinery application. Extensive studies suggest that our framework significantly lowers the risks of turbine failure, extends equipment lifetime, decreases the cost of maintenance, and increases the profitability of operations. These are the metrics suggested by IEEE taskforce on maintenance [40].

The key to success in wind farm maintenance lies in integration of the complex wind farm dynamics into smart sensor-driven systems. In this research, we propose a unified framework that adapts to the real-time sensor data to optimize operations and maintenance (O&M) efforts for the entire wind farm. At the turbine level, we leverage the real-time degradation data to predict their remaining life distribution. In contrast to the diagnostic systems that estimate the current state of turbine health, our models use online statistical learning to predict the future trajectory of health, thus providing ample response time and visibility for failure related risks. To reach an optimal solution for the entire wind farm, we develop a mixed integer optimization model, which considers i) the sensor-updated failure risks from each turbine, ii) the operational factors such as electricity price and forecasted wind speed, and iii) the significant cost reductions resulting from grouping the turbine maintenances together; a concept called opportunistic maintenance. Last objective refers to the common practice of minimizing the number of maintenance crew visits to wind farms by i) scheduling the maintenance of highly degraded turbines together, and ii) waiting for other preventive maintenances before repairing a failed turbine. This concept is important for wind farms since the maintenance crew visits require significant initial investment [31]. This initial cost is particularly significant for off-shore wind farms. Our operational decisions relate to the turbine dispatch that provides the operational revenue. There is significant coupling between maintenance decisions and dispatch. Firstly, a turbine cannot produce while under maintenance. Secondly, any turbine that fails unexpectedly can stay in a failed state until a corrective maintenance is

scheduled. Thus another tradeoff occurs in terms of when to schedule the corrective maintenance. The optimization model determines whether i) it is more profitable to conduct maintenance right away so that the wind turbine can start generating electricity, or ii) it would make more sense to delay maintenance so the maintenance can be grouped with other turbines as well. Depending on the electricity price, forecasted wind speed, and the sensor-updated failure risks, our model automatically determines how aggressive it should group the maintenances of turbines; thus providing an optimal maintenance policy that can adapt to the operators requirements. We also consider the optimal maintenance policies for cases where single maintenance crew is responsible for multiple wind farm locations. We present extensive computational experiment results to show proposed framework achieve significant improvements in profitability and reliability.

The remainder of the chapter proceeds as follows. Section II provides the integrated maintenance-operations model that considers the interactions between maintenance, and dispatch decisions. Section III presents the experimental framework and experimental results. The conclusions are provided in Section IV.

## ***5.2 Sensor-Driven Adaptive Scheduling of Maintenance and Operations***

In this section, we propose a novel mixed-integer optimization model for the sensor-driven adaptive opportunistic maintenance and operations scheduling (AOMO) of wind farms. A key aspect of our framework is the link between predictive analytics and the wind farm maintenance and operations scheduling. To connect them, a discretized form of the sensor updated dynamic maintenance cost from every wind turbine in field is incorporated into the objective function. In order to ensure the optimal scheduling of maintenance and operations for the entire farm, we consider various constraints and interdependencies, such as i) the limits on the maintenance crew capacity, ii) the operational factors dependent on electricity price and forecasted

wind speed, and iii) the significant cost reductions resulting from grouping the wind turbine maintenances together. This is accomplished by coupling the operations and maintenance in two different scenarios. Firstly, a wind turbine under maintenance does not produce power. Secondly, any wind turbine that fails unexpectedly can stay in a failed state until a corrective maintenance is scheduled. Thus a tradeoff occurs in terms of when to schedule the corrective maintenance. The optimization model determines whether it is more profitable to conduct maintenance right away so that the wind turbine can start generating electricity, or if it would make more sense to delay maintenance so that the maintenance can be grouped with other wind turbines as well. Depending on the electricity price, forecasted wind speed, and the sensor-updated failure risks, our model automatically determines how aggressive it should group the maintenances of wind turbines; thus providing an optimal maintenance policy that can adapt to the operators requirements.

We also extend our model for cases where a single maintenance crew can handle a number of different locations. For these cases, we consider the factors such as travel time, and differing costs of site visits. Difference in the site visit costs are associated with the remoteness of the location and the distance to the shore for on-shore and off-shore farms, respectively.

### 5.2.1 Decision Variables and Associated Costs

We denote the set of maintenance epochs by  $\mathcal{T}$  and the set of wind turbines by  $\mathcal{G}$ . The set  $\mathcal{G}$  can be further partitioned into two subsets of wind turbines at the time of planning  $t_p$ . The first subset, denoted by  $\mathcal{G}_o$ , includes the wind turbines that are either operational or under maintenance at  $t_p$ . The second subset of  $\mathcal{G}$ , denoted as  $\mathcal{G}_f$ , includes those wind turbines that are in failed state at  $t_p$ . An operational turbine can undergo preventive maintenance. For this, we let the binary variable  $\mathbf{z}$  determine the start time of *preventive* maintenance, thus  $z_t^i = 1$  if the maintenance of an operational

turbine  $i$  starts at period  $t$ . There is a dynamic maintenance cost associated with these decisions as discussed in Section III.

A failed turbine can only experience corrective maintenance. We use binary variables  $\mathbf{v}$  to determine the start time of *corrective* maintenance, thus  $v_t^i = 1$  if turbine  $i$  experiences a corrective maintenance at period  $t$ . There is no time-dependent maintenance cost associated with  $\mathbf{v}$ .

Moreover,  $\mathbf{x}$  is a binary decision variable, whereby  $x_t^\ell = 1$  means that the maintenance crew visits wind farm location  $\ell$  at period  $t$ . There is a significant crew deployment cost  $C_t^{v,\ell}$  associated with this variable. Each period  $t$  in  $\mathcal{T}$  is divided to constituent subperiods  $\mathcal{S}$  in order to model wind farm operations in more detail. More specifically,  $y_{s,t}^i \in \mathbb{R}_+^n$  denotes the generation level from wind turbine  $i$  during period  $t \in \mathcal{T}$  and subperiod  $s \in \mathcal{S}$ .

### 5.2.2 Objective function

The objective in the AOMO model is to maximize the net profit of maintaining and operating a wind farm:

$$\begin{aligned} \max_{\mathbf{z}, \mathbf{v}, \mathbf{x}, \mathbf{y}} \quad & \underbrace{\sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} y_{s,t}^i \cdot \pi_{s,t}}_{\text{operational revenue}} - \underbrace{\sum_{\ell \in \mathcal{L}} \sum_{t \in \mathcal{T}} x_t^\ell \cdot C_t^{v,\ell}}_{\text{crew deployment cost}} \\ & - \underbrace{\xi_m \sum_{i \in \mathcal{G}_o} \sum_{t \in \mathcal{T}} z_t^i \cdot C_{t_i^o,t}^i}_{\text{expected turbine maintenance cost}}, \end{aligned} \quad (36)$$

where  $\pi_{s,t}$  is the electricity price at period  $t$ , subperiod  $s$ , and  $\xi_m$  is the maintenance criticality coefficient.

The objective function (36) evaluates the operational revenue as well as two sources of expenditures: crew deployment cost and turbine maintenance cost. Evaluation of the first two terms is easy. The last term, the turbine preventive maintenance cost, corresponds to the dynamic maintenance cost associated with a turbine maintenance. Notice that the dynamic maintenance costs  $C_{t_i^o,t}^i$ 's are computed from the

RLDs of operating wind turbines, which are updated based on sensor observations. In this way, the objective function (36) adapts to these dynamic sensor updates over time.

### 5.2.3 Constraints

#### 5.2.3.1 Wind turbine maintenance coordination

Constraint (37) ensures that a wind turbine's preventive maintenance is scheduled within the time limit  $\zeta_i$ , which is defined as the first time that its sensor-updated reliability falls below a control threshold  $\eta$ . More specifically,  $\zeta_i := \min\{t \in \mathcal{T} : P(R_{t_i}^i > t) < \eta\}$ . Constraint (38) limits the number of corrective maintenances within the planning horizon to at most one per wind turbine.

$$\sum_{t=1}^{\zeta_i} z_t^i = 1, \quad \forall i \in \mathcal{G}_o. \quad (37)$$

$$\sum_{t \in \mathcal{T}} v_t^i \leq 1, \quad \forall i \in \mathcal{G}_f. \quad (38)$$

The following constraints ensure that maintenance crew visits the wind farm  $\ell$  if any of the wind turbines within that wind farm is scheduled for preventive (39) or corrective maintenance (40).

$$z_t^i \leq x_t^\ell, \quad \forall \ell \in \mathcal{L}, i \in \mathcal{G}_o^\ell, t \in \mathcal{T}, \quad (39)$$

$$v_t^i \leq x_t^\ell, \quad \forall \ell \in \mathcal{L}, i \in \mathcal{G}_f^\ell, t \in \mathcal{T}, \quad (40)$$

where  $\mathcal{G}_o^\ell$  and  $\mathcal{G}_f^\ell$  are the sets of operational, and failed wind turbines at location  $\ell$ , respectively.

#### 5.2.3.2 Maintenance crew coordination

Constraint (41) limits the maintenance crew visits to only one of the wind farm locations during a single maintenance epoch. Constraint (42) ensures that if the

weather conditions are harsh at wind farm location  $\ell$ , then the maintenance crew cannot conduct maintenance at that location.

$$\sum_{\ell \in \mathcal{L}} x_t^\ell \leq 1, \quad \forall t \in \mathcal{T}, \quad (41)$$

$$x_t^\ell = 0, \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}_w^\ell, \quad (42)$$

where  $\mathcal{T}_w^\ell$  is the set of times when a crew cannot visit the wind farm  $\ell$  due to extreme weather conditions.

The following constraint considers the distance between wind farm locations  $\ell$  and  $\ell'$ , and ensures that a maintenance cannot be initiated before the required travel time  $\theta_{\ell,\ell'}$  passes. For every pair of locations  $\{\ell, \ell'\}$ , we enforce (43).

$$x_t^{\ell'} + x_\tau^\ell \leq 1, \quad \forall t \in \{\theta_{\ell,\ell'} + 1, \dots, \mathcal{T}\}, \tau \in \{t - \theta_{\ell,\ell'}, \dots, t\}. \quad (43)$$

### 5.2.3.3 Maintenance capacity

The following constraint (44) ensures that the number of ongoing maintenances at time  $t$  does not exceed a limit on maintenance labor capacity per period at location  $\ell$ , namely  $M_t^\ell$ . For onshore and offshore wind farms, this limitation may depend on the labor capacity, or the number of available workboats and helicopters, respectively.

$$\sum_{i \in \mathcal{G}_o^\ell} z_t^i + \sum_{i \in \mathcal{G}_f^\ell} v_t^i \leq M_t^\ell, \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}. \quad (44)$$

### 5.2.3.4 Operational considerations

The maintenance decision variables  $\mathbf{z}, \mathbf{v}$  are coupled with the operational decisions  $\mathbf{y}$ .

Constraint (45) ensures that i) an operational turbine  $i$  produces electricity within its available capacity at epoch  $t$ , namely  $p_{s,i}^t$ , which depends on the forecasted wind

power at period  $t$ , subperiod  $s$ ; and ii) a wind turbine under maintenance can not produce electricity.

$$y_{s,i}^t \leq p_{s,i}^t \cdot (1 - z_i^i), \quad \forall i \in \mathcal{G}_o, t \in \mathcal{T}, s \in \mathcal{S}. \quad (45)$$

Constraint (46) stipulates that a failed wind turbine should be scheduled for corrective maintenance before it can start producing electricity. This constraint, along with (38), allows the model to dynamically determine whether or not to schedule a failed wind turbine for corrective maintenance within the planning horizon. When scheduled, it also determines the time of corrective maintenance. Both of these decisions are driven by the potential loss in production revenue.

$$y_{s,i}^t \leq p_{s,i}^t \cdot \sum_{j=1}^{t-1} v_j^i, \quad \forall i \in \mathcal{G}_f, t \in \mathcal{T}, s \in \mathcal{S}. \quad (46)$$

In summary, the AOMO model is given as

$$\begin{aligned} (AOMO) \quad & \min_{\mathbf{z}, \mathbf{v}, \mathbf{x}, \mathbf{y}} \quad (36) \\ & \text{s.t.} \quad (37) - (46) \\ & \mathbf{z}, \mathbf{v}, \mathbf{x} \text{ binary, } \mathbf{y} \geq \mathbf{0}. \end{aligned}$$

### 5.3 *Experimental Results*

In this section we present three studies to highlight the performance of AOMO. In the first study, we perform a benchmark analysis. We also present the impact of different crew deployment costs on the maintenance schedule. In the second study, we analyze how different electricity prices affect the resulting maintenance schedule of AOMO. In the third study, we consider a scenario with multiple wind farm locations. The first two studies schedule the maintenance of a single wind farm with 100 wind turbines, whereas the last study considers wind farms in three different locations with 100 wind

turbines in the first location, and 50 wind turbines in each of the second and third locations.

To test the performance of a maintenance policy, we designed an experimental framework. Our experimental framework involves two modules: planning module, and an execution module. In the *planning module*, we solve an optimization model to schedule the maintenance and operations of the wind turbines for a 200 day planning horizon, given the dynamic maintenance costs of the operational wind turbines. We use Gurobi 5.6.0 [51]. In the *execution module*, we fix the maintenance schedule for the first 16 days (freeze period). We then model the chain of events during this period. We use the degradation data from a real-world rotating machinery application as representative of the degradation observed in the wind turbines. We ensure that the expected lifetime corresponds to wind turbine statistics provided by [96]. For each day within the freeze period, we determine which wind turbines experience an ongoing maintenance (preventive or corrective maintenance as dictated by the fixed schedule of the optimization model), an unexpected failure or an idle period. For every wind turbine  $i \in \mathcal{G}_o$ , an unexpected failure occurs when the degradation function of the wind turbine reaches failure threshold before the time of its scheduled preventive maintenance. The remaining wind turbines  $i \in \mathcal{G}_f$  stay idle until a reactive maintenance occurs. Once the execution module reaches to the end of the freeze period, we update the dynamic maintenance costs for each operational wind turbine based on the most recent sensor observations (as in Section III.B). We also update the list  $\mathcal{G}_o$  and  $\mathcal{G}_f$ . During this execution module, for each subperiod, we keep track of the following metrics:

- *Revenues*: Based on the availability of each wind turbine, wind profile and electricity price, we calculate the resulting operational revenue.
- *Expenditures*: We obtain the wind turbine maintenance cost by the sum of the number of preventive actions and the unexpected failures multiplied by  $c^p$  and

$c^f$ , respectively. We obtain the crew deployment cost by multiplying the crew visit instances by their associated deployment costs.

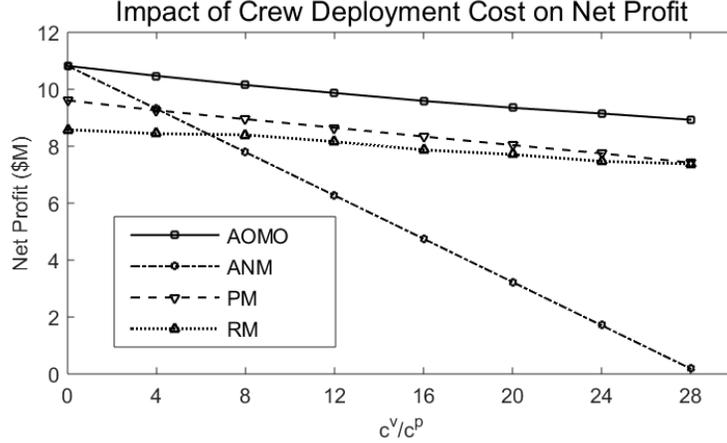
- *Maintenance Metrics:* We record the number of crew visits, unexpected failures, and preventive and reactive maintenances. We also register the total idle time of wind turbines.

We execute the experimental process 20 times in a rolling horizon fashion to cover a period of 320 days. To have a fair comparison, we repeat this experimental procedure 10 times with different initial wind turbine ages, and calculate the metrics by taking the average of the corresponding metrics from these experiments. The age of the wind turbines at the start of experiments is obtained by running them for a warm-up period. We next present the results of our experiments.

### 5.3.1 Comparative Study on AOMO, and the Impact of Crew Deployment Cost

In this study, we first perform a comparative study for AOMO. To do so, we compare the cost and maintenance metrics of AOMO, with three benchmark models:

- *Adaptive Non-opportunistic Model (ANM):* The ANM model is identical to our proposed model AOMO, except that in ANM the crew visits do not have an associated cost, namely  $C_t^{v,\ell} = 0 \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}$  in (36). ANM generalizes single turbine maintenance policies in the literature [18, 19] to cases with multiple wind turbines.
- *Periodic Model (PM):* The PM model differs from AOMO in two aspects: i) it does not benefit from the sensor-driven dynamic maintenance costs, thus we set  $C_{t_i^o,t}^{d,i} = 0 \quad \forall i \in \mathcal{G}_o, t \in \mathcal{T}$ , and ii) it includes a set of constraints to ensure that the wind turbine’s preventive maintenance occurs when the wind turbine’s age is between 130 and 142 days. Depending on the age and type of the wind turbine, periodic maintenance frequencies of wind turbines differ



**Figure 10:** Comparative Study on Net Profit of Maintenance Policies subjected to Different Crew Deployment Costs.

between 3 months to a year [80]. The period presented herein is obtained using the degradation database and the traditional approach presented in [5].

- *Reactive Model (RM)*: The RM model does not schedule any preventive actions, however it is identical to AOMO in terms of how it schedules the corrective maintenances. To do so, we replace constraint (37) with  $z_t^i = 0 \quad \forall t \in \mathcal{T}, \ell \in \mathcal{L}, i \in \mathcal{G}^\ell$ .

Figure 10 provides the net profits of the four policies under different crew deployment cost profiles. Net profit is defined by the difference between the operational revenue and expenditures (crew deployment and turbine maintenance). We let the price of electricity be \$25/MWh, and use the yearly wind data from [81]. We let  $c^f = 4 \times c^p = \$16K$ , and fix the deployment cost to a constant value, i.e.  $c^v = C_t^{v,\ell} \quad \forall \ell \in \mathcal{L}, t \in \mathcal{T}$ . We note that AOMO always provides a better net profit than the benchmark models, since:

- *AOMO adapts to the crew deployment costs*: When the cost  $c^v = 0$ , AOMO becomes identical to ANM. However, as  $c^v$  increases, AOMO significantly outperforms ANM, as cost incentives in AOMO dynamically integrate the benefits of the opportunistic maintenance. In fact, for higher values of  $c^v$ , ANM provides

a worse performance compared to the more basic models like PM and RM. This clearly demonstrates that ad-hoc maintenance policies driven by single wind turbine analysis, even if they use sophisticated sensor-driven predictive models, perform poorly as the crew deployment cost increases. To obtain the full benefit of sensor-driven maintenance, a policy should integrate the dynamics within the maintenance and operations of the wind farm as a whole.

- *AOMO is driven by sensor information:* In contrast to PM and RM, AOMO detects the condition of the wind turbines using sensor observations, and adapts the schedule accordingly. The differences in revenue represents the economic value of this sensor information.

We next analyze the value of sensor information in detail by considering the effect of crew deployment cost on different maintenance policies. Tables 5, 6, and 7 compare the cost and maintenance metrics associated with AOMO, PM and RM, respectively. All the maintenance metrics presented in the tables refer to the entire farm. For instance, “# preventive actions” is a measure of the total number of preventive actions experienced by all the wind turbines in the wind farm. Recall that some wind turbines that experience an unexpected failure would stay in a failed state until their corrective maintenance is scheduled. The total duration of time spent in this failed state is denoted as idle days. We note that, *regardless* of the crew deployment cost  $c^v$ , AOMO provides the following advantages:

- *Improve reliability while decreasing the turbine maintenance cost:* AOMO uses the sensor-driven predictive models to detect when the wind turbine condition becomes critical, and performs maintenance when needed. This significantly decreases the number of failure instances, and provides considerable savings in wind turbine maintenance cost. For instance, when  $c^v = 12c_p$ , AOMO decreases failure instances by 70.6% and 85.2% compared to PM and RM respectively.

Reductions in wind turbine maintenance cost correspond to 44.2% and 57.3% of the costs in PM and RM respectively.

- *Increase availability and operational revenue:* Decreasing the number of failure instances reduces the number of idle days, which ensures that more wind turbines are available at any time, making the most of the available generation capacity. As a result, the operational revenue increases in AOMO (e.g. by 2.8% and 8.3% compared to PM and RM respectively, when  $c^v = 12c_p$  ).
- *Decrease crew visits:* The AOMO schedule experiences fewer number of outages (failures and preventive maintenances). Consequently, it also significantly decreases the need for frequent crew visits (in comparison to PM) for cases when  $c^v > 0$  (e.g. decrease by 18.1% and 7.3% in outages and crew visits, respectively, when  $c^v = 12c_p$ ).

**Table 5:** Benchmark for Adaptive Opportunistic Maintenance & Operation (AOMO)

$c^v / c^p :=$	<b>0</b>	<b>4</b>	<b>8</b>	<b>12</b>	<b>16</b>	<b>20</b>	<b>24</b>	<b>28</b>
<b>Net Profit</b>	\$10.83 M	\$10.41 M	\$10.15 M	\$9.87 M	\$9.62 M	\$9.38 M	\$9.15 M	\$8.94 M
<b>Operational Revenue</b>	\$11.82 M	\$11.81 M	\$11.80 M	\$11.79 M	\$11.78 M	\$11.78 M	\$11.77 M	\$11.75 M
<b>Expenditures</b>	\$0.99 M	\$1.40 M	\$1.65 M	\$1.92 M	\$2.15 M	\$2.40 M	\$2.62 M	\$2.81 M
· <b>Turbine Maintenance</b>	\$0.99 M	\$1.03 M	\$1.04 M	\$1.06 M	\$1.08 M	\$1.09 M	\$1.10 M	\$1.14 M
· <b>Crew Deployment</b>	\$0 M	\$0.37 M	\$0.60 M	\$0.85 M	\$1.08 M	\$1.30 M	\$1.52 M	\$1.67 M
<b># Preventive Actions</b>	185.9	177.2	173.1	173.8	179.5	178.8	178.4	172.6
<b># Turbine Failures</b>	15.2	19.8	21.8	23.0	22.5	23.5	24.3	27.9
<b># Crew Visits</b>	95	23.1	18.9	17.8	16.8	16.3	15.8	14.9
<b># Idle Days</b>	104.4	134.4	158.6	180.2	208.2	212.2	224.8	272.6

**Table 6:** Benchmark for Opportunistic Periodic Maintenance (PM)

$c^v / c^p :=$	<b>0</b>	<b>4</b>	<b>8</b>	<b>12</b>	<b>16</b>	<b>20</b>	<b>24</b>	<b>28</b>
<b>Net Profit</b>	\$9.61 M	\$9.26 M	\$8.95 M	\$8.65 M	\$8.34 M	\$8.04 M	\$7.75 M	\$7.42 M
<b>Operational Revenue</b>	\$11.51 M	\$11.49 M	\$11.46 M	\$11.47 M				
<b>Expenditures</b>	\$1.89 M	\$2.23 M	\$2.51 M	\$2.82 M	\$3.13 M	\$3.43 M	\$3.72 M	\$4.05 M
· <b>Turbine Maintenance</b>	\$1.89 M	\$1.89 M	\$1.89 M	\$1.90 M	\$1.88 M	\$1.89 M	\$1.89 M	\$1.89 M
· <b>Crew Deployment</b>	\$0 M	\$0.34 M	\$0.62 M	\$0.92 M	\$1.25 M	\$1.54 M	\$1.83 M	\$2.16 M
<b># Preventive Actions</b>	160.4	162.9	162.1	162.1	161.0	162.2	160.7	161.1
<b># Turbine Failures</b>	78.3	77.6	77.6	78.2	77.5	77.8	77.8	77.8
<b># Crew Visits</b>	47.1	21.3	19.5	19.2	19.5	19.2	19.1	19.3
<b># Idle Days</b>	823.4	1122.8	1238.8	1232.0	1236.2	1246.2	1243.6	1252.0

**Table 7:** Benchmark for Opportunistic Reactive Maintenance (RM)

$c^v / c^p :=$	<b>0</b>	<b>4</b>	<b>8</b>	<b>12</b>	<b>16</b>	<b>20</b>	<b>24</b>	<b>28</b>
<b>Net Profit</b>	\$8.58 M	\$8.44 M	\$8.40 M	\$8.16 M	\$7.87 M	\$7.71 M	\$7.47 M	\$7.38 M
<b>Operational Revenue</b>	\$11.11 M	\$11.13 M	\$11.07 M	\$10.89 M	\$10.66 M	\$10.54 M	\$10.22 M	\$10.12 M
<b>Expenditures</b>	\$2.53 M	\$2.62 M	\$2.68 M	\$2.74 M	\$2.79 M	\$2.84 M	\$2.75 M	\$2.73 M
· <b>Turbine Maintenance</b>	\$2.53 M	\$2.52 M	\$2.51 M	\$2.48 M	\$2.44 M	\$2.44 M	\$2.39 M	\$2.38 M
· <b>Crew Deployment</b>	\$0 M	\$0.10 M	\$0.17 M	\$0.25 M	\$0.35 M	\$0.40 M	\$0.36 M	\$0.36 M
<b># Preventive Actions</b>	-	-	-	-	-	-	-	-
<b># Turbine Failures</b>	158.1	157.8	157.0	155.2	152.5	152.2	149.2	148.5
<b># Crew Visits</b>	38.0	6.2	5.2	5.3	5.4	5.0	3.8	3.2
<b># Idle Days</b>	1754.0	1806.2	2102.4	2621.2	3277.6	3616.8	4499.4	4776.4

**Table 8:** Impact of Electricity Price on Maintenance Schedule (AOMO)

Electricity Price (\$/MWh)	<b>12.5</b>	<b>25</b>	<b>37.5</b>	<b>50</b>	<b>62.5</b>	<b>75</b>	<b>87.5</b>	<b>100</b>
<b>Net Profit</b>	\$4.01 M	\$9.88 M	\$15.77 M	\$21.68 M	\$27.55 M	\$33.44 M	\$39.35 M	\$45.24 M
<b>Expenditures</b>	\$1.89 M	\$1.91 M	\$1.91 M	\$1.92 M	\$1.94 M	\$1.95 M	\$1.95 M	\$1.96 M
· <b>Turbine Maintenance</b>	\$1.06 M	\$1.06 M	\$1.05 M	\$1.05 M	\$1.05 M	\$1.05 M	\$1.04 M	\$1.04 M
· <b>Crew Deployment</b>	\$0.83 M	\$0.85 M	\$0.87 M	\$0.87 M	\$0.88 M	\$0.90 M	\$0.90 M	\$0.92 M
<b># Preventive Actions</b>	174.4	175.1	178.8	178.8	178.9	180.7	179.6	180.3
<b># Turbine Failures</b>	22.5	22.5	20.7	20.7	21.1	20.4	20.1	20.0
<b># Crew Visits</b>	17.3	17.8	18.1	18.1	18.4	18.8	18.8	19.0
<b># Idle Days</b>	184.7	181.3	168.0	166.0	166.6	156.3	154.2	152.8

We next analyze the impact of the crew visit cost on AOM. Table 5 shows that as the crew deployment cost increases, the cost factors also increase, causing a rise in crew deployment cost and a decrease in net profit as a clear consequence. However there are a number of other changes that are not as obvious. With increasing  $c^v$ , AOMO groups the maintenance of wind turbines more aggressively, thus decreasing the crew visits, and the associated crew deployment cost. This inevitably deviates the maintenance policy from the optimal maintenance suggested by the sensor-driven approach, leading to a slight increase in the number of failures. This also corresponds to an increase in the turbine maintenance cost. Increasing  $c^v$  also leads to more idle days. As it becomes progressively more expensive to schedule a visit, AOMO waits for more wind turbines to degrade before fixing a failed wind turbine. We note however, that AOMO dynamically determines how to alter its schedule to find the optimal policy under different  $c^v$  scenarios. By doing so, AOMO accurately considers the tradeoff between the optimal wind turbine maintenance policy, and the significant cost reductions attained by limiting the number of crew visits; thus AOMO result in a significantly better net profit value.

### 5.3.2 Impact of Electricity Price on AOMO

We next analyze the impact of electricity price on the schedule of AOMO (Table 8). To do so, we consider a farm with 100 wind turbines, and fix the costs  $c^v = 3 \times c^f = 12 \times c^p = \$48K$ . We change the electricity price from \$12.5/MWh to \$100/MWh to study the impact of electricity price on the maintenance and operational metrics. We can clearly detect that increasing the electricity price increases the operational revenue, and therefore the net profit. In addition, we note that there is a significant dependency between the length of the idle time, and the price of electricity. If a failed wind turbine is maintained early on, the revenue from their production would not be lost. However, if the reactive maintenance can be postponed, then the number of

crew visits can be decreased. As the electricity prices rise, the opportunity cost of lost revenue also increases, allowing the maintenance policy to schedule more crew visits to minimize the loss of production. As crew visits increase, the need to postpone the preventive maintenances decreases, leading to less number of failure instances. This leads to a slight increase in expenditure (increase in crew deployment cost and decrease in wind turbine maintenance cost). However, the increase in expenditure is outweighed by the production revenues.

### 5.3.3 Multiple Location Performance of AOMO

In the last study, we analyze a scenario where a single maintenance crew is responsible for 3 wind farm locations. The first location has 100 wind turbines, while the second and third locations have 50 wind turbines each. As in the first experimental study, the price of electricity is \$25/MWh, and the wind turbine maintenance costs are  $c^f = 4 \times c^p = \$16K$ . For the first and the second locations, we fix the crew deployment costs as certain multiples of the preventive maintenance cost. However, we make the crew deployment cost of the third location significantly more expensive,  $c^{v,3} = 10 \times c^{v,1}$ . We also enforce that it takes one maintenance period to go to location 3 from location 1 or 2, and vice versa. The results are presented in Table 9.

We first analyze some of the interesting dynamics between the second and the third locations. We note that since the number of wind turbines are the same, for the case where  $c^{v,3} = c^{v,2} = 0$ , the maintenance metrics are similar. However as the crew deployment cost increases, location 3 crew deployment cost becomes significantly larger than that of location 2. AOMO optimizes the maintenance over all the locations, thus provides a schedule that is much more proactive in location 2. Evidently, location 3 experiences more failures, and idle days, and significantly less crew visits and preventive maintenances, in comparison to location 2.

Lastly, we compare locations 1 and 2. We note that the crew deployment cost for

**Table 9:** Multiple Locations Performance of AOMO

$c^{v,1} / c^p :=$	0	4	8	12	16
<b><i>Location 1: 100 Wind Turbines, Nominal Crew Deployment Cost</i></b>					
# Preventive Actions	178.3	173.6	172.6	171.9	170.3
# Turbine Failures	19.8	22.9	22.9	25.2	28.2
# Crew Visits	47.1	23.1	18.5	16.5	14.7
# Idle Days	160.8	215.4	217.2	266.6	319.0
<b><i>Location 2: 50 Wind Turbines, Nominal Crew Deployment Cost</i></b>					
# Preventive Actions	87.0	84.3	84.0	82.0	77.6
# Turbine Failures	10.0	12.5	15.7	16.4	19.2
# Crew Visits	32.1	16.9	13.9	12.0	10.5
# Idle Days	111.2	155.0	192.4	210.8	276.2
<b><i>Location 3: 50 Wind Turbines, Expensive (10×) Crew Deployment Cost</i></b>					
# Preventive Actions	88.4	43.8	39.0	38.2	36.8
# Turbine Failures	12.2	41.1	45.1	45.8	47.0
# Crew Visits	27.7	5.4	5.0	4.7	4.4
# Idle Days	149.4	1237.8	1403.8	1588.6	1733.8

both locations remain the same. However, as the crew deployment cost increases, location 1 becomes more efficient than location 2. When  $c^v = 12c^p$ , 100 wind turbines in location 1 stay idle for a total of 266.6 days, whereas 50 wind turbines in location 2 stay idle for 210.8 days. This means that a wind turbine in location 2 is expected to stay idle significantly longer than a wind turbine in location 1. This happens because one would have to wait longer to group multiple wind turbine maintenances together in a location with a smaller number of wind turbines. Thus the schedule in location 2 deviates more from the optimal CBM policy to get the same benefits of the opportunistic maintenance. This inevitably leads to more failures. When failures occur, wind turbines wait longer for their corrective maintenance to be grouped with other wind turbines.

## 5.4 Conclusions

In this chapter, we propose an integrated framework that utilizes the critical information provided by sensor-driven analytics in order to enhance wind farm maintenance

and operational decisions. Unlike the traditional methods, the proposed framework effectively uses the sensor information coming from wind turbines to learn their unique degradation patterns and to dynamically estimate the remaining life distribution; this information is then incorporated into an optimal predictive maintenance and operations model. In contrast to many existing sensor driven wind turbine maintenance policies, the proposed method considers the complex interdependencies between wind turbines within a wind farm, and captures specific maintenance requirements. We conduct extensive experiments using real rotating machinery vibration signals. The results demonstrate significant improvements in terms of both reliability and profitability for large-scale wind farm maintenance.

## CHAPTER VI

# SENSOR-DRIVEN CONDITION-BASED GENERATION MAINTENANCE SCHEDULING: MODELING LOAD DEPENDENT DEGRADATION

### *6.1 Introduction*

Increasing congestion and volatility in power systems, aging generator fleet, and the minimal investments on the power infrastructure have been pushing the generators to operate closer to their operational limits. The severity of operational load on generators, in turn, impact how fast they degrade. For instance, factors such as the level of production, the frequency of start-up and shut-down cycles, and the climatic conditions, can potentially shorten the lifetime of the generators by an order of magnitude. Thus, operators often observe that a generator that typically survives a year under regular operating environments, require maintenance much more frequently while operating at harsher environments. This phenomena of load dependent degradation is well studied in degradation modeling literature [10, 11, 32]. However, while the loading conditions significantly impact the degradation of the generators, there has been no comprehensive fleet maintenance and operations scheduling policy that adapts to the changes in the loading, and the degradation in the generators. In this chapter, we expand upon the predictive analytics in Chapter 2, and the adaptive predictive scheduling model in Chapters 3-4 to provide i) an accurate load dependent generator degradation model, and ii) a flexible framework whereby the scheduler gains some control on how fast the generators are degrading.

The load dependent framework is composed of two stages: i) predictive analytics, and ii) adaptive optimization model. In the predictive analytics stage, we provide

an accurate prediction of the remaining life distribution of the engineering systems under varying environmental conditions. To accomplish this characterization, we use the degradation sensors installed on the generators, and implement sensor-driven Bayesian learning to update our predictions on the generators' remaining life distribution. These sensor observations also help us characterize the rate of degradation present in the generators under different loading environments. The load dependency is a significant factor for determining generator fleet maintenance and operations due to the following two main reasons:

- *Load significantly affects the remaining life and optimal maintenance decisions:*  
For complex systems operating under time-varying load environments, the environmental profile significantly affects how long their constituent components survive. Ignoring this direct interaction can cause significant errors in life prediction, and can introduce an increased number of early maintenances and failures.
- *Deciding on the operational decisions of the generators, allows a maintenance planner to significantly alter the optimal maintenance time of the generators:*  
In most maintenance operations, planners are obliged to abide by a number of strict requirements in both maintenance and operations. In the maintenance side, such requirements include the labor capacity, maintenance dependencies between generators such as inclusion and exclusion, and separations between consecutive maintenances. In the operational side, it is often not economical or feasible to maintain many generators at their optimal maintenance time, since their generation capacity might be required to satisfy the peak demand in the system. The load dependent framework presented herein, provides the maintenance planners with additional flexibility, whereby they have the option of i) lowering the load on the cheap generators and postpone the preventive maintenance to after the demand peak, or else ii) increasing the load on the cheap generators and use more juice from the available generators before their

early maintenance. Alternatively, maintenance planner can also apply different dispatch policies on different generators to spread out the maintenance schedule.

In the remainder of this chapter, we introduce our framework that provides a direct link between the operational requirements of the grid, and the degradation rates in the generators. Section II and III provide the predictive analytics framework and the dynamic maintenance cost under load dependent regime. Section IV presents the load dependent adaptive predictive maintenance (LDAPM) model that consider the negative impacts of increasing load in terms of i) escalating the operational burden on the grid, and ii) accelerating the rate of degradation in the power generators. Therefore, we ensure that the LDAPM decreases the real time operational cost, as well as the maintenance and reliability burden in the power grids. Section V introduces the algorithm that we use to solve LDAPM. In Section VI, we present the experimental framework and the results of numerical studies. Lastly, we provide the conclusions in Section VII.

## ***6.2 Load Dependent Degradation Modeling and Prognosis***

In this section, we develop a parametric model to characterize the degradation experienced by a generator operating in a time-varying load environment. To do so, we expand upon the degradation model introduced in Section 2.1. The model consists of deterministic and stochastic parameters  $\kappa$  and  $\theta_i$  that characterize the population characteristics of the generators, and the stochastic degradation parameter characterizing the unit-to-unit variance across the generator populations, respectively. In addition, we assume that the rate of the degradation is affected by the operational conditions under which the generator is operating. In other words, the rate at which the observed degradation signal,  $D_i(t)$ , grows over time depends on the loading conditions. These loading conditions are dependent on the maintenance and operational decisions, here collectively denoted by  $\boldsymbol{\chi}_i$ . We let  $\Psi_i(t, \boldsymbol{\chi}_i) \in \{0, 1 \dots L\}$  define the

load profile applied to the unit, where  $L$  is the most severe loading level of the generator. Using this characterization, we define the degradation signal as a continuous time continuous state stochastic function  $D_i(t)$ , subject to loading conditions  $\boldsymbol{\chi}_i$ , whereby:

$$D_i(t, \boldsymbol{\chi}_i) = D_i(t_o) + \int_0^t R_i(\kappa, \theta_i, \boldsymbol{\chi}_i) ds + \int_0^t V_i(\kappa, \theta_i, \boldsymbol{\chi}_i) dW(s) \quad (47)$$

where  $D_i(t_o)$ ,  $R_i(\kappa, \theta_i, \boldsymbol{\chi}_i)$ , and  $V_i(\kappa, \theta_i, \boldsymbol{\chi}_i)$  denote the initial degradation level, degradation drift function, and the error function, respectively. Since the maintenance decisions are discrete (typically in weeks), we let  $\Psi_i(t, \boldsymbol{\chi}_i)$  be a piecewise constant function of time  $t$ . We let  $1, 2, \dots, H$  represent the maintenance periods, such that  $[t_{j-1}, t_j)$  is the  $j^{\text{th}}$  maintenance period. For any  $t \in [j-1, j)$ ,  $\Psi_i(t, \boldsymbol{\chi}_i) = \Psi_j$ , where  $\Psi_j$  is a constant driven by the loading conditions  $\boldsymbol{\chi}_i$  at time  $t$ . Then, the discretized form of the degradation function for every maintenance epoch  $t$  is given as:

$$D_i(t, \boldsymbol{\chi}_i) = D_i(t_o) + \sum_{s=1}^t (\mu_0 \cdot \Psi_i(s, \boldsymbol{\chi}_i)) + \sum_{s=1}^t \sigma_0 \cdot \Psi_i(s, \boldsymbol{\chi}_i) \cdot (W(s) - W(s-1)) \quad (48)$$

We define the time of failure  $\tau_i = \min(t > 0 : D_i(t, \boldsymbol{\chi}_i) \geq \Lambda_i)$ , where  $\Lambda_i$  is a failure threshold. We note that for the case of constant stress, namely  $\Psi_i(t, \boldsymbol{\chi}_i) = 1, \forall t \in \mathcal{T}$ , the degradation model becomes identical to the exponential base case introduced in Section 2.3. For this degradation function, we showed that the failure time follows an Inverse Gaussian distribution with mean parameter  $\xi_i = \frac{\Lambda - D_i(t_o)}{\mu_{\beta'}^i}$  and shape parameter  $\alpha_i = \frac{(\Lambda - D_i(t_o))^2}{\sigma^2}$ , where  $\mu_{\beta'}^i$  is the sensor-updated drift of the degradation in generator  $i$  [38].

Our next objective is to find the best approximation to this stopping time when the environmental condition  $\Psi_i(t, \boldsymbol{\chi}_i)$  is time-varying. To do so, we adopt a proposition from [32] as follows:

**Proposition 2.** *Given that the degradation function is defined as (48), the distribution of the failure time  $\tau_i$  is  $F_i(t|\xi_i, \alpha_i, \chi_i) = IG(\tau_i(t)|\xi_i, \alpha_i)$ ,  $t > 0$ , where*

$$\tau(t) = \sum_{s=1}^t \Psi_i(s, \boldsymbol{\chi}_i) \quad (49)$$

We illustrate the main idea of the proposition as follows. Assume that the same generator experiences the following two operational conditions:

1. In case 1, the generator operates for a period of one week under harsh loading conditions where  $\Psi_i(1) = 2$ ,
2. In case 2, the generator operates for a period of two weeks under nominal loading conditions where  $\Psi_i(1) = \Psi_i(2) = 1$ ,

Then the probability of failure at the end of these cases (after a week in case 1, and after 2 weeks in case 2) would be identical. Evidently, more important than the duration of the period  $t$ , is the transformed time  $\tau(t)$  under which the generator operates. We note that  $\tau(t)$  in both cases is 2, thus their failure in the transformed time scale follows the same distribution. This is why, we denote the transformed time  $\tau(t)$  as the degradation equivalent time.

Capitalizing on this property, we can then characterize the remaining life distribution as an inverse Gaussian function in the time transformed scale. More specifically, we can find the probability of failure at time  $t$  for a generator subjected to the loading conditions  $\Psi_i(\cdot)$ , as follows:

$$\begin{aligned}
 F_i(t|\xi_i, \alpha_i, \Psi_i(t)) &= P\{\tau_i = \tau(t) = \sum_{s=1}^t \Psi_i(s, \mathbf{x}_i)\} \\
 &= \sqrt{\frac{\alpha_i}{2\pi\tau(t)^3}} \exp\left\{-\frac{\alpha_i(\tau(t) - \xi_i)^2}{2\xi_i^2\tau(t)}\right\}.
 \end{aligned} \tag{50}$$

In what follows, we transform this sensor updated predictions of the remaining life distribution into dynamic cost functions that will adapt to real time changes in the sensor-observations.

### ***6.3 Load Dependent Dynamic Maintenance Cost***

A key aspect of our methodology is linking our predictive model with the optimization framework. Unlike the dynamic maintenance cost function outlined in Section II, in

this chapter we impose a two way interaction. More specifically, we ensure that i) the operational decisions which affect the loading on the generators are passed to the predictive analytics stage, so that the distribution of the failure time can be estimated using (50), and ii) the sensor updated probability of failure for specific loading conditions is communicated with the optimization model.

We note that the optimization model which determines the loading condition  $\Psi_i(\cdot)$ , cannot be solved without the remaining life distribution estimates that requires  $\Psi_i(\cdot)$ . To circumvent this problem, we rewrite the dynamic maintenance cost in terms of the transformed time  $\tau(t)$ , which does not require the loading condition information. We then let the optimization model determine the right  $\tau(t)$  given the maintenance and operational decisions.

The dynamic maintenance cost function that models the tradeoff between the cost of preventive maintenance (early repair before failure) versus the cost of unexpected failure, can be presented as follows:

$$K_{\tau(t_i^o), \tau(t)}^{d,i} = \frac{c_i^p P(R_{t_i^o}^{i'} > \tau(t)) + c_i^f P(R_{t_i^o}^{i'} \leq \tau(t))}{\int_0^{\tau(t)} P(R_{t_i^o}^{i'} > z) dz + \tau(t_i^o)}, \quad (51)$$

which is the cost rate associated with conducting generator maintenance  $\tau(t)$  transformed time periods after the transformed time of observation  $\tau(t_i^o)$ ;  $R_{t_i^o}^{i'}$  is the time of failure in the degradation equivalent time,  $c_i^p$  and  $c_i^f$  are the costs of planned maintenance and failure replacement, respectively. The probability  $P(R_{t_i^o}^{i'} > \tau(t))$  in this function is derived from the RLDs evaluated by expression (50). In essence, the dynamic cost functions are directly related to the RLDs and hence the degradation states of each generator in the time transformed domain.

Since certain generators can be scheduled for maintenance multiple times, it would be beneficial to characterize the associated maintenance cost of a new generator that has just completed its maintenance. We define  $\tau_i'$  as the time of failure in the degradation equivalent time. For a new generator, the maintenance cost function  $K_t^{n,i}$  takes

the following form:

$$K_{\tau(t)}^{\tau^{n,i}} = \frac{c_i^p P(\tau_i' > \tau(t)) + c_i^f P(\tau_i' \leq \tau(t))}{\int_0^{\tau(t)} P(\tau_i' > z) dz}. \quad (52)$$

The dynamic cost functions help identify the optimal time to repair generators based on their most recently updated RLD. In the following section, we discuss an optimization model that finds the optimal maintenance and operations scheduling for a fleet of generators by considering the sensor updated failure probabilities, as well as the coupling between the generator degradation and the loading. To do so, the optimization model integrates this load dependent dynamic cost into its objective function. Thus, it provides a mapping between the transformed time when the preventive maintenance is scheduled, and the dynamic maintenance cost presented herein.

#### ***6.4 Load Dependent Adaptive Predictive Maintenance***

We start by introducing the decision variables, sets and constants used by our optimization model.

##### **Decision Variables:**

$\nu_{t,i,k} \in \{0, 1\}$        $\nu_{t,i,k} = 1$  iff the  $k^{th}$  maintenance of generator  $i$  starts at maintenance epoch  $t$ . If a certain maintenance  $k$  is not scheduled, then  $\nu_{t,i,k} = \nu_{t,i,k_\ell}$  for all  $t \in \mathcal{T}$ , where  $k_\ell$  is the last scheduled maintenance.

$\gamma_{t,i,\ell} \in \{0, 1\}$        $\gamma_{t,i,\ell} = 1$  if the loading environment is at a level harsher than or equal to  $\ell$  for generator  $i$  at time  $t$ .

$\gamma_{t,i}^o \in \{0, 1\}$        $\gamma_{t,i}^o = 1$  if last scheduled maintenance outage of generator  $i$  ended before time  $t$ , in other words, if the last maintenance of generator  $i$  started before time  $t - T_i^M$ .

$z_{t,i,k} \in \{0, 1\}$	$z_{t,i,k} = 1$ iff the duration between the start of the $k^{th}$ and the $(k - 1)^{th}$ maintenances of generator $i$ is $t$ maintenance epochs.
$z_{i,k}^o \in \{0, 1\}$	$z_{i,k}^o = 0$ iff the $k^{th}$ maintenance is scheduled for generator $i$ within the planning horizon.
$x_{s,i}^t \in \{0, 1\}$	$x_{s,i}^t = 1$ iff generator $i$ is committed in day $s$ within maintenance epoch $t$ .
$y_{s,i}^t \in \mathbb{R}_+^n$	Generation output of generator $i$ in day $s$ within maintenance epoch $t$ .
$\psi_{s,p}^{DC,t} \in \mathbb{R}_+^n$	Demand curtailment in day $s$ within maintenance epoch $t$ at demand bus $p$ .
$\psi_{s,\ell}^{TL,t} \in \mathbb{R}_+^n$	Transmission line slack variable in day $s$ within maintenance epoch $t$ at line $\ell$ .

**Sets:**

$\mathcal{N}$	Set of loads.
$\mathcal{G}$	Set of generators.
$\mathcal{K}_i$	Set of possible maintenances for generator $i$ .
$\mathcal{S}$	Set of days within one maintenance epoch.
$\mathcal{T}$	Set of maintenance epochs within the planning horizon.

**Constants:**

$B_{s,i}^t$	Generation cost of generator $i$ in day $s$ within maintenance epoch $t$ .
$\delta_s^t$	Demand vector at day $s$ in maintenance epoch $t$ .

$T_i^M$	Maintenance duration for generator $i$ .
$f_l^{max}$	Flow limit on transmission line $l$ .
$H$	Planning horizon in terms of maintenance epochs.
$L$	Maximum number of generators that can be under maintenance simultaneously.
$M_i$	Maximum number of maintenances to be scheduled for generator $i$ within the planning horizon.
$p_i^{max}$	Maximum production level of generator $i$ .
$p_i^{min}$	Minimum production level of generator $i$ .
$P_i^R$	Reward per maintenance period for postponing the preventive maintenance of generator $i$ .
$P_{DC}$	Penalty cost for unit unsatisfied demand.
$P_{TL}$	Penalty cost for unit overload on a transmission load.
$\mathbf{P}_d^D$	Network incidence matrix for loads.
$\mathbf{P}_p^G$	Network incidence matrix for generators.
$R_i$	Remaining time required for maintenance of generator $i$ at the start of the planning horizon.
$\mathbf{r}_l$	Network shift factor vector for line $l$ .
$V_{s,i}^t$	No-load cost of generator $i$ in day $s$ within maintenance epoch $t$ .

$\zeta_i^d$	Period within which at least one maintenance should be scheduled to start for degraded generator $i$ .
$\zeta_i^n$	Period within which at least one maintenance should be scheduled to start for new generator $i$ .

This chapter focuses on a sensor driven framework for determining the optimal scheduling for a fleet of generators, at the presence of a two way interaction between maintenance and operations. In the maintenance problem, we leverage the condition monitoring information coming from generation assets to decide on both the time and the number of maintenances to be scheduled within a planning horizon. This problem is subject to constraints on labor capacity, maintenance dependencies between generators such as inclusion and exclusion, and separations between consecutive maintenances. In the operations problem, unit commitment problem is solved for a fleet of generators. More specifically, the commitment and dispatch level of the generators are determined, in order to satisfy the electricity demand requirements, and network feasibility. The first interaction between these problems, namely the effect of maintenance on operations, has been well studied in literature. This natural interaction occurs because a generator under maintenance cannot produce any electricity. In such cases, the demand for electricity should be provided by the remaining active generators. This interaction is captured through coupling constraints between the generator maintenance and commitment decisions. The second interaction occurs because the level of load on the generator affects its rate of degradation. When a generator is not committed, or dispatches a minimal amount of power, the rate of degradation remains at its base/minimal rate. However as the load increases, generators' degradation accelerates. Thus a critical dilemma occurs: would it make more sense i) to increase the dispatch level of a cheap generator to satisfy the electricity demand less costly now in expense of an accelerated degradation, or ii) to provide

some of the demanded electricity from a more expensive generator to delay the next maintenance on the cheap generator. There are a large number of factors that may affect this decision, including but not limited to the future electricity demand, remaining life and the level of degradation present in the generators, and the sensor driven dynamic maintenance costs for the entire fleet. Similar interactions can also apply to factors other than the dispatch limit, such as the number of shutdown startup cycles, and fast rampings of the generators.

We capture the effects of operations on maintenance by using the concept of degradation equivalent time. More specifically, the loading level in the generators are divided into  $L$  degradation states. Depending on the operational decisions (i.e the level of dispatch), the rate of the degradation in generators can be significantly faster than the base rate. The optimization model couples the dispatch level and maintenance decisions with the corresponding degradation equivalent time  $\tau(t)$  at the time of scheduled preventive maintenance through use of additional variables. In an attempt to solve this problem to optimality, both the maintenance and the operations decisions are optimized together, so that both problems will be considered along with their two way interaction.

The objective is to minimize the dynamic maintenance and the operational cost of the generator fleet:

$$\begin{aligned} \xi_m & \left[ \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_{\tau(t_i^e), t - R_i}^{d,i} \cdot z_{t,i,1} + \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{k=2}^{M_i} K_{t - T_i^M}^{n,i} \cdot z_{t,i,k} \right] - \sum_{i \in \mathcal{G}} P_i^R \left( \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} \right) \\ & + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} \sum_{s \in \mathcal{S}} (V_{s,i}^t \cdot x_{s,i}^t + B_{s,i}^t \cdot y_{s,i}^t) \\ & + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \left( \sum_{p \in \mathcal{D}} (P_{DC} \cdot \psi_{s,p}^{DC,t}) + \sum_{\ell \in \mathcal{L}} (P_{TL} \cdot \psi_{s,\ell}^{TL,t}) \right), \end{aligned}$$

where  $\xi_m$  is the maintenance criticality coefficient. We note that the first line of the objective function picks the dynamic cost corresponding to  $\tau(t)$  at the time of

preventive maintenance, here captured by the variable  $\mathbf{z}$ . In other words, the relationship between the degradation equivalent time  $\tau(t)$  at the start of the preventive maintenance, and  $\mathbf{z}$  can be presented as  $\tau(t) = \sum_{t=1}^H t \cdot z_{t,i,1}$ . The last expression in the first line evaluates the reward for operating the generators for longer time periods before scheduling them for maintenance. Lastly, the second and the third lines of the objective function provides the operational cost due commitment & dispatch, and demand curtailment & line capacity penalty, respectively.

The objective function is subject to a number of constraints in the maintenance and operations side:

**Enforcing Maintenance Time Limits:**

- The first constraint (53) ensures that the first maintenance occurs within  $\zeta_i^d$  maintenance epochs.  $\zeta_i^d$  depends on the remaining life distribution of the  $i^{th}$  generator. Depending on the application,  $\zeta_i^d$  can be set to a limiting period, when the sensor updated cumulative failure probability exceeds a specific control threshold.

$$z_{i,1}^o + \sum_{t=1}^{\zeta_i^d} z_{t,i,1} = 1 \tag{53}$$

- Likewise, constraint (54) limits the duration between the start time of two consecutive maintenances.

$$\sum_{t \in \mathcal{T}} t \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \nu_{t,i,k-1} \leq \zeta_i^n, \tag{54}$$

$$\forall i \in \mathcal{G}, \forall k \in \{2, \dots, M_i\}$$

**Coordinating Same Type Maintenance Variables:**

- The presented model allows a number of maintenances to be scheduled within the planning horizon. Constraint (55) ensures that for every such maintenance, a start time is selected.

$$\sum_{t \in \mathcal{T}} \nu_{t,i,k} = 1, \quad \forall i \in \mathcal{G}, \forall k \in \mathcal{K}_i \quad (55)$$

- Constraint (56) controls two factors. Firstly, for generator  $i$ , it dictates whether the  $k^{th}$  maintenance is scheduled within  $H$  (namely,  $z_{i,k}^o = 0$ ), or is projected to take place beyond  $H$  ( $z_{i,k}^o = 1$ ). Secondly, for any maintenance that is scheduled within  $H$ , it ensures that a certain time is selected to register the difference between two consecutive maintenances.

$$z_{i,k}^o + \sum_{t \in \mathcal{T}} z_{t,i,k} = 1, \quad \forall i \in \mathcal{G}, \forall k \in \mathcal{K}_i \setminus \{1\} \quad (56)$$

- Constraint (57) provides an ordering between consecutive maintenances. Constraint (58) ensures that if a maintenance is not scheduled, its start time is the same as the last scheduled maintenance. Since it is impossible for two maintenances to start at the same time, these instances indicate that the subsequent maintenance has not been scheduled.

$$\sum_{t \in \mathcal{T}} t \nu_{t,i,k} \geq \sum_{t \in \mathcal{T}} t \nu_{t,i,k-1}, \quad (57)$$

$$\forall i \in \mathcal{G}, \forall k \in \{2, \dots, M_i\}$$

$$\begin{aligned} & \sum_{t=1}^{H-\zeta_i^n} H \nu_{t,i,k-1} + \sum_{t=H-\zeta_i^n+1}^H t \nu_{t,i,k-1} \\ & \geq \sum_{t=1}^{H-\zeta_i^n} H \nu_{t,i,k} + \sum_{t=H-\zeta_i^n+1}^H t \nu_{t,i,k}, \end{aligned} \quad (58)$$

$$\forall i \in \mathcal{G}, \forall k \in \{2, \dots, M_i\}$$

- The following set of constraints ensure that a unit maintenance cannot be started if there is an ongoing maintenance. The following constraints represent

this relationship for the first maintenance and the consecutive maintenances.

$$\sum_{t=1}^{R_i} \nu_{t,i,1} = 0, \quad \forall i \in \mathcal{G}. \quad (59)$$

$$H \cdot z_{i,k}^o + \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \cdot \nu_{t,i,k-1} \geq T_i^M + 1 \quad (60)$$

$$\forall i \in \mathcal{G}, \quad k \in \{2, \dots, M_i\}.$$

- Constraint (61) ensures the  $k$ -th maintenance is scheduled only if the  $(k-1)$ -th maintenance is scheduled.

$$z_{i,k}^o \geq z_{i,k-1}^o, \quad \forall i \in \mathcal{G}, \quad k \in \{2, \dots, M_i\}. \quad (61)$$

### Coordinating Different Type Maintenance Variables:

- Constraints (62,63,64) coordinates the degradation equivalent measure  $z$  with the decision variables  $\gamma$ . More specifically, the following constraints provide a mapping between the loading conditions at each time period  $t$ , with the degradation equivalent time when the preventive maintenance is scheduled. More specifically,  $\sum_{\ell=0}^L Q_{\ell,i} \gamma_{t,i,\ell}$  gives the loading condition at time  $t$ ,  $\Psi(t)$ . By summing  $\sum_{\ell=0}^L Q_{\ell,i} \gamma_{t,i,\ell} = \Psi(t)$  over time periods until the first maintenance, we can get the degradation equivalent time during the first preventive maintenance, given by  $\sum_{t \in \mathcal{T}} t z_{t,i,1}$ .

We note that our model allows  $M_i$  number of possible maintenances for each generator  $i$ . However in our formulation, the loading variable  $\gamma$  does not have an index for the maintenance number. Therefore we need to enforce a coupling between time periods and the maintenance numbers. To do so, we use (62) to provide a lower bound for the degradation equivalent time using a forward induction argument:

$$\sum_{e=1}^t \sum_{\ell=0}^L Q_{\ell,i} \gamma_{e,i,\ell} \leq \sum_{n=1}^k \sum_{e \in \mathcal{T}} e z_{e,i,n} + \sum_{e=1}^t (\nu_{e,i,k} + z_{i,k}^o) \cdot L \cdot T \quad (62)$$

$$\forall k \in \mathcal{K}_i, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

For any time  $t$ , maintenance  $k$  and generator  $i$ , if i) the  $k^{\text{th}}$  maintenance is scheduled before time  $t$ , or if ii) the  $k^{\text{th}}$  maintenance is not scheduled; the constraint becomes redundant. Otherwise, it ensures that the sum of the loading environments until time  $t$ , namely  $\sum_{e=1}^t \sum_{\ell=0}^L Q_{\ell,i} \gamma_{e,i,\ell}$ , provides a lower bound for the sum of the degradation equivalent times for all maintenances  $k' \leq k$ . Note that the constraint goes from small to large in terms of both the time and the maintenance indices.

We next execute the same logic from reverse. (63) provides a lower bound for the degradation equivalent time using a backward induction argument:

$$\sum_{e=t}^H \sum_{\ell=0}^L Q_{\ell,i} \gamma_{e,i,\ell} \leq \sum_{n=k}^{M_i} \sum_{e \in \mathcal{T}} e z_{e,i,n} + \sum_{e=t}^H (\nu_{e,i,k-1} + z_{i,k}^o) \cdot L \cdot T \quad (63)$$

$$\forall k \in \mathcal{K}_i \setminus \{1\}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

In this constraint, for any time  $t$ , maintenance  $k$  and generator  $i$ , if i) the  $(k-1)^{\text{th}}$  maintenance is scheduled after time  $t$ , or if ii) the  $k^{\text{th}}$  maintenance is not scheduled; the constraint becomes redundant. Otherwise, it ensures that the sum of the loading environments after time  $t$ , namely  $\sum_{e=t}^H \sum_{\ell=0}^L Q_{\ell,i} \gamma_{e,i,\ell}$ , provides a lower bound for the sum of the degradation equivalent times for all maintenances  $k' \geq k$ . We note that unlike the previous constraint, constraint (63) goes from large to small in terms of both the time and the maintenance indices.

In (64), we impose an equality between the sum of all loading conditions within the planning horizon and the transformed times for each maintenance:

$$\sum_{e=1}^H \sum_{\ell=0}^L Q_{\ell,i} \gamma_{e,i,\ell} = \sum_{k=1}^K \sum_{e \in \mathcal{T}} e z_{e,i,k} \quad (64)$$

$$\forall i \in \mathcal{G}$$

We next ensure some logical constraints on the loading variables. In (65), we enforce that generator  $i$  cannot have any loading (thus remains offline) at time  $t$ , if there is an ongoing maintenance:

$$\gamma_{t,i}^o + \gamma_{t,i,\ell} \leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{T_i^M - 1} \nu_{t-e,i,k} \quad (65)$$

$$\forall \ell \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

In (66), we ensure that if the loading of the generator at time  $t$  is  $\ell$ , then the  $\gamma$  variables for the  $\ell^{\text{th}}$  level and all the levels before  $\ell$  gets the value 1, or more specifically  $\gamma_{t,i,\ell'} = 1$  for all  $\ell' \leq \ell$ :

$$\gamma_{t,i,\ell} \leq \gamma_{t,i,\ell-1} \quad (66)$$

$$\forall \ell \in \mathcal{L}/\{0\}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

Lastly, we ensure that  $\gamma_{t,i}^o$  is zero for all time periods before the last scheduled maintenance (67), and the loading variables  $\gamma_{t,i,\ell}$  cannot be 1 for any time period  $t$  that is after the last scheduled maintenance (68):

$$\gamma_{t,i}^o \leq \sum_{e=1}^t \nu_{e,i,M_i} \quad (67)$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

$$\gamma_{t,i,\ell} \leq \sum_{e=t}^H \nu_{e,i,M_i} \quad (68)$$

$$\forall \ell \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

**Limits on Maintenance Crew:**

- Following constraints ensure that the available labor capacity,  $Y$ , is not exceeded at any time. To do so, we consider the number of ongoing maintenances at time  $t$  by looking back in time and checking if a maintenance is initiated for generator  $i$  at time  $\{t - D_i + 1, \dots, t\}$ . Note that when we analyze maintenances, if  $t \in \{1, \dots, H - \zeta_n(i)\}$  we need to check for every maintenance  $k$ , however for  $t \in \{\dots, H - \zeta_n(i) + 1, \dots, H\}$  we check only the last maintenance  $M_i$  to eliminate double counting.

$$\begin{aligned}
& \sum_{i \in \mathcal{G}} \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{T_i^{M-1}} \nu_{t-e,i,k} \leq Y, \quad \forall t \in \{1, \dots, H - \zeta_n(i)\} \\
& \sum_{i \in \mathcal{G}} \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{D}_i^1} \nu_{t-e,i,k} + \sum_{i \in \mathcal{G}} \sum_{e \in \mathcal{D}_i^2} \nu_{t-e,i,M_i} \leq Y, \\
& \quad \forall t \in \{H - \zeta_n(i) + 1, \dots, H - \zeta_n(i) + T_i^M\} \\
& \sum_{i \in \mathcal{G}} \sum_{e=0}^{T_i^{M-1}} \nu_{t-e,i,k} \leq Y, \quad \forall t \in \{H - \zeta_n(i) + T_i^M + 1, \dots, H\}
\end{aligned} \tag{69}$$

where the sets  $\mathcal{D}_i^1 = \{t - H - \zeta_n(i), \dots, T_i^M\}$  and  $\mathcal{D}_i^2 = \{0, \dots, t - H - \zeta_n(i) - 1\}$

### Coordination for O&M:

- In cases where a certain generator is under maintenance at the start of the planning horizon, the corresponding commitment variables  $\mathbf{x}$ , are set to zero.

$$\begin{aligned}
x_{s,i}^t &= 0 \quad \forall i \in \mathcal{G}, \quad \forall s \in \mathcal{S} \\
& \forall t \in \{1, \dots, R_i\} \text{ if } R_i > 0
\end{aligned} \tag{70}$$

- In this set of constraints, we couple the maintenance decision variable  $\nu$  with generator commitment variables  $\mathbf{x}$ . Constraint (71) ensures that if a unit is under maintenance during maintenance epoch  $i$ , it cannot be committed in any of the days within that epoch. To verify that unit  $i$  is not under maintenance

at time  $t$ , it suffices to check that a maintenance activity on unit  $i$  has not been initiated during any of the following maintenance epochs  $\{t - T_i^M + 1, \dots, t\}$ .

$$\begin{aligned}
x_{s,i}^t &\leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{T_i^M-1} \nu_{t-e,i,k} \\
\forall i \in \mathcal{G}, \quad \forall t \in \{1, \dots, H - \zeta_n(i)\} \\
x_{s,i}^t &\leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{D}_i^1} \nu_{t-e,i,k} + \sum_{e \in \mathcal{D}_i^2} \nu_{t-e,i,M_i} \\
\forall i \in \mathcal{G}, \\
\forall t \in \{H - \zeta_n(i) + 1, \dots, H - \zeta_n(i) + T_i^M - 1\} \\
x_{s,i}^t &\leq 1 - \sum_{e=0}^{T_i^M-1} \nu_{t-e,i,k} \\
\forall i \in \mathcal{G}, \quad \forall t \in \{H - \zeta_n(i) + T_i^M - 1, \dots, H\}
\end{aligned} \tag{71}$$

where the sets  $\mathcal{D}_i^1$  and  $\mathcal{D}_i^2$  are defined in a similar fashion to constraint (69).

- We next couple the load dependency variables  $\gamma$  with the unit commitment variables. We provide the modeling for a number of cases, where the load severity depends on: i) dispatch level of the generator, ii) the number of turn-on turn-off instances, and iii) the number of sudden changes in ramping.

1. We first characterize the load dependency in regards to the level of production:

$$\sum_{s \in \mathcal{S}} y_{s,i}^t \geq S \left[ \Gamma_0^L \cdot \gamma_{t,i,0} + \sum_{l \in \mathcal{L}/0} ((\Gamma_l^L - \Gamma_{l-1}^L) \gamma_{t,i,l}) \right] \tag{72}$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

$$\sum_{s \in \mathcal{S}} y_{s,i}^t \leq S \left[ \sum_{l \in \mathcal{L}} ((\Gamma_{l+1}^L - \Gamma_l^L) \gamma_{t,i,l}) + p_i^{max} \cdot \gamma_{t,i}^o \right] \tag{73}$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

where  $\Gamma_l^L$  is the average load level to reach to degradation regime  $l$ .

2. Second load dependency is in regards to the number of shut-down start-up instances:

$$\sum_{s \in \mathcal{S}} (\pi_{s,i}^{U,t} + \pi_{s,i}^{D,t}) \geq S \left[ \Gamma_0^{UD} \cdot \gamma_{t,i,0} + \sum_{l \in \mathcal{L}/0} ((\Gamma_l^{UD} - \Gamma_{l-1}^{UD}) \gamma_{t,i,l}) \right] \quad (74)$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

$$\sum_{s \in \mathcal{S}} (\pi_{s,i}^{U,t} + \pi_{s,i}^{D,t}) \leq S \left[ \sum_{l \in \mathcal{L}} ((\Gamma_{l+1}^{UD} - \Gamma_l^{UD}) \gamma_{t,i,l}) + p_i^{max} \cdot \gamma_{t,i}^o \right] \quad (75)$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

where  $\Gamma_l^{UD}$  is the number of shut-down, start-up instances required to go to degradation regime  $l$ , and  $\pi_{s,i}^{U,t}, \pi_{s,i}^{D,t}$  are the binary variables turn-on and shut-down, respectively.

3. Third load dependency is in regards to the number of sudden changes in ramping:

$$\sum_{s \in \mathcal{S}} (\psi_{s,i}^{U,t} + \psi_{s,i}^{D,t}) \geq S \left[ \Gamma_0^{RL} \cdot \gamma_{t,i,0} + \sum_{l \in \mathcal{L}/0} ((\Gamma_l^{RL} - \Gamma_{l-1}^{RL}) \gamma_{t,i,l}) \right] \quad (76)$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

$$\sum_{s \in \mathcal{S}} (\psi_{s,i}^{U,t} + \psi_{s,i}^{D,t}) \leq S \left[ \sum_{l \in \mathcal{L}} ((\Gamma_{l+1}^{RL} - \Gamma_l^{RL}) \gamma_{t,i,l}) + p_i^{max} \cdot \gamma_{t,i}^o \right] \quad (77)$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G}$$

$$y_{s,i}^t - y_{s-1,i}^t \leq \zeta_i^l + (RAMP_{max}^i - \zeta_i^l) \cdot \psi_{s,i}^{U,t} \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}/1, \quad (78)$$

$$y_{s-1,i}^t - y_{s,i}^t \leq \zeta_i^l + (RAMP_{max}^i - \zeta_i^l) \cdot \psi_{s,i}^{D,t} \quad \forall i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}/1,$$

where  $\Gamma_l^{RL}$  is the number of extreme ramping instances required to go to degradation regime  $l$ , and  $\psi_{s,i}^{U,t}, \psi_{s,i}^{D,t}$  are the binary variables indicating instances of sudden ramping in the upwards and downwards direction respectively.

In the modeling framework, one can either choose one of these options, or include all/subset of these options, and define a separate  $\gamma$  variable that would be the weighted sum of  $\gamma$  variables coming from different options considered by the model.

## 6.5 Algorithm and Decomposition

In this section, we briefly present an algorithm that is inspired by the solution methodology presented in Chapter IV. Thus, we will highlight the differences in terms of the model decomposition, but will not reintroduce the algorithm. Interested reader is referred to Chapter 4.5 for the algorithm, and Appendices A & B for the proofs on  $\epsilon$ -optimality and finite convergence of the algorithm. We first present the load dependent maintenance and operations model with dispatch driven load-dependency in its compact form as follows:

$$\min_{z, \nu, \gamma, x, y} \mathbf{c}^\top \mathbf{z} + \mathbf{v}^\top \mathbf{x} + \mathbf{b}^\top \mathbf{y} \quad (79a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} + \mathbf{L}\boldsymbol{\gamma} \leq \mathbf{g} \quad (79b)$$

$$\mathbf{B}\boldsymbol{\gamma} + \mathbf{E}\mathbf{y} \leq \mathbf{h} \quad (79c)$$

$$\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{y} \leq \boldsymbol{\ell} \quad (79d)$$

where  $\mathbf{A}\mathbf{z} + \mathbf{K}\boldsymbol{\nu} + \mathbf{L}\boldsymbol{\gamma} \leq \mathbf{g}$  represents the maintenance constraints,  $\mathbf{B}\boldsymbol{\gamma} + \mathbf{E}\mathbf{y} \leq \mathbf{h}$  captures the coupling between the dispatch level and degradation load state,  $\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{y} \leq \boldsymbol{\ell}$  are operational constraints within a certain week. Decomposing the problem into the master maintenance problem, and operational subproblem yields:

$$\min_{z, \nu, \gamma} \mathbf{c}^\top \mathbf{z} + q(\gamma) \quad (80a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{z} + \mathbf{K}\nu + \mathbf{L}\gamma \leq \mathbf{g} \quad (80b)$$

where the operational problem  $q(\gamma)$ , and its relaxed lower bounding problem attained by relaxing the binary variables, namely  $q^R(\gamma)$ , have the following relationship:

$$q(\gamma) \geq q^R(\gamma) = \sum_{t \in \mathcal{T}} q_t^R(\gamma) \quad (81)$$

where,  $q_t(\cdot)$  and  $q_t^R(\cdot)$  are the exact and the relaxed operational problems for the maintenance period  $t$ . These problems can be represented as follows:

$$q_t(\gamma) = \min_{\mathbf{x}_t, \mathbf{y}_t} \mathbf{v}^\top \mathbf{x}_t + \mathbf{b}^\top \mathbf{y}_t \quad (82a)$$

$$\text{s.t.} \quad \mathbf{E}_t \mathbf{y}_t \leq \mathbf{h}_t - \mathbf{B}_t \gamma_t \quad (82b)$$

$$\mathbf{F}_t \mathbf{x}_t + \mathbf{G}_t \mathbf{y}_t \leq \boldsymbol{\ell}_t \quad (82c)$$

$$x \in \{0, 1\} \quad (82d)$$

$$q_t^R(\gamma) = \min_{\mathbf{x}_t, \mathbf{y}_t} \mathbf{v}^\top \mathbf{x}_t + \mathbf{b}^\top \mathbf{y}_t \quad (83a)$$

$$\text{s.t.} \quad \mathbf{E}_t \mathbf{y}_t \leq \mathbf{h}_t - \mathbf{B}_t \gamma_t \quad (83b)$$

$$\mathbf{F}_t \mathbf{x}_t + \mathbf{G}_t \mathbf{y}_t \leq \boldsymbol{\ell}_t \quad (83c)$$

$$x \in [0, 1] \quad (83d)$$

We next use the method outlined in Chapter IV.  $\sum_{t \in \mathcal{T}} \phi_t$  along with the benders cuts  $BC_t^h(\gamma_t) \leq \phi_t$  will recover the cost from  $q_t^R(\cdot)$ ,  $\mathbf{e}\boldsymbol{\eta}$  along with the total cost recovery constraints  $TCR_t^j(\gamma_t) \leq \eta_t^j$  will recover the cost  $q_t(\cdot) - q_t^R(\cdot)$  for all maintenance periods  $t$ .

$$\min_{z, \nu, \gamma, \mathbf{x}, \mathbf{y}} \mathbf{c}^\top \mathbf{z} + \sum_{t \in \mathcal{T}} \phi_t + \mathbf{e}\boldsymbol{\eta} \quad (84a)$$

$$\text{s.t.} \quad \mathbf{Az} + \mathbf{K}\boldsymbol{\nu} + \mathbf{L}\boldsymbol{\gamma} \leq \mathbf{g} \quad (84b)$$

$$BC_t^h(\boldsymbol{\gamma}_t) \leq \phi_t \quad \forall t \in \mathcal{T}, h \in \mathcal{H} \quad (84c)$$

$$TCR_t^\omega(\boldsymbol{\gamma}_t) \leq \eta_t^\omega \quad \forall t \in \mathcal{T}, \omega \in \Omega_t \quad (84d)$$

where  $\Omega_t$  is the partial set of maintenance scenarios in week  $t$ . Given the formulation of these subproblems, we then apply the algorithm outlined in Chapter 4.5.

## 6.6 Experiments

In this section, we present two studies to highlight the performance of LDAPM. In both studies, we schedule the maintenance and operations of 6 generators using the IEEE 30-Bus case. To test the performance of our framework, we expand upon the framework presented in Chapters 3 – 5 in order to capture the load dependent nature of degradation. More specifically, our experimental framework is composed of two modules: optimization module, and the execution module. In the optimization module, we use the time transformed version of the dynamic sensor-updated cost functions to obtain the optimal maintenance and operations decisions. In the execution module, we model the chain of events that occur during a freeze period. To do so, we first evaluate the loading conditions on each generator using the results of LADPM. More specifically, we use the optimal decision for the variable  $\boldsymbol{\gamma}$ , to model the rate of degradation for each generator. We then determine whether an unexpected failure or a successful maintenance have occurred during any time point within the freeze period. If a preventive maintenance is experienced, we take the generator offline for 3 weeks. Otherwise, if the generator fails unexpectedly before the time of its scheduled maintenance, then the generator stays offline for the duration of 6 weeks.

For every time period within the planning horizon, we solve a unit commitment model with the available generators (those that are not undergoing a preventive or corrective maintenance), in order to obtain the operational cost. We also evaluate the maintenance cost by finding the number of preventive and corrective maintenances

and multiplying those instances by the cost of preventive maintenance  $c_i^p$  and corrective maintenance  $c_i^f$ , respectively. In all our experiments, we fix these costs across generators, and let  $c_i^f = 4 \cdot c_i^p = \$800,000$ . We make the maintenance decisions weekly, and the unit commitment decisions daily (at the time of peak demand). Planning horizon for every problem is 80 weeks, and the maintenance and operations scheduling is updated every  $\tau_R = 8$  weeks. The experiments are solved using Gurobi [51].

In order to make a fair comparison, we perform benchmark analysis for LDAPM against two conventional methods in literature, namely the periodic model (PM), and the reliability based model (RBM). These approaches are population driven (therefore do not capture the sensor information), and are not adaptive to the loading conditions. For these problems, we solve the APMII (from Chapter IV) using the periodic and RBM models, and use the resulting loading environment to simulate the generator degradation at the execution stage. More specifically, for the PM case, we enforce a constraint to ensure the preventive maintenance takes place at a specific age range for every generator, with the objective of minimizing total operational cost. We look at the overall demand and the available generator capacities to adjust the optimal period. We therefore devise a more intelligent periodic policy that is not extremely conservative. For the RBM case, we use the optimization model of APMII, however the cost function for this scenario is derived using a Weibull distribution. We derive a Weibull estimate using the failure times from the rotating machinery application  $F_W(t)$  subject to the most severe loading environment. This provides a conservative estimate for the time of failure. We then condition this distribution on the time of survival to estimate the remaining life distribution and the associated maintenance costs.

In every case study, we execute the implementation for a period of 160 weeks, thus covering 20 rolling horizons. We repeat this process 10 times using different generators in different ages. We obtain the age of the generators at the start of the

experiments by running the generators for a warming period. For every metric that we present, we take the average of the results coming from these 10 runs.

### 6.6.1 Comparative Study on LDAPM with two loading levels: 30-Bus System

In this section we present the experimental results for LDAPM. In our analyses we use the IEEE 30-Bus system, which is composed of 30 buses and 41 branches. In this study, we assume that all the generators have two loading levels: i) nominal loading, and ii) severe loading. In the nominal loading case, we let  $\Psi(\cdot) = 1$ , whereas in the severe loading case we accelerate degradation by a factor of two, or more specifically we set  $\Psi(\cdot) = 2$ . We couple the loading level with the dispatch level of the generator. If the generator's average weekly production exceeds 80% of its maximum capacity  $p_i^{max}$ , it switches to the harsh loading environment. More specifically, for every generator  $i$ , we let  $\Gamma_0^L = p_i^{min}$  and  $\Gamma_0^L = 0.8 \cdot p_i^{max}$ .

Table 10 presents the reliability and cost metrics for the three policies considered in this study. We note that LDAPM has a number of advantages over the conventional methods, PM and RBM: 1) LDAPM leverages on the sensor information to have an accurate estimation on the remaining life distribution of the generators, in addition 2) LDAPM captures the interaction between the operational decisions and degradation, therefore i) it uses sensor data along with the information on the loading profile in order to schedule maintenance, ii) and it optimizes the operational profile to minimize the impact of the harsh loading environments. Evidently, LDAPM provides a maintenance schedule that performs significantly less number of outages (reduction of %15.7 and %42.2 for PM and RBM, respectively), and incurs minimal number of unexpected failures (reduction of %84.2 and %72.7 for PM and RBM, respectively), while also ensuring that the mean loading level is kept close to the nominal value of 1.

We can also see that LDAPM provides significant savings in terms of the incurred

maintenance costs. Perhaps more importantly, LDAPM also minimizes the impact of maintenance onto the operations because of the following factors:

- Generators age slower in LDAPM, because the model typically operates them in nominal loading unless there is a significant advantage in using the full capacity of the generators.
- LDAPM captures the dependency of load and sensor information into its life prediction, therefore incurs less unexpected failures while executing a more liberal maintenance policy (evident by less number of preventive actions).
- LDAPM has significantly more flexibility for delaying the optimal maintenance time of the generator. Thus, it can control the production level and minimize the risk of multiple failures occurring simultaneously.

As a result, we see that LDAPM provides %67.1 and %69.1 savings compared to the operational costs of PM and RBM. A similar trend is apparent in terms of the total cost as well (reduction by %66.6 and %68.7 for PM and RBM, respectively)

**Table 10:** Benchmark for LDAPM - IEEE 30-Bus Case with  $L = 2$

	<b>Periodic</b>	<b>RBM</b>	<b>LDAPM</b>
<b># Preventive</b>	15.2	25.5	15.4
<b># Failures</b>	3.8	2.2	0.6
<b># Total Outages</b>	19.0	27.7	16.0
<b>Mean Loading</b>	1.48	1.41	1.18
<b>Maintenance Cost</b>	\$6.08 M	\$6.86 M	\$3.56 M
<b>Operations Cost</b>	\$324.14 M	\$345.93 M	\$ 106.69 M
<b>Total Cost</b>	\$330.22 M	\$352.79 M	\$ 110.25 M

### 6.6.2 Comparative Study on LDAPM with three loading levels: 30-Bus System

We next consider a more interesting scenario whereby we increase the number of loading levels to 3. The very first level  $\ell = 0$  covers the loading environment where

a generator does not produce any power (turned off) during the entire week. In this case, we assume that the generator does not experience any degradation during that week. The loading case  $\ell = 1$  is the nominal case, whereby we let  $\Psi(\cdot) = 1$ , whereas in the severe loading case, like in the previous study, we accelerate degradation by a factor of two, or more specifically we set  $\Psi(\cdot) = 2$ .

We note that, similar trends also apply to this study. We can see that LDAPM decreases the number of unexpected failures, outages, as well as the costs associated with maintenance (decreasing the cost of maintenance by %75.9 and %76.9 compared to PM and RBM, respectively) and operations (this time reducing by %64.8 and %78.6 compared to PM and RBM, respectively). However we see in this scenario that LDAPM performs significantly better than it did in the 2 level case. This is because, allowing to turn generators off and postponing their maintenance that way, has significantly enhanced the flexibility of our model. As a result, we see that the mean loading has decreased from 1.18 in the LDAPM with 2 degradation levels, to 0.54 in this 3-level scenario with the option of turning generators off.

**Table 11:** Benchmark for LDAPM - IEEE 30-Bus Case with  $L = 3$

	<b>Periodic</b>	<b>RBM</b>	<b>LDAPM</b>
<b># Preventive</b>	15.9	26.4	6.6
<b># Failures</b>	3.3	1.0	0.1
<b># Total Outages</b>	19.2	27.4	6.7
<b>Mean Loading</b>	0.83	0.86	0.54
<b>Maintenance Cost</b>	\$5.82 M	\$6.08 M	\$1.4 M
<b>Operations Cost</b>	\$288.84 M	\$475.82 M	\$ 101.65 M
<b>Total Cost</b>	\$294.66 M	\$481.89 M	\$ 103.05 M

## ***6.7 Conclusion***

In this chapter, we considered the interaction between the operational load on the generators, and their corresponding rate of degradation. This interaction is particularly important since it significantly affects the remaining life of the generators and the optimal maintenance decisions. Also, by deciding on the dispatch level of the generators, a maintenance planner can intentionally alter the optimal maintenance time (i.e. by lowering the load on the cheap generators and postponing their preventive maintenance, or by increasing the load on the cheap generators and using more capacity before early maintenance). We proposed optimal maintenance and dispatch decisions for generators operating in this environment. Our experiments on IEEE 30-Bus case showed that considering the load dependency along with the sensor information, can provide significant savings in both maintenance and operations cost, while ensuring a more reliable electricity system.

## CHAPTER VII

### CONCLUSION

Classical reliability methods estimate the failure risks by analyzing population-specific properties, i.e. failure times of similar assets. In reality, however, even identical assets exhibit significant variation in their failure times due to unit-specific properties, such as metallurgical variations and manufacturing imperfections. The optimization models for generation maintenance presented in this thesis are the first fleet scheduling models in maintenance literature that leverages on predictive degradation modeling to capture these unit-specific properties. To date, maintenance decisions in generation maintenance are still based on time-based schedules that do not use sensor information. The approach we propose is important due to the following reasons:

Classical reliability mechanisms and time-based policies, do not account for the actual condition of the asset, and therefore should not be used to anticipate failures. Accurate prediction of failure is crucial since failure instances increase the cost of asset maintenance drastically, and can lead to human fatalities. More importantly, electric power networks rely on uninterrupted operation of their constituent components. Unexpected failures of generators can cause significant deviations in grid frequency and trigger major power blackouts in the network.

If implemented in a conservative fashion, time-based policies still drive up the cost of maintenance and operations due to frequent unnecessary maintenances. Using the sensor information allows for an effective use of the generation resources and further improves system reliability and profitability.

In this thesis we proposed sensor driven maintenance and operations scheduling policies for conventional power plant fleets, and wind farms. In the penultimate

chapter, we expanded upon these two models to couple the loading level of the power plants, with the optimal decisions for maintenance and operations scheduling.

The presented methods build on an automated sensor acquisition system that evaluates the equipment health without the need for any stoppage, or disassembly. The condition of the asset and the prediction of the failure time are performed continuously while the equipment is operational in the field. Experimental studies conducted in laboratory provide insights for a more accurate mathematical characterization of degradation, and enhance the credibility of our remaining life predictions.

The integrated framework presented in this thesis will be used as a basis for a number of sensor-driven applications in maintenance and operations scheduling in power systems. In what follows, we highlight a few of those research directions. Firstly, we plan to extend the framework in Chapters II-IV by integrating the market operations into this framework. More specifically, we envision a coordination framework between generator companies and the independent system operator (ISO). In this framework, we foresee that every generation company monitors its own generators, and depending on the condition of the generators and the market, bids for an optimal maintenance slot. ISO then gets the bids from the generator companies, and coordinates the maintenance to maximize the reliability of the system.

We next plan to tackle the difficulties in the scalability of our framework. In this thesis, we devised specialized algorithms to solve the scheduling problems in the IEEE 30-Bus and the IEEE 118-Bus cases. However, many practical networks extend much beyond these cases. Therefore, to ensure scalability of our approach, we plan to work on a distributed optimization framework that will breakdown the computational workload across generators (or subsets of generators) that can run in parallel and coordinate among themselves to reach to a good system-wide solution. A key issue is in controlling the level and synchronicity of information exchange to keep the communication overhead low.

We also plan to focus on a more detailed view of the generator. Generators are typically composed a large number of constituent subcomponents. A failure occurs when any of the critical subcomponents fail (competing risks characterization). In this line of work, dynamic reliability assessment of a generator is envisioned to be a function of the failure risks of its subsequent subcomponents. A sensor-driven subcomponent maintenance policy is planned to be provided, which can: i) attain a desired level of generator and grid reliability, ii) benefit from opportunistic maintenance among the subcomponents of the same generator, and iii) provide a more accurate characterization of the sensor driven failure risks.

Lastly, we plan to focus on the applications of our framework to a more general class of problems in power systems. Predictive degradation models, and digital control mechanisms can be used to improve the current state-of-the-art in general class of power network reliability problems, including cascade control, contingency analysis and demand response. The last problem, for instance, plays a pivotal role in aligning the power grid operations with the decisions taken by the customers. The incentive mechanisms suggested to date typically attempt to decrease the overall production cost. The goal in this work would be to predict an accurate characterization of the impact of customer actions on the overall reliability of the power grid using predictive modeling, and devise appropriate price based policies to improve network reliability.

## APPENDIX A

### PROOF OF LEMMA 1

The APMII model (26) can be represented as follows:

$$\min_{\mathbf{z}^o, \boldsymbol{\nu}} q(\boldsymbol{\nu}) + \sum_{i \in \mathcal{G}} \sum_{k \in \mathcal{K}_i} \min_{\mathbf{z}_{:,i,k}} \{ \mathbf{c}_{i,k}^\top \mathbf{z}_{:,i,k} \mid \mathbf{z}_{:,i,k} \in \mathcal{P}_k^i(\mathbf{z}^o, \boldsymbol{\nu}) \},$$

where, given feasible  $\mathbf{z}^o, \boldsymbol{\nu}$ , the APMII's constraints  $\mathcal{P}_k^i(\mathbf{z}^o, \boldsymbol{\nu})$  over  $\mathbf{z}_{:,i,k}$ 's are decoupled for each  $i, k$ . We want to show that for any fixed  $\{\mathbf{z}^o, \boldsymbol{\nu}\}$ , if we relax  $\mathbf{z}_{:,i,k}$  to be in  $[0, 1]$ , the relaxed problem still has a binary optimal solution in  $\mathbf{z}_{:,i,k}$ . For  $k = 1$ ,  $\mathbf{z}_{:,i,1} = \boldsymbol{\nu}_{:,i,1}$  has to be binary due to (16). In the following, we focus on  $k \geq 2$ . For any  $i \in \mathcal{G}, k \geq 2$ , if  $z_{i,k}^o = 1$ , then  $z_{t,i,k} = 0 \forall t$  by (12). If  $z_{i,k}^o = 0$ , then  $\sum_{t \in \mathcal{T}} z_{t,i,k} = 1$  by (12) and  $\sum_{t \in \mathcal{T}} t z_{t,i,k} = \sum_{t \in \mathcal{T}} t \nu_{t,i,k} - \sum_{t \in \mathcal{T}} t \nu_{t,i,k-1} =: b_{ik}$  by (17), which, together with constraint (14), ensures that  $b_{ik}$  is a nonnegative integer. Denote the  $k$ -th maintenance cost of generator  $i$  at time  $t$  as  $\phi_{i,k}(t) = C_t^{n,i}$ , which is convex in  $t$  given by (7). Since  $\sum_{t \in \mathcal{T}} z_{t,i,k} = 1$  and  $z_{t,i,k} \geq 0$ , then for any  $z_{t,i,k}$  feasible for the relaxed problem, the Jensen's inequality suggests

$$\begin{aligned} \phi_{i,k}(b_{ik}) &= \phi_{i,k}(z_{1,i,k} + 2z_{2,i,k} + \cdots + Hz_{H,i,k}) \\ &\leq z_{1,i,k}\phi_{i,k}(1) + z_{2,i,k}\phi_{i,k}(2) + \cdots + z_{H,i,k}\phi_{i,k}(H) = \mathbf{c}_{i,k}^\top \mathbf{z}_{:,i,k}. \end{aligned}$$

Thus,  $q(\boldsymbol{\nu}) + \sum_{i \in \mathcal{G}} \mathbf{c}_{i,1}^\top \mathbf{z}_{:,i,1} + \sum_{i,k \geq 2} \phi_{i,k}(b_{ik})$  is a lower bound to the optimal cost of the relaxed problem for the fixed  $\mathbf{z}^o, \boldsymbol{\nu}$ . In fact, this lower bound can be achieved by the solution  $z_{t,i,k} = 1$  if  $t = b_{ik}$  and 0 otherwise for  $k \geq 2$ . This binary solution together with  $\mathbf{z}_{:,i,1} = \boldsymbol{\nu}_{:,i,1} \forall i \in \mathcal{G}$  is feasible for the relaxed problem, therefore also optimal for APMII.

Lastly, we let  $q(\boldsymbol{\nu}) \leftarrow 0$ , and APMII reduces to AMPI. Thus the lemma also applies to AMPI. This completes the proof.

## APPENDIX B

### PROOF OF THEOREM 1

*Finite convergence:* Recall that all the coupling variables in the first stage problem are binary, thus for each step  $h$  of Algorithm 1, the Benders' decomposition procedure would iterate a finite number of times before it revisits a certain solution, at which point, Benders' convergence would be guaranteed for any tolerance  $\epsilon_b$ . Evidently, Benders' decomposition terminates in finite steps. Then, if the condition in line 10 of TCR is true, Algorithm 1 terminates. Otherwise, TCR augments the set  $\Omega$  by at least one different maintenance status. Since the number of all possible statuses is  $H2^{|\mathcal{G}|}$ , TCR is executed at most  $H2^{|\mathcal{G}|}$  number of times, at which point  $\Omega$  becomes  $\bar{\Omega}$  and  $\rho_k^* + \delta^h = \rho_k^*$ , thus terminating the algorithm.

*$\epsilon$ -Optimality:* (i) We first prove that  $\rho_k^* \leq \rho^*$ , where  $\rho_k^*$  is the optimal cost of the final RMP solved before Algorithm 1 terminates. We note that for any feasible maintenance solution  $\{\mathbf{z}, \boldsymbol{\nu}\}$ , we have

$$\mathbf{c}^\top \mathbf{z} + \min_{\boldsymbol{\eta}, \boldsymbol{\varphi}} \left\{ \mathbf{e}^\top \boldsymbol{\eta} + \sum_{t \in \mathcal{T}} \varphi^t \mid s.t. : (34c), (34d) \right\} \leq \mathbf{c}^\top \mathbf{z} + q(\boldsymbol{\nu}).$$

Let  $\{\mathbf{z}', \boldsymbol{\nu}'\}$  be the optimal solution of APMII, i.e:  $\rho^* = \rho(\mathbf{z}', \boldsymbol{\nu}')$ . Then we have

$$\begin{aligned} \rho_k^* &= \min_{\mathbf{z}, \boldsymbol{\nu} \in \mathcal{I}} \left\{ \mathbf{c}^\top \mathbf{z} + \min_{\boldsymbol{\eta}, \boldsymbol{\varphi}} \left\{ \mathbf{e}^\top \boldsymbol{\eta} + \sum_{t \in \mathcal{T}} \varphi^t \mid s.t. : (34c), (34d) \right\} \right\} \\ &\leq \mathbf{c}^\top \mathbf{z}' + \min_{\boldsymbol{\eta}, \boldsymbol{\varphi}} \left\{ \mathbf{e}^\top \boldsymbol{\eta} + \sum_{t \in \mathcal{T}} \varphi^t \mid s.t. : (34c), (34d) \right\} \\ &\leq \mathbf{c}^\top \mathbf{z}' + q(\boldsymbol{\nu}') = \rho^*, \end{aligned}$$

where  $\mathcal{I}$  denotes the feasible set for the APMII problem.

(ii) Next, we claim that  $\rho_k^* \leq \rho(\mathbf{z}^*, \boldsymbol{\nu}^*) \leq \rho_k^*(1 + \epsilon)$ . The first inequality holds because  $\rho_k^* \leq \rho^*$  and  $\rho^* \leq \rho(\mathbf{z}^*, \boldsymbol{\nu}^*)$ .

If the condition in Algorithm 1, line 12, does not hold, i.e. if  $\sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}_k) \leq (1 + \epsilon^b) \cdot \sum_{t \in \mathcal{T}} \phi_k^t$ , then:

$$\rho_k^* \leq \mathbf{c}^\top \mathbf{z}_k + \mathbf{e}^\top \boldsymbol{\eta}_k + \sum_{t \in \mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}_k) + \delta^h = \rho(\mathbf{z}^*, \boldsymbol{\nu}^*) \quad (86a)$$

$$\leq (1 + \epsilon^b) \left( \mathbf{c}^\top \mathbf{z}_k + \mathbf{e}^\top \boldsymbol{\eta}_k + \sum_{t \in \mathcal{T}} \phi_k^t + \delta^h \right), \quad (86b)$$

where (86b) =  $(1 + \epsilon^b)(\rho_k^* + \delta^h)$ . Since  $\rho_k^* + \delta^h \leq \rho_k^*(1 + \epsilon^c)$ :

$$\rho_k^* \leq (86b) \leq (1 + \epsilon^b)(1 + \epsilon^c) \rho_k^* = (1 + \epsilon) \rho_k^*.$$

Using (i) and (ii), we have  $\rho^* \leq \rho(\mathbf{z}^*, \boldsymbol{\nu}^*) \leq (1 + \epsilon) \rho^*$ . This concludes the proof.

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