Accomplishments

* What are the major goals of the project?

Study several fundamental problems in extremal combinatorics, many of them motivated by problems in theoretical computer science.

* What was accomplished under these goals (you must provide information for at least one of the 4 categories below)?

Major Activities: 1. A Wowzer Type Lower Bound for the Strong Regularity Lemma

The regularity lemma of Szemerédi asserts that one can partition every graph into a bounded number of quasi-random bipartite graphs. In some applications however, one would like to have a strong control on how quasi-random these bipartite graphs are. Alon, Fischer, Krivelevich and Szegedy obtained a powerful variant of the regularity lemma, which allows one to have an arbitrary control on this measure of quasi-randomness. However, their proof only guaranteed to produce a partition where the number of parts is given by the Wowzer function, which is the iterated version of the Tower function. We show here that a bound of this type is unavoidable by constructing a graph $H$, with the property that even
if one wants a very mild control on the quasi-randomness of a
regular partition, then the number of parts in any such partition of
H must be given by by a Wowzer-type function.

2. Color-Critical Graphs Have Logarithmic Circumference

A graph G is k-critical if every proper subgraph of G is
(k-1)-colorable, but the graph G itself is not. We prove that every
k-critical graph on n vertices has a cycle of length at least
log(n)/100log(k), improving a bound of Alon, Krivelevich and
Seymour from 2000. Examples of Gallai from 1963 show that the
bound cannot be improved to exceed 2(k-1)log(n)/log(k-2). We
thus settle the problem of bounding the minimal circumference of
k-critical graphs, raised by Dirac in 1952 and Kelly and Kelly in
1954.

3. The Quasi-Randomness of Hypergraph Cut Properties

Let a_1,...,a_k satisfy a_1+...+a_k=1 and suppose a k-uniform
hypergraph on n vertices satisfies the following property; in any
partition of its vertices into k sets A_1, ..., A_k of sizes a_1*n, ..., a_k*n,
the number of edges intersecting A_1, ..., A_k is
(asymptotically) the number one would expect to find in a random
k-uniform hypergraph. Can we then infer that H is quasi-random?
We show that the answer is negative if and only if a_1 =...= a_k =
1/k. This resolves an open problem raised in 1991 by Chung and
Graham [J. AMS '91]. While hypergraphs satisfying the property
corresponding to a_1 =...= a_k = 1/k are not necessarily quasi-
random, we manage to find a characterization of the hypergraphs
satisfying this property. Somewhat surprisingly, it turns out that
(essentially) there is a unique non quasi-random hypergraph
satisfying this property. The proofs combine probabilistic and
algebraic arguments with results from the theory of association
schemes.

Key outcomes or
Other
achievements:

* What opportunities for training and professional development has the project provided?

PI Shapira was the PhD advisor of Subrahmanyam Kalyanasundaram who completed his PhD in 2011. The project also funded travel to many conferences where the PI gave lectures and interacted with other researchers.

* How have the results been disseminated to communities of interest?
Several project results have appeared in general mathematical journals that is, journals that publish results in all areas of mathematics. Lectures in many conferences and workshops were also given.

Products

Journals

Status = PUBLISHED; Acknowledgment of Federal Support = No; Peer Reviewed = Yes


Status = PUBLISHED; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


Status = PUBLISHED; Acknowledgment of Federal Support = No; Peer Reviewed = Yes


Status = PUBLISHED; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes

I. Benjamini, O. Schramm, A. Shapira (4/10/10). Every Minor-Closed Property of Sparse Graphs is Testable. *Advances in Math*. 223 (6), 2200-2218.

Status = PUBLISHED; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


Status = PUBLISHED; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


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Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes


Status = PUBLISHED; Acknowledgment of Federal Support = Yes ; Peer Reviewed = Yes

S. Kalyanasundaram, A. Shapira (7/1/14). A Note on Even Cycles and Quasi-Random Tournaments. *Journal of Graph Theory*. TBD TBD.
Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


Status = AWAITING_PUBLICATION; Acknowledgment of Federal Support = Yes; Peer Reviewed = Yes


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**Books**

**Book Chapters**

**Thesis/Dissertations**

**Conference Papers and Presentations**

**Other Publications**

**Technologies or Techniques**
Nothing to report.

**Patents**
Nothing to report.

**Inventions**
Nothing to report.

**Licenses**
Nothing to report.

**Websites**
Nothing to report.

**Other Products**
Nothing to report.

**Participants**

Research Experience for Undergraduates (REU) funding

What individuals have worked on the project?

<table>
<thead>
<tr>
<th>Name</th>
<th>Most Senior Project Role</th>
<th>Nearest Person Month Worked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asaf Shapira</td>
<td>PD/PI</td>
<td>2</td>
</tr>
<tr>
<td>Prasad Tetali</td>
<td>PD/PI</td>
<td>1</td>
</tr>
</tbody>
</table>

What other organizations have been involved as partners?  
Nothing to report.

Have other collaborators or contacts been involved?  
N

**Impacts**

What is the impact on the development of the principal discipline(s) of the project?

Some important open problems were solved during the three years of the project. Some of them have been open for more than 20 years. A number of studies were initiated that have the potential of having many follow up papers.

What is the impact on other disciplines?  
Nothing to report.

What is the impact on the development of human resources?

Under PI Tetali’s direction, graduate students Ioannis Panageas and Arindam Khan were afforded the opportunity to attend conferences that included the Abel Conference at the IMA in honor of Endre Szemeredi and the ACM-SIAM annual Symposium on Discrete Algorithms.

What is the impact on physical resources that form infrastructure?  
Nothing to report.

What is the impact on institutional resources that form infrastructure?  
Nothing to report.

What is the impact on information resources that form infrastructure?  
Nothing to report.

What is the impact on technology transfer?  
Nothing to report.
What is the impact on society beyond science and technology?
Nothing to report.

Changes

Changes in approach and reason for change
Nothing to report.

Actual or Anticipated problems or delays and actions or plans to resolve them
Nothing to report.

Changes that have a significant impact on expenditures
Nothing to report.

Significant changes in use or care of human subjects
Nothing to report.

Significant changes in use or care of vertebrate animals
Nothing to report.

Significant changes in use or care of biohazards
Nothing to report.