PERFORMANCE CHARACTERIZATION OF TURBOSHAFT ENGINES: WORK POTENTIAL AND SECOND-LAW ANALYSIS

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ABSTRACT

This paper develops and describes work potential analysis methods applicable to turboshaft engine flow-fields. These methods are based on the second-law of thermodynamics and enable a unified, comprehensive assessment of performance at the part, component, and engine levels. The focus herein is on using gas specific power as a work potential figure of merit in analyzing turboshaft engines. This is shown to be a useful tool for assessing local performance potential in a gas turbine flow-field. The fundamental relationships between heat, work, and irreversibility in turboshaft engines are developed and the relationship of flow irreversibility to engine performance losses is discussed. These theoretical ideas are then formulated in a method that enables characterization of performance losses in terms of engine spatial location and loss mechanism. This method is then demonstrated at the engine system level via cycle analysis, at the component level via quasi 1-D analysis, and at the part level via a stator row multi-dimensional CFD simulation analyzed in terms of irreversible entropy production.

NOMENCLATURE

ρ	fluid density
T	fluid temperature
P	fluid pressure
R	gas constant
W,w	work interaction, flow-specific work
Q,q	heat interaction
ghp	gas (specific) power
S,s	entropy, flow-specific entropy
U	velocity
M	Mach
H,h	enthalpy, flow-specific enthalpy
C_P	specific heat
γ	ratio of specific heats
A	cross sectional area
c	circumference
δw	differential work interaction
δq	differential heat interaction
τ_w	wall shear stress
η	second law effectiveness (work term)
J	η for compressors, $1/\eta$ for turbines
ΔW_{comp}	compressor-turbine shaft work per unit mass
ΔW_{shaft}	power turbine shaft work per unit mass
ΔQ	heat interaction per unit mass
ΔS_{irr}	entropy generation per unit mass

subscripts:

i	engine inlet station
e	engine exit station
4	maximum temperature (combustor exit)
t	total condition

INTRODUCTION

The use of work potential methods in the design and optimization of aerospace gas turbine engines has recently received increased attention in the aerospace propulsion community [1, 2, 3, 4]. This is because full understanding and optimization gas turbine engine performance inherently requires characterization and quantification second-law effects on engine performance and operability [5]. Furthermore, in order to assess engine performance and losses in a truly meaningful way, one must evaluate and measure the actual performance losses relative to the performance of the best possible (ideal) engine of the same type [6]. Hence, the performance and performance losses of a turboshaft engine are most usefully assessed against the ideal turboshaft engine performance (in terms of the fundamental criteria of delivered shaft power). Similarly, the performance of a turbojet should be assessed against the ideal turbojet engine (in terms of thrust power produced). The objective of this paper is to show that second law analysis (or work potential analysis), *when applied within these critical constraints*, is a consistent and highly useful assessment tool for turboshaft engine analysis.

The concept of work potential has application at all levels of propulsion system analysis, ranging from the

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simplistic flow station representations used for cycle analysis to highly detailed flowfield simulations common in component detail design. This is illustrated in Figure 1 which shows the various levels of analysis detail commonly used in propulsion system design. In this case, the compressor is used as an example to illustrate how an actual system is typically modeled.

The first level idealizes the compressor as two discrete flow stations wherein the average properties at the compressor entrance and exit are of interest while the compressor itself is modeled as a "black box." Work potential methods can be used in conjunction with cycle analysis to estimate total loss inside the compressor and compare this to losses in other components such that the performance of the whole system can be optimized.

The second level is a quasi-1-D flow analysis across the compressor. In this analysis, the flow is analyzed in terms of its average properties at each axial cross-section. Work potential analysis at this level can yield additional information as to the differential losses occurring at each stage in the compressor and is useful for balancing loads across stages to minimize total component loss, etc.

The third and most detailed level is a full simulation of the thermodynamic processes undergone by each fluid element passing through the compressor. The fluid elements starting at station "a" each have a unique thermodynamic state and collectively form a cloud of points whose centroid is equivalent to the average station properties. Each fluid element undergoes a unique thermodynamic process, some passing near the mid-span of the compressor blades and experiencing little loss, while others pass near the blade tip or are entrained in the boundary layer, leading to significant loss. Work potential methods applied to CFD solutions help identify and eliminate those fluid elements that contribute the most to loss. One can then backtrack to specific flow interactions inside the compressor that led to the loss.

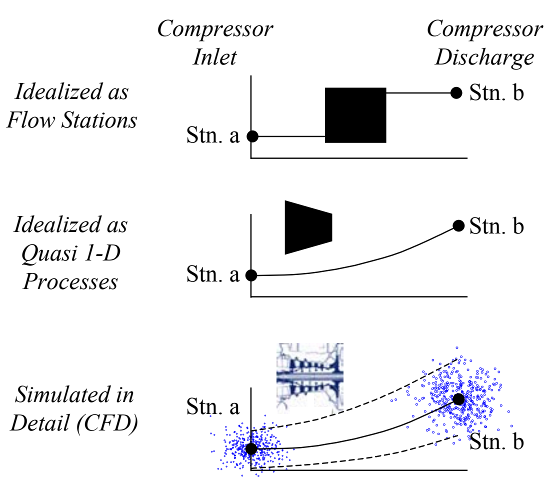

Fig. 1: Application of work potential methods at various levels of analysis fidelity.

This paper describes and demonstrates the application of work potential methods at all three levels of analysis detail. The first part defines and discusses the theoretical underpinnings of thermodynamic work potential (with emphasis on gas specific power, or simply gas horsepower) from the standpoint of turboshaft engine cycle analysis. This is shown to enable rapid and comprehensive engineering assessment of losses in all engine components by providing a common "currency" to measure loss that applies to any component in a turboshaft engine [7].

The second part of this paper focuses on development and application of methods explicitly relating engine shaft power and shaft power losses to flow irreversibility. This quasi 1-D analysis enables the identification and quantification of losses in terms of engine spatial location as well as in terms of specific loss mechanism. It is more detailed and comprehensive than the cycle analysis technique, as it directly accounts for downstream interactions and their effects on engine performance within a given engine flowfield. Finally, a second law analysis of a representative three-dimensional stator flow field is discussed.

THERMODYNAMIC WORK POTENTIAL DEFINED

The concept of thermodynamic work potential is quite old but has taken a surprisingly long time to penetrate into mainstream thermodynamic knowledge. It is rooted in the second law of thermodynamics and the concept of entropy. Various work potential figures of merit (FoMs) have been developed over the years, the most general and best-known of these being exergy [8, 9, 10]. However, if an aircraft engine is optimized to produce maximum exergy output for a given fuel input, the result is usually a sub-optimal engine design, as noted by Riggins [7]. Consequently, exergy analysis is somewhat esoteric if one is *a priori* constrained to use a particular cycle, notably the Brayton cycle.

A more useful work potential FoM for analysis of turboshaft engines is gas horsepower. Gas horsepower (GHP) is defined as the shaft work that would be obtained in expanding a gas from a prescribed temperature and pressure to a reference (usually ambient) pressure in an imaginary power turbine. It is often used to measure theoretical power output of gas generators. Expressed mathematically:

$$ghp \equiv h(T_i, P_i) - h(P = P_{ref}, s = s_i) \qquad (2)$$

where subscript 'i' denotes the thermodynamic state of the gas at the initial condition. Gas horsepower is actually a special case of exergy in which only pressure equilibrium with the environment is enforced [11]. Therefore, GHP is a thermodynamic property (just like temperature, pressure, enthalpy, etc.) describing the maximum work that could be extracted from a substance in taking it from a prescribed initial state at constant velocity into pressure equilibrium with the environment.

The GHP loss inside any arbitrary system can be calculated by summing the GHP fluxes into and out of the system. The difference between the fluxes in and out is equal to the sum of the power output and the GHP loss rate:

$$g\dot{h}p_{in} - g\dot{h}p_{out} = \dot{w}_{out} + g\dot{h}p_{loss} \,. \qquad (3)$$

Note that when heat is added in the combustor, a component of this heat energy becomes available as gas work potential. In this case, Eq. 3 must necessarily include another term. This concept provides a consistent and comprehensive framework by which to measure the performance of turboshaft engine components and systems.

TURBOSHAFT CYCLE ANALYSIS INCORPORATING WORK POTENTIAL METHODS

The first application of work potential analysis methods will consider a typical turboshaft cycle analysis for an engine of roughly 2,000 shp output and having low and high-pressure compressors coupled through a single shaft to a two-stage gas generator turbine followed by a two-stage power turbine. A simplified method for calculating GHP is presented and then used to calculate GHP at every engine flow station. This information is then used to calculate GHP loss in each component.

Method

Gas horsepower analysis of turboshaft engines is a relatively simple exercise in repeated application of Eqs. 2 and 3 to all stations and components in a turboshaft engine. The first step in the process is to apply Eq. 1 to calculate GHP at every flow station in the engine. Since gas horsepower is a thermodynamic property made up of other properties, one need only have a comprehensive tabulation of the properties for fuel-air mixtures at various temperatures and pressures. Any number of thermodynamic properties software packages can be used to do this. The most convenient presentation of the data for use in engine analysis is to plot contours of mass-specific gas horsepower as a function of temperature, pressure, and fuel-air ratio. Fig. 2 and Fig. 3 are two such contour plots showing gas horsepower per pound-mass of fluid, expressed in horsepower per lbm/s of fluid flow. Fig. 2 shows GHP for pure air over a range of temperature and pressure, while Fig. 3 shows GHP for stoichiometric mixtures of fuel and air. Thus, if temperature, pressure, flow rate, and fuel-air ratio are known at every flow station (as from cycle analysis), then GHP can also be calculated at every flow station by simply interpolating from these contour plots. A much more comprehensive set of figures and tables suitable for engine analysis is available in Ref. [12].

The second step is to calculate the total loss in each component. Once GHP at every flow station is known, it is fairly simple to do this using Eq. 2. Note that this requires detailed knowledge of the cycle model, cooling flow circuits, power take-off, and so on. The following discussion will illustrate this idea in detail.

Typical Results

As an illustrative example of how these ideas can be applied to practical analysis of a turboshaft engine, consider the example mentioned in the previous section. The actual engine is typically idealized as a collection of flow stations, components, and flow circuits, as shown in Figure 4. This cycle model can be used to calculate temperature, pressure, fuel-air ratio, and flow rate at every station in the engine, as well as total fuel consumption, shaft power output, etc. This cycle analysis model also contains various loss mechanisms, as shown in Figure 4. These typically are due to component losses (modeled as "efficiencies"), pressure drops, leakage, cooling losses, and mechanical friction. The flow station information produced by the cycle model can be used in conjunction with the method described in the previous section to develop a comprehensive picture of all losses in the engine at any flight condition and power setting.

To illustrate, consider the performance of a typical turboshaft engine operating at sea-level static (SLS) conditions on a standard (58 ^{o}F) day at full throttle. The data from the cycle deck can be used to directly calculate the GHP at each station in the engine and thereby the GHP loss inside each component. Consider the low-pressure compressor as an example. The inlet is station 2 and the discharge is station 25. The flow rate at station 2 is 10.15

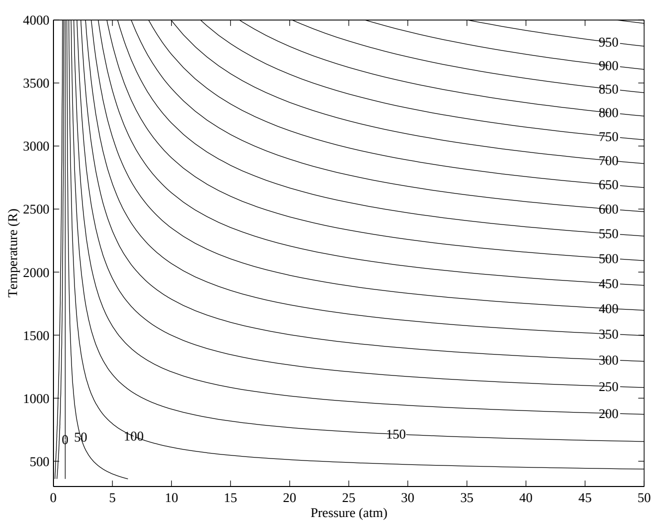

Fig. 2: Contours of mass-specific gas horsepower (HP/pps) for air.

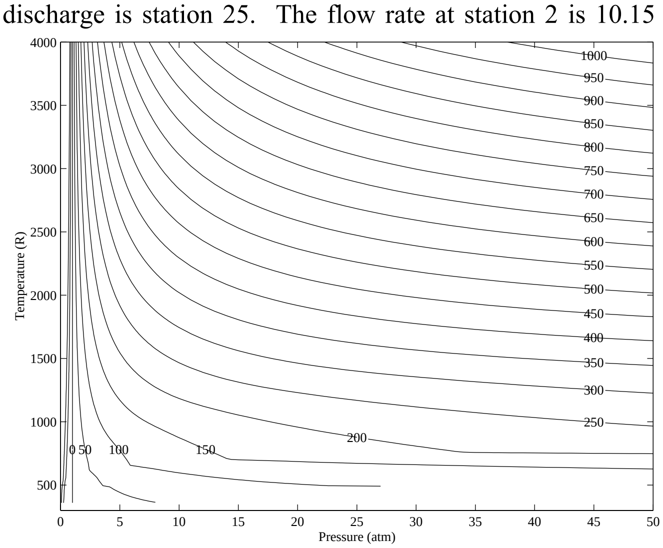

Fig. 3: Contours of mass-specific gas horsepower (HP/pps) for stoichiometric mixtures of Jet-A and air.

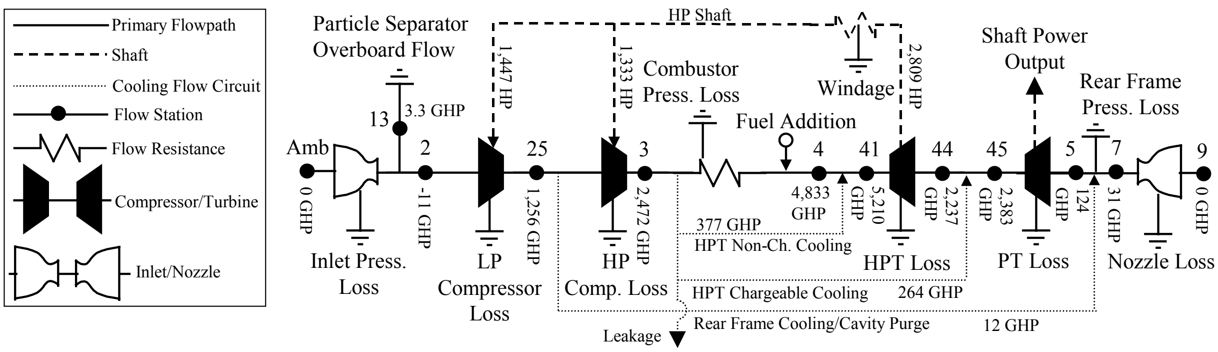

Fig. 4: Cycle model schematic with annotation of major loss mechanisms and estimates on GHP at each flow station for SLS full power operating conditions.

lbm/s at 14.3 psia and 519 ^{o}R. Fig. 2 shows that these conditions correspond to a flow-specific GHP at station 2 of –1.07 HP/pps, yielding a total GHP at station 2 of –11 HP. The flow is discharged to station 25 at 84.8 psia and 937 ^{o}R. The flow-specific GHP is found from Fig. 2 as 125 HP/pps. At a flow rate of 10.05 pps, this yields a GHP of 1,256 HP. The shaft power into the low-pressure compressor is calculated by the cycle deck to be 1,447 HP. Therefore, the low pressure compressor GHP loss is the difference between GHP flux in and out of the compressor: 1,447 HP + (-11 HP) – 1,256 HP = 180 HP loss. GHP loss in other components can be calculated in like manner, shown in Table 1.

These results illustrate in an intuitive way the relative magnitudes of component losses. The largest GHP losses clearly occur in the turbomachinery, though rear frame pressure losses are high, also. The theoretical GHP addition in the combustor is roughly 3,036 HP, with the net average engine shaft power delivered being 2,020 HP. The difference is destroyed through various loss mechanisms in the engine. Finally, note that if this procedure were used to calculate component loss at every operating condition, the result would be a comprehensive description of engine performance known as a "loss deck" [13].

Table 1: GHP losses in a typical turboshaft engine at SLS standard day conditions.

Loss Mechanism	*Loss (HP)*	*% Total*
Inlet Pressure Loss	14	1.3
Particle Separator Ovrbrd Flow	3.3	0.3
LP Compressor Loss	180	17.7
HP Compressor Loss	105	10.3
Combustor Pressure Loss	24	2.3
HPT Loss	164	16.1
LPT Loss	239	23.5
HPT Chargeable Cooling	118	11.6
Rear Frame Pressure Loss	105	10.3
Nozzle Loss	31	3.0
Leakage	3.7	0.3
Shaft Windage	29	2.9
Total Loss	1,016	100
Net Shaft Power Output	2,020	
GHP Addition in Combustor	+3,036	

TURBOSHAFT ENGINE QUASI 1-D ANALYSIS

This section focuses on the development and application of methodology and techniques for explicitly relating incremental engine shaft power and shaft power losses to flow irreversibility. This methodology enables the identification and quantification of losses in terms of engine spatial location as well as in terms of specific loss mechanism. The following discussion first develops the rationale and methodology relating flow irreversibilities to losses in shaft power. This method is then demonstrated for a simplified turboshaft flow-field by utilizing quasi 1-D flow analysis coupled with second law concepts.

This analysis is based on application of the steady-flow quasi-one-dimensional flow equations. These equations allow flux-consistent flow simulations from which entropy distributions and audits can be easily extracted for use in the analysis. The quasi 1-D compressible flow equations in differential form are:

$$\frac{d\rho}{\rho}+\frac{du}{u}+\frac{dA}{A}=0 \qquad (4)$$

$$\frac{dP}{P}+udu=\frac{-\tau_w c(dx)}{\rho A}+J\cdot\delta w \qquad (5)$$

$$C_p dT+udu=\delta w+\delta q \qquad (6)$$

$$\frac{dP}{P}=\frac{d\rho}{\rho}+\frac{dT}{T} \qquad (7)$$

where for the compressor: $J = \eta$, and for the turbine: $J = 1/\eta$. Here δq and δw denote the differential heat and shaft work per mass added to a differential element across dx.

Note that the shaft work term in the momentum equation and the associated second-law effectiveness, η, represent a body force model of work interaction [14]. The departure of η from unity therefore represents irreversibilities associated with the transfer of energy to the flow from an external shaft. These irreversibilities could be properly subdivided into frictional and thermal losses if enough information is provided (i.e. from multi-dimensional CFD). However from the standpoint of the quasi 1-D analysis presented in this part of the study, these irreversibilities are considered (and book-kept) in a separate lumped category associated with all work interaction losses.

Lost Shaft Work and Irreversibilities

In order to clarify the relationship between shaft work losses and flow irreversibilities, first consider an axial flow compressor stage in a turboshaft engine. A streamtube processed by that stage has total pressure increased by the introduction of swirl kinetic energy associated with the angular motion of the rotor blades. From the standpoint of the axial bulk flow through the stage, a portion of this kinetic energy is realized as pressure work within the overall stage, hence raising total pressure. This is also manifested in the momentum equation (Eq. 5) by an increase in stream thrust (defined in [15] as ρu^2A+PA) through the stage. In an ideal compressor, the fluid would experience the maximum possible increase in stream thrust (or total pressure) for a given amount of external work interaction to the flow. In a non-ideal compressor (one in which irreversibilities occur) with the same amount of delivered shaft work across the boundary, the fluid would experience an increase in streamthrust (or total pressure) due to work interaction *less* than that experienced by the ideal compressor. This decrease in net stream thrust (or energy received by the flow as pressure work) is due to flow irreversibilities that convert energy initially delivered from the shaft as work to energy received internally in the fluid as heat.* This means that *irreversibilities occurring in the engine are intimately related to work potential and inevitably act to decrease the potential shaft work output available from the power turbine*.

One can establish a direct linkage between flow irreversibilities and lost shaft work by examining the entropy increase due to irreversibilities in a given engine. If the entropy increase is compared to the difference between delivered power turbine shaft work and ideal shaft work for a given engine (presuming the same external energy interactions in the gas turbine core and with the same engine geometry), one will find that they are proportional. Thus, if quasi 1-D analysis is used to estimate entropy change in each differential element, this can be directly converted into a loss in work potential. This enables one to book-keep all losses in a turboshaft engine both in terms of loss mechanism (mass, momentum, or energy transfer, non-equilibrium kinetics, shocks, etc.) and in terms of engine location.

As an example, consider a given turboshaft engine flow-field with fixed exit area and given exit static pressure. Presume that a detailed differential description of entropy increases associated with flow irreversibilities is known throughout this flow-field via quasi 1-D analysis. The second-law analysis begins with the flow-field for the ideal engine (flow-path with no irreversibilities). All external shaft work interactions between compressor and HP turbine as well as burner heat interaction are equal to those that occur in the actual engine. Engine exit area and exit static pressure (P_e) are also the same between ideal and actual engines. This latter requirement, when coupled with reversible flow processes, results in additional shaft work obtained from the power turbine for the ideal engine as discussed above.

To assess the loss distribution in the engine, begin by starting at the engine exit plane of the actual engine. Let us remove the irreversiblity associated with the first upstream differential step of the *actual* engine flow-field. The resulting power output from the (now slightly more ideal) engine can be re-calculated and compared to the actual engine. The difference represents the loss shaft work due to the differential irreversibility occurring in the flow over the first upstream differential step. This process is repeated successively, removing irreversibilities progressively from the back to the front of the engine until the ideal engine flow-field is obtained (i.e. all irreversibilities associated with the actual engine flow-field have been consistently deleted from the actual engine flow-field).

As the progression is made from the actual engine to the ideal engine, a summation can be made of the lost shaft work due to all losses in each component. In addition, a summation is made of the lost shaft work due to each separate loss mechanism. This methodology can be illustrated using a *T-S* diagram (Fig. 5) on which both ideal and actual engine conditions are shown schematically, including the locus of points describing the intermediate engine flow-field conditions. Note that this technique allows the complete loss analysis of the engine both in terms of loss mechanism and engine location since the differential entropy increase for each mechanism at each step is also known.

This process logically defines a consistent integration path in which a family of successively more reversible

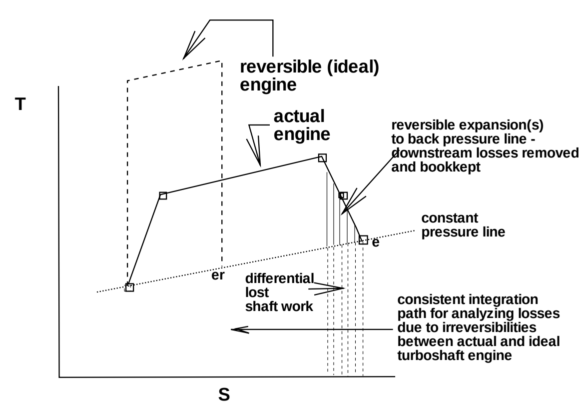

Fig. 5 T-S diagram showing actual and ideal engine cycles with intermediate loss cycles.

* Note that second law considerations also dictate that energy added in the burner as heat cannot be fully extracted as shaft work by the power turbine. This is because the combustion process is typically highly irreversible, with a consequent large loss in work potential. The more irreversible this heat addition process is, the less work can ultimately be produced in the turbine.

turboshaft engine flow-fields is developed, all intermediate between ideal and actual flow-fields. Note that the direction of the loss-buildup process is critical (i.e. the sequential deletion of irreversibilities from downstream to upstream) and is mandated here by the assumption of strong coupling of downstream irreversibilities with upstream irreversibilities. Due to this strong coupling, there is only one thermodynamically consistent integration path for moving from the actual engine to the ideal engine and consistently assigning lost shaft work increments to irreversibilities. However, note that although this method provides a snapshot of the lost shaft power increments associated with irreversibilities for a given engine flow-field, the entropy-shaft work relationship is in fact non-linear and is strongly coupled with any changes in actual fluid dynamics. In other words, removing an irreversibility from the middle of the actual engine (say through a design change) will impact the balance of shaft work losses associated with other irreversibilities occurring elsewhere in the engine. In this sense, the target for optimization (for given gas turbine external interactions) should be maximizing overall power turbine shaft work or equivalently minimizing overall entropy production throughout the engine.

Analytical Relationships Between Shaft Work and Irreversibility in a Turboshaft Engine

This section summarizes the analytical relationship between shaft work delivered by the power turbine and irreversibility. Reference [16] gives details of the equation development. Figure 6 shows a schematic of the turboshaft engine and heat and work nomenclature used in this section.

It can be shown from the second law of thermodynamics and the governing equations of fluid dynamics that the total pressure increase from state i to e is directly related to the differential work interactions occurring between stations 'i' and 'e' and the overall entropy increase due solely to irreversibility:

$$\frac{P_{te}}{P_{ti}} = e^{\int_{i}^{e} \frac{\delta w}{RT_t} - \frac{\Delta S_{irr}}{R}} . \qquad (8)$$

Here δw is the differential external-to-flowpath work interaction per mass (defined positive to the fluid) and ΔS_{irr} is the total irreversibility occurring between 'i' and 'e.'

Utilizing this expression and applying conservation of mass and energy throughout the engine, the following

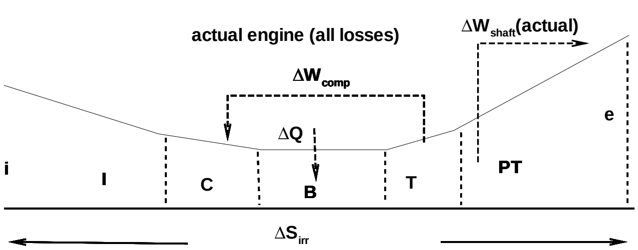

Fig. 6 Turboshaft engine schematic and energy interactions.

equation is obtained which directly relates shaft work obtained in the power turbine to irreversibilities occurring in the engine flowpath:

$$\frac{\Delta W_{shaft}}{C_p T_{ti}} = 1 + \frac{\Delta Q}{C_p T_{ti}} - \frac{\left(1+\frac{\gamma-1}{2}M_e^2\right)}{\left(1+\frac{\gamma-1}{2}M_i^2\right)} \left(\frac{P_e}{P_i}\right)^2 \left(\frac{A_e}{A_i}\right)^2 \left(\frac{M_e}{M_i}\right)^2 \qquad (9)$$

where:

$$\left(\frac{M_e}{M_i}\right)^2 = \left[\frac{\left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}} \left(1+\frac{\Delta W_{comp}}{C_p T_{ti}} + \frac{\Delta Q}{C_p T_{ti}}\right)}{\left(e^{\frac{-\Delta S_{irr}}{R}}\right)^{\frac{\gamma-1}{\gamma}} \left(1+\frac{\Delta W_{comp}}{C_p T_{ti}}\right)}\right] \left(\frac{P_i}{P_e}\right)^2 \left(\frac{A_i}{A_e}\right)^2 \qquad (10)$$

. ΔQ is the overall heat per unit mass added in the burner, and ΔW_{comp} is the shaft work interaction occurring between compressor and HP turbine in the gas turbine core. P_e and A_e are the exit pressure and area of the engine; similarly P_i and A_i are the inlet pressure and area of the engine. It is assumed for simplicity that there are no mechanical losses between compressor and HP turbine. Also thermodynamic properties are assumed to be constant and the mass of fuel is assumed negligible relative to the mass of air. These restrictions can be relaxed without modifying the basic loss analysis technique presented here.

Note the functional dependence of power turbine shaft power implicit in these relationships:

$$\frac{\Delta W_{shaft}}{C_p T_{ti}} = f\left(\frac{P_e}{P_i}, \gamma, \frac{\Delta W_{comp}}{C_p T_{ti}}, \frac{\Delta Q}{C_p T_{ti}}, \frac{\Delta S_{irr}}{R}, M_i, \frac{A_e}{A_i}\right) \quad (11)$$

These expressions relate the total irreversible entropy increase in the engine flow to the shaft work produced by the power turbine. The value of actual shaft work obtained for a given irreversibility from inlet to exit when compared to the ideal shaft work (also obtained utilizing this equation with ΔS_{irr} = 0) yields the lost shaft work due to the total irreversibility in the engine. Equation 9 can also be used to easily facilitate the previously described loss buildup methodology by applying it for each differential irreversible entropy increase in the engine in a manner consistent with the previous discussion, and using the integration path shown in Fig. 5.

A simplified form of the lost shaft work equation can be derived by assuming that inlet and exit pressures are equal and that Mach numbers are small:

$$\frac{\Delta W_{shaft}}{C_p T_{ti}} = 1 + \frac{\Delta Q}{C_p T_{ti}} - \frac{\left(1+\frac{\Delta W_{comp}}{C_p T_{ti}} + \frac{\Delta Q}{C_p T_{ti}}\right)}{\left(\left(e^{\frac{-\Delta S_{irr}}{R}}\right)^{\frac{\gamma-1}{\gamma}}\right)\left(1+\frac{\Delta W_{comp}}{C_p T_{ti}}\right)} . \qquad (12)$$

Note also the simplified functional dependence for the power turbine shaft work:

$$\frac{\Delta W_{shaft}}{C_p T_{ti}} = f\left(\gamma, \frac{\Delta W_{comp}}{C_p T_{ti}}, \frac{\Delta Q}{C_p T_{ti}}, \frac{\Delta S_{irr}}{R}\right). \qquad (13)$$

The fundamental relationships between heat, work, and irreversibility in turboshaft engines are analyzed by setting pressure ratios and area ratios constant in Eqs. 9 and 10 and varying the remaining parameters. As a result, two sets of contours are produced. The first set of contours (Fig. 7) depicts constant shaft work curves for different combinations of compressor/HP turbine work interaction and burner heat addition. Here the irreversible entropy increase through the engine is fixed as indicated. Included on this contour plot are lines that denote combinations of compressor/HP turbine work interaction and burner heat addition that lead to a constant value of $T_{t(max)}$ (max temperature) at the end of the burner. The values for the irreversible entropy increase, area ratio, and inlet Mach number are shown on the figure.

By differentiating the expression relating maximum temperature to work and heat interactions, an equation is obtained which expresses the maximum possible shaft work output as a unique combination of compressor/HP turbine work interaction and burner heat interaction:

$$\frac{\Delta Q}{C_p T_{t_i}} = \frac{\left(1+\frac{\Delta W_{Comp}}{C_p T_{t_i}}\right)^2}{\left(e^{\frac{\Delta S_{irr}}{R}}\right)^{\frac{\gamma-1}{\gamma}}} - \frac{\Delta W_{Comp}}{C_p T_{t_i}} - 1 \quad (14)$$

This equation is based on the following expression for the maximum temperature in the engine:

$$\frac{T_{t(\max)}}{T_{ti}} = 1 + \frac{\Delta W_{comp}}{C_p T_{ti}} + \frac{\Delta Q}{C_p T_{ti}} \quad (15)$$

Essentially, this describes the relationship between cycle pressure ratio and combustor equivalence ratio (for a given maximum allowable turbine inlet temperature) necessary to achieve the maximum shaft power output. Figure 8 shows the curve formed by the collection of points generated by this expression.

The second set of contours that can be obtained from Eq. 9 also depicts lines of constant power turbine shaft work (see Fig. 9). In this case, however, the engine maximum temperature at the end of the burner (T_{t4}) was held constant such that heat addition in the burner is determined by the compressor work interaction. The compressor/HP turbine work interaction and total irreversibility increase through the engine are then varied.

Irreversibility Description for Quasi-One-Dimensional Model of Turboshaft Engine

In order to post-process an engine flow-field and apply the technique described in previous sections for analyzing shaft work losses, it is necessary to develop expressions for the various irreversible entropy increases occurring in the flow-field. The total differential entropy increase can be divided into two contributions—entropy increases due to reversible heat addition and entropy increases due to irreversibilities. Note that for any positive heat interaction to the flow, no matter how irreversible, there is a minimum entropy increase associated with that heat addition had it occurred reversibly. The irreversible entropy associated with heat addition is that increment of entropy generated in addition to this reversible entropy increase. Irreversible entropy increases can be described as resulting from specific loss mechanisms in the flow. The most general description of these loss mechanisms involves losses due to mass diffusion (chemical species gradients), momentum diffusion (friction and velocity gradients), thermal diffusion (heat transfer across temperature gradients), and non-equilibrium chemical kinetics. These mechanisms are most properly viewed as enveloping shocks, although depending on numerical resolution issues and modeling issues, the Euler entropy jump implicit in the governing equations of fluid dynamics may be treated as a 'separate' loss mechanism.

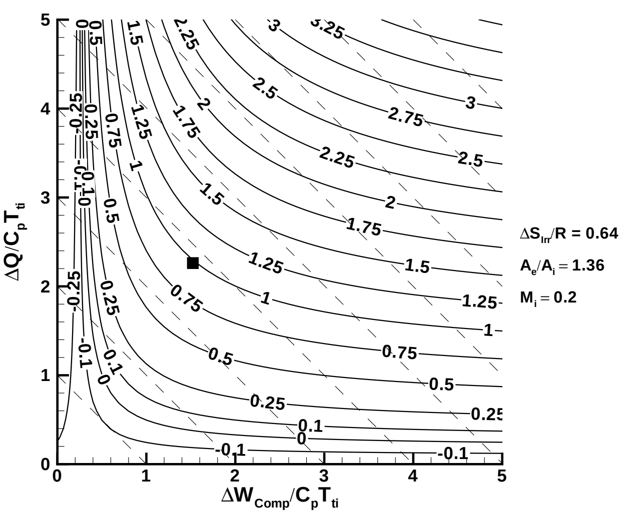

Fig. 7 Burner heat addition and compressor work addition shaft work contours ($\Delta W_{shaft}/C_pT_{ti}$).

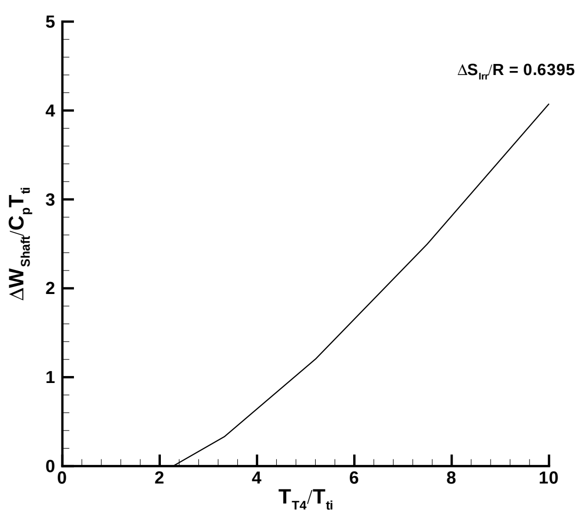

Fig. 8 Maximum shaft work versus burner exit total temperature ratio for a constant irreversibility.

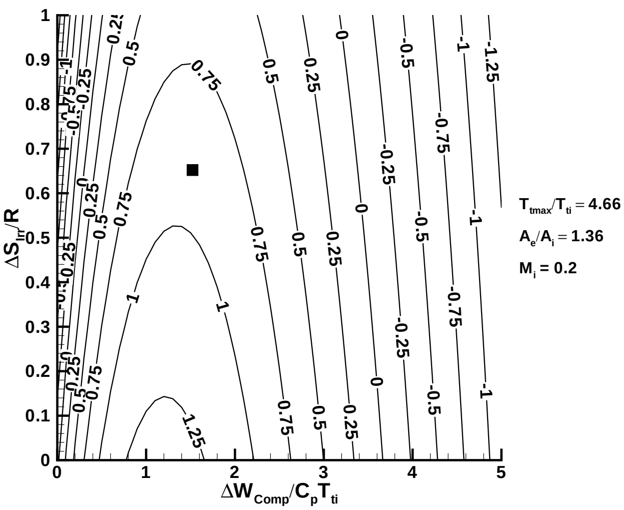

Fig. 9 Irreversible entropy increase and compressor work addition shaft work contours.

The following relationships provide a summary of the three loss mechanisms (irreversibilities) considered in this paper: heat transfer, circumferential friction, and a lumped loss inclusive of the external shaft work interaction with the flow. Note, as previously discussed in other sections, this last loss mechanism is actually composed of friction and heat transfer losses (as well as possible shock losses) which must be provided to the quasi-one-dimensional based analysis used here by higher order simulation methods such as multi-dimensional computational fluid dynamics. A preview of the entropy generation due to specific mechanism in an example three-dimensional stator flow-field will be provided later in this paper. Such information could be rapidly folded into the quasi-one-dimensional analysis.

Heat Transfer entropy is given by:

$$\Delta S_{irr\,HT} = \frac{\delta q}{T} - \frac{\delta q}{T_t} \tag{16}$$

Here T_t is the local total temperature of the flow and T is the local static temperature of the flow. The heat transfer δq is simply specified in the present model, hence allowing a Rayleigh heat addition process to model fuel-air heat release. Circumferential Wall Friction entropy is given by:

$$\Delta S_{irr\,Fr} = \frac{\tau_w c(dx)}{\rho A T} \tag{17}$$

and shaft work interaction irreversibility is given by:

$$\Delta S_{irr\,w} = (1 - J) \cdot \frac{\delta w}{T} \tag{18}$$

Code Description

A simple quasi 1-D code assuming perfect gas and constant-specific heats was developed for this study in order to simulate the flow-field through a generic turboshaft engine. This code is based on the differential quasi-1-D equations described in Eqs. 4-7 and yields a complete axial differential description of the flow-field through the entire engine rather than a component-only (station-wise) description of the flow as provided by cycle analysis. The differential description provides a logical bridge between the analysis/methodology described in this paper and multi-dimensional simulations (which can provide very detailed descriptions of losses). After the flow-field is calculated using this code, all irreversibilities and associated lost shaft work increments are calculated and book-kept in a post-processing routine based directly on the relationships and techniques described in earlier sections. Note that the purpose of this differential analysis (DA) code is intended solely as proof-of-concept and hence is applied to a relatively simple engine flow-field without provision for detailed engine models such as cooling flow circuits or variable specific heats. However, the methodology developed in this work is completely generic and can be applied to any type of flow-field.

Code Results

Table 2 provides a description of the major features of the specific engine geometry and flow conditions modeled in order to demonstrate the technique described in the previous sections. This represents a generic engine flowpath and is not directly comparable to the results obtained earlier using the gas horsepower method, which was applied to a more complex engine cycle model with variable specific heats, cooling flows, etc. Note also that linear distributions of flow-path cross-sectional areas were assumed between given axial stations. Also, linear work and heat interactions with axial distance were assumed within the relevant components.

As discussed above, the DA code determines the complete differential description of the flow-field properties throughout the engine. However, the parameter distribution of interest for loss analysis is irreversible entropy increase through the engine, shown in Fig. 10. The distribution is segmented by component with lines drawn at the inlet/exit of each component. This shows, as expected, that entropy increases due to irreversible heat addition occur in the burner. Likewise, irreversibilities due to work addition/extraction are only present in the compressor, gas turbine, and power turbine. Friction is present in all components. Note that the contribution due to the 'lumped' irreversibilities associated with work addition/extraction is significantly greater than axial friction or heat transfer and therefore represents the greatest opportunity for improvements in performance. The best way to resolve this 'lumped' loss into its constituent components is through detailed loss analysis using CFD simulations.

These results are summarized in Table 3. This clearly shows the components and specific loss mechanisms that contribute the most to the total lost shaft work. In this case, it is again apparent that the inefficiencies associated with the work addition process in the turbines and compressors dominate other losses. Figure 11 presents this data in the form of a pie graph showing the various fractions of shaft work lost due to various irreversibilities.

Table 2 Engine parameters.

Compressor pressure ratio = 17.07
Compressor polytropic efficiency = 0.90
HP turbine polytropic efficiency = 0.90
Power turbine polytropic efficiency = 0.89
Inlet Mach number = 0.2
Inlet static temperature = 27° C
Inlet static pressure = 1 atm
Exit static pressure = 1 atm
Maximum total temperature = 1127° C
Cross-sectional areas
Inlet face = 0.0056 ft^2
Burner entrance = 0.0009 ft^2
HP turbine entrance = 0.0009 ft^2
Power turbine entrance = 0.0027 ft^2
Engine exit = 0.0076 ft^2
Work addition (compressor) = 2917 hp
Heat addition (burner) = 285 BTU/lbm
Shaft power delivered by power turbine = 1836.39 hp
Total irreversible entropy increase/R = .64
Ideal gas properties (constant through engine)
γ = 1.4 (air properties used throughout)

Table 3 Component/Mechanism Lost Work (hp).

	Mechanism			
Component	Total	Friction	Work interaction	Heat
Total	762	40.9	712	10
Inlet	3.5	3.5	0	0
Compressor	366	6.0	360	0
Burner	21.5	11.6	0	10
HP Turbine	192	11	181	0
Power Turbine	180	9.4	171	0

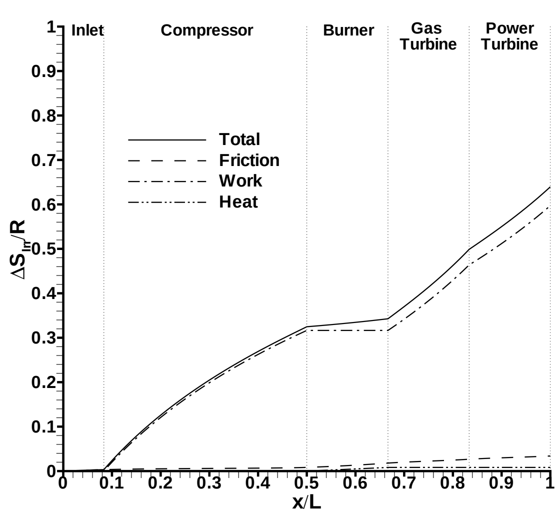

Fig. 10 Mechanism and component irreversible entropy increases.

LOSS ANALYSIS APPLIED TO MULTI-DIMENSIONAL CFD-GENERATED FLOW-FIELDS

The *greatest potential benefit* of applying the methodology described in this paper is to use it in the context of multi-dimensional CFD where spatially detailed and comprehensive losses are calculated with high fidelity. Furthermore, these loss distributions can be appropriately incorporated into the methodology/analysis described in this paper. Specifically, information relevant to the third loss mechanism (the lumped work interaction irreversibility as treated in the earlier quasi-one-dimensional analysis example) can be completely determined.

A preliminary examination of a representative three-dimensional CFD generated flow-field in a gas turbine stator row has been conducted as part of this investigation. The flowfield solution used here is based on NASA's Stator Row 37 test case [17]. Irreversible entropy distributions due to friction and heat transfer have been extracted from the flow-field variables. The 3-D domain analyzed here extends from stator blade leading edge to trailing edge, from hub to outer casing, and from blade suction surface to adjacent blade pressure surface. The grid in the stator region was 49(axial) by 51(transverse) by 41(radial). Note that this effort involved post-processing this particular region of a larger CFD-generated flow-field. It is analyzed in this study in order to demonstrate representative audits of irreversible entropy generation in a sample engine flow-path. This analysis, in fact, follows earlier work described in References [3] and [7] in which similar techniques were applied to multi-dimensional simulations of high-speed (ram/scramjet) flow-fields.

Figure 12 shows pressure contours at a slice (constant radius from hub) of the flow-field across the stator row. Figure 13 provides the relative contributions of friction irreversibility and heat transfer irreversibility to the overall irreversible entropy generation along the axis of the stator row. These irreversibilities are inclusive of all friction and heat transfer between all adjacent streamtubes in this multi-dimensional simulation. As can be seen, friction dominates heat transfer and is responsible for 80% of all losses through the stator row.

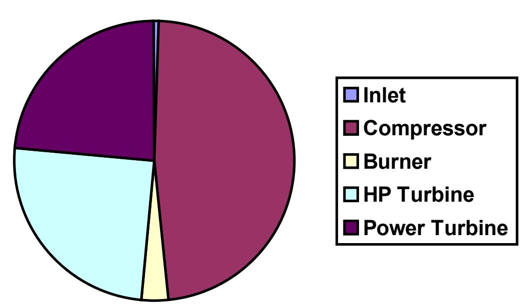

Fig. 11 Percent of lost shaft work in power turbine (sorted by component).

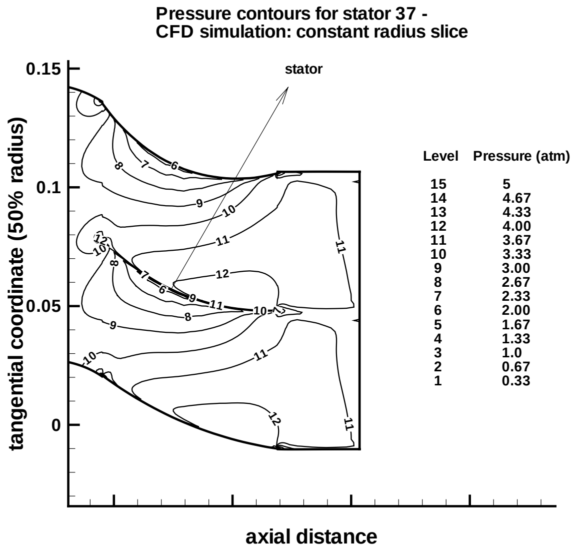

Fig. 12 Pressure contours from stator row 37 (constant radius view).

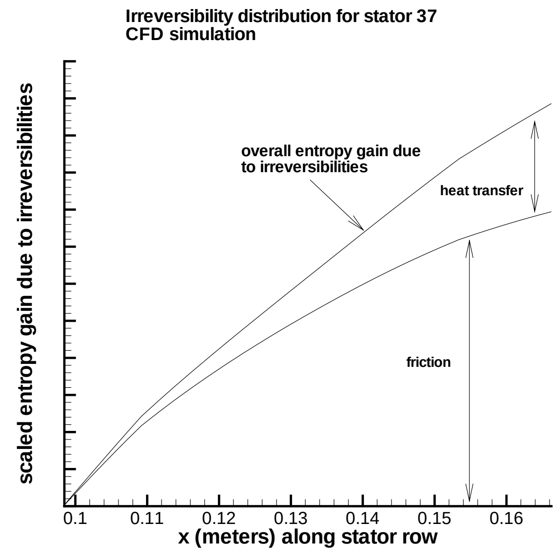

Fig. 13 Entropy distribution due to irreversibilities along stator row 37.

APPLICATION TO VEHICLE SYSTEMS

Though it is not the subject of this paper, the concepts discussed herein have much broader application than just engine analysis. The principles discussed are completely general and are therefore applicable to analysis of the entire vehicle itself [18]. If losses in other parts of the vehicle are known, they can be added to the data in Table 3 to obtain a comprehensive picture of the losses throughout all vehicle systems. For instance, the engine losses could be stacked up against rotor drivetrain losses, main rotor parasite power, induced power, airframe drag power required, tail rotor power required, etc. Furthermore, *the total losses in all vehicle systems are proportional to the total fuel consumption of the vehicle* [19], meaning that thermodynamic losses can be directly related to vehicle mass properties. Figure 11 suggests that the vast majority of fuel consumption due to losses in the engine can be traced to the power turbine. The loss data given in Fig. 11 can be viewed at the vehicle level as being proportional to the engine fuel consumption increments required in order to overcome the various irreversibilities within the engine flowpath.

Extension of work potential (second law) methods beyond engine-only assessment would conceptually produce a truly optimized vehicle for a given mission. For such a vehicle, all vehicle systems and sub-systems are optimized with respect to the fundamental mission criteria subject only to the physical limits of the vehicle itself. Finally, appropriate use of second law methods may assist engine/vehicle designers in making revolutionary advances in technology, allowing them to think ‘outside of the box’, to break away from current design paradigms.

The application of work potential methods to turboshaft engines and helicopter airframes could ultimately enable: 1) the development of rotorcraft vehicles with extended range and endurance, 2) the design of smaller, lighter, and more powerful powerplants, 3) lower initial research, development, and production costs, and 4) lower life-cycle costs for rotorcraft vehicles. It can provide direction in planning and quantify necessary resource allocations regarding design changes that may be mandated by technology development. It also enables the full utilization of numerical simulations in terms of diagnostics and flow-field evaluation [7, 14] for both external and internal flows.

SUMMARY

The use of second law methods in the design and evaluation of turboshaft engines can produce significant benefits in terms of correctly evaluating design features and flow losses, generating detailed loss audits in terms of flow loss mechanisms and engine location, and ultimately relating fuel usage to irreversibilities. This paper described the basis and use of the concept of work potential in the form of gas specific horsepower for assessing the local state of the flow in the context of shaft work potential. Such a parameter can be configured in engineering charts that provide designers/analysts with relevant information concerning performance gains and losses. This information is highly useful because it is a figure of merit based on a single ‘universal’ currency that is valid across all components in the engine. In addition, methodology is developed which directly relates lost shaft work to flow irreversibilities for assessing actual engine flow-fields. This provides a means for quantifying performance losses in a given engine.

Lastly, irreversible entropy production from a multi-dimensional CFD simulation of flow through a stator row is analyzed in order to determine the relative contributions of internal friction and heat transfer to the overall loss. Very little second law information is currently utilized from large-scale CFD simulations and related post-processing efforts.

However, it can be argued that second-law information embedded in existing and future high-fidelity simulations is actually the most valuable information available from such simulations and may one day provide the critical tool needed to enable continued improvements in turbomachine performance.

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