A COMPARISON OF THERMODYNAMIC LOSS MODELS APPLIED TO THE J-79 TURBOJET ENGINE

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Abstract

The objective of this paper is to clarify the relationships between various work loss figures of merit that have been proposed for application to gas turbine propulsion. The intent is to provide a working comparison by way of a pedagogical example using the J-79 turbojet engine. The results of this analysis are then used to draw inferences as to what applications each work potential figure of merit is best suited. Finally, a general "work exclusion principal" is suggested as a guide to which of the various loss figures of merit is most appropriate for a given application.

Nomenclature

Note: lower case letters denote mass-specific quantities
A = Cross-Sectional Area (ft^2)
Ae = Available Energy (BTU)
C_{FG} = Nozzle Thrust Coefficient
c_p = Constant Pressure Specific Heat (0.24 BTU/lbm-R)
Ex = Exergy (BTU)
F_{Net} = Net Thrust (lbf)
g = Gravitational Acceleration, 32.17 ft/sec-sec
H = Enthalpy (BTU)
I = Impulse Function (lbf)
J = Work Equivalent of Heat, 778 ft-lb/BTU
M = Gas Mach Number
$\dot{m}$ = Mass Flow Rate (lbm/s)
P = Pressure (atm)
R = Gas Constant (0.069 BTU/lbm-R)
Sa = Stream Thrust (lbf/lbm)
T = Temperature (R)
V = Gas Velocity (ft/s)
W_P = Thrust Work Potential (HP or BTU/s)
$\dot{W}_{out}$ = Power Output (HP)
$\Delta P/P$ = Combustor Pressure Drop (%)
γ = Ratio of Specific Heats (1.4)
η = Efficiency
ρ = Gas Density ($slug/ft^3$)

Subscripts
amb = Ambient Conditions
exp = Isentropically Expanded to Ambient Pressure
Th = Thermal Cycle
C = Compressor
T = Turbine

Engine Station Designations (Per SAE ARP755B)
2 Engine Front Face (assumed atmospheric)
3 Compressor Discharge
4a Turbine Inlet (sans heat addn.)
4 Turbine Inlet
5 Turbine Exit
9 Nozzle Exit Plane

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Introduction

The past several decades have witnessed considerable research and development of methods for estimation of loss in work potential for thermodynamic systems. These methods are based on the combined first and second laws of thermodynamics, and have proven to be very powerful analysis tools for estimation of losses for various applications, particularly in cryogenics. As pointed out by Bejan,[1] the applications in which combined first and second law methods have their strongest potential are those wherein thermodynamic losses (and minimization thereof) play a pivotal role in determining the design of the system. One such example of this is vehicle propulsion, particularly aircraft jet propulsion. In fact, from a thermodynamic standpoint, *an aircraft in cruising flight produces nothing but loss.* The crux of the aircraft cruise optimization problem is optimal partitioning of these losses between engine internal losses and drag work (loss) such that the total loss is minimized. The potential for improved propulsion system designs based on methods using the combined first and second law of thermodynamics is the underlying motivation prompting this investigation into methods for estimating loss in work potential.

These methods have the long-term potential to change the way propulsion systems are analyzed and designed. Their chief merit is that they provide powerful insight as to the true thermodynamic cost of each loss source. The first law of thermodynamics is misleading in this regard because it can only measure the *quantity* of energy, not the *quality* (work-producing potential). The second strength of this approach is that it puts all losses on an equal (directly comparable) footing. That is to say that all losses are quantified in terms of a loss in work potential, which is a thermodynamic property of the working fluid and not a function of the machine or component itself. This is a powerful tool to supplement the typical approach whereby component efficiencies are perturbed to obtain sensitivities for specific fuel consumption, thrust, etc.

Combined first and second law methods have received little attention in the aerospace propulsion community beyond the publication of a few key papers, such as references 2, 3, and 4. This can be attributed to at least two reasons. The first is lack of widespread understanding within the propulsion community as to the theory and application of these methods. The second relates to the special circumstances surrounding the design of jet propulsion systems.

One of these special circumstances is that propulsion system mass carries an implied loss that is on the same order of magnitude as the thermodynamic losses internal to the engine. Consequently, *the optimal solution must be a balance between internal performance and weight (mass).* A second special circumstance is that much of the work potential in the fuel is *inherently unavailable* when used for jet propulsion, which calls to question how one should allocate chargeability for these losses, as will be discussed later in this paper.

The purpose of this paper is to demonstrate the application of several thermodynamic loss figures of merit (these being exergy, available energy, stream thrust, and thrust work potential) and evaluate their usefulness for jet propulsion design and analysis. This is done primarily through the use of a simple pedagogical example modeled on the General Electric J-79 turbojet engine. This example has been intentionally simplified so that the reader can easily verify the results using simple hand calculations and thereby reinforce understanding of the material presented here. This paper is closely related to a second paper that focuses on the theory and definitions of the above four loss figures of merit.[5,6] The authors assume some familiarity on the part of the reader with combined law methods of system analysis. Lacking this, the interested reader is referred to the particularly lucid presentation given in reference 7.

Simplified J-79 Analytical Model

As previously mentioned, the application used as a basis for comparison is a simplified turbojet engine cycle model based loosely on the J-79.[‡] The assumptions used in this analysis are enumerated in Table I, and a schematic of the assumed engine configuration and station nomenclature is shown in Figure 1.[§] Two operating conditions are considered here for analysis: sea level static military power and Mach 0.9, 20,000 ft military power. These two operating conditions were chosen because they are nearly identical in that the engine front face experiences the same temperature and nearly the same static pressure for both cases. Thus, if Reynolds number and nozzle pressure ratio effects are ignored, the component efficiencies are the same for both conditions.

‡ The analysis given herein treats the J-79 as a pedagogical example on which to apply loss methods, and makes no mention of the historical significance of this engine and the tremendously talented people who created it. The interested reader is urged to examine references 8, 9, and 10 for a detailed account of the fascinating history surrounding this machine.

§ Station 4a is an imaginary station immediately before heat addition but after the combustor pressure drop, inserted to artificially separate the combustor heat addition and pressure drop processes.

The model used here considers only four sources of component loss: compressor efficiency, turbine efficiency, nozzle thrust coefficient, and combustor pressure drop. The presence of secondary cooling flow circuits, mechanical losses, leakage, customer power take-off, customer bleed, etc. is ignored as these would merely distract from the objective. It is relatively straightforward to incorporate these effects into the cycle model if a detailed loss analysis is desired.

Table I: Assumptions Used for Analysis of Simplified J-79 Cycle Model.

Engine Cycle	Overall Pressure Ratio = 13.5
	Turbine Inlet Temperature = 1830 F (2290 R)
	Airflow = 170 lbm/s (132.3 lbm/s @ 20K, M0.9)
Efficiencies	Compressor Efficiency = 0.85
	Turbine Efficiency = 0.90
	Combustor Pressure Drop = 5%
	Nozzle Thrust Coefficient = 0.985
	All Other Components Are Perfect
	No Internal Cooling Flow Circuits
Other	No Installation Effects, AIA Ram Recovery
	Dry (Unaugmented) Operation Only
	Calorically Perfect Gas (γ = Constant = 1.4)
	Constant Gas Composition (No Vitiation)
	Fuel Mass Is Negligible Relative to Air Mass
	Gas Kinetic Energy Inside Engine Is Negligible
	Reference Conditions Are Local Ambient
	Nozzle Exit Pressure Is Local Ambient
	c_p = 0.24 BTU/lbm, Fuel Heat = 18,400 BTU/lbm

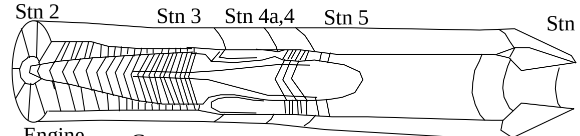

Figure 1: Simplified J-79 Cycle Model Configuration and Station Designations.

Based on these assumptions, a standard cycle analysis using conservation of mass, momentum, and energy yields the J-79 performance estimates given in Table II, and the internal temperatures and pressures listed in Table III. These results show that the thermal efficiency of the J-79 engine estimated for sea level static conditions is 40.5%, which is typical for gas turbine engines of this vintage. This efficiency appears to be quite low, leading one to believe that the configuration is a poor design from an energy utilization standpoint and could be improved upon tremendously. However, this is misleading, as it only describes the distribution of energy, not the distribution of *work potential.* In fact, as will be shown using the various second-law methods, the actual power output for this engine is on the order of 75-80% of the thermodynamic ideal available using the Brayton cycle with a 13.5 pressure ratio. Also, it should be pointed out that the specific fuel consumption of this example engine is optimistic relative to the actual machine due to the simplifying assumptions used in the analysis.

The thermal efficiency of the engine operating at M0.9, 20,000 ft is somewhat higher due to the lower ambient temperature as well as the higher overall cycle pressure ratio imparted via ram compression in the inlet. Note that the engine produces more power (gas horsepower) at Mach 0.9 than sea level static. Thrust horsepower is measured relative to an Earth-fixed reference frame, as is net thrust horsepower for all cases discussed herein.

Table II : J-79 Performance Estimates Based on Standard Cycle Analysis.

Figure of Merit	*Sea Level Static*	*M0.9, 20,000 ft*
Air Flow Rate	170.0 lbm/s	132.3 lbm/s
Compressor Power Rq'd	161.4 BTU/lbm	162.0 BTU/lbm
Thermal Energy Input	263.9 BTU/lbm	262.8 BTU/lbm
Fuel/Air Ratio	0.0148	0.0147
Fuel Flow Rate	9,047 lbm/hr	7,010 lbm/hr
Specific Fuel Consumpt.	0.740 lbm/lbf-hr	0.974 lbm/lbf-hr
Thermal Efficiency	40.5%	54.8 %
Gross Thrust (Dry)	12,227 lbf	11,034 lbf
Ram Drag	0 lbf	3,837 lbf
Net Thrust	12,227 lbf	7,198 lbf
Gas Power Output**	25,717 HP	26,923 HP
Gross Thrust Power	0 HP	18,724 HP
Net Thrust Power	0 HP	12,214 HP
Propulsive Efficiency	0 %	45.4 %
Overall Efficiency	0 %	24.8 %

**Assuming Turbine Expansion to Ambient Pressure in Vehicle-Fixed Frame

Table III: Engine Internal Temperature, Pressure, and Entropy for Each Flow Station.

		Sea Level Static		*M0.9, 20,000 ft*	
Description	*Stn*	*Temp (R)*	*Press (atm)*	*Temp (R)*	*Press (atm)*
Freestream	0	519	1.00	447	0.46
Inlet Flange	2	519	1.00	519	0.78
Comp. Discharge	3	1,191	13.50	1,195	10.50
Turbine Inlet	4a	1,191	12.83	1,195	10.00
Turb. Inlet (w/ $\Delta P/P$)	4	2,290	12.83	2,290	10.00
Turb. Discharge	5	1,617	3.23	1,615	2.50
Nozzle Exit Plane	9	1,172	1.00	1,015	0.46

Perturbation Estimates for Component Loss

The most common technique for gauging the relative importance of losses in gas turbine engines is a general class of methods referred to here as perturbation methods. These methods involve the analysis of a family of cycles each derived from a common baseline cycle. The objective is to perturb the baseline one parameter at a time and analyze the impact that this has on performance. The change in perturbed performance relative to the baseline is then used to deduce the total loss contributed by an individual loss mechanism.

There are two broad categories of perturbation method, the first involving infinitesimal perturbations from a baseline to obtain what are commonly referred to as sensitivity derivatives. The other method involves large (usually nonlinear) perturbations such that component interactions cannot be considered insignificant. Small perturbation methods cannot be used to *directly* calculate the absolute magnitude of losses. However, it is possible to directly gauge the *relative importance* of loss mechanisms through the use of sensitivities. This consists of a one-variable-at-a-time numerical derivative on engine performance with respect to each component efficiency. Calculation of this derivative necessarily implies some constraints on the way the cycle is re-balanced in the perturbed state. For present purposes, compressor discharge temperature and turbine inlet temperature are held constant, the reason being that these parameters are fundamental to defining the nominal cycle of interest and should therefore be fixed.

Table IV shows the sensitivity of engine performance to a one-point delta in the various component efficiencies. These results indicate that the most sensitive component efficiency is the nozzle thrust coefficient, with the least sensitive efficiency being the combustor pressure drop. It is also interesting to observe that the specific fuel consumption becomes increasingly sensitive to component efficiencies the further downstream the component lies. Note that since the sensitivities are a one-variable-at-a-time perturbation, they do not account for interactions amongst component efficiencies.

Absolute magnitudes of losses can be roughly estimated by simply extrapolating the sensitivities back to 100% component efficiency, as shown in

Table V. Using this approach, the compressor has the greatest impact on thrust and thermal efficiency, while the turbine has the greatest impact on specific fuel consumption. However, this does not account for component interactions or non-linear behavior of the sensitivities for large deviations from the design point.

Table IV: Sensitivity of Engine Performance to Changes in Component Efficiencies (M0.9, 20K).

Component Loss	*Change in Thrust*	*Change in SFC*	*Change in η_{TH}*
Compressor Eff: +15 pt	+65	-0.0054	+0.0046
Combustor $\Delta P/P$: –5%	+26	-0.0036	+0.0027
Turbine Eff: +10 pt	+47	-0.0064	+0.0048
Nozzle C_{fg}: +1.5 pt	+112	-0.0149	+0.0112

Table V: Extrapolation of Sensitivities to Estimate Approximate Absolute Impact of Component Losses (M0.9, 20K).

Component Loss	*Change in Thrust*	*Change in SFC*	*Change in η_{TH}*
Compressor Eff: +15 pt	+975	-0.081	+0.069
Combustor $\Delta P/P$: –5%	+130	-0.018	+0.014
Turbine Eff: +10 pt	+470	-0.064	+0.048
Nozzle C_{fg}: +1.5 pt	+168	-0.022	+0.017

Another way that ordinary cycle analysis methods can be used to obtain an estimate of the absolute magnitudes of component losses is to simply re-set one component efficiency at a time to 1.0 and re-balance the cycle, the results of which are shown in Table VI. The caveat to this large perturbation method is that it changes the cycle of the machine. For instance, if compressor efficiency is set to 1.0, the cycle will re-balance either at a higher pressure ratio for the same shaft power, or will have reduced shaft power for the same pressure ratio. Either way, the cycle is fundamentally changed and so *the loss estimate obtained via this method is part due to component loss and part due to a change in the cycle* (this applies to the small perturbation method also). The assumption used here is that the compressor discharge temperature should be held constant while the cycle pressure ratio is allowed to vary. Note that these results are similar to the sensitivity extrapolation and show that compressor losses are dominant, followed by turbine, nozzle, and combustor losses.

Table VI: Approximate Absolute Impact of Engine Component Losses Based on Perturbed Component Efficiencies at M0.9, 20,000 ft.

Component Loss	*Change in Thrust*	*Change in SFC*	*Change in η_{TH}*
Compressor Eff: 100 %	+750.7	-0.090	+0.0757
Combustor $\Delta P/P$: 0%	+127.3	-0.017	+0.0117
Turbine Eff: 100 %	+417.2	-0.052	+0.0409
Nozzle C_{fg}: 100 %	+170.0	-0.022	+0.0158

Exergy

The most widely known and fully developed measure of work potential is exergy. It is a comprehensive work potential figure of merit that is a function only of temperature and pressure at two points: the engine station of interest and ambient conditions. An extensive body of literature exists describing the background, theory, and application of this method, and the reader is referred to reference 7 for more information on these aspects. Suffice it to say that for this simplified analysis, exergy is calculated at each engine station using the ideal gas exergy expression given in reference 5:

$$ex = c_p\left(T - T_{amb}\right) - c_p T_{amb}\ln\left(T/T_{amb}\right) + RT_{amb}\ln\left(P/P_{amb}\right) \quad (1)$$

The results of this analysis are shown in Table VII and Table VIII. Note that *the largest exergy losses are non-equilibrium (irreversible) combustion and exhaust heat rejection, both of which are due to the nature of the Brayton cycle itself* rather than an imperfection in the machine.[**] The only way these losses can be reduced is through a change in the cycle parameters (in this case, overall pressure ratio and turbine inlet temperature), or a change in the basic configuration of the machine (such as the addition of a regenerator). This result clearly shows that there is considerable room for improvement relative to the theoretical ideal. Amongst the component-specific losses, the compressor is the most significant contributor, followed by the turbine. This result is not surprising because 1) the compressor efficiency is lower, and 2) it is well known that inefficiencies at low temperature tend to be more significant in an exergy sense that those at high temperature.

The overall exergy input for the SLS cycle is approximated as 263.9 BTU/lbm and the power output (in the form of exhaust kinetic energy) is 106.9 BTU/lbm, yielding a thermal efficiency of 40.5%. However, the largest losses are due to the inherent nature of the Brayton cycle itself, rather than inefficiency in the engine. In other words, *even a perfect Brayton cycle would appear to have large losses when viewed in an exergy sense*, even though exhaust exergy is completely unavailable to a machine using the Brayton cycle.[††] One could argue that since irreversible combustion and exhaust heat losses are inevitable for the Brayton cycle, they should not be chargeable against the machine itself. Viewed in this light, the effective exergy input is the total exergy input less the irreversible combustion and exhaust heat losses (127.1 BTU/lbm), yielding a "Brayton corrected" thermal efficiency of roughly 84%. In effect, this means that 84% of the exergy that is theoretically accessible using the Brayton cycle was converted into exhaust kinetic energy, with the remainder being dissipated as exhaust heat (this is neglecting the fact that component inefficiency contributes to exhaust heat also).

Table VII: Exergy at Each Engine Station Relative to the Vehicle-Fixed Reference Frame (BTU/lbm).

	Sea Level Static		*M0.9, 20,000 ft*	
Station	***Ex***	***ΔEx***	***Ex***	***ΔEx***
freestream	0	- - -	17.4	- - -
2	0	- - -	17.4	0.0
3	150.4	+150.4	169.9	+152.5
4a	148.6	-1.8	168.3	-1.6
4	331.1	+182.6	361.3	+193.0
5	163.8	-167.3	194.2	-167.1
9	162.4	-1.5	192.2	-2.0
Atmo. Diffusion	0	-162.4	0	-126.9

[**] For the present analysis, the exergy available in the fuel is approximated as being equal to the lower heating value of the fuel.

[††] This concept is explained in detail in reference 12.

Table VIII: Sources of Exergy Loss.

	Sea Level Static			M0.9, 20,000 ft		
Component	***Exergy Loss (BTU/lbm)***	***BTU/lbm***	***Loss/Ex$_{in}$***	***Exergy Loss (BTU/lbm)***	***BTU/lbm***	***Loss/Ex$_{in}$***
Compressor	161.4-150.4 =	11.0	4.2 %	162.0-(169.9-17.4) =	9.5	3.6 %
Combustor ΔP/P	150.4-148.6 =	1.8	0.7 %	169.9-168.3 =	1.6	0.6 %
Irreversible Comb.	263.9-182.6 =	81.3	30.8 %	262.8-193.0 =	69.8	26.6 %
Turbine	(331.1-163.8)-161.4 =	5.9	2.2 %	361.3-194.2-162.0 =	5.1	1.9 %
Nozzle	163.8-162.4 =	1.4	0.5 %	194.2-192.2 =	2.0	0.8 %
Exhaust Heat	162.4-106.9 =	55.5	21.0 %	192.2-143.9 =	48.3	18.4 %
Exh. Residual KE**	106.9-0 =	106.9	40.5 %	143.9-17.4-65.3 =	61.2	23.3 %
Thrust Work**	12,212 lbf * 0 ft/s =	0.0	0.0 %	7,198 lbf * 933 ft/s	65.3	24.8 %
Work Out/Exergy In	0/263.9 =	0 %		65.3/262.8 =	24.8 %	

**As Viewed in the Earth-Fixed Reference Frame

It is interesting to note that if the J-79 example were used as a topping cycle in a combined cycle plant, it would then be possible to convert the exhaust heat exergy from the J-79 powerplant into useful work produced by the bottoming cycle. In this case, it would clearly be appropriate to count exhaust exergy as a loss because it is theoretically available to the machine, though the exergy loss due to irreversible combustion would still be non-chargeable. In this case, the "combined cycle corrected" efficiency would be 106.9/(263.9-81.3) or 58.5%.

Finally, if this same fuel were combusted in a fuel cell instead of a gas turbine engine, it would be possible to "combust" the fuel at near-equilibrium conditions and thereby avoid the losses due to irreversible combustion in a gas turbine engine. The total exergy theoretically accessible by the fuel-cell powerplant is roughly equal to the Gibbs free energy of the reaction, which typically approaches 90% or more of the total heating value. In this scenario, the first-law cycle efficiency of 40.5% begins to have real meaning because it is theoretically possible to approach 100% efficiency with such a machine. This line of reasoning suggests that *the appropriate choice of thermal efficiency depends in some measure on the nature of the machines being used and the intent of the analyst*.

Available Energy

A second figure of merit of potential utility for gas turbine applications is available energy or "gas horsepower." This is an intuitive metric that measures the shaft work available in expanding the flow isentropically to ambient pressure, as described in reference 5. For the purposes of this analysis, available energy is estimated using the ideal gas assumption:

$$ae = c_p T\left[1-\left(P_{amb}/P\right)^{\frac{\gamma-1}{\gamma}}\right] \quad (2)$$

The available energy results shown in Table IX and Table X indicate that the turbine incurs the greatest loss from an available energy point of view, followed by the compressor, combustor pressure loss, and finally the nozzle. Not surprisingly, the component losses estimated using the available energy method are larger than their exergy counterparts. This is because losses contribute to increased exhaust gas temperature, and therefore, the exhaust exergy of the flow. Thus, a portion of the loss is recoverable in the exhaust heat and is not bookkept as a loss using the exergy method. All results for available energy are assumed to be in the vehicle-fixed reference frame (i.e., the reference frame of the observer moving with the propulsion system).

Table IX: Available Energy at Each Engine Internal Station (BTU/lbm).

	Sea Level Static		M0.9, 20,000 ft	
Station	***ae***	***Δae***	***ae***	***Δae***
freestream	0.0	- - -	0.0	- - -
2	0.0	- - -	17.4	+17.4
3	149.9	+149.9	169.4	+152.0
4a	147.9	-2.0	167.7	-1.7
4	284.5	+136.6	321.4	+153.7
5	110.2	-174.2	148.3	-173.1
9	106.9	-3.3	143.9	-4.4
atmo. dfsn.	0	-106.9	0.0	-143.9

Table X: Loss in Available Energy.

	Sea Level Static			M0.9, 20,000 ft		
Component	***ae Loss (BTU/lbm)***	***BTU/lbm***	***Loss/ae$_{in}$***	***ae Loss (BTU/lbm)***	***BTU/lbm***	***Loss/ae$_{in}$***
Compressor	161.4-149.9 =	11.5	8.4 %	162.0-(169.4-17.4) =	10.0	6.5 %
Combustor ΔP/P	149.9-147.9 =	3.9	1.5 %	169.4-167.7 =	1.7	1.1 %
Non-Equilib Comb.	Non-chargeable	- - -	- - -	Non-chargeable	- - -	- - -
Turbine	174.2-161.4 =	12.8	9.4 %	321.4-148.3-162.0 =	11.1	7.2 %
Nozzle	110.3-106.9 =	3.4	2.5 %	148.3-143.9 =	4.4	2.9 %
Exhaust Heat	Non-chargeable	- - -	- - -	Non-chargeable	- - -	- - -
Residual KE**	106.9-0 =	106.9	78.3 %	143.9-65.3-17.4 =	61.2	39.8 %
Thrust Work**	12,227 lbf * 0ft/s =	0.0	0.0 %	7,198 lbf * 933 ft/s =	65.3	42.5 %
Work Out/ae In	0/136.6	0.0		65.3/153.7	0.425	

**As Viewed in the Earth-Fixed Reference Frame

Note that irreversible combustion and exhaust heat losses do not appear in this analysis (they are listed in Table X as "non-chargeable" to denote that they are not charged against the machine efficiency). If all component efficiencies in this engine were perfect, the total available energy loss would be zero, and the total power output would be equal to the ideal cycle. Based on this observation, *it is reasonable to view available energy as a Brayton figure of merit because available energy is completely accessible within the confines of the simple Brayton cycle*.

The implication of the previous statement is that available energy is a good figure of merit for engines designed to produce shaft horsepower such as turboshaft and aeroderivative ground power generation units. However, for propulsive devices designed to produce thrust by action on a body of fluid it is not possible to use all of the available energy produced by the thermal cycle for the production of thrust work (except in the limit of infinitesimal delta velocity acting on an infinite mass flow rate). Thus, aircraft engines not only have an inherent "thermal unavailability" due to the cycle, they also have an inherent "propulsive unavailability" as well.[13] It therefore seems intuitive that a good FoM for jet propulsion must necessarily involve thrust itself.

Stream Thrust

Stream thrust is a force-based FoM that measures loss in equivalent thrust available in expansion to ambient pressure as a measure of thrust potential inherent in a flow. It was shown in reference 5 that stream thrust is related to available energy by: $Sa = 6.955\sqrt{ae}$ where Sa is in lbf/lbm and ae is in BTU/lbm for the vehicle-fixed reference frame. Using this equation in conjunction with the results of Table IX yields the stream thrust at every flow station, shown in Table XI. Interestingly, the results show that the compressor makes the largest contribution to increasing stream thrust for both cases examined, and is one of the rare instances where nature appears to give back more than was paid out. In addition, the combustion process adds a further 34.6 lbf/lbm, and both of these effects are highly desirable if the objective is to generate thrust.

Table XI: Gross Stream Thrust at Each Flow Station for Sea Level Static Operation.

	Sea Level Static		*M0.9, 20,000 ft*	
Station	***Sa (lbf/lbm)***	***ΔSa (lbf/lbm)***	***Sa (lbf/lbm)***	***ΔSa (lbf/lbm)***
ambient	0	- - -	0	- - -
2	0	- - -	29.0	+29.0
3	85.2	+85.2	90.5	61.5
4a	84.6	-0.6	90.1	-0.4
4	117.3	+32.7	124.7	+34.6
5	73.0	-44.3	84.7	-40.0
9	71.9	-1.1	83.4	-1.3

Direct estimation of stream thrust losses based on the results of Table XI is somewhat ambiguous because there is no "conservation of stream thrust" principle as there is for exergy and available energy. The method suggested by Riggins[4] for estimating loss in stream thrust is to progressively remove the losses from downstream to upstream, re-balancing the cycle after each loss is removed to find change in engine net stream thrust. This method (called the lost thrust method) assumes that the change in specific thrust after each step is due to the loss just removed. The logic of this approach can be explained as follows. Losses produced by most components are a function of the upstream flow conditions feeding the component. Therefore, if the objective is to evaluate the loss contributed by a particular component, that component must be evaluated relative to its ideal *at the actual inlet conditions*. This dictates that losses must be removed from back to front, as the opposite direction would cause changes in the cycle to propagate downstream and interfere with loss estimates for downstream components.

A drawback to using this method for gas turbine engines is that the loss estimate obtained depends on the assumptions used in re-balancing the cycle. For instance, assume that the cycle pressure ratio and turbine inlet temperature are held constant as losses are removed. When the cycle is re-balanced after removing the compressor losses, the combustor heat input will change because the compressor discharge temperature decreased. Thus, the change in specific thrust is due to not only the compressor loss, but also due to the change in combustor heat addition. Alternatively, if compressor discharge and turbine inlet temperatures are assumed to define the nominal cycle, the compressor loss estimate is due to changes in compressor loss and overall cycle pressure ratio. It is therefore important to choose these assumptions such that they are physically meaningful to the problem at hand.

The latter assumption was chosen as being more physically meaningful for this problem, and the results for the lost thrust method are as shown in Table XII. Note that this method indicates that the compressor causes the greatest loss in stream thrust, followed by the turbine. The combustor pressure drop and nozzle loss are a distant third and fourth. The relative magnitude of stream thrust losses is very similar to the perturbation estimates for loss given in
Table V and Table VI. An important point here is that non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy losses are transparent as viewed from a stream thrust perspective.

Table XII: Loss in Stream Thrust at Each Flow Station.

	Sea Level Static			*M0.9, 20,000 ft*		
Component	***Sa Loss (lfbf/lbm)***	***lbf/lbm***	***Loss/Ideal***	***Sa Loss (lfbf/lbm)***	***lbf/lbm***	***Loss/Ideal***
Compressor	84.9 – 78.4 =	6.5	7.7 %	93.8-88.7 =	5.1	7.9 %
Combustor ΔP/P	78.4 – 77.2 =	1.2	1.4 %	88.7-87.8 =	0.9	1.4 %
Non-Equilib Comb.	Non-chargeable	- - -		Non-chargeable	- - -	
Turbine	77.2 – 73.0 =	4.1	4.8 %	87.8-84.7 =	3.1	4.8 %
Nozzle	73.0 – 71.9 =	1.1	1.3 %	84.7-83.4 =	1.3	2.0 %
Exhaust Heat	Non-chargeable	- - -		Non-chargeable	- - -	
Residual KE	Non-chargeable	- - -		Non-chargeable	- - -	
Sa Output	- - -	71.9	84.7 %	83.4-29.0 =	54.4	84.0 %
Sa Out/Ideal	71.9/(71.9+Loss)=	84.7%		54.4/(54.4+Loss)=	84.0%	

Thrust Work Potential

Thrust work potential is directly related to stream thrust through flight velocity and is related to available energy through propulsive efficiency, as explained in reference 5. In effect, it is a measure of the thrust work obtainable in a simple isentropic expansion. Results for the thrust work analysis at M0.9, 20,000 ft are shown in Table XIV. Note that sea level static conditions reduce to the trivial case for this figure of merit, and therefore are not shown in this table.

Loss in thrust work potential is calculated by applying the procedure previously outlined whereby irreversibilities are removed from downstream to upstream and the cycle re-balanced after each pass. This results in what Riggins refers to as the “lost thrust potential” method.[4] This approach is subject to the same strengths and weaknesses as that described for the stream thrust method in that it is capable of isolating loss sources, but the loss estimates depend on the assumptions used in re-balancing the cycle.

The results for the lost thrust potential analysis are shown in Table XIV. Not surprisingly, the relative magnitudes of thrust work potential losses calculated using this method are the same as for the stream thrust losses. Inspection of these loss estimates also reveals that they are very similar in relative magnitude to the approximate first law estimates given in

Table V and Table VI. If one were to define efficiency as the ratio of the actual thrust work potential divided by that of the perfect (no component losses) machine, the efficiency of the J-79 example is 83.8%.

Table XIII : Specific Gross Thrust Work Potential for M0.9, 20,000 ft Flight Conditions.

Station	***Wp (BTU/lbm)***	***ΔWp (BTU/lbm)***
ambient	0.0	- - -
2	34.8	+34.8
3	108.4	+73.6
4a	107.9	-0.5
4	149.6	+41.7
5	101.7	-47.9
9	100.1	-1.6
Net Wp	100.1-34.8 =	65.3

Table XIV: Loss in Specific Thrust Work Potential for M0.9, 20,000 ft Flight Conditions.

Component	***Wp Loss (BTU/lbm)***	***BTU/lbm***
Compressor	78.0-71.8 =	6.2 (7.9 %)
Combustor ΔP/P	71.8-70.6 =	1.2 (1.4 %)
Non-Equilib Comb.	Non-chargeable	- - -
Turbine	70.6-66.9 =	3.7 (4.8 %)
Nozzle	66.9-65.4 =	1.5 (2.0 %)
Exhaust Heat	Non-chargeable	- - -
Residual KE	Non-chargeable	- - -
Useful wp	- - -	65.4
wp Out/Ideal	65.4/(65.4+Wp Lost)	83.8%

Comparison of Results

Based on the discussion up to this point, one can now make a direct comparison of the various figures of merit and use this to draw inferences as to the relative merits of each for use in gas turbine loss estimation. Comparative results for the loss stack-up of engine work potential are shown in Figure 2. Note that irreversible combustion and exhaust heat losses only appear in the exergy analysis and are non-chargeable when using the available energy and thrust work potential methods. Exhaust residual kinetic energy is bookkept as a loss for the exergy and available energy methods, but is non-chargeable when using the stream thrust and thrust work potential methods. Based on this comparison, it is clear that the stream thrust and thrust work potential methods yield loss stack-ups that closely match the loss stack-up obtained using perturbation methods. Although stream thrust and thrust work potential are not analytically equivalent to today’s perturbation methods for loss estimation, they yield results that are essentially equivalent.

It is clear from this comparison that there is much more work potential available in the fuel when viewed from an exergy or available energy point of view than from a thrust work potential or cycle perturbation point of view. For instance, the exergy input of the fuel is approximately 263 BTU per lbm air, while the analogous figure for thrust work potential is only 78.0 BTU per lbm air. The results of this analysis show that irreversible combustion, exhaust heat, and residual kinetic energy losses are roughly *an order of magnitude* greater than individual component losses. Yet, this fact

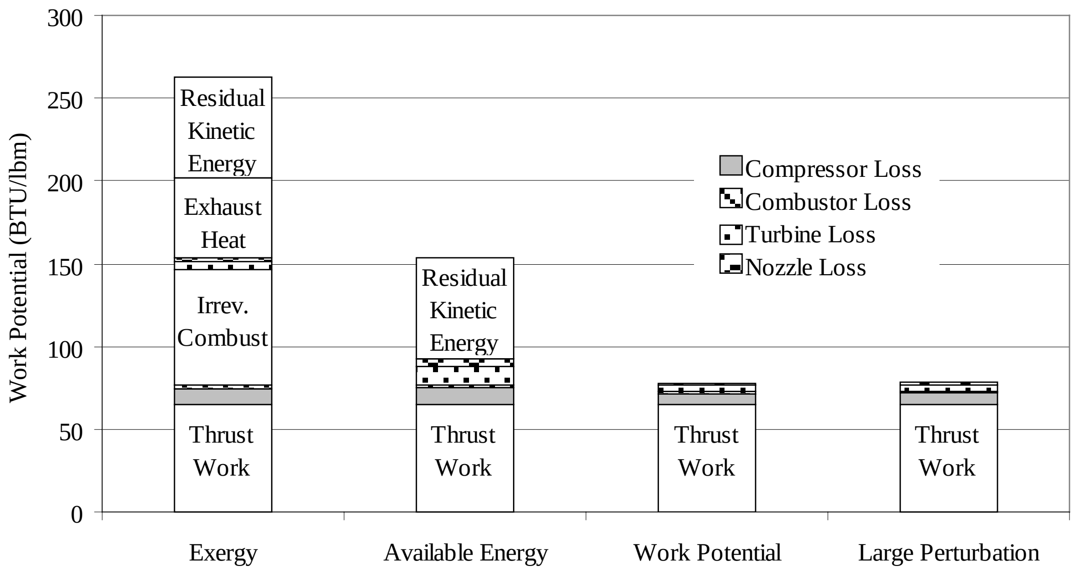

Figure 2: Absolute Specific Work Output (and Loss) per Unit Airflow for the J-79 Turbojet Engine at M0.9, 20K.

Table XV: Effective Fuel Work Availability for Estimated Using Each Analysis Method (M0.9, 20,000 ft).

	Exergy	Avail. Energy	Stream Thr.	Work Pot.	Lg. Perturb.
Fuel Heating Value (BTU/lbm)	18,400	18,400	18,400	18,400	18,400
Ideal Work (BTU/lbm)	~18,400	10,456	5,305	5,305	5,333
Actual Thrust Work (BTU/lbm)	4,442	4,442	4,442	4,442	4,442
Lost Work (BTU/lbm)	13,958	6,014	863	863	891

is not widely recognized, primarily because these losses do not appear when using typical perturbation methods to estimate loss. This inherent recognition of all sources of loss is a strong argument for application of exergy methods to propulsion systems analysis.

It was mentioned earlier that available energy losses due to component inefficiency will always be larger than their corresponding exergy losses due to the higher “heat recovery” ability of exergy. Examination of Figure 2 reveals that thrust work potential losses are always less than their corresponding available energy losses. This can be explained as follows: production of jet thrust necessarily implies a loss due to exhaust residual kinetic energy. Therefore, not all of the available energy can be converted into thrust work and some is therefore inherently unavailable to jet propulsive cycles. Consequently, a loss in available energy will result in a proportionately smaller loss in thrust work potential simply because not all available energy can be converted into thrust work (except in the limit of vanishing specific thrust acting on infinite mass flow).

Another way to view the results of these analyses is to normalize the results such that the sum of losses and useful work equals 100%, as shown in Figure 3. This figure clearly shows that the partitioning of total losses is considerably different between the various methods. The last three are very similar and show that losses account for only ~15% of the total work potential. Residual kinetic energy appears as a significant loss in the available energy analysis, and component losses are completely dominated by cycle effects in the exergy analysis. If the nominal specific work is divided by fuel/air ratio, the result is work available per pound of fuel, as shown in Table XV. It is a simple matter to define fuel flow chargeability based on the loss breakdowns shown in Figure 3 and Table XV.

Although the loss estimation methods discussed herein have only been applied to a single-stream turbojet engine, they are general enough to be applied to multiple-flow machines that have secondary flowpaths, and numerous loss sources. The exergy and available energy methods have already been demonstrated in reference 12 for a mixed flow turbofan engine. The stream thrust and lost thrust potential methods can also be applied to reference 12, but their application will require careful consideration as to the assumptions used in re-balancing the cycle. The logical approach would be to apply the lost thrust method by starting with the outermost stream and working inwards towards the core engine, always moving from back to front when calculating losses within a given flow stream.

Selection of Loss Figures of Merit

The foregoing analyses clearly show that each of the loss FoMs investigated herein is a special case of another, more general FoM. Each differs from the next

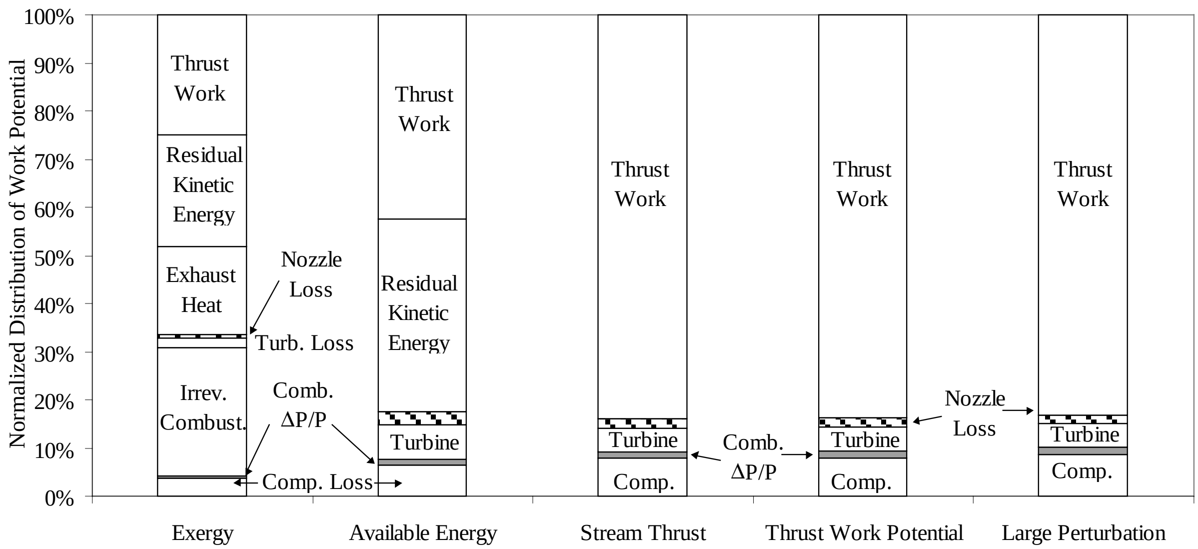

Figure 3: Comparison of Loss Chargeability Figures of Merit for a J-79 at M0.9, 20,000 ft.

primarily in the way "useful work potential" is defined, and thus each is well-suited for a particular application. For instance, in designing a combined cycle power plant, it would clearly be desirable to mimize the loss of exergy in the gas turbine topping cycle such that the sum of exhaust exergy and shaft work are maximized. However, for a simple gas turbine power generation unit, the direct objective is minimization of losses in available energy, thereby maximizing shaft horsepower produced per pound of fuel. For jet propulsion applications, the immediate objective is production of thrust. Thus, maximization of net thrust work potential per pound of fuel is the ultimate goal.

These observations suggest that *the proper choice of loss figure of merit really depends on how one defines "useful work."* In other words, it may not be appropriate to charge a loss source against the machine efficiency if that loss is inherently unavailable to the machine (as exhaust heat is for a gas turbine engine). If a portion of the work potential is inherently unavailable to a particular machine, then it is irrelevent to the ultimate performance of the machine.

Another argument in favor of this "work potential exclusion principal" can be presented by considering the work potential accessible by tapping the energy available in the nuclear bonds of the fuel itself. It is well-known that elements lighter than iron can be fused into heavier elements yielding net energy output while elements heavier than iron can be split to obtain lighter products in addition to net energy output. Moreover, the work potential contained in the nuclear bonds of the fuel is far larger than that contained in the chemical bonds of the molecules. Therefore, the work potential of the fuel is clearly far larger than suggested by the lower heating value alone. However, it is absurd to optimize a gas turbine engine for maximum nuclear exergy output because it is inherently unavailable to the machine, and therefore, irrelevent to its design, optimization, and operation. Likewise, latent heat in the exhaust stream is irrelevant to the design and optimization of a simple gas turbine because it is inherently unusable within the confines of the Brayton cycle.

Conclusions

The loss estimation methods investigated here present a robust suite of tools that can be easily applied to the analysis of propulsion systems, each well suited to a particular application. Their application to the J-79 example revealed that the largest losses in work potential are due to irreversible combustion, exhaust heat, and exhaust residual kinetic energy. From an available energy standpoint, the largest loss is due to exhaust residual kinetic energy, with turbine and compressor losses being a distant second and third, respectively. As viewed using stream thrust or thrust work potential, the largest losses are due to the compressor, followed by the turbine. The results obtained for this last method are very similar to those obtained using perturbation methods, and show that roughly 85% of the total work theoretically available in a Brayton cycle having a 13.5 pressure ratio is actually realized in the J-79 engine. Based on these results, one could argue that the most appropriate figure of merit for jet propulsion purposes is stream thrust or thrust work potential, at least for component optimization purposes.

Finally, the results given here suggest that the proper choice of thermodynamic figure of merit is dependent on the type of machine being analyzed and the intentions of the analyst. If the objective is to understand the absolute loss relative to the maximum work allowed by the second law of thermodynamics, then exergy is an appropriate tool to use. If the objective is component optimization for minimum loss, then the focus should be on minimization of loss in *useful* work potential (available energy for a simple gas turbine, and thrust work potential for propulsors). Inherently unavailable work potential should not be bookkept as a loss chargeable to the machine when optimizing to deliver a particular type of work output.

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